x-dependent hadron structure from lattice QCD

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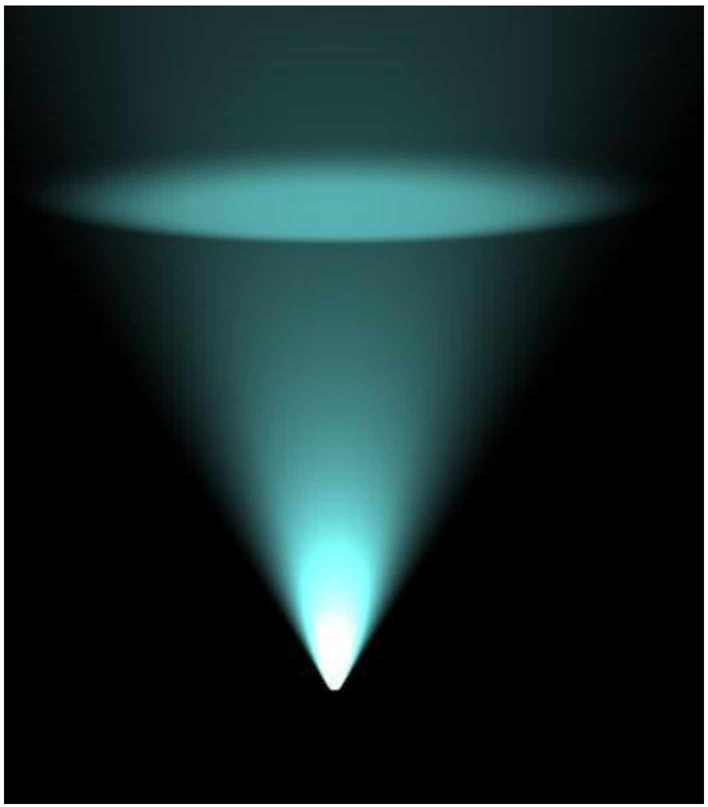
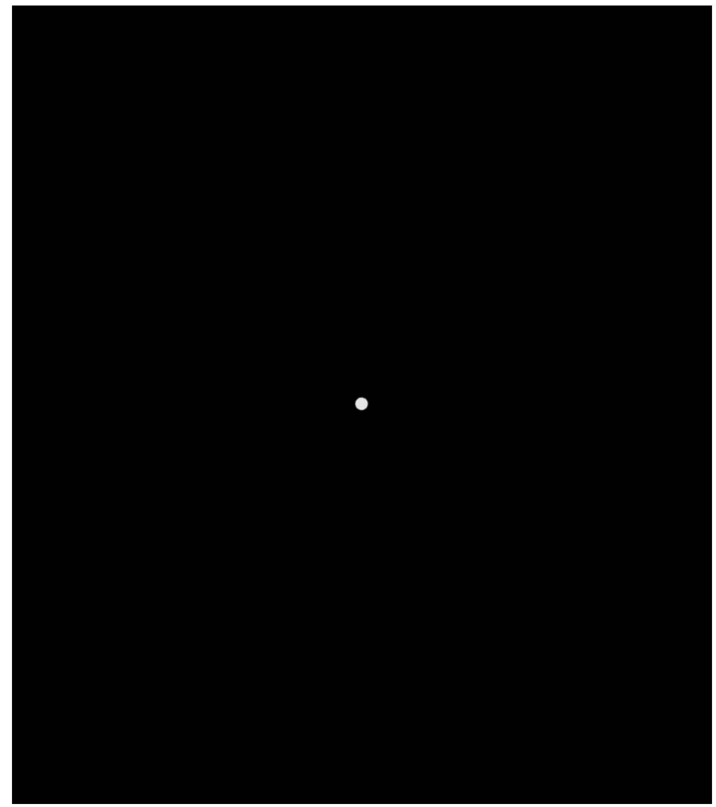
Light ~~Cone~~ Point 2018

Lattice QCD is formulated in Euclidean spacetime





Euclidean approaches to light-cone physics

Distribution function approaches

Quasi and pseudo PDFs and ``lattice cross-sections”

Position-space correlators

Structure function approaches

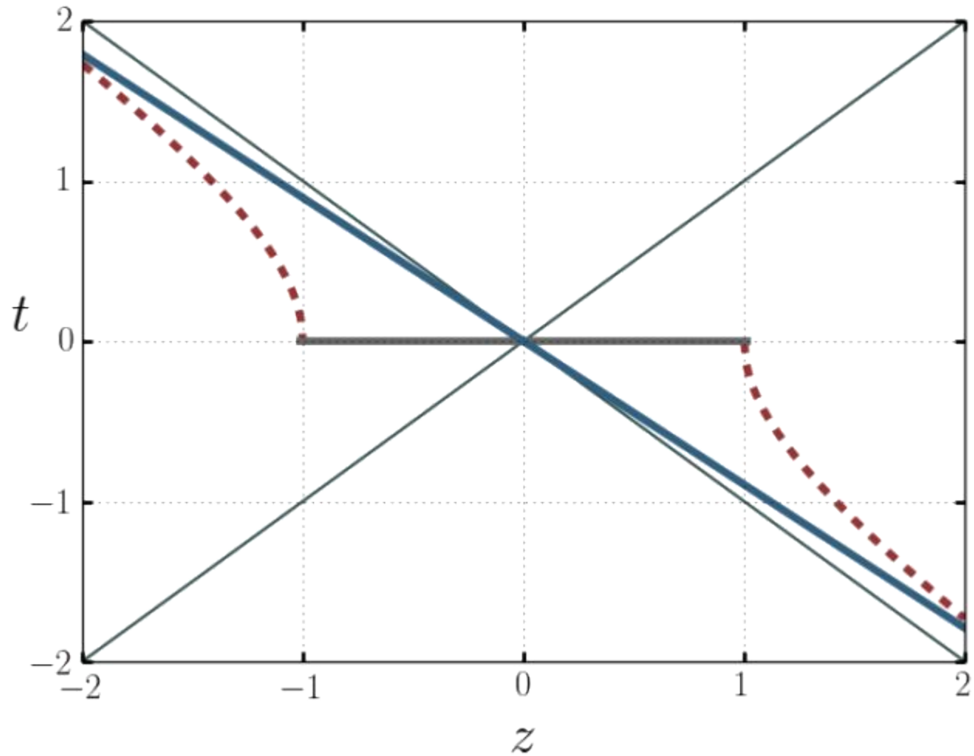
Euclidean hadronic tensor

Compton amplitude

A recent renaissance

Quasi and pseudo PDFs: systematic approach to the light cone

Ji, PRL 110 (2013) 262002
Radyushkin, PRD 96 (2017) 034025



Ioffe-time distribution

$$I^{(0)}(\zeta = P \cdot n, n^2) = \frac{1}{2P^\mu} \left\langle P \left| \overline{\psi}(n) W(n,0) \Gamma_\mu \psi(0) \right| P \right\rangle$$

$$W(n(u),0) = \mathcal{P} \exp\left[-ig_0 \int_0^u \mathrm{d}v \frac{\mathrm{d}y^\mu}{\mathrm{d}v} A_\mu^a(y(v)) T^a \right]$$

Radyushkin, PRD 96 (2017) 034025

See A. Radyushkin's talk at 11:05

A panoply of distributions

PDFs

$$\zeta = -P^+\xi^- \qquad n^2 = 0$$

ξ^-

Quasi PDFs

P^z

$$\zeta = P^z z \qquad n^2 = z^2 < 0$$

$$I^{(0)}(\zeta = P\cdot n, n^2) = \frac{1}{2P^\mu}\left\langle P\left|\overline{\psi}(n)W(n,0)\Gamma_\mu\psi(0)\right|P\right\rangle$$

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ζ

Pseudo PDFs

$$\zeta = P^z z \qquad n^2 = z^2 < 0$$

Fourier transforms generate distribution functions

See A. Radyushkin's talk at 11:05

A panoply of distributions

PDFs

Quasi PDFs

Pseudo PDFs

ξ^-

P^z

ζ

$$I^{(0)}(\zeta = P \cdot n, n^2) = \frac{1}{2P^\mu} \langle P \left| \overline{\psi}(n) W(n,0) \Gamma_\mu \psi(0) \right| P \rangle$$

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Fourier transforms generate distribution functions

Factorisation theorems relate distribution functions

Quasi and pseudo distributions

Ji, PRL 110 (2013) 262002
Ji et al., NPB 924 (2017) 326
Izubuchi et al., 1801.03917

Factorisation theorems

$$\widetilde{f}_{j/H}(\xi, P^z, \mu_{\rm R}) = \int_{-1}^{1} \frac{{\rm d}y}{|y|} C^{(\widetilde{f})}\left(\frac{\xi}{y}, \frac{\mu_{\rm R}}{P^z}, \frac{\mu}{p^z}\right) f_{j/H}(y,\mu) + \mathcal{O}\left(\frac{M^2}{(P^z)^2}, \frac{\Lambda^2_{\rm QCD}}{(P^z)^2}\right)$$

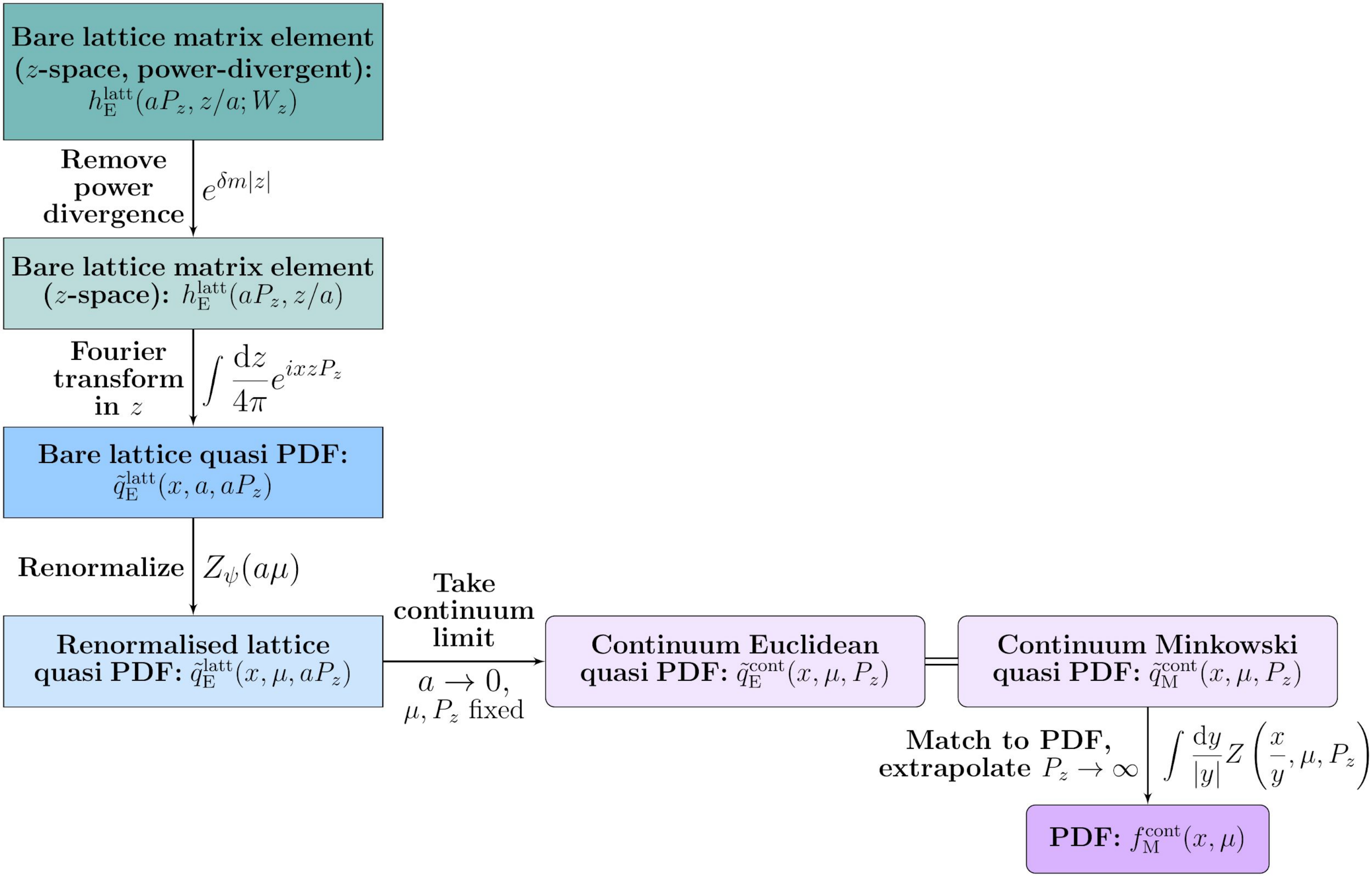
$$\widetilde{p}_{j/H}(\xi, z^2\mu_{\rm R}^2) = \int_{-1}^{1} \frac{{\rm d}y}{|y|} C^{(\widetilde{p})}\left(\frac{\xi}{y}, \frac{\mu_{\rm R}^2}{\mu^2}, \mu^2 z^2\right) f_{j/H}(y,\mu) + \mathcal{O}\left(M^2 z^2, \Lambda^2_{\rm QCD} z^2\right)$$

Examples of ``good lattice cross-sections”

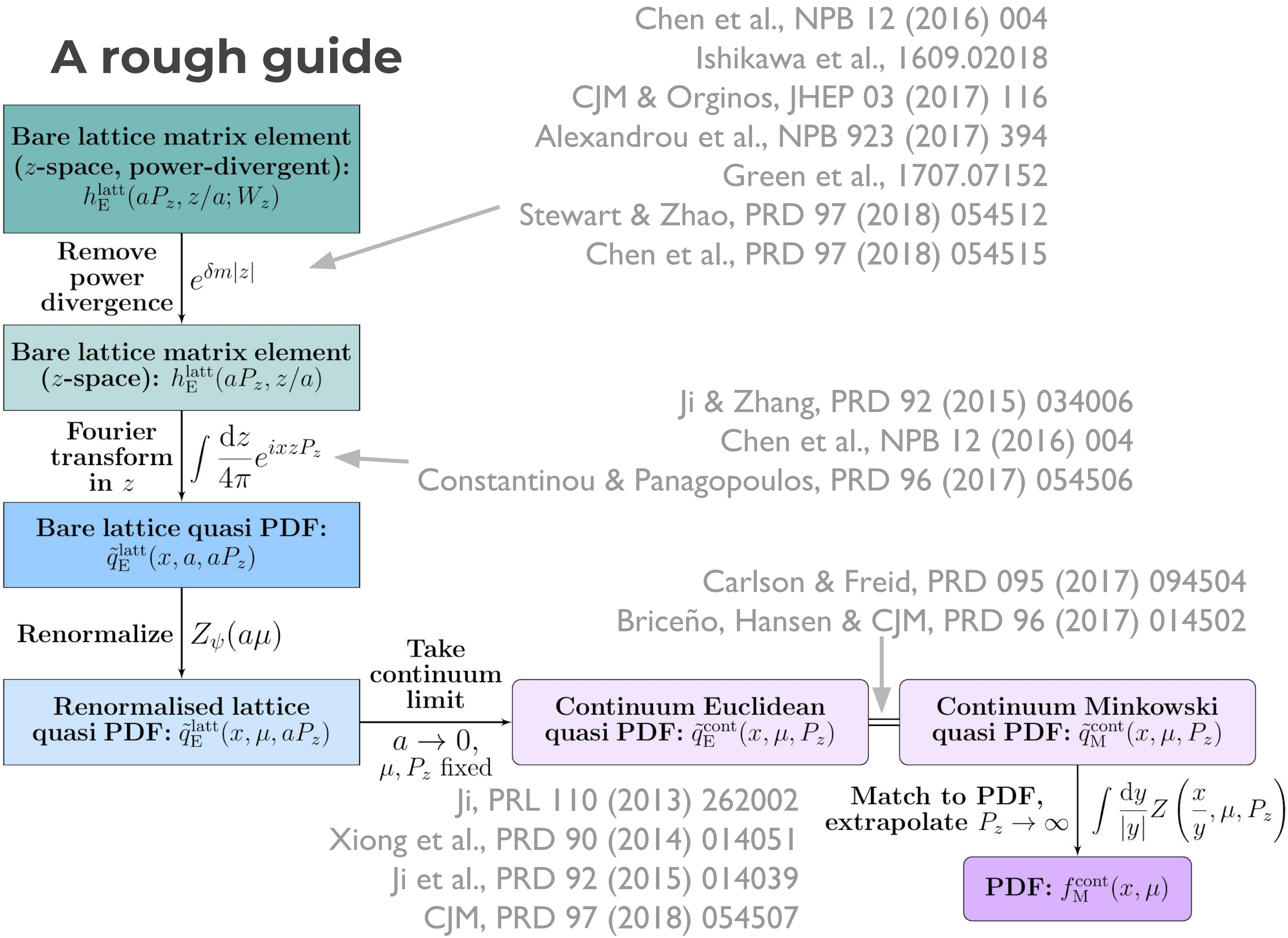
See R. Sufian’s talk Tues 14:00

Ma & Qiu, PRL 120 (2018) 022003
Ma & Qiu, 1404.6860

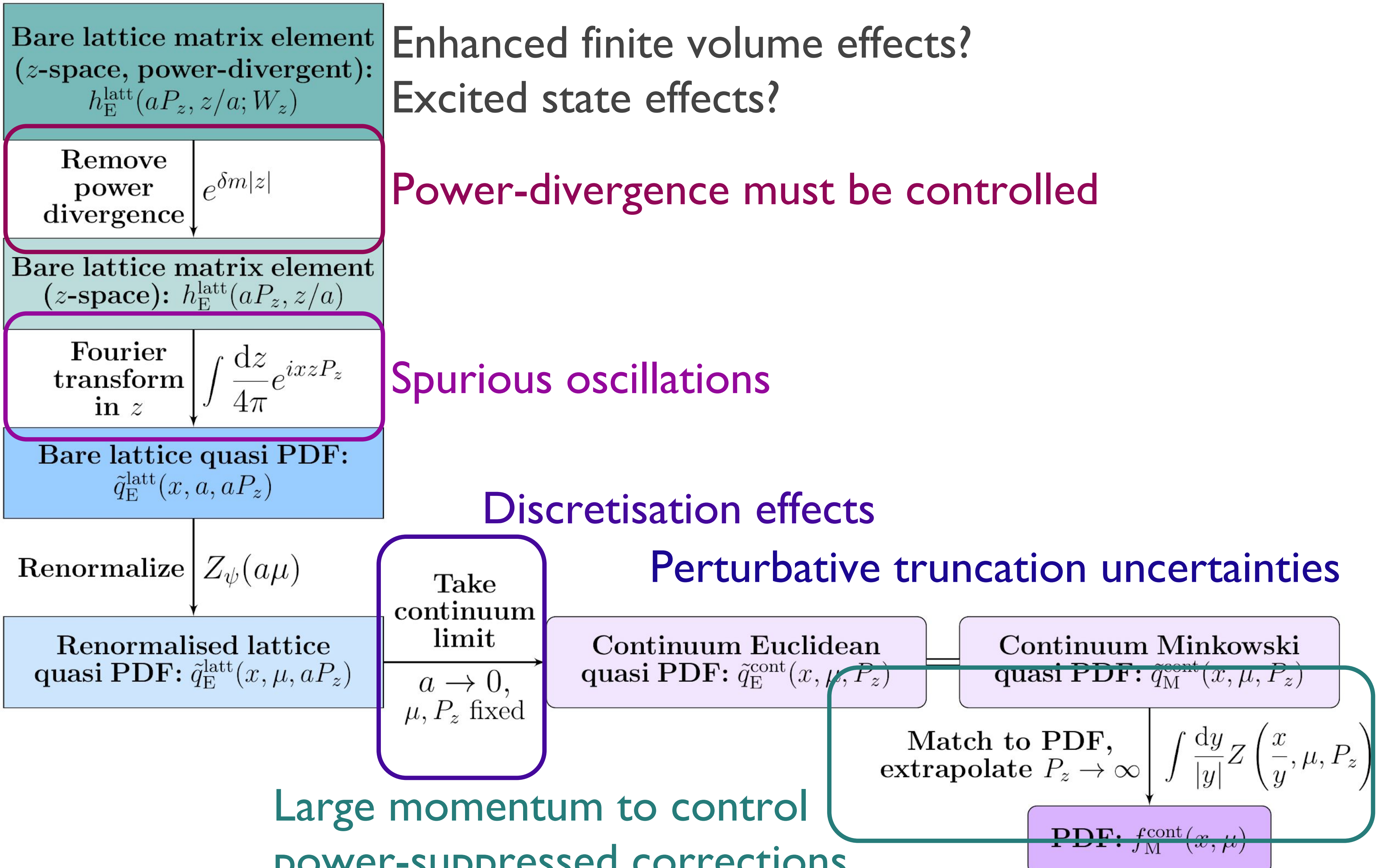
A rough guide



A rough guide



A rough guide (to systematic uncertainties)



Enhanced finite volume effects?
Excited state effects?

Power-divergence must be controlled

Spurious oscillations

Discretisation effects

Perturbative truncation uncertainties

Large momentum to control power-suppressed corrections

A rough guide (to systematic uncertainties)

Bare lattice matrix element (z-space, power-divergent): $h_{\mathrm{E}}^{\mathrm{latt}}(aP_z, z/a; W_z)$

Enhanced finite volume effects?
Excited state effects?

↓ **Remove power divergence** $e^{\delta m|z|}$

Power-divergence must be controlled

Bare lattice matrix element (z-space): $h_{\mathrm{E}}^{\mathrm{latt}}(aP_z, z/a)$

Moments of quasi distributions suffer power-divergent mixing
Rossi & Testa, PRD 96 (2017) 014507
Rossi & Testa, to appear soon

↓ **Fourier transform in z** $\int \frac{\mathrm{d}z}{4\pi} e^{ixzP_z}$

Spurious oscillations

Bare lattice quasi PDF: $\tilde{q}_{\mathrm{E}}^{\mathrm{latt}}(x, a, aP_z)$

Discretisation effects

↓ **Renormalize** $Z_\psi(a\mu)$

Perturbative truncation uncertainties

Renormalised lattice quasi PDF: $\tilde{q}_{\mathrm{E}}^{\mathrm{latt}}(x, \mu, aP_z)$

→ **Take continuum limit** $a \to 0$, μ, P_z fixed

Continuum Euclidean quasi PDF: $\tilde{q}_{\mathrm{E}}^{\mathrm{cont}}(x, \mu, P_z)$ = **Continuum Minkowski quasi PDF:** $\tilde{q}_{\mathrm{M}}^{\mathrm{cont}}(x, \mu, P_z)$

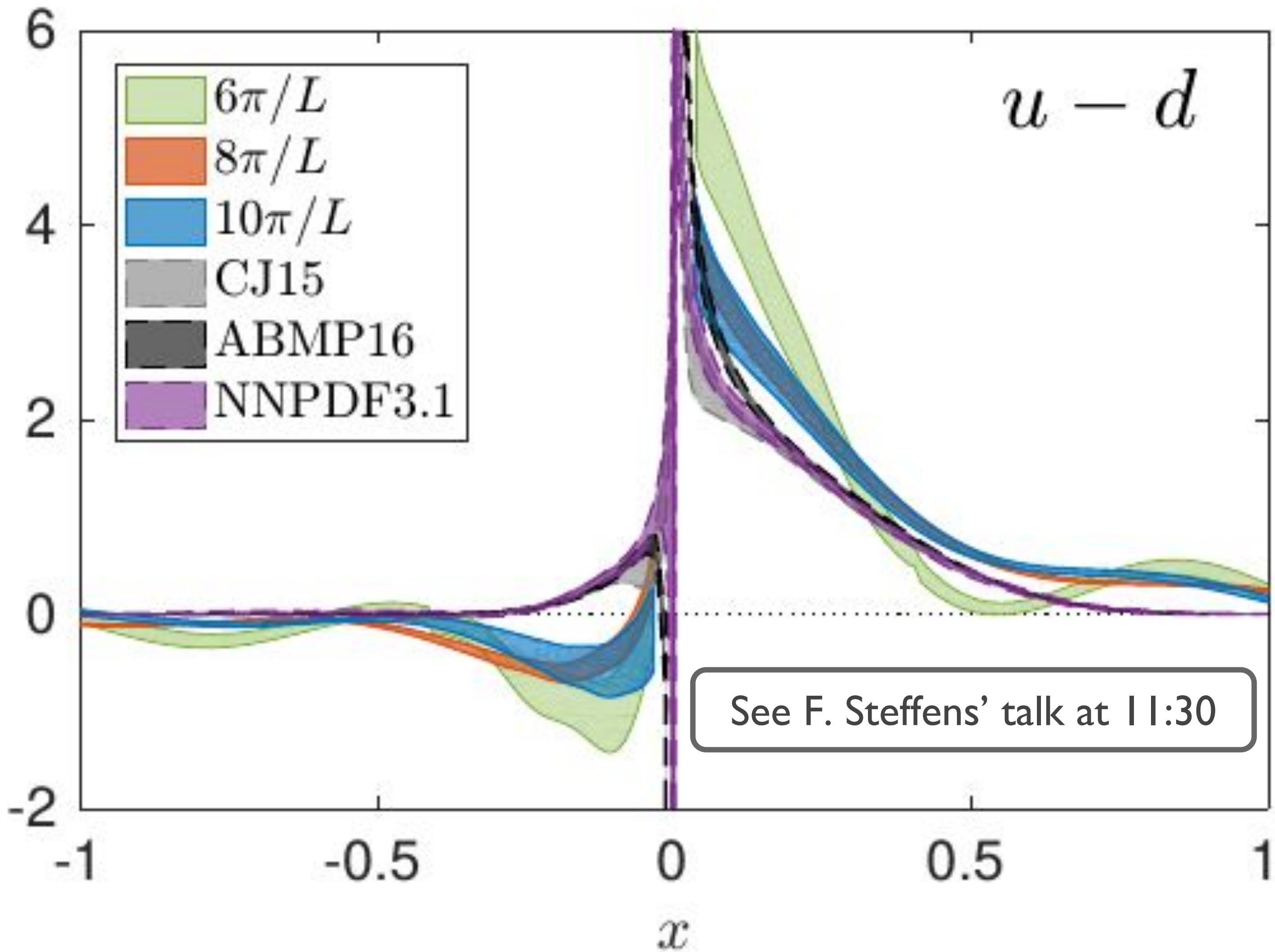
↓ **Match to PDF, extrapolate $P_z \to \infty$** $\int \frac{\mathrm{d}y}{|y|} Z\left(\frac{x}{y}, \mu, P_z\right)$

PDF: $f_{\mathrm{M}}^{\mathrm{cont}}(x, \mu)$

Large momentum to control power-suppressed corrections

Alexandrou et al., 1803.02685

Highlights from the ETM Collaboration

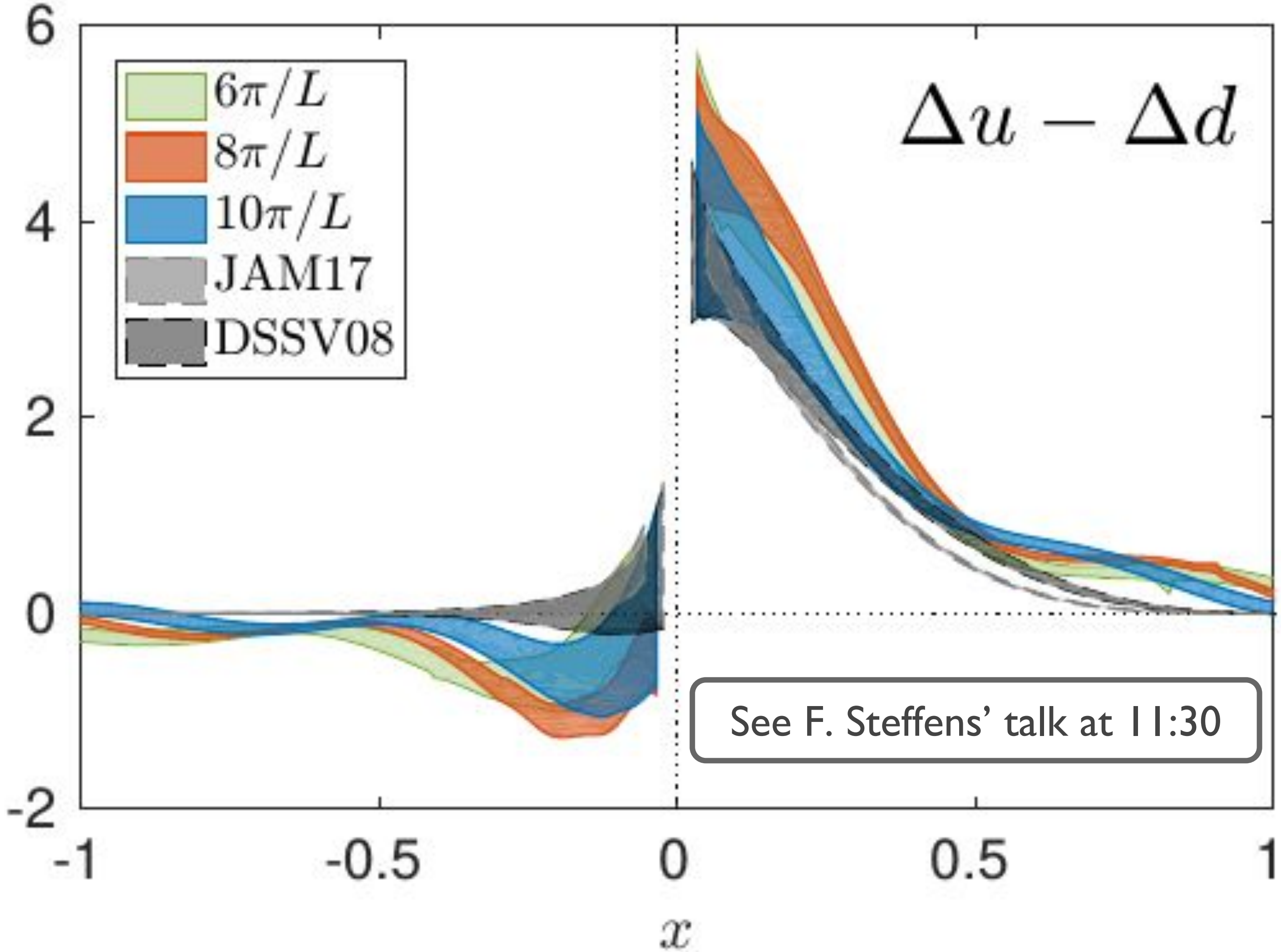


Two flavours of twisted mass quarks
Nonperturbative RI' renormalisation

Lattice spacing ~ 0.09 fm
Pion mass ~ 130 MeV

Alexandrou et al., 1803.02685

Highlights from the ETM Collaboration

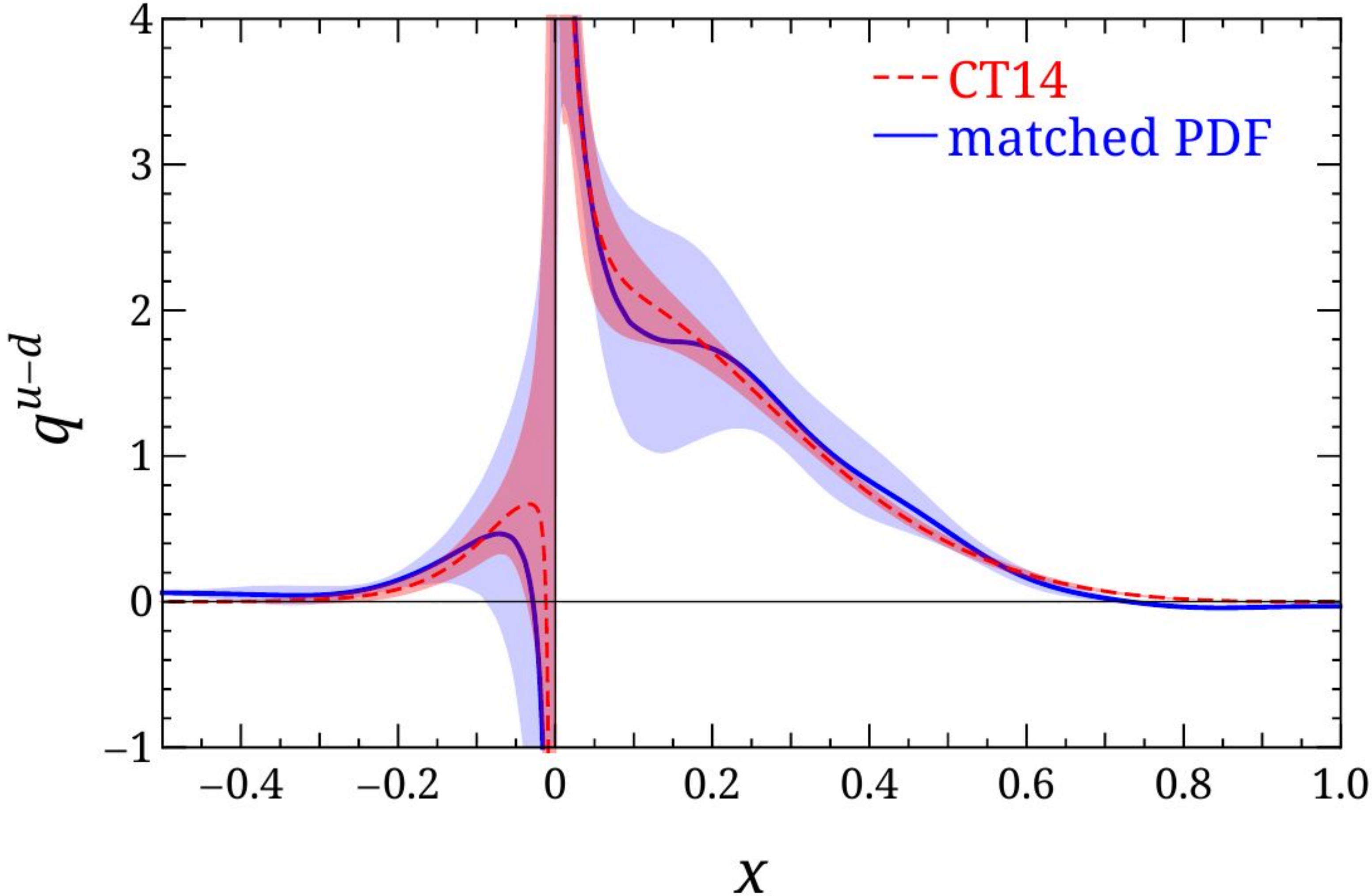


Two flavours of twisted mass quarks
Nonperturbative RI' renormalisation

Lattice spacing ~ 0.09 fm
Pion mass ~ 130 MeV

Chen et al., 1803.04393

Highlights from the LP^3 Collaboration



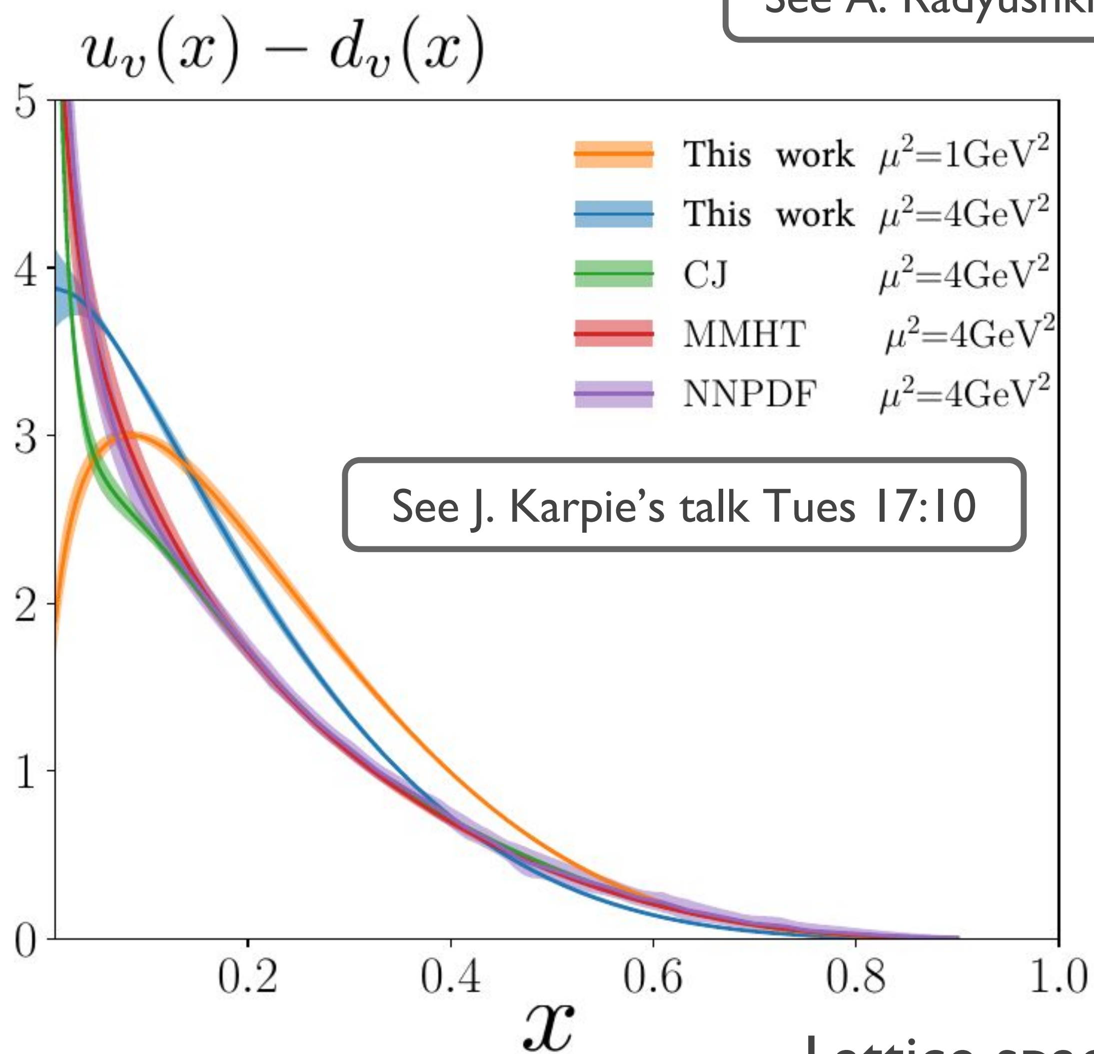
2+1+1 flavours of staggered quarks
Nonperturbative RI/MOM renormalisation

Lattice spacing ~ 0.09 fm
Pion mass ~ 135 MeV

Karpie et al., EPJ Conf 175 (2018) 06032
Orginos et al., PRD 96 (2017) 094503

Highlights from JLab

See A. Radyushkin's talk at 11:05



Quenched approximation

Lattice spacing ~ 0.09 fm
Pion mass ~ 600 MeV

Euclidean approaches to light-cone physics

Distribution function approaches

Quasi and pseudo PDFs and ``lattice cross-sections”

Position-space correlators

Direct structure function approaches

Euclidean hadronic tensor

Compton amplitude

Position-space correlators

Braun & Müller, EPJ C55 (2008) 349
Bali et al., EPJ C78 (2018) 217

Position-space correlation functions
- encode all necessary information
- avoid challenges of renormalisation

Match lattice matrix element to perturbative QCD factorisation

$$\Phi_\pi(p\cdot z) = \frac{2(\pi z^2)^2}{f_\pi p\cdot z}\langle \pi^0(\mathbf{p})|[\overline{u}q](z/2)[\overline{q}\gamma_5 u](-z/2)|0\rangle$$

$$\Phi_\pi(p\cdot z) = \int_0^1 \mathrm{d}x\, e^{i(x-1/2)p\cdot z}\phi_\pi(x)$$

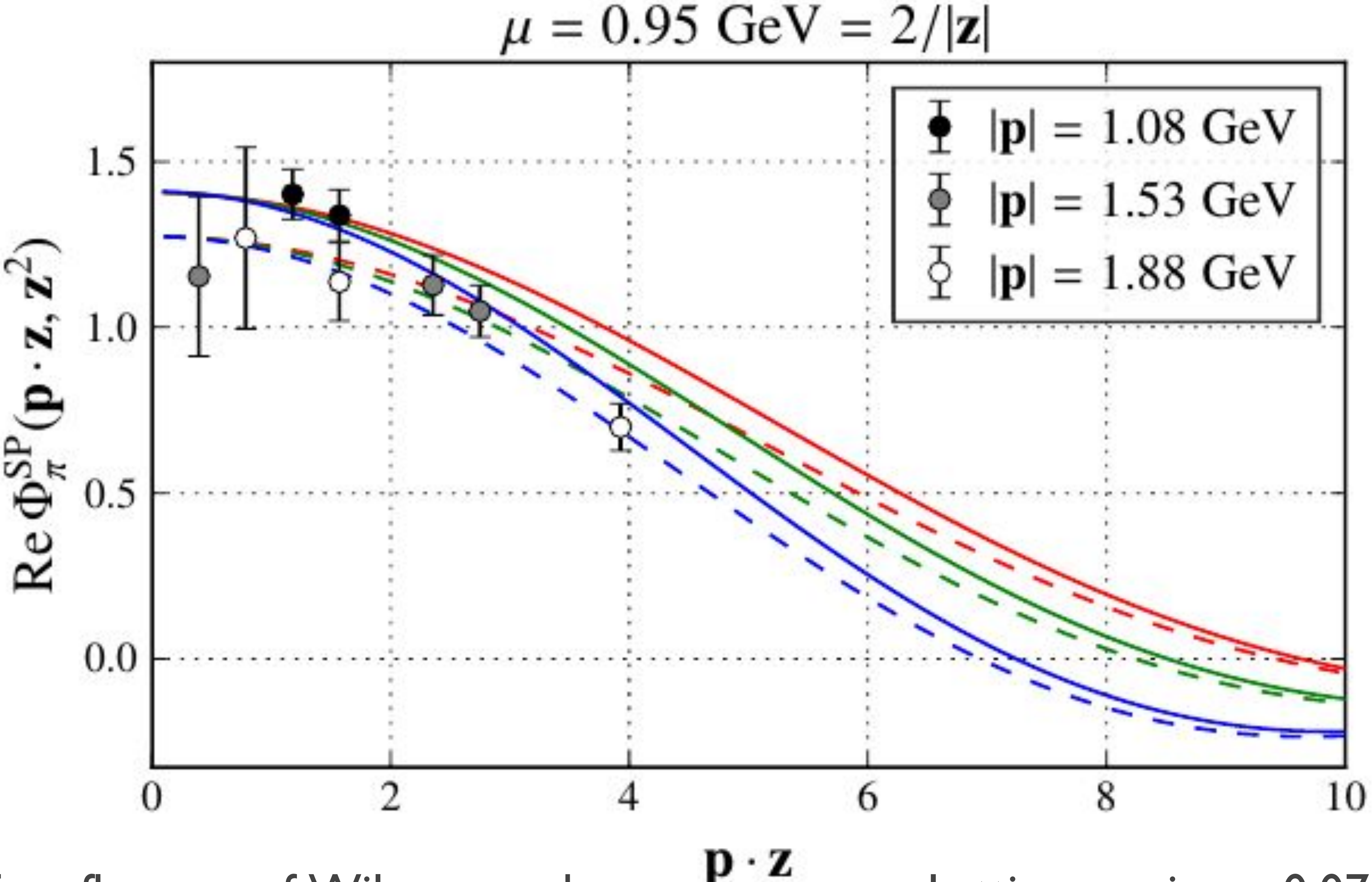
Require
- small z^2 for factorisation
- large p.z only to distinguish pion DA shapes

May suffer from enhanced finite-volume effects?

Briceño, Guerrero, Hansen & CJM, 1805.01034

See J. Guerrero's talk Tues 16:50

Pion PDA

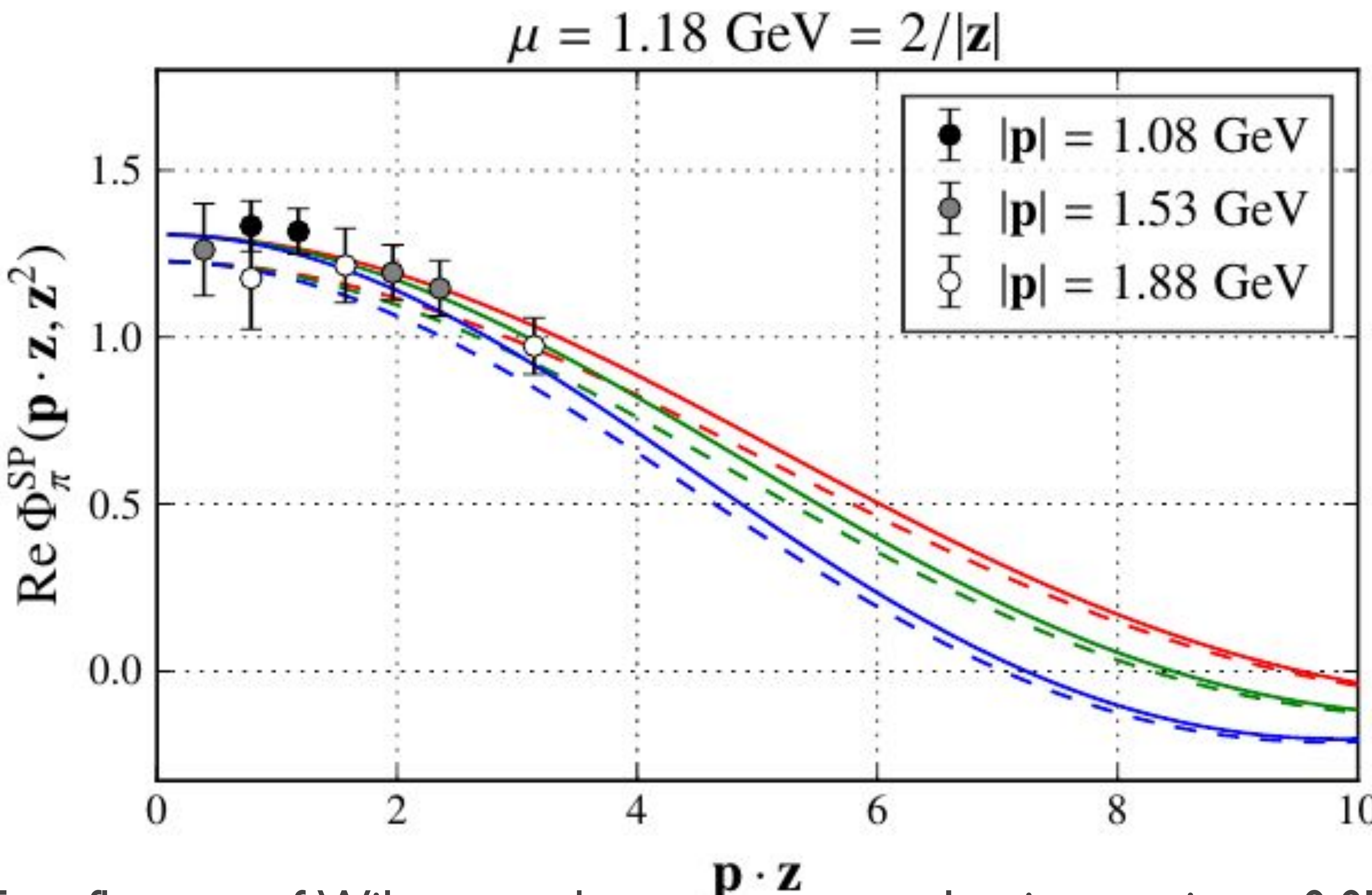


Two flavours of Wilson quarks
Nonperturbative renormalisation

Lattice spacing ~ 0.071 fm
Pion mass ~ 295 MeV

Bali et al., EPJ C78 (2018) 217

Pion PDA



Two flavours of Wilson quarks
Nonperturbative renormalisation

Lattice spacing ~ 0.071 fm
Pion mass ~ 295 MeV

Euclidean approaches to light-cone physics

Distribution function approaches

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Position-space correlators

Structure function approaches

Euclidean hadronic tensor

Compton amplitude

Euclidean hadronic tensor

Liu & Dong, PRL 72 (1994) 1790
Liu, PRD 62 (2000) 074501
Liu, PoS(LATTICE 2015) 115

Formulate hadronic tensor in Euclidean path-integral formalism

- renormalisation straightforward
- frame invariant

$$W_{\mu\nu}(\mathbf{q},\tau) = \frac{1}{4\pi}\sum_{\mathbf{x}} e^{-i\mathbf{q}\cdot\mathbf{x}}\langle P|J_\mu(\mathbf{x},\tau)J_\nu(0,0)|P\rangle$$

$$W_{\mu\nu}(q^2, q\cdot P) = \frac{1}{i}\int_{c-i\infty}^{c+i\infty} \mathrm{d}\tau\, e^{q\cdot P\tau} W_{\mu\nu}(\mathbf{q},\tau)$$

First results expected soon

Challenges:

- requires calculation of four-point function
- difficult inverse problem to convert to Minkowski spacetime

Compton amplitude

Chambers et al., PRL 118 (2017) 242001

Calculate Compton amplitude via a Feynman-Hellman method

- avoids renormalisation and mixing issues
- with sufficient data can disentangle higher-twist contributions

$$T_{\mu\nu}(P,q) = \int \mathrm{d}^4x\, e^{iq\cdot x} \langle P|\mathcal{T} J_\mu(x) J_\nu(0)|P\rangle$$

$$T_{33}(P,q) = \sum_{n=2,4,\ldots}^{\infty} 4\omega^n \int_0^1 \mathrm{d}x\, x^{n-1} F_1(x,q^2) \qquad \omega = \frac{2P\cdot q}{q^2}$$

$$T_{33}(P,q) = 4\omega \int_0^1 \mathrm{d}x \frac{\omega x}{1-(\omega x)^2} F_1(x,q^2)$$

Challenges

- must reconstruct inverse Mellin transform

See G. Schierholz's talk at 11:55

Outlook

Outlook

Much has been understood:
factorisation theorems
nonperturbative renormalisation
some early issues resolved

Preliminary results encouraging:
multiple complementary approaches
lattice inputs to global PDF fits

Outlook

Much has been understood:
factorisation theorems
nonperturbative renormalisation
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Systematic uncertainties unexplored
large momentum
spurious oscillations
excited state effects?
enhanced finite volume effects?

Preliminary results encouraging:
multiple complementary approaches
lattice inputs to global PDF fits

Unresolved theoretical issues?

Thank you

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