Power corrections to TMD factorization for particle production

I. Balitsky

JLAB & ODU

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Power corrections to TMD factorization

A typical factorization formula for production of a particle with a small transverse momentum in hadron-hadron collisions:

$$\frac{d\sigma}{d\eta d^2q_\perp} = \sum_f \int d^2b_\perp e^{i(q,b)_\perp} \mathcal{D}_{f/A}(x_A, b_\perp, \eta)\mathcal{D}_{f/B}(x_B, b_\perp, \eta)\sigma(ff \to H)$$
$$+ \text{power corrections} + \text{Y} - \text{terms}$$

When we increase transverse momentum $q_\perp^2$ of the produced particle:

- At first the leading power TMD analysis with (nonperturbative) TMDs applies,
- then at some point power corrections kick in,
- and finally at $q_\perp^2 \sim Q^2$ they are transformed into so-called Y-term making smooth transition to collinear factorization formulas.

In this talk I try to answer the question about the first transition, namely at what $q_\perp^2$ power corrections become significant.

Higgs production by gluon fusion in pp scattering

Suppose we produce a scalar particle (Higgs) in a gluon-gluon fusion.

For simplicity, assume the vertex is local:

$$\mathcal{L}_\Phi \;=\; g_\Phi \int dz\, \Phi(z) F^2(z), \qquad F^2 \equiv F^a_{\mu\nu} F_a^{\mu\nu}$$

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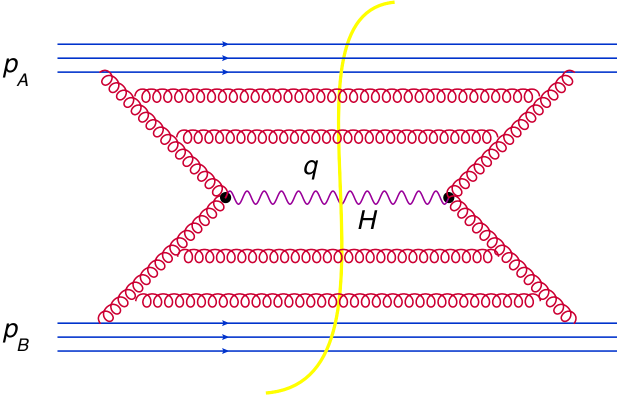


$s >> Q^2 >> q_\perp^2$

$q^2 = \; Q^2 = M_H^2$

Matrix element between hadron states $\Rightarrow \sum_X = 1$

"Hadronic tensor"

$$W(p_A,p_B,q) \stackrel{\text{def}}{=} \sum_X \int d^4x\, e^{-iqx}\langle p_A,p_B|F^2(x)|X\rangle\langle X|F^2(0)|p_A,p_B\rangle$$
$$= \int d^4x\, e^{-iqx}\langle p_A,p_B|F^2(x)F^2(0)|p_A,p_B\rangle$$

Double functional integral for W

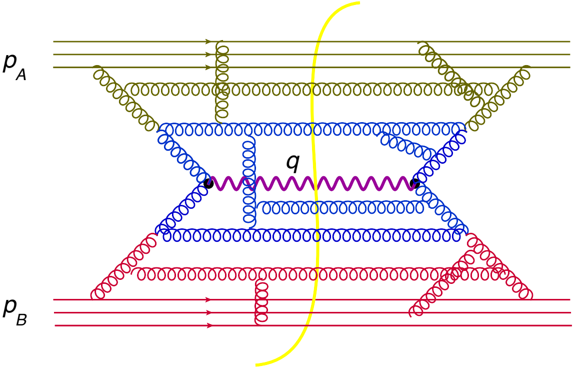
$$W(p_A,p_B,q) = \sum_X \int d^4x\, e^{-iqx}\langle p_A,p_B|F^2(x)|X\rangle\langle X|F^2(0)|p_A,p_B\rangle$$
$$= \lim_{\substack{t_f\to\infty\\ t_i\to-\infty}} \int d^4x\, e^{-iqx}\int^{\tilde{A}(t_f)=A(t_f)} D\tilde{A}_\mu DA_\mu \int^{\tilde{\psi}(t_f)=\psi(t_f)} D\tilde{\bar{\psi}}D\tilde{\psi}D\bar{\psi}D\psi\ \Psi^*_{p_A}(\vec{\tilde{A}}(t_i),\tilde{\psi}(t_i))$$
$$\times\ \Psi^*_{p_B}(\vec{\tilde{A}}(t_i),\tilde{\psi}(t_i))e^{-iS_{\rm QCD}(\tilde{A},\tilde{\psi})}e^{iS_{\rm QCD}(A,\psi)}\tilde{F}^2(x)F^2(y)\Psi_{p_A}(\vec{A}(t_i),\psi(t_i))\Psi_{p_B}(\vec{A}(t_i),\psi(t_i))$$

"Left" A,ψ fields correspond to the amplitude $\langle X|F^2(0)|p_A,p_B\rangle$,
"right" fields $\tilde{A},\tilde{\psi}$ correspond to amplitude $\langle p_A,p_B|F^2(x)|X\rangle$
The boundary conditions $\tilde{A}(t_f)=A(t_f)$ and $\tilde{\psi}(t_f)=\psi(t_f)$ reflect the sum over intermediate states X.

Rapidity factorization for particle production

Sudakov variables:

$$p \;=\; \alpha p_1 + \beta p_2 + p_\perp, \qquad p_1 \simeq p_A, \;\; p_2 \simeq p_B, \;\;\; p_1^2 = p_2^2 = 0$$



We integrate over “central” fields in the background of projectile and target fields.

Rapidity factorization for particle production

After integration over C fields

$$
\begin{aligned}
&W(p_A,p_B,q)\\
&= \int d^4x\, e^{-iqx}\int^{\tilde{A}(t_f)=A(t_f)} D\tilde{A}_\mu DA_\mu \int^{\tilde{\psi}_a(t_f)=\psi_a(t_f)} D\bar{\psi}_a D\psi_a D\bar{\tilde{\psi}}_a D\tilde{\psi}_a\\
&\quad\times e^{-iS_{\rm QCD}(\tilde{A},\tilde{\psi}_a)}e^{iS_{\rm QCD}(A,\psi_a)}\Psi^*_{p_A}(\vec{\tilde{A}}(t_i),\tilde{\psi}_a(t_i))\Psi_{p_A}(\vec{A}(t_i),\psi(t_i))\\
&\quad\times \int^{\tilde{B}(t_f)=B(t_f)} D\tilde{B}_\mu DB_\mu \int^{\tilde{\psi}_b(t_f)=\psi_b(t_f)} D\bar{\psi}_b D\psi_b D\bar{\tilde{\psi}}_b D\tilde{\psi}_b\\
&\quad\times e^{-iS_{\rm QCD}(\tilde{B},\tilde{\psi}_b)}e^{iS_{\rm QCD}(B,\psi_b)}\Psi^*_{p_B}(\vec{\tilde{B}}(t_i),\tilde{\psi}_b(t_i))\Psi_{p_B}(\vec{B}(t_i),\psi_b(t_i))\\
&\quad\times e^{S_{\rm eff}(U,V,\tilde{U},\tilde{V})}\mathcal{O}(q,x,y;A,\psi_a,\tilde{A},\tilde{\psi}_a;B,\psi_b,\tilde{B},\tilde{\psi}_b)
\end{aligned}
$$

$\mathcal{O}$ - sum of the *connected* diagrams for $F^2(x)F^2(0)$ in the background fields

$S_{\rm eff}$ - effective action (sum of disconnected diagrams = $e^{S_{\rm eff}}$).

Approximations for projectile and target fields

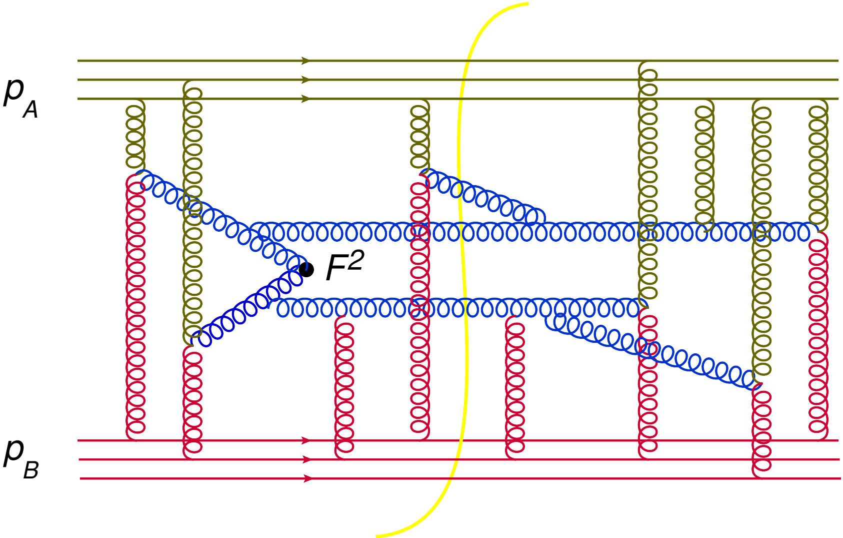
At the tree level $\beta=0$ for $A,\tilde{A}$ fields and $\alpha=0$ for $B,\tilde{B}$ fields $\Leftrightarrow$
$A=A(x_-,x_\perp),\ \tilde{A}=\tilde{A}(x_-,x_\perp)$ and $B=B(x_+,x_\perp),\ \tilde{B}=\tilde{B}(x_-,x_\perp)$.

NB: because of boundary conditions $\tilde{A}(t_f)=A(t_f)$ and $\tilde{\psi}(t_f)=\psi(t_f)$ for the purpose of calculating the integral over central fields one can set

$$A(x_-,x_\perp)=\tilde{A}(x_-,x_\perp),\quad \psi_a(x_-,x_\perp)=\tilde{\psi}_a(x_-,x_\perp)$$
$$\text{and}$$
$$B(x_+,x_\perp)=\tilde{B}(x_+,x_\perp),\quad \psi_b(x_+,x_\perp)=\tilde{\psi}_b(x_+,x_\perp).$$

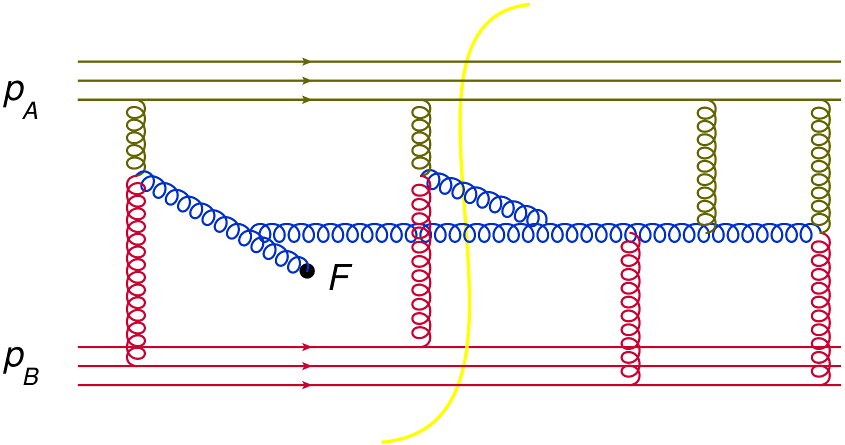
The fields A,ψ and $\tilde{A},\tilde{\psi}$ do not depend on $x_+ \Rightarrow$
if they coincide at $x_+=\infty \Rightarrow$ they coincide everywhere.
Similarly,
B,ψ_b and $\tilde{B},\tilde{\psi}_b$ do not depend on $x_- \Rightarrow$
if they coincide at $x_-=\infty$ they should be equal.

$F^2_{\mu\nu}(C)$ in the tree approximation



$F_{\mu\nu}(C)$ = sum of tree diagrams in external A and B fields

$F_{\mu\nu}(C)$ in the tree approximation



$F_{\mu\nu}(C)$ = sum of tree diagrams in external $\tilde{A}, A$ and $\tilde{B}, B$ fields with sources $\tilde{J}_\mu = D^\mu F_{\mu\nu}(\tilde{A} + \tilde{B})$ and $J_\mu = D^\mu F_{\mu\nu}(A + B)$

Since $\tilde{A} = A$ and $\tilde{B} = B$ the sources and background fields are the same to the left and to the right of the cut

$\Rightarrow$

$F_{\mu\nu}(C)$ is a sum of diagrams with *retarded* Green functions

(F. Gelis, R. Venugopalan)

$F_{\mu\nu}(C)$ in the tree approximation

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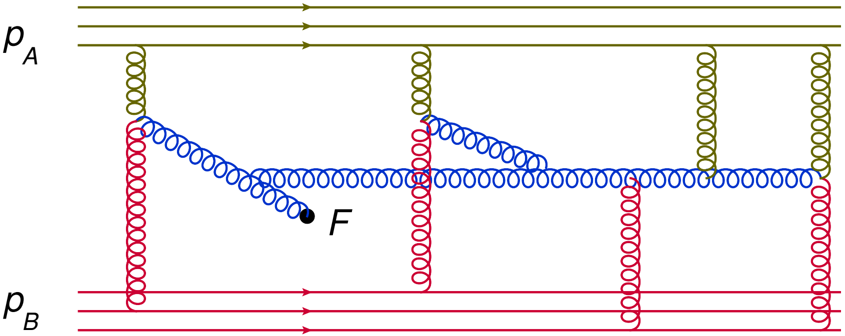

Classical solution

The sum of diagrams with retarded Green functions ⇔ solution of classical YM equations

$$D^\nu F^a_{\mu\nu} = \sum_f g\bar{\psi}^f t^a \gamma_\mu \psi^f, \quad (\not{P} + m_f)\psi^f = 0$$

with boundary conditions

$$A_\mu(x) \overset{x_+ \to -\infty}{=} \bar{A}_\mu(x_-, x_\perp), \quad \psi(x) \overset{x_+ \to -\infty}{=} \psi_a(x_-, x_\perp)$$
$$A_\mu(x) \overset{x_- \to -\infty}{=} \bar{B}_\mu(x_+, x_\perp), \quad \psi(x) \overset{x_- \to -\infty}{=} \psi_b(x_+, x_\perp)$$

following from $C_\mu, \psi_c \overset{t \to -\infty}{\to} 0$.
The projectile and target fields satisfy YM equations

$$D^\nu F^a_{\mu\nu} = \sum_f g\bar{\psi}^f_a t^a \gamma_\mu \psi^f_a, \quad (\not{P} + m_f)\psi^f_a = 0$$
$$D^\nu F^a_{\mu\nu} = \sum_f g\bar{\psi}^f_b t^a \gamma_\mu \psi^f_b, \quad (\not{P} + m_f)\psi^f_b = 0$$

Method of solution: start with $\bar{A}_\mu + \bar{B}_\mu$ and correct by computing Feynman diagrams (with retarded propagators) with a source $J_\nu = D^\mu F^{\mu\nu}(U + V)$

Classical solution in $A_+^{\text{projectile}} = A_-^{\text{target}} = 0$ gauge

Convenient gauge: $A_+ = 0$ for the projectile and $A_- = 0$ for the target.

$$
\begin{aligned}
U_i(x_-, x_\perp) \;\sim\; m_\perp, \quad U_-(x_-, x_\perp) \;\sim m_\perp^2/\sqrt{s}, \quad U_+ \;=\; 0 \\
V_i(x_+, x_\perp) \;\sim\; m_\perp, \quad V_+(x_+, x_\perp) \;\sim\; m_\perp^2/\sqrt{s}, \quad V_- \;=\; 0
\end{aligned}
$$

and we have to solve

$$
D^\nu F^a_{\mu\nu} \;=\; \sum_f g\bar{\psi}^f t^a \gamma_\mu \psi^f, \qquad (\not{P} + m_f)\psi^f \;=\; 0
$$

with boundary conditions

$$
\begin{aligned}
A_\mu(x) \overset{x_+ \to -\infty}{=} U_\mu(x_-, x_\perp), \qquad \psi(x) \overset{x_+ \to -\infty}{=} \Sigma_a(x_-, x_\perp) \\
A_\mu(x) \overset{x_- \to -\infty}{=} V_\mu(x_+, x_\perp), \qquad \psi(x) \overset{x_- \to -\infty}{=} \Sigma_b(x_+, x_\perp)
\end{aligned}
$$

Now we start with $U_\mu + V_\mu$ and compute Feynman diagrams (with retarded propagators) with a source $J_\nu = D^\mu F^{\mu\nu}(U + V) \;\sim\; m_\perp^3$

Expansion at small momentum transfer

The solution of YM equations in general case (scattering of two "color glass condensates") is yet unsolved problem.
Fortunately, for our case of particle production with $\frac{q_\perp}{Q} \ll 1$ we can use this small parameter and construct the approximate solution as a series in $\frac{q_\perp}{Q}$.

Example:

$$A_- = U_- + \int dz (x|\frac{1}{p^2 + i\epsilon p_0} p_-|z)[U_j, V^j](z) = U_- + \frac{1}{2}\int dz (x|\frac{1}{\alpha - \frac{p_\perp^2}{\beta s} + i\epsilon}|z)[U_j, V^j](z)$$

The characteristic $\alpha \geq \alpha_q$ and $\beta \geq \beta_q$ so $\alpha \gg \frac{p_\perp^2}{\beta s}$

$$\Rightarrow (x|\frac{1}{\alpha - \frac{p_\perp^2}{\beta s} + i\epsilon}|z) = (x|\frac{1}{\alpha + i\epsilon}|z) + (x|\frac{1}{\alpha + i\epsilon}\frac{p_\perp^2}{\beta s}\frac{1}{\alpha + i\epsilon}|z) + \dots$$

and in the leading order in $p_\perp/p_\parallel$ we get

$$\begin{aligned} A_-(x) &= U_-(x_-, x_\perp) + \frac{1}{2}\int dz (x|\frac{1}{\alpha + i\epsilon}|z)[U_j, V^j](z) \\ &= U_-(x_-, x_\perp) - \frac{i}{2}\int_{-\infty}^{x_-} dx'_- \, [U_j(x'_-, x_\perp), V^j(x_+, x_\perp)] \end{aligned}$$

Gluon fields in the leading order in $p_\perp^2/p_\parallel^2 \sim q_\perp^2/Q^2$

With the expansion

$$\frac{1}{p^2+i\epsilon p_0} = \frac{1}{p_\parallel^2 - p_\perp^2 + i\epsilon p_0} = \frac{1}{p_\parallel^2} - \frac{1}{p_\parallel^2+i\epsilon p_0}p_\perp^2\frac{1}{p_\parallel^2+i\epsilon p_0} + \dots$$

the dynamics in transverse space is trivial.

$$\begin{aligned}
\text{Gluon fields :} \qquad & F_{-i}^{(-1)} = V_{-i}, \quad F_{+i}^{(-1)} = U_{+i}, \\
& F_{+-}^{(-1)} = U_{+-} + V_{+-} - iU_j^{ab}V^{bj} \\
& F_{-i}^{(0)a} = U_{-i}^a - iU_-^{ab}V_i^b - \frac{i/\sqrt{s}}{2(\alpha+i\epsilon)}\tilde{L}_i^{(0)} - \mathcal{D}_i^{ab}V_j^{bc}\frac{1/\sqrt{s}}{2(\alpha+i\epsilon)}U^{cj}, \\
& F_{+i}^{(0)a} = V_{+i}^a - iV_+^{ab}U_i^b - \frac{i/\sqrt{s}}{2(\beta+i\epsilon)}\tilde{L}_i^{(0)} - \mathcal{D}_i^{ab}U_j^{bc}\frac{1/\sqrt{s}}{2(\beta+i\epsilon)}V^{cj}, \\
& F_{ik}^{(0)} = U_{ik} + V_{ik} - i[U_i, V_k] - i[V_i, U_k],
\end{aligned}$$

where $F^{(n)}$ denotes term of n-th order in expansion in powers of $\frac{p_\perp^2}{p_\parallel^2} \sim \frac{q_\perp^2}{Q^2}$.

$$\begin{aligned}
L_i^{(0)a} \equiv\; & -iU^{jab}V_{ji} - iV^{jab}U_{ji} - i\mathcal{D}_j^{ab}(U^{jbc}V_i^c + V^{jbc}U_i^c) \\
& - i\big(U_{+-}^{ab}V_i^b - V_{+-}^{ab}U_i\big) + \bar{\Sigma}_a t^a\gamma_i\Sigma_b + \bar{\Sigma}_b t^a\gamma_i\Sigma_a
\end{aligned}$$

Gluon fields in the leading order in $p_\perp^2/p_\parallel^2 \sim q_\perp^2/Q^2$

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F_{+i}^{(0)a} &= V_{+i}^a - iV_+^{ab}U_i^b - \frac{i/\sqrt{s}}{2(\beta+i\epsilon)}\tilde{L}_i^{(0)} - \mathcal{D}_i^{ab}U_j^{bc}\frac{1/\sqrt{s}}{2(\beta+i\epsilon)}V^{cj},\\
F_{ik}^{(0)} &= U_{ik} + V_{ik} - i[U_i, V_k] - i[V_i, U_k],
\end{aligned}$$

We integrate over α without cutoff $\alpha > \sigma$ since the contour over α can be removed from the pole to the region of large α (if there is no pinch). Similarly, we integrate over all β's.

(Different from SCET where they keep the cutoffs $\alpha > \sigma_b$ and $\beta > \sigma_a$).

TMD factorization with power corrections (A. Tarasov and I.B.)

At the tree level

$$F^2(x) \;=\; 4U^{ai}_{+}(x)V^{a}_{-i}(x) + 2f^{mac}f^{mbd}\Delta^{ij,kl}U^a_i(x)U^b_j(x)V^c_k(x)V^d_l(x) \;+\; \dots$$

$$\Delta^{ij,kl} \;\equiv\; g^{ij}g^{kl} - g^{ik}g^{jl} - g^{il}g^{jk}$$

$\Rightarrow$ in the region $s \gg Q^2 \gg Q_\perp^2$

$$W(p_A,p_B,q) \;=\; \frac{16}{N_c^2-1}\int d^2x_\perp\; e^{i(q,x)_\perp}\frac{2}{s}\int dx_- dx_+\; e^{-i\alpha_q x_- - i\beta_q x_+}$$

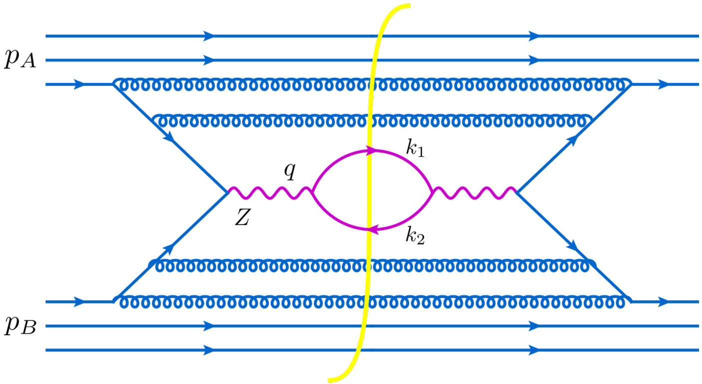
$$\times\;\Big\{\langle p_A|U^{mi}_{+}(x_-,x_\perp)U^{mj}_{+}(0)|p_A\rangle\langle p_B|V^{n}_{-i}(x_+,x_\perp)V^{n}_{-j}(0)|p_B\rangle$$

$$-\;\frac{N_c^2}{N_c^2-4}\frac{\Delta^{ij,kl}}{Q^2}\int_{-\infty}^{x_-} d\frac{2}{s}x'_-\; d^{abc}\langle p_A|U^{a}_{+i}(x_-,x_\perp)U^{b}_{+j}(x'_-,x_\perp)U^{c}_{+r}(0)|p_A\rangle$$

$$\times\int_{-\infty}^{x_-} d\frac{2}{s}x'_+\; d^{mnl}\langle p_B|V^{m}_{-k}(x_+,x_\perp)V^{n}_{-l}(x'_+,x_\perp)V^{n}_{-r}(0)|p_B\rangle \;+\; x\leftrightarrow 0\Big\}$$

The correction is $\sim \frac{Q_\perp^2}{Q^2}$.

Z-boson production in pp scattering



$$\frac{d\sigma}{dQ^2 dy dq_\perp^2} = \frac{e^2 Q^2}{192 s s_W^2 c_W^2} \frac{1 - 4s_W^2 + 8s_W^4}{(m_Z^2 - Q^2)^2 + \Gamma_Z^2 m_Z^2} [-W_Z(p_A, p_B, q)],$$

$$W_Z(p_A, p_B, q) \equiv \frac{1}{(2\pi)^4} \sum_X \int d^4x\, e^{-iqx} \langle p_A, p_B | J_\mu(x) | X \rangle \langle X | J^\mu(0) | p_A, p_B \rangle$$

Leading-N_c power corrections

Power corrections are $\sim$ leading twist $\times \frac{q_\perp^2}{Q^2} \times \left(1 + \frac{1}{N_c} + \frac{1}{N_c^2}\right)$.

(Pleasant) surprise: terms not suppressed by $\frac{1}{N_c}$ are determined by the leading-twist terms due to QCD equations of motion

Leading twist:

$$\frac{1}{8\pi^3 s}\int dx_- d^2x_\perp \; e^{-i\alpha x_- + i(k,x)_\perp} \; \langle A|\hat{\bar{\psi}}_f(x_-, x_\perp) \; \not{p}_2 \hat{\psi}_f(0)|A\rangle \; = \; f_{1f}(\alpha, k_\perp^2)$$

Power correction:

$$\frac{g}{8\pi^3 s}\int dx_- dx_\perp \; e^{-i\alpha_q x_- + i(k,x)_\perp}$$

$$\times \; \langle A|\hat{\bar{\psi}}^f(x_-, x_\perp) \; \not{p}_2[\hat{U}_i(x_-, x_\perp) - i\gamma_5 \hat{\tilde{U}}_i(x_-, x_\perp)]\hat{\psi}^f(0)|A\rangle$$

$$= \; -k_i f_{1f}(\alpha_q, k_\perp^2) \; + \; O(\alpha_q).$$

(Mulders & Tangerman, 1996)

Power corrections to Z-boson production (A. Tarasov and I.B.)

Result:

$$
\begin{aligned}
W_Z(p_A,p_B,q) \;&=\; -\frac{e^2}{8s_W^2c_W^2N_c}\int d^2k_\perp \\
&\times \Bigg[\Big\{(1+a_u^2)\Big[1-2\frac{(k,q-k)_\perp}{Q^2}\Big] f_{1u}(\alpha_z,k_\perp)\bar{f}_{1u}(\beta_z,q_\perp-k_\perp) \\
&+ 2(a_u^2-1)\frac{k_\perp^2(q-k)_\perp^2}{m_N^2Q^2} h_{1u}^\perp(\alpha_z,k_\perp)\bar{h}_{1u}^\perp(\beta_z,q_\perp-k_\perp) + (\alpha_z\leftrightarrow\beta_z)\Big\} \\
&+ \Big\{u\leftrightarrow c\Big\} + \Big\{u\leftrightarrow d\Big\} + \Big\{u\leftrightarrow s\Big\}\Bigg]\Big(1 \;+\; O\big(\frac{1}{N_c}\big)\Big).
\end{aligned}
$$

$$a_{u,c} = (1-\tfrac{8}{3}s_W^2), \quad a_{d,s} = (1-\tfrac{4}{3}s_W^2)$$

Power correction is $\sim \frac{q_\perp^2}{Q^2}$.

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&+2(a_u^2-1)\frac{k_\perp^2\,(q-k)_\perp^2}{m_N^2Q^2}h_{1u}^\perp(\alpha_z,k_\perp)\bar{h}_{1u}^\perp(\beta_z,q_\perp-k_\perp)\;+\;(\alpha_z\leftrightarrow\beta_z)\Big\} \\
&+\Big\{u\leftrightarrow c\Big\}+\Big\{u\leftrightarrow d\Big\}+\Big\{u\leftrightarrow s\Big\}\Big]\Big(1\;+\;O\big(\frac{1}{N_c}\big)\Big).
\end{aligned}
$$

$$a_{u,c}=(1-\tfrac{8}{3}s_W^2),\quad a_{d,s}=(1-\tfrac{4}{3}s_W^2)$$

Power correction is $\sim\frac{q_\perp^2}{Q^2}$.

($\frac{1}{N_c}$ and $\frac{1}{N_c^2}$ terms involve twist-3 quark-quark-gluon TMDs which do not reduce to leading-twist distributions).

Estimate of power corrections

If $Q^2 \gg k_\perp^2 \gg m_N^2$ we can approximate

$$f_1(\alpha_z, k_\perp^2) \simeq \frac{f(\alpha_z)}{k_\perp^2}, \quad h_1^\perp(\alpha_z, k_\perp^2) \simeq \frac{m_N^2 h(\alpha_z)}{k_\perp^4}$$

$$\Rightarrow W_Z(p_A, p_B, q) \simeq -\frac{e^2}{8s_W^2 c_W^2 N_c} \int d^2k_\perp \frac{1}{k_\perp^2 (q-k)_\perp^2} \Big[1 - 2\frac{(k, q-k)_\perp}{Q^2}\Big]$$
$$\times \sum_f (1 + a_f^2) [f_f(\alpha_z) \bar{f}_f(\beta_z) + \bar{f}_f(\alpha_z) f_f(\beta_z)]$$

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$$\Rightarrow W_Z(p_A, p_B, q) \simeq -\frac{e^2}{8 s_W^2 c_W^2 N_c} \int d^2k_\perp \frac{1}{k_\perp^2 (q-k)_\perp^2} \Big[1 - 2\frac{(k, q-k)_\perp}{Q^2}\Big]$$
$$\times \sum_f (1 + a_f^2) [f_f(\alpha_z)\bar{f}_f(\beta_z) + \bar{f}_f(\alpha_z) f_f(\beta_z)]$$

With logarithmic accuracy

$$W_Z(p_A, p_B, q) = -\frac{\pi e^2}{4 s_W^2 c_W^2 N_c} \Big[\frac{1}{q_\perp^2} \ln \frac{q_\perp^2}{m_N^2} + \frac{1}{Q^2} \ln \frac{Q^2}{q_\perp^2}\Big]$$
$$\times \sum_f (1 + a_f^2) [f_f(\alpha_z)\bar{f}_u(\beta_z) + \bar{f}_f(\alpha_z) f_f(\beta_z)]$$

$\Rightarrow$ power correction reaches 10% level at $q_\perp \sim \frac{1}{4} Q \sim$ 20 GeV

Conclusions

1. Conclusions
   - Higher-twist power correction to H and Z production at $s \gg q^2 \gg q_\perp^2$ are calculated. The estimate gives 10% corrections at $q_\perp \sim \frac{1}{4}Q$.
2. Outlook
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Thank you for attention!