Fermion-antifermion phenomenology in Minkowski space

Jorge H. A. Nogueira

Università di Roma 'La Sapienza' and INFN, Sezione di Roma (Italy)
Instituto Tecnológico de Aeronáutica, (Brazil)

Supervisors: Profs. T. Frederico (ITA) and G. Salmè (INFN)

Collaborators: Dr. E. Ydrefors, Prof. W. de Paula and Dr. C. Mezrag

Light Cone 2018
Jefferson Lab, Newport News/US
May 15, 2018

Outline

General goals

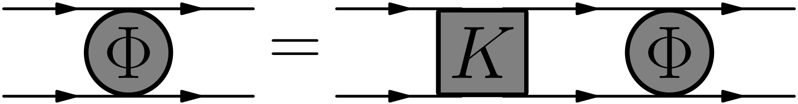
- Bethe-Salpeter equation to study non-perturbative systems;
- Fully covariant relativistic description in Minkowski space;
- Understand step-by-step the degrees of freedom;
- How bad is to ignore the crosses in the BSE kernel?
- Introducing color factors and the large N_c limit;
- Make the numerics feasible;
- No Fock space truncation;
- Phenomenological studies within the approach;

Bethe-Salpeter equation

- The BSE for the bound state with total four momentum $p^2 = M^2$, composed of two scalar particles of mass m reads

$$\Phi(k,p) = S(p/2+k)S(p/2-k)\int \frac{d^4k'}{(2\pi)^4} iK(k,k',p)\Phi(k',p),$$

$$S(k) = \frac{i}{k^2 - m^2 + i\epsilon} \quad \text{: Feynman propagator}$$



- The kernel K is given as a sum of irreducible Feynman diagrams (ladder, cross-ladder, etc).

E. E. Salpeter and H. A. Bethe, Phys. Rev. 84, 1232 (1951)

N. Nakanishi, Graph Theory and Feynman Integrals (Gordon and Breach, New York, 1971)

Nakanishi integral representation

- General representation for N-leg transition amplitudes;
- 2-point correlation function: Kallen-Lehmann spectral representation;
- For the vertex function (Bound state) - 3-leg amplitude:

$$\Phi(k,p) = \int_{-1}^{1} dz' \int_{0}^{\infty} d\gamma' \frac{g(\gamma', z'; \kappa^2)}{(\gamma' + \kappa^2 - k^2 - (p \cdot k)z' - i\epsilon)^3}, \quad \kappa^2 = m^2 - M^2/4$$

where $\gamma \equiv |\mathbf{k}_\perp|^2 \in [0, \infty)$ and $z \equiv 2\xi - 1 \in [-1, 1]$ with $\xi \in [0, 1]$

- All dependence upon external momenta in the denominator;
- Allows to recognize the singular structure and deal with it analytically;
- Weight function $g(\gamma', z')$ is the unknown quantity to be determined numerically;

T. Frederico, G. Salme and M. Viviani, Phys. Rev. D 85, 036009 (2012)

Light-front projection

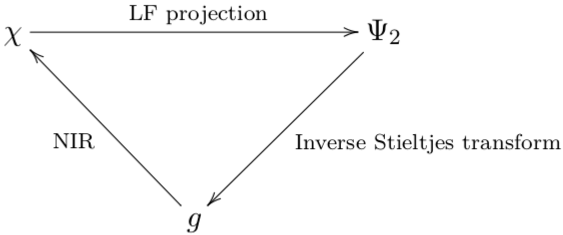
- Much easier to treat Minkowski space poles properly;
- Simpler dynamics of the propagators/amplitudes within LF (See talk by Prof. Ji);
- Easy connection with LFWF:
  - Introduce the LF variables $k_\pm = k_0 \pm k_z$;
  - Valence LFWV from the BS amplitude:

$$\psi_{n=2/p}(\xi, \mathbf{k}_\perp) = \frac{p^+}{\sqrt{2}}\, \xi\, (1-\xi) \int_{-\infty}^{\infty} \frac{dk^-}{2\pi}\, \Phi(k,p),$$

  - Corresponding to eliminate the relative LF time $t + z = 0$;
- Essential in this approach to solve BSE directly in Minkowski space;

Relations: LF, NIR and BS amplitude

- The Nakanishi integral representation (NIR) gives the Bethe-Salpeter amplitude χ (BSA) through the weight function g;
- The Light-Front projection of the BSA gives the valence light-front wave function (LFWF) Ψ_2;
- The inverse Stieltjes transform gives g from the valence LFWF;



Carbonell, Frederico, Karmanov Phys.Lett. B769 (2017) 418-423

BSE in Minkowski space

- Applying the NIR on both sides of the BSE and integrating over k_- leads to the integral equation:

$$\int_0^\infty d\gamma' \frac{g(\gamma', z; \kappa^2)}{[\gamma + \gamma' + z^2 m^2 + (1 - z^2)\kappa^2]^2} =$$
$$\int_0^\infty d\gamma' \int_{-1}^{1} dz' V(\alpha; \gamma, z, \gamma', z') g(\gamma', z'; \kappa^2)$$

where V is expressed in terms of the BS interaction kernel.

  - Ladder approx. - agreement among different groups [1];
  - Cross-ladder impact; suppression with color dof [2];
  - Scattering length; Spectroscopy and LF momentum distributions of the excited states [3];
  - Agreement with BSE in Euclidean space [4];

[1] Carbonell, Karmanov EPJA 27 (2006) 1; EPJA 46 (2010) 387; Frederico, Salmè, Viviani PRD 89 (2014) 016010
[2] Carbonell, Karmanov EPJA 27 (2006) 11; Gigante, JHAN, Ydrefors, Gutierrez, Karmanov, Frederico PRD 95 (2017) 056012; JHAN, Chueng-Ryong Ji, Ydrefors, Frederico Phys.Lett. B777 (2018) 207-211
[3] Frederico, Salmè, Viviani EPJC 75 (2015) 398; Gutierrez et al PLB 759 (2016) 131
[4] Gigante, JHAN, Ydrefors, Gutierrez, Int.J.Mod.Phys.Conf.Ser. 45 (2017) 1760055; Gutierrez et al PLB 759 (2016) 131

One example to support the hypothesis

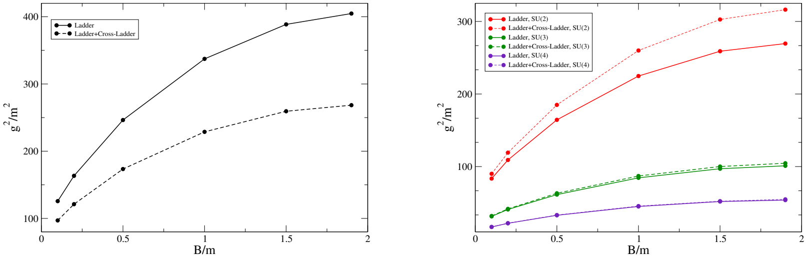


Figure: Coupling constant for various values of the binding energy B obtained by using the Bethe-Salpeter ladder (L) and ladder plus cross-ladder (CL) kernels, for an exchanged mass of $\mu = 0.5m$. In the upper panels are shown the results computed with no color factors. Similarly, in the lower panels are compared the results for $N = 2, 3$ and 4 colors.

- Suppression is already pretty good for $N_c = 3$ - might support the truncation at the ladder...at least within this system.

JHAN, C.-R. Ji, E. Ydrefors and T. Frederico, Phys.Lett. B777 (2018) 207-211

Fermion-antifermion BSE in Minkowski space

- Introducing spin

$$\Phi(k,p) = S(p/2+k) \int d^4k' \, F^2(k-k') \, iK(k,k') \, \Gamma_1 \, \Phi(k',p) \, \hat{\Gamma}_2 \, S(k-p/2)$$

where $\Gamma_1 = \Gamma_2 = 1 \ (scalar), \ \gamma_5 \ (pseudo), \ \gamma^\mu \ (vector)$

$$iK_V^{\mu\nu}(k,k') = -i\, g^2 \frac{g^{\mu\nu}}{(k-k')^2 - \mu^2 + i\epsilon}, \quad F(k-k') = \frac{(\mu^2 - \Lambda^2)}{[(k-k')^2 - \Lambda^2 + i\epsilon]}$$

- Taking benefit from orthogonality properties for the decomposition

$$\Phi(k,p) = \sum_{i=1}^{4} S_i(k,p)\phi_i(k,p)$$

where the spin dependent structures are $S_1 = \gamma_5$, $S_2 = \frac{\not{p}}{M}\gamma_5$, $S_3 = \frac{k\cdot p}{M^3}\not{p}\,\gamma_5 - \frac{1}{M}\not{k}\gamma_5$ and $S_4 = \frac{i}{M^2}\sigma^{\mu\nu}p_\mu k_\nu\,\gamma_5$

- The scalar amplitudes ϕ_i are represented by the NIR;
  - In the equal mass case, symmetry under the exchange of the particles simplifies the problem;
  - $g_j(\gamma', z'; \kappa^2)$ expanded as Laguerre(γ) $\times$ Gegenbauer(z);

Extra singular contribution of the fermionic system

- The coupled integral equation system is given by

$$\psi_i(\gamma,z) = g^2 \sum_j \int_{-1}^{1} dz' \int_0^\infty d\gamma' \, g_j(\gamma',z';\kappa^2) \, \mathcal{L}_{ij}(\gamma,z,\gamma',z';p)$$

  - S_i operators + fermionic propagators: $(k^-)^n$ extra singularities;
  - Singularities have generic form:

$$\mathcal{C}_n = \int_{-\infty}^{\infty} \frac{dk^-}{2\pi} (k^-)^n \, \mathcal{S}(k^-,v,z,z',\gamma,\gamma') \qquad n = 0,1,2,3$$

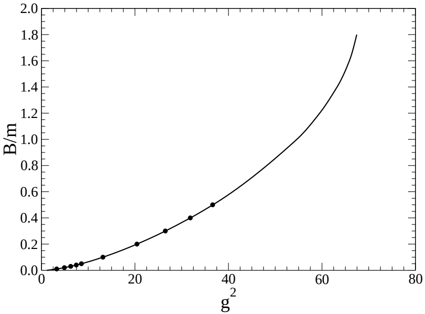
  - End-point singularities can be analytically treated by

$$\mathcal{I}(\beta,y) = \int_{-\infty}^{\infty} \frac{dx}{[\beta x - y \mp i\epsilon]^2} = \pm \frac{2\pi i \delta(\beta)}{[-y \mp i\epsilon]}$$

de Paula, Frederico, Salmè, Viviani PRD 94 (2016) 071901; EPJC 77 (2017) 764
Yan et al PRD 7 (1973) 1780
Pole-dislocation method: de Melo et al. NPA631 (1998) 574C, PLB708 (2012) 87

Coupling Constants

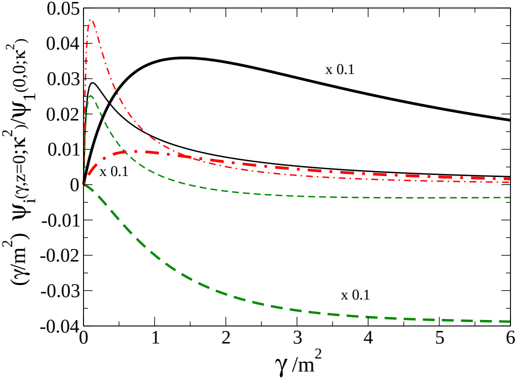
- Vector coupling as a function of the binding energy for $\mu/m = 0$;



- Dots: Kernel regularized by a cutoff;
  - No analytical treatment of the singularities;
- Agreement also with results in Euclidean space (for the scalar exchange) - see [2];

[1] Carbonell, Karmanov EPJA 46 (2010) 387
[2] de Paula, Frederico, Salmè, Viviani PRD 94 (2016) 071901, EPJC 77 (2017) 764

High-momentum tails



- LF amplitudes ψ_i times γ/m^2 at fixed $z = 0$ ($\xi = 1/2$);
  - Thin lines $B/m = 0.1$ and thick 1.0 ;
  - Solid: $i = 1$, Dashed: $i = 2$, dash-dot: $i = 4$, $\psi_3 = 0$ for $z = 0$;
  - As expected for the pion valence amplitude;

X. Ji et al, PRL 90 (2003) 241601; Brodsky, Farrar PRL 31 (1973) 1153

Valence probabilities

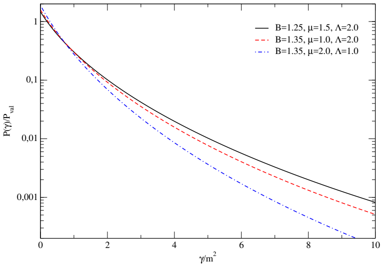
- By properly normalizing the BSE we can study the valence probabilities of the bound states;
- Taking, for instance, $\mu/m = 0.15$ and a cutoff $\Lambda/m = 2$ for the vertex form factor (fermonic case):

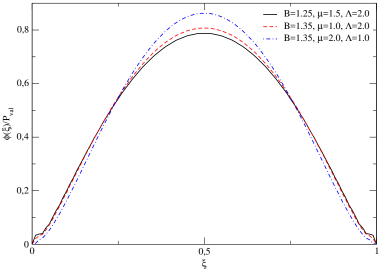
B/m	$\mathcal{P}^F_{val}$	$\mathcal{P}^B_{val}$
0.01	0.96	0.94
0.1	0.78	0.80
1.0	0.68	0.67

- Results are similar for massless vector exchange;
- Very low $\mathcal{P}^F_{val}$: higher Fock components are extremely important;
- Lack of color confining kernel might be playing a role;

Mock pion

- Guideline for the mock pion input parameters:
  - Gluon effective mass ≈ 500 MeV (Landau Gauge LQCD) [1];
  - $m_q \approx 250$ MeV [2]; $M_\pi = 140$ MeV fixed;
  - $\Lambda/m = 1, 2, 3$
  - $\alpha_s = g^2/(4\pi)(1-\mu^2/\Lambda^2)^2$; Reasonable rescaled coupling constant in the infrared region [3];
- Transverse and longitudinal momentum valence distributions for different sets of parameters:

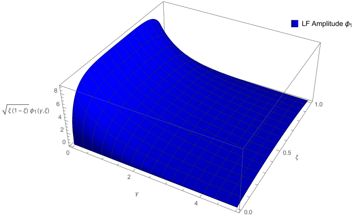


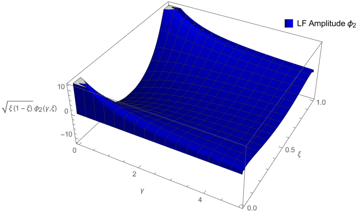


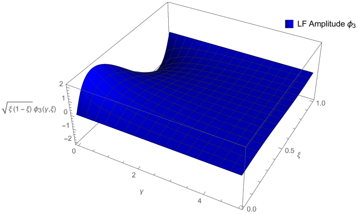
[1] Oliveira, Bicudo, JPG 38 (2011) 045003; Duarte, Oliveira, Silva, Phys. Rev. D 94 (2016) 01450240
[2] Parappilly, et al, PR D73 (2006) 054504
[3] A. Deur, S.J. Brodsky, G.F. de Teramond, Prog. Part. Nucl. Phys. 90, 1 (2016)
[4] de Paula, TF, Pimentel, Salmè, Viviani, EPJC 77 (2017) 764; de Paula et al, in preparation

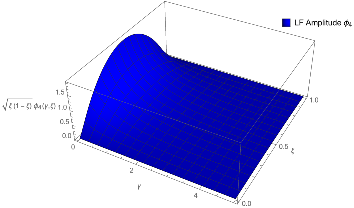
3D LF amplitudes

- Dynamical observables: the LFWF components;
  - ($B/m = 1.35$, $\mu/m = 2.0$, $\Lambda/m = 1.0$, m_q=215 MeV): $f_\pi = 96$ MeV, $P_{val} = 0.68$
  - Other observables are straightforward to compute once you have BS amplitude solution;









Conclusions

- NIR + LF projection $\rightarrow$ Showing to be essential tools to deal with the BSE in Minkowski space;
  - Many systems already treated and several subtle points under control;
  - Approach is even more robust - now able to deal properly with all singularities within fermionic systems;
  - Valence is far from enough once spin dof is included;
  - Approach gives you all the information beyond the valence;
  - Ladder approx. supported by the color suppression of the non-planar diagrams;
- Simple and direct connection with physical observables;
- More sensitive numerics - e.g. derivatives of the basis;
  - Exploration of new numerical methods is important;
- Still the most stable method within Minkowski space (see talk by E. Ydrefors);
- Essential features need to be included;
  - Confining kernel;
  - Self-energies and vertex corrections;

Outlook

- Address color confinement to the kernel;
  - 1st step: ansatz that connects to the superconformal LFH confining potential - simply (and unique) harmonic oscillator;
  - $q\bar{q}$ bound state in 1+1 dimensions and how it matches with DLCQ;
    - Can bring some understanding of some features within the approach;
  - More formal possibility: summation of H graphs in the kernel;
- Self-energies, vertex corrections;
  - Other approaches (LFH, LQCD, DSE) can inspire a first step;
  - Start by simpler phenomenological ways of implementing it, such as by models inspired by Lattice QCD (see talk by Prof. Frederico);
  - DSE fully in Minkowski space by means of spectral representation;
- Unequal mass case with possible applications for other mesons;
- Form factors, PDFs, TMDs, Fragmentation functions...
- Fermion-boson BSE was solved (in preparation);
  - Possible phenomenological applications for baryons;

A. V. Smirnov, V. A. Smirnov and M. Steinhauser, Phys. Rev. Lett. 104, 112002 (2010)

Thank you!