RECONSTRUCTING PARTON DISTRIBUTION FUNCTIONS FROM THEIR COORDINATE SPACE BEHAVIOR

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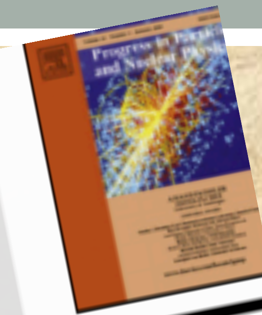

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Review

Parton distributions and lattice QCD calculations: A community white paper

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In collaboration with A. Rajan

The Review poses (and possibly answers some of) the following questions:

1. What information from PDF fits is relevant to constrain/test/validate lattice calculations?

2. What PDF-related quantities we would like lattice QCD to compute?

The aim of our work is to pose (and possibly answer) the following questions:

1. Can we quantify the accuracy that lattice calculations should have in order to have a direct impact on PDF fits

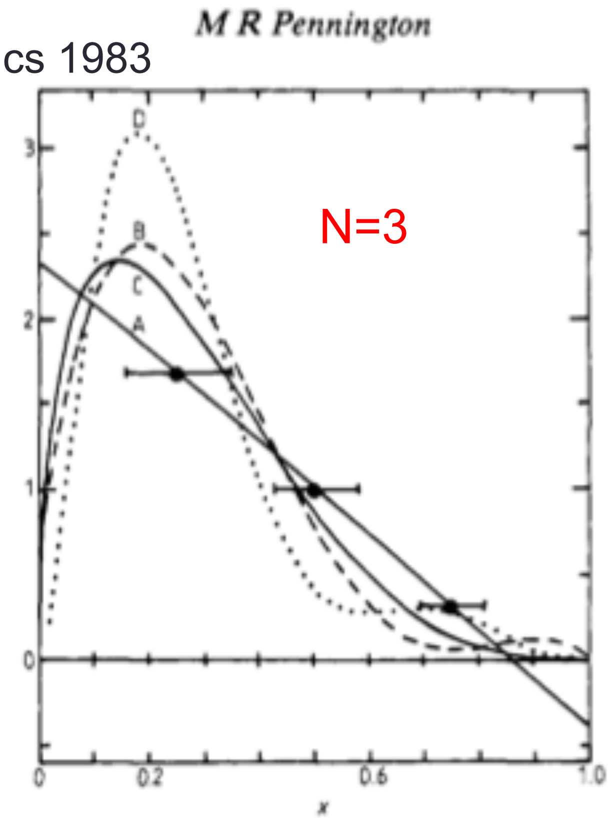
2. To which extent available lattice results agree with the results of global PDF fits?

….extend to Generalized Parton Distributions!

1. DEFINITIONS

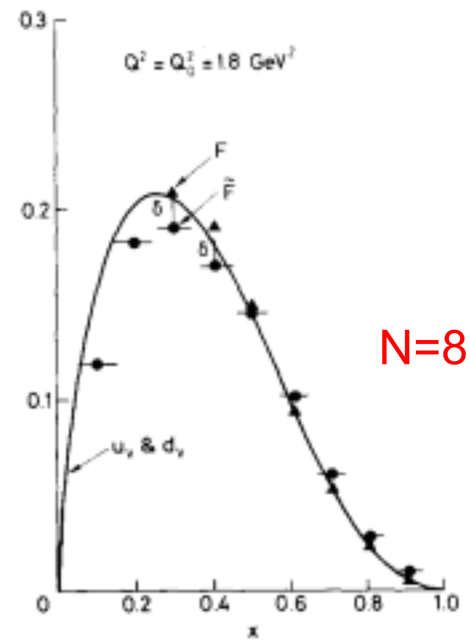
Early attempts: Reconstructing DIS structure functions from their Mellin moments, using the orthogonal polynomials technique

Rep. Prog. In Physics 1983



Mellin Moment

$$M_n(Q^2) = \int_0^1 dx\, x^{n-2} F_2(x, Q^2)$$



F. Yndurain, PLB 1978

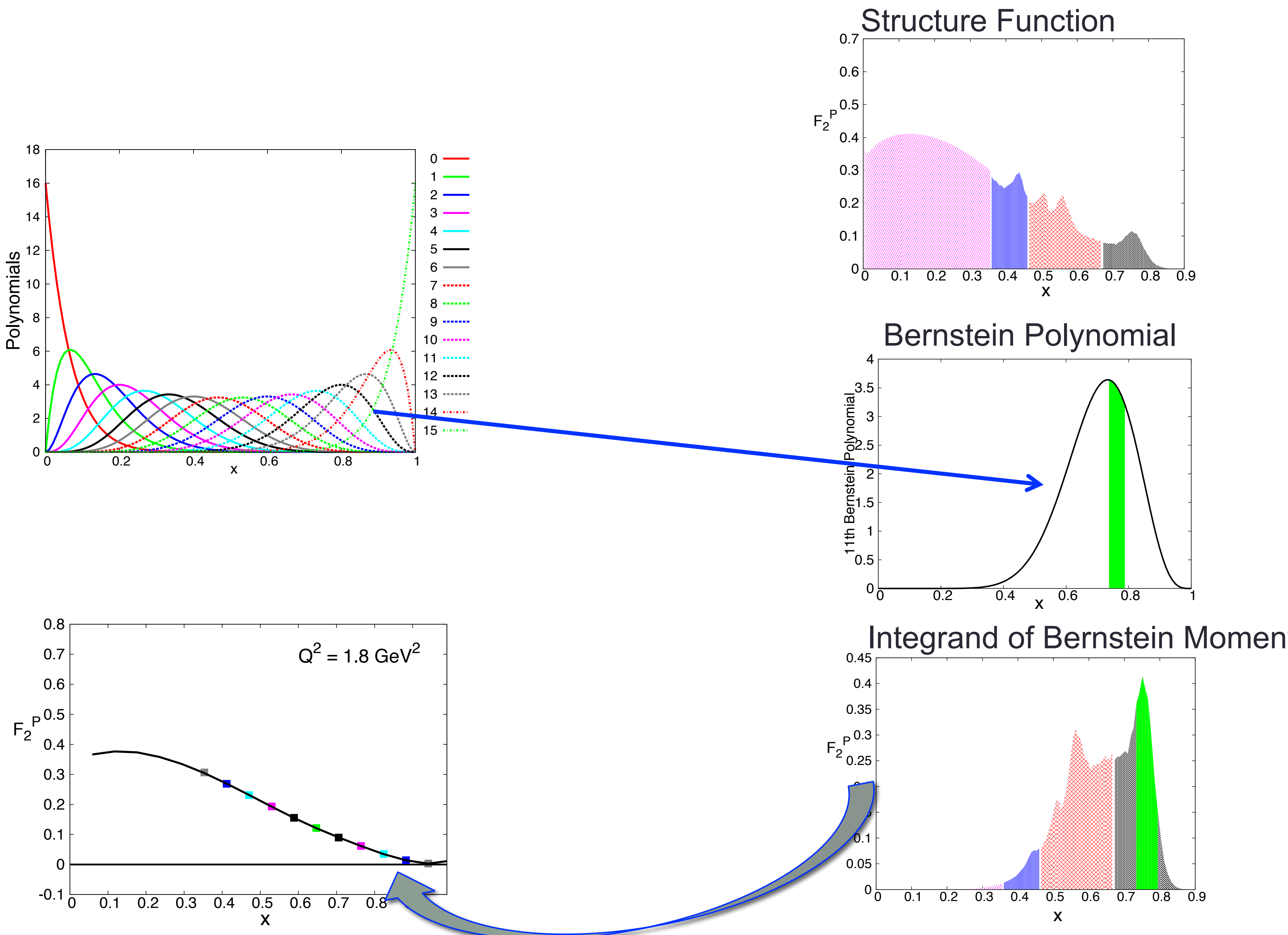
In a nutshell….

… replace Mellin Moment with Orhtogonal Polynomial Moment

$$M_n(Q^2) = \int_0^1 dx\, x^{n-2} F_2(x, Q^2)$$

$$\Updownarrow$$

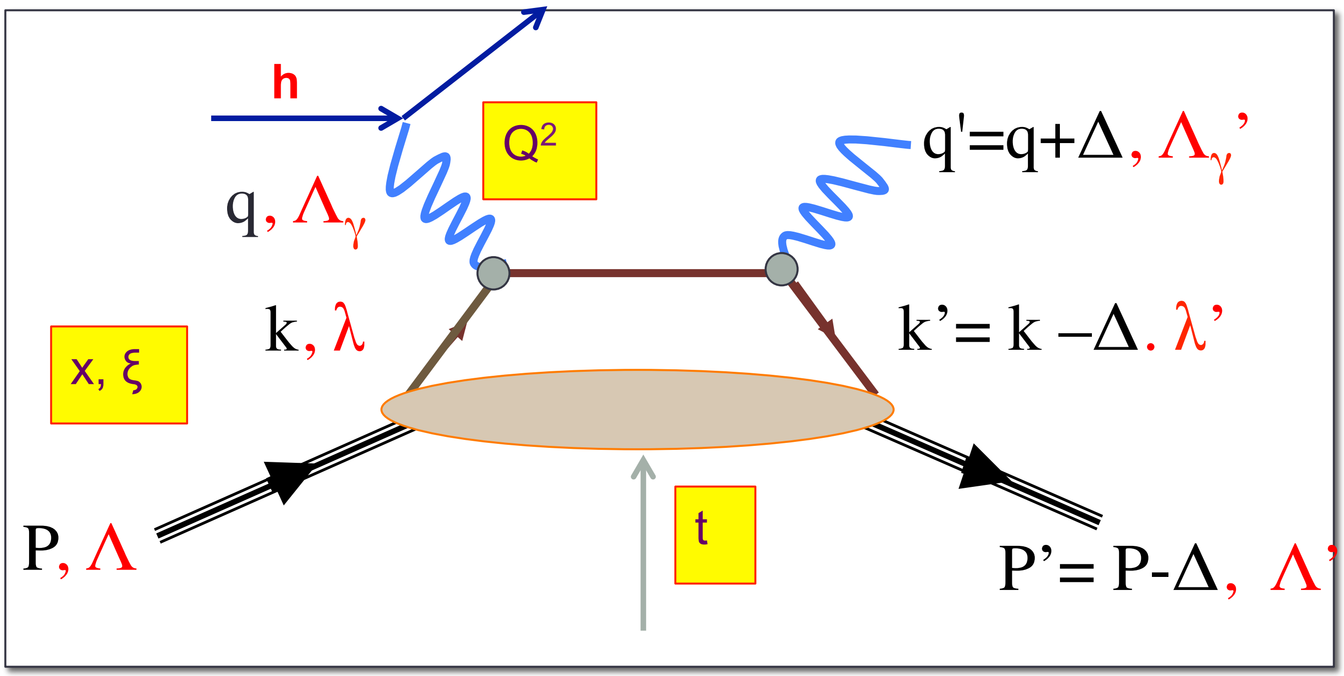
$$\int_0^1 dx\, B_k^m(x) F_2(x, Q^2) = \frac{(m+1)!}{k!} \sum_{l=0}^{m-k} \frac{(-1)^l}{l!(m-k-l)!} M_{k+l+2}(Q^2)$$



E. Askanazi and S.L., arXiv:1710.02592

GPD/PDF CORRELATION FUNCTION

$$W^{\Gamma}_{\Lambda'\Lambda}(x,\xi,\Delta) = \frac{1}{2}\int \frac{dz^-}{(2\pi)} e^{ixP^+z^-} \langle P-\Delta,\Lambda' \mid \bar{q}(0)\,\Gamma\mathcal{W}\left(0,z^-\right) q\left(z^-\right) \mid P,\Lambda\rangle\Big|_{z_T=0,z^+=0}$$



GPDs involve two types of distance

$$H^q(x, 0, \Delta) = \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle P - \Delta, \Lambda' \mid \bar{q}(0)\gamma^+ q(z^-) \mid P, \Lambda \rangle_{\mathbf{z}_T = 0}$$

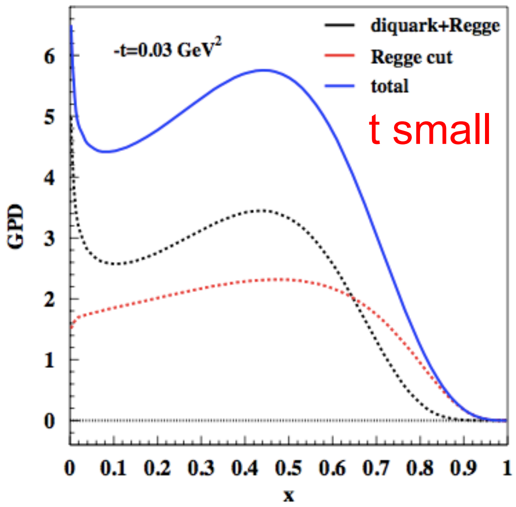
x distribution ➡ **Fourier transform** of **non-diagonal** density distribution in **z⁻**

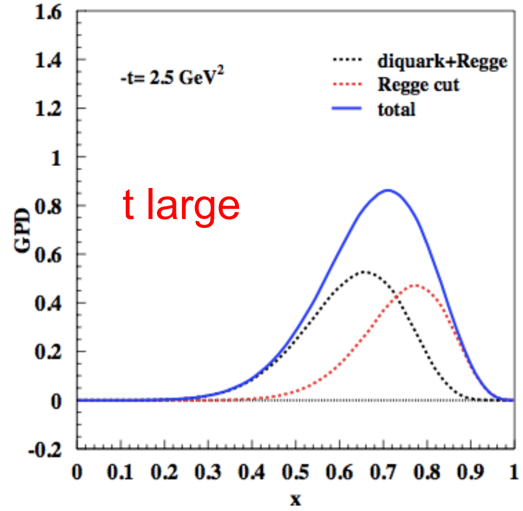
Δ distribution ➡ **Fourier transform** of **diagonal** density distribution in **b**

$$\bar{q}_+^\dagger(0, b) q_+(z^-, b) \rightarrow \rho(0, b; z^-, b)$$

$H^u(x,0,\Delta)$

x dependence

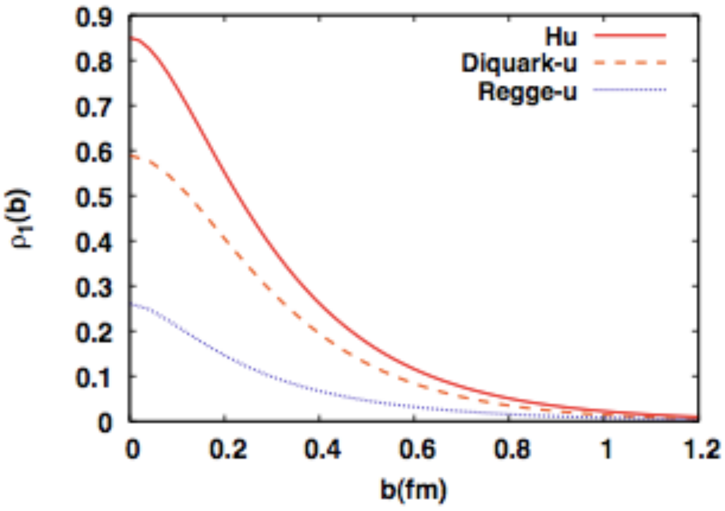




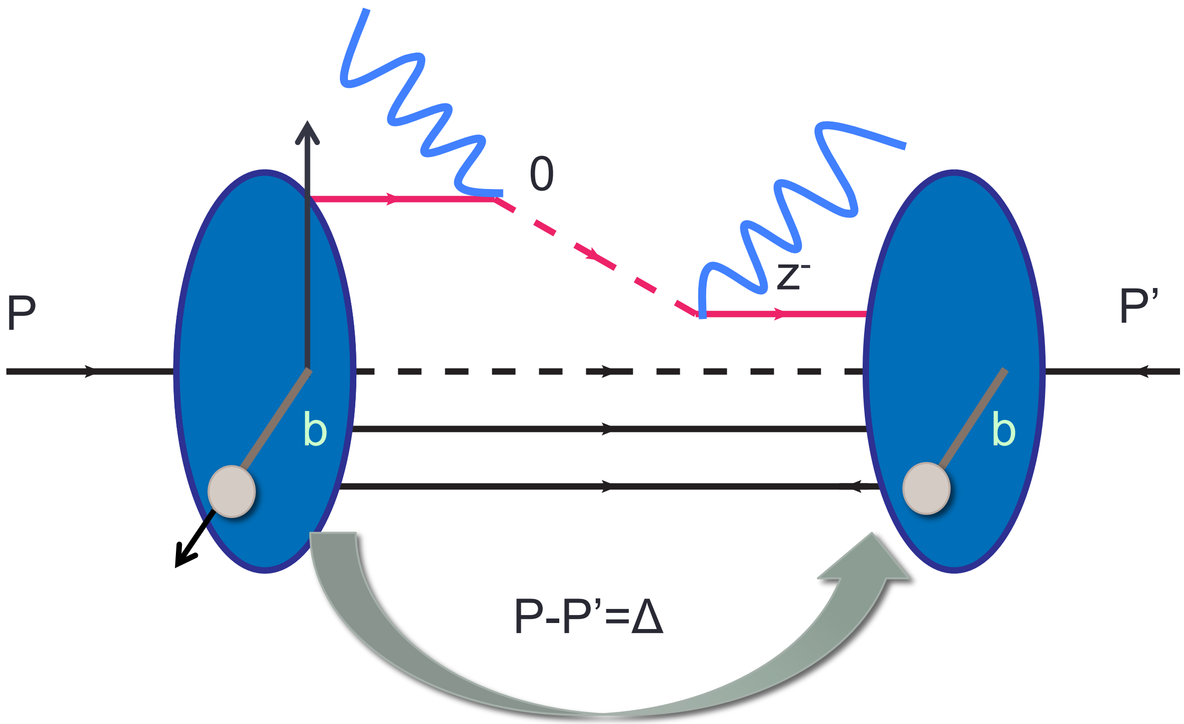
O. Gonzalez-Hernandez et al., PRC88 (2013)

density distribution on transverse plane

$$F^q(t \equiv \Delta^2) = \int dx H^q(x, 0, \Delta) \rightarrow \rho^q(b)$$



Space-time picture of the off-forward correlation function



$$\langle P-\Delta,\Lambda' \mid \bar{q}(0)\gamma^{+}\mathcal{W}(0,z)q(z^{-}) \mid P,\Lambda\rangle_{\mathbf{z}_T=0}$$

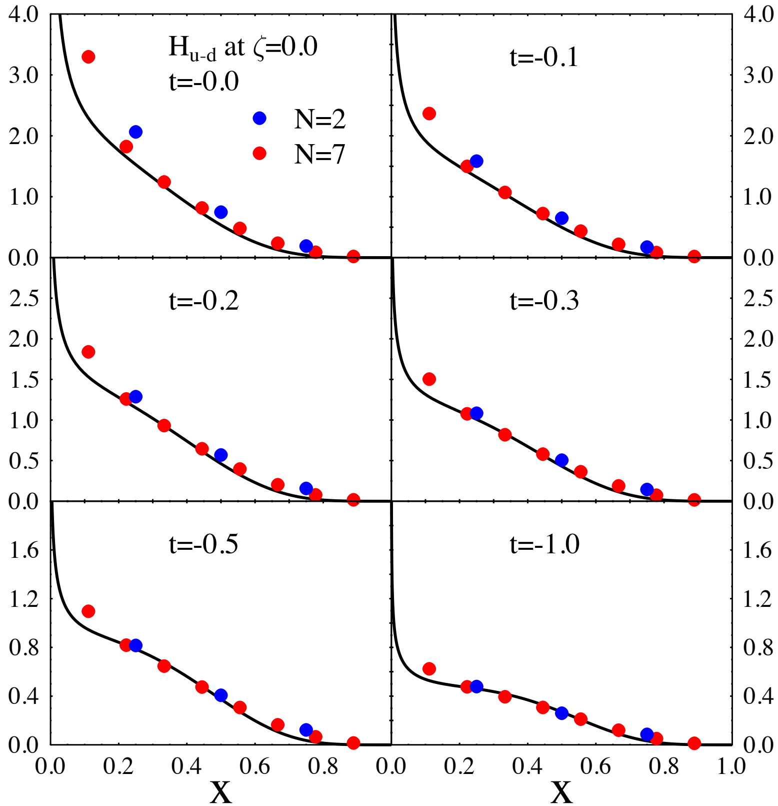
Reconstructing GPDs from a finite number of moments

(Using lattice moments from LHPC, Ph. Hägler et al., 2005, 2006, 2008)

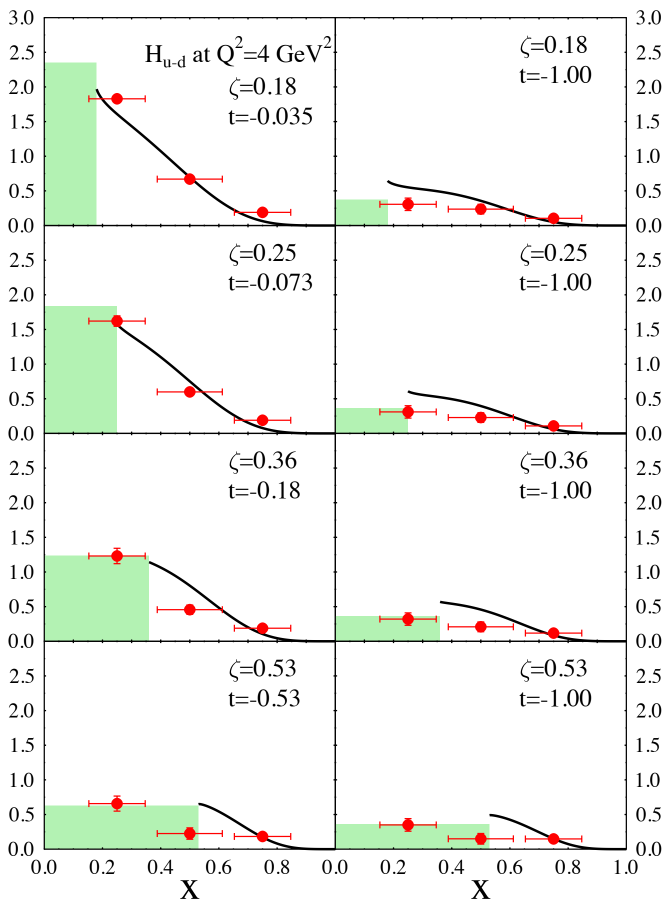
Bernstein polynomials based analysis

H. Honkanen et al., EPJC(2007)

8 moments from param.



3 moments from LHPC



2. DETERMINATION OF PDF FROM COORDINATE SPACE BEHAVIOR

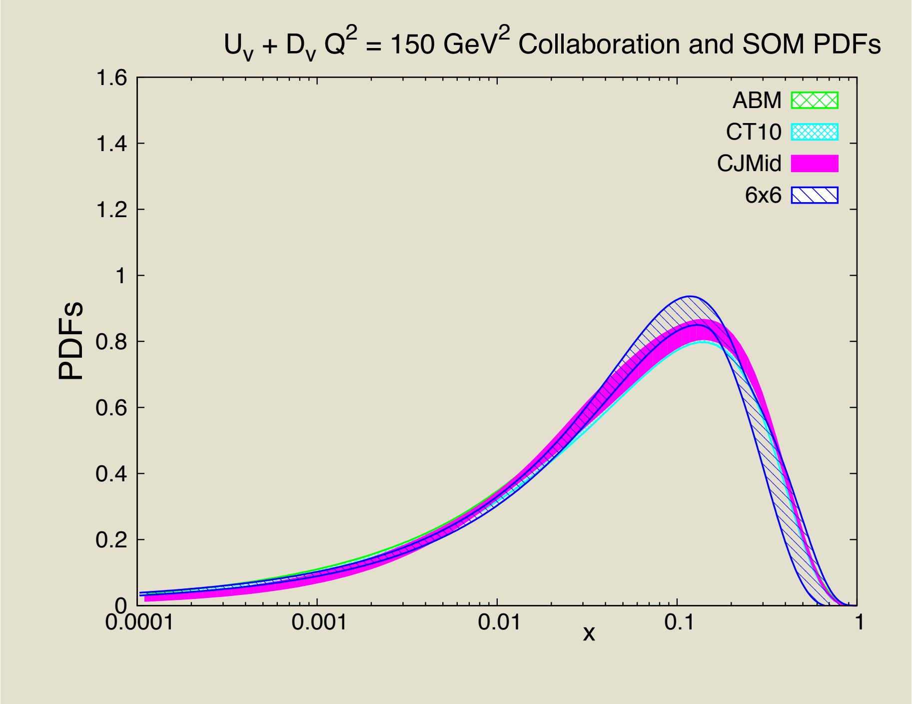
Example: Unpolarized PDFs

$$f_1^q(x) = \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle P,\Lambda \mid \bar{q}(0)\gamma^+ q(z^-) \mid P,\Lambda\rangle_{\mathbf{z}_T=0}$$

Parton x-momentum distribution is the Fourier transform of a non-diagonal one body density distribution in coordinate z-space

$$\bar{q}_+^\dagger(0)q_+(z^-) \to \rho(0,z^-)$$

$$F_2(x) = \sum_q x f_1^q(x)$$



E. Askanazi, K. Holcomb, S. L., J. Phys. G 42, no. 3, 034030 (2015) [arXiv:1411.2487 [hep-ph]].

Symmetry: C-parity

Collins and Soper, '80s

$$\bar{q}(x) = -q(-x)$$

C-even $$q(x) + \bar{q}(x)$$ Anti-symmetric with respect to x=0

C-odd $$q(x) - \bar{q}(x)$$ Symmetric with respect to x=0

$$\overbrace{\langle P,\Lambda' \mid \bar{q}(0)\,\gamma^{+}q\left(z^{-}\right) \mid P,\Lambda\rangle + \langle P,\Lambda' \mid \bar{q}(0)\,\gamma^{+}q\left(-z^{-}\right) \mid P,\Lambda\rangle}^{\mathrm{T_s}} = \frac{i}{2P^{+}}\int_0^1 dx q_s(x)\sin(xz)$$

$$\overbrace{\langle P,\Lambda' \mid \bar{q}(0)\,\gamma^{+}q\left(z^{-}\right) \mid P,\Lambda\rangle - \langle P,\Lambda' \mid \bar{q}(0)\,\gamma^{+}q\left(-z^{-}\right) \mid P,\Lambda\rangle}^{\mathrm{T_c}} = \frac{1}{2P^{+}}\int_0^1 dx q_v(x)\cos(xz)$$

$$T_s(z) \quad = \frac{i}{2M}\int_0^1 dx q(x)\sin(xz) = \frac{i}{2M}\left[M_2 z + \frac{1}{3!}M_4 z^3 + ...\right]$$

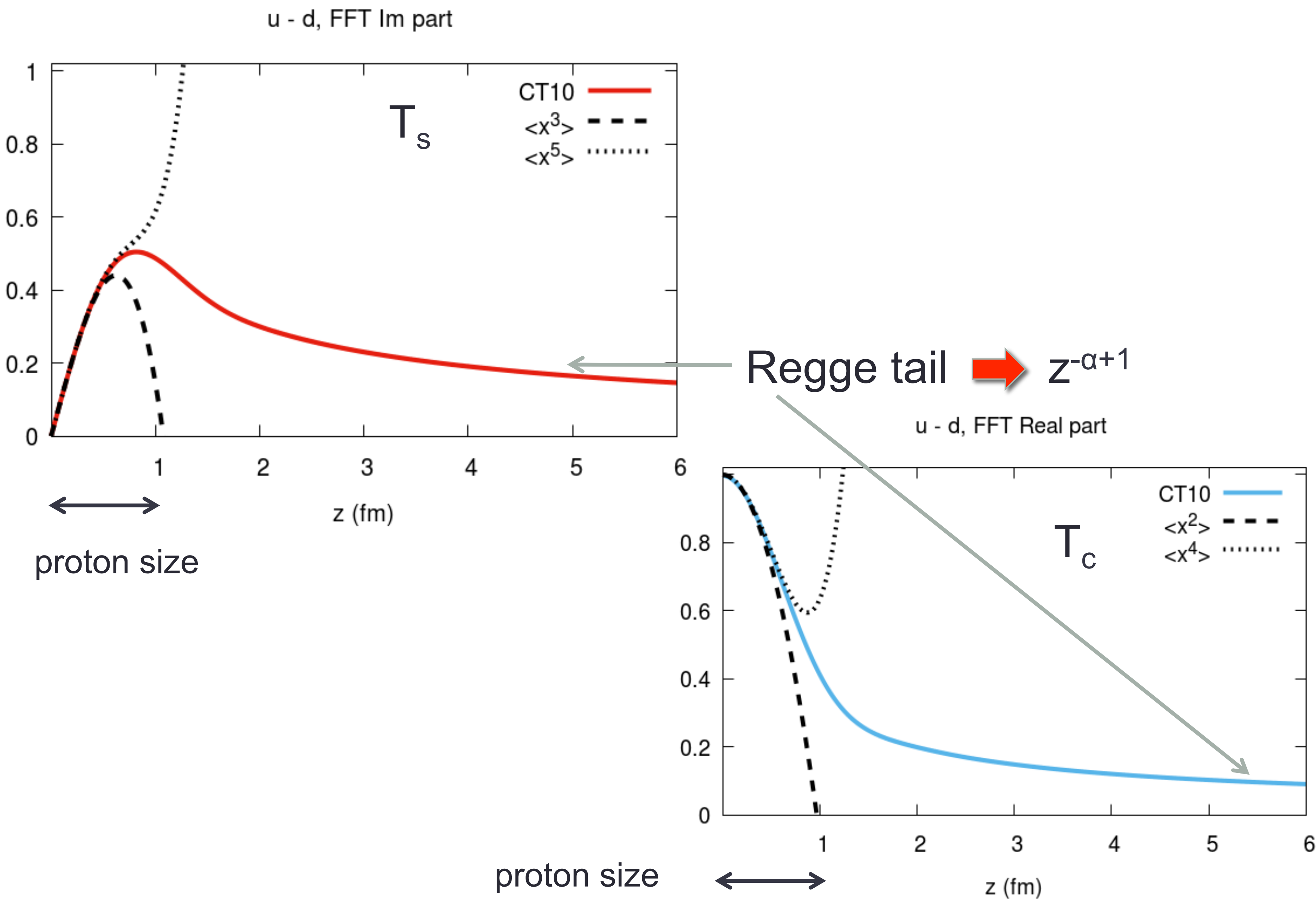
$$M_2 = \int_0^1 dx x\, q(x)$$

$$M_4 = \int_0^1 dx x^3 q(x)$$

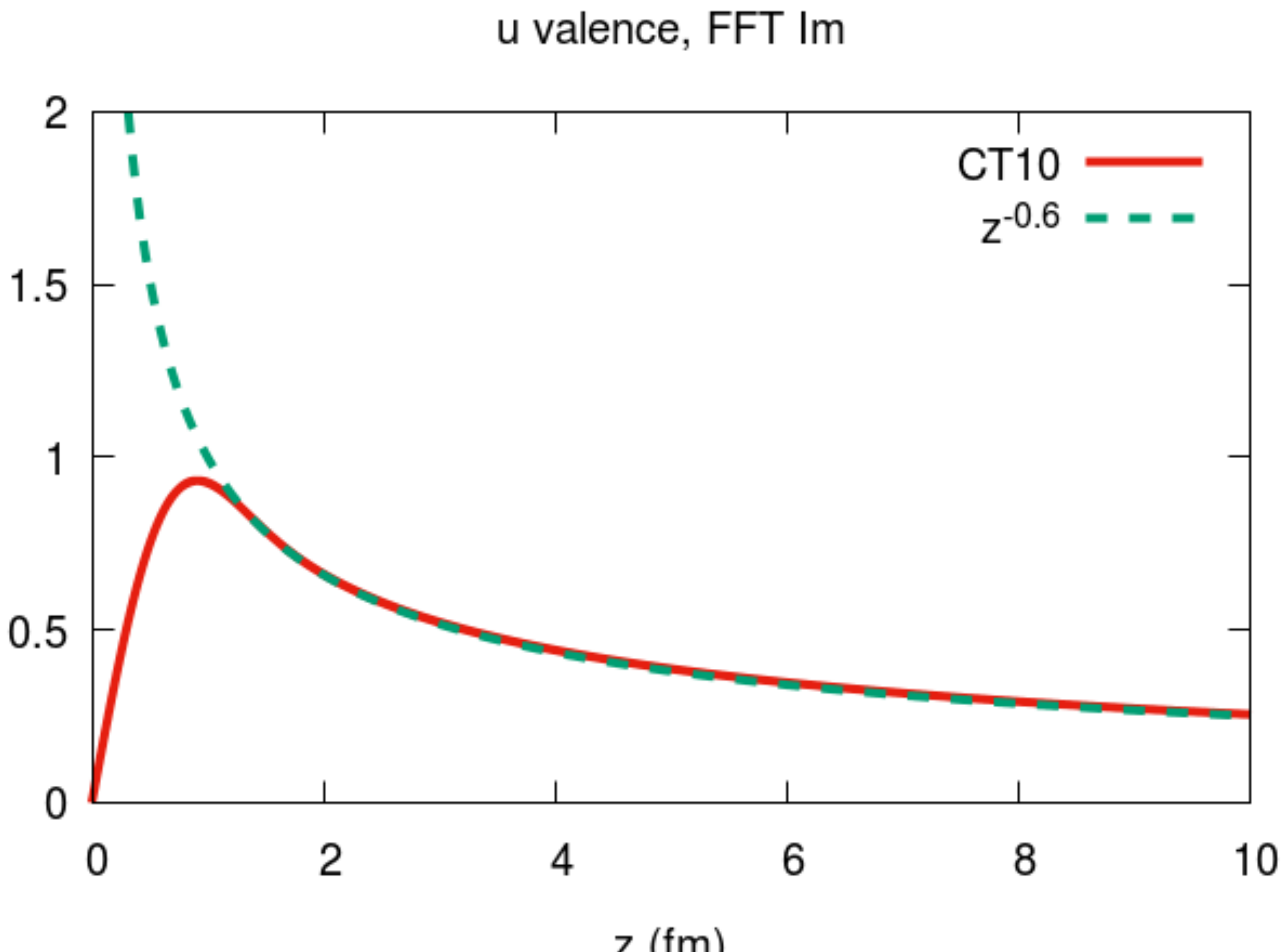
$$T_c(z) \quad = \frac{1}{2M}\int_0^1 dx q_s(x)\cos(xz) = \frac{i}{2M}\left[M_1 + \frac{1}{2}M_3 z^2 + ...\right]$$

$$M_1 = \int_0^1 dx \;\, q(x)$$

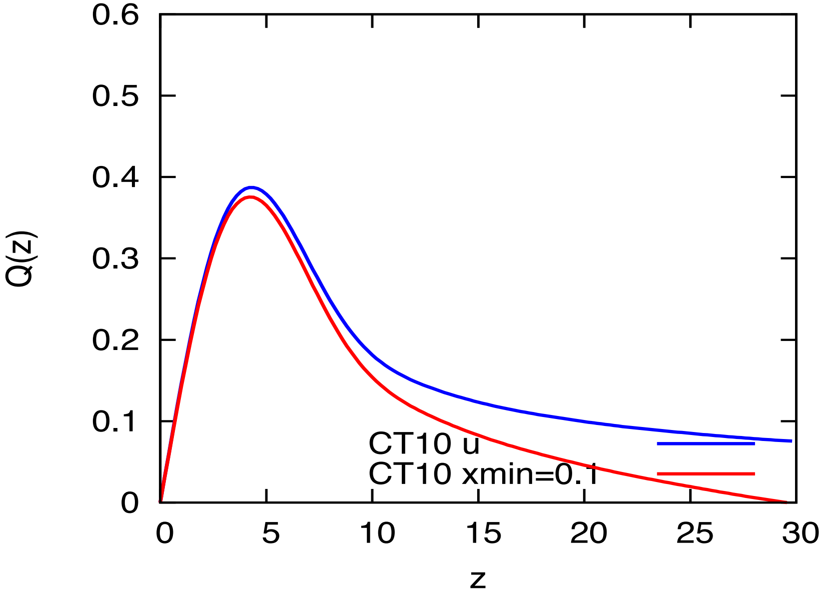
$$M_3 = \int_0^1 dx x^2 q(x)$$


u - d, FFT Im part
CT10
<x3>
<x5>
Ts
z (fm)
proton size
Regge tail
z-α+1
u - d, FFT Real part
CT10
<x2>
<x4>
Tc
z (fm)
proton size

Regge term



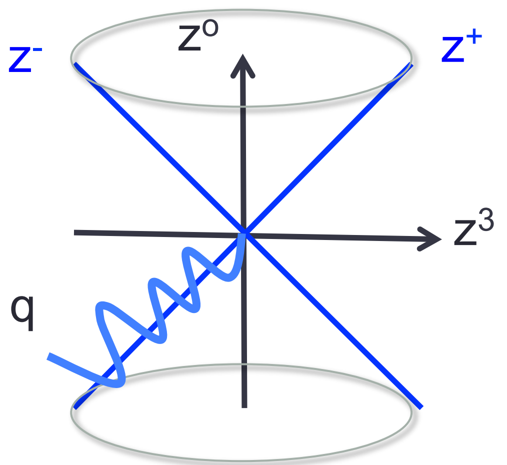
Effect of Regge term



Space-time picture of the correlation function

- At small distances - $z^- < R$ - the virtual photon scatters incoherently from the individual partons. This region is dominated by bulk properties such as the average momentum.

- At larger distances - $z^- >> R$ - the interaction is given by coherent scattering over several partons. The virtual photon converts into a q-qbar pair covering a distance z^-



3. HOW WELL CAN WE RECONSTRUCT THE X DEPENDENT FUNCTION (PDF/ GPD)?

2), 3) 4)

Information from lattice: Mellin Moments

$$M_n(Q^2) = \int_0^1 dx\, x^{n-2} F_2(x, Q^2)$$

u-d

W. Detmold et al., Eur.Phys.J.3 (2001), Mod.Phys.Lett. A18 (2003)

Moment u-d μ^2= 4 GeV^2	Linear extrapolation	Chiral extrapolation	Phenomenology CT10
M_1	1	1	1
M_2	0.262	0.18(3)	0.169
M_3	0.0843	0.05(2)	0.0536
M_4	0.0340	0.02(1)	0.0221

LHPC (Ph. Hägler et al.) Phys.Rev. D77 (2008)

u-d

Moment u-d μ^2= 4 GeV^2	m_π=352 MeV	Chiral extrapolation	Phenomenology CT10
M_1	1	1	1
M_2	0.206(14)	0.157(10)	0.169
M_3	0.078(16)	/	0.0536
M_4	/	/	0.0221

G. Bali et al. PoS LATTICE2015 (2016)

Moment u-d μ^2= 4 GeV^2	Linear extrapolation	Chiral extrapolation	Phenomenology CT10
M_1	1	1	1
M_2	/	0.200(-7/+9)	0.169
M_3	/	/	0.0536
M_4	/	/	/

ETMC (C. Alexandrou et al., Phys Rev D93, 2016)

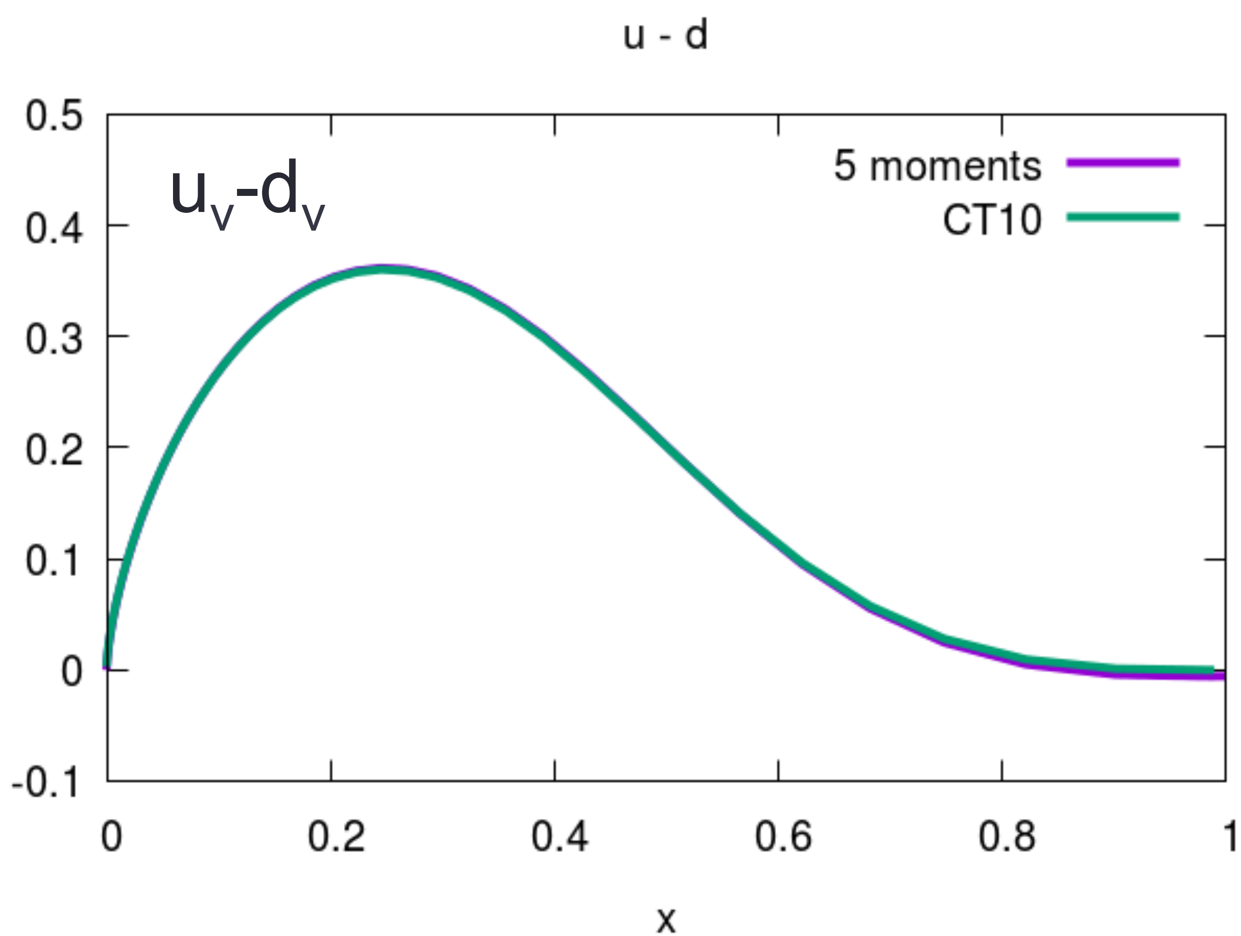
u-d

Moment u-d $\mu^2 = 4\ GeV^2$	**m_π=physical (largest source sink)**	**Hägler: Chiral extrapolation**	**Phenomenology CT10**
M_1	1	1	1
M_2	0.208(24)	0.157(10)	0.169
M_3	/	/	0.0536
M_4	/	/	0.0221

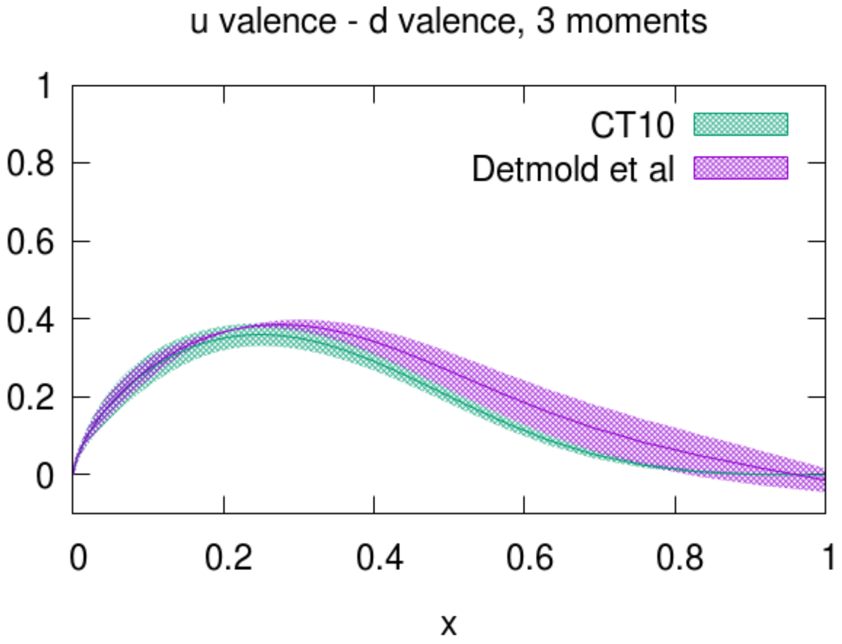
LHPC (J. Green et al., Phys. Lett. B, 2014)

Moment u-d $\mu^2 = 4\ GeV^2$	**m_π=physical**	**Hägler: Chiral extrapolation**	**Phenomenology CT10**
M_1	1	1	1
M_2	0.140(21)	0.157(10)	0.169
M_3	/	/	0.0536
M_4	/	/	0.0221

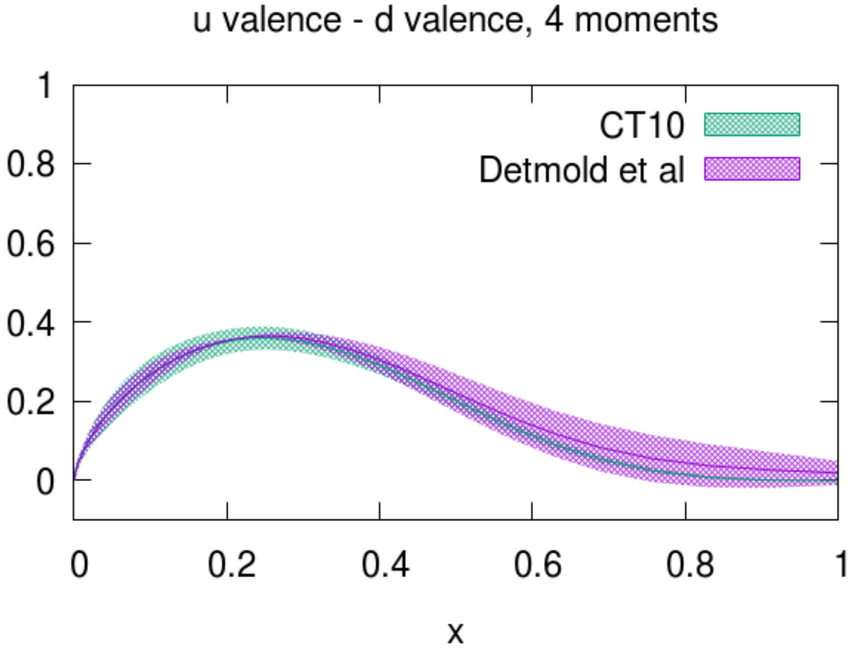
Reconstructed pdf: using its own moments up to x^5



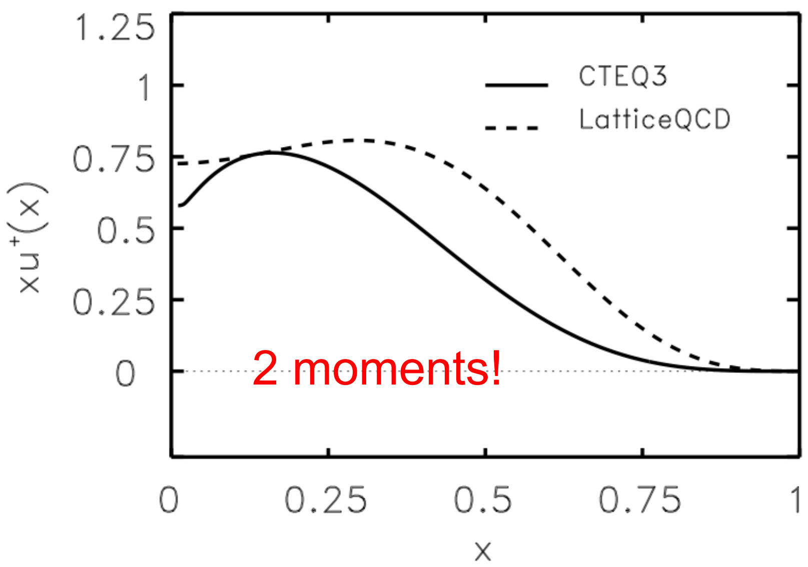
no error shown

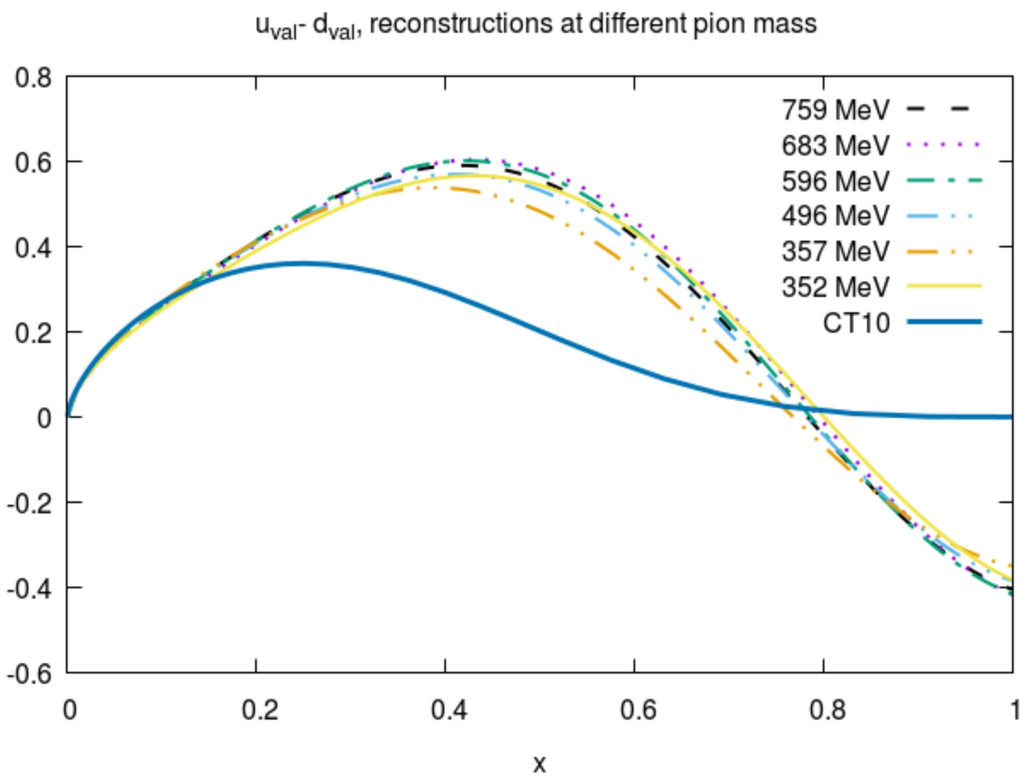


Using Detmold et al.



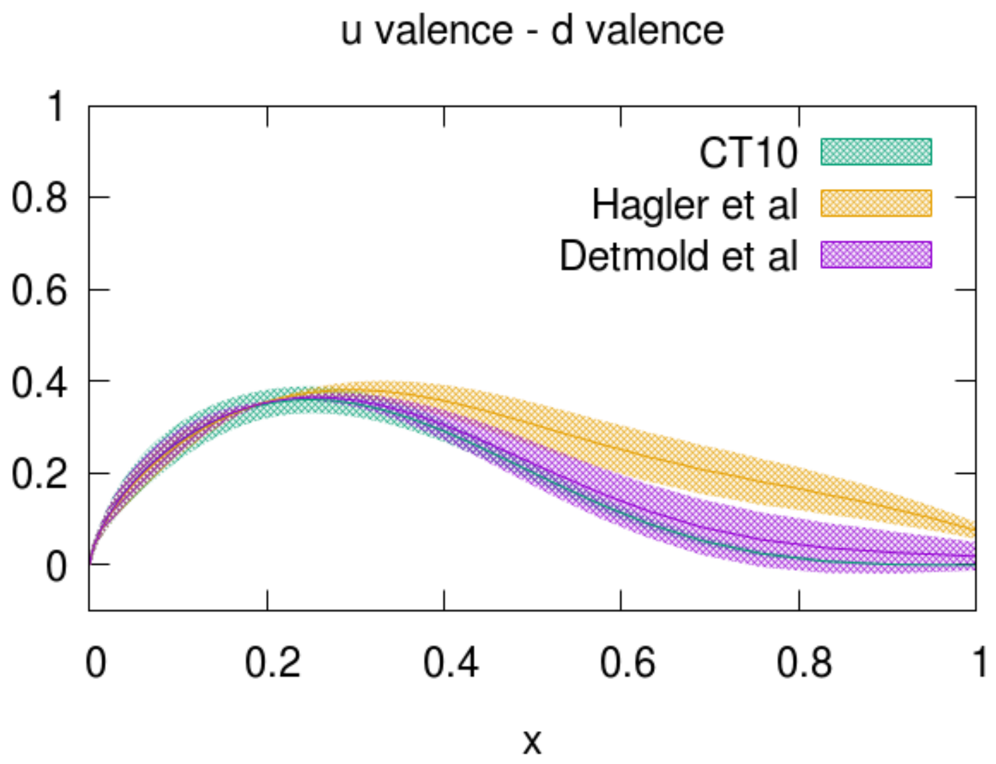
Weigl and Mankiewicz, PLB 1996





LHPC (Ph. Hägler et al.) Phys.Rev. D77 (2008)

Comparison Detmold and Hägler

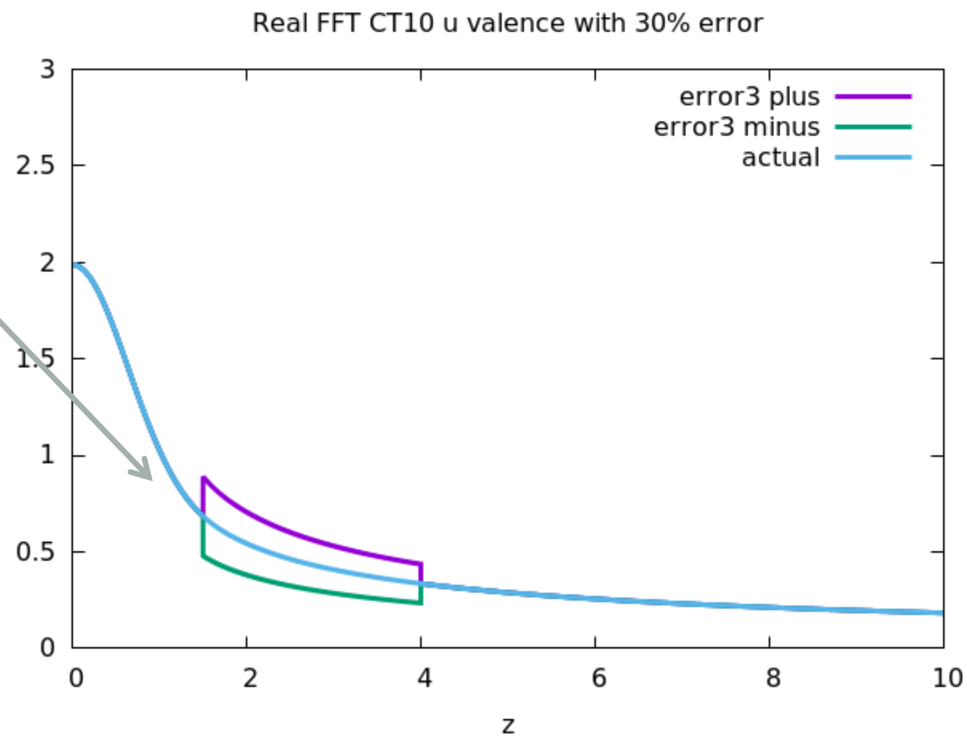


Error analysis in progress

ii) What PDF-related quantities would we like lattice QCD to compute?

Sources of error:

- lattice moments
- Regge tail (CT10)
- unknown region



A careful analysis needs to be done and it's on its way!

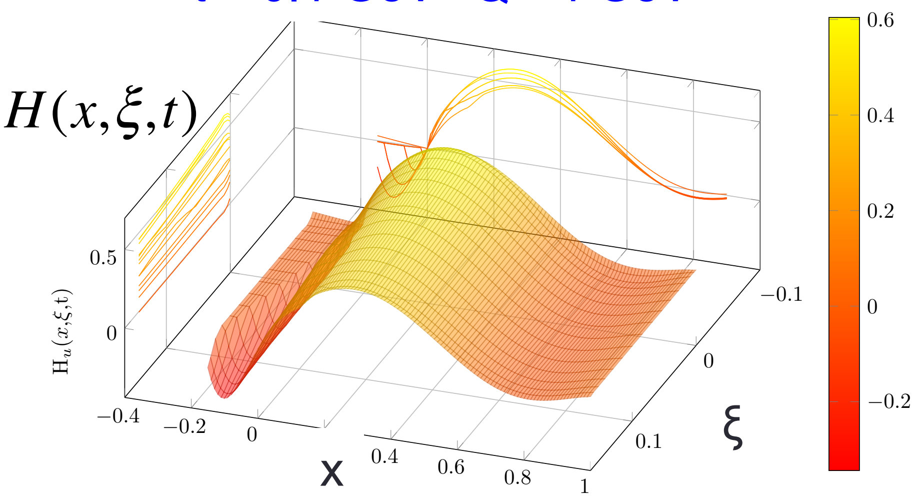
First stab at GPDs…

$$\overline{H}_{02}(X_{02}) = 3A_{10} - 6A_{20} + 3\left[A_{30} + \left(\frac{2\zeta}{2-\zeta}\right)^2 A_{32}\right],$$

$$\overline{H}_{12}(X_{12}) = 6A_{20} - 6\left[A_{30} + \left(\frac{2\zeta}{2-\zeta}\right)^2 A_{32}\right],$$

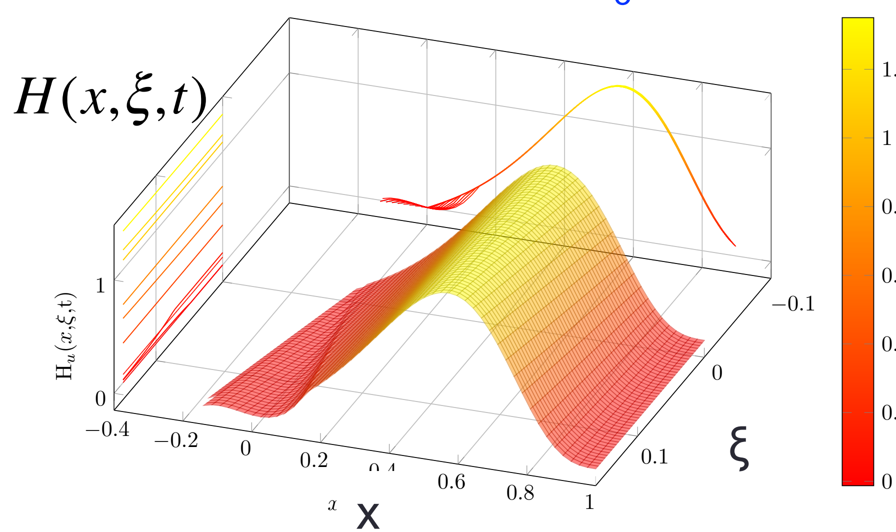
$$\overline{H}_{22}(X_{22}) = 3A_{30} + \left[\left(\frac{2\zeta}{2-\zeta}\right)^2 A_{32}\right].$$

t = 0.1 GeV^2, Q^2=4 GeV^2



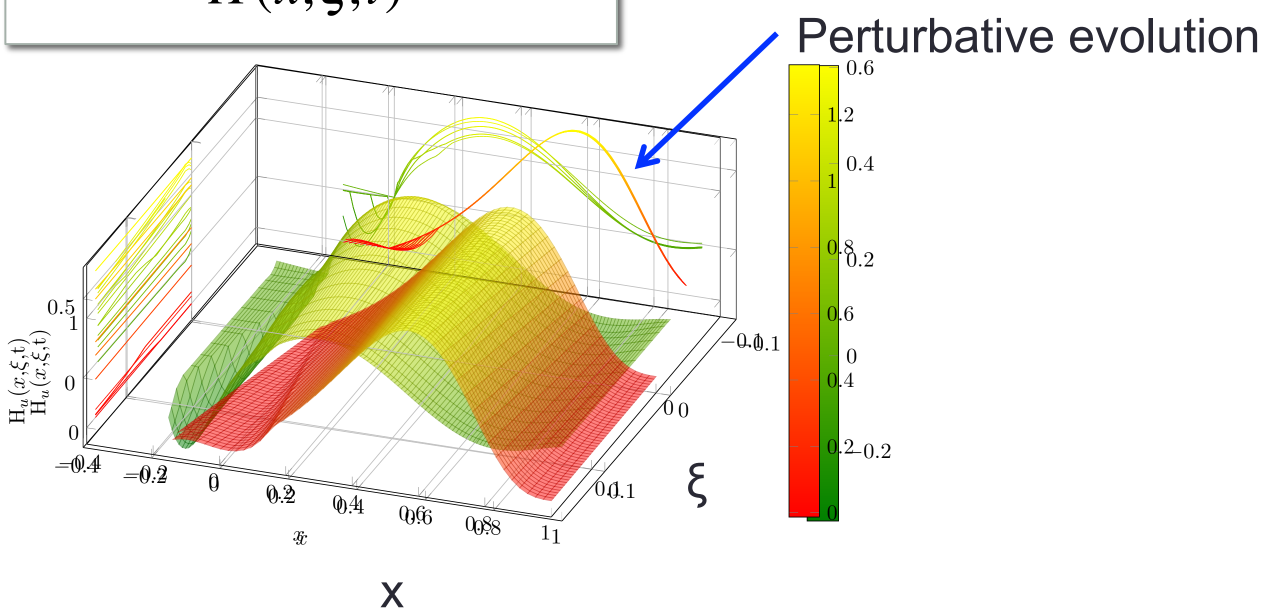
Reconstructed

t = 0.1 GeV^2, Q_o^2= 0.1 GeV^2



Model

$$H(x,\xi,t)$$



4.PSEUDO-PDFS

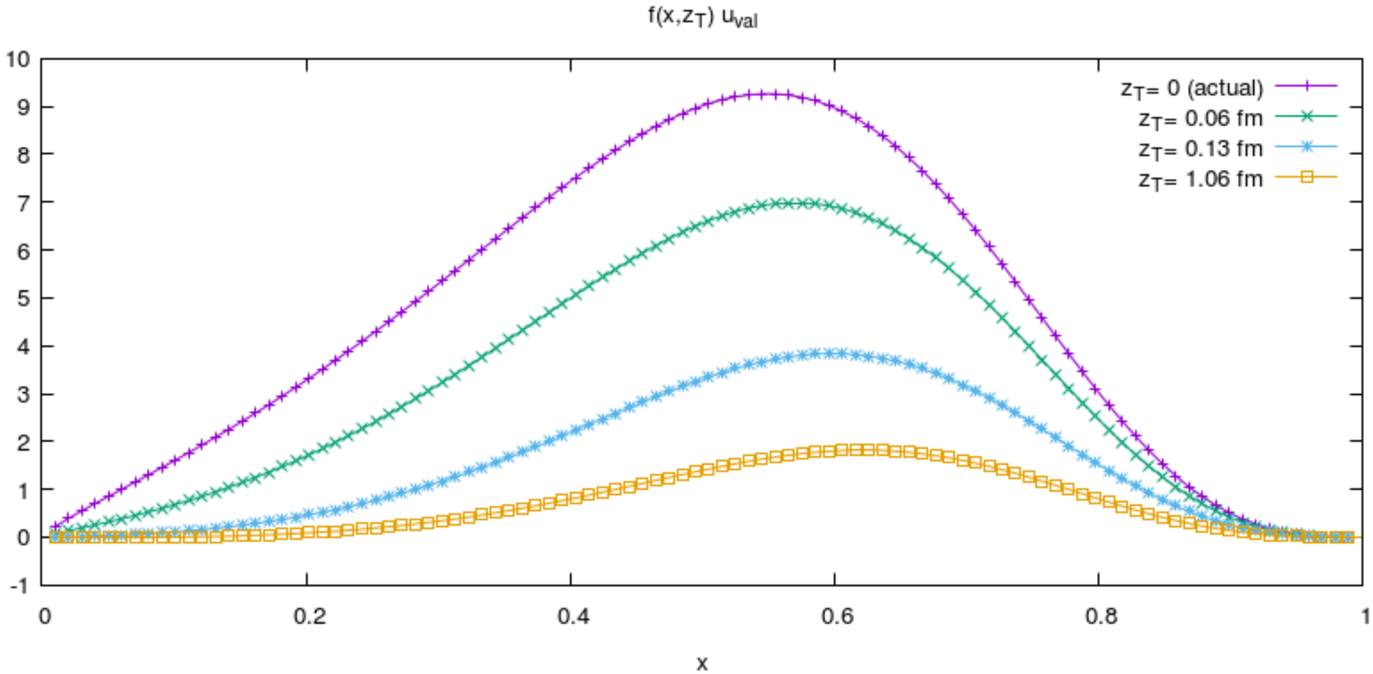
- ✓ Pseudo-PDFs interpolate between PDFs evaluated on the LF and lattice QCD space-like correlations

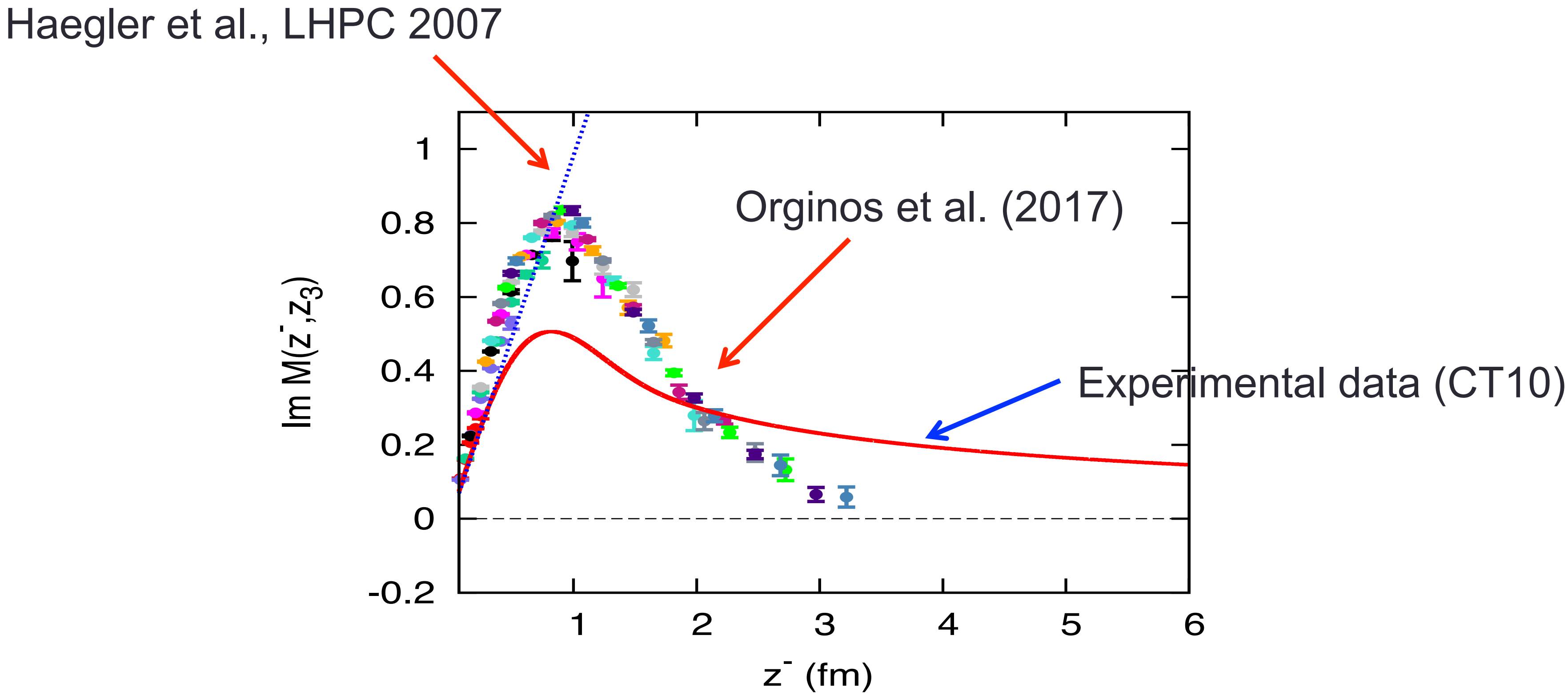
- ✓ Pseudo-PDFs are the Fourier transforms of $z_T(z_3)$-dependent generalized Ioffe time distributions, $\mathcal{M}(\nu, z_3^2)$

- ✓ For $z_3 \to 0$ the ratio $\dfrac{\mathcal{M}(\nu, z_3^2)}{\mathcal{M}(0, z_3^2)}$ reduces to $\mathcal{M}(\nu, 0)$

- ✓ $\mathcal{M}(\nu, 0)$ is the Ioffe time distribution conjugate to $f(x)$

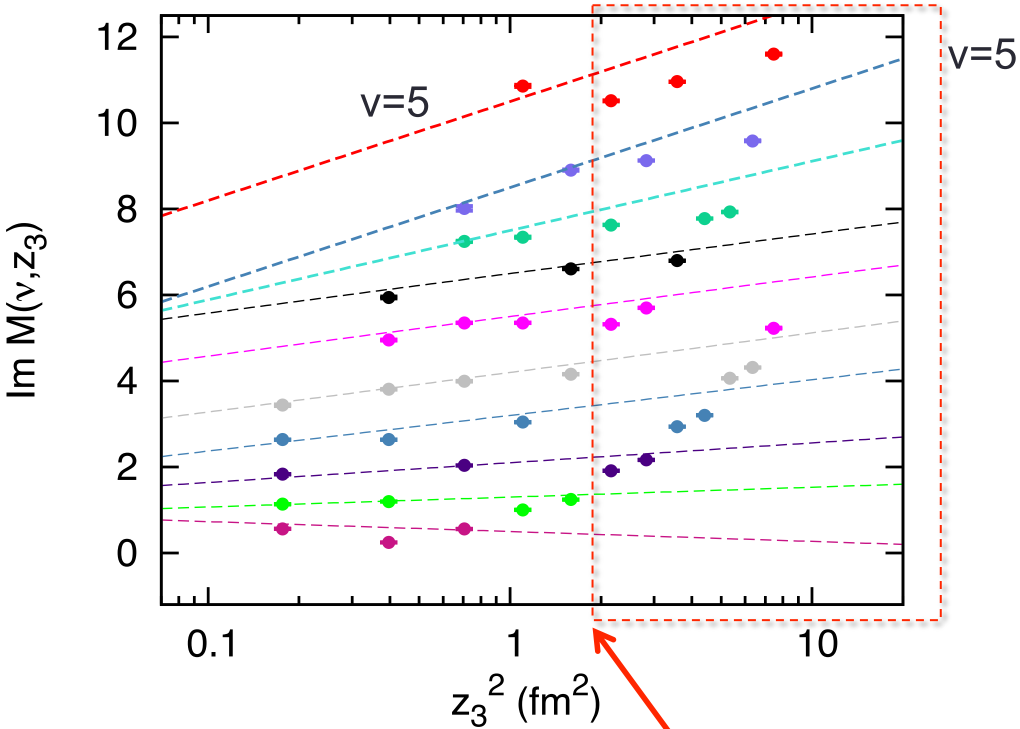
A plan for phenomenological studies…



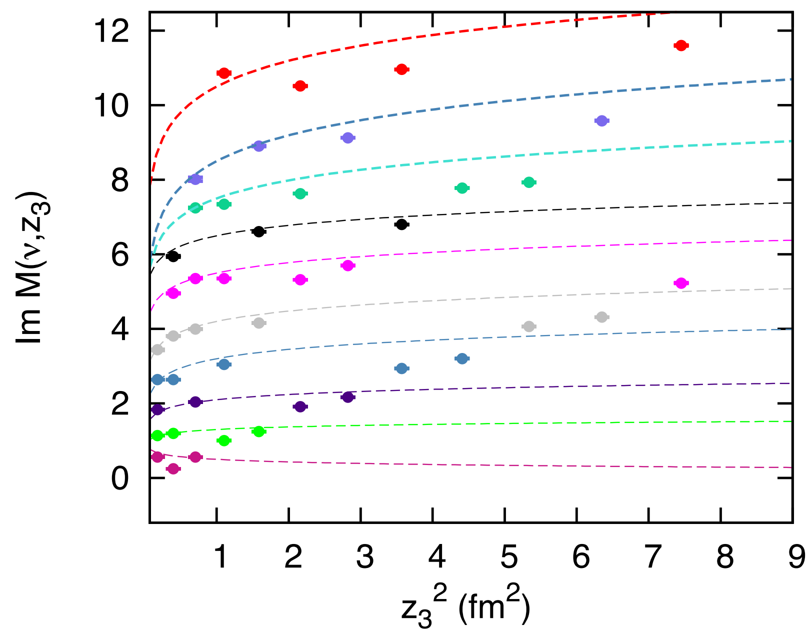

Haegler et al., LHPC 2007
Orginos et al. (2017)
Experimental data (CT10)
Im M(z⁻,z₃)
z⁻ (fm)
1
0.8
0.6
0.4
0.2
0
-0.2
1
2
3
4
5
6

Evolution in z_T

Non-perturbative region



transition



Conclusions and Outlook

- ✓ We presented results on the reconstruction of PDFs from their Mellin moments which are available in lattice QCD
- ✓ Lattice QCD moments evaluation complements pseudo-PDFs analysis
- ✓ Both methods need assumptions on small x behavior
- ✓ Lattice QCD moments evaluation can be extended to GPDs
- ✓ Error analysis is crucial

FIRST GLANCE AT GPDS FROM AB INITIO CALCULATIONS!