Parton distributions in nonlocal chiral effective field theory

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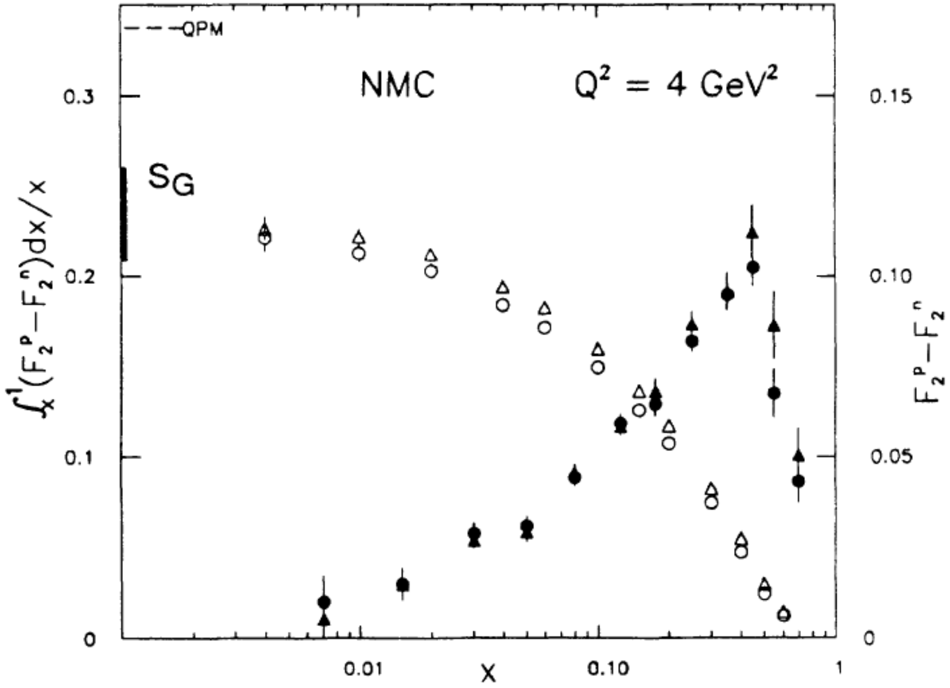
Collaboration with C. R. Ji, W. Melnitchouk, Y. Salamu and A. W. Thomas

Newport News, May 14-18, 2018

Introduction

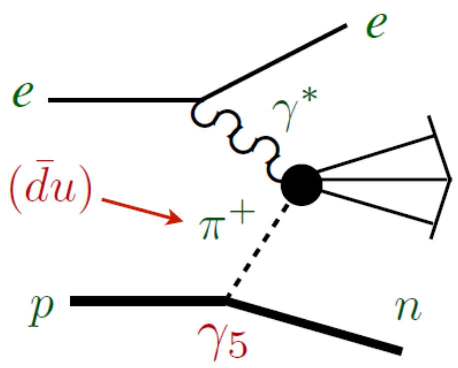
Gottifried sum rule (GSR):

$$\int_0^1 \left[F_2^p(x) - F_2^n(x)\right] \frac{dx}{x} = \frac{1}{3} - \frac{2}{3}\int_0^1 \left[\bar{d}_p(x) - \bar{u}_p(x)\right] \mathrm{d}x$$



New Muon Collaboration, PRD **50**, 1 (1994)

→ Sullivan process

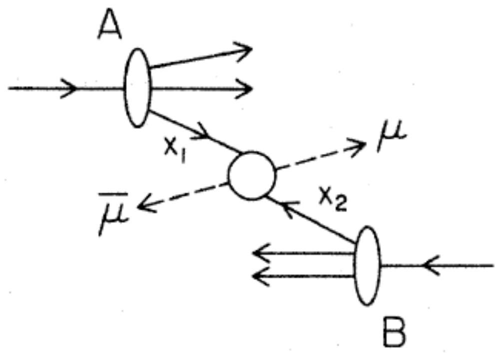


$S_G = 0.235 \pm 0.026$

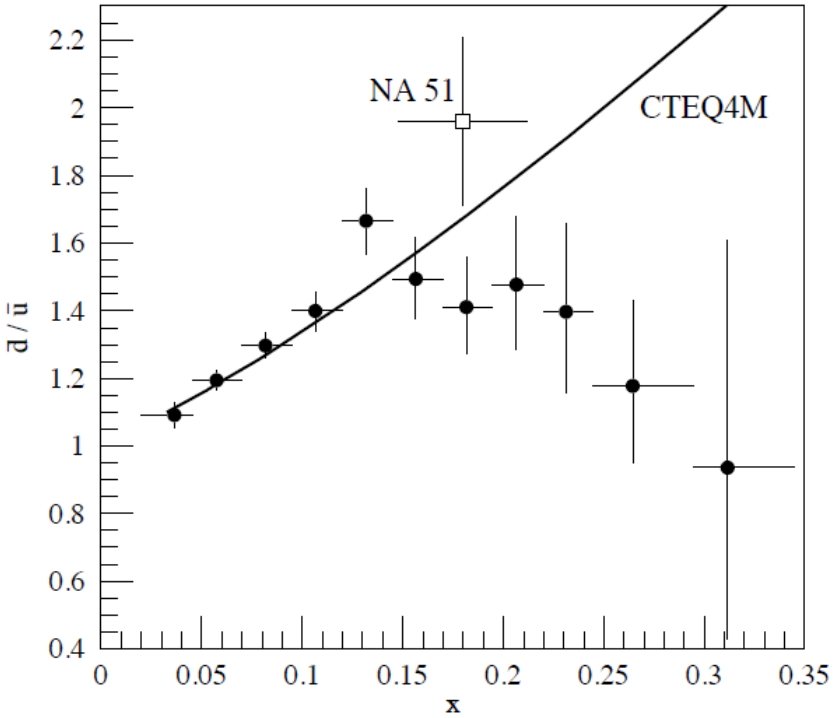
$$\int_0^1 \left[\bar{d}_p(x) - \bar{u}_p(x)\right] dx = 0.147 \pm 0.039$$

Introduction

→ Drell-Yan process



$$\frac{d^2\sigma}{dx_b dx_t} = \frac{4\pi\alpha^2}{9Q^2}\sum_q e_q^2\left(q(x_b)\bar{q}(x_t) + \bar{q}(x_b)q(x_t)\right)$$



Fermilab E866, PRL ***80****, 3715 (1998)*

$$\frac{\sigma^{pd}}{2\sigma^{pp}} \approx \frac{1}{2}\left(1 + \frac{\bar{d}(x_t)}{\bar{u}(x_t)}\right) \quad \rightarrow \quad \int_0^1 dx\,(\bar{d} - \bar{u}) = 0.118 \pm 0.012$$

Introduction

BEBC, CDHS and CDHSW experiments concluded that the s-quark PDF was somewhat harder than the sbar.

Beyond extractions from individual experiments, global QCD analyses of charged lepton and neutrino DIS, along with other high energy scattering data, have generally found positive values for S-.

Taking into account some of these uncertainties, the phenomenological analysis of Bentz *et al.* concluded that S- = (0 + - 2) X 0.001.

Catani et al., showed that perturbative three-loop effects can induce nonzero negative S- values ~ -0.0005, through Q2 evolution of symmetric s-sbar distributions from a low input scale Q ~ 0.5 GeV.

$$S^- \equiv \langle x(s-\bar{s})\rangle = \int_0^1 dx\, x\,(s(x)-\bar{s}(x))$$

In 2001 NuTeV collaboration, using ν DIS, measured:

- $\sin^2\theta_W = 0.2277 \pm 0.0013(\text{stat}) \pm 0.0009(\text{syst})$
- G. P. Zeller *et al.* Phys. Rev. Lett. **88**, 091802 (2002)

World average (not including NuTeV):

- $\sin^2\theta_W = 0.2227 \pm 0.0004$

3 σ discrepancy!!! $\Longrightarrow$ "NuTeV anomaly"

Nonlocal effective Lagrangian

- Flavour asymmetries can arise from the chiral loops $\longrightarrow$ chiral EFT
- Why nonlocal ?

Motivation:

Finite-range-regularisation $\longrightarrow$ Relativistic regulator

Heavy baryon EFT $\longrightarrow$ Covariant EFT

Gauge symmetry distroyed $\longrightarrow$ Gauge symmetry restored

Ward identity, Charge conservation, Renormalised nucleon charge

Solution:

Gauge invariant nonlocal chiral effective Lagrangian

Regulator derived from the Lagrangian

Nonlocal effective Lagrangian

Chiral effective Lagrangian

$$\mathcal{L} = i Tr\, \bar{B}\gamma_\mu \not{D} B - m_B Tr\, \bar{B}B + \bar{T}^{abc}_\mu (i\gamma^{\mu\nu\alpha} D_\alpha - m_T \gamma^{\mu\nu}) T^{abc}_\nu + \frac{f^2}{4} Tr\, \partial_\mu \Sigma \partial^\mu \Sigma^+ + D\, Tr\, \bar{B}\gamma_\mu\gamma_5 \{A_\mu, B\}$$
$$+ F\, Tr\, \bar{B}\gamma_\mu\gamma_5 [A_\mu, B] + \left[\frac{\mathcal{C}}{f}\epsilon^{abc}\bar{T}^{ade}_\mu (g^{\mu\nu} + z\gamma_\mu\gamma_\nu) B^e_c \partial_\nu \phi^d_b + H.C\right],$$

Local interaction including pi meson

$$\mathcal{L}_\pi{}^{local} = \int dx \frac{D+F}{\sqrt{2}f} \bar{p}(x)\gamma^\mu\gamma_5\, n(x)(\partial_\mu + ie\,\mathscr{A}_\mu(x))\pi^+(x)$$

Corresponding nonlocal Lagrangian

$$\mathcal{L}_\pi{}^{nl} = \int dx \int dy \frac{D+F}{\sqrt{2}f} \bar{p}(x)\gamma^\mu\gamma_5 n(x) F(x-y) \exp[ie \int_x^y dz_\nu \int da\, \mathscr{A}^\nu(z-a)F(a)]$$
$$\times (\partial_\mu + ie \int da\, \mathscr{A}_\mu(y-a)F(a))\pi^+(y),$$

Local EM interaction

$$\mathcal{L}^{local}_{EM} = -e\bar{p}(x)\gamma^\mu p(x)\mathscr{A}_\mu(x) + \frac{(c_1-1)e}{4m_N}\bar{p}(x)\sigma^{\mu\nu}p(x)F_{\mu\nu}(x)$$

Corresponding nonlocal EM interaction

$$\mathcal{L}^{nl}_{EM} = -e\int da \bar{p}(x)\gamma^\mu p(x)\mathscr{A}_\mu(x-a)F_1(a) + \frac{(c_1-1)e}{4m_N}\int da \bar{p}(x)\sigma^{\mu\nu}p(x)F_{\mu\nu}(x-a)F_2(a)$$

Nonlocal effective Lagrangian

The Lagrangian is gauge invariant under the following transformation:

$$\pi^+(y) \to e^{i\alpha(y)}\pi^+(y), \quad p(x) \to e^{i\alpha(x)}p(x), \quad \mathscr{A}_\mu(x) \to \mathscr{A}_\mu(x) - \frac{1}{e}\partial_\mu \alpha'(x)$$

$$\alpha(x) = \int da\alpha'(x-a)F(a)$$

EM currents including pi meson:

$$\mathcal{L}^{nor} = ie\int dx \int dy \frac{D+F}{\sqrt{2}f}\bar{p}(x)\gamma^\mu\gamma_5 n(x)F(x-y)\pi^+(y)\int da\, \mathscr{A}_\mu(y-a)F(a)$$

$$\mathcal{L}^{add} = ie\int dx \int dy \frac{D+F}{\sqrt{2}f}\bar{p}(x)\gamma^\mu\gamma_5 n(x)F(x-y)\int_x^y dz_\nu \int da\, \mathscr{A}^\nu(z-a)F(a)\partial_\mu\pi^+(y)$$

Nonlocal effective Lagrangian

Parameters:

$D = 0.76$ and $F = 0.50$ ($g_A = D + F = 1.26$) $\mathcal{C} = 1$

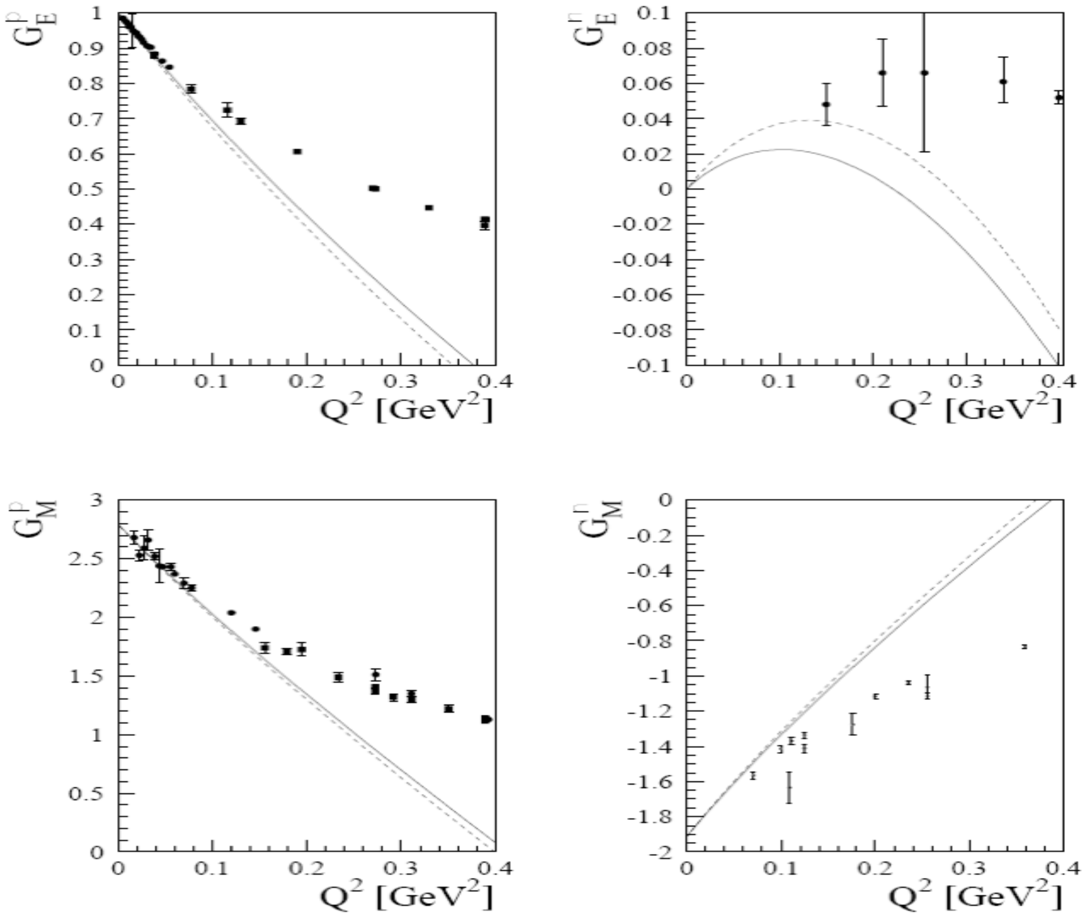
c1 , c2 are determined by the proton and neutron magnetic moments 2.79 and -1.91.

Nonlocal effective Lagrangian $\longrightarrow$ correlation function: $\tilde{F}(k) = \dfrac{1}{(1 - k^2/\Lambda^2)^2}$

Traditional ChPT $\longrightarrow$ four more LECs: c3, c4, c5, c6

Nonlocal effective Lagrangian

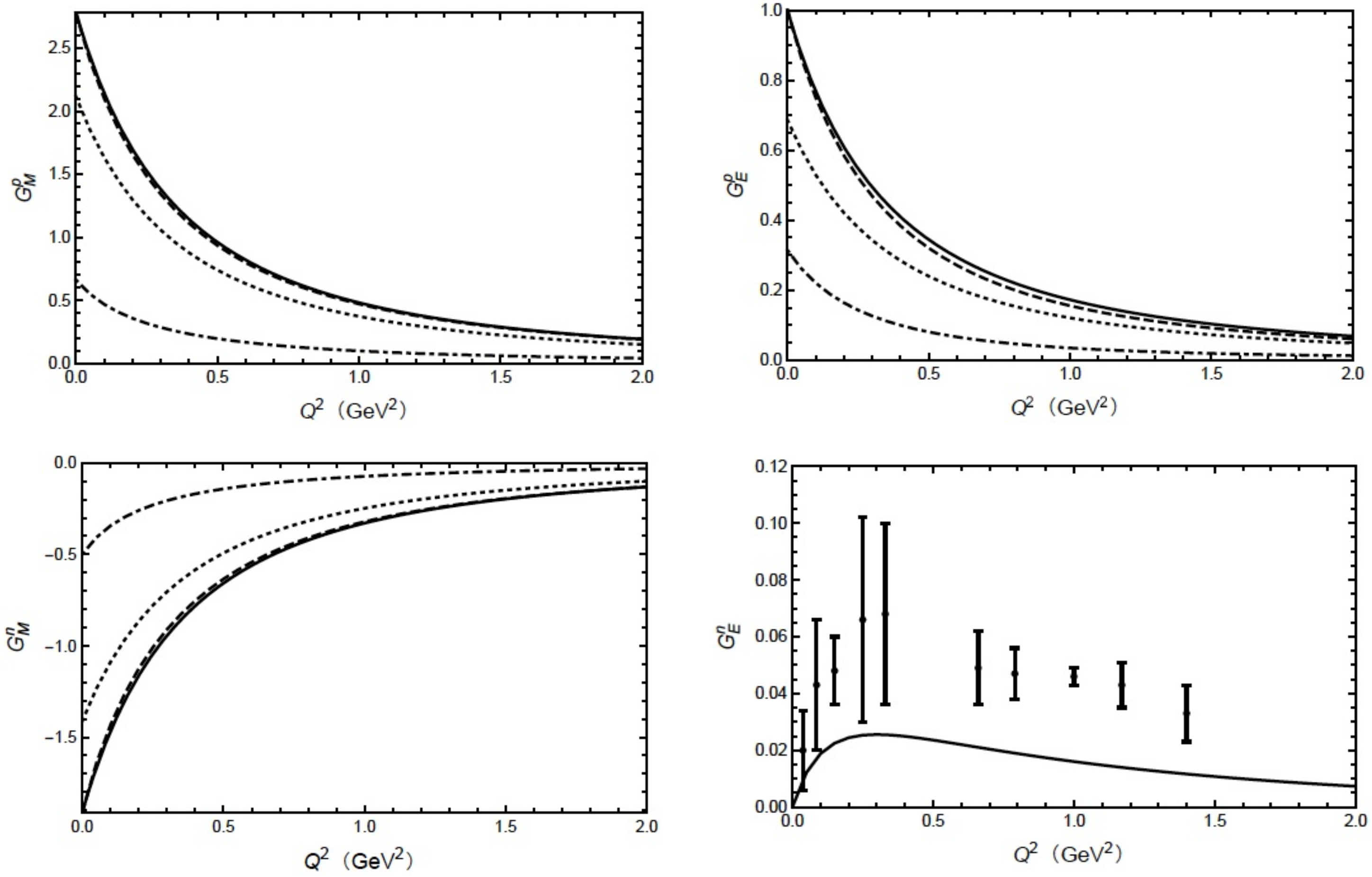
Nucleon Form Factors



T. Fuchs, J. Gegelia, S. Scherer, J. Phys. G30 (2004) 1407

Nonlocal effective Lagrangian

Nucleon Form Factors



Λ = 0.85 GeV

$\langle r_M \rangle_p$ = 0.848 (0.836) fm

$\langle r_E \rangle_p$ = 0.857 (0.847) fm

$\langle r_M \rangle_n$ = 0.867 (0.889) fm

$\langle r^2_E \rangle_n$ = -0.077 (-0.113) fm^2

F. C. He and P. Wang,
Phys. Rev. D97 (2018) 036007

- Generate the regulator which makes the loop integral convergent.
- Generate correct tree-level contribution which makes the total one close to the experiments.

Nonlocal effective Lagrangian

Nonlocal Lagrangian:

$$\begin{aligned}\mathcal{L}^{(\mathrm{nonloc})}(x) &= \bar{B}(x)(i\gamma^\mu \mathscr{D}_{\mu,x} - M_B)B(x) + \overline{T}_\mu(x)(i\gamma^{\mu\nu\alpha}\mathscr{D}_{\alpha,x} - M_T\gamma^{\mu\nu})T_\nu(x) \\ &+ \bar{p}(x)\left[\frac{C_{B\phi}}{f}\gamma^\mu\gamma^5 B(x) + \frac{C_{T\phi}}{f}\Theta^{\mu\nu}T_\nu(x)\right] \\ &\qquad \times \int d^4a\, \mathcal{G}_\phi^q(x, x+a)F(a)\ \mathscr{D}_{\mu,x+a}\phi(x+a) + \mathrm{h.c.} \\ &+ \frac{iC_{\phi\phi^\dagger}}{2f^2}\bar{p}(x)\gamma^\mu p(x)\int d^4a \int d^4b\ \mathcal{G}_\phi^q(x+b, x+a)F(a)F(b) \\ &\qquad \times \left[\phi(x+a)(\mathscr{D}_{\mu,x+b}\phi)^\dagger(x+b) - \mathscr{D}_{\mu,x+a}\phi(x+a)\phi^\dagger(x+b)\right] \\ &+ \mathscr{D}_{\mu,x}\phi(x)(\mathscr{D}_{\mu,x}\phi)^\dagger(x) + \cdots ,\end{aligned}$$

Gauge link: $\mathcal{G}_\phi^q(x,y) = \exp\left[-ie_\phi^q \int_x^y dz^\mu \mathscr{A}_\mu(z)\right]$

Nonlocal strong Interaction:

$$\begin{aligned}\mathcal{L}_{\mathrm{had}}^{(\mathrm{nonloc})}(x) &= \bar{p}(x)\left[\frac{C_{B\phi}}{f}\gamma^\mu\gamma^5 B(x) + \frac{C_{T\phi}}{f}\Theta^{\mu\nu}T_\nu(x)\right]\int d^4a\, F(a)\,\partial_\mu\phi(x+a) + \mathrm{h.c.} \\ &+ \frac{iC_{\phi\phi^\dagger}}{2f^2}\bar{p}(x)\gamma^\mu p(x)\int d^4a \int d^4b\ F(a)F(b) \\ &\qquad \times \left[\phi(x+a)\partial_\mu\phi^\dagger(x+b) - \partial_\mu\phi(x+a)\phi^\dagger(x+b)\right],\end{aligned}$$

Nonlocal effective Lagrangian

Normal interaction from minimal substitution:

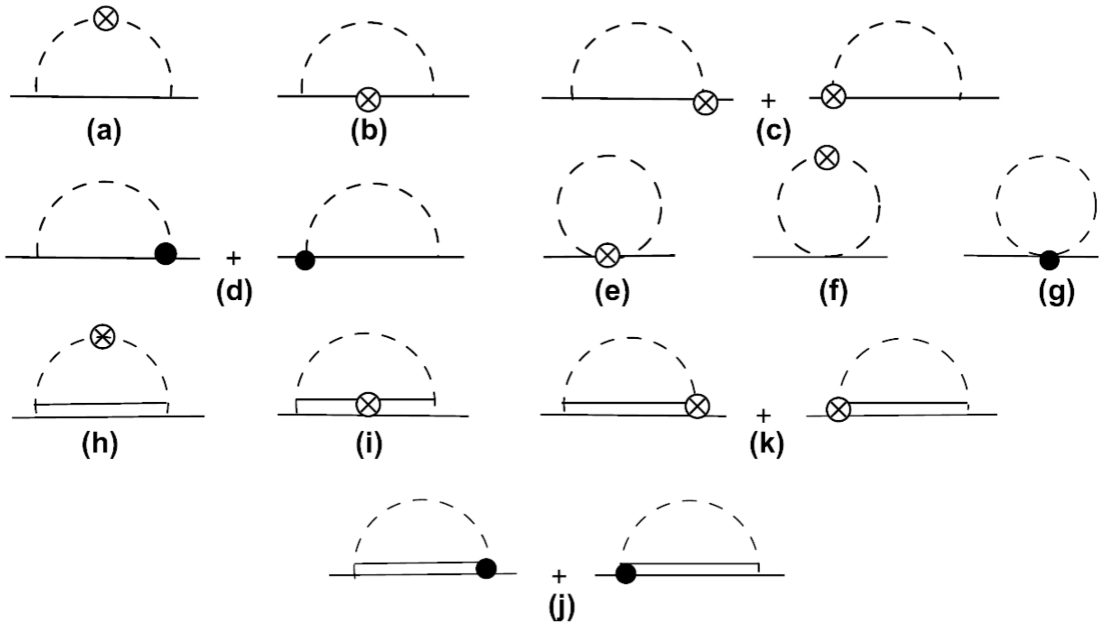
$$\begin{aligned}
\mathcal{L}_{\rm em}^{\rm (nonloc)}(x) &= e_B^q\,\bar{B}(x)\gamma^\mu B(x)\,\mathscr{A}_\mu(x) \;+\; e_T^q\,\overline{T}_\mu(x)\gamma^{\mu\nu\alpha}T_\nu(x)\,\mathscr{A}_\alpha(x)\\
&+ ie_\phi^q\left[\partial^\mu\phi(x)\phi^\dagger(x)-\phi(x)\partial^\mu\phi^\dagger(x)\right]\mathscr{A}_\mu(x)\\
&- ie_\phi^q\,\bar{p}(x)\left[\frac{C_{B\phi}}{f}\,\gamma^\mu\gamma^5 B(x)+\frac{C_{T\phi}}{f}\,\Theta^{\mu\nu}T_\nu(x)\right]\\
&\qquad\qquad \times\int d^4a\,F(a)\,\phi(x+a)\mathscr{A}^\mu(x+a)+\text{h.c.}\\
&- \frac{e_\phi^q C_{\phi\phi^\dagger}}{2f^2}\,\bar{p}(x)\gamma^\mu p(x)\int d^4a\,F(a)\int d^4b\,F(b)\\
&\qquad\qquad \times\,\phi(x+a)\phi^\dagger(x+b)\left[\mathscr{A}^\mu(x+a)+\mathscr{A}^\mu(x+b)\right]
\end{aligned}$$

Additional interaction from expansion of gauge link:

$$\begin{aligned}
\mathcal{L}_{\rm link}^{\rm (nonloc)}(x) &= -ie_\phi^q\,\bar{p}(x)\left[\frac{C_{B\phi}}{f}\,\gamma^\rho\gamma^5 B(x)+\frac{C_{T\phi}}{f}\,\Theta^{\rho\nu}T_\nu(x)\right]\\
&\qquad \times\int_0^1 dt\int d^4a\,F(a)\,a^\mu\,\partial_\rho\phi(x+a)\mathscr{A}_\mu(x+at)+\text{h.c.}\\
&+ \frac{e_\phi^q C_{\phi\phi^\dagger}}{2f^2}\,\bar{p}(x)\gamma^\rho p(x)\int_0^1 dt\int d^4a\int d^4b\,F(a)\,F(b)\,(a-b)^\mu\\
&\quad \times\left[\phi(x+a)\partial_\rho\phi^\dagger(x+b)-\partial_\rho\phi(x+a)\phi^\dagger(x+b)\right]\mathscr{A}_\mu\big(x+at+b(1-t)\big)
\end{aligned}$$

Splitting functions

One loop diagrams:



$$\Gamma^{\mu}_{\phi B}\,(2\pi)^4\delta^{(4)}(p-p) = \langle p|i^2\int d^4x\,d^4y\,d^4z\,\mathcal{L}^{(\text{nonloc})}_{\text{had}(B)}(x)\,J^{\mu}_{q,\text{em}}(y)\,\mathcal{L}^{(\text{nonloc})}_{\text{had}(B)}(z)|p\rangle$$

$$= \frac{i^2C^2_{B\phi}}{f^2}\langle p|\int d^4x\,d^4y\,d^4z\int d^4a\,F(a)\int d^4b\,F(b)\,\bar{p}(x)\,\gamma^{\nu}\gamma^5B(x)\,\partial_{\nu}\phi(x+a)$$

$$\times\,\left(-i[\phi(y)\partial^{\mu}\phi^{\dagger}(y)-\phi^{\dagger}(y)\partial^{\mu}\phi(y)]\right)\bar{B}(z)\gamma^{\rho}\gamma^5p(z)\,\partial_{\rho}\phi^{\dagger}(z+b)\,|p\rangle,$$

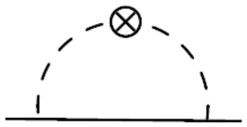
$$\equiv \int\frac{d^4k}{(2\pi)^4}\widetilde{\Gamma}^{\mu}_{\phi B}$$

Splitting functions

$$f_{\phi B}^{(\mathrm{rbw})}(y) = \frac{M}{p^+} \int \frac{d^4k}{(2\pi)^4}\, \widetilde{\mathbf{\Gamma}}^+_{\phi B}\, \delta\left(y - \frac{k^+}{p^+}\right)$$

Dipole form factor:

$$\widetilde{F}(k) = \left(\frac{\overline{\Lambda}}{D_\Lambda}\right)^2 \qquad D_\Lambda = k^2 - \Lambda^2 + i\varepsilon \qquad \overline{\Lambda}^2 \equiv \Lambda^2 - m_\phi^2$$



(a)

$$f_{\phi B}^{(\mathrm{rbw})}(y) = \frac{iC_{B\phi}^2 \overline{\Lambda}^8}{f^2} \int \frac{d^4k}{(2\pi)^4} \left[\frac{y\overline{M}^2(\Delta^2 - m_\phi^2)}{D_\phi^2\, D_B\, D_\Lambda^4} - \frac{y\overline{M}^2}{D_\phi\, D_B\, D_\Lambda^4} + \frac{y(\overline{M}\Delta - 2p\cdot k)}{D_\phi^2\, D_\Lambda^4}\right] \times \delta\left(y - \frac{k^+}{p^+}\right),$$

$$f_{\phi B}^{(\mathrm{rbw})}(y) = \frac{C_{B\phi}^2 \overline{M}^2}{(4\pi f)^2}\left[f_B^{(\mathrm{on})}(y) + f_B^{(\delta)}(y) - \delta f_B^{(\delta)}(y)\right]$$

On-shell contribution: $f_B^{(\mathrm{on})}(y) = \overline{\Lambda}^8 \int dk_\perp^2\, \frac{y\left[k_\perp^2 + (yM + \Delta)^2\right]}{\bar{y}^2\, D_{\phi B}^2\, D_{\Lambda B}^4}$

$$D_{\phi B} = -\frac{k_\perp^2 + yM_B^2 - y\,\bar{y}\,M^2 + \bar{y}\,m_\phi^2}{\bar{y}}$$

$$D_{\Lambda B} = -\frac{k_\perp^2 + yM_B^2 - y\,\bar{y}\,M^2 + \bar{y}\,\Lambda^2}{\bar{y}}.$$

Splitting functions

$$\text{when } \Lambda \to \infty, \qquad f^{(\mathrm{on})}_{\pi^+ n}(y) \longrightarrow \int dk_\perp^2 \frac{y\left(k_\perp^2 + y^2 M^2\right)}{\left[k_\perp^2 + y^2 M^2 + \bar{y}\, m_\pi^2\right]^2}$$

Delta function terms:

$$f_B^{(\delta)}(y) = -\frac{\overline{\Lambda}^8}{\overline{M}^2} \int dk_\perp^2 \int_0^1 dz \, \frac{z^3}{(k_\perp^2 + \Omega)^4} \, \delta(y)$$

$$= \frac{1}{\overline{M}^2} \int dk_\perp^2 \left[\log \frac{\Omega_\phi}{\Omega_\Lambda} + \frac{\overline{\Lambda}^2 (11\, \Omega_\Lambda^2 - 7\, \Omega_\Lambda \Omega_\phi + 2\, \Omega_\phi^2)}{6 \Omega_\Lambda^3} \right] \delta(y)$$

$$\delta f_B^{(\delta)}(y) = -\frac{\overline{\Lambda}^8}{\overline{M}^2} \int dk_\perp^2 \int_0^1 dz \, \frac{z^4}{(k_\perp^2 + \Omega)^4} \, \delta(y)$$

$$= \frac{1}{\overline{M}^2} \int dk_\perp^2 \left[-4 \frac{\Omega_\phi}{\overline{\Lambda}^2} \log \frac{\Omega_\phi}{\Omega_\Lambda} - \frac{3\Omega_\Lambda^3 + 13 \Omega_\Lambda^2 \Omega_\phi - 5 \Omega_\Lambda \Omega_\phi^2 + \Omega_\phi^3}{3 \Omega_\Lambda^3} \right]$$

$$\Omega_\phi = k_\perp^2 + m_\phi^2 \,, \qquad \Omega_\Lambda = k_\perp^2 + \Lambda^2$$

when $\Lambda \to \infty$, the delta function term is the same as that in the local chiral EFT.

Splitting functions

General regulator: $F(t,\ u) \equiv F[k^2,\ (p-k)^2] = \left(\frac{\Lambda_t - m^2}{\Lambda_t - t}\right) \cdot \left(\frac{\Lambda_u - M^2}{\Lambda_u - u}\right) = \frac{a}{d_N}\frac{b}{d_\pi}$

$$f_{n\pi^+}(y) = \frac{-i\cdot(F+D)^2}{4f^2} \cdot \int \frac{d^4k}{(2\pi)^4}\left(\frac{4\ (p.k)}{D_\pi^2} + \frac{8\ M^2}{D_\pi D_N} + \frac{8\ M^2\ m^2}{D_\pi^{\ 2} D_N}\right)\cdot F^2(t,\ u)\cdot y\cdot\delta\left(y - \frac{k^+}{p^+}\right)$$

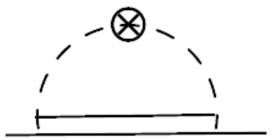
$$\left(\left(-\frac{\pi\ i}{\Lambda_t} - \frac{9\ \pi\ i\ M^2}{a^2} - \frac{3\ \pi\ i}{a}\right)\ln(m^2)\right)\ m^4 + \left(-\pi\ i\ \ln(m^2)\right)\ m^2$$

$$\left(\left(\frac{3}{2}\ \frac{\pi\ i\ M^2}{a^2} - \frac{1}{4}\ \frac{\pi\ i}{M^2}\right)\ln(m^2)\right)\ m^4 - \frac{1}{2}\ \frac{\pi^2\ i\ m^3}{M} + \left(\frac{1}{2}\ \pi\ i\ \ln(m^2)\right)\ m^2$$

$$\left(\left(\frac{3\ \pi\ i\ M^2}{a^2} + \frac{\pi\ i}{\Lambda_t} - \frac{1}{2}\ \frac{\pi\ i}{M^2}\right)\ln(m^2)\right)\ m^4 - \frac{3}{4}\ \frac{\pi^2\ i\ m^3}{M} + \left(\frac{1}{2}\ \pi\ i\ \ln(m^2)\right)\ m^2$$

- LNAs do not depend on the regulators.
- LNAs are the same in relativistic and heavy baryon EFT.

Splitting functions



(h)

$$f_{\phi T}^{(\mathrm{rbw})}(y) = \frac{iC_{T\phi}^2\,\overline{\Lambda}^8}{6M_T^2 f^2}\int\frac{d^4k}{(2\pi)^4}\left[\frac{y(\overline{M}_T^2-m_\phi^2)^2(\Delta_T^2-m_\phi^2)}{D_\phi^2\,D_T\,D_\Lambda^4}\right.$$

$$-\frac{y(\overline{M}_T^2-m_\phi^2)(\overline{M}_T^2+2\Delta_T^2-3m_\phi^2)}{D_\phi\,D_T\,D_\Lambda^4}+\frac{y(2\overline{M}_T^2+\Delta_T^2-k^2-2m_\phi^2)}{D_T\,D_\Lambda^4}$$

$$+\frac{y}{D_\phi^2\,D_\Lambda^4}\Big(4(p\cdot k)^2-2(\overline{M}_T^2-k^2)\,p\cdot k+(M_T^2-k^2)^2$$

$$\left.+M(2M_T^3-M^3-2M^2M_T)-2Mk^2(2M+M_T)\Big)\right]\delta\left(y-\frac{k^+}{p^+}\right)$$

$$f_{\phi T}^{(\mathrm{rbw})}(y) = \frac{C_{T\phi}^2\overline{M}_T^2}{(4\pi f)^2}\left[f_T^{(\mathrm{on})}(y)+f_T^{(\mathrm{on\,end\,pt})}(y)-\frac{1}{18}f_T^{(\delta)}(y)+\frac{\overline{M}_T^2-m_\phi^2}{6M_T^2}\Big(f_\phi^{(\delta)}(y)-\delta f_\phi^{(\delta)}(y)\Big)\right]$$

On-shell contribution: $f_T^{(\mathrm{on})}(y) = \dfrac{\overline{\Lambda}^8}{6M_T^2\,\overline{M}_T^2}\displaystyle\int dk_\perp^2\,\frac{y\,(\overline{M}_T^2-m_\phi^2)}{\bar{y}}\left[\frac{(\overline{M}_T^2-m_\phi^2)(\Delta_T^2-m_\phi^2)}{D_{\phi T}^2\,D_{\Lambda T}^4}-\frac{3(\Delta_T^2-m_\phi^2)+4MM_T}{D_{\phi T}\,D_{\Lambda T}^4}\right]$

$$\text{when }\Lambda\to\infty,\qquad f_\Delta^{(\mathrm{on})}(y) \;=\; C_\Delta\int dk_\perp^2\,\frac{y\,(\overline{M}^2-m_\pi^2)}{1-y}$$

$$\times\left[\frac{(\overline{M}^2-m_\pi^2)(\Delta^2-m_\pi^2)}{D_{\pi\Delta}^2}-\frac{3(\Delta^2-m_\pi^2)+4MM_\Delta}{D_{\pi\Delta}}\right]$$

Y.Salamu, C. R. Ji, W. Melnitchouk and P. Wang, Phys. Rev. Lett. 114 (2015) 122001.

Splitting functions

End-point term: $$f_T^{(\text{on end pt})}(y) = \frac{\overline{\Lambda}^8}{6M_T^2\,\overline{M}_T^2}\int dk_\perp^2\,\frac{y}{\bar{y}^2\,D_{\Lambda T}^4} \times\Big[k_\perp^2 + y^2M^2 + 3\overline{M}_T^2 - 4MM_T - 2y(\overline{M}_T^2 - M\Delta_T) - 2\bar{y}\,m_\phi^2\Big]$$

Local limit: $$f_T^{(\text{on end pt})}(y) \xrightarrow[\Lambda\to\infty]{} \frac{1}{6M_T^2\,\overline{M}_T^2}\int dk_\perp^2\,\delta(\bar{y})\left\{\Big[\Omega_T - 2(\Delta_T^2 - m_\phi^2) - 6MM_T\Big]\log\frac{\Omega_T}{\mu^2} - \Omega_T + 2(\Delta_T^2 - m_\phi^2) + 6MM_T\right\}.$$

Delta term: $$f_T^{(\delta)}(y) = \frac{\overline{\Lambda}^8}{M_T^2\overline{M}_T^2}\int dk_\perp^2\int_0^1 dz\,\frac{z^3}{(k_\perp^2+\Omega)^3}\,\delta(y)$$
$$= \frac{1}{M_T^2\overline{M}_T^2}\int dk_\perp^2\,\frac{1}{2\Omega_\Lambda^2}\left[6\Omega_\Lambda^2\Omega_\phi\log\frac{\Omega_\phi}{\Omega_\Lambda} + (\Omega_\phi-\Omega_\Lambda)(\Omega_\phi^2 - 5\Omega_\phi\Omega_\Lambda - 2\Omega_\Lambda^2)\right]\delta(y)$$
$$\Omega_\phi = k_\perp^2 + m_\phi^2\,, \qquad \Omega_\Lambda = k_\perp^2 + \Lambda^2$$

when $\Lambda\to\infty$, all terms are the same as those in the local chiral EFT.

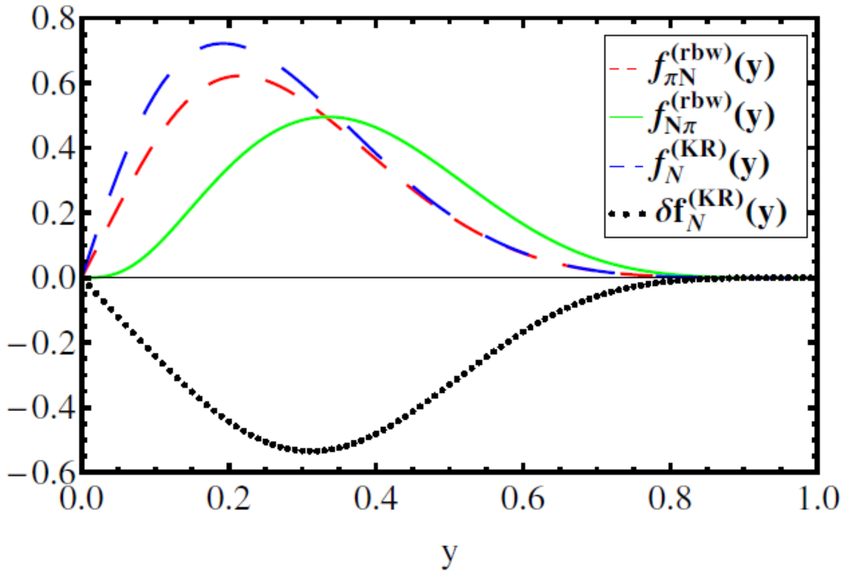
Splitting functions

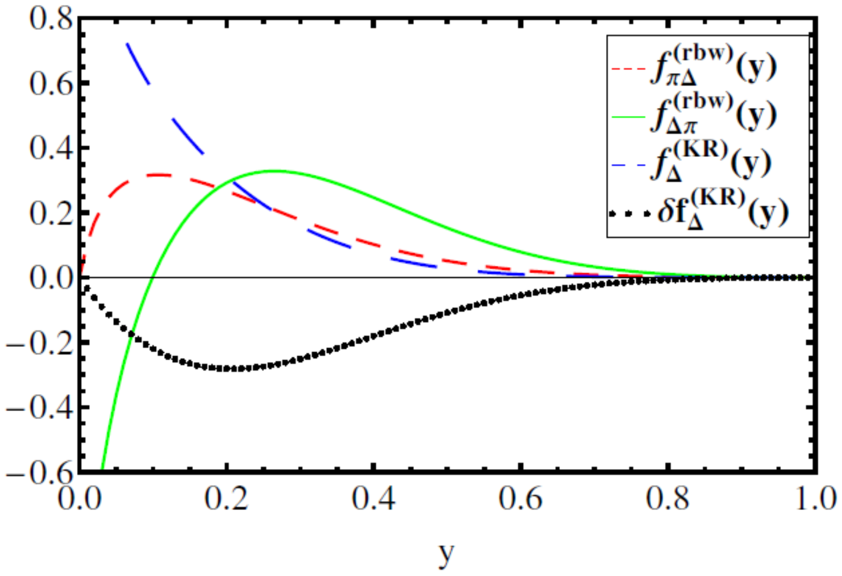
Relationships between splitting functions:

$$f_{\phi B}^{(\mathrm{rbw})}(y) = f_{B\phi}^{(\mathrm{rbw})}(y) + f_{B}^{(\mathrm{KR})}(y) + \delta f_{B}^{(\mathrm{KR})}(y)$$

$$f_{\phi}^{(\mathrm{bub})}(y) = f_{\phi}^{(\mathrm{tad})}(y) + \delta f_{\phi}^{(\mathrm{tad})}(y)$$

Gauge invariance of nonlocal Lagrangian, charge conversation restoration.





Numerical results of PDFs

Match quark operator to hadron operator:

$$\mathcal{O}_q^{\mu_1\cdots\mu_n} = \sum_j c_{q/j}^{(n)}\, \mathcal{O}_j^{\mu_1\cdots\mu_n} \qquad c_{q/j}^{(n)} = \int_{-1}^{1} dx\, x^{n-1}\, q_j(x) \equiv \langle x^{n-1}\rangle_{q/j}$$

$$\langle N(p)|\mathcal{O}_q^{\mu_1\cdots\mu_n}|N(p)\rangle = 2\,\langle x^{n-1}\rangle_q\, p^{\mu_1}\cdots p^{\mu_n} \qquad \langle N(p)|\mathcal{O}_j^{\mu_1\cdots\mu_n}|N(p)\rangle = 2\, f_j^{(n)}\, p^{\mu_1}\cdots p^{\mu_n}$$

$$\int_{-1}^{1} dx\, x^{n-1} q(x) = \int_{-1}^{1} dx\, x^{n-1} \sum_j \int_0^1 dy\, f_j(y) \int_0^1 dz\, \delta(x-yz)\, q_j^v(z) \longrightarrow q(x) = \sum_j (f_j \otimes q_j^v)(x) \equiv \sum_j \int_0^1 dy \int_0^1 dz\, \delta(x-yz)\, f_j(y)\, q_j^v(z)$$

dbar-ubar asymmetry:

$$\bar{d}(x) - \bar{u}(x) = [f_{\pi^+ n}(y) + f_{bub,\pi^+}(y) + f_{\pi^+\Delta^0}(y) - f_{\pi^-\Delta^{++}}(y)] \otimes \bar{q}_\pi(x)$$

s-sbar asymmetry:

$$\begin{aligned} s(x) - \bar{s}(x) = \sum_{B,T,\phi} \Big[& f_{B\phi}^{(\mathrm{rbw})}(y) \otimes s_B(x) + f_B^{(\mathrm{KR})}(y) \otimes s_B^{(KR)}(x) + \delta f_B^{(\mathrm{KR})}(y) \otimes s_B^{\delta}(x) \\ & - f_{\phi B}^{(\mathrm{rbw})}(y) \otimes \bar{s}_\phi(x) + f_{T\phi}^{(\mathrm{rbw})}(y) \otimes s_T(x) + f_T^{(\mathrm{KR})}(y) \otimes s_T^{(KR)}(x) \\ & + \delta f_T^{(\mathrm{KR})}(y) \otimes s_T^{\delta}(x) - f_{\phi T}^{(\mathrm{rbw})}(y) \otimes \bar{s}_\phi(x) + f_\phi^{(\mathrm{tad})} \otimes s_\phi^{(tad)}(x) \\ & + \delta f_\phi^{(\mathrm{tad})}(y) \otimes s_\phi^{\delta}(x) - f_\phi^{(\mathrm{bub})}(y) \otimes \bar{s}_\phi(x) \Big], \end{aligned}$$

Numerical results of PDFs

Input PDFs of valence at tree level:

valence quark in mesons:

$$u_{\pi^+}(x) = \bar{d}_{\pi^+}(x) = \bar{u}_{\pi^-}(x) = d_{\pi^-}(x) = \bar{s}_{K^+}(x) = \bar{s}_{K^0}(x) = \bar{q}_{\pi}(x)$$

valence quark in baryons:

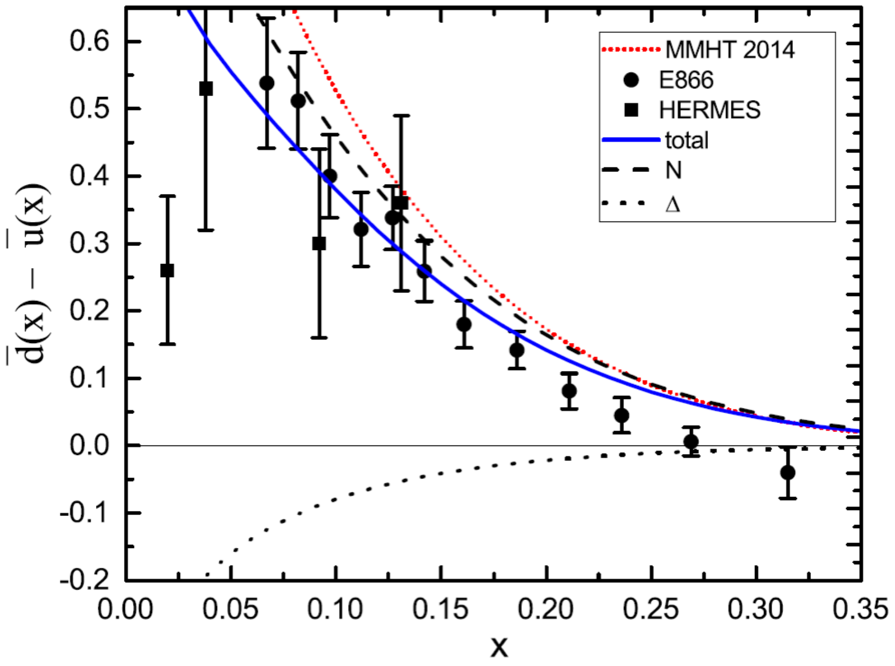
$$u_n(x) = d(x), u_{\Sigma^+}(x) = 2u_{\Sigma^0} = u(x), u_{\Lambda}(x) = \frac{4\,d(x) + u(x)}{6}, u_{\Delta^0}(x) = \frac{1}{2}u_{\Delta^+}(x) = \frac{1}{3}\,u_{\Delta^{++}}(x)$$
$$= u_{\Sigma^{*0}}(x) = \frac{1}{2}u_{\Sigma^{*+}}(x) = d(x), d_n(x) = u(x), 2\,d_{\Sigma^0}(x) = d_{\Sigma^-}(x) = u(x), d_{\Lambda}(x) = \frac{4d(x) + u(x)}{6}$$
$$\frac{1}{3}d_{\Delta^-}(x) = \frac{1}{2}\,d_{\Delta^0}(x) = d_{\Delta^+}(x) = \frac{1}{2}d_{\Sigma^{*-}}(x) = d_{\Sigma^{*0}}(x) = d(x), s_{\Lambda}(x) = \frac{2\,u(x) - d(x) + 2s(x)}{3},$$
$$s_{\Sigma^+}(x) = s_{\Sigma^0}(x) = d(x), s_{\Sigma^{+*}}(x) = s_{\Sigma^{0*}}(x) = d(x),$$

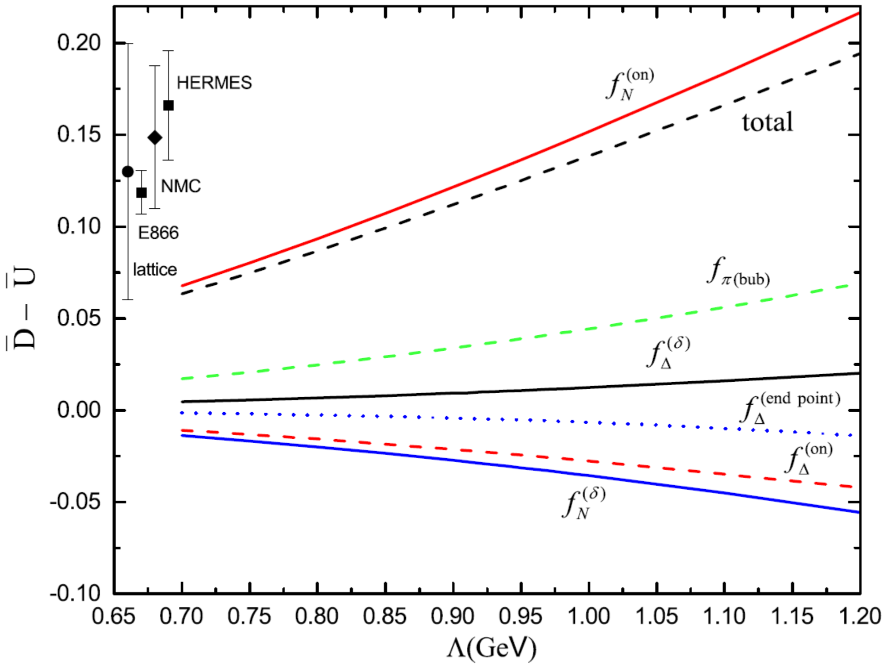
quark distributions for KR diagrams:

$$u_n^{(KR)}(x) = \frac{\Delta u(x) - \Delta d(x)}{F + D}, u_{\Sigma^0}^{(KR)}(x) = \frac{\Delta d(x)}{F - D}, u_{\Lambda}^{(KR)} = \frac{2\Delta u(x) - \Delta d(x)}{3F + D}$$
$$u_{\Delta^0}^{(KR)}(x) = u_{\Delta^{++}}^{(KR)}(x) = u_{\Sigma^{*0}}^{(KR)}(x) = \frac{2\Delta d(x) - \Delta u(x)}{-2D},$$
$$d_n^{(KR)}(x) = \frac{\Delta u(x) - \Delta d(x)}{F + D}, d_{\Sigma^+}^{(KR)}(x) = \frac{\Delta d(x)}{F - D},$$
$$d_{\Delta^0}^{(KR)}(x) = d_{\Delta^{++}}^{(KR)}(x) = d_{\Sigma^{*+}}^{(KR)}(x) = \frac{2\Delta d(x) - \Delta u(x)}{-2D},$$
$$s_{\Sigma^+}^{(KR)}(x) = s_{\Sigma^0}^{(KR)}(x) = \frac{\Delta d(x)}{F - D}, s_{\Lambda}^{(KR)}(x) = \frac{2\Delta u(x) - \Delta d(x)}{3F + D},$$
$$s_{\Sigma^{*+}}^{(KR)}(x) = s_{\Sigma^{*0}}^{(KR)}(x) = \frac{2\Delta d(x) - \Delta u(x)}{-2D},$$

Numerical results of PDFs

dbar — ubar asymmetry:

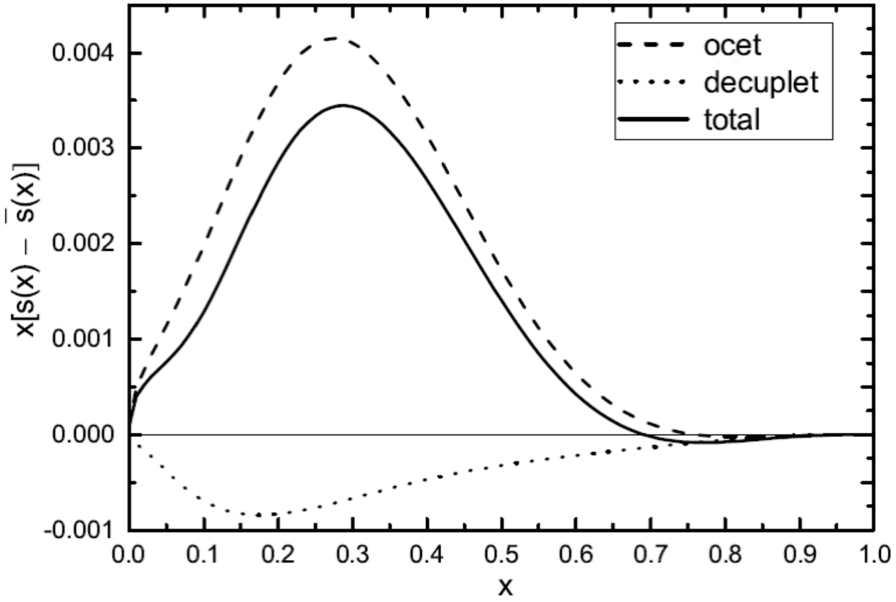


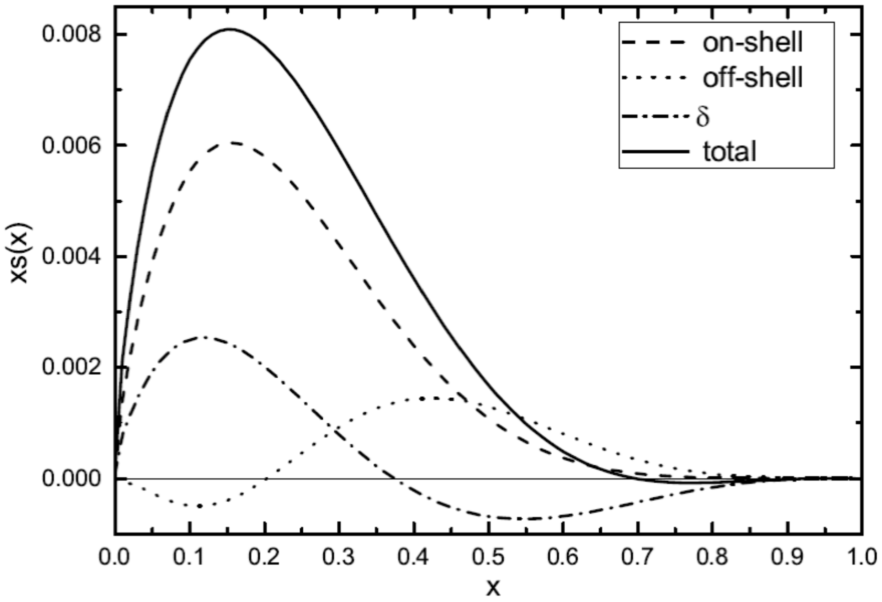


- Contribution from octet intermediate states is more important than that from decuplet intermediate states.
- The sum of the on-shell contribution is close to the total result.

Numerical results of PDFs

s — sbar asymmetry:





Again:

- Contribution from octet intermediate states is more important than that from decuplet intermediate states.
- The sum of the on-shell contribution is close to the total result.

Numerical results of PDFs

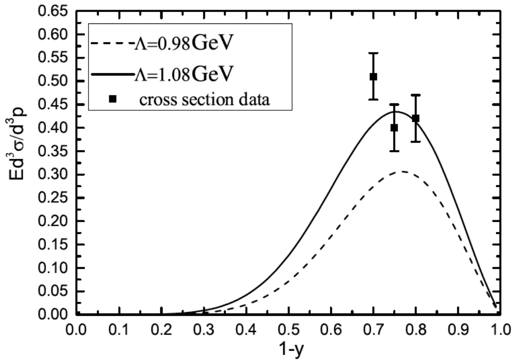
First moment (10^-2): $\int_0^1 dx[s_p(x) - \bar{s}_p(x)] = 0$

Λ (GeV)	a	b	c	d	e	f	g	$S^{(0)}_{oct}$	h	i	j	k	$S^{(0)}_{dec}$	$S^{(0)+}$
1.08	0.776	1.587	0.670	−1.481	4.446	1.442	−3.004	2.218	0.299	0.789	0.169	−0.658	0.299	5.035
0.98	0.468	0.928	0.517	−0.977	2.700	0.836	−1.864	1.305	0.169	0.419	0.144	−0.394	0.169	2.947
0.91	0.308	0.614	0.379	−0.685	1.793	0.536	−1.257	0.844	0.106	0.250	0.114	−0.258	0.106	1.899

Second moment (10^-3): $\int_0^1 dx x[s(x) \pm \bar{s}(x)]$

Λ (GeV)	a	b	c	d	e	g	$S^{(1)}_{oct}$	h	i	j	k	$S^{(1)}_{dec}$	S^{-}	S^{+}
1.08	1.204	2.070	−0.271	−1.367	7.212	−4.873	2.770	0.417	0.718	0.013	−0.659	0.071	1.220	4.463
0.98	0.683	1.227	0.049	−0.986	4.380	−3.023	1.648	0.219	0.387	0.092	−0.422	0.056	0.801	2.606
0.91	0.429	0.799	0.136	−0.730	2.908	−2.039	1.074	0.130	0.233	0.100	−0.290	0.043	0.557	1.676

$$E\frac{d^3\sigma}{d^3p} = \frac{C^2_{K^+\Lambda}\overline{M}^2}{16\pi^3 f^2_\phi}\frac{y\,[k^2_\perp + (My+\Delta)^2]}{(1-y)D^2_{K^+\Lambda}}F^{(\mathrm{on})}(y,k^2_\perp)\,\sigma^{pK^+}_{\mathrm{tot}}(sy)$$



Summary

- Nonlocal Lagrangian is constructed by introducing the correlation function and the gauge link.
- Correlation function generates the regulator and tree level contribution.
- Gauge link guarantees the locally gauge invariance.
- The total contribution includes on-shell, off-shell, end-point and delta contributions.
- When Lambda goes to infinity, all the formulas are the same as those in the local case.
- At finite Lambda, the LNAs are the same as those in the local case.
- For both ubar-dbar and s-sbar asymmetry, the contribution from the octet part is dominant.
- The on-shell contribution is close to the total result.
- Without fine-tuning the parameters, the agreement between the calculation and experiment is very good.

The End !