Trident pair production in lightfront quantization

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V. Dinu and GT, Phys. Rev. D **97**, no. 3, 036021 (2018)

Physics at high intensity

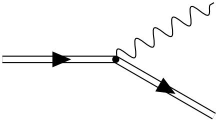
- High-intensity laser facilities $I: 10^{22} \nearrow 10^{26}\text{W/cm}^2$?

  ELI, XCELS,...

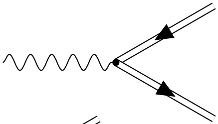
- cf. Schwinger limit $I \sim (m^2/e)^2 \sim 10^{29}\text{W/cm}^2$

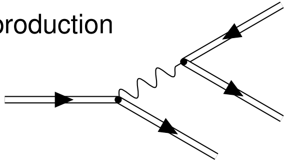
- New physics even for $I \ll 10^{29}\text{W/cm}^2$

  Review: A. Di Piazza *et al.* Rev. Mod. Phys (2012)



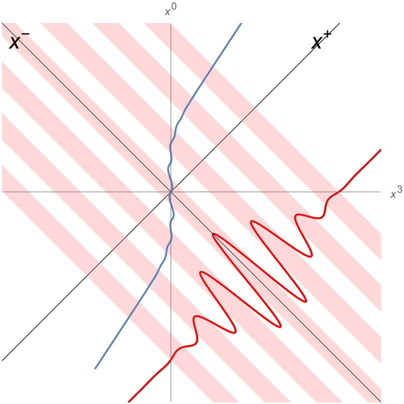
- Basic processes
  - nonlinear Compton scattering
  - nonlinear Breit-Wheeler pair production
  - trident pair production
  - cascades





Model high-intensity lasers with pulsed plane waves

- $a_0 = \frac{eE}{m\omega} > 1 \to$ treat field exactly
- $eF_{\mu\nu}(x^+) = k_\mu a'_\nu - k_\nu a'_\mu$
- Lightfront coordinates: $x^\pm = t \pm z$
- $k^2 = 0,\ k\cdot x = \omega x^+,\ k\cdot a(x^+) = 0$
- Pulsed plane waves: $a'(\pm\infty) = 0$
- Sol. to Lorentz force eq.



$$m\ddot{x}^\mu = eF^{\mu\nu}\dot{x}_\nu:\ \pi_\mu(x^+) := m\dot{x}_\mu = p_\mu - a_\mu + \frac{2ap - a^2}{2k\cdot p}k_\mu$$

Volkov solution and Furry picture

- Treat background field exactly in QED

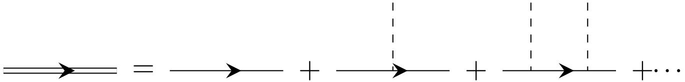
$$(\mathscr{D}^2+m^2)\varphi=0 \qquad \mathscr{D}_\mu=\partial_\mu+ia_\mu(x^+)$$

- Simple solution for arbitrary $a_\perp(x^+)$: $\qquad \varphi(x)=e^{-ipx-i\int^{x^+}\frac{2ap-a^2}{2k\cdot p}}$

Volkov (1935)

- Furry picture: $H=H_{\text{“free”}}[a]+H_{\text{int}}[a]$

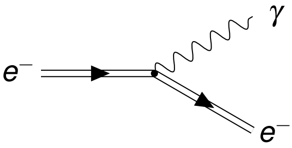
- Volkov solutions describe Lorentz force: $\quad i\mathscr{D}_\mu\varphi(x)=\pi_\mu(x^+)\varphi(x)$

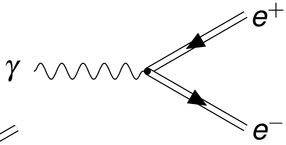


Basic processes

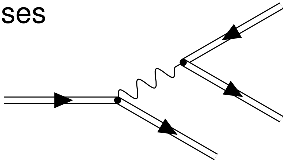
- First order processes
  - Compton scattering
  - Breit-Wheeler pair production
  - Observed at SLAC (1999)

    Bamber *et al.* PRD (1999)





- Higher order processes
  - Trident

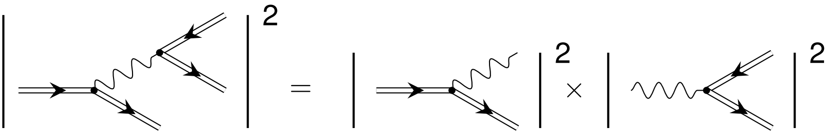


Baier, Katkov, & Strakhovenko (1972); Ritus (1972); Hu, Müller & Keitel PRL (2010); Ilderton PRL (2011); King & Ruhl (2013)

Trident - two-step and one-step

- Higher orders from sequence of first orders

  Baier, Katkov, & Strakhovenko (1972); Ritus (1972); King & Ruhl (2013)



$$+ \text{“one-step” terms} = \mathbb{P}_{\rm two} + \mathbb{P}_{\rm one}$$

- Particle-in-cell simulations at high intensity

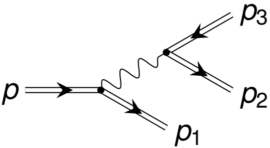
  Review: Gonoskov *et al.* PRE (2015)

  - $a_0 = \frac{eE}{m\omega} \gg 1 \qquad \to \qquad |\mathbb{P}_{\rm one}| \ll \mathbb{P}_{\rm two}$

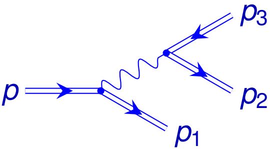
- We are interested in $\mathbb{P}_{\rm one}$

Direct and Exchange

- Exchange of identical particles in final state

$\Bigg|$ 

$- \quad (p_1 \leftrightarrow p_2) \Bigg|^2 \quad =$

$\Bigg|$ 

$\Bigg|^2 \quad + \quad (p_1 \leftrightarrow p_2) \quad +$ "cross term" $=$

"direct part" $+$ "exchange part" $= \mathbb{P}_{\text{dir}} + \mathbb{P}_{\text{ex}}$

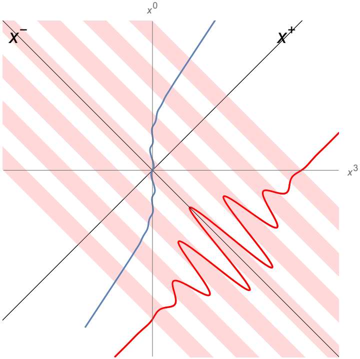
Direct and Exchange

- $\mathbb{P}_{\text{ex}}$ more difficult than $\mathbb{P}_{\text{dir}}$
- $\mathbb{P}_{\text{ex}}$ neglected in previous analytical studies
- Expect $|\mathbb{P}_{\text{ex}}| \ll \mathbb{P}_{\text{dir}}$ for $\chi \gg 1$ where $\chi := a_0 b_0 = \frac{eE}{m\omega}\frac{kp}{m^2}$
- But for how large χ?
- And what about $\chi \lesssim 1$?
- $\mathbb{P}_{\text{ex}}$ does not contribute to $\mathbb{P}_{\text{two}}$
- How does $\mathbb{P}_{\text{ex}}$ contribute to $\mathbb{P}_{\text{one}}$?

Lightfront quantisation

- $a_\mu(x^+), \pi_\mu(x^+), \varphi = e^{-ipx} f(x^+)$
- Use $x^+ = t + z$ instead of t
- Lightfront quantisation + Furry picture for plane waves

  Neville & Rohrlich 1971

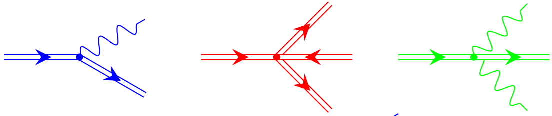


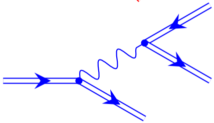
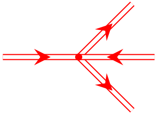
Lightfront Hamiltonian

- LF Hamiltonian $H = P_+ = (P_0 + P_3)/2$ for evolution in $x^+ = x^0 + x^3$: $|\,\psi; x^+\rangle = T_+ e^{-i\int^{x^+} H_{\rm int}}\,|\,{\rm in}\rangle$

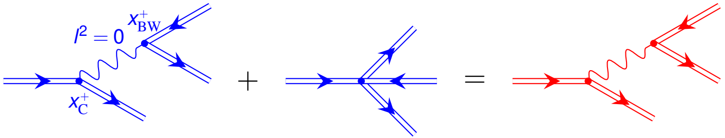
- "Instantaneous" terms in H. $p^2 = m^2$ in Ψ and $l^2 = 0$ in A_μ

$$H_{\rm int} = \frac{1}{2}\int \mathrm{d}\bar{x}\; ejA + \frac{e^2}{2} j_- \frac{1}{(i\partial_-)^2} j_- + e^2 \bar{\Psi} \not{A} \frac{\gamma^+}{4i\partial_-} \not{A} \Psi \qquad j^\mu = \bar{\Psi}\gamma^\mu \Psi\,,$$



trident amplitude =  + 

On-shell



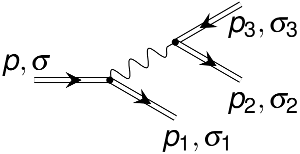
- LF Hamiltonian formalism
- On-shell: $l^2 = 0 \implies l_+ = \frac{l_\perp^2}{4l_-}$
- Instantaneous terms
- x^+ ordered: $x^+_{\mathrm{BW}} > x^+_{\mathrm{C}}$
- cf. two-step & one-step

- standard covariant formalism
- Start: off-shell l_+-integral
- LF gauge: $\frac{g_{\mu\nu} - \frac{k_\mu l_\nu + l_\mu k_\nu}{kl}}{l^2 + i\varepsilon}$
- l_+ integral $\rightarrow$ terms with $\theta(x^+_{\mathrm{BW}} - x^+_{\mathrm{C}})$ and $\delta(x^+_{\mathrm{BW}} - x^+_{\mathrm{C}})$

Pair production probability

- Sum over momenta and spin

$$\mathbb{P} = \sum \Bigg|$$

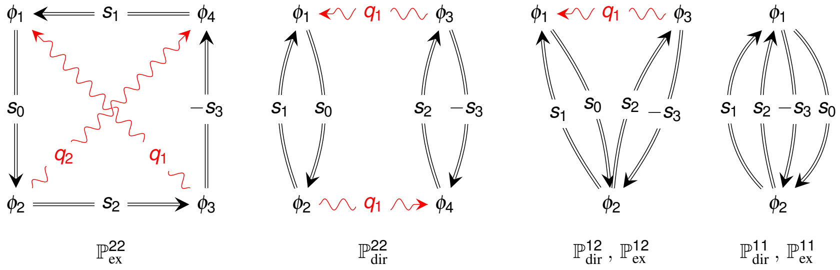


$$- \quad (p_1,\sigma_1 \leftrightarrow p_2,\sigma_2)\Bigg|^2$$

- $F_{\mu\nu}(x^+) \quad \to \quad \delta^3_{-,\perp}(p_1+p_2+p_3-p)$
- Integrate over Gaussian $p_1^\perp, p_2^\perp$ integrals
- $P_- = P_0 + P_3 > 0 \quad \to \quad (p-p_1-p_2)_- > 0$
- Probability density: $\mathbb{P} = \int_0^1 \mathrm{d}s_1 \mathrm{d}s_2 \theta(1-s_1-s_2)\mathbb{P}(s), \qquad s_i = \frac{p_{i-}}{p_-}$

Exact probability for arbitrary field shape

- LF formalism → 3 direct + 3 exchange terms



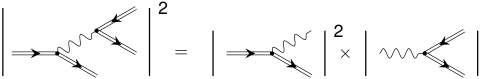
- Integrals over $\phi_i = kx_i = \omega x_i^+$. Long. momenta $s_i = \frac{kp_i}{kp}$, $q_i = 1 - s_i$

- Symmetries: $\mathbb{P}^{22}_{\rm ex}$: $\phi_1 \to \phi_2 \to \phi_3 \to \phi_4, s_1 \to -s_0 \to s_2 \to s_3 \to s_1$

- Compact expressions for arbitrary plane waves $a_\perp(x^+)$

V. Dinu & GT PRD (2018)

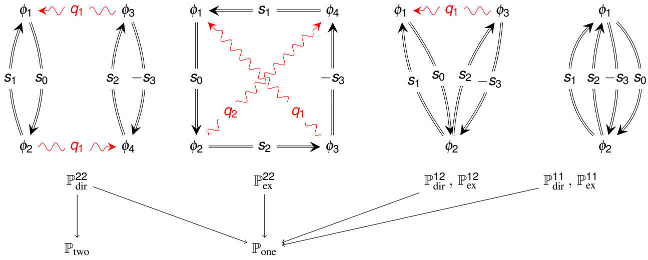
- Two-step and one-step separation:



$+$ "one-step" terms $= \mathbb{P}_{\text{two}} + \mathbb{P}_{\text{one}}$

- LF separation:

V. Dinu & GT PRD (2018)



- From now on: $\mathbb{P}_{\text{two}}$, $\mathbb{P}^{\text{dir}}_{\text{one}}$ and $\mathbb{P}^{\text{ex}}_{\text{one}}$

$a_0 \gg 1$ and the locally constant field approximation

- Constant fields: $\mathbb{P}_{\text{two}} \sim (\Delta x^+)^2$ and $\mathbb{P}_{\text{one}}^{\text{dir}} \sim \Delta x^+$

  Baier, Katkov, and Strakhovenko (1972); Ritus (1972); King and Ruhl (2013)

- $a_0 = \frac{eE}{m\omega} \gg 1$: $\qquad \mathbb{P} = a_0^2 P_2 + a_0 P_1 + P_0 + \dots$

  V. Dinu & GT PRD (2018)

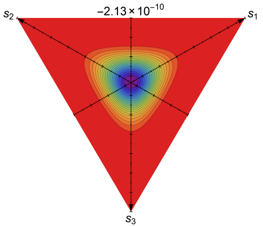
- $\mathbb{P}_{\text{two}} = a_0^2 P_2 + \mathcal{O}(a_0^0) \qquad \mathbb{P}_{\text{one}} = a_0 P_1 + \dots$

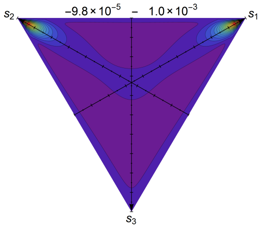
- Avoid large volume factors and include higher orders

- Both constant and non-constant fields

- Both $\mathbb{P}_{\text{dir}}$ and $\mathbb{P}_{\text{ex}}$

Constant field, $a_0 \gg 1$ and $\chi \ll 1$

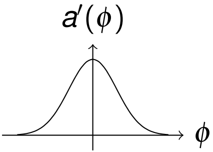




- Longitudinal momenta $s_i = kp_i/kp$ for $\chi = 1/2$ and $\chi = 16$
- Use saddle-point approx. for $\chi \ll 1$
- Constant field: $\mathbb{P}_{\rm two} \approx \alpha^2 \frac{(a_0\Delta\phi)^2}{64} e^{-\frac{16}{3\chi}}$ $\quad \mathbb{P}^{\rm dir}_{\rm one} \approx -\alpha^2 \frac{a_0\Delta\phi\sqrt{\chi}}{16\sqrt{6\pi}} e^{-\frac{16}{3\chi}}$ $\quad \mathbb{P}^{\rm ex}_{\rm one} \approx \frac{13}{18}\mathbb{P}^{\rm dir}_{\rm one}$
- $\mathbb{P}_{\rm two}$ and $\mathbb{P}^{\rm dir}_{\rm one}$ agree with literature. $\mathbb{P}^{\rm ex}_{\rm one}$ is new. V. Dinu & GT PRD (2018)
- $\mathbb{P}^{\rm ex}_{\rm one}$ as important as $\mathbb{P}^{\rm dir}_{\rm one}$

Pulsed Fields with $a_0 \gg 1$ and $\chi \ll 1$

- Pulsed plane wave $a(\phi) = a_0 f(\phi)$, $\phi = \omega x^+$, $f^{(3)}(0) = -\zeta$



- $\mathbb{P}_{\text{two}} = \alpha^2 \frac{\pi\sqrt{3}}{128} \frac{a_0^2 \chi}{\zeta} e^{-\frac{16}{3\chi}}$ $\qquad$ $\mathbb{P}^{\text{dir}}_{\text{one}} = -\alpha^2 \frac{a_0 \chi}{64\sqrt{\zeta}} e^{-\frac{16}{3\chi}}$ $\qquad$ $\mathbb{P}^{\text{ex}}_{\text{one}} = \frac{13}{18} \mathbb{P}^{\text{dir}}_{\text{one}}$

  V. Dinu & GT PRD (2018)

- $\mathbb{P}^{\text{ex}}_{\text{one}}$ on the same order as $\mathbb{P}^{\text{dir}}_{\text{one}}$ in general for $a_0 \gg 1$ and $\chi \ll 1$

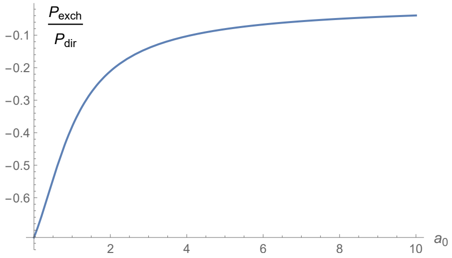
$a_0 \sim 1$ and $\chi \ll 1$

- Generalize from LCF regime ($a_0 \gg 1$) to $a_0 \sim 1$
- Sauter pulse $a'(\phi) = a_0 \text{sech}^2\phi$:

V. Dinu & GT PRD (2018)

$$\mathbb{P}^{\text{dir}}_{\text{one}} = -\frac{2}{\pi}\arctan\sqrt{1-\frac{a_0}{(1+a_0^2)\text{arccot}a_0}}\,\mathbb{P}_{\text{two}} \qquad \mathbb{P}^{\text{ex}}_{\text{one}} = \frac{13}{18}\mathbb{P}^{\text{dir}}_{\text{one}}$$

- $a_0 \sim 1$: $\mathbb{P}^{\text{dir}}_{\text{one}} \sim \mathbb{P}^{\text{ex}}_{\text{one}} \sim \mathbb{P}^{\text{dir}}_{\text{two}}$ $\qquad$ $a_0 \gg 1$: $\mathbb{P}^{\text{dir}}_{\text{one}} \sim \mathbb{P}^{\text{ex}}_{\text{one}} \ll \mathbb{P}^{\text{dir}}_{\text{two}}$



Monochromatic field

- $a'(\phi) = a_0 \cos\phi$, $a_0 \sim 1$, $\chi \ll 1$: $\quad \mathbb{P}_{\text{two}} \sim N\mathbb{P}^{\text{dir}}_{\text{one}} \sim N\mathbb{P}^{\text{ex}}_{\text{one}}$
- $\mathbb{P} = \text{prefactor} \exp\left\{-\frac{4a_0}{\chi}\left([2+a_0^2]\text{arcsinh}\frac{1}{a_0} - \sqrt{1+a_0^2}\right)\right\}$

  V. Dinu & GT PRD (2018)
- Compare with SLAC experiment:

  $\mathbb{P} \sim e^{-\frac{\sqrt{2}c}{\chi}} \qquad c_{\text{SLAC}} = 2.4 \pm 0.1(\text{stat.})\,^{+0.2}_{-0.6}(\text{syst.}) \qquad c_{\text{we}} \approx 2.46$
- Agreement
- However: too large error bars
- and too close to perturbative $\mathbb{P} \sim a_0^{8/kp}$

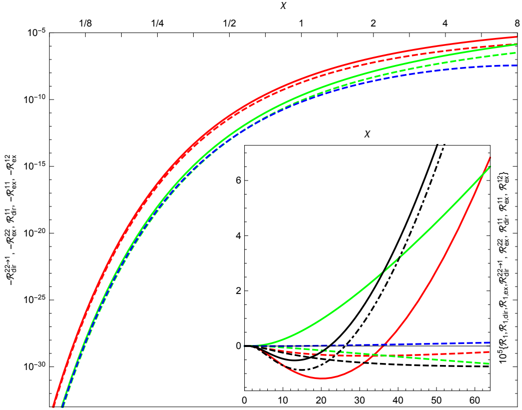
$a_0 \gg 1$ and general χ

- Consider again $a_0 \gg 1$, but larger χ

V. Dinu & GT PRD (2018)

- $\mathbb{P}^{\mathrm{ex}}_{\mathrm{one}} < 0$ for all χ
- $\mathbb{P}^{\mathrm{dir}}_{\mathrm{one}} < 0$ for $\chi \lesssim 20$
- $|\mathbb{P}^{\mathrm{ex}}_{\mathrm{one}}| > |\mathbb{P}^{\mathrm{dir}}_{\mathrm{one}}|$ for $17 \lesssim \chi \lesssim 26$
- $\mathbb{P}^{\mathrm{dir}}_{\mathrm{one}} \gg |\mathbb{P}^{\mathrm{ex}}_{\mathrm{one}}|$ for $\chi \gg 30$
- Expect α expansion not valid for $\chi \gg 1$

Narozhnyi (1980); Fedotov (2017)



Conclusions

- Strong fields $\rightarrow$ plane waves $\rightarrow$ lightfront formalism
- LF $\rightarrow$ compact $\mathbb{P}$ for arbitrary field shapes V. Dinu & GT PRD (2018)
- All terms, both $\mathbb{P}_{\mathrm{dir}}$ and $\mathbb{P}_{\mathrm{ex}}$
- Previously neglected $\mathbb{P}_{\mathrm{ex}} \sim \mathbb{P}_{\mathrm{dir}}^{\mathrm{one}}$
  - Analytically for $\chi \ll 1$ and $a_0 \gg 1$ or $a_0 \sim 1$
  - Numerically for $a_0 \gg 1$ and quite large χ
- Can apply these methods to double nonlinear Compton scattering