

Particles, fields, dynamics and the light-front vacuum

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Outline

- **Light-front representations of particles.**
- **Light-front dynamics of particles - finite number of DOF.**
- **Particles \rightarrow fields.**
- **Light-front operator algebra.**
- **Noether's theorem and light-front dynamics of fields.**
- **Zero modes and the light-front vacuum.**

Notation

Poincaré group - inhomogeneous $SL(2, \mathbb{C})$

$$X = x^\mu \sigma_\mu = \begin{pmatrix} x^+ & x_\perp^* \\ x_\perp & x^- \end{pmatrix} \quad x^\mu = \frac{1}{2} \text{Tr}(\sigma_\mu X)$$

$$\det(X) = -(x^0)^2 + \mathbf{x}^2$$

$$X' = AXA^\dagger + B \quad B = B^\dagger \quad \det(A) = 1$$

Light-front:

$$X \rightarrow \begin{pmatrix} 0 & x_\perp^* \\ x_\perp & x^- \end{pmatrix} \quad \tilde{\mathbf{x}} = (x^-, \mathbf{x}_\perp)$$

Kinematic subgroup, preserves $x^+ = 0$, (7 parameters):

$$A = \begin{pmatrix} \alpha e^{i\phi} & 0 \\ (\beta_1 + i\beta_2) & e^{-i\phi}/\alpha \end{pmatrix} \quad B = \begin{pmatrix} 0 & b_\perp^* \\ b_\perp & b^- \end{pmatrix}$$

Relativistic particles

$$U(\Lambda, a)$$

↓

$$\{P^\mu, M^{\mu\nu}\}$$

↓

Commuting observables:

$$|(m, j)\tilde{\mathbf{p}}, \mu\rangle_f \quad \tilde{\mathbf{p}} := (p^+, \mathbf{p}_\perp) \quad p^- = \frac{\mathbf{p}_\perp^2 + m^2}{p^+}$$

$$j_{fi} = \frac{1}{2} \epsilon_{ijk} B_f^{-1}(P/M)^i{}_\mu B_f^{-1}(P/M)^j{}_\nu M^{\mu\nu}$$

Mass m spin j irreducible unitary representation of P.G.:

↓

$$U(\Lambda, a)|(m, j)\tilde{\mathbf{p}}, \mu\rangle_f =$$

$$\sum e^{i\Lambda p \cdot a} |(m, j)\tilde{\Lambda p}, \nu\rangle_f D_{\nu\mu}^j [B_f^{-1}(\Lambda p/m) \Lambda B_f(p/m)] \sqrt{\frac{(\Lambda p)^+}{p^+}}$$

Free-particle dynamics: $U(I, x^+)$.

$(\Lambda, b) \in$ **kinematic subgroup**

\Downarrow

$$\left\{ e^{i\Lambda p \cdot b}, R_{wf}(\Lambda, p) := B_f^{-1}(\Lambda p/m) \Lambda B_f(p/m) \right\}, \sqrt{\frac{(\Lambda p)^+}{p^+}}$$

independent of mass (m).

Special light-front properties:

$$D_{\mu\nu}^j[R_{wf}(B_f(p'/m), p)] = \delta_{\mu\nu} \quad D_{\mu\nu}^j[R_{wf}(R, p)] \neq D_{\mu\nu}^j[R]$$

$$P^+ \geq 0 \quad \text{kinematic}$$

$$P^+ = P^- = M \quad \text{rest frame dynamical}$$

Systems of free particles:

$$|(m_1, j_1)\tilde{\mathbf{p}}_1, \mu_1\rangle_f \cdots |(m_n, j_n)\tilde{\mathbf{p}}_n, \mu_n\rangle_f$$

Infinitesimal generators: $G_0 = \sum_n G_n \otimes I_{1n}$.

Commuting observables (system):

$$\{M_0, \mathbf{j}^2, P_0^+, \mathbf{P}_{0\perp}, \mathbf{j}_{0f} \cdot \hat{\mathbf{z}}, \mathbf{d}\}$$

Irreducible basis states:

$$|(m_0, j_0)\tilde{\mathbf{p}}_0, \mu_0; \mathbf{d}\rangle_{0f}$$

\mathbf{d} = invariant degeneracy parameters

$$U(\Lambda, b)|m_0, j_0\rangle_{0f} |\tilde{\mathbf{p}}_0, \mu; \mathbf{d}\rangle_f = \sum e^{i\Lambda P_0 \cdot b} |(m_0, j_0)\tilde{\Lambda} p_0, \nu; \mathbf{d}\rangle_f D_{\nu\mu}^j [R_{wf}(\Lambda, p_0)] \sqrt{\frac{(\Lambda p_0)^+}{p_0^+}}$$

Dynamics - Finite DOF

Problem: Add non-trivial interactions to M and \mathbf{j}^2 that satisfy $[M, \mathbf{j}^2] = 0$ and commute with all kinematic generators.

Bakamjian-Thomas solution (finite DOF)
set $\mathbf{j} = \mathbf{j}_0$, add interactions to M .

$$M^2 = M_0^2 + V$$

$$[V, \mathbf{j}_0] = [V, \mathbf{E}_{\perp 0}] = [V, K_0^3] = [V, P_0^+] = [V, \mathbf{P}_{\perp 0}] = 0$$

$$M > 0 \quad M = M^\dagger$$

Diagonalize $M^2 = M_0^2 + V$ in non-interacting irreducible basis:

$$|(m_0, j_0) \tilde{\mathbf{p}}_0, \mu_0; l_0, s_0\rangle \quad \mathbf{d} = \{l_0, s_0\}$$

Dynamics: This gives a complete set of irreducible eigenstates with a light-front kinematic subgroup that is consistent with the dynamics of M^2 .

$$U(\Lambda, b) |(m_I, j_0) \tilde{\mathbf{p}}_0, \mu; \mathbf{d}_I \rangle_f = \sum e^{i\Lambda p \cdot b} |(m_I, j_0) \tilde{\Lambda} p, \nu; \mathbf{d}_I \rangle_f D_{\nu\mu}^j [R_{wf}(\Lambda, p)] \sqrt{\frac{(\Lambda p)^+}{p^+}}$$

Interacting angular momentum:

$$\mathbf{J}_\perp = \frac{1}{P_0^+} \left[\frac{1}{2} (P_0^+ - P^-) (\hat{\mathbf{z}} \times \mathbf{E}_0) - (\hat{\mathbf{z}} \times \mathbf{P}_0) K_0^3 + \mathbf{P}_\perp \omega_{f0}^3 + M \mathbf{j}_{f\perp 0} \right]$$

$$N \geq 3$$

- The construction can be generalized to any number of particles.
- Cluster properties require a dynamical spin:

$$U_{ij}(\Lambda, a) \otimes U_k(\Lambda, a) \rightarrow \mathbf{j}_{(ij)\otimes k} \neq \mathbf{j}_0$$

- Key tool: Tensor product representations are related to representations with non-interacting spin by S -matrix-preserving unitary transformations.
- Cluster properties require all dynamical operators are many-body operators.
- Dirac's other forms of dynamics - same construction; start with different sets of commuting observables.
- Dynamics in all of Dirac's other forms of dynamics related by an S -matrix preserving unitary transformation.

Particles \rightarrow fields:

Lorentz covariant particle states:

$$\begin{aligned} U(\Lambda, 0)| (m, j) \tilde{\mathbf{p}}, \mu \rangle_f &= \\ \sum | (m, j) \tilde{\Lambda} p, \nu \rangle_f D_{\nu\mu}^j [B_f^{-1}(\Lambda p) \Lambda B_f(p)] \sqrt{\frac{(\Lambda p)^+}{p^+}} & \\ \Downarrow & \\ U(\Lambda, 0)| (m, j) \tilde{\mathbf{p}}, \mu \rangle_f D_{\nu\mu}^j [B_f^{-1}(p)] \sqrt{p^+} &= \\ \sum | (m, j) \tilde{\Lambda} p, \nu \rangle_f D_{\nu\alpha}^j [B_f^{-1}(\Lambda p)] \sqrt{(\Lambda p)^+} D_{\alpha\mu}^j [\Lambda] & \\ U(\Lambda, 0)| (m, j) \tilde{\mathbf{p}}, \mu \rangle_f D_{\nu\mu}^j [B_f^\dagger(p)] \sqrt{p^+} &= \\ \sum | (m, j) \tilde{\Lambda} p, \nu \rangle_f D_{\nu\alpha}^j [B_f^\dagger(\Lambda p)] \sqrt{(\Lambda p)^+} D_{\alpha\mu}^j [(\Lambda^\dagger)^{-1}] & \end{aligned}$$

Particles \rightarrow free fields

Local right or left handed, mass m spin j light-front **free** fields can be constructed from the occupation number representation using the Lorentz-covariant single-particle basis:

$$\begin{aligned}\Psi_{rf\mu}(x) = & \frac{Z}{(2\pi)^{3/2}} \int \frac{d\tilde{\mathbf{p}}\theta(p^+)}{\sqrt{p^+}} (e^{ip\cdot x} D_{\mu\nu}^j(B_f(p/m)) a_f(\tilde{\mathbf{p}}, \nu) \\ & + e^{-ip\cdot x} D_{\mu\nu}^j(B_f(p/m)\sigma_2) b_f^\dagger(\tilde{\mathbf{p}}, \mu))\end{aligned}$$

$$\begin{aligned}\Psi_{lf\mu}(x) = & \frac{Z}{(2\pi)^{3/2}} \int \frac{d\tilde{\mathbf{p}}\theta(p^+)}{\sqrt{p^+}} (e^{ip\cdot x} D_{\mu\nu}^j(B_f^{\dagger-1}(p/m)) a_f(\tilde{\mathbf{p}}, \nu) \\ & + e^{-ip\cdot x} D_{\mu\nu}^j(B_f^{\dagger-1}(p/m)\sigma_2) b_f^\dagger(\tilde{\mathbf{p}}, \mu))\end{aligned}$$

Free field dynamics:

$$U(\Lambda, a)\Psi_{rf\mu}(x)U^\dagger(\Lambda, a) = \sum D_{\mu\nu}^j[\Lambda^{-1}]\Psi_{rf\nu}(\Lambda x + a)$$

$$U(\Lambda, a)\Psi_{lf\mu}(x)U^\dagger(\Lambda, a) = \sum D_{\mu\nu}^j[\Lambda^\dagger]\Psi_{lf\nu}(\Lambda x + a)$$

- Free fields constructed using different Lorentz covariant irreducible bases (instant form, point front) are **identical** to these.

Interacting fields

Light-front quantization:

- Free field operators restricted to the light-front hyperplane are irreducible (i.e. they can be used to represent any operator).
- The spectral condition on P^+ **suggests** that the physical vacuum is identical to the free-field vacuum.
- Noether's theorem gives **formal** expressions for the Poincaré generators as operators integrated over the light-front hyperplane.

**The conventional wisdom about the spectral condition on P^+
in interacting theories:**

$$[V, P^+] = 0$$

$$P^+ V|0\rangle = VP^+|0\rangle = 0$$

$$V|0\rangle = 0 \quad \text{or}$$

$$V|0\rangle \neq 0$$

is an eigenstate of P^+ with eigenvalue 0

**This is the origin of the assertion that the light-front vacuum
is the free Fock vacuum.**

Irreducibility - free scalar field case

$$\phi(x) = \frac{1}{(2\pi)^{3/2}} \int \frac{d\tilde{\mathbf{p}}\theta(p^+)}{\sqrt{p^+}} (e^{ip \cdot x} a(\tilde{\mathbf{p}}) + e^{-ip \cdot x} a^\dagger(\tilde{\mathbf{p}}))$$

Restriction to light front irreducible:

$$\phi(\tilde{\mathbf{p}}) = \frac{1}{2(2\pi)^{3/2}} \int_{x^+=0} e^{-i\tilde{\mathbf{x}} \cdot \tilde{\mathbf{p}}} \phi(\tilde{\mathbf{x}}, 0) d\tilde{\mathbf{x}}$$

$$a(\tilde{\mathbf{p}}) = \sqrt{p^+} \theta(p^+) \phi(\tilde{\mathbf{p}})$$

$$a^\dagger(\tilde{\mathbf{p}}) = \sqrt{p^+} \theta(p^+) \phi(-\tilde{\mathbf{p}})$$

$[O, \phi(\tilde{\mathbf{x}}, 0)] = 0 \forall \tilde{\mathbf{x}}$ implies O is proportional to the identity (exception $p^+ = 0$ contributions).

$p^+ = 0$ is strongly suppressed for operators constructed out of 4d smeared fields.

Noether generators - dynamical theory - (first problem)

$$P^+ = 4 \int_{x^+=0} d\tilde{\mathbf{x}} : \frac{\partial\phi(x)}{\partial x^-} \frac{\partial\phi(x)}{\partial x^-} :$$

$$P^i = 2 \int_{x^+=0} d\tilde{\mathbf{x}} : \frac{\partial\phi(x)}{\partial x^-} \frac{\partial\phi(x)}{\partial x^i} :$$

$$J^3 = 2 \int_{x^+=0} d\tilde{\mathbf{x}} : \frac{\partial\phi(x)}{\partial x^-} (x^j \frac{\partial\phi(x)}{\partial x^i} - x^i \frac{\partial\phi(x)}{\partial x^j}) :$$

$$E^i = 2 \int_{x^+=0} d\tilde{\mathbf{x}} : \frac{\partial\phi(x)}{\partial x^-} (2x^i \frac{\partial\phi(x)}{\partial x^-} - x^+ \frac{\partial\phi(x)}{\partial x^i}) :$$

$$K^3 = 4 \int_{x^+=0} d\tilde{\mathbf{x}} : \frac{\partial\phi(x)}{\partial x^-} (\frac{\partial\phi(x)}{\partial x^-} x^- - \frac{\partial\phi(x)}{\partial x^+} x^+) :$$

$$P^- = 4 \int_{x^+=0} d\tilde{\mathbf{x}} : \frac{\partial\phi(x)}{\partial x^-} \frac{\partial\phi(x)}{\partial x^+} :$$

$$F^i = 2 \int_{x^+=0} d\tilde{\mathbf{x}} : \frac{\partial\phi(x)}{\partial x^-} (2x^i \frac{\partial\phi(x)}{\partial x^+} - x^- \frac{\partial\phi(x)}{\partial x^i}) :$$

- Noether's generators on the light front + an irreducible set of fields on the light front are **insufficient** to define the dynamics (characteristic initial-value surface problem).
 - Additional assumptions are needed to define self-adjoint dynamical generators of unitary one-parameter subgroups.
 - Physical constraint - dynamics should lead to the Wightman functions of a local field theory.
- ∴ Additional input: Use the field theory - field equations + integration by parts:

$$\frac{\partial^2 \phi(x)}{\partial x^- \partial x^+} = (\nabla_{\perp}^2 - m^2)\phi(x) - \frac{d\mathcal{V}}{d\phi(x)}$$

$$P^- = 4 \int_{x^+=0} d\tilde{\mathbf{x}} : \frac{\partial\phi(x)}{\partial x^-} \frac{\partial\phi(x)}{\partial x^+} :=$$

$$-4 \int_{x^+=0} d\tilde{\mathbf{x}} : \phi(x) \frac{\partial^2\phi(x)}{\partial x^- \partial x^+} :=$$

$$4 \int_{x^+=0} d\tilde{\mathbf{x}} : \phi(x) (-\nabla_{\perp}^2 \phi(x) + m^2 \phi(x) + \frac{d\mathcal{V}}{d\phi}(x)) :$$

$$F^i = 2 \int_{x^+=0} d\tilde{\mathbf{x}} : \frac{\partial\phi(x)}{\partial x^-} (2x^i \frac{\partial\phi(x)}{\partial x^+} - x^- \frac{\partial\phi(x)}{\partial x^i}) :=$$

$$-2 \int_{x^+=0} d\tilde{\mathbf{x}} : \phi(x) (2x^i \frac{\partial^2\phi(x)}{\partial x^- \partial x^+} - x^- \frac{\partial^2\phi(x)}{\partial x^- \partial x^i}) :=$$

$$2 \int_{x^+=0} d\tilde{\mathbf{x}} : \phi(x) (2x^i (-\nabla_{\perp}^2 \phi(x) + m^2 \phi(x) + \frac{d\mathcal{V}}{d\phi}(x))) - x^- \frac{\partial^2\phi(x)}{\partial x^- \partial x^i} :$$

- Dynamical generators now **formally** defined as operators on the irreducible light-front algebra.

Free field case

- Free field

$$P^- = \int d\tilde{\mathbf{p}} a^\dagger(\tilde{\mathbf{p}}) \frac{\mathbf{p}_\perp^2 + m^2}{p^+} a(\tilde{\mathbf{p}})$$

- Light-front Heisenberg equations generate x^+ dependence:

$$\frac{d\phi(x)}{dx^+} = i[P^-, \phi(x)]$$

- The solution can be expressed in terms of the light-front creation and annihilation operators which can be expressed in terms of the irreducible algebra of fields on the light front.

$$\phi(x) = \int F_m(x, \tilde{\mathbf{y}}) \phi(\tilde{\mathbf{y}}, 0) d\tilde{\mathbf{y}}$$

$$F_m(x, \tilde{\mathbf{y}}) = \frac{1}{2(2\pi)^3} \int d\tilde{\mathbf{p}} e^{-\frac{i}{2} \frac{\mathbf{p}_\perp^2 + m^2}{p^+} x^+ + i\tilde{\mathbf{p}} \cdot (\tilde{\mathbf{x}} - \tilde{\mathbf{y}})}$$

F_m : local 4d field algebra \rightarrow light-front field algebra.

$$\frac{1}{2(2\pi)^3} \int d^4x g(x^+, \tilde{\mathbf{x}}) e^{-\frac{i}{2} \frac{\mathbf{p}_\perp^2 + m^2}{p^+} x^+ + i\tilde{\mathbf{p}} \cdot \tilde{\mathbf{x}}} = g_{ft} \left(\frac{\mathbf{p}_\perp^2 + m^2}{p^+}, \tilde{\mathbf{p}} \right)$$

- $g_{ft} \left(\tilde{\mathbf{p}}, \frac{\mathbf{p}_\perp^2 + m^2}{p^+} \right)$ vanishes strongly near $p^+ = 0$.

All spectral information is lost for $x^+ = 0$.

$$\begin{aligned}
& \langle 0 | \phi(x) \phi(y) | 0 \rangle = \\
& \int F_m(x, \tilde{\mathbf{z}}) F_m(y, \tilde{\mathbf{w}}) \langle 0 | \phi(\tilde{\mathbf{z}}, 0) \phi(\tilde{\mathbf{w}}, 0) | 0 \rangle d\tilde{\mathbf{z}} d\tilde{\mathbf{w}} = W_{20}(s) = \\
& -i \frac{\epsilon(s^0) \delta(s_0^2 - \mathbf{s}^2)}{4\pi} \\
& + \frac{im\theta(s_0^2 - \mathbf{s}^2)}{8\pi \sqrt{(s_0^2 - \mathbf{s}^2)}} (\epsilon(s^0) J_1(m\sqrt{(s_0^2 - \mathbf{s}^2)}) \\
& \quad - iN_1(m\sqrt{(s_0^2 - \mathbf{s}^2)})) \\
& - \frac{m\theta(\mathbf{s}^2 - s_0^2)}{4\pi^2 \sqrt{\mathbf{s}^2 - s_0^2}} K_1(m\sqrt{\mathbf{s}^2 - s_0^2})
\end{aligned}$$

Gives correct 2-point Wightman function using light-front Fock vacuum.

Limit $z^+ \rightarrow 0$:

$$\langle 0 | \phi(x) \phi(y) | 0 \rangle \rightarrow -i \frac{\epsilon(z^-) \delta(\mathbf{z}_\perp^2)}{4\pi} - \frac{m}{4\pi^2 \sqrt{\mathbf{z}_\perp^2}} K_1(m \sqrt{\mathbf{z}_\perp^2}).$$

(Lorentz invariant - remembers mass)

Direct calculation on light front

$$\langle 0 | \phi(\tilde{\mathbf{x}}) \phi(\tilde{\mathbf{y}}) | 0 \rangle = \frac{1}{(2\pi)^3} \int \frac{d\tilde{\mathbf{p}} \theta(p^+)}{2p^+} e^{i\tilde{\mathbf{p}} \cdot \tilde{\mathbf{z}}}.$$

(not Lorentz invariant - does not know about mass)

(all information about spectrum lost)

limit of integral \neq integral of limit

Remark on Haag's theorem

Different mass free fields restricted to the light front are unitarily equivalent (Schlieder and Seiler).

Extensions to local $4d$ algebra with different F_m are inequivalent.

$$f \sim_{m_1} g \quad \phi_{m_1}(f) = \phi_{m_1}(g)$$

$$f \not\sim_{m_2} g \quad \phi_{m_2}(f) \neq \phi_{m_2}(g)$$

Representations of Heisenberg fields in the light-front algebra ($F_m(x, \tilde{\mathbf{y}})$ for interacting fields)

Hagg expansion - irreducibility of in(out) fields:

$$\phi(x) = \sum_{n=0}^{\infty} \frac{1}{n!} \int d^4x_1 \cdots d^4x_n L_n(x; x_1, \cdots, x_n) : \phi_{in}(x_1) \cdots \phi_{in}(x_n) : .$$

$$\phi(x) = \sum_{n=0}^{\infty} \frac{1}{n!} \int d^4x_1 \cdots d^4x_n L_n(x; x_1, \cdots, x_n) \times$$

$$F_{m_1}(x_1^+; \tilde{\mathbf{x}}_1 - \tilde{\mathbf{y}}_1) \cdots F_{m_n}(x_n^+; \tilde{\mathbf{x}}_n - \tilde{\mathbf{y}}_n) \times$$

$$d\tilde{\mathbf{y}}_1 \cdots d\tilde{\mathbf{y}}_n : \tilde{\phi}_{10}(\tilde{\mathbf{y}}_1) \cdots \tilde{\phi}_{n0}(\tilde{\mathbf{y}}_n) :$$

- **Maps local Heisenberg field algebra into light-front Fock algebra.**

Problem 2: Interacting fields, zero modes and renormalization

Observations:

- **Changing the orientation of the light front should not change the physics.**
- **Invariance under change of light-front orientation = rotational invariance.**
- **$p^+ \rightarrow 0$ with one light front corresponds to $p_i \rightarrow -\infty$ in a theory with a different light front.**
- **Dynamical generators in local field theories involve products of fields at the same point, which are not well defined.**
- **Ultraviolet and infrared renormalization are constrained by rotational covariance.**

Is the light front vacuum really the Fock vacuum?

Example : $\phi^4(\tilde{\mathbf{x}})$:

$$V|0\rangle = \int d\tilde{\mathbf{x}} : \phi^4(\tilde{\mathbf{x}}) : |0\rangle =$$
$$2(2\pi)^2 \int p^+ \theta(p^+) \delta(p^+) dp^+ \prod d\mathbf{p}_{i\perp} \frac{d\xi_1 d\xi_2 d\xi_3 d\xi_4 (1 - \sum_n \xi_n)}{\sqrt{\xi_1 \xi_2 \xi_3 \xi_4}} \times$$
$$\delta(\sum \mathbf{p}_{i\perp}) \prod a^\dagger(\xi_i \mathbf{p}^+, \mathbf{p}_{i\perp}) |0\rangle + \dots$$

which looks like the interaction should annihilate the vacuum, **however**

$$\|V|0\rangle\|^2 \rightarrow \int \frac{\delta(p^+)^2 \theta(p^+)}{p^+} \dots$$

is not defined of the free field Fock vacuum!

- The assertion that V annihilates the vacuum is misleading. Quantities like $\delta(p^+)^2$, $\delta(p^+)\theta(p^+)$ are not even distributions. The singular terms occur at $p^+ = 0$.
- The light front Hamiltonian should be a well-defined operator of the physical Hilbert space.

$$[V, P^+] = 0 \quad \rightarrow \quad V = V(P^+)$$

it could still have a complicated kernel in transverse relative momenta and momentum fractions.

- $P^+ \geq 0$ implies that any modifications to the vacuum are associated with $p^+ = 0$.

The light-front vacuum generating functional

$$\phi(\tilde{f}) = \phi_+(\tilde{f}) + \phi_-(\tilde{f})$$

$$\phi_+(\tilde{f}) \text{ increases } p^+ \quad \phi_-(\tilde{f}) \text{ decreases } p^+$$

Algebraic normal ordering on the free field light-front algebra
(**independent of mass spectrum**):

$$: e^{i\phi(\tilde{f})} := e^{i\phi_+(\tilde{f})} e^{i\phi_-(\tilde{f})}$$

$$e^{i\phi(\tilde{f})} =: e^{i\phi(\tilde{f})} : e^{\frac{1}{2}(\tilde{f}, \tilde{f})}$$

$$(\tilde{f}, \tilde{g}) = \int \frac{d\tilde{\mathbf{p}} \theta(p^+)}{p^+} \tilde{f}(-\tilde{\mathbf{p}}) \tilde{g}(\tilde{\mathbf{p}})$$

Light-front Fock vacuum

$$\langle 0 | : e^{i\phi(\tilde{f})} : | 0 \rangle = 1$$

$$E_{\text{Fock}}[\tilde{f}] := \langle 0 | e^{i\phi(\tilde{f})} | 0 \rangle = e^{\frac{1}{2}(\tilde{f}, \tilde{f})}$$

Non-Fock vacuum - free light front algebra

Spectral condition: non-zero contributions from $p^+ = 0$:

$$\langle 0 | : e^{i\phi(\tilde{f})} : | 0 \rangle = 1 + \sum \langle 0 | : \phi(f)^n : | 0 \rangle$$

Cluster properties imply:

$$\langle 0 | : e^{i\phi(\tilde{f})} : | 0 \rangle = e^{\sum \frac{i^n}{n!} w_{cn}(f)}$$

$$E[\tilde{f}] = E_{Fock}[\tilde{f}] e^{\sum \frac{i^n}{n!} w_{cn}(f)}$$

- w_{cn} **multilinear connected functionals. They can only depend on $p_{i\perp}$ and test functions and their derivatives at $p^+ = 0$.**

- **Translational invariance of vacuum** $\rightarrow P^-$ densely defined self-adjoint operator satisfying:

$$P^-|0\rangle = 0$$

$$:: \phi^n(\tilde{x}) :: \rightarrow \lim_{y_i \rightarrow 0} \left(\prod \phi(\tilde{x} + y_i) - \sum B_k(\tilde{x}) C_k(y_1 \cdots y_n) \right)$$

- **The requirement** $:: \phi^n(\tilde{x}) :: |0\rangle = 0$ **relates the coefficients** $C_k(y_1 \cdots y_n)$ **to the vacuum.**
- **Non-trivial contributions are due to** $p^+ = 0$
- **The structure of vacuum, the definition of** P^- , **and rotational covariance must be treated simultaneously.**

Summary

- **There are no issues with ill-posed initial value problems with light-front treatments of interacting particles. All forms of dynamics are equivalent.**
- **Free fields are independent of Lorentz covariant basis.**
- **Both particle and field dynamics require constructing self-adjoint dynamical generators on the Hilbert space.**
For fields on the light front this requires (1) extending the Noether generators to operators in the light-front algebra, and (2) a consistent treatment of the vacuum, renormalization of local operator products on the light front in the dynamical generators and rotational covariance.
- **Extension should lead to same theory as canonical quantization.**

Open questions
(for the students in the audience)

- **Can light-front field theories be formulated in terms of the free field algebra (with a non-Fock vacuum)?**
- **Role of rotational covariance in relating infrared and ultraviolet divergences?**
- **Is there a systematic way to simultaneously construct a self-adjoint P^- and compatible vacuum?**
- **More information (answers?): See next four talks by Collins, Brodsky, Beane and Hiller**

Thanks to the conference organizers and sponsors!