Covering both spacelike and timelike regions of transition form factors in the light-front quark model

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in collaboration with H.-Y. Ryu and C.-R. Ji

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Light Cone 2108, May 14-18, 2018, JLab, Newport News, VA

Outline

1. Motivation

2. Methodology for $P \to \gamma^*\gamma \; (P = \pi^0, \eta, \eta', \cdots)$ transitions

Exactly solvable covariant model

↓ Covariant vs LF calculations; Dispersion relations etc.

Frame-independent LF formulation

↓ Process-indepenent matching condition

Realistic LF Quark Model (LFQM)

3. Numerical Results

4. Conclusion

1. Motivation

- **Meson-photon transitions** $P(\pi^0, \eta, \eta') \rightarrow \gamma^* \gamma$ **:**
- Simplest exclusive processes involving the strong interaction
- Significant role for both the low- and high-energy precision tests of the SM

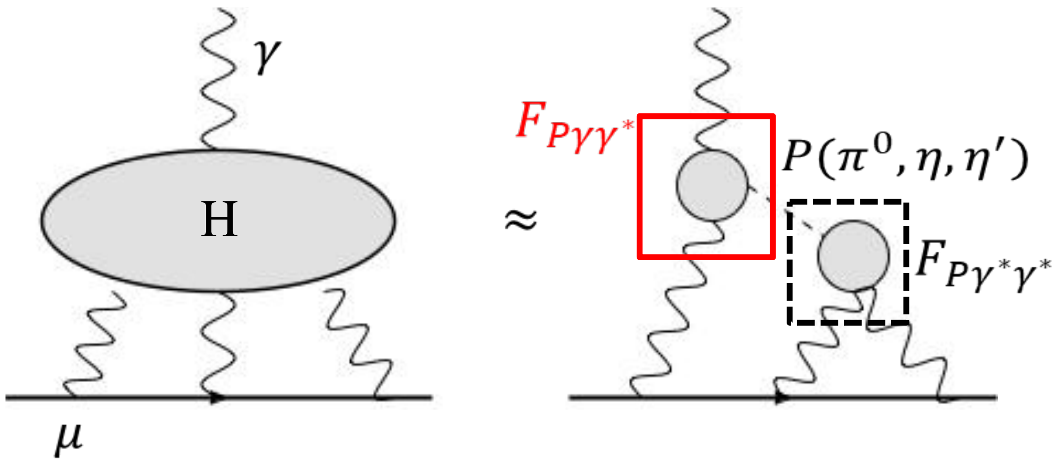
1. Motivation

- **Meson-photon transitions $P(\pi^0,\eta,\eta') \to \gamma^*\gamma$:**
- Simplest exclusive processes involving the strong interaction
- Significant role for both the low- and high-energy precision tests of the SM

1) For the low-energy regime:

The transition form factors(TFFs) enter the prediction of important observables such as $P \to \ell\bar{\ell}(\ell = e,\mu)$ decays and the Hadronic Light by Light scattering (HLbL) contribution to the muon $(g-2)_\mu$:

e.g.) Pseudoscalar-pole contribution to HLbL.



$a_\mu = (g-2)/2$

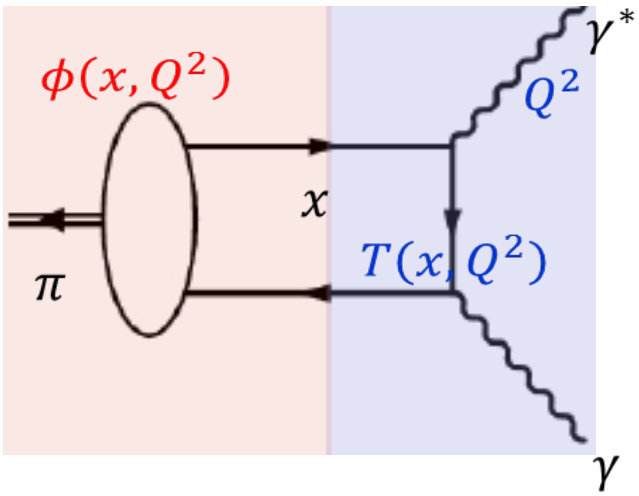
[Exp. –Th.(SM)]($\sim 3\sigma$)
$= (278 \pm 88) \times 10^{-11}$

HLbL
$= (116 \pm 40) \times 10^{-11}$

A. Nyffeler, PRD (2016)

2) For the high-energy regime: TFFs can be calculated from pQCD

e.g.) $\pi \to \gamma^* \gamma$ TFF



At leading twist:

$$F_{\pi\gamma}(Q^2) = \int T(x, Q^2)\phi(x, Q^2)dx + \cdots$$

T: Hard scattering amplitude for $\gamma^*\gamma \to q\bar{q}$ transition which is calculable in pQCD

ϕ: Nonperturbative meson DA describing $P \to q\bar{q}$ transition

$$\phi(x, \mu) \propto \int_{|\boldsymbol{k}_\perp|^2 \le \mu^2} d^2\boldsymbol{k}_\perp \psi(x, \boldsymbol{k}_\perp)$$

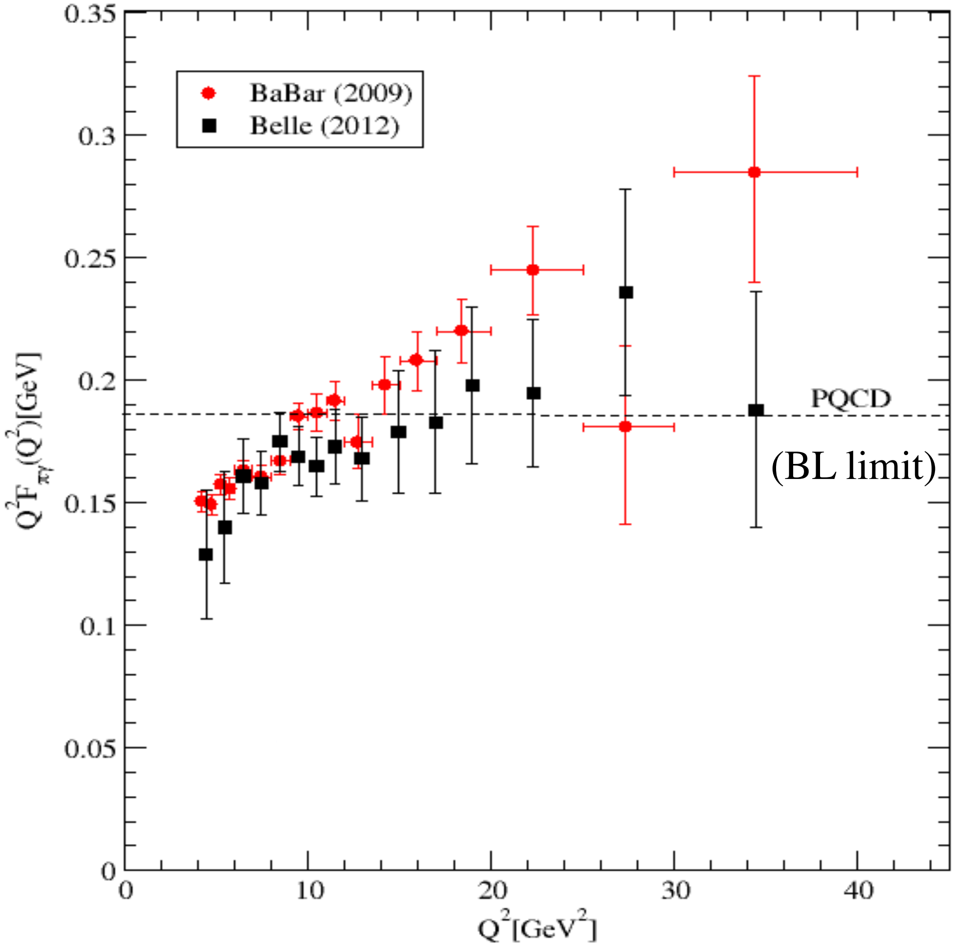
$$\xrightarrow{\mu \to \infty} 6x(1-x): \text{"Asymptotic DA"}$$

In theory (pQCD):

$$Q^2 F_{\pi\gamma} = f_\pi \sqrt{2} \sim 0.185 \text{ GeV}$$

: Brodsky-Lepage(BL) limit [BL PRD (1980)]

- **Experimental status for $F_{\pi\gamma}(Q^2)$ from $e^+e^- \to e^+e^-\pi^0$**



1) BarBar vs. Belle?

2) What about timelike region?

e.g.) some data for $e^+e^- \to \gamma^* \to \eta(\eta')\gamma$

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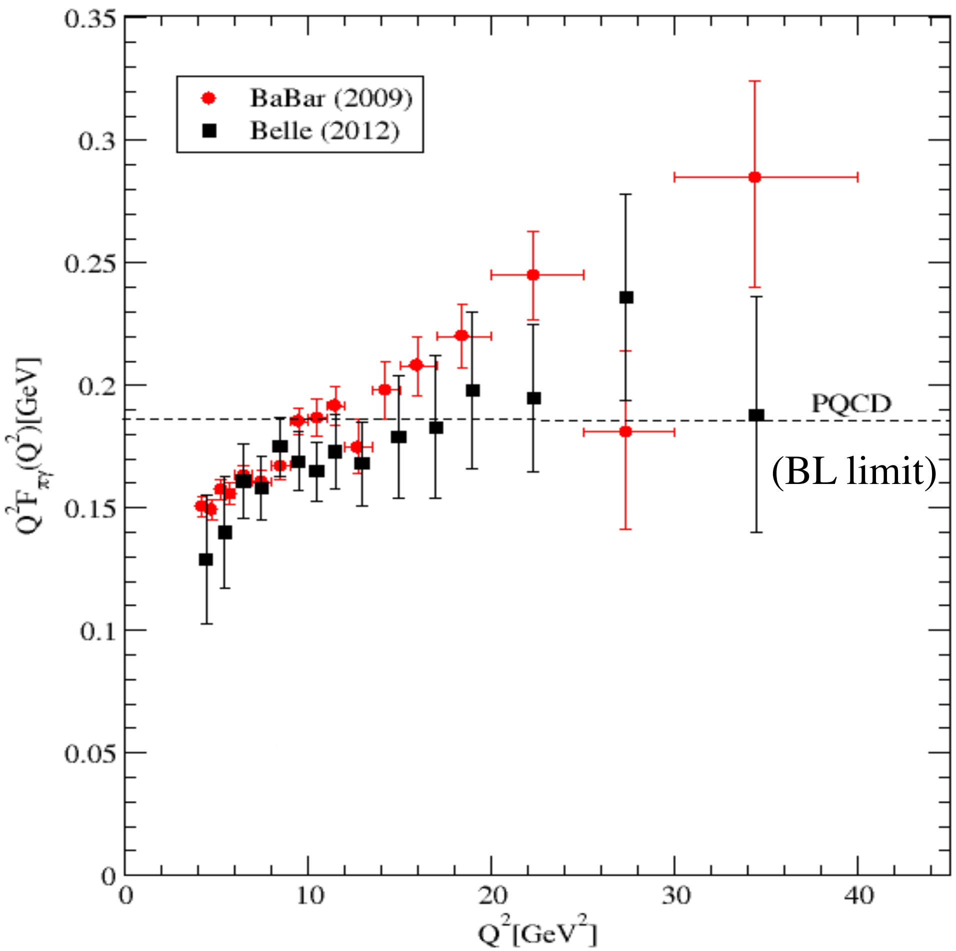


1) BarBar vs. Belle?

2) What about timelike region?

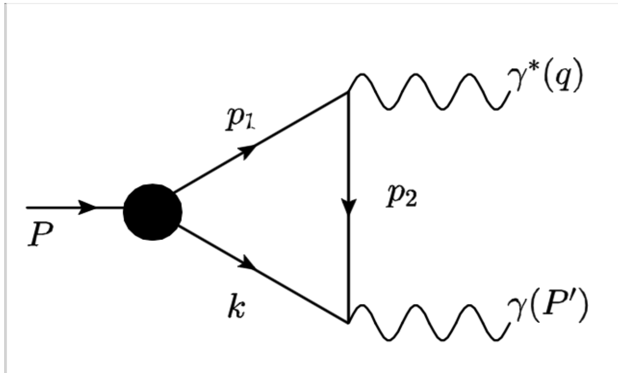
e.g.) some data for $e^+e^- \to \gamma^* \to \eta(\eta')\gamma$

Analyzing both the spacelike and timelike regions appears important to examine the issue of scaling behavior.

We shall explore both spacelike and timelike regions using the LFQM!

2. Manifestly Covariant Model for Meson-Photon Transition

$$\Gamma^{\mu} = \langle \gamma(P-q) | J^{\mu}_{em} | P(P) \rangle = i e^2 F_{P\gamma}(Q^2) \epsilon^{\mu\nu\varrho\sigma} P_{\nu} \epsilon_{\rho} q_{\sigma}$$



(a)

$$\Gamma^{\mu}_{(a)} = i e_Q e_{\bar{Q}} N_c \int \frac{d^4k}{(2\pi)^4} \frac{H_0}{N_{p_1} N_k N_{p_2}} S^{\mu}$$

$$N_{p_j} = p_j^2 - m_Q^2 + i\epsilon \ (j = 1,2)$$

$$N_k = k^2 - m_{\bar{Q}}^2 + i\epsilon \quad (m_Q = m_{\bar{Q}})$$

$$S^{\mu} = \mathrm{Tr}[\gamma_5 (\not{p}_1 + m_Q) \gamma^{\mu} (\not{p}_2 + m_Q) \not{\epsilon} (-\not{k} + m_Q)] \qquad H_0(p^2, k^2) = g$$

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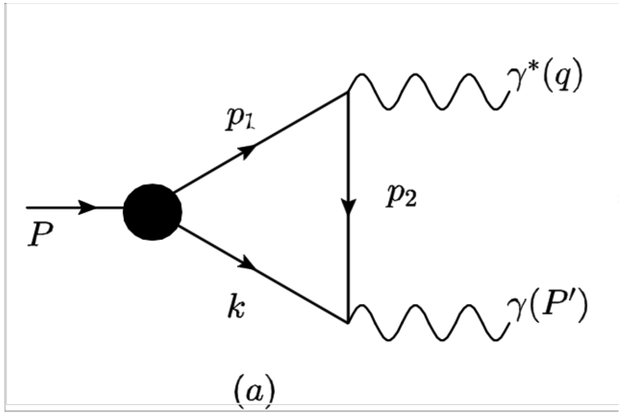

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1) Covariant Calculation

$$\Rightarrow \quad F_{\pi\gamma}(q^2) \propto \int_0^1 dx \int_0^{1-x} dy \frac{m_Q}{x(1-x-y)M^2 + xyq^2 - m_Q^2}$$

2) Light-Front Calculation

(I) $q^+ \neq 0$ frame : $\left(0 < \alpha = \dfrac{q^+}{P^+} < 1\right)$

$$P = \left(P^+, \frac{M^2}{P^+}, \mathbf{0}_\perp\right),\ q = \left(\alpha P^+, \frac{M^2}{P^+}, \mathbf{0}_\perp\right),\ P' = ((1-\alpha)P^+, 0, \mathbf{0}_\perp)$$

$\alpha = 0$

$\alpha = 1$

(II) $q^+ = 0$ frame

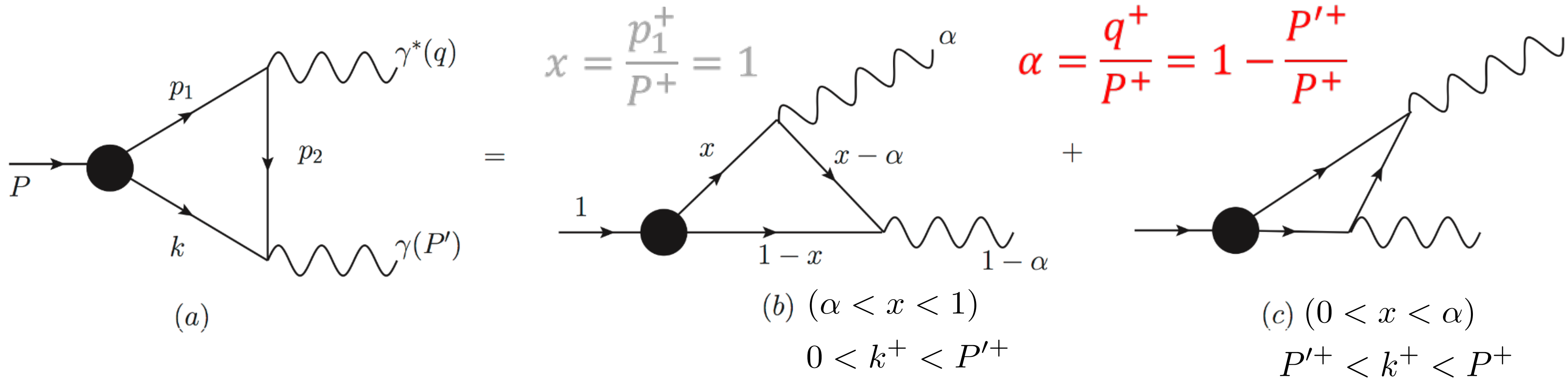
$$q = (0, q^-, \mathbf{q}_\perp)$$

$$q^2 = -\mathbf{q}_\perp^2 = -Q^2$$

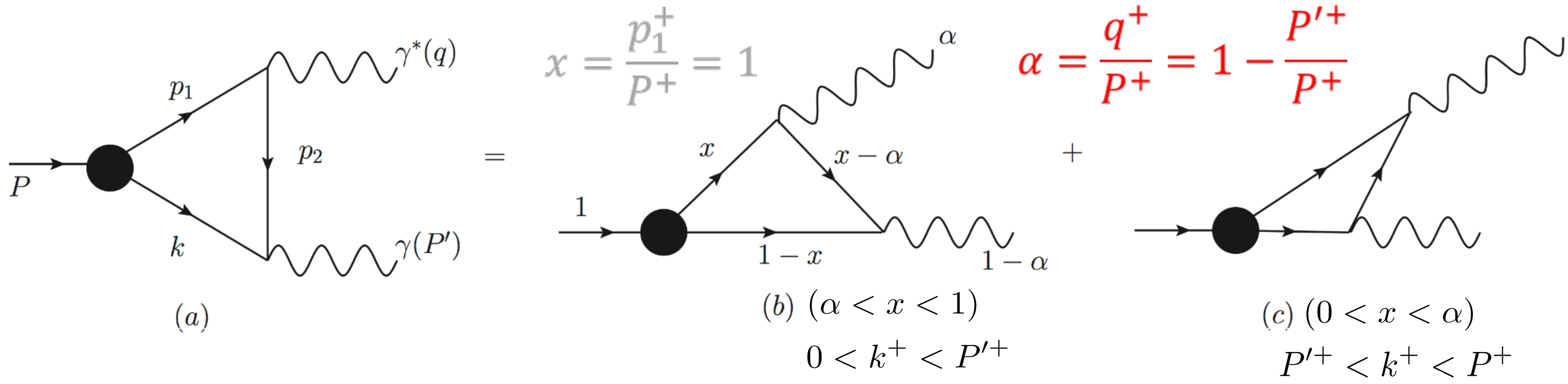
(III) $q^+ \neq 0$ frame

$$q = \left(P^+, \frac{q^2}{P^+}, \mathbf{0}_\perp\right)$$

$$P' = \left(0, \frac{M^2 - q^2}{P^+}, \mathbf{0}_\perp\right)$$



2-I) Light-Front Calculation ($q^+ \neq 0$ frame with $0 < \alpha < 1$)



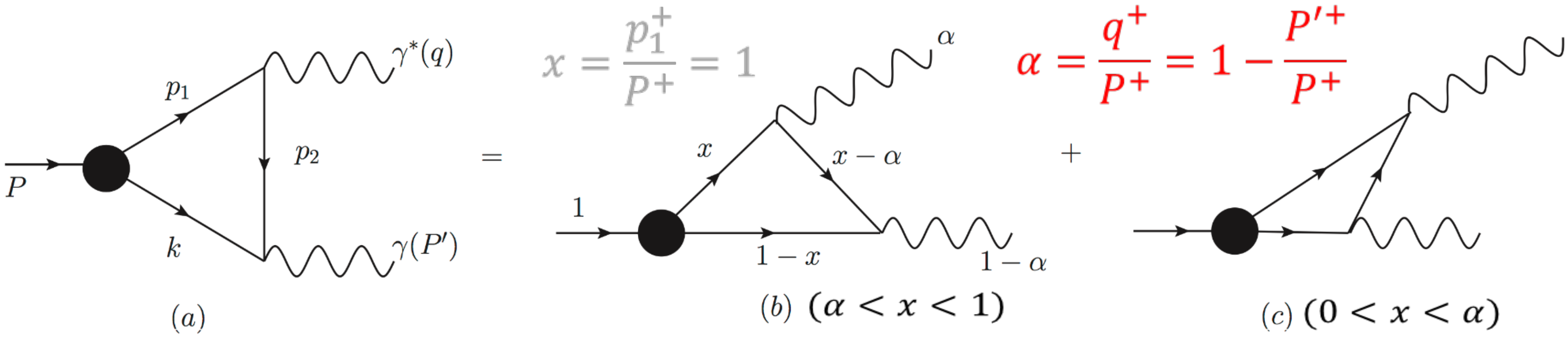
2-I) Light-Front Calculation ($q^+ \neq 0$ frame with $0 < \alpha < 1$)

$$[F_{\pi\gamma}]^{\rm LF}_{(b)} \propto \int_0^1 \frac{dx}{(1-x)} \int d^2\mathbf{k}_\perp \frac{m_Q}{(\alpha-1)M_0^2}\chi(x,\mathbf{k}_\perp)$$

$$[F_{\pi\gamma}]^{\rm LF}_{(c)} \propto \int_0^1 \frac{x' dx}{(1-x)} \int d^2\mathbf{k}_\perp \frac{m_Q}{x'(1-x')M^2 - x(1-x)M_0^2}\chi(x,\mathbf{k}_\perp)$$

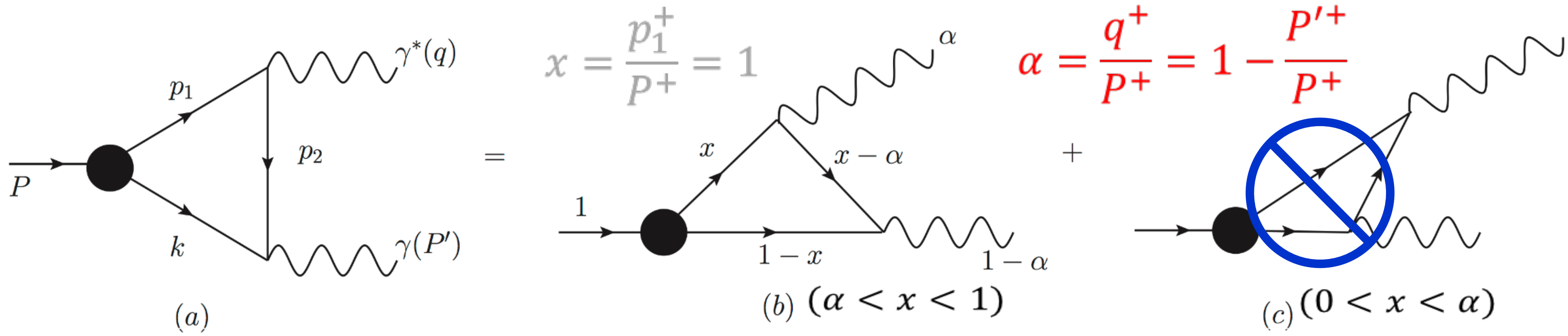
where $x' = x/\alpha$

$$\chi(x,\mathbf{k}_\perp) = \frac{g}{x(M^2 - M_0^2)},\ M_0^2 = \frac{\mathbf{k}_\perp^2 + m_Q^2}{x(1-x)}$$



3) Equivalence between Covariant Calculation and Light-Front Calculation

Covariant Calculation	LF Calculations in different reference frames
Diagram (a)=	(b) + (c) for $0 < \alpha < 1$ ($q^+ \neq 0$)



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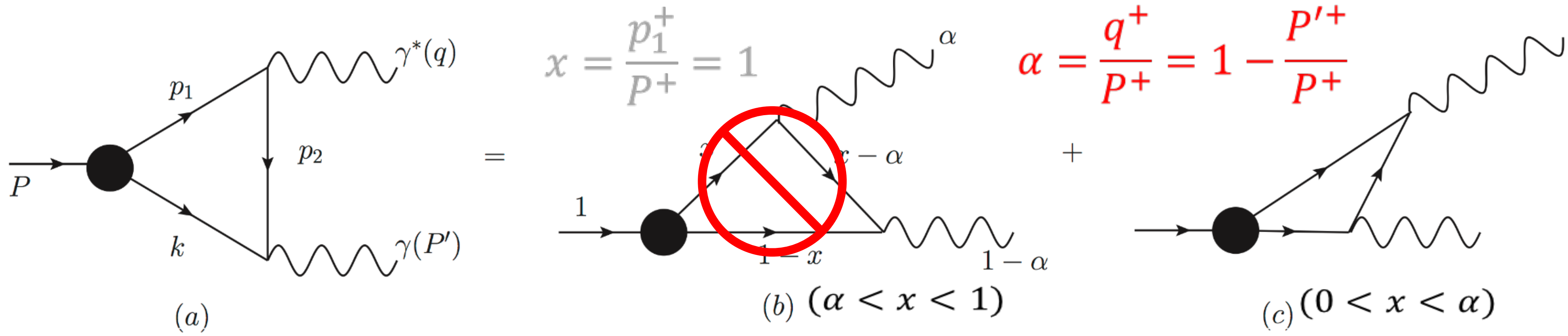
Covariant Calculation	LF Calculations in different reference frames
Diagram (a)=	(b) + (c) for $0 < \alpha < 1$ ($q^+ \neq 0$)
	(b) for $\alpha = 0\ (q^+ = 0)$: defined in $q^2 < 0$

In $q^+ = 0$ frame: $q = (0, q^-, \mathbf{q}_\perp) \to q^2 = -\mathbf{q}_\perp^2 \equiv -Q^2$

$$[F_{\pi\gamma}]^{\rm LF}_{\alpha=0}(Q^2) \propto \int_0^1 \frac{dx}{x(1-x)} \int d^2\mathbf{k}_\perp \frac{m_Q}{M_0'^2} \chi(x, \mathbf{k}_\perp),$$

$$M_0' = M_0(\mathbf{k} \to \mathbf{k}'_\perp) \text{ with } \mathbf{k}'_\perp = \mathbf{k}_\perp + (1-x)\mathbf{q}_\perp$$



3) Equivalence between Covariant Calculation and Light-Front Calculation

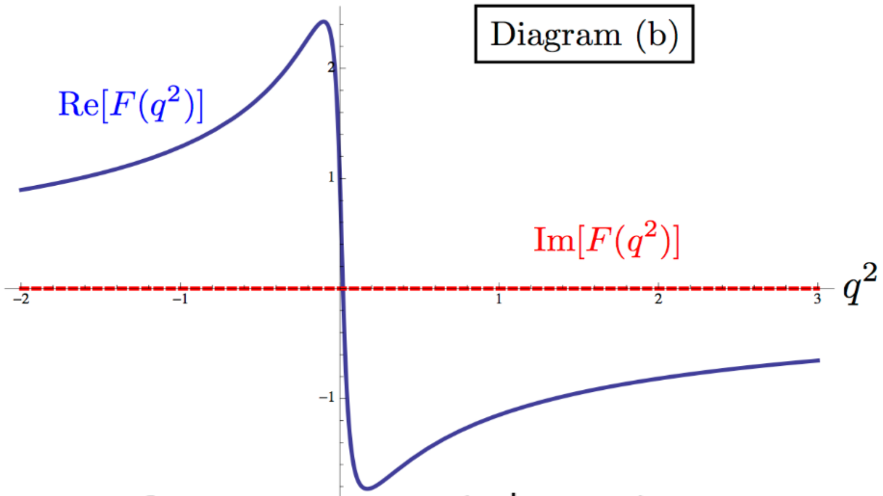
Covariant Calculation	LF Calculations in different reference frames
Diagram (a)=	(b) + (c) for $0 < \alpha < 1$ $(q^+ \neq 0)$
	(b) for $\alpha = 0$ $(q^+ = 0)$: defined in $q^2 < 0$
	(c) for $\alpha = 1 (q^+ \neq 0)$: defined in $q^2 > 0$
$F^{Cov}_{(a)}(q^2) = [F^{LF}_{(b)} + F^{LF}_{(c)}]_{0<\alpha<1} = [F^{LF}_{(b)}]_{\alpha=0} = [F^{LF}_{(c)}]_{\alpha=1}$	

In $\alpha = 1$ frame : $q = (P^+, \frac{q^2}{P^+}, \mathbf{0}_\perp)$

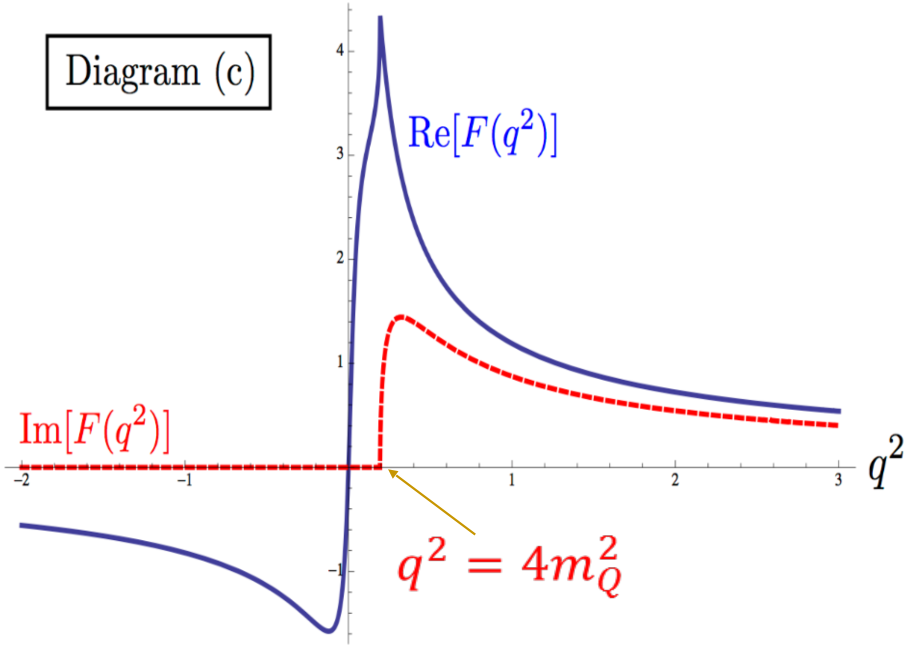
Our New findings!

$$[F^{LF}_{(c)}]_{\alpha=1}(Q^2) \propto \int_0^1 \frac{dx}{(1-x)^2} \int d^2\boldsymbol{k}_\perp \frac{m_Q}{M_0^2 - q^2} \chi(x, \boldsymbol{k}_\perp)$$

4) Direct vs. DR Calculations



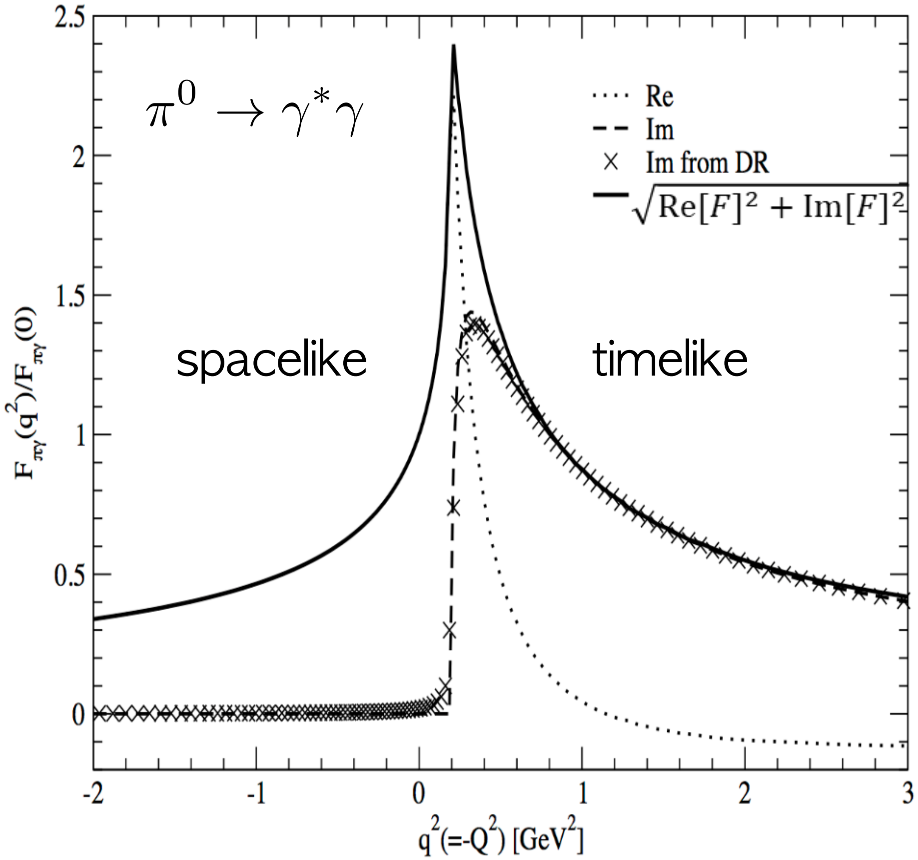
for $0 < \alpha < 1$ ($q^+ \neq 0$)



Dispersion Relation(DR) for $F(q^2) = \mathrm{Re}\, F(q^2) + i\mathrm{Im}\, F(q^2)$:

$$\mathrm{Re}\, F(q^2) = \frac{1}{\pi} P \int_{-\infty}^{\infty} \frac{\mathrm{Im}\, F(q'^2)}{q'^2 - q^2} dq'^2$$

$$\mathrm{Im}\, F(q^2) = -\frac{1}{\pi} P \int_{-\infty}^{\infty} \frac{\mathrm{Re}\, F(q'^2)}{q'^2 - q^2} dq'^2$$



$$F^{Cov}_{(a)}(q^2) = [F^{LF}_{(b)} + F^{LF}_{(c)}]_{0<\alpha<1} = [F^{LF}_{(b)}]_{\alpha=0} = [F^{LF}_{(c)}]_{\alpha=1}$$

3. Application to Light-Front Quark Model (LFQM)

Manifestly Covariant Model :

$$\chi(x,\boldsymbol{k}_\perp) = \frac{g}{x\,(M^2 - M_0^2)}$$

LFQM : $\Psi^{SS_z}_{\lambda_1\lambda_2}(x,\boldsymbol{k}_\perp) = R^{SS_z}_{\lambda_1\lambda_2}(x,\boldsymbol{k}_\perp)\phi_R(x,\boldsymbol{k}_\perp)$

$$\phi_R(x,\boldsymbol{k}_\perp) = C(x,\boldsymbol{k}_\perp)\exp(-\frac{M_0^2}{8\beta^2})$$

$$1 = \int_0^1 dx \int \frac{d^2\boldsymbol{k}_\perp}{16\pi^3} |\phi_R(x,\boldsymbol{k}_\perp)|^2$$

Process-independent correspondence:

$$\sqrt{2N_c}\,\frac{\chi}{1-x} = \frac{\phi_R}{\sqrt{m_Q^2 + \boldsymbol{k}_\perp^2}}$$

CJ: PRD91,014018(15) ,
PRD89,033011(14)

$$[F_{\pi\gamma}]^{\rm SLF}_{\alpha\to 1} = \frac{\sqrt{2N_c}}{4\pi^3}\int_0^1 \frac{dx}{(1-x)}\int d^2\boldsymbol{k}_\perp\, \frac{m_Q}{M_0^2 - q^2}\,\frac{\phi_R}{\sqrt{m_Q^2 + \boldsymbol{k}_\perp^2}}$$

$$[F_{\pi\gamma}]^{\rm SLF}_{\alpha\to 0} = \frac{\sqrt{2N_c}}{4\pi^3}\int_0^1 \frac{dx}{x(1-x)}\int d^2\boldsymbol{k}_\perp\, \frac{m_Q}{M_0'^2}\,\frac{\phi_R}{\sqrt{m_Q^2 + \boldsymbol{k}_\perp^2}}$$

- **Summary for the LFQM Calculation of $F_{\pi\gamma}(q^2)$**

$$[F_{\pi\gamma}]^{\rm SLF}_{\alpha\to 1} = \frac{\sqrt{2N_c}}{4\pi^3}\int_0^1 \frac{dx}{(1-x)}\int d^2\boldsymbol{k}_\perp \frac{m_Q}{M_0^2-q^2}\frac{\phi_R}{\sqrt{m_Q^2+\boldsymbol{k}_\perp^2}}$$

$$[F_{\pi\gamma}]^{\rm SLF}_{\alpha\to 0} = \frac{\sqrt{2N_c}}{4\pi^3}\int_0^1 \frac{dx}{x(1-x)}\int d^2\boldsymbol{k}_\perp \frac{m_Q}{M_0'^2}\frac{\phi_R}{\sqrt{m_Q^2+\boldsymbol{k}_\perp^2}}$$

At sufficiently high Q^2

$$F_{\pi\gamma}(q^2) \approx \frac{f_\pi\sqrt{2}}{3}\int_0^1 \frac{dx}{(1-x)Q^2}\phi_{2;\pi}(x) + O(1/Q^4) \qquad (q^2=-q_\perp^2=-Q^2)$$

where

$$\phi_{2;\pi}(x) = \frac{\sqrt{2N_c}}{f_P 8\pi^3}\int d^2\boldsymbol{k}_\perp \frac{\phi_R(x,\boldsymbol{k}_\perp)}{\sqrt{\boldsymbol{k}_\perp^2+m_Q^2}} m_Q \quad \text{: Twist-2 DA}$$

For the $(\boldsymbol{\eta}, \boldsymbol{\eta}') \to \boldsymbol{\gamma\gamma}^*$ transitions:

Use $\eta - \eta'$ mixing scheme in the quark-flavor basis

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} \eta_q \\ \eta_s \end{pmatrix} \qquad \eta_q = \frac{u\bar{u} + d\bar{d}}{\sqrt{2}}, \eta_s = s\bar{s}$$

⇩

Transition form factor $\boldsymbol{F_{P\gamma}}$ mixing scheme for $\boldsymbol{P \to \gamma\gamma^*}$ $(\boldsymbol{P = \pi^0, \eta, \eta'})$

$$F_{\pi\gamma}(q^2) = \frac{(e_u^2 - e_d^2)}{\sqrt{2}} I_{\rm tot}^{m_{u(d)}}$$

$$F_{\eta\gamma}(q^2) = \cos\phi \frac{(e_u^2 + e_d^2)}{\sqrt{2}} I_{\rm tot}^{m_{u(d)}} - \sin\phi \, e_s^2 \, I_{\rm tot}^{m_s}$$

$$F_{\eta'\gamma}(q^2) = \sin\phi \frac{(e_u^2 + e_d^2)}{\sqrt{2}} I_{\rm tot}^{m_{u(d)}} + \cos\phi \, e_s^2 \, I_{\rm tot}^{m_s}$$

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Quadratic(linear)

Gell-Mann-Okubo $\phi = [44.7^o, 31.7^o]$

mass formula:

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$$F_{\eta'\gamma}(q^2) = \sin\phi \frac{(e_u^2 + e_d^2)}{\sqrt{2}} I_{\rm tot}^{m_{u(d)}} + \cos\phi \, e_s^2 \, I_{\rm tot}^{m_s}$$

We shall use

$\phi = (37 \pm 5)^o$

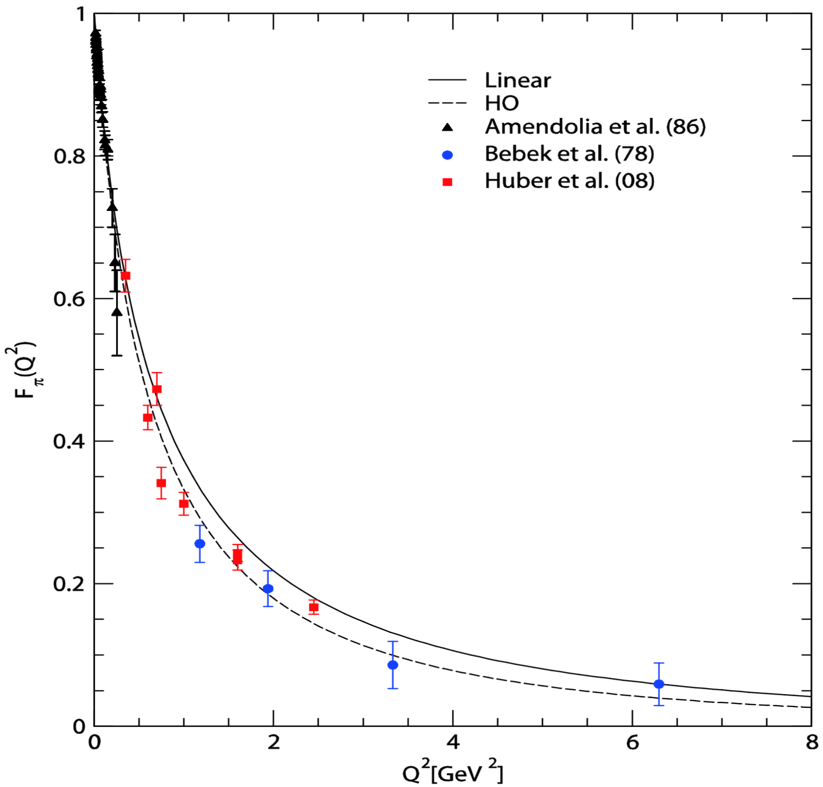
4. Numerical Results

(in unit of GeV)

Model	m_q	m_s	$\beta_{q\bar{q}}$	$\beta_{s\bar{s}}$
Linear	0.22	0.45	0.3659	0.4128

CJ: PRD59, 074015(99); PLB460, 461(99)

1) Pion E&M form factor



CJ: PRD91,014018(15)

2) Pion Charge Radius

	LFQM	Exp.
$\langle r_\pi^2 \rangle^{1/2}$ [fm]	0.652	0.672(8)

3) Decay Constants

	LFQM	Exp.
f_π [MeV]	130	130.41(23)
f_0	$1.16 f_\pi$	$1.17 f_\pi$ [1] $1.25 f_\pi$ [2]
f_8	$1.32 f_\pi$	$1.26 f_\pi$ [1] $1.28 f_\pi$ [2]

[1] Feldmann, Kroll, Stech, PRD58, 114006(98)

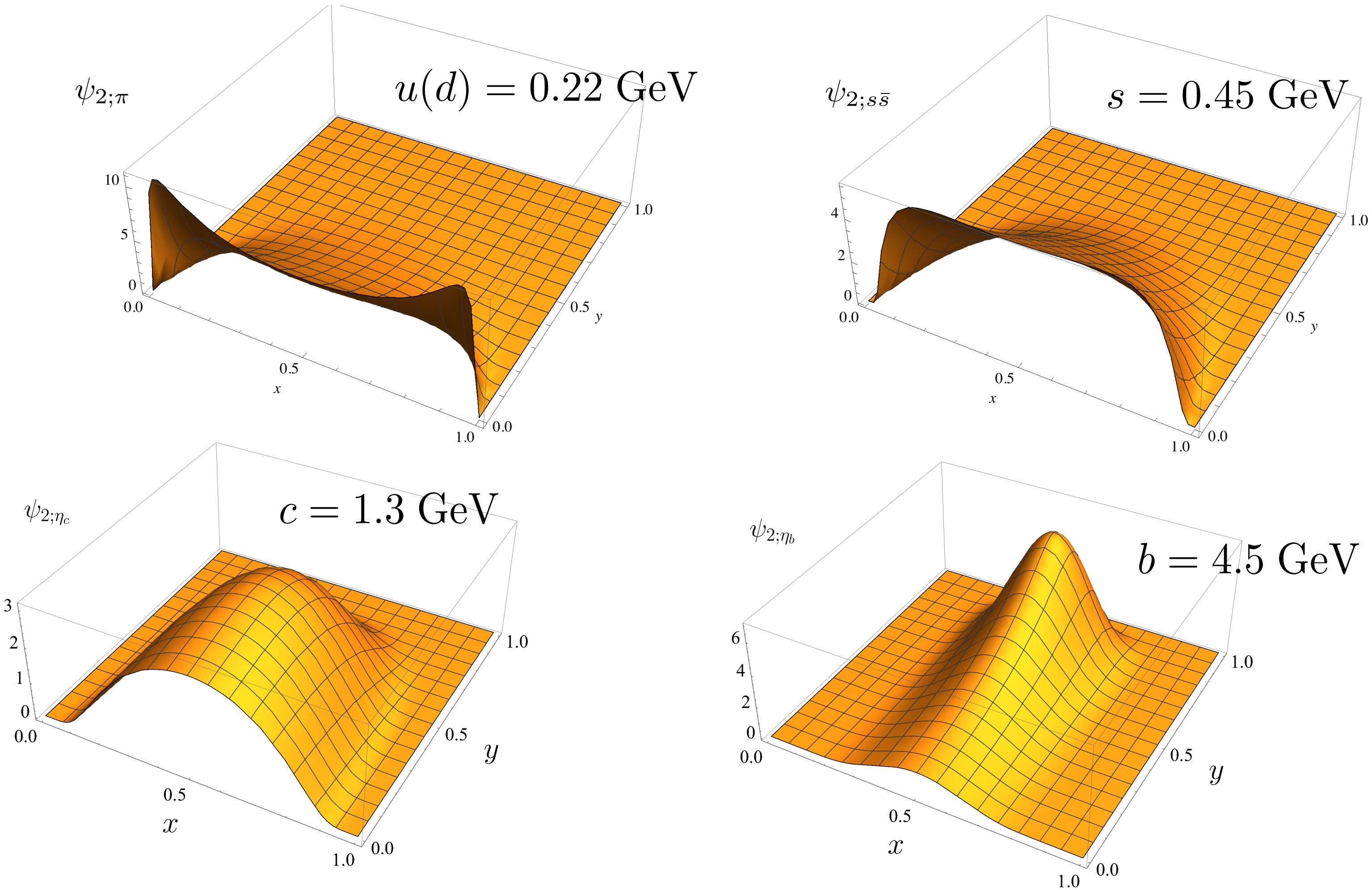
[2] Leutwyler, Nucl. Phys. B (Proc. Suppl.) 64,223(98)

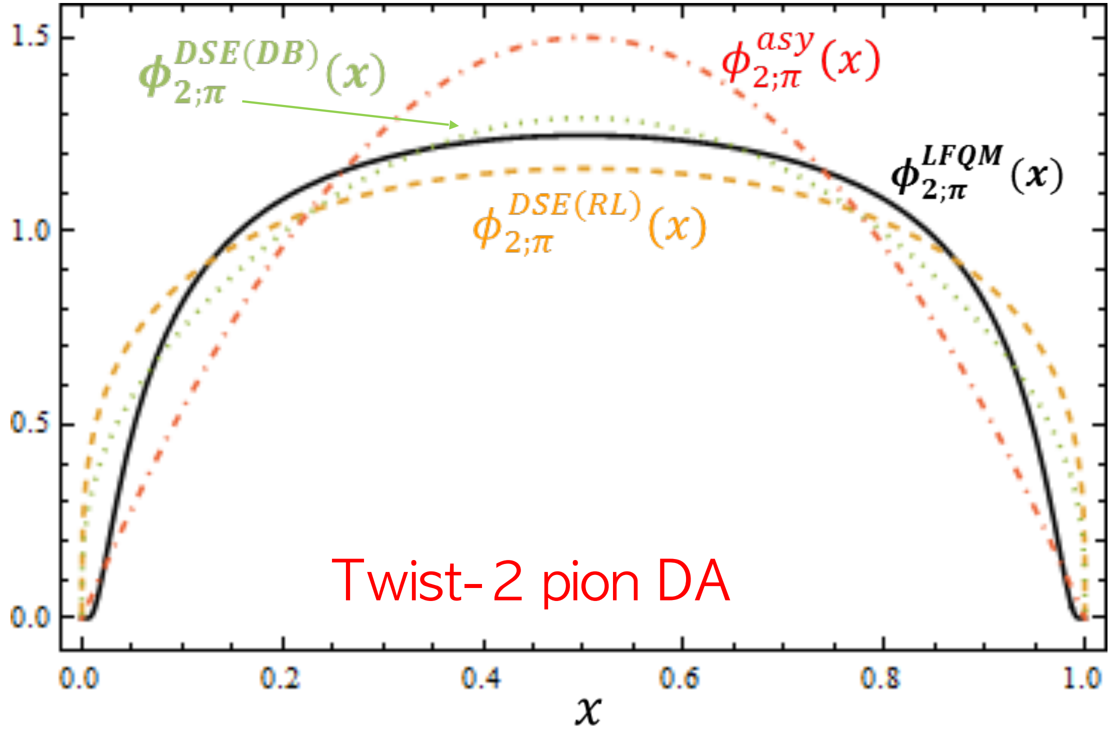
Transverse momentum dependent DA (TDMA) $\psi_{2;P}(x, \mathbf{k}_\perp)$:

Radyushkin, PLB(2014)

$$\mathbf{k}_\perp^2 = \frac{y}{1-y}$$

$$\phi_{2;P}(x) = \frac{\sqrt{2N_c}}{f_P 8\pi^2} \int d^2\mathbf{k}_\perp \frac{\phi_R(x, \mathbf{k}_\perp)}{\sqrt{\mathbf{k}_\perp^2 + m_Q^2}} m_Q \equiv \int d^2\mathbf{k}_\perp\, \psi_{2;P}(x, \mathbf{k}_\perp) = \int_0^1 dy\, \psi_{2;P}(x, y)$$





CJ, PRD(2007)

$$\langle \xi^n \rangle = \int_0^1 dx\, \xi^n \phi_{2;\pi}(x)$$

where $\xi = x - (1-x)$

$\langle \xi^2 \rangle$; measure of the width of the DA

$$\langle \xi^2 \rangle_\pi^{LFQM} = 0.24$$

$\langle \xi^2 \rangle_\pi^{RL(DB)} = 0.28\ (0.25)$ [Chang et al. 13]

$\langle \xi^2 \rangle_\pi^{asy} = 0.20\ (\phi_{2;\pi}^{asy} = 6x(1-x))$

$\langle \xi^2 \rangle_\pi^{LAT} = 0.27 \pm 0.04$ [Braun et al. 06]

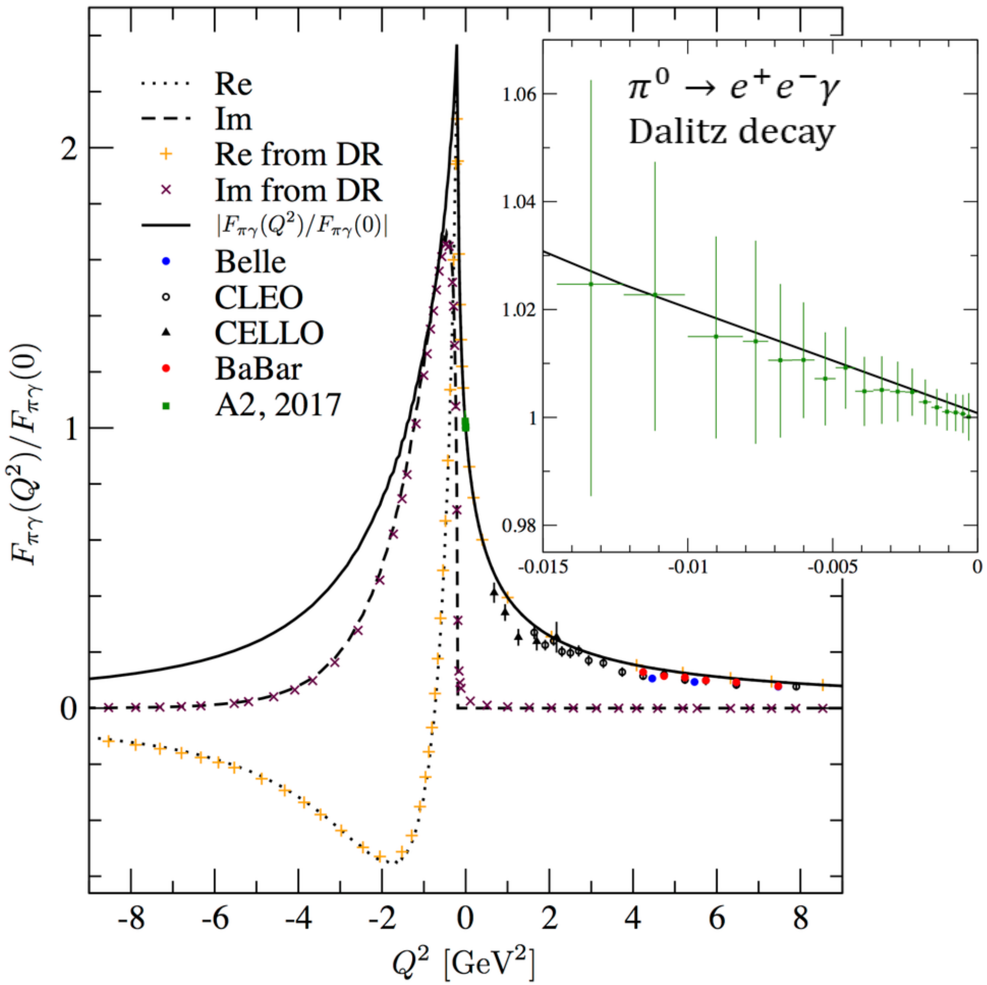
$\langle \xi^2 \rangle_\pi^{flat} = 1/3 (\phi_{2;\pi}^{flat} = 1)$

$\langle \xi^2 \rangle_\pi^{Ads/QCD} = 0.25 (\phi_{2;\pi}^{Ads/QCD} = \frac{8}{\pi}\sqrt{x(1-x)})$

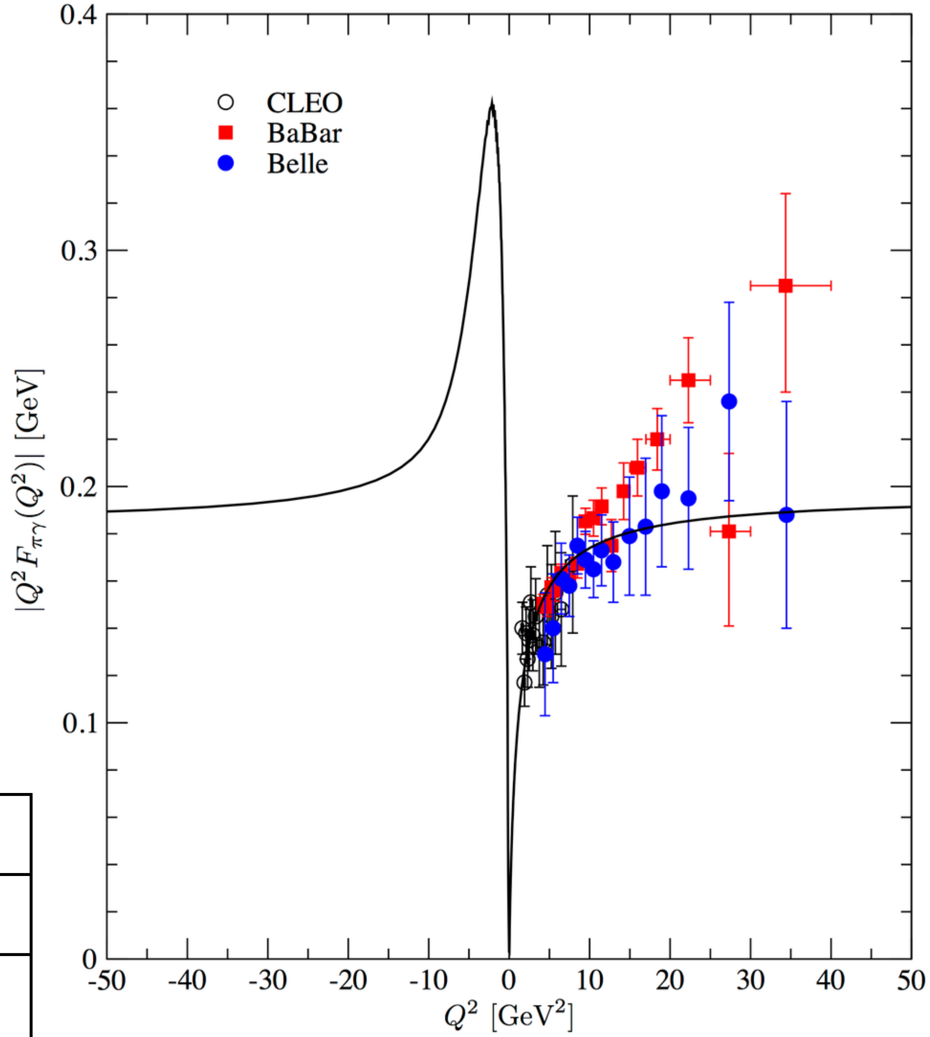
[Brodsky et al. 11]

$\langle \xi^2 \rangle_\pi^{delta} = 0\ (\phi_{2;\pi}^{delta} = \delta(x - \frac{1}{2}))$

Results for $F_{\pi\gamma}(q^2)$



$$F(m_{ll} = q) = \frac{1}{1 - \frac{m_{ll}^2}{\Lambda^2}} \approx 1 + a_\pi \frac{m_{ll}^2}{m_\pi^2}$$

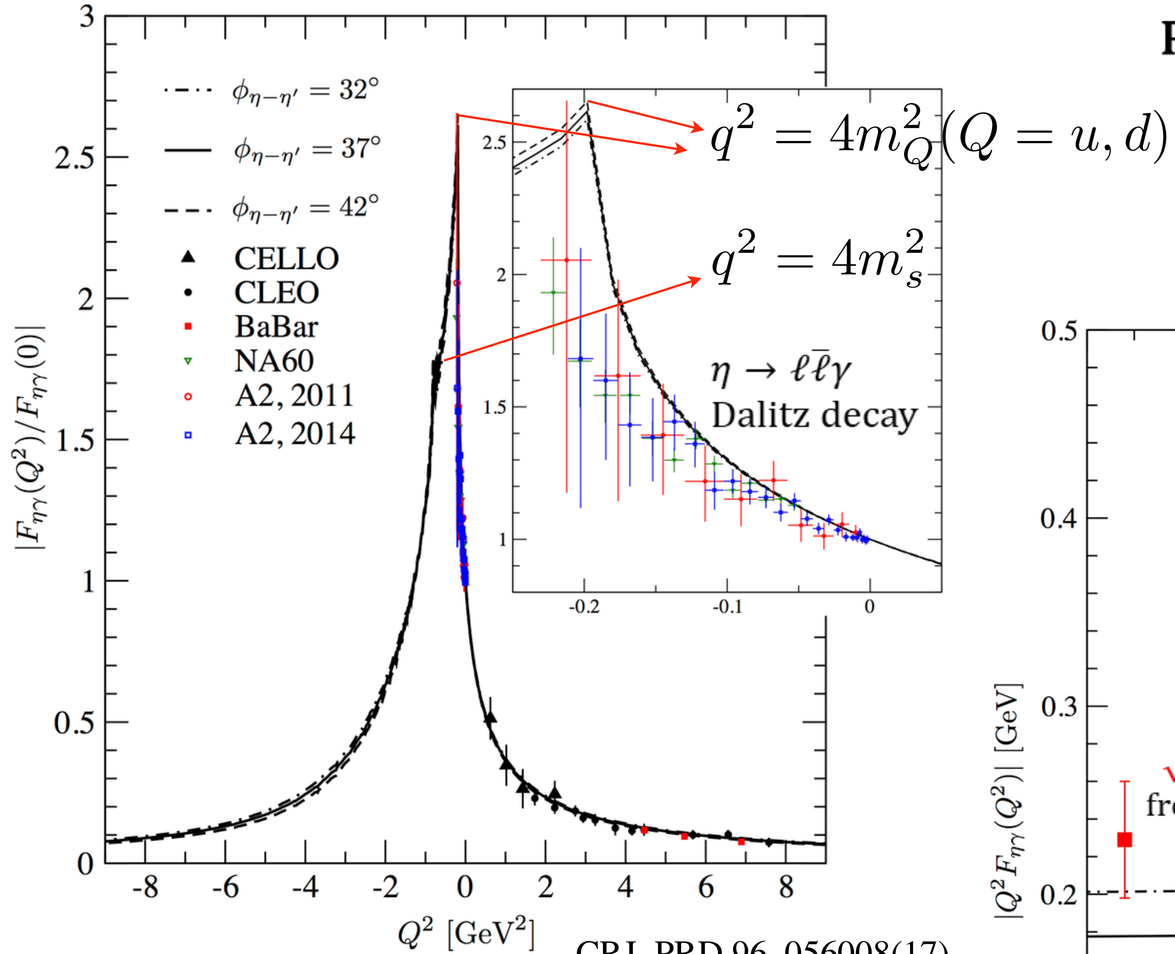


Slope parameter a_π: CRJ, PRD 96, 056008(17)

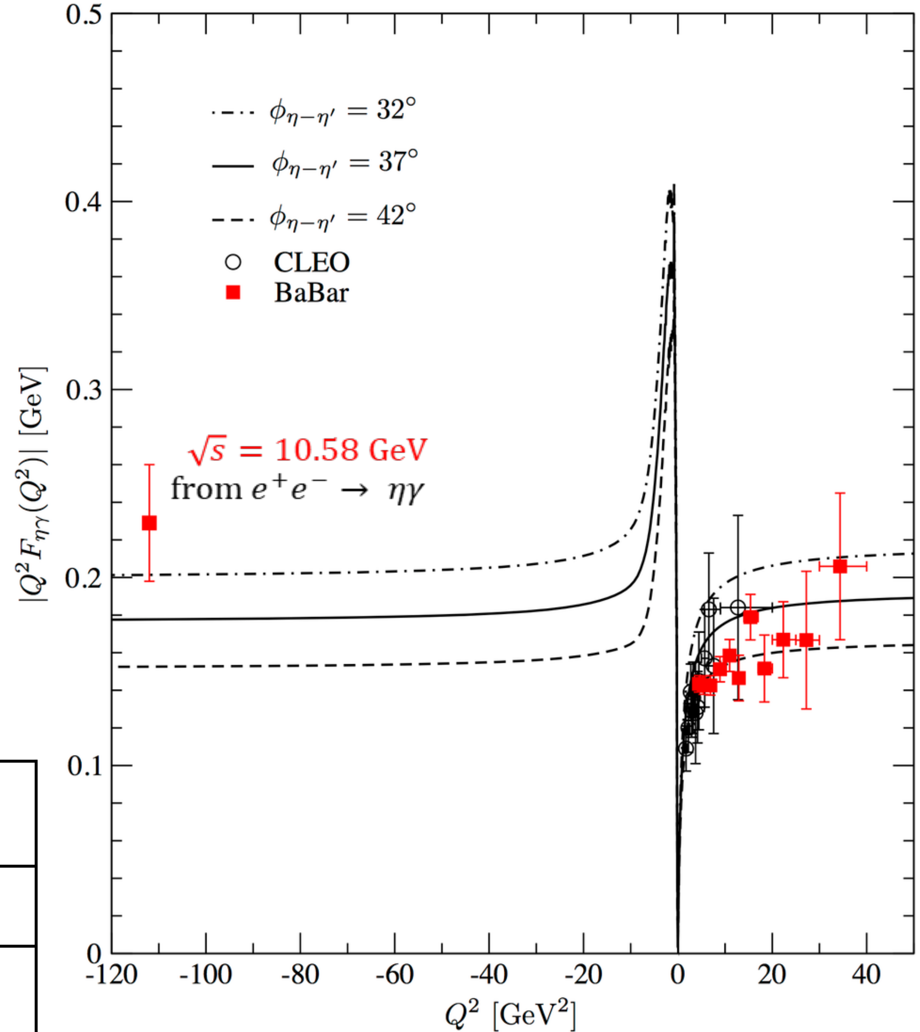
Ours	0.0355
A2 at MAMI(16)	0.030 ± 0.010
World average(PDG)	0.032 ± 0.004

Results for $F_{\eta\gamma}(q^2)$

$$F(m_{ll}) = \frac{1}{1 - \frac{m_{ll}^2}{\Lambda^2}}$$



CRJ, PRD 96, 056008(17)

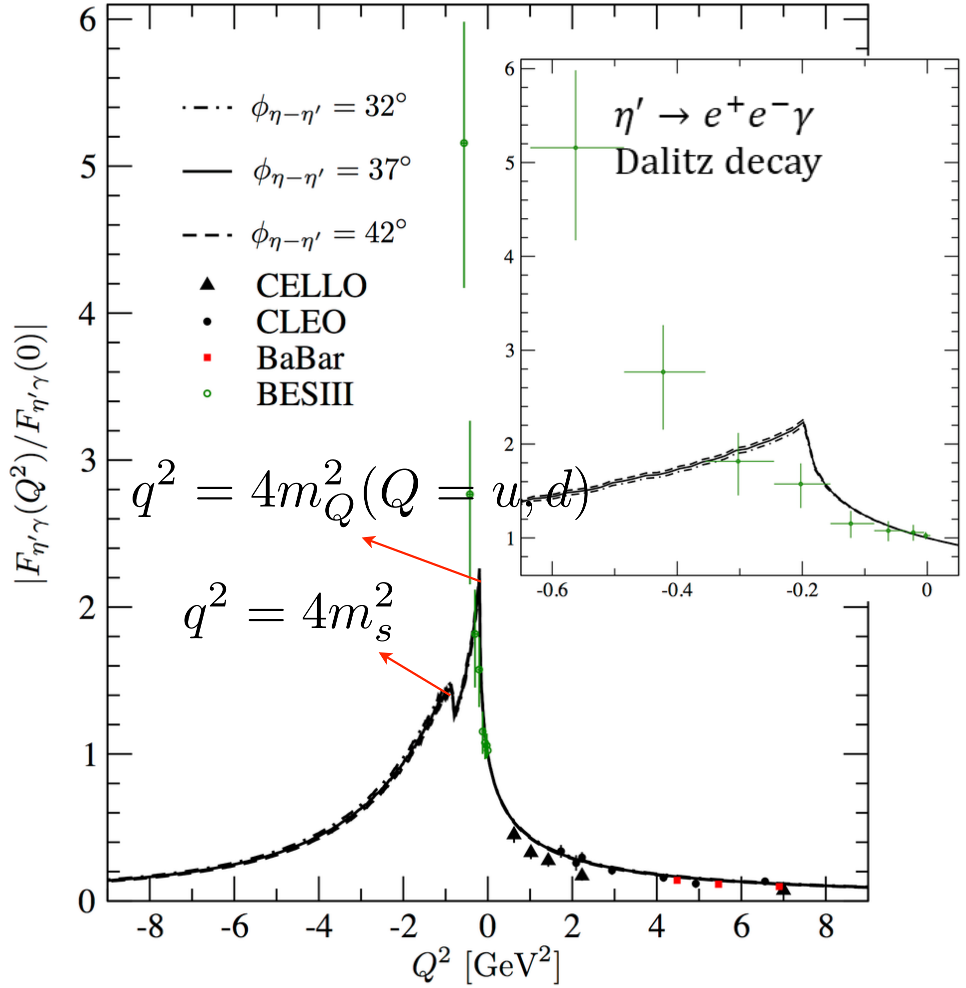


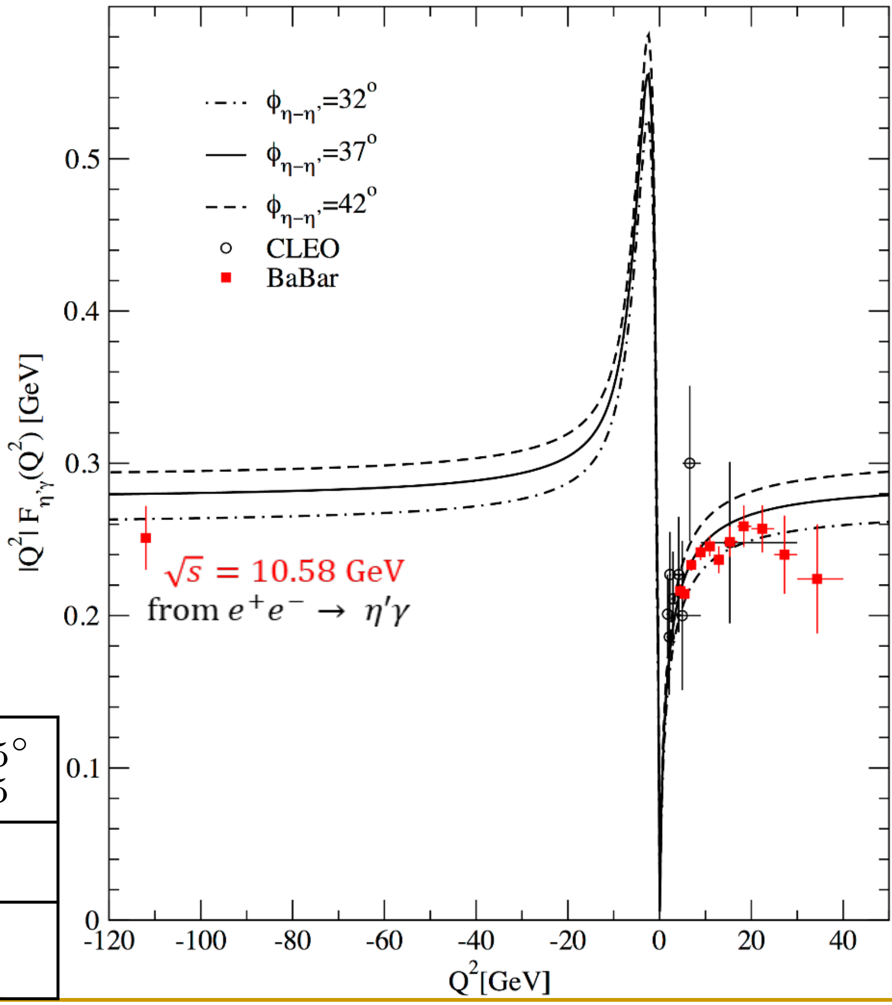
Slope parameter Λ^{-2} [GeV^{-2}]:

Ours	$2.112^{-0.031}_{+0.038}$ for $\phi = 37^{-5°}_{+5}$
A2 at MAMI	$1.95 \pm 0.15 \pm 0.10$
NA2 at CERN	$1.95 \pm 0.17 \pm 0.05$

Results for $F_{\eta'\gamma}(q^2)$

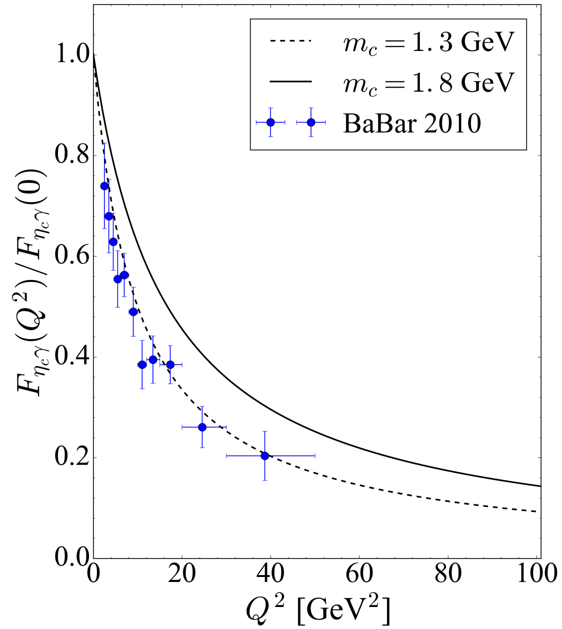
$$F(m_{ll}) = \frac{1}{1 - \frac{m_{ll}^2}{\Lambda^2}}$$

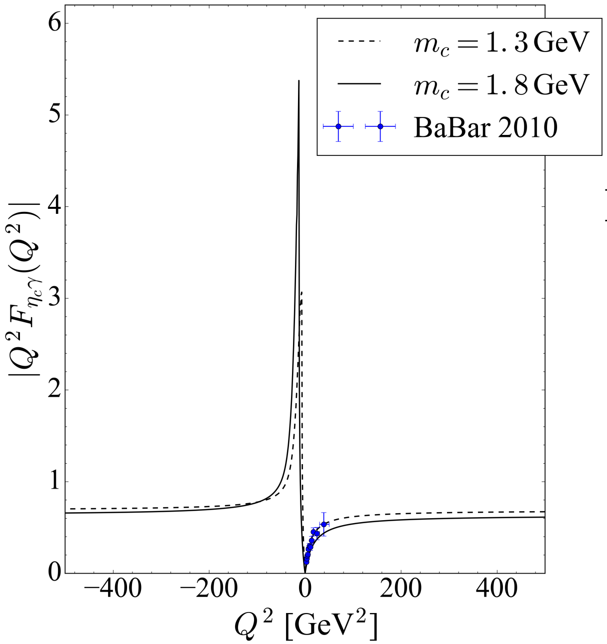




Slope parameter Λ^{-2} [GeV^{-2}]:

Ours	$1.732^{-0.035}_{+0.031}$ for $\phi = 37^{-5°}_{+5}$
BESIII (2015)	1.60 ± 0.25
Lepton-Col.(1979)	1.7 ± 0.4



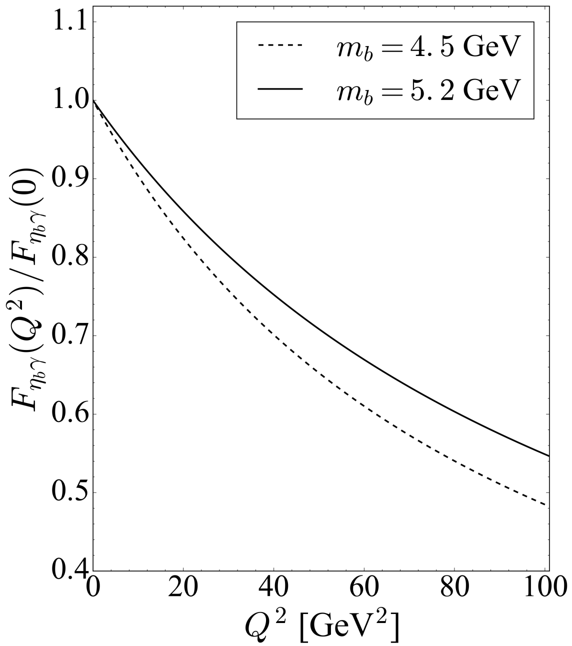


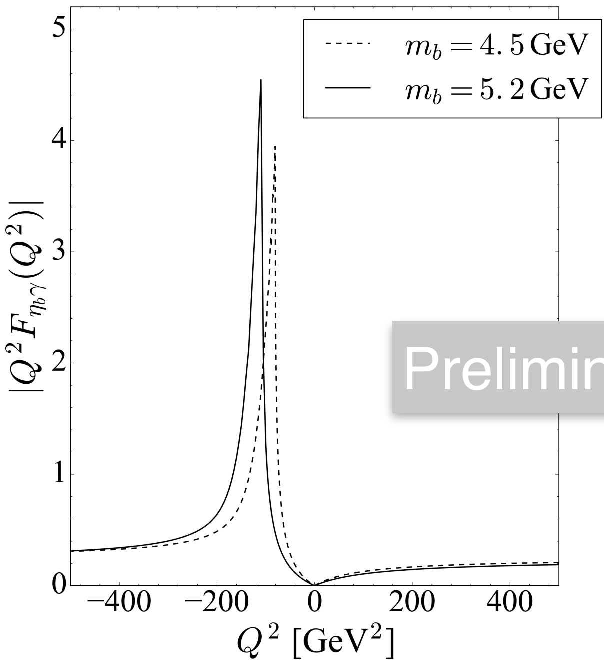
$$\Gamma_{P\to\gamma\gamma} = \frac{\pi}{4}\alpha^2 M_P^3 |F_{P\gamma}(0)|^2$$

$$\Gamma_{\eta_c\to\gamma\gamma} = 1.55\ [4.88]\ \text{keV}$$

for $m_c = 1.8\ [1.3]$ GeV

$$\Gamma^{\text{exp}}_{\eta_c\gamma\gamma} = 5.1 \pm 0.4\ \text{keV}$$





Preliminary

$$\Gamma_{\eta_b\to\gamma\gamma} = 0.128\ [0.239]\ \text{keV}$$

for $m_b = 5.2\ [4.5]$ GeV

5. Conclusion

- We investigate meson-photon transitions for both spacelike and timelike regions using the LFQM

  - Find the new direct method to explore for both timelike and spacelike region.

$$[F_{\pi\gamma}]^{\rm SLF}_{\alpha\to 1} = \frac{\sqrt{2N_c}}{4\pi^3}\int_0^1 \frac{dx}{(1-x)}\int d^2\boldsymbol{k}_\perp \frac{m_Q}{M_0^2 - q^2}\frac{\phi_R}{\sqrt{m_Q^2+\boldsymbol{k}_\perp^2}}$$

  - Observe the resonance peaks corresponding to ρ-type pole for $F_{\pi\gamma}$ and (ρ, ϕ) meson type poles for $F_{(\eta,\eta')\gamma}$

  - Our results for $Q^2 F_{P\gamma}(Q^2)$ are consistent with the PQCD prediction showing a scaling behavior for both timelike and spacelike regions.

- Future works:

  - Apply our new method to other timelike form factor calculations!