Orbital angular momentum in the nucleon's chiral periphery

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Outline

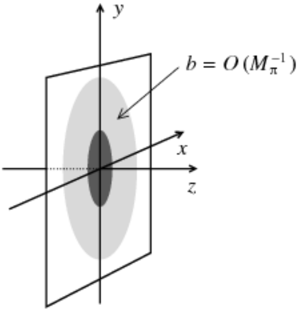
- Motivation
- Transverse Densities
  Definitions, properties
- Orbital angular momentum in impact parameter space
  Spectral Functions, ChPT calculation
- Light front formulation
  Wave functions and χGPDs
- Transverse densities of orbital angular momentum

Motivation

We produce distributions of orbital angular momentum for the nucleon from first principles in the peripheral region and present them in the light front formalism.

- Explore definitions of orbital angular momentum
- Provide a mechanical picture of OAM distributions. Light-front formulation of dynamics in the chiral periphery.

Transverse Densities



$$\rho(b) = \int \frac{d^2\mathbf{\Delta_T}}{(2\pi)} F(-\Delta_T^2)$$

Connect Form Factors and GPDs to nucleon intrinsic spacial structure

G.A. Miller,ARNPS 60 (2010)
M.Burkardt,PRD62(2000)

- Boost invariants;
- Light front wave function overlap for multiparticle systems.
- Distance b as parameter

$$[b \to M_\pi^{-1}]$$

Chiral periphery.

M.Strikman, C.Weiss PRC82(2010)
CG, C.Weiss JHEP 1401 (2014)

Transverse density of orbital angular momentum

- From form factors of energy-momentum tensor

$$\begin{aligned}\langle N_2|T_{\mu\nu}(0)|N_1\rangle &= \bar{u}_2\left[A(\Delta^2)\gamma_{(\mu}\,p_{\nu)} + B(\Delta^2)p_{(\mu}\,i\sigma_{\nu)\alpha}\frac{\Delta^\alpha}{2M_N}\right.\\ &\left. +C(\Delta^2)\left(\frac{\Delta_\mu\Delta_\nu - \Delta^2 g_{\mu\nu}}{M_N}\right) + \tilde{C}(\Delta^2)M_N g_{\mu\nu}\right]u_1,\end{aligned}$$

and the condition,

$$\int d^2b\rho_I(b) = A(0) + B(0) = 1.$$

- Various alternatives,

M. V. Polyakov, PLB **555**, 57 (2003)

L. Adhikari and M. Burkardt, PRD **94**, no. 11, 114021 (2016)

$$\rho_J(b) \equiv -\frac{1}{2}b\frac{d}{db}\left(\rho_A(b) + \rho_B(b)\right)$$

- We look at $\rho_J(b)$ in the Chiral periphery!

Transverse density of orbital angular momentum

- Study longitudinal component

$$\langle L_z(\boldsymbol{b})\rangle = \frac{1}{2p^+}\langle \boldsymbol{b} \times T^{+T}(\boldsymbol{b})\rangle \cdot \boldsymbol{e}_z,$$

- Matrix elements

$$\langle L_z(\boldsymbol{b})\rangle = -\frac{\langle S_z\rangle}{2} b \frac{d}{db}\left(\rho_A(b) + \rho_B(b)\right).$$

- One can then identify the quantity,

$$\rho_{l_z}(b) \equiv -\frac{1}{2} b \frac{d}{db}\left(\rho_A(b) + \rho_B(b)\right)$$

as a transverse density of longitudinal orbital angular momentum. It satisfies the condition,

$$\int d^2 b \rho_{l_z}(b) = A(0) + B(0).$$

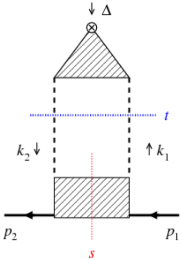
EMT form factors

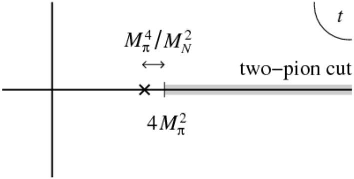
$$\begin{aligned}\langle N_2|T_{\mu\nu}(0)|N_1\rangle &= \bar{u}_2\left[A(\Delta^2)\gamma_{(\mu}\, p_{\nu)} + B(\Delta^2)p_{(\mu}\, i\sigma_{\nu)\alpha}\frac{\Delta^\alpha}{2M_N}\right.\\ &\left.+C(\Delta^2)\left(\frac{\Delta_\mu\Delta_\nu - \Delta^2 g_{\mu\nu}}{M_N}\right) + \tilde{C}(\Delta^2)M_N g_{\mu\nu}\right]u_1,\end{aligned}$$

Working in the $\Delta^+ = 0$ reference frame, one can write the EMT form factors in terms of matrix elements of light front components of the EMT. For the combination $A + B$,

$$A(-\mathbf{\Delta}_T^2) + B(-\mathbf{\Delta}_T^2) = \frac{4}{2p^+}\,\frac{i(\mathbf{\Delta}_T \times \mathbf{e}_z)}{\mathbf{\Delta}_T^2}\cdot\sum_{\sigma_1\sigma_2}\langle N_2|\, T^{+T}(0)\,|N_1\rangle S_z(\sigma_1,\sigma_2)$$

TD from dispersive representations of form factors in χPT





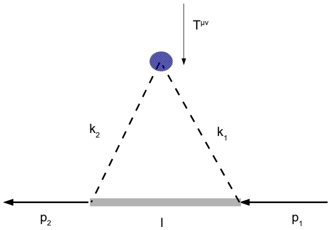
Two-pion cut dispersion relation

$$F(t) = \int_{4M_\pi^2}^{\infty} \frac{dt'}{t'-t} \frac{\mathrm{Im}F(t')}{\pi}$$

Filter high momentum contributions

$$\rho(b) = \int_{4M_\pi^2}^{\infty} \frac{dt}{2\pi} K_0(\sqrt{t}b) \frac{\mathrm{Im}F(t+i0)}{\pi}$$

Energy momentum tensor and OAM in χPT



- Use $\mathcal{L}_{eff}$ and

$$T_{\mu\nu} = \mathrm{Tr}\left(\partial_\mu\pi\partial_\nu\pi - \frac{1}{2}g_{\mu\nu}(\partial_\sigma\pi)^2\right)$$

to obtain

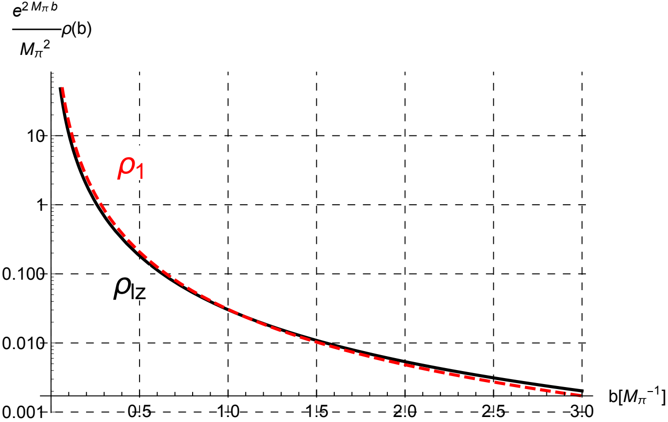
$$\left\langle N_2 | T^{N\pi}_{\mu\nu}(0) | N_1 \right\rangle = -\frac{3}{4}\frac{g_A^2}{F_\pi^2}\int\frac{d^4 l}{(2\pi)^4}\left[-i\bar{u}_2 \not{k}_2\gamma^5(\not{l} + M_N)\not{k}_1\gamma^5 u_1 D_N(l)\right.$$
$$\left.(k_{2\mu}k_{1\nu} + k_{1\mu}k_{2\nu} - g_{\mu\nu}k_2 k_1)D_\pi(k_2)D_\pi(k_1)\right].$$

From imaginary part of $T^{\pi N}$,

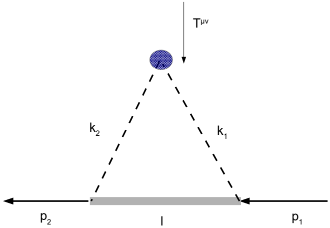
$$\frac{1}{\pi}\mathbf{Im}(A(t)+B(t)) = \frac{3}{4}\frac{g_A^2}{F_\pi^2}M_N^2\frac{(t/2 - M_\pi^2)^3}{(4\pi)^2\sqrt{P^2}^5\sqrt{t}}\left(\frac{2}{3}x^3 + x - \left(x^2+1\right)\mathbf{arctan}(x)\right) \quad (1)$$

Energy momentum tensor and OAM in χPT

$$\rho_{lz}(b) = \int_{4M_\pi^2}^{\infty} \frac{dt}{2\pi} \frac{\sqrt{t}b}{2} K_1(\sqrt{t}b) \frac{\boldsymbol{Im}[A+B](t+i0)}{\pi}$$



Energy momentum tensor and OAM in χPT



- Use $\mathcal{L}_{eff}$ and

$$T_{\mu\nu} = \mathrm{Tr}\left(\partial_\mu\pi\partial_\nu\pi - \frac{1}{2}g_{\mu\nu}(\partial_\sigma\pi)^2\right)$$

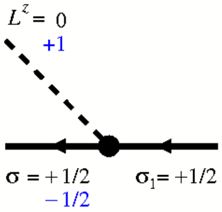
to obtain

$$\begin{aligned}\left\langle N_2 | T^{N\pi}_{\mu\nu}(0) | N_1 \right\rangle &= -\frac{3}{4}\frac{g_A^2}{F_\pi^2}\int \frac{d^4 l}{(2\pi)^4}\left[-i\bar{u}_2 \not{k}_2\gamma^5(\not{l} + M_N)\not{k}_1\gamma^5 u_1 D_N(l)\right. \\ &\left.(k_{2\mu}k_{1\nu} + k_{1\mu}k_{2\nu} - g_{\mu\nu}k_2 k_1)D_\pi(k_2)D_\pi(k_1)\right].\end{aligned}$$

In LF variables ($v^\pm = v^0 \pm v^z$),

$$\begin{aligned}\left\langle N_2 | \frac{T^{++}_{N\pi}(0)}{2p^+} | N_1 \right\rangle &= \tfrac{3}{4}p^+ \int \tfrac{dy}{y\bar{y}}\tfrac{d^2\boldsymbol{k}_T}{(2\pi)^3} y \Psi^*\left(y, \boldsymbol{k}_T + \bar{y}\tfrac{\boldsymbol{\Delta}_T}{2};\right)\Psi\left(y, \boldsymbol{k}_T - \bar{y}\tfrac{\boldsymbol{\Delta}_T}{2}\right), \\ \left\langle N_2 | \frac{T^{+T}_{N\pi}(0)}{2p^+} | N_1 \right\rangle &= \tfrac{3}{4}\int \tfrac{dy}{y\bar{y}}\tfrac{d^2\boldsymbol{k}_T}{(2\pi)^3}\boldsymbol{k}_T \Psi^*\left(y, \boldsymbol{k}_T + \bar{y}\tfrac{\boldsymbol{\Delta}_T}{2};\right)\Psi\left(y, \boldsymbol{k}_T - \bar{y}\tfrac{\boldsymbol{\Delta}_T}{2}\right)\end{aligned}$$

Light front wave functions



$$\Psi(y, \widetilde{\boldsymbol{k}}_T = \boldsymbol{k}_T + y p_{1T}) \equiv \frac{\Gamma(y, \widetilde{\boldsymbol{k}}_T)}{\underbrace{\Delta\mathcal{M}^2(y, \widetilde{\boldsymbol{k}}_T)}_{\text{Inv. Mass difference}}}$$

while in transverse coordinate space,

$$\begin{aligned}\Phi(y, r_T) &= \int \frac{d^2\widetilde{\boldsymbol{k}}_T}{(2\pi)^2} e^{\widetilde{\boldsymbol{k}}_T \cdot \boldsymbol{r}_T} \Psi(y, \widetilde{\boldsymbol{k}}_T) \\ &= -2i\left[U_0(y, r_T) S^z + i\,\frac{U_1(y, r_T)\boldsymbol{r}_T \cdot \boldsymbol{S}_T}{r_T}\right]\end{aligned}$$

Eigenfunctions of LF Hamiltonian.

Allow quantum mechanical description of peripheral dynamics.

Computable at leading order in chiral periphery

C.G , C. Weiss, JHEP **1507**, 170 (2015)

$$\begin{aligned}\Gamma(y, \widetilde{\boldsymbol{k}}_T) &\approx \frac{g_A M_N}{F_\pi} \bar{u}(y, \boldsymbol{k}_T) i\gamma_5 u(p_{1T}) \\ &= \frac{2i g_A M_N^2}{F_\pi \sqrt{\bar{y}}}\left[y \boldsymbol{S}_z + \frac{\widetilde{\boldsymbol{k}}_T \cdot \boldsymbol{S}_T}{M_N}\right]\end{aligned}$$

with radial functions,

$$\left.\begin{array}{c} U_0(y, r_T) \\ U_1(y, r_T) \end{array}\right\} = \frac{g_A M_N\, y\sqrt{\bar{y}}}{2\pi F_\pi} \left\{ \begin{array}{c} y M_N \;\; K_0(M_T r_T) \\ M_T \;\; K_1(M_T r_T) \end{array}\right. ,$$

and transverse mass

$$M_T^2 = \bar{y}^2 M_\pi + y^2 M_N^2$$

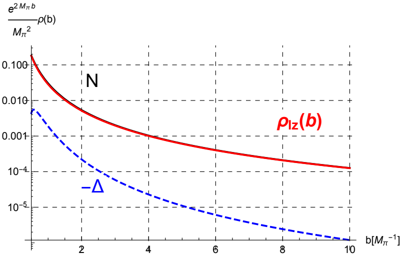
OAM in Chiral Periphery

$$\rho_{l_z}(b) = \frac{3}{2}\frac{1}{2\pi}\int \frac{dy}{y\bar{y}^3}\bar{y}\mathrm{Tr}\left[S_z\Phi^\dagger(y,\boldsymbol{r}_T)\left(\boldsymbol{r}_T\times(-i)\frac{\partial}{\partial\boldsymbol{r}_T}\right)\Phi(y,\boldsymbol{r}_T)\right]|_{\boldsymbol{r}_T=\boldsymbol{b}/\bar{y}},$$

Validates a probabilistic quantum-mechanical interpretation of $\rho_{lz}(b)$ as a density of OAM in the nucleon's periphery! This cannot be achieved with other definitions of $\rho_{lz}(b)$

$$\begin{aligned}
\rho_{l_z,N}(b) &= \frac{D_N}{2}\frac{1}{2\pi}\int\frac{dy}{y\bar{y}^2}U_1(y,\boldsymbol{b}/\bar{y})^2. \\
\rho_{l_z,\Delta}(b) &= \frac{D_\Delta}{2}\frac{1}{6\pi}\int\frac{dy}{y\bar{y}^2} \\
&\quad \left(2V_1(y,\boldsymbol{b}/\bar{y})^2 + 2V_1(y,\boldsymbol{b}/\bar{y})W_1(y,\boldsymbol{b}/\bar{y})\right. \\
&\quad \left. -W_1(y,\boldsymbol{b}/\bar{y})^2 + 3W_2(y,\boldsymbol{b}/\bar{y})^2\right).
\end{aligned}$$

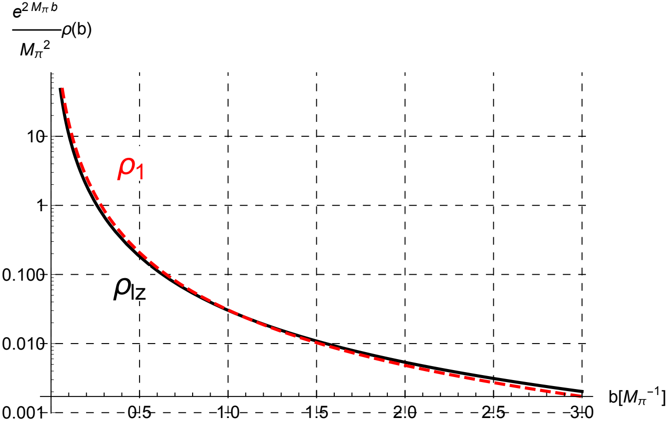
OAM in Chiral Periphery



- Relative (-) sign between intermediate N and *Delta*
- Large N_c cancelation

$$
\begin{aligned}
\rho_{l_z,\Delta}(b) &\approx -\frac{D_\Delta}{2}\frac{1}{6\pi}\int\frac{dy}{y\bar{y}^2}W_1(y,\boldsymbol{b}/\bar{y})^2 \\
&\approx -\frac{D_\Delta}{2}\frac{1}{6\pi}\int\frac{dy}{y\bar{y}^2}\left(\frac{3}{2}U_1(y,\boldsymbol{b}/\bar{y})\right)^2 \\
&\approx -\rho_{l_z,N}(b)
\end{aligned}
$$

Energy momentum tensor and OAM in χPT



$$\rho_{1,N}(b) = \frac{1}{2\pi}\int \frac{dy}{y\bar{y}^3}\left(U_0(y,\mathbf{b}/\bar{y})^2 + U_1(y,\mathbf{b}/\bar{y})^2\right),$$

$$\rho_{l_z,N}(b) = \frac{3}{2}\frac{1}{2\pi}\int \frac{dy}{y\bar{y}^2} U_1(y,\mathbf{b}/\bar{y})^2.$$

Summary and Outlook

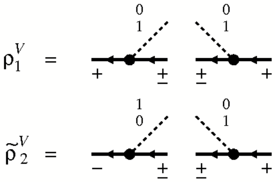
Transverse densities computed in Chiral EFT were used in a model independent approach to quantify the distribution of orbital angular momentum in the periphery of the nucleon.

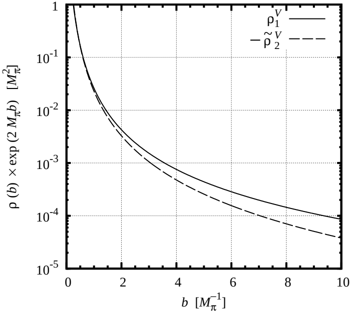
By using LF variables, a propose density of angular momentum is written as a proper density in LF-quantum mechanics of a pion-nucleon fluctuation of a nucleon.

This framework opens the possibility of fully exploring the role that chiral dynamics plays constraining the nucleons internal structure.

- Transverse densities associated with form factors of the energy momentum tensor.Distributions of matter in impact parameter space.
- Expand on different intermediate baryons. (Ongoing) work on intermediate Δ probes large Nc limit properties of LCWF and allows the study of higher orbital modes

Charge and magnetization densities from LF dynamics





C.G , C. Weiss, JHEP **1507**, 170 (2015)

Light front current matrix as wavefunction overlap,

$$\frac{J(b)}{2p^+} = \frac{1}{2\pi}\int \frac{dy}{y\bar{y}}\,\Phi^\dagger\left(y,\frac{b}{\bar{y}}\right)\Phi\left(y,\frac{b}{\bar{y}}\right),$$

then in terms of radial functions

$$\left.\begin{array}{c}\rho_1^V(b)\\[1ex] \tilde{\rho}_2^V(b)\end{array}\right\} = \frac{1}{2\pi}\int\frac{dy}{y\bar{y}^3}\left\{\begin{array}{c}[U_0(y,b/\bar{y})]^2 + [U_1(y,b/\bar{y})]^2\\[1ex] -2\,U_0(y,b/\bar{y})\,U_1(y,b/\bar{y})\end{array}\right\}.$$

Inequality and positive definiteness of light front current,

$$|\rho_1(b)| > \tilde{\rho}_2(b) \Rightarrow J^+(b) > 0$$

weakly bound pions.

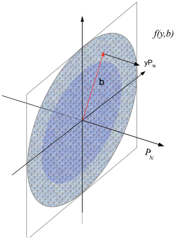
Near parametric equality,

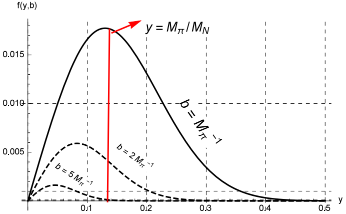
$\rho_1(b) \approx -\tilde{\rho}_2(b) \Rightarrow$ relativistic pion-nucleon system

Explain through left right asymmetry in Transverse densities .

CG, C. Weiss, PRC **92**, no. 2, 025206 (2015)

Charge magnetization densities and χGPDs





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then in terms of radial functions

$$\left.\begin{array}{c} \rho_1^V(b) \\ \widetilde{\rho}_2^V(b) \end{array}\right\} = \frac{1}{2\pi}\int \frac{dy}{y\bar{y}^3} \left\{ \begin{array}{c} [U_0(y, b/\bar{y})]^2 \; + \; [U_1(y, b/\bar{y})]^2 \\ -2\, U_0(y, b/\bar{y})\; U_1(y, b/\bar{y}) \end{array} \right\}.$$

Integrands correspond to Fourier trans. of GPDs, $H(y,t)$ and $E(y,t)$, i.e., $f_1(y,b)$, $f_2(y,b)$ respectively,

$$\left.\begin{array}{c} f_1^V(y,b) \\ \widetilde{f}_2^V(y,b) \end{array}\right\} = \frac{1}{2\pi}\,\frac{1}{y\bar{y}^3} \left\{ \begin{array}{c} [U_0(y, b/\bar{y})]^2 + [U_1(y, b/\bar{y} \\ -2U_0(y, b/\bar{y})U_1(y, b/\bar{y} \end{array} \right.$$