Overview of hadron structure from Lattice QCD

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Material provided by



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OUTLINE OF TALK





Motivation

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LQCD meets Nature



- ★ 4-D discretization, ab initio formulation of QCD
- ★ Make contact with well-known experimental data
- Provide input for quantities not easily accessible in experiments
- Guide New Physics searches

Rich experimental activities in major facilities

JLAB (12GeV Upgrade)



Argonne (ATLAS)



RHIC (BNL)



JPARC



ALICE Investigation of baryon & meson structure

- * Origin of mass and spin
- New physics searches
- proton radius puzzle
- the list is long... *



BES III



COMPASS





PSI

MAMI



Lattice QCD receiving attention

Lattice-Related Talks @ Light-Cone 2018

- This talk
- ★ "The Bjorken-x dependence of hadron structure from LQCD", C. Monahan
- ★ "Pseudo-PDFs and quasi-PDFs structure", A. Radyushkin
- ★ "An LQCD computation of quark PDFs at the physical point", F. Steffens
- ★ "Full nucleon structure functions from the lattice", G. Schierholz
- ★ "Lattice Field Theory in Real Time (and finite density)", P. Bedaque
- ★ "Recent results in lattice QCD spectroscopy", B. Hoerz
- ★ "Nucleon PDFs in small boxes", J. Guerrero
- ★ "Pion Valence Distributions from Hadronic Lattice Cross Sections", R. Sufian
- ★ "Pseudo Distributions from Lattice QCD", J. Karpie
- ★ "Direct calculation of slope of form factors in lattice QCD", **D. Richards**
- ★ "Excited-state contributions to nucleon charges in LQCD", C. Egerer
- ★ "Prospects for studying the structure of the excited states via LQCD", A. Baroni
- ★ "Nuclear physics on the lattice", W. Detmold
- ★ "New developments in nuclear lattice simulations", D. Lee
- \star "A percent level determination of the nucleon g_A from LQCD", A. Walker-Loud
- ★ "Total decay and transition rates from lattice QCD", **D. Robaina**

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Lattice Talks @ Light-Cone 2013 - 2017: 14 total !

Probing Nucleon Structure



Parton Distribution Functions

- Universal quantities for the description of the nucleon's structure (non-perturbative nature)
- ★ 1-dimensional picture of nucleon structure
- ★ Distribution functions are necessary for the analysis of Deep inelastic scattering data
- ★ Parametrized in terms of off-forward matrix of light-cone operators
- ★ Not directly accessible in a euclidean lattice

★ Moments of PDFs easily accessible in lattice QCD

- one relies on OPE to reconstruct the PDFs
- reconstruction difficult task:
 - \Rightarrow signal-to-noise is bad for higher moments
 - \Rightarrow n > 3: operator mixing (unavoidable!)

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Alternative approaches to access PDFs

Hadronic Tensor [K.F. Liu, Dong, PRL 72 (1994) 1790, K.F. Liu, PoS(LATTICE 2015) 115] OPE without an OPE

• the forward Compton amplitude

 $T_{\mu\nu}(p,q) = \rho_{\lambda\lambda'} \int d^4x e^{iq \cdot x} \langle p, \lambda' | T J_{\mu}(x) J_{\nu}(0) | p, \lambda \rangle$

overcomes issues of renormalization and operator mixing

see talk by G. Schierholz

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Direct access to PDFs

- quasi-PDFs
- pseudo-PDFs
- good lattice cross-sections

[X. Ji, arXiv:1305.1539]

[A. Radyushkin, arXiv:1705.01488]

[Y-Q Ma&J. Qiu, arXiv:1709.03018]

see talks by C. Monahan, A. Radyushkin, F. Steffens, J. Guerrero, J. Karpie, R.Sufian



Introduction

to Lattice QCD

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Lattice formulation of QCD

- Space-time discretization on a finite-sized
 4-D lattice
 - Quark fields on lattice points
 - Gluons on links
- ★ Serves as a regulator
 - UV cut-off: inverse lat. spacing
 - IR cut-off: inverse lattice size



courtesy: USQCD

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Technical Aspects

- Parameters (define cost of simulations):
 - quark masses (aim at physical values)
 - lattice spacing (ideally fine lattices)
 - lattice size (need large volumes)

★ Discretization not unique:

- Wilson, Clover, Twisted Mass, Staggered, Overlap, Domain Wall
- Mixed actions









$$R^{\mu}_{\mathcal{O}}(\Gamma, \vec{q}, t) = \frac{G_{\mathcal{O}}(\Gamma, \vec{q}, t)}{G(\vec{0}, t_f)} \times \sqrt{\frac{G(-\vec{q}, t_f - t)G(\vec{0}, t)G(\vec{0}, t_f)}{G(\vec{0}, t_f - t)G(-\vec{q}, t)G(-\vec{q}, t_f)}}$$





Plateau Method:

Summation Method:

2-state fits:

 $R_{\mathcal{O}}(\Gamma, \vec{q}, t) \xrightarrow{t_f - t \to \infty} \Pi^{\mu}(\Gamma, \vec{q})$

 $t - t_i \rightarrow \infty$





4. Renormalization:

connection to experiments

$$\Pi^R(\Gamma, \vec{q}) = \mathbb{Z}_{\mathcal{O}} \,\Pi(\Gamma, \vec{q})$$

Simpler case!



connection to experiments

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Simpler case!

5. Extraction of form factors e.g. Axial current:

$$A_{\mu}^{3} \equiv \bar{\psi} \gamma_{\mu} \gamma_{5} \frac{\tau^{3}}{2} \psi \Rightarrow \bar{u}_{N}(p') \left[\mathbf{G}_{\mathbf{A}}(\mathbf{q}^{2}) \gamma_{\mu} \gamma_{5} + \mathbf{G}_{\mathbf{p}}(\mathbf{q}^{2}) \frac{q_{\mu} \gamma_{5}}{2 m_{N}} \right] u_{N}(p)$$

Inherited Uncertainties

Laborious effort to eliminate uncertainties

Statistical errors significantly increase with:

- ★ decrease of pion mass
- \star increase of momentum transfer Q^2 between source-sink
- \star increase of source-sink separation ($T_{\rm sink}$)

Systematic

- ★ Cut-off Effects due to finite lattice spacing
- ★ Finite Volume Effects
- ★ Contamination from other hadron states
- ★ Chiral extrapolation for unphysical pion mass
- ★ Renormalization and mixing

Also Talk by A. Walker-Loud, C. Egerer



Nucleon

Structure

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Nucleon Charges



Systematic Uncertainties

Systematics must be addressed at the physical point

ETM Collaboration: 3 ensembles at the physical point:

 $\begin{array}{rrrr} N_f{=}2 & 48^3 \times 64 & a{=}0.092 {\rm fm} \\ N_f{=}2 & 64^3 \times 128 & a{=}0.082 {\rm fm} \\ N_f{=}2{+}1{+}1 & 64^3 \times 128 & a{=}0.08 {\rm fm} \end{array}$

★ Study of volume, quenching effects and excited states



- Agreement with experiment at T_{sink} =1.5 1.65fm
- ★ Volume effects are small
- Combination of volume and quenching non-negligible

Momentum Fraction



Momentum Fraction



Parton Momentum Fraction

 χ QCD Collaboration: Quark spin: overlap on RBC/UKQCD domain-wall ensembles, m_{π} =170, 300, 330MeV Connected diagram: improved axial current [J. Liang et al., Phys. Rev. D 96 (2017) 034519] Disconnected diagram: check of anomalous Ward identity Results (PRELIMINARY):

 $\Delta_u = 0.846(18)(32), \Delta_d = -0.410(16)(18), \Delta_s = -0.0353(83)(71)$

Gluom momentum fraction:

[Y. Yang et al., arXiv:1805.00531]

- ★ Various valence quarks linear extrapolation
- ★ Non-perturbative renormalization
- ★ Various levels of HYP smearing and 2 definitions of operator



- Renormalized results independent of the smearing
- Systematic related to renormalization must be addressed
- Mixing with quark operator must be eliminated

DIS experiment (1988) show 20-30% of spin carried by valence quarks

"... $g_1(x)$ for the proton has been determined and its integral over x found to be $0.114\pm0.012\pm0.026$, in disagreement with the Ellis-Jaffe sum rule. ... These values for the integrals of g_1 lead to the conclusion that the total quark spin constitutes a rather small fraction of the spin of the nucleon." [J. Ashman et al., Phys. Lett., vol. B206 (1988) 364]

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naive non-relativistic SU(6) quark model: ΔΣ=1, L_q=0, J_q=0

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Necessary computations:

- ★ Axial Charge
- ★ Quark Momentum Fraction
- ★ Gluon Momentum Fraction

The proton spin puzzle from Lattice QCD



The proton spin puzzle from Lattice QCD



Satisfaction of spin and momentum sum rule is not forced



Hadron

Form Factors

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Nucleon Electromagnetic Form Factors



Nucleon Electromagnetic Form Factors



25

Meson Electromagnetic Form Factors



Electromagnetic Transition Form Factors





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Great progress over the last years:

★ Simulations at the physical point

Talk by P.Bedaque on real time simulations

- ★ Careful assessment of systematic uncertainties
 - Volume effects
 - quenching effect (strange and charm)
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★ Beyond hadron structure

- Investigations to Nuclear Effects (Talks by W. Detmold, D. Lee) [E. Chang et al. (NPLQCD), Phys. Rev. Lett. 120, 152002 (2018), arXiv:1712.03221]
- Spectroscopy and exotic states (Talks by B. Hoerz, A.Baroni)

THANK YOU



TMD Topical Collaboration



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BACKUP SLIDES