Mesons and Baryons in Basis Light Front Quantization

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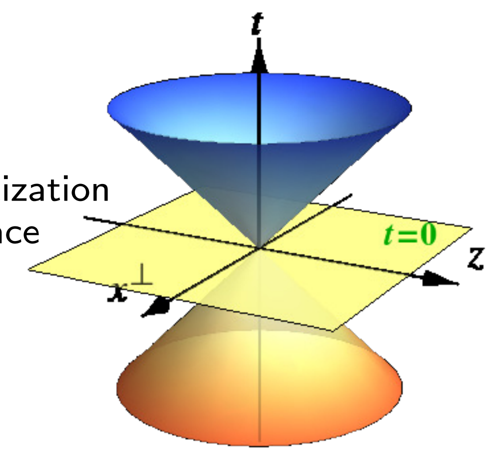
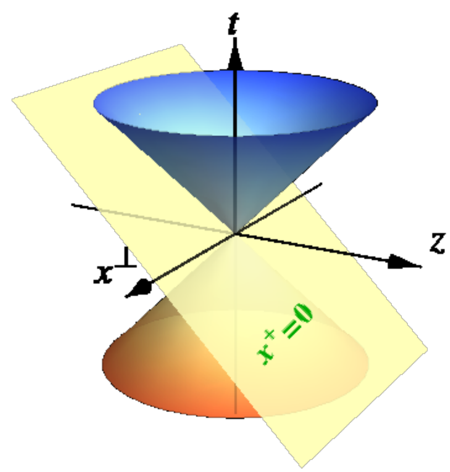
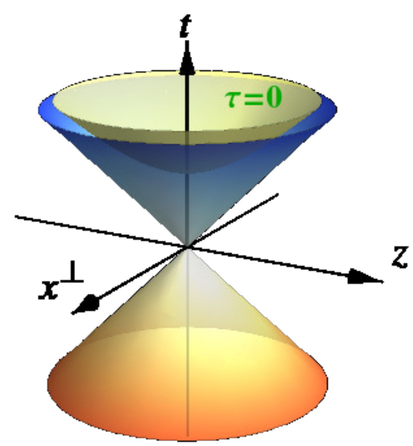
Dirac's forms of relativistic dynamics [Dirac, Rev. Mod. Phys. **21**, 392 1949]

Instant form is the well-known form of dynamics starting with $x^0 = t = 0$

$K^i = M^{0i}$, $J^i = {}^1\!/_2\, \varepsilon^{ijk} M^{jk}$, $\varepsilon^{ijk} = (+1,-1,0)$ for (cyclic, anti-cyclic, repeated) indeces

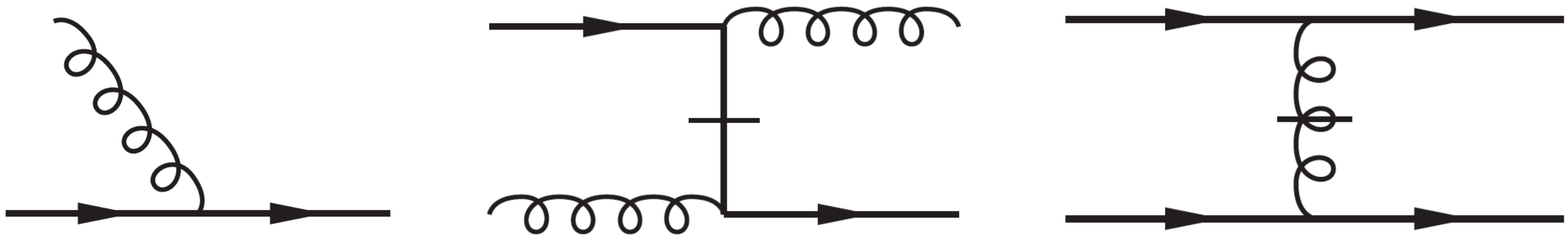
Front form defines relativistic dynamics on the light front (LF): $x^+ = x^0 + x^3 = t + z = 0$

$$P^\pm \triangleq P^0 \pm P^3,\ \vec{P}^\perp \triangleq (P^1, P^2),\ x^\pm \triangleq x^0 \pm x^3,\ \vec{x}^\perp \triangleq (x^1, x^2),\ E^i = M^{+i},$$
$$E^+ = M^{+-},\ F^i = M^{-i}$$

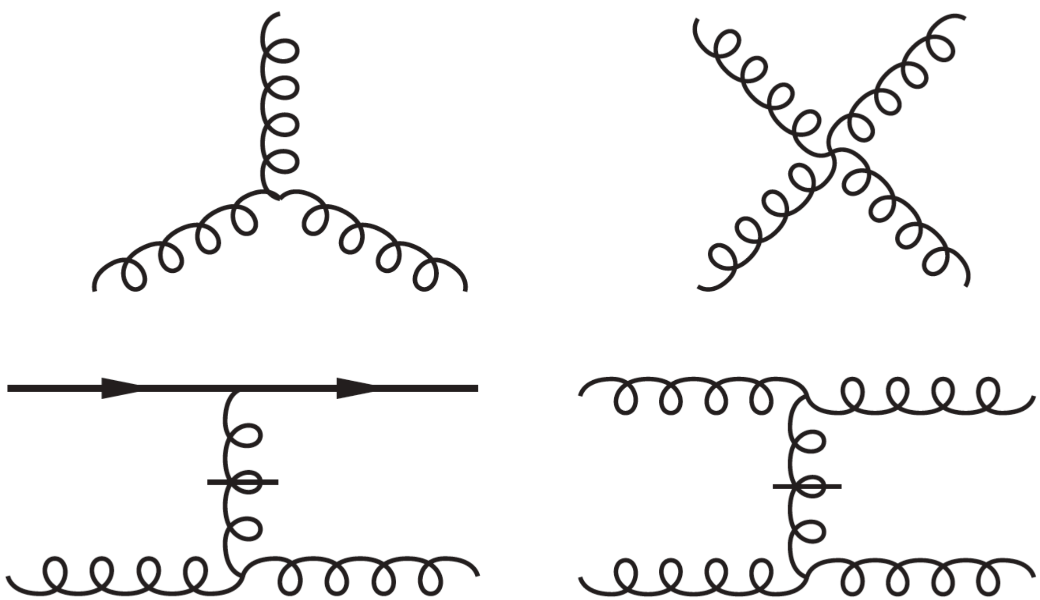
	instant form	front form	point form
time variable	$t = x^0$	$x^+ \triangleq x^0 + x^3$	$\tau \triangleq \sqrt{t^2 - \vec{x}^2 - a^2}$
quantization surface			
Hamiltonian	$H = P^0$	$P^- \triangleq P^0 - P^3$	P^μ
kinematical	$\vec{P}, \vec{J}$	$\vec{P}^\perp, P^+, \vec{E}^\perp, E^+, J^-$	$\vec{J}, \vec{K}$
dynamical	$\vec{K}, P^0$	$\vec{F}^\perp, P^-$	$\vec{P}, P^0$
dispersion relation	$p^0 = \sqrt{\vec{p}^2 + m^2}$	$p^- = (\vec{p}_\perp^2 + m^2)/p^+$	$p^\mu = m v^\mu \ (v^2 = 1)$

Adapted from talk by Yang Li

Light Front (LF) Hamiltonian Defined by its Elementary Vertices in LF Gauge



QED & QCD



QCD

Light-Front Wavefunctions (LFWFs)

$$|\psi_h(P,j,\lambda)\rangle = \sum_n \int [\mathrm{d}\mu_n]\, \psi_{n/h}(\{\vec{k}_{i\perp}, x_i, \lambda_i\}_n) |\{\vec{p}_{i\perp}, p_i^+, \lambda_i\}_n\rangle$$

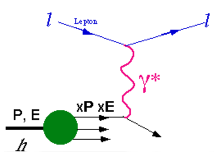
LFWFs are *frame-independent* (boost invariant) and depend only on the relative variables: $x_i \equiv p_i^+/P^+, \vec{k}_{i\perp} \equiv \vec{p}_{i\perp} - x_i \vec{P}_\perp$

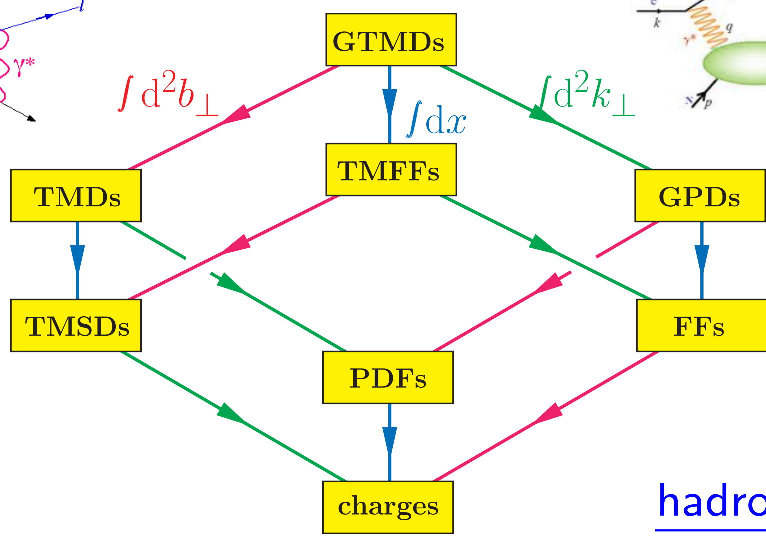
LFWFs provide intrinsic information of the structure of hadrons, and are indispensable for exclusive processes in DIS [Lepage '80]

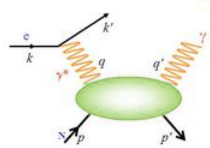
- Overlap of LFWFs: structure functions (e.g. PDFs), form factors, ...
- Integrating out LFWFs: light-cone distributions (e.g. DAs)

"Hadron Physics without LFWFs is like Biology without DNA!"

— Stanley J. Brodsky

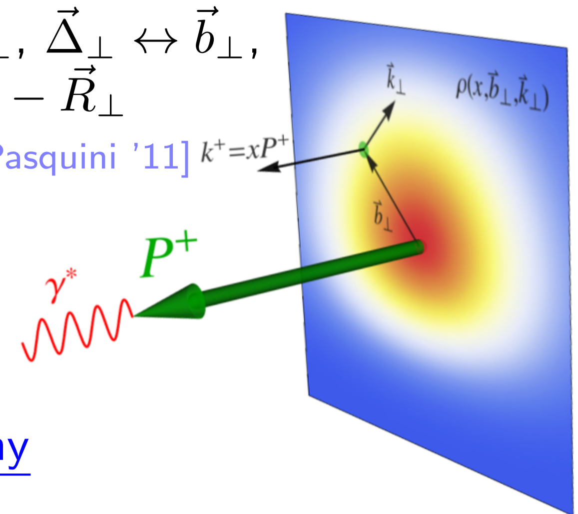






$\vec{k}_\perp \leftrightarrow \vec{r}_\perp,\ \vec{\Delta}_\perp \leftrightarrow \vec{b}_\perp,$
$\vec{b}_\perp = \vec{r}_\perp - \vec{R}_\perp$

[Lorcé & Pasquini '11]



hadron tomography

Discretized Light Cone Quantization

[H.C. Pauli & S.J. Brodsky, PRD32 (1985)]

$\Downarrow$

Basis Light Front Quantization

[J.P. Vary, et al., PRC81 (2010)]

$$\phi\left(\vec{k}_\perp, x\right) = \sum_\alpha \left[f_\alpha\left(\vec{k}_\perp, x\right) a_\alpha + f_\alpha^*\left(\vec{k}_\perp, x\right) a_\alpha^\dagger \right]$$

where $\{a_\alpha\}$ satisfy usual (anti-) commutation rules.

Furthermore, $f_\alpha(\vec{x})$ are arbitrary except for conditions:

Orthonormal: $$\int f_\alpha\left(\vec{k}_\perp, x\right) f_{\alpha'}^*\left(\vec{k}_\perp, x\right) \frac{d^2 k_\perp dx}{(2\pi)^3 2x(1-x)} = \delta_{\alpha\alpha'}$$

Complete: $$\sum_\alpha f_\alpha\left(\vec{k}_\perp, x\right) f_\alpha^*\left(\vec{k}'_\perp, x'\right) = 16\pi^3 \sqrt{x(1-x)}\,\delta^2\left(\vec{k}_\perp - \vec{k}'_\perp\right)\delta\left(x - x'\right)$$

For mesons we adopt (later extended to baryons): [Y. Li, et al., PLB758 (2016)]

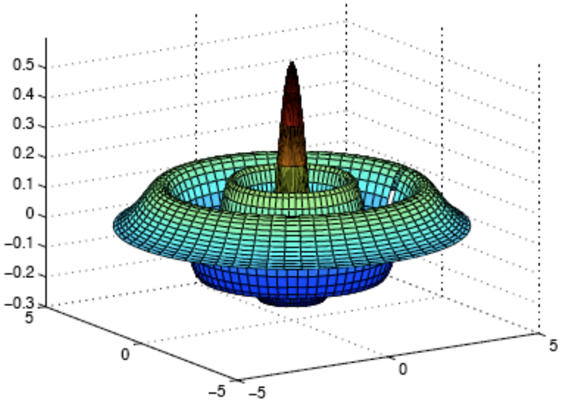
$$f_{\alpha=\{nml\}}\left(\vec{k}_\perp, x\right) = \phi_{nm}\left(\vec{k}_\perp / \sqrt{x(1-x)}\right) \chi_l(x)$$

ϕ_{nm} 2D-HO functions

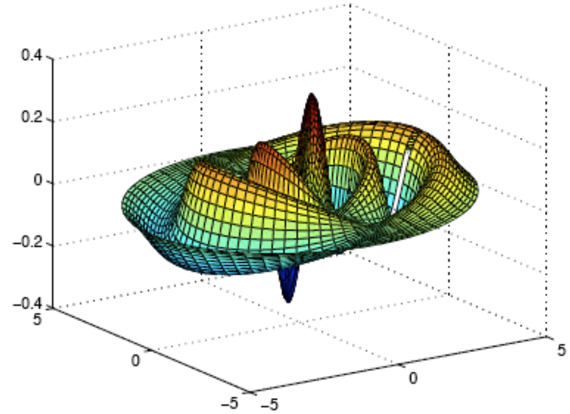
χ_l Jacobi polynomials times $x^a(1-x)^b$

Set of Transverse 2D HO Modes for n=4

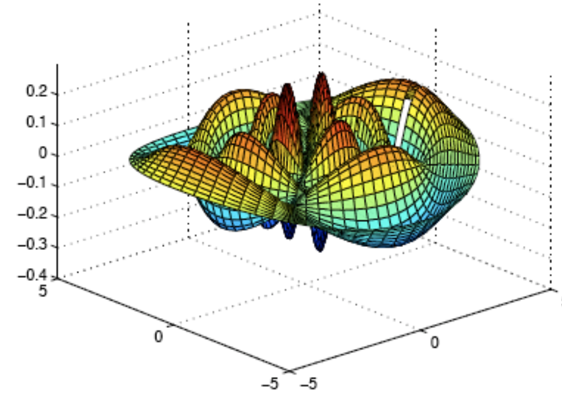
m=0



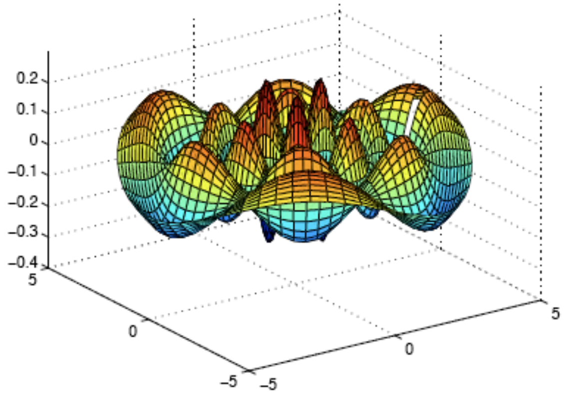
m=1



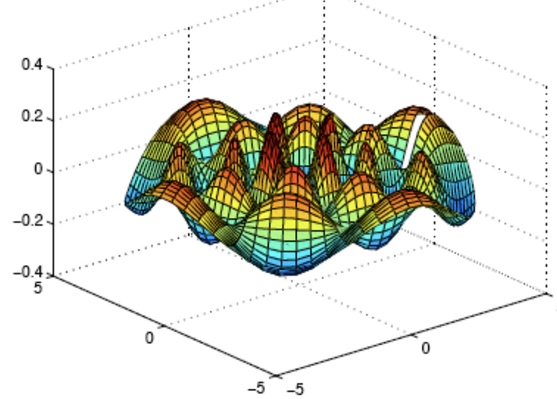
m=2



m=3



m=4

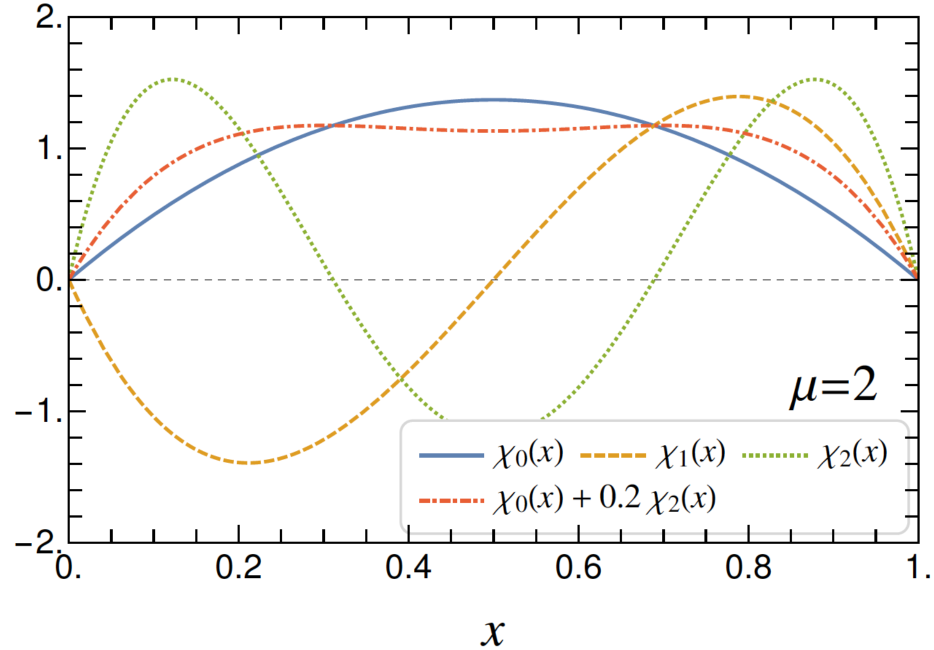


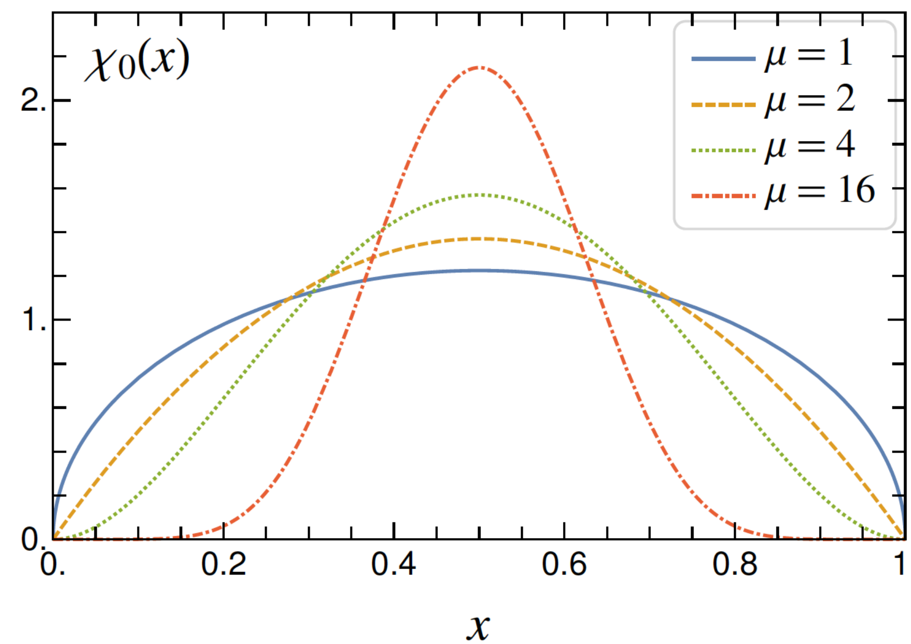
J.P. Vary, H. Honkanen, J. Li, P. Maris, S.J. Brodsky, A. Harindranath,
G.F. de Teramond, P. Sternberg, E.G. Ng and C. Yang, PRC 81, 035205 (2010)

Normalized longitudinal basis functions

$$\chi_l(x) = \frac{\sqrt{4\pi l!(2l+2\mu+1)\Gamma(l+2\mu+1)}}{\Gamma(l+\mu+1)} x^{\frac{\mu}{2}}(1-x)^{\frac{\mu}{2}} P_l^{(\mu,\mu)}(2x-1)$$

Jacobi polynomials





Applications of the BLFQ to Mesons and Baryons at LC2018

Guangyao Chen, Yang Li, Pieter Maris, Kirill Tuchin and James P. Vary, "Diffractive charmonium spectrum in high energy collisions in the basis light-front quantization approach," Phys. Letts. B **769**, 477 (2017); arXiv: 1610.04945. [Talk on Thursday morning]

Sofia Leitao, Yang Li, Pieter Maris, M.T. Pena, Alfred Stadler, James P. Vary and **Elmar P. Biernat**, "Comparison of two Minkowski-space approaches to heavy quarkonia", Eur. Phys. J. C 77, 696 (2017); arXiv: 1705:06178 [Talk on Friday]

Meijian Li, Yang Li, Pieter Maris and James P. Vary, "Radiative transitions between 0-+ and 1-- heavy quarkonia on the light front," arXiv:1803.11519 [Talk on Thursday in session 3B]

Shaoyang Jia, et al., "Basis Light-Front Quantization for the Light Mesons," in preparation [Talk on Thursday in session 3B]

Chandan Mondal, et al., "Baryons in a BLFQ approach," in preparation [Talk on Thursday in session 3B]

BLFQ
Symmetries & Constraints

All $J \geq J_z$ states in one calculation

Baryon number $\sum_i b_i = B$

Charge $\sum_i q_i = Q$

Angular momentum projection (M-scheme) $\sum_i (m_i + s_i) = J_z$

Longitudinal momentum (Bjorken sum rule) $\sum_i x_i = \sum_i \frac{k_i}{K} = 1$

Finite basis regulators

Transverse mode regulator (2D HO) $\sum_i (2n_i + |m_i| + 1) \leq N_{\max}$

Longitudinal mode regulator (Jacobi) $\sum_i l_i \leq L$

Global Color Singlets (QCD)

Light Front Gauge

Optional Fock-Space Truncation

$H \rightarrow H + \lambda H_{CM}$

Preserve transverse boost invariance

Light-Front Regularization and Renormalization Schemes

1. Regulators in BLFQ (N_{max}, L)
2. Additional Fock space truncations (if any)
3. Counterterms identified/tested*
4. Sector-dependent renormalization**
5. RGPEP (Glazek, Gomez-Rocha, and others, e.g. arXiv:1805.03436)
6. SRG & OLS in NCSM*** - adapted to BLFQ (future)

*D. Chakrabarti, A. Harindranath and J.P. Vary,
Phys. Rev. D **69**, 034502 (2004)
*P. Wiecki, Y. Li, **X. Zhao**, P. Maris and J.P. Vary,
Phys. Rev. D **91**, 105009 (2015)

**__V. A. Karmanov__, J.-F. Mathiot, and A. V. Smirnov,
Phys. Rev. D **77**, 085028 (2008); Phys. Rev. D **86**, 085006 (2012)
Y. Li, **V.A. Karmanov, P. Maris and J.P. Vary,
Phys. Letts. B. 748, **278** (2015); arXiv: 1504.05233
****X. Zhao**, invited talk at noon on Thursday at LC2018

***B.R. Barrett, P. Navratil and J.P. Vary,
Prog. Part. Nucl. Phys. **69**, 131 (2013)

Heavy Quarkonia [Y.Li,PLB758,2016; PRD96,2017]

- Effective Hamiltonian in the $q\bar{q}$ sector

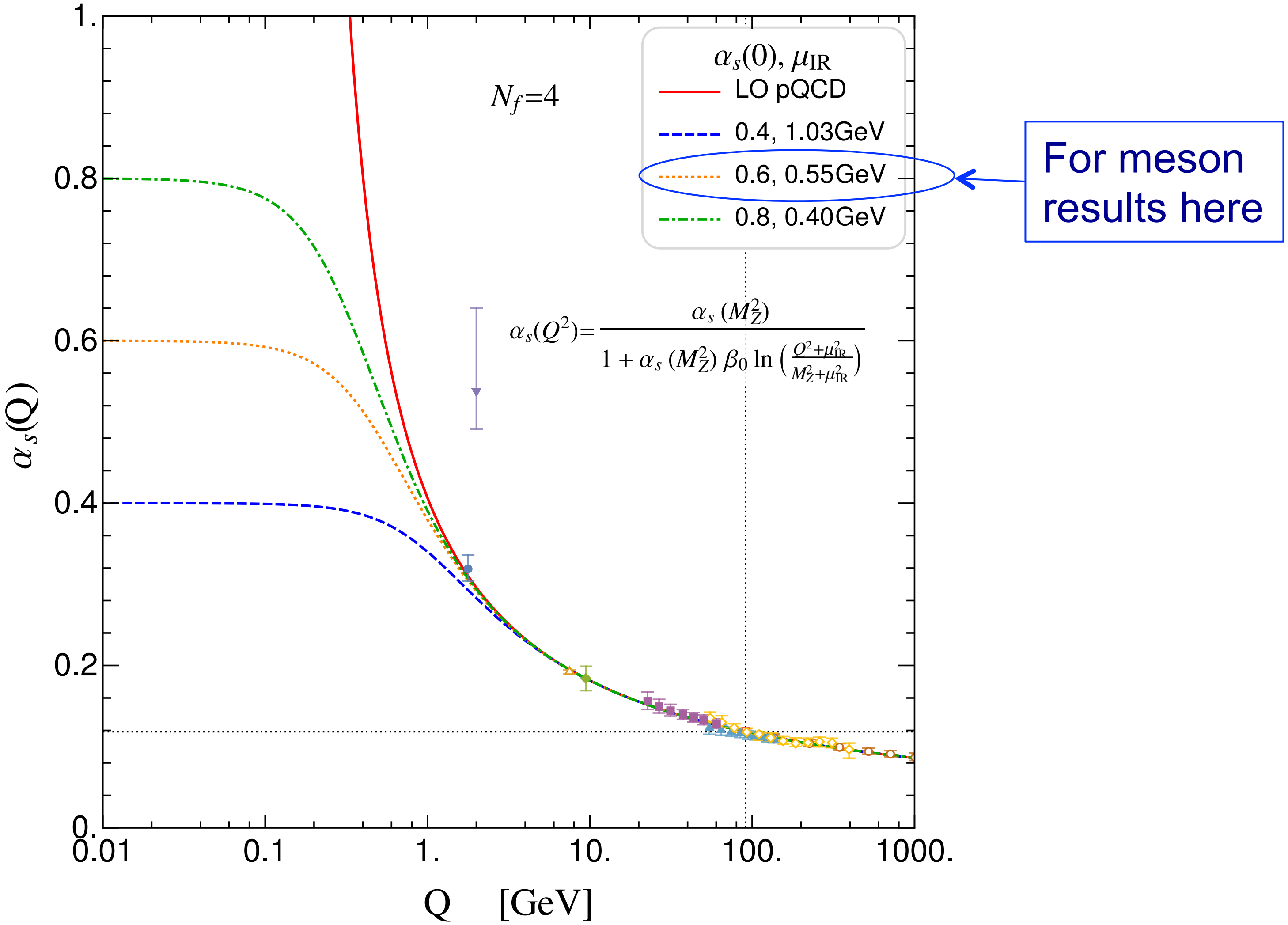
$$H_{\text{eff}} = \overbrace{\underbrace{\frac{\vec{k}_\perp^2 + m_q^2}{x} + \frac{\vec{k}_\perp^2 + m_{\bar{q}}^2}{1-x}}_{\text{LF kinetic energy}} + \underbrace{\kappa^4 x(1-x)\vec{r}_\perp^2 - \frac{\kappa^4}{(m_q+m_{\bar{q}})^2}\frac{\partial}{\partial x}\left(x(1-x)\frac{\partial}{\partial x}\right)}_{\text{confinement}}}^{H_0} + \underbrace{V_g}_{\text{one-gluon exchange}}$$

where $x = p_q^+/P^+$, $\vec{k}_\perp = \vec{p}_{q\perp} - x\vec{P}_\perp$, $\vec{r}_\perp = \vec{r}_{q\perp} - \vec{r}_{\bar{q}\perp}$.

  - Confinement
    transverse holographic confinement [S.J.Brodsky,PR584,2015]
    longitudinal confinement [Y.Li,PLB758,2016]
  - One-gluon exchange with running coupling
    $V_g = -\frac{4}{3}\frac{4\pi\alpha_s(Q^2)}{Q^2}\bar{u}_{\sigma'}\gamma^\mu u_\sigma \bar{v}_s \gamma_\mu v_{s'}$

- Basis representation
  - valence Fock sector: $|q\bar{q}\rangle$
  - basis functions: eigenfunctions of H_0 (LF kinetic energy+ confinement)

Adopt a running coupling with regulated IR behavior: improves UV properties of BLFQ applications

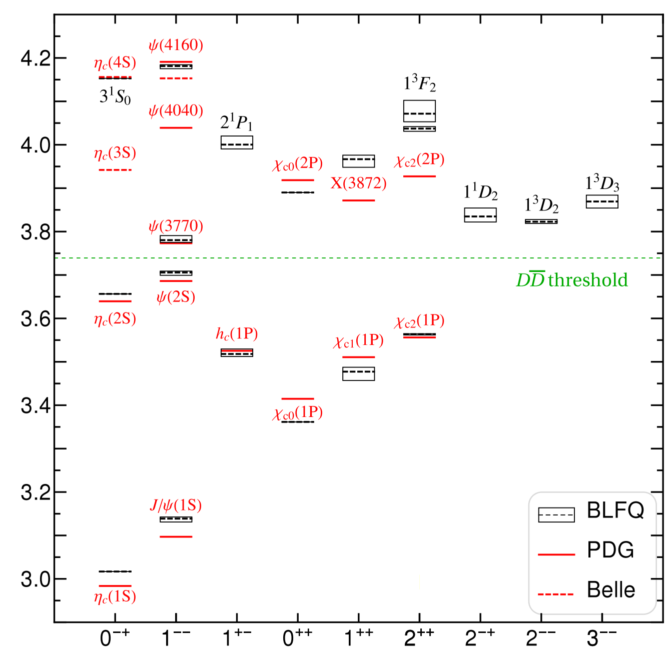


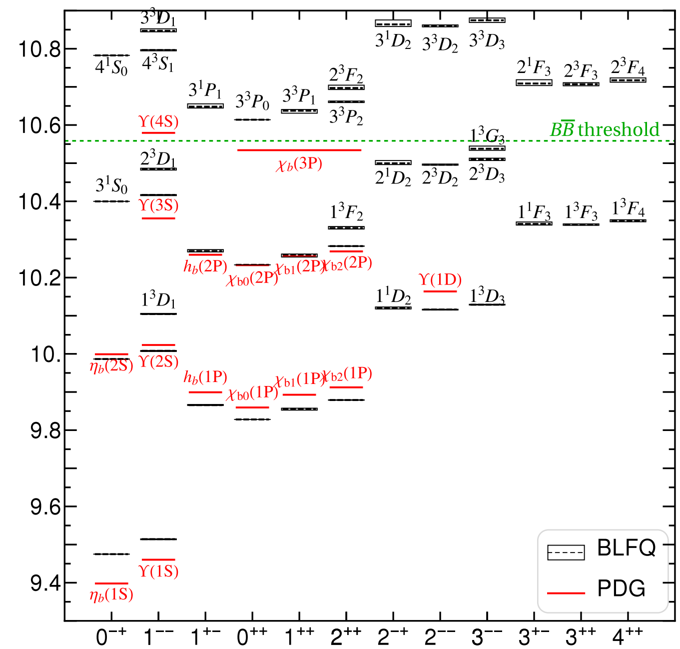
Q = average 4-momentum transfer between q and $\bar{q}$

Y. Li, P. Maris and J.P. Vary, PRD96, 016022 (2017)

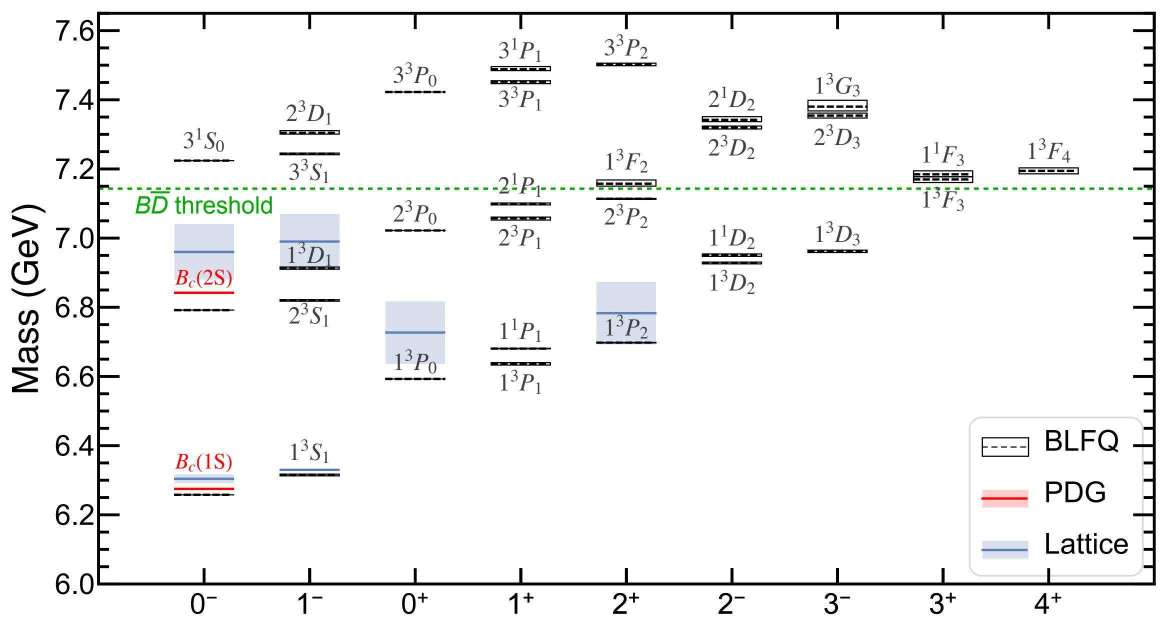
Spectroscopy

[Li, Maris & Vary, PRD ’17; Tang, Li, Maris & Vary, in preparation]





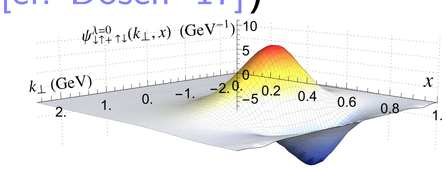
Heavy mesons: rms deviations 31 – 38 MeV

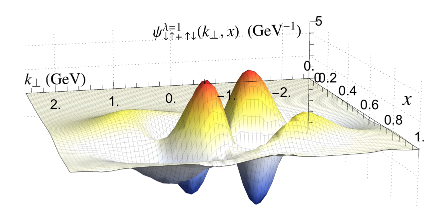


fitting parameters:

$m_c, m_b, \kappa_{qQ} = c\sqrt{M_{qQ}}$

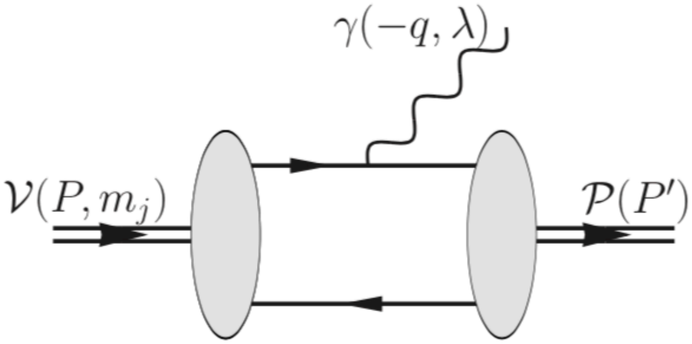
(HQET, [cf. Dosch ’17])





Radiative transitions between 0^{-+} and 1^{--} heavy quarkonia

Meijian Li, et al.; arXiv: 1803.11519
LC2018 talk on Thursday 3:00 pm

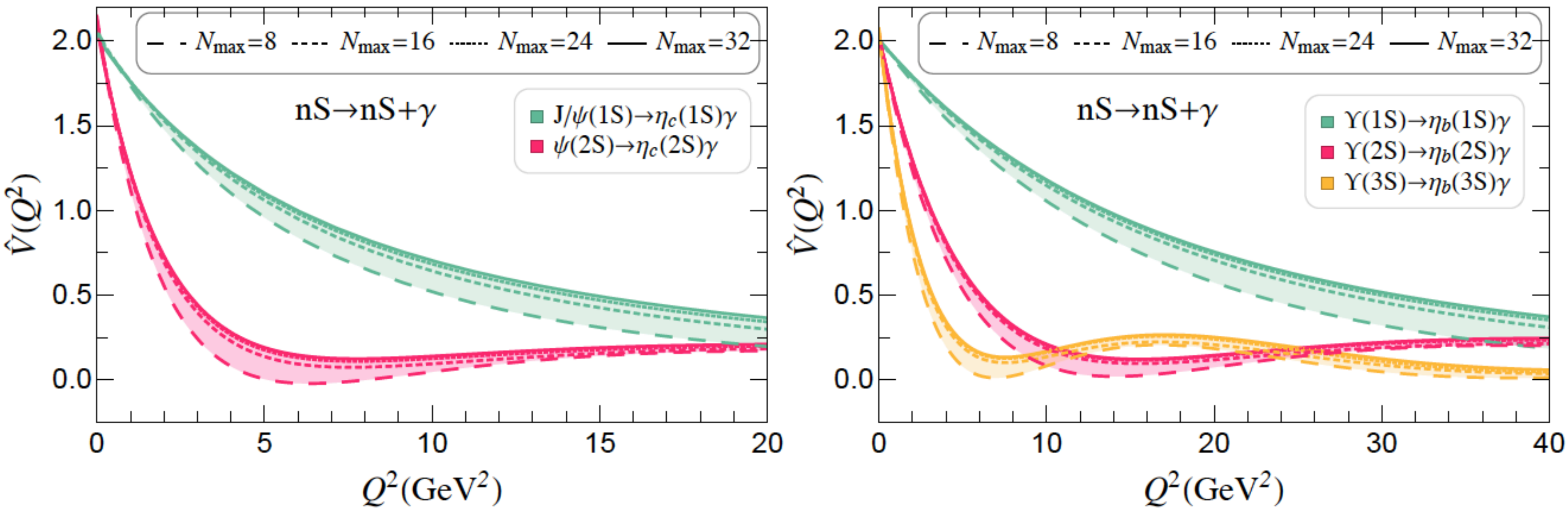


$\mathcal{V} \to \mathcal{P} + \gamma$ (or $\mathcal{P} \to \mathcal{V} + \gamma$):

$$\langle \mathcal{P}(P')| J^\mu(0) |\mathcal{V}(P,m_j)\rangle = \frac{2V(Q^2)}{m_\mathcal{P} + m_\mathcal{V}} \epsilon^{\mu\alpha\beta\sigma} P'_\alpha P_\beta e_\sigma(P,m_j)$$

momentum transfer: $q^\mu = P'^\mu - P^\mu,\ Q^2 \equiv -q^2$

Impulse approximation : $V(Q^2) = 2eQ_f \hat{V}(Q^2)$, Q_f is the quark charge.



Light-front wavefunctions: Yang Li, Pieter Maris, James P. Vary. Phys. Rev. D 96, 016022 (2017)

Radiative transitions between 0^{-+} and 1^{--} heavy quarkonia

Meijian Li, et al.; arXiv: 1803.11519
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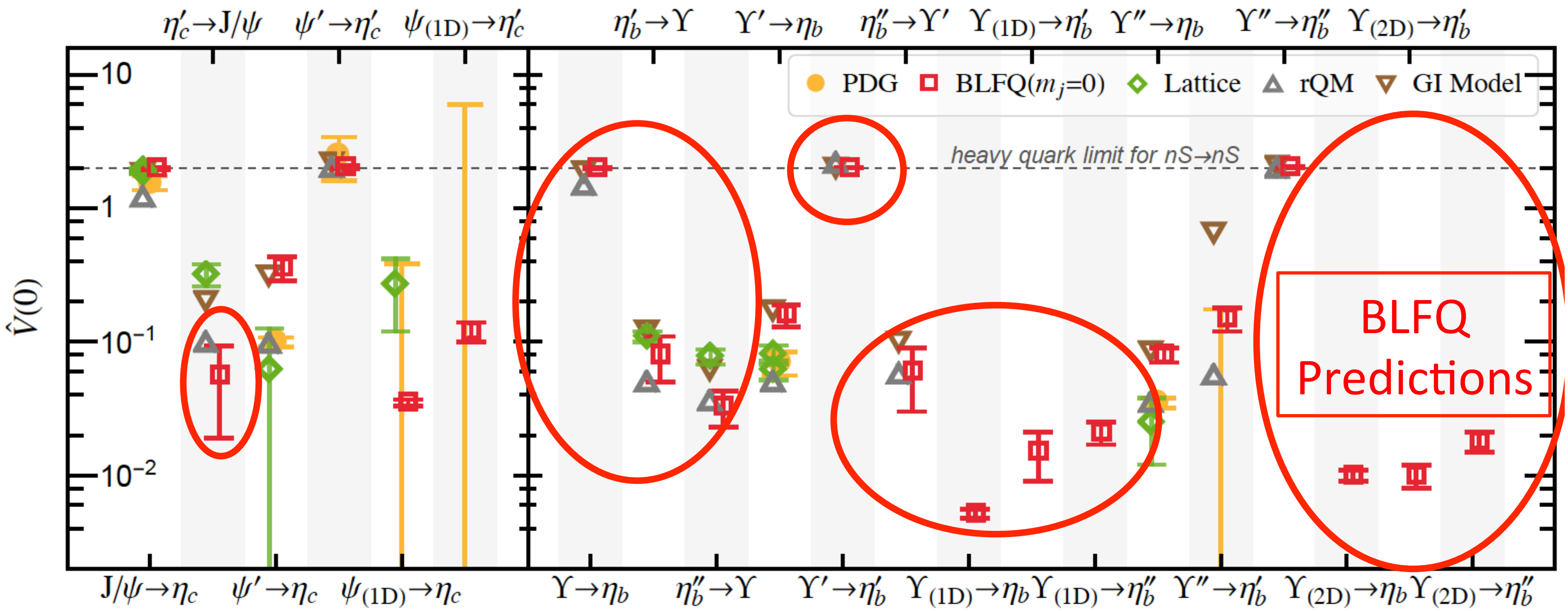
Decay width:

$$\Gamma(\mathcal{V} \to \mathcal{P} + \gamma) = \int d\Omega_q \frac{1}{32\pi^2} \frac{|\vec{q}|}{m_{\mathcal{V}}^2} \frac{1}{2J_{\mathcal{V}}+1} \sum_{m_j,\lambda} |\mathcal{M}_{m_j,\lambda}|^2 = \frac{(m_{\mathcal{V}}^2 - m_{\mathcal{P}}^2)^3}{(2m_{\mathcal{V}})^3 (m_{\mathcal{P}} + m_{\mathcal{V}})^2} \frac{|V(0)|^2}{(2J_{\mathcal{V}}+1)\pi}$$

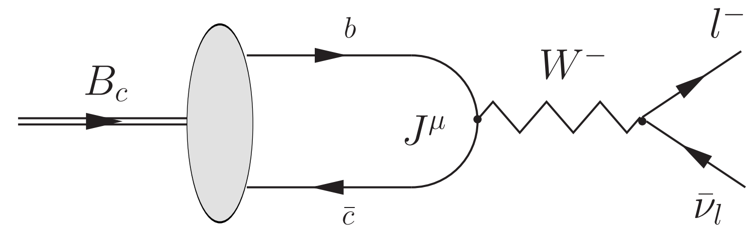


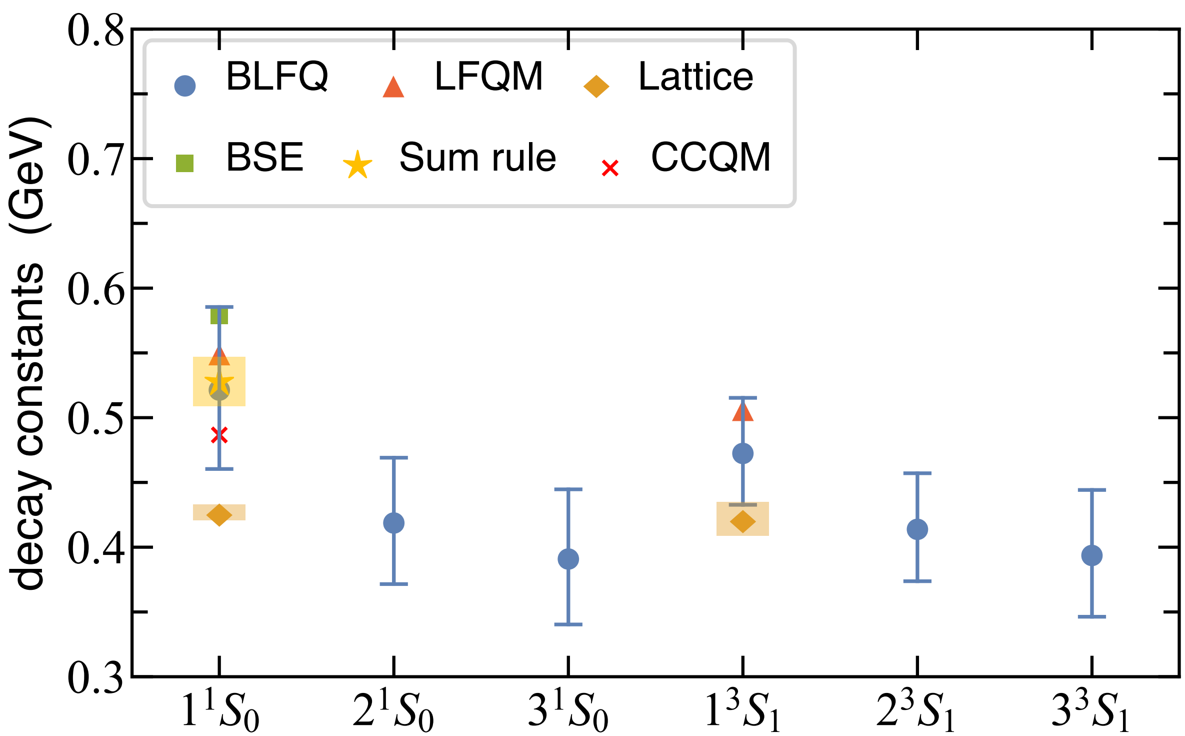
[PDG] C.Patrignani, et al., CPC40,2016.
[Lattice] J. J. Dudek, et al., PRD73,2006; PRD79, 2009. D. Bečirević, et al., JHEP01,2013; JHEP05,2015. C. Hughes, et al., PRD92,2015. R.Lewis, et al.,PRD86,2012.
[relativistic Quark Model (rQM)] D.Ebert, et al., PRD67, 2013.
[Godfrey-Isgur Model (GI Model)] T.Barnes, et al., PRD72,2005; S.Godfrey, et al., PRD92, 2015.

B_c Meson System in BLFQ

Decay Constants

$$\frac{f_{P,V}}{2\sqrt{2N_c}} = \int_0^1 \frac{dx}{2\sqrt{x(1-x)}} \int \frac{d^2k_\perp}{(2\pi)^3} \psi^{(\lambda=0)}_{\uparrow\downarrow\mp\downarrow\uparrow}(x, \vec{k}_\perp)$$





Decay constants for vector and pseudo-scalar Bc mesons. The results are obtained with $N_{\max}=L_{\max}=32$, corresponding to UV regulators:

$$\Lambda_{\rm UV} \triangleq \kappa\sqrt{N_{\max}} \approx m_b + m_{\bar{c}}$$

The length of BLFQ error bars:

$$\Delta f_{b\bar{c}} = 2|f_{b\bar{c}}(N_{\max}=32) - f_{b\bar{c}}(N_{\max}=24)|$$

which is taken to indicate the sensitivity to basis truncation.

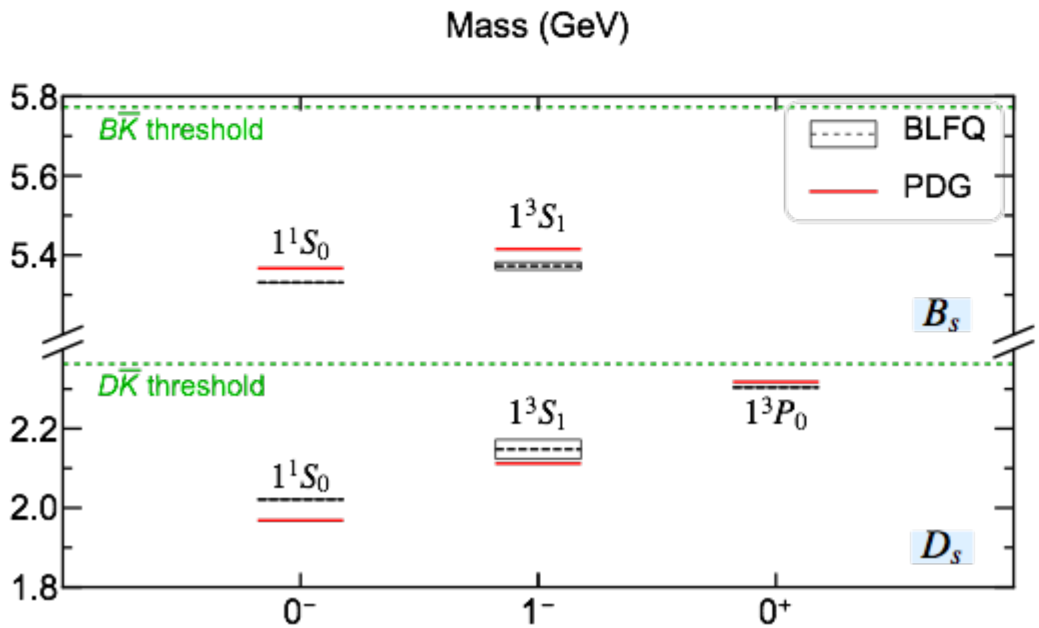
BLFQ: S. Tang, et al., in preparation
LFQM: H.-M. Cho and C.R. Ji, (2009)
Lattice: B. Colquhoun, et al., (2015)

Heavy-Light systems (preliminary)

S. Tang, et al., in preparation

	N_f	κ (GeV)	m_c (GeV)	m_b (GeV)	m_s (GeV)	rms (MeV)	$N_{\max} = L_{\max}$
$D_s(c\bar{s})$	4	0.783	1.603	–	0.597	38	32
$B_s(b\bar{s})$	4	1.054	–	4.902	0.597	39	32

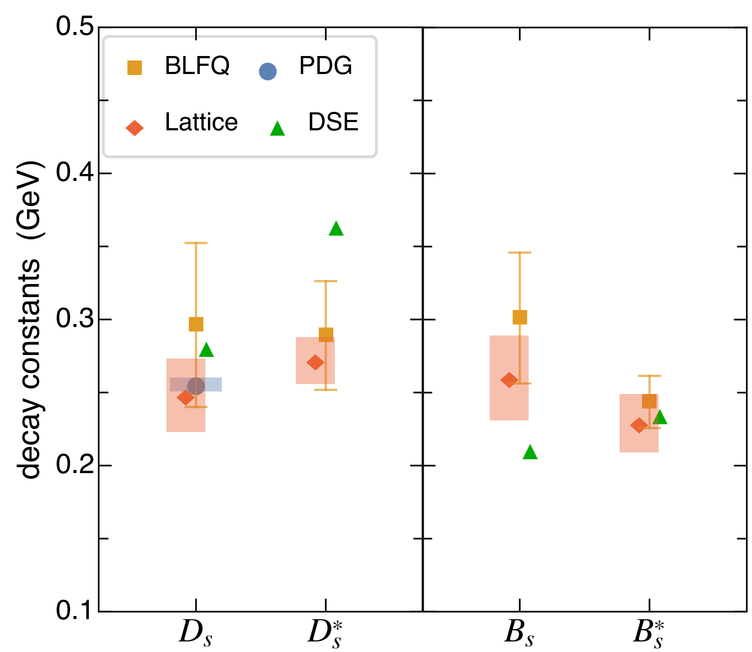
$$\kappa = \sqrt{(\kappa^2_{b\bar{b}/c\bar{c}} + \kappa^2_{s\bar{s}})/2}, \text{ with } \kappa_{s\bar{s}} = 0.54 \text{ GeV}$$



Mass spectrum of heavy-light systems. States under open flavor threshold confirmed by experiments.

Decay constants calculated with $N_{\max}$ = 8 for D_s, $N_{\max}$ = 32 for B_s, corresponding to UV cutoffs:

$$\Lambda_{\mathrm{UV}} \triangleq \kappa\sqrt{N_{\max}} \approx m_q + m_a$$



Hadron Tomography

[Adhikari, et al., Phys. Rev. C 93, 055202 (2016); & in preparation]

- Generalized parton distributions (GPDs) [Ji '97 & '98]

$$H(x,\zeta,t) = \tfrac{1}{2}\int \frac{\mathrm{d}z^-}{2\pi} e^{ixP^+z^-} \langle P'|\overline{\psi}(-\tfrac{1}{2}z)\gamma^+\psi(+\tfrac{1}{2}z)|P\rangle\Big|_{z^+=z^\perp=0}$$

$q = P' - P$, $\zeta = q^+/P^+$, $t = q^2$.

  - DVCS, SIDIS, ..., spin physics

- Impact parameter dependent GPDs: [Burkardt '01]

$$q(x,\vec{b}_\perp) = \int \frac{\mathrm{d}^2\Delta_\perp}{(2\pi)^2} e^{i\vec{\Delta}_\perp\cdot\vec{b}_\perp} H(x,\zeta=0,t=-\Delta_\perp^2).$$

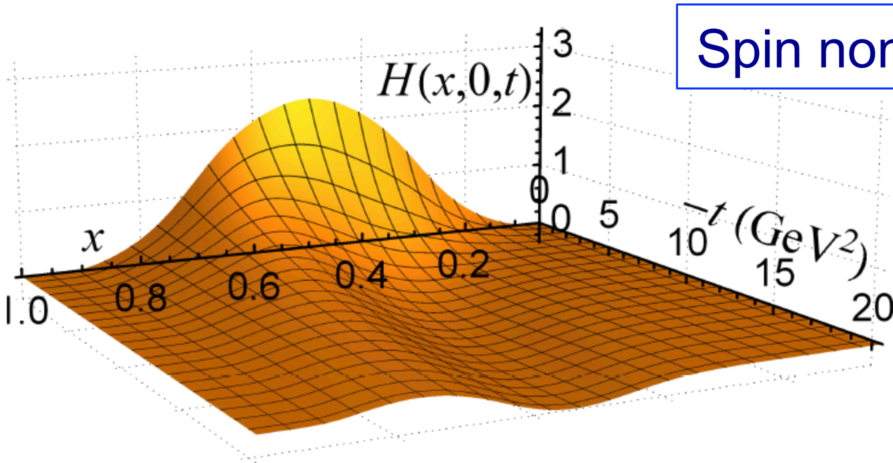
  - partonic interpretation: $\int d^2b_\perp \int_0^1 dx\, q\big(x,\vec{b}_\perp\big) = 1.$

Spin non-flip GPDs $H(x,0,t)$

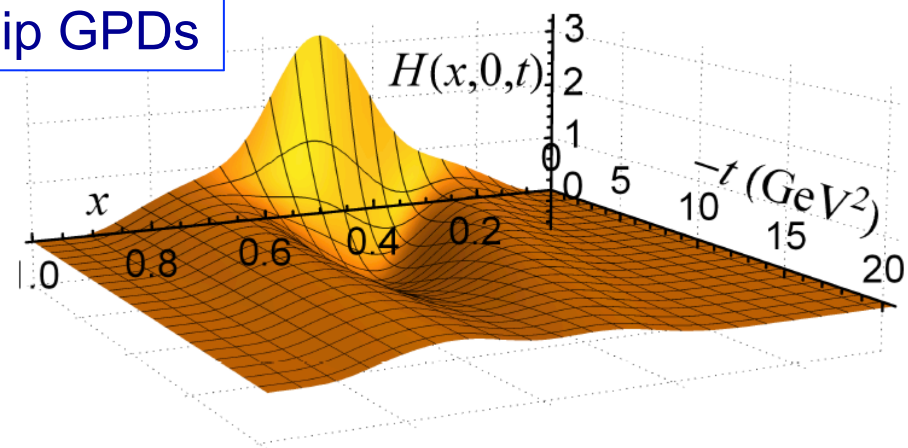
$$H(x,\xi=0,t=-\vec{\Delta}_\perp^2) = \sum_{\lambda_q,\lambda_{\bar{q}}} \int d^2\vec{k}_\perp\, \psi^*(\vec{k}'_\perp,x,\lambda_q,\lambda_{\bar{q}})\psi(\vec{k}_\perp,x,\lambda_q,\lambda_{\bar{q}}).$$

Charmonium Tomography

Spin non-flip GPDs

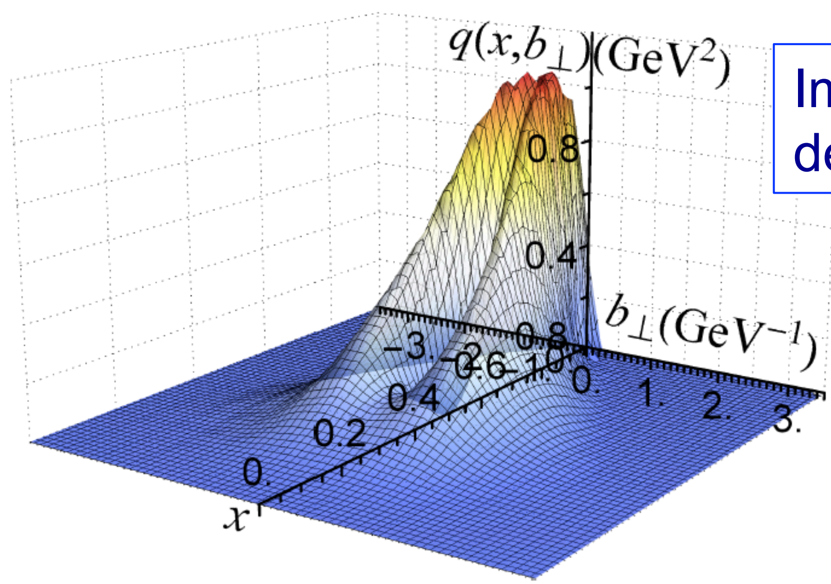


(a) $\chi_{c0} : 1^3P_0(0^{++})$

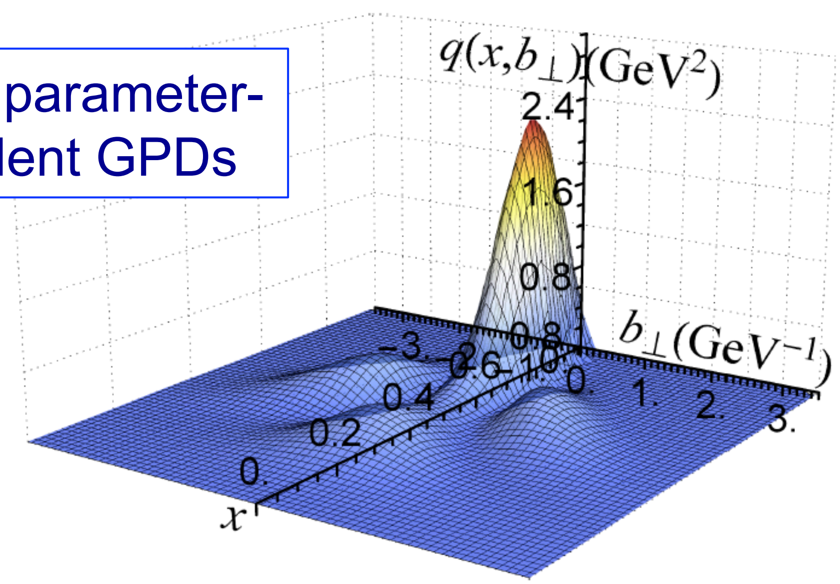


(c) $\eta_c' : 2^1S_0(0^{-+})$

Impact parameter-dependent GPDs



(b) $\chi_{c0} : 1^3P_0(0^{++})$

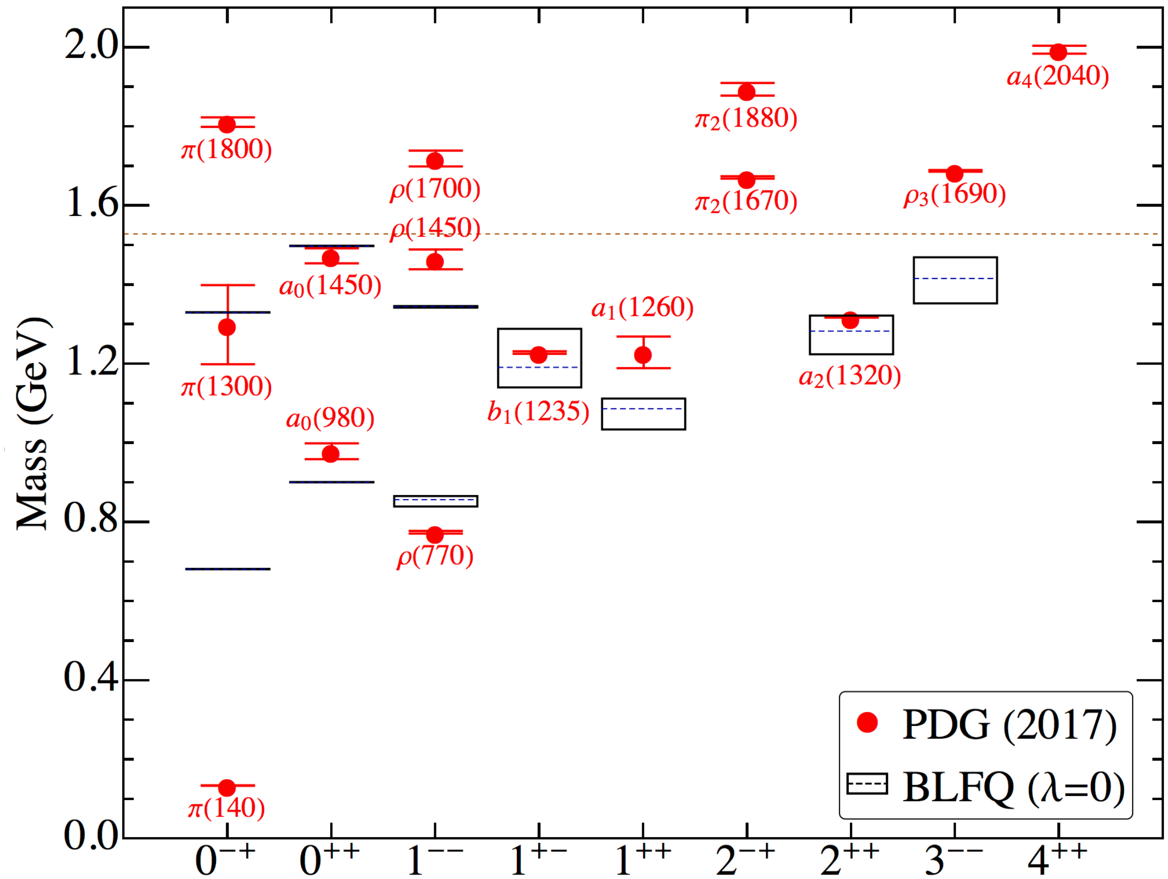


(d) $\eta_c' : 2^1S_0(0^{-+})$

L. Adhikari, et al., in preparation

Moving to light mesons – role of chiral symmetry

Spectroscopy: BLFQ with one-gluon dynamics



Confining strength and quark mass obtained by fitting the lowest PDG masses excluding pion

BLFQ mass uncertainty due to slight violation of rotational symmetry

r.m.s. deviation (9 states): 202 MeV

Model parameters:

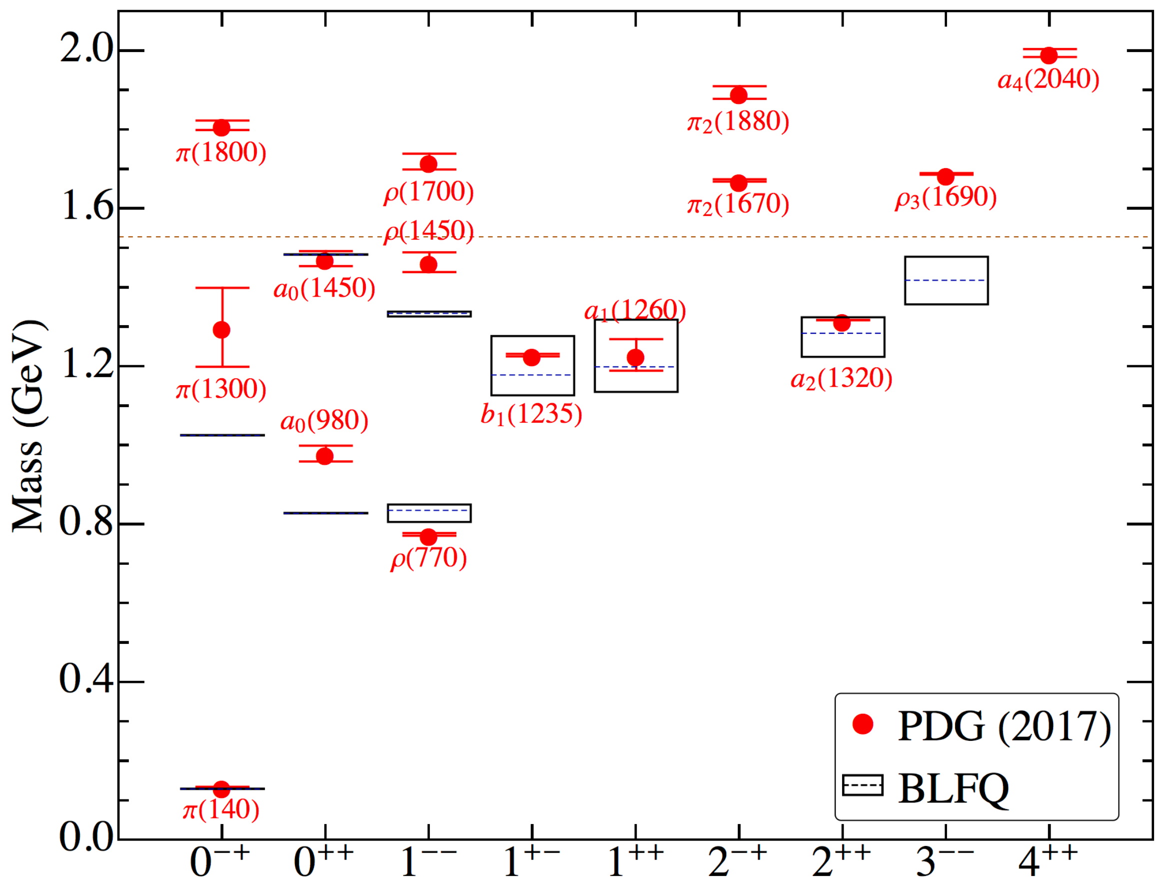
$$\kappa = 0.54 \text{ GeV}$$

$$m_q = m_{\bar{q}} = 330 \text{ MeV}$$

Wenyang Qian, et al., In preparation

Wenyang Qian, et al., In preparation

Spectroscopy: BLFQ with one-gluon & H_{γ_5} interactions



$$H_{\gamma_5} = \lambda \bar{\psi}\gamma^5\psi\bar{\psi}\gamma^5\psi$$

Coupling strength λ obtained by fitting the pion mass

r.m.s. deviation (9 states): 119 MeV

Model parameters:

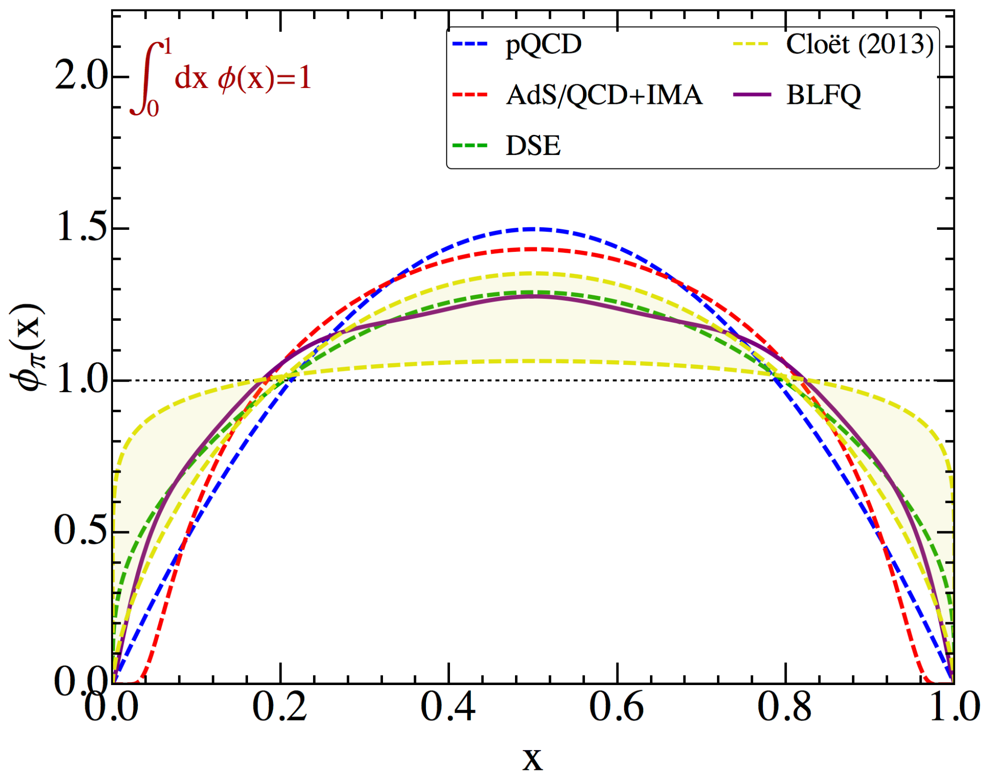
$\lambda = 4.91 \text{ GeV}^{-2}$

$\kappa = 0.54 \text{ GeV}$

$m_q = m_{\bar{q}} = 330 \text{ MeV}$

Wenyang Qian, et al., In preparation

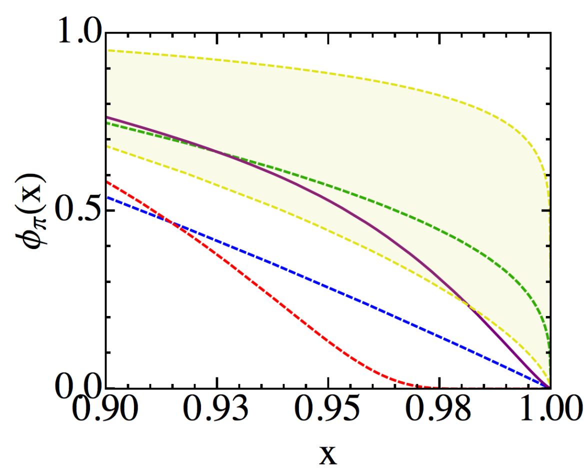
Parton distribution amplitudes for the pion



DSE: Lei Chang et al, PRL110, 132001(2013)
Cloët(2013): Cloët et al, PRL111, 092001(2013)
AdS/QCD + IMA: Brodsky et al, PhysRep548, 1(2015)

Exclusive processes at large momentum transfer

$$\phi_{\mathcal{P},\mathcal{V}}(x,\mu) \sim \frac{1}{f_{\mathcal{P},\mathcal{V}}\sqrt{x(1-x)}} \times \int \frac{d^2\boldsymbol{k}_\perp}{(2\pi)^3}\psi^{(m_j=0)}_{\uparrow\downarrow\pm\downarrow\uparrow}(x,\boldsymbol{k}_\perp)$$



BLFQ for the light mesons with the Nambu-Jona-Lasinio model

- Valence Fock sector for the mesons $\pi^+, \rho^+, K^+, K^{*+}$.
- Same diagonal Hamiltonian as in the heavy quarkonium calculations. Same set of basis functions.
- Contact interactions from the NJL model:

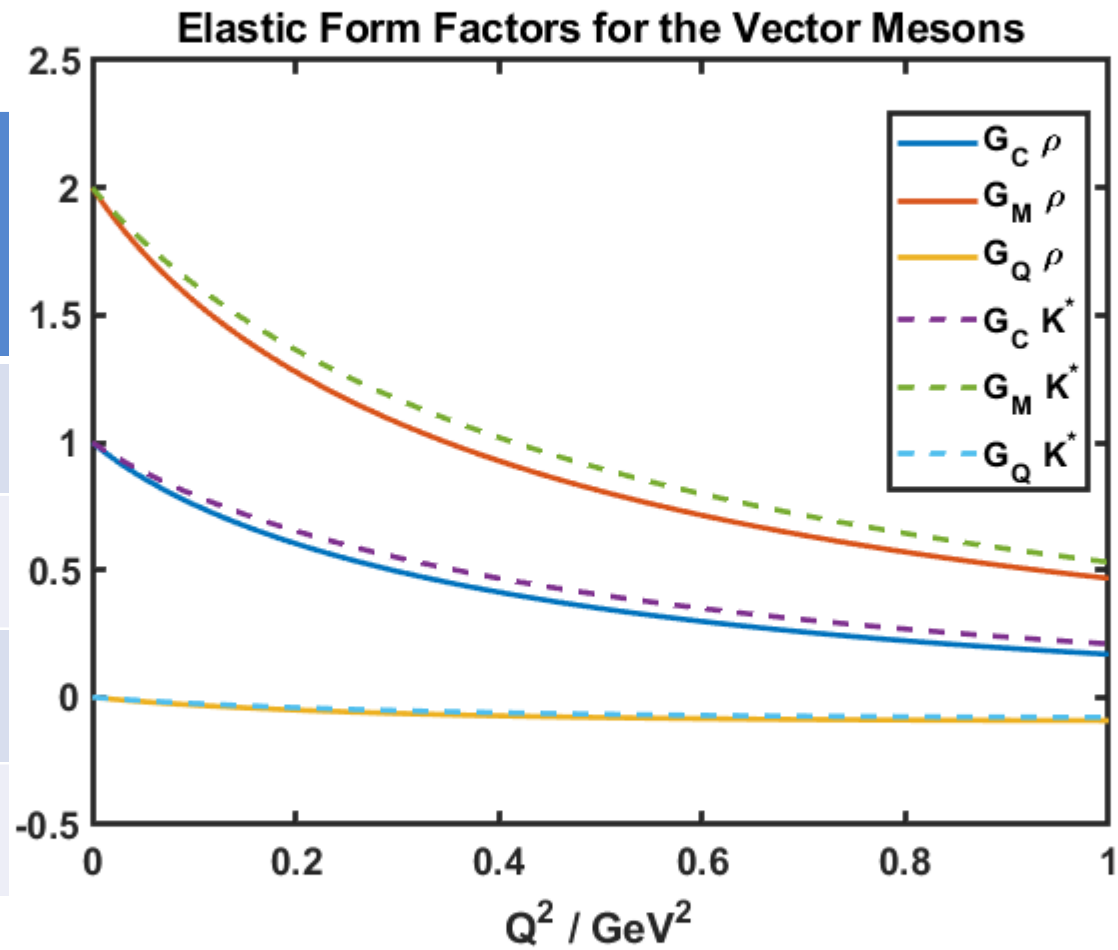
$$H_{NJL}^{\pi\rho} = -\frac{1}{2} G_{P\pi} P^+ [(\bar{\psi}\psi)^2 + \sum(\bar{\psi} i\gamma_5 \tau_a \psi)^2]$$

$$H_{NJL}^{K} = -G_{PK} P^+ \sum[(\bar{\psi}\lambda_a\psi)^2 + (\bar{\psi} i\gamma_5 \lambda_a \psi)^2].$$

Shaoyang Jia, parallel session 3B, Thursday 14:40 to 15:00

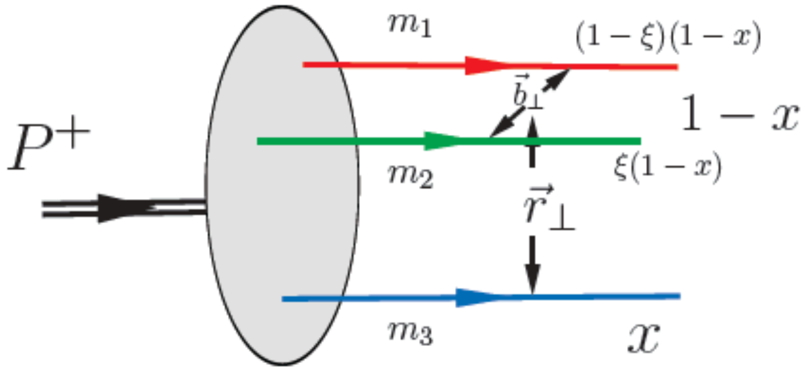
BLFQ NJL Model Parameters					
κ	369.5 MeV	N_{max}	8	$G_{P\pi}$	6.340 GeV^{-2}
m_l	222.2 MeV	M_{max}	2	G_{PK}	7.326 GeV^{-2}
m_s	398.8 MeV	L_{max}	8		

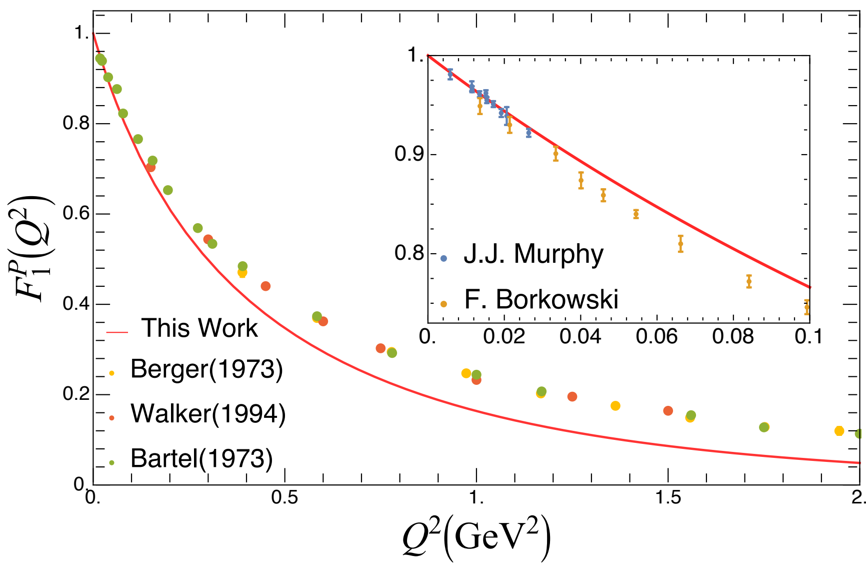
	Mass (MeV)	Decay Constant (MeV)	Charge Radius (fm²)
π^+	139.57	148.08	0.263
ρ^+	775.26	121.02	0.773
K^+	493.68	177.77	0.250
K^{*+}	891.76	106.30	0.608

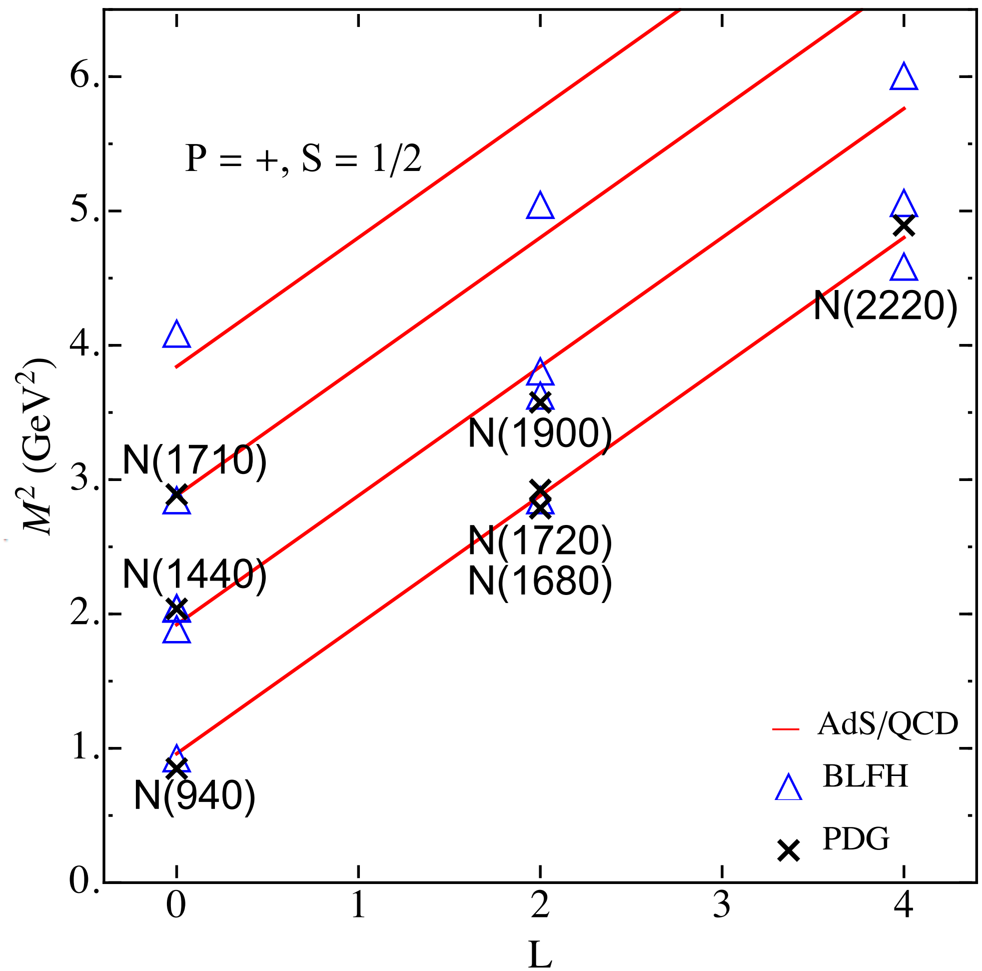


Baryons

Anji Yu, et al., in preparation







$$M^2_{\mathbf{n}_1,\mathbf{m}_1,\mathbf{n}_2,\mathbf{m}_2,L,l} = (\mathrm{m}_3 + \mathrm{M}_L)^2 + 2\kappa^2(2\mathbf{n}_1 + |\mathbf{m}_1| + 2\mathbf{n}_2 + |\mathbf{m}_2| + 2)$$

$$+\frac{\mathrm{M}_L + \mathrm{m}_3}{\mathrm{m}_1 + \mathrm{m}_2 + \mathrm{m}_3}\kappa^2(2l+1) + \frac{\kappa^4}{(\mathrm{m}_1 + \mathrm{m}_2 + \mathrm{m}_3)^2} l(l+1) + \mathrm{const.},$$

$$\mathrm{M}_L^2 = (\mathrm{m}_1 + \mathrm{m}_2)^2 + \frac{\mathrm{m}_1 + \mathrm{m}_2}{\mathrm{m}_1 + \mathrm{m}_2 + \mathrm{m}_3}\kappa^2(2L+1) + \frac{\kappa^4}{(\mathrm{m}_1 + \mathrm{m}_2 + \mathrm{m}_3)^2}L(L+1)$$

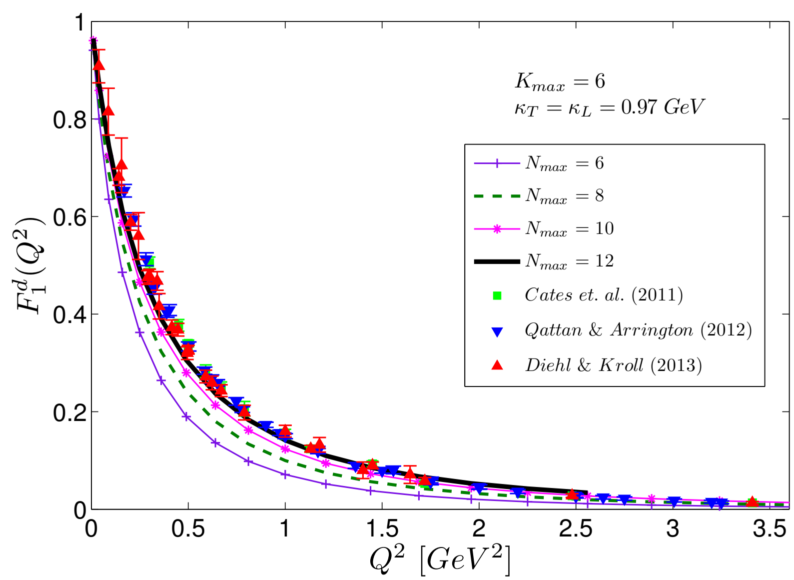
Flavor form factor & GPD in BLFQ Chandan Mondal, talk on Thursday

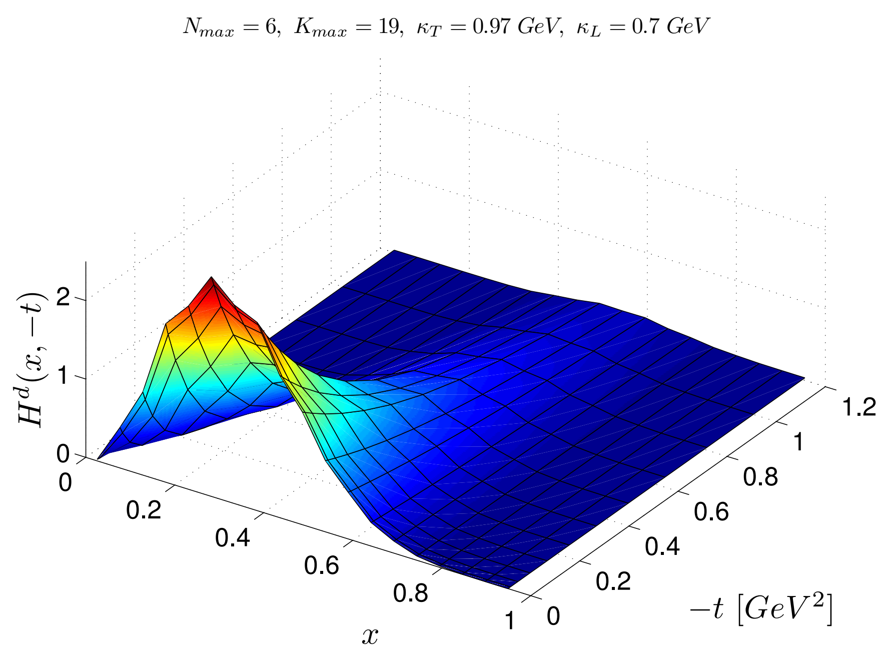
☐ Dirac form factor (F_1) in light-front [with $q^+ = 0$] for the proton

$$F_1(-q^2) = \langle P+q;\Lambda|\frac{J^+(0)}{2P^+}|P;\Lambda\rangle; \qquad F_1^q(-q^2) = \int dx\; H^q(x,-q^2).$$

✓ In terms of overlap of light-front WFs:, $H^q(x,-q^2)$:

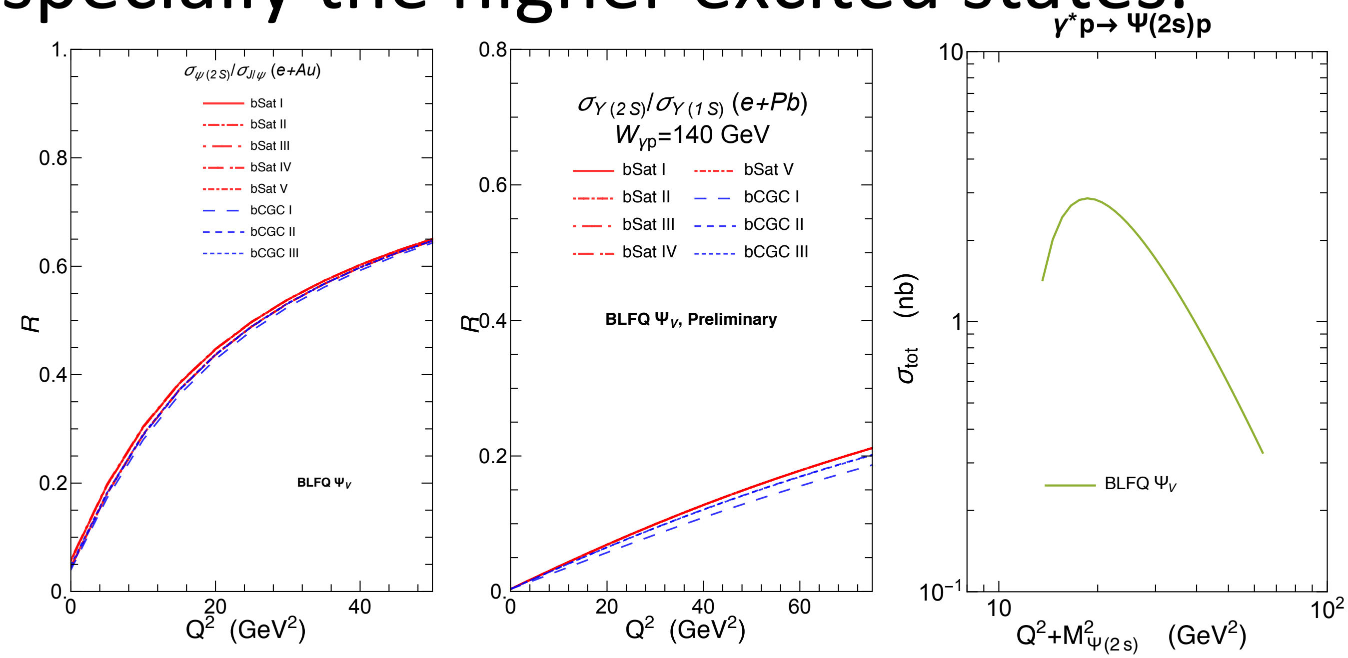
$$\sum_{\lambda_i}\int \prod_{i=1}^{3} dx_i\, d^2\mathbf{k}_{\perp i}\delta(1-\sum x_j)\delta(x-x_1)\delta^2(\sum \mathbf{k}_{\perp j})\Psi^{\Lambda *}_{\lambda_i}(x_i,\mathbf{k}'_{\perp i})\Psi^{\Lambda}_{\lambda_i}(x_i,\mathbf{k}_{\perp i})$$





Probing small-x gluons in high-energy nuclear collisions through vector-meson production

- Electron Ion Collider--high luminosity, wide kinematic range.
- Enable precision measurement of VM LFWF, especially the higher excited states.

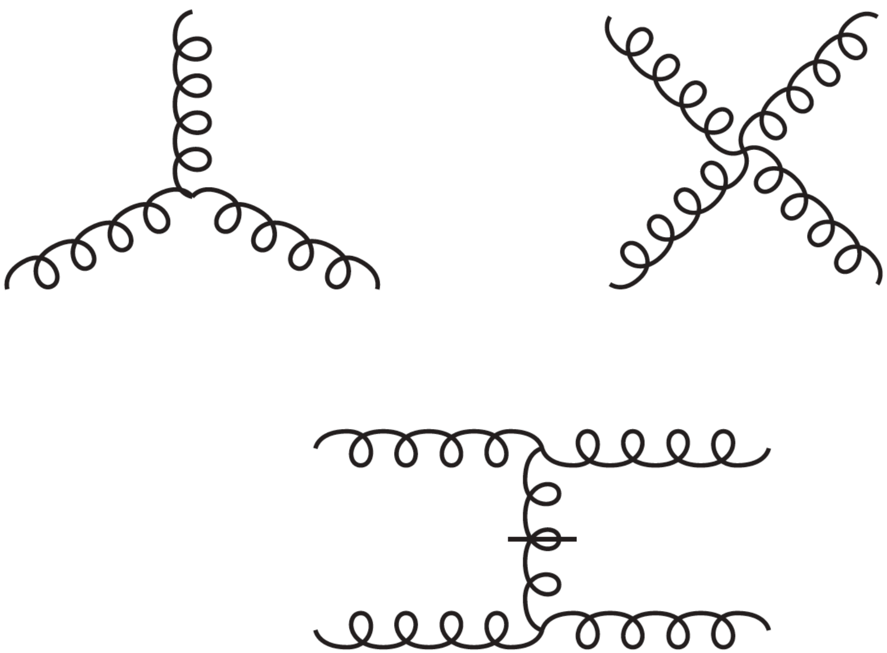


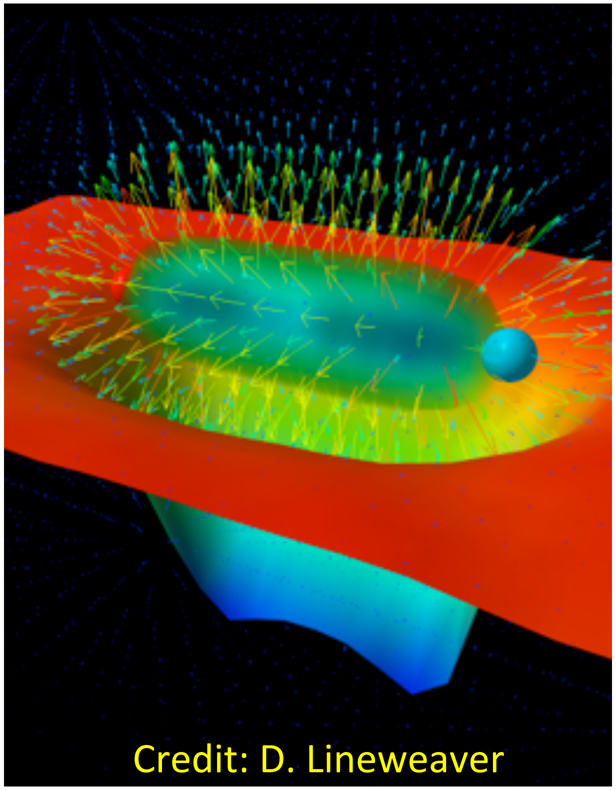
Chen, Li, Maris, Tuchin and Vary, PLB 769, 477, 2017

Guangyao Chen's talk on Thursday

Full Basis Light-Front Quantization (FBLFQ)
UV and IR regulators but no additional Fock space truncation

Pure glue sector QCD – Glueballs?



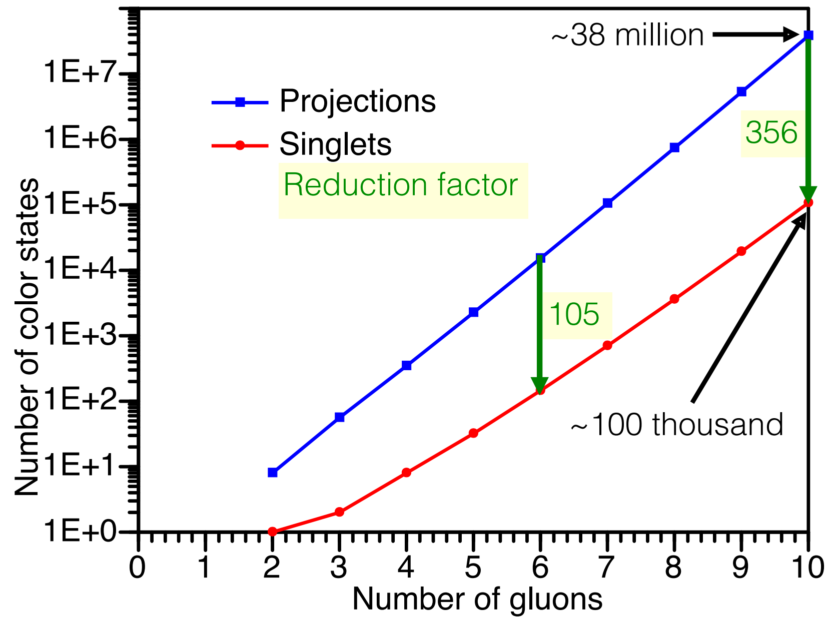


Retain only the triple gluon vertex in initial test application

PRELIMINARY RESULTS

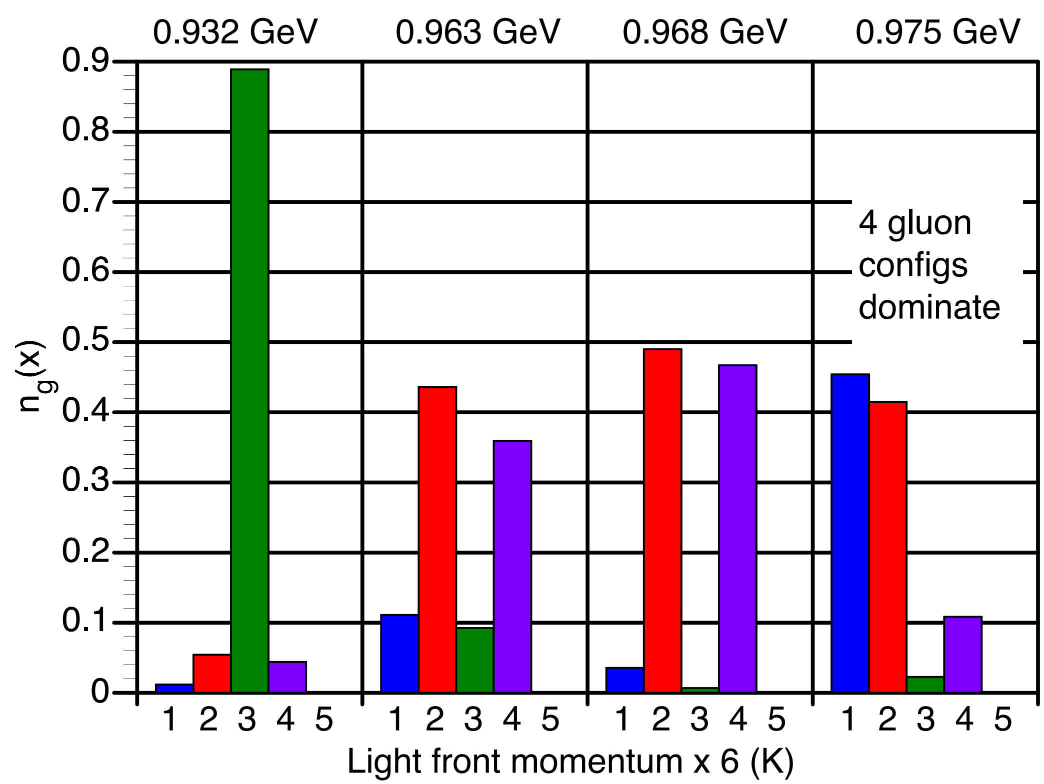
Color basis space dimensions of each multi-gluon space-spin configuration

Figure extended from Vary, et al., 2010



Distribution functions for
4 lowest mass eigenstates

$N_{max} = K = 6$
$b=0.5$ GeV
$g_S=0.5$
$m_g = 0.25$ GeV



Hamiltonian matrix dimension ~ 2000;
Calculation runs in 3 mins on laptop.

Next step: add 2 other vertices

Summary and Outlook

Hamiltonian Light-Front QCD is producing descriptions and predictions with modeled confinement

- Bound states and transitions of the hadrons are described
- Time-dependent scattering applications are underway
- Future – expand the Fock spaces (e.g. F-BLFQ)
- Future – renormalization, counterterms and regulators
- Future – additional methods for emergent phenomena
- Efficient utilization of supercomputing resources

Thank you for your attention