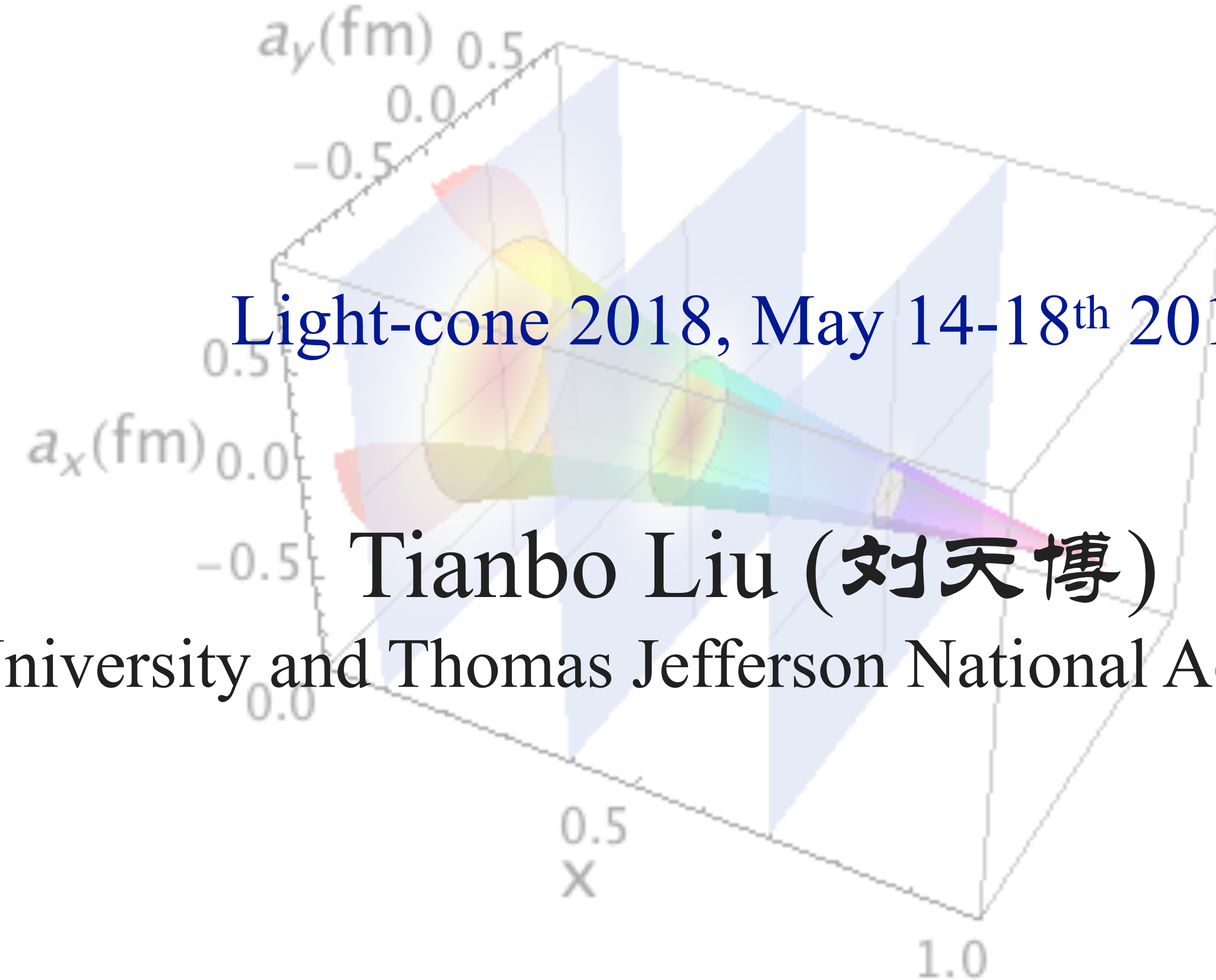
Generalized Parton Distributions in Light-Front Holographic QCD



Light-cone 2018, May 14-18th 2018

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Gauge/Gravity Duality and LF holography

Maldacena’s conjecture

J.M. Maldacena, Int. J. Theor. Phys. 38, 1113 (1999).

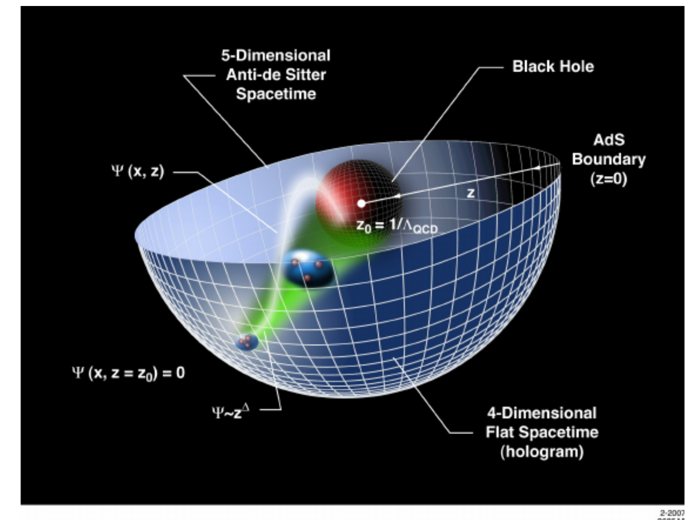
Gauge field theory in d-dim

$\Updownarrow$

Gravitational theory in $d+1$-dim

A realization: AdS / CFT

semiclassical gravity approximation to strongly coupled QFTs



[Brodsky & de Téramond]

Light-front holographic QCD

QCD: conformal symmetry is broken by quark masses and quantum effects

Asymptotic freedom

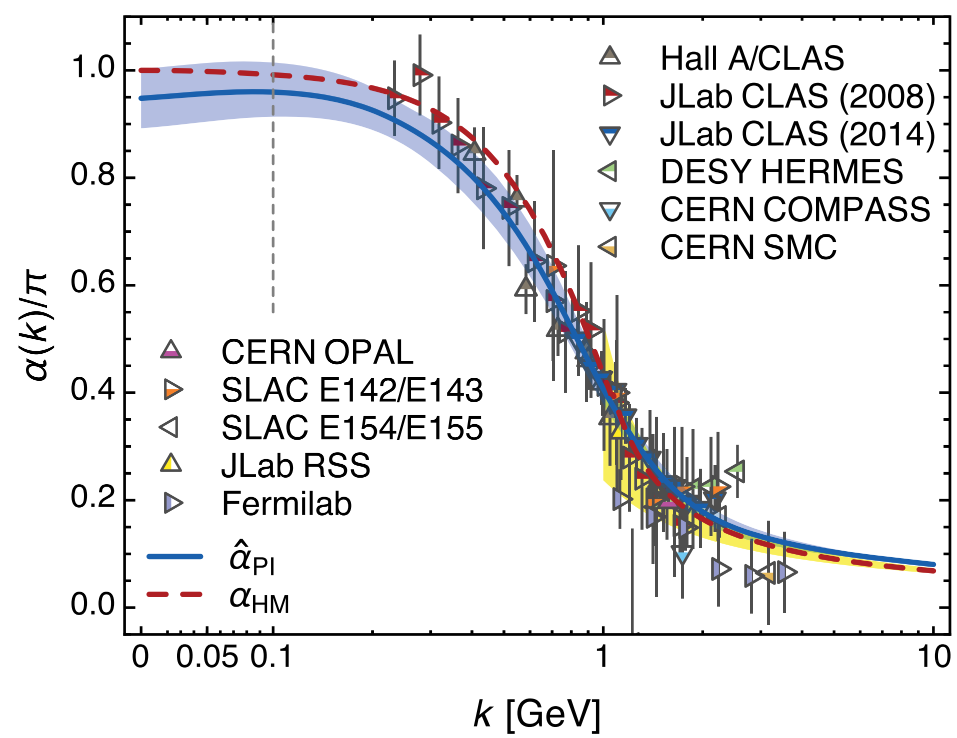
Confinement and an infrared fixed point

“bottom-up” approach: modify the background AdS space

impact LF variable-$\zeta \Leftrightarrow z$ holographic variable in AdS

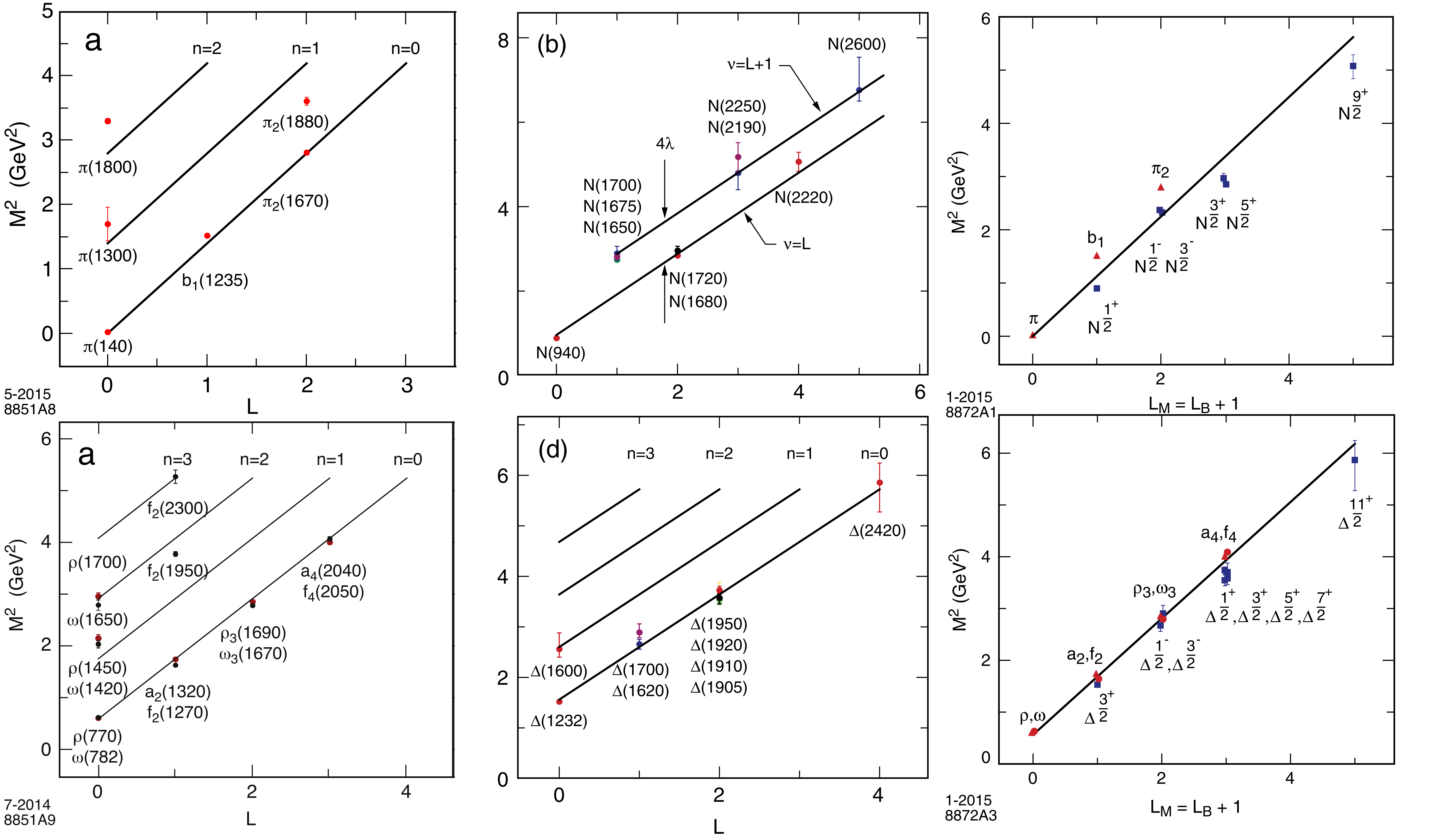
measuring the separation of partons in a hadron

G.F. de Téramond and S.J. Brodsky, Phys. Rev. Lett. 102, 081601 (2009);
S.J. Brodsky and G.F. de Téramond, Phys. Rev. Lett. 96, 201601 (2006);
Phys. Rev. D 77, 056007 (2008); Phys. Rev. D 78, 025032 (2008).



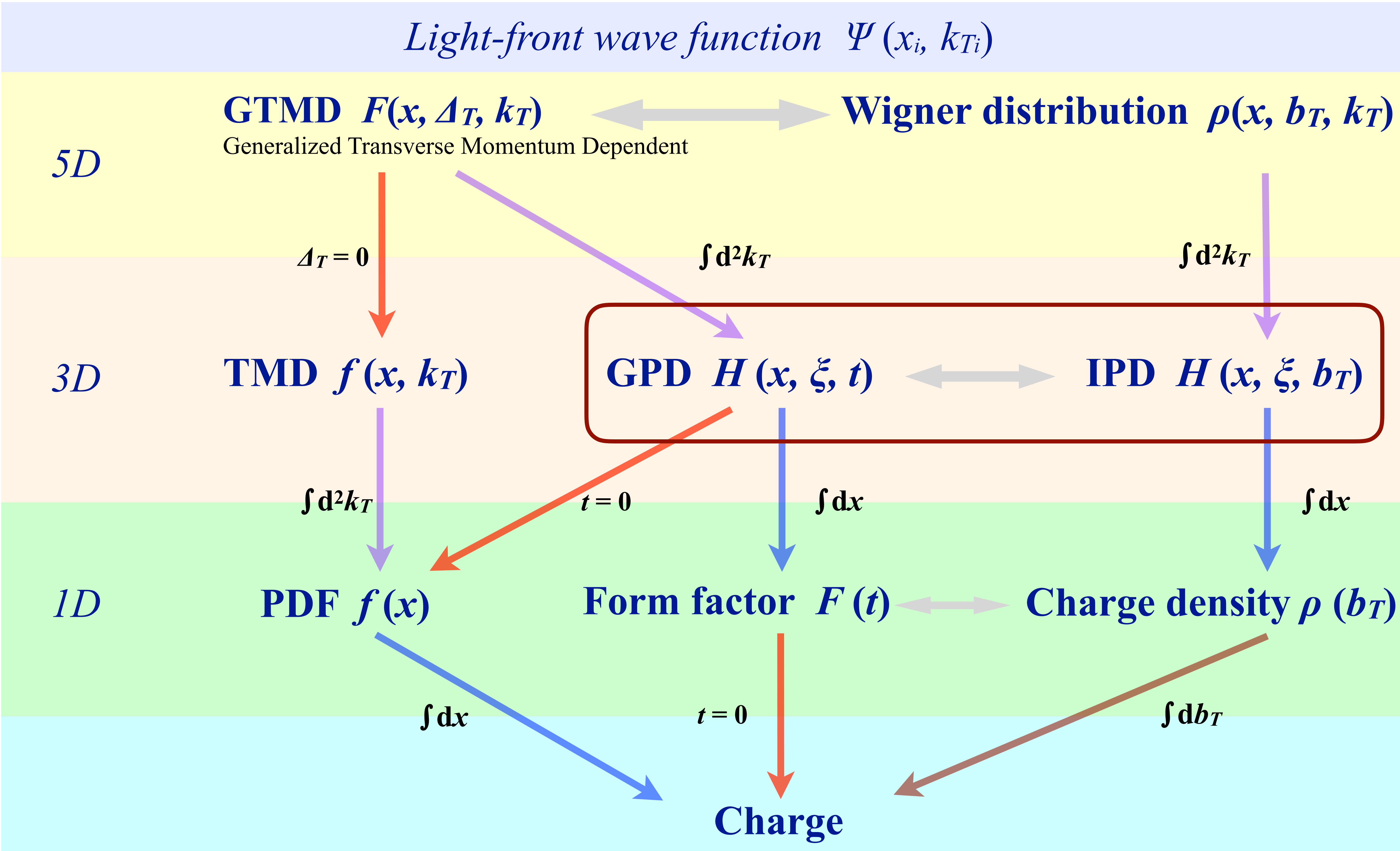
D. Binosi, C. Mezrag, J. Papavassiliou, C.D. Roberts, J. Rodríguez-Quintero, Phys. Rev. D 96, 054026 (2017).

Hadron Spectrum in LF Holography



S.J. Brodsky, G.F. de Téramond, H.G. Dosch, J. Erlich, Phys. Rep. 584, 1 (2015);
H.G. Dosch, G.F. de Téramond, S.J. Brodsky, Phys. Rev. D 91, 045040 (2015); D91, 085016 (2015).

Unified View of Partonic Structures



Form Factors in Holographic QCD

Form factor of a spinless hadron

$$\int d^4x dz \sqrt{g}\,\Phi^*_{P'}(x,z)\overleftrightarrow{\partial}_M \Phi_P(x,z) A^M(x,z)$$ → The coupling of an external EM field propagating in AdS space to a hadron mode

$$\sim (2\pi)^4 \delta^4(P'-P-q)\epsilon_\mu (P+P')^\mu F(q^2)$$ → EM form factor in physical spacetime

$$x^M = (x^\mu, z) \qquad \sqrt{g} = (R/z)^5$$

J. Polchinski and M.J. Strassler, JHEP 05 (2003) 012.

$$\Phi_P(x,z) = e^{iP\cdot x}\Phi(z) \qquad A_\mu(x,z) = e^{iq\cdot x} V(q^2,z)\epsilon_\mu(q), \qquad A_z = 0$$

Extracting the momentum conservation factor

$$F(Q^2) = \int \frac{dz}{z^3} V(Q^2,z)\Phi^2_\tau(z)$$

For hadron modes scale as $\Phi_\tau \sim z^\tau$ at small $z \sim 1/Q$

$$F_\tau(Q^2) \sim \left(\frac{1}{Q^2}\right)^{\tau-1}$$ recover the hard-scattering power scaling

S.J. Brodsky and G.R. Farrar, Phys. Rev. Lett. 31, 1153 (1973).

V.A. Matveev, R.M. Muradian, A.N. Tevkhelidze, Lett. Nuovo Cimento 7, 719 (1973).

Form Factors in Holographic QCD

Hadron wave function of twist-τ (soft-wall)

$$\Phi_\tau(z) = \sqrt{\frac{2}{\Gamma(\tau-1)}}\kappa^{\tau-1} z^\tau e^{-\kappa^2 z^2/2}$$

EM current

$$V(Q^2, z) = \kappa^2 z^2 \int_0^1 \frac{dx}{(1-x)^2} x^{Q^2/4\kappa^2} e^{-\kappa^2 z^2 x/(1-x)} = 4\kappa^4 z^2 \sum_{n=0}^{\infty} \frac{L_n^1(\kappa^2 z^2)}{M_n^2 + Q^2}$$

H.R. Grigoryan and A.V. Radyushkin, Phys. Rev. D 76, 095007 (2007).

have poles at $-Q^2 = M_n^2 = 4\kappa^2(n+1)$

compare with LFHQCD spectral formula $M_n^2 = 4\kappa^2\left(n + \frac{J+L}{2}\right)$

corresponding to the Regge trajectory $J = L = 1$

ρ meson trajectory is $J = L + 1 = 1$ shift poles to $-Q^2 = M_{\rho_n}^2 = 4\kappa^2\left(n + \frac{1}{2}\right)$

S.J. Brodsky, G.F. de Téramond, H.G. Dosch, J. Erlich, Phys. Rep. 584, 1 (2015).

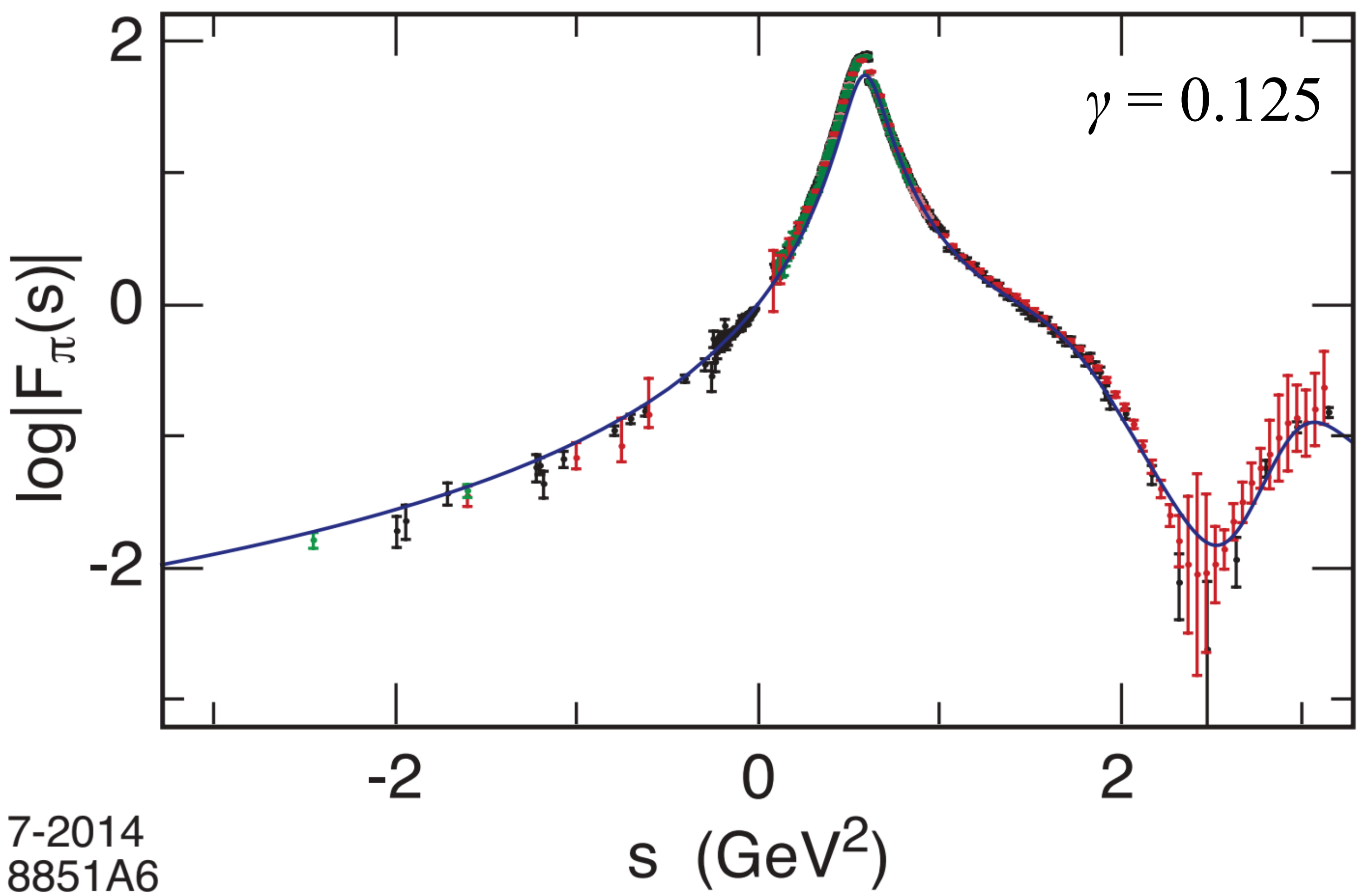
Form factor

$$F_\tau(Q^2) = \frac{1}{\left(1 + \frac{Q^2}{M^2_{\rho_{n=0}}}\right)\left(1 + \frac{Q^2}{M^2_{\rho_{n=1}}}\right)\cdots\left(1 + \frac{Q^2}{M^2_{\rho_{n=\tau-2}}}\right)}$$

S.J. Brodsky and G.F. de Téramond, Phys. Rev. D 77, 056007 (2008).

Pion Form Factor

Pion form factor compared with data



$$F_\pi(t) = \sum_\tau P_\tau F_\tau(t) \qquad \sum_\tau P_\tau = 1$$

Truncated at twist-τ = 4

$$F_\pi(q^2) = (1-\gamma)F_{\tau=2}(q^2) + \gamma F_{\tau=4}(q^2)$$

G.F. de Téramond and S.J. Brodsky, Proc. Sci. LC2010 (2010) 029.
S.J. Brodsky, G.F. de Téramond, H.G. Dosch, J. Erlich, Phys. Rep. 584, 1 (2015). [Sec. 6.1.5]

Form Factors in Holographic QCD

Nucleon form factor: spin-nonflip

$$\int d^4x dz \sqrt{g} \bar{\Psi}_{P'}(x,z) e_A^M \Gamma^A A_M(x,z) \Psi_P(x,z)$$ → The coupling of an external EM field propagating in AdS space to a nucleon mode

$$\sim (2\pi)^4 \delta^4(P' - P - q) \epsilon_\mu \bar{u}(P') \gamma^\mu F_1(q^2) u(P)$$ → Dirac form factor in physical spacetime

$$\{\Gamma^A, \Gamma^B\} = 2\eta^{AB} \qquad \Gamma^A = (\gamma^\mu, -i\gamma^5) \qquad e_A^M = \left(\tfrac{z}{R}\right)\delta_A^M$$

Nucleon wave function

$$\Psi_+(z) \sim z^{\tau+1/2} e^{-\kappa^2 z^2/2}, \qquad \Psi_-(z) \sim z^{\tau+3/2} e^{-\kappa^2 z^2/2}$$

$$(L=0) \qquad\qquad (L=1)$$

Dirac form factor

G.F. de Téramond, H.G. Dosch, S.J. Brodsky, Phys. Rev. D 87, 075005 (2013).

$$F_1^N(Q^2) = \sum_\pm g_\pm^N \int \frac{dz}{z^4} V(Q^2, z) \Psi_\pm^2(z)$$

$g_\pm$ are determined by the spin-flavor structure

SU(6) symmetry: $g_+^p = 1, \quad g_-^p = 0, \quad g_+^n = -\frac{1}{3}, \quad g_-^n = \frac{1}{3}.$

Form Factors in Holographic QCD

Nucleon form factor: spin-flip

$$\int d^4xdz\sqrt{g}\bar{\Psi}_{P'}(x,z)e^M_A e^N_B[\Gamma^A,\Gamma^B]F_{MN}(x,z)\Psi_P(x,z)$$

→ Effective spin-flip amplitude in AdS space of an external EM field coupling to a nucleon

$$\sim (2\pi)^4\delta^4(P'-P-q)\epsilon_\mu \bar{u}(P')\frac{\sigma^{\mu\nu}q_\nu}{2M_N}F_2(q^2)u(P),$$

→ Pauli form factor in physical spacetime

Z. Abidin and C.E. Carlson, Phys. Rev. D 79, 115003 (2009).

Pauli form factor

$$F_2^N(Q^2) = \chi_N \int \frac{dz}{z^3}\Psi_+(z)V(Q^2,z)\Psi_-(z)$$

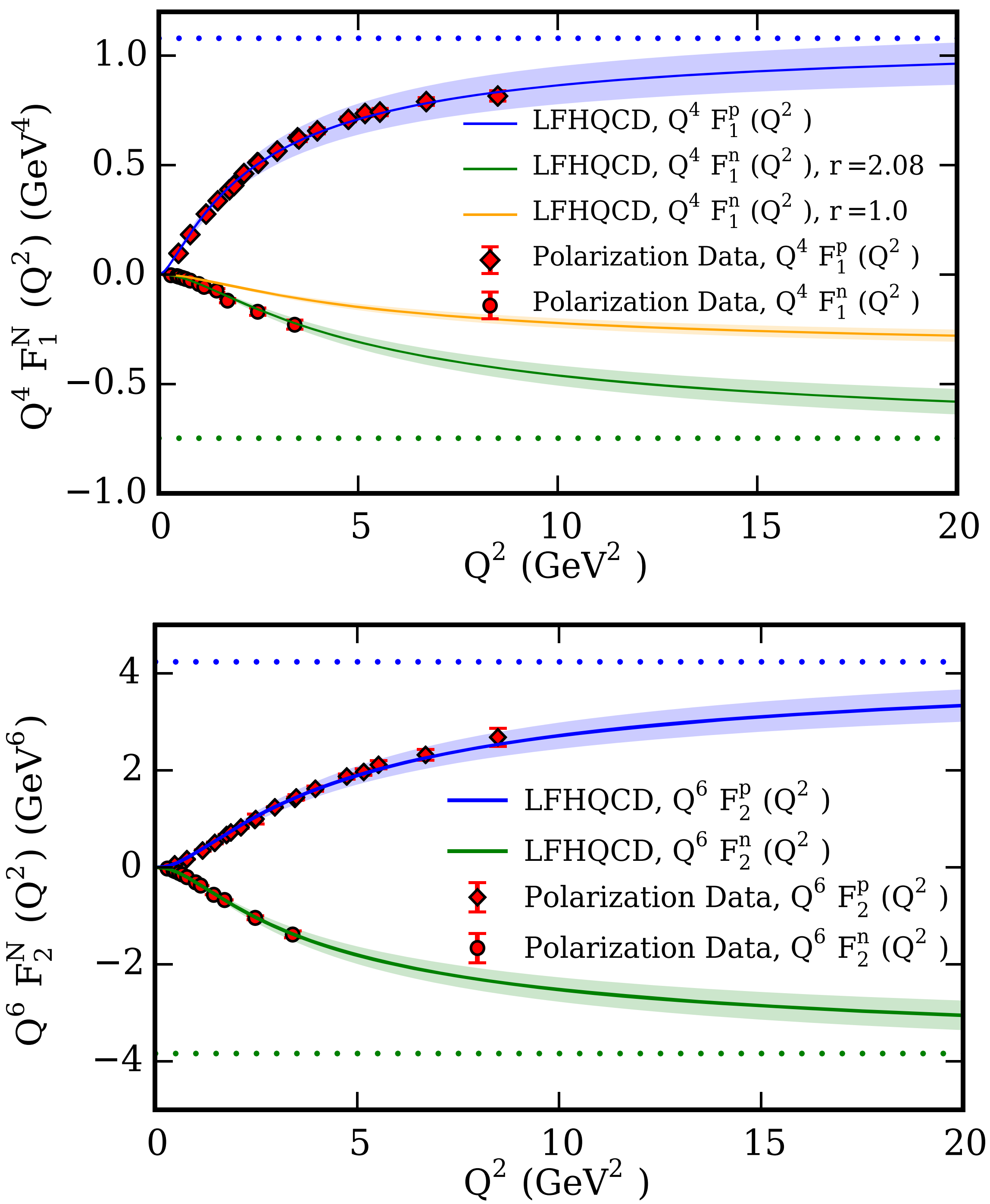
normalized to anomalous magnetic moments

Scaling: additional power of z in the wave function product of Ψ_+ and Ψ_-

the leading scaling of the Pauli form factor has additional power of $1/Q^2$

Nucleon Form Factors

Nucleon form factors compared with data



$$F_1^p(Q^2) = F_{\tau=3}(Q^2)$$

$$F_1^n(Q^2) = -\frac{r}{3}[F_{\tau=3}(Q^2) - F_{\tau=4}(Q^2)]$$

$$F_2^p(Q^2) = \chi_p[(1-\gamma_p)F_{\tau=4}(Q^2) + \gamma_p F_{\tau=6}(Q^2)]$$

$$F_2^n(Q^2) = \chi_n[(1-\gamma_n)F_{\tau=4}(Q^2) + \gamma_n F_{\tau=6}(Q^2)]$$

$$\gamma_p = 0.27 \qquad \gamma_n = 0.38$$

R.S. Sufian, G.F. de Téramond, S.J. Brodsky, A. Deur, H.G. Dosch, Phys. Rev. D 95 014011 (2017).

From Form Factors to GPDs

Form factor for arbitrary twist-τ

S.J. Brodsky and G.F. de Téramond, Phys. Rev. D 77, 056007 (2008);

S.J. Brodsky, G.F. de Téramond, H.G. Dosch, J. Erlich, Phys. Rep. 584, 1 (2015).

$$F_\tau(t) = \frac{1}{N_\tau} B\left(\tau - 1, \frac{1}{2} - \frac{t}{4\lambda}\right) \qquad N_\tau = \frac{\sqrt{\pi}\,\Gamma(\tau - 1)}{\Gamma(\tau - \frac{1}{2})}$$

For integer τ, it gives the pole structure

$$F_\tau(Q^2) = \frac{1}{\left(1 + \frac{Q^2}{M_0^2}\right)\left(1 + \frac{Q^2}{M_1^2}\right) \cdots \left(1 + \frac{Q^2}{M_{\tau-2}^2}\right)}$$

$M_n^2 = 4\lambda(n + \frac{1}{2}), n = 0, 1, 2, \ldots, \tau - 2$

ρ vector meson and its radial excitations

It has the same structure of Veneziano amplitude $B(1 - \alpha(s), 1 - \alpha(t))$
with the s-channel dependence replaced by a fixed pole $1 - \alpha(s) \to \tau - 1$

$$\alpha(t) = \frac{t}{4\lambda} + \frac{1}{2} \qquad \rho \text{ trajectory}$$

The mass scale λ is fixed by the ρ meson mass: $\sqrt{\lambda} = \kappa = m_\rho/\sqrt{2} = 0.548\,\mathrm{GeV}$

For large $Q^2 = -t$, it has the scaling behavior $F_\tau(Q^2) \sim \left(\frac{1}{Q^2}\right)^{\tau-1}$

From Form Factors to GPDs

Writing the form factor in terms of valence GPD at zero skewness

$$F^q(t) = \int_0^1 dx\, H_v^q(x,t)$$

Express the form factor with the Euler integral representation

$$F_\tau(t) = \frac{1}{N_\tau} B\left(\tau - 1, \frac{1}{2} - \frac{t}{4\lambda}\right) \qquad B(u,v) = \int_0^1 dy\, y^{u-1}(1-y)^{v-1} \qquad y = w(x)$$

$w(x)$ is a reparametrization function

$$\begin{aligned} H^q(x,t) &= \frac{1}{N_\tau}[1 - w(x)]^{\tau-2} w(x)^{-\frac{1}{2}} w'(x) e^{(t/4\lambda)\log[1/w(x)]} \\ &= q_\tau(x)\exp[t f(x)], \end{aligned}$$

$$q_\tau(x) = \frac{1}{N_\tau}[1 - w(x)]^{\tau-2} w(x)^{-\frac{1}{2}} w'(x) \qquad f(x) = \frac{1}{4\lambda}\log\left(\frac{1}{w(x)}\right)$$

The collinear distribution $q(x)$ and the profile function $f(x)$ are related by a universal τ-*independent* reparametrization function $w(x)$.

Constraints on w(x)

Mathematical constraints:

$$B(u,v) = \int_0^1 dy\, y^{u-1}(1-y)^{v-1} \qquad\qquad y = w(x)$$

$$w(0) = 0, \qquad w(1) = 1, \qquad w'(x) \geq 0 \quad \text{for} \quad x \in [0,1]$$

Physical requirements:

Small-x behavior: $H_v^q(x,t) \sim x^{-t/4\lambda} q_v(x)$ Regge theory motivated ansatz

$$w(x) \sim x$$

Large-x behavior: $q_\tau(x) \sim (1-x)^{2\tau-3}$ Drell-Yan inclusive counting rule

$$w(x) = 1 - (1-x)w'(1) + \frac{1}{2}(1-x)^2 w''(1) + \cdots$$

$$w'(1) = 0 \quad \text{and} \quad w''(1) \neq 0$$

$$f(x) = \frac{1}{4\lambda}\log\left(\frac{1}{w(x)}\right) \qquad\qquad f'(1) = 0 \;\; \text{and} \;\; f''(1) \neq 0$$

Pion and Nucleon GPDs

Pion:

$$H_v^{u,\bar{d}}(x,t) = (1-\gamma)H_{\tau=2}(x,t) + \gamma H_{\tau=4}(x,t)$$

Nucleon:

Spin-nonflip:

$$H_v^u(x,t) = \left(2 - \frac{r}{3}\right)H_{\tau=3}(x,t) + \frac{r}{3}H_{\tau=4}(x,t)$$

$$H_v^d(x,t) = \left(1 - \frac{2r}{3}\right)H_{\tau=3}(x,t) + \frac{2r}{3}H_{\tau=4}(x,t)$$

Spin-flip:

$$E_v^u(x,t) = \chi_u[(1-\gamma_u)H_{\tau=4}(x,t) + \gamma_u H_{\tau=6}(x,t)]$$

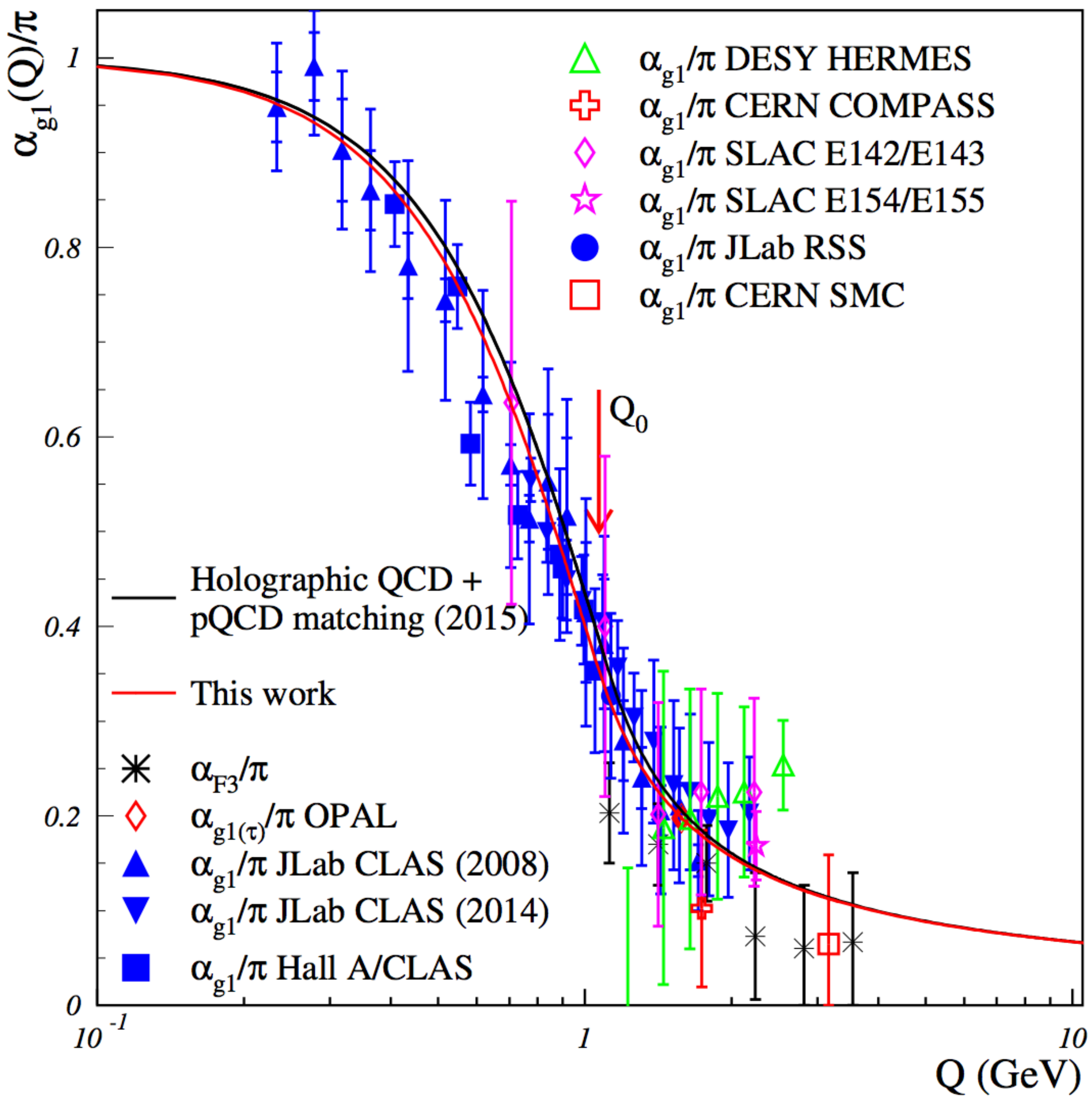
$$E_v^d(x,t) = \chi_u[(1-\gamma_d)H_{\tau=4}(x,t) + \gamma_d H_{\tau=6}(x,t)]$$

$$\gamma_u \equiv \frac{2\chi_p\gamma_p + \chi_n\gamma_n}{2\chi_p + \chi_n}, \qquad \gamma_d \equiv \frac{2\chi_n\gamma_n + \chi_p\gamma_p}{2\chi_n + \chi_p}$$

strange quark contribution is neglected

Matching Scale

Matching the couplings from LFHQCD and pQCD



Bjorken sum rule:

$$\frac{\alpha_{g_1}(Q^2)}{\pi} = 1 - \frac{6}{g_A}\int_0^1 dx\, g_1^{p-n}(x, Q^2)$$

Effective coupling in LFHQCD (valid at low-Q^2)

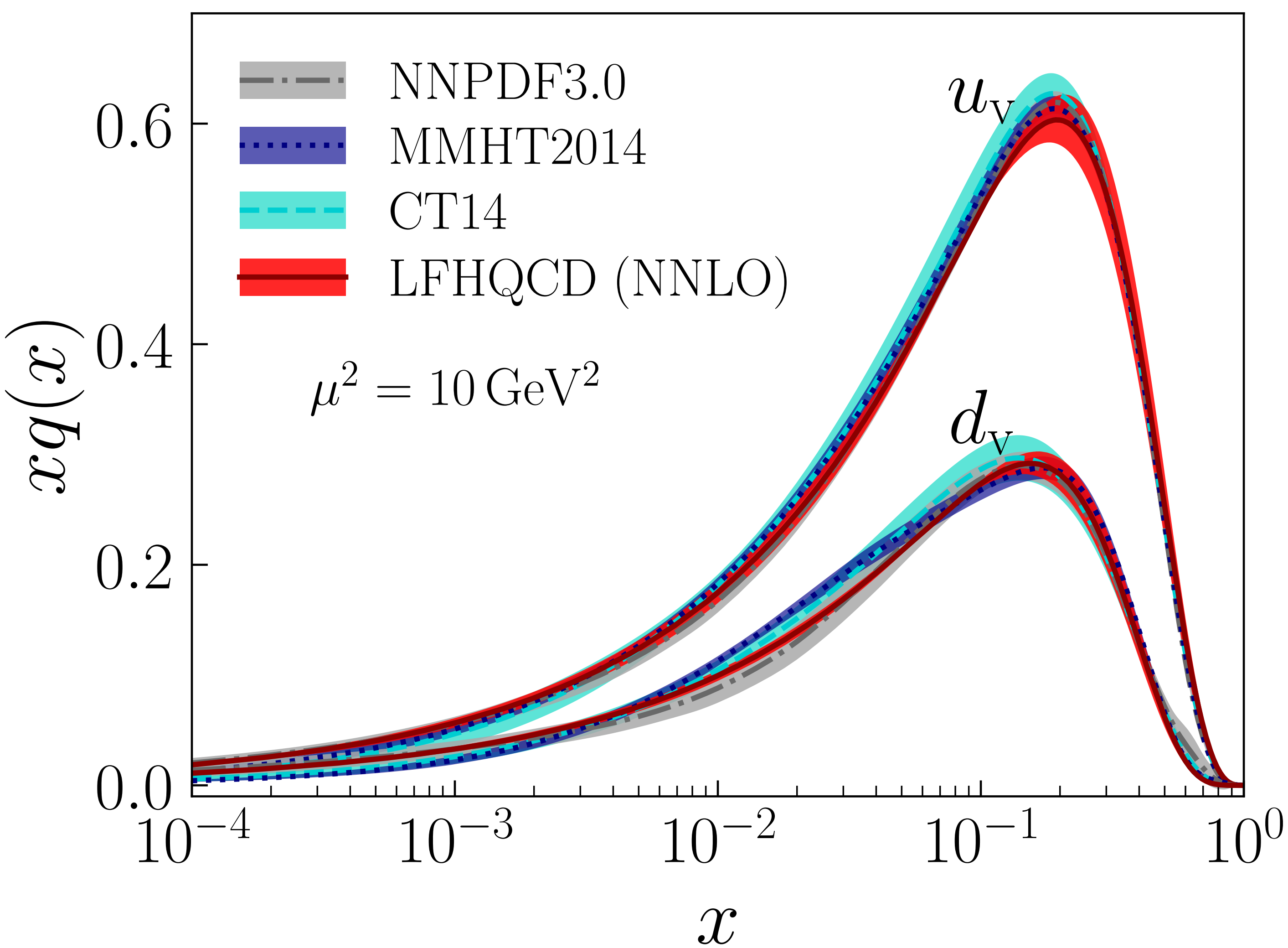
$$\alpha_{g_1}^{AdS}(Q^2) = \pi \exp\left(-Q^2/4\kappa^2\right)$$

Imposing continuity for α and its first derivative

A. Deur, S.J. Brodsky, G.F. de Téramond,
Phys. Lett. B 750, 528 (2015); J. Phys. G 44, 105005 (2017).

Nucleon PDFs

Nucleon PDFs in comparison with global fits



$$f(x) = \frac{1}{4\lambda}\left[(1-x)\log\left(\frac{1}{x}\right) + a(1-x)^2\right]$$

$$w(x) = x^{1-x} e^{-a(1-x)^2}$$

Parameter "a" is fixed by the first moment $a = 0.531 \pm 0.037$

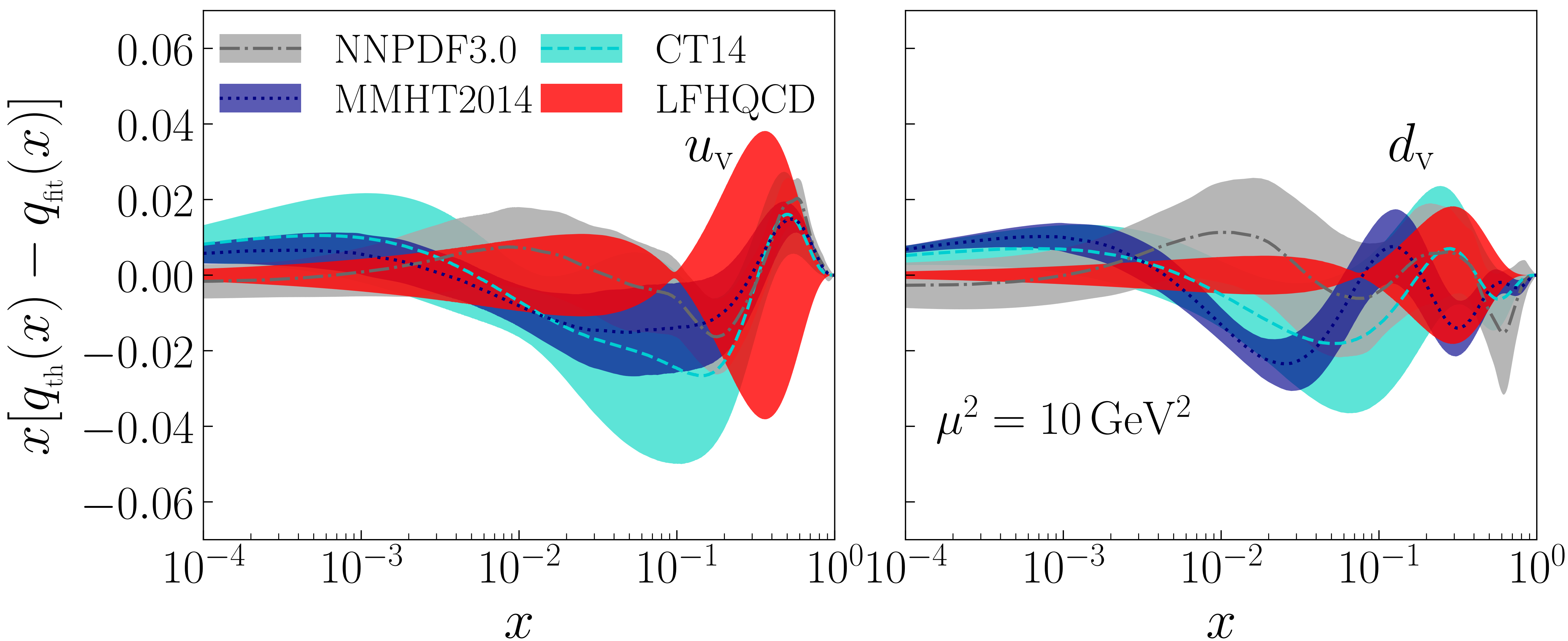
Evolved from the matching scale 1.06 ± 0.15 GeV

Red bands: the uncertainties of the matching scale and the parameter "a".

G.F. de Téramond, TL, R.S. Sufian, H.G. Dosch, S.J. Brodsky, A. Deur, Phys. Rev. Lett. 120, 182001 (2018).

Nucleon PDFs

Difference between LFHQCD results and global fits



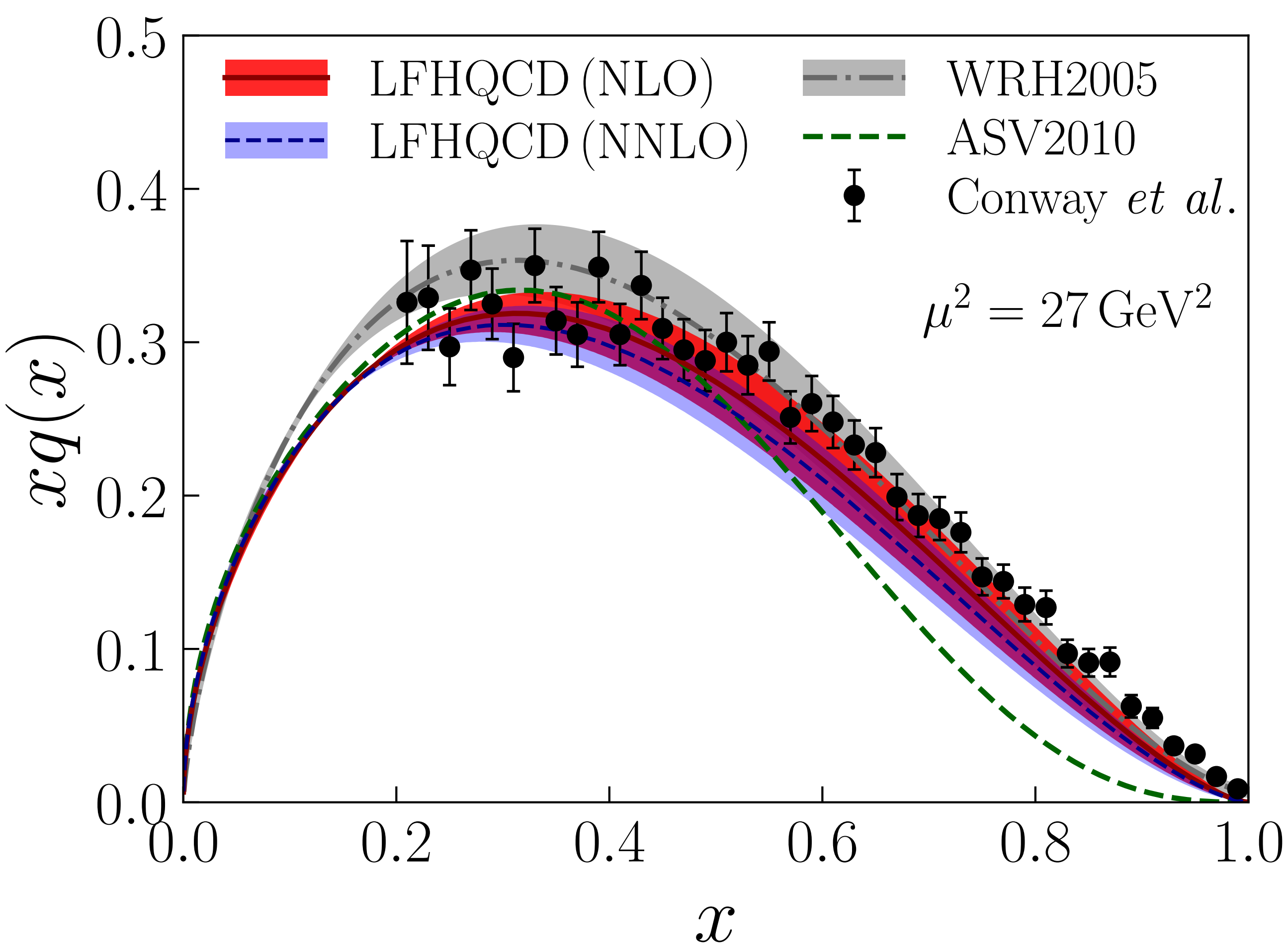
Red bands: the uncertainties of the matching scale and the parameter "*a*".

G.F. de Téramond, TL, R.S. Sufian, H.G. Dosch, S.J. Brodsky, A. Deur, Phys. Rev. Lett. 120, 182001 (2018).

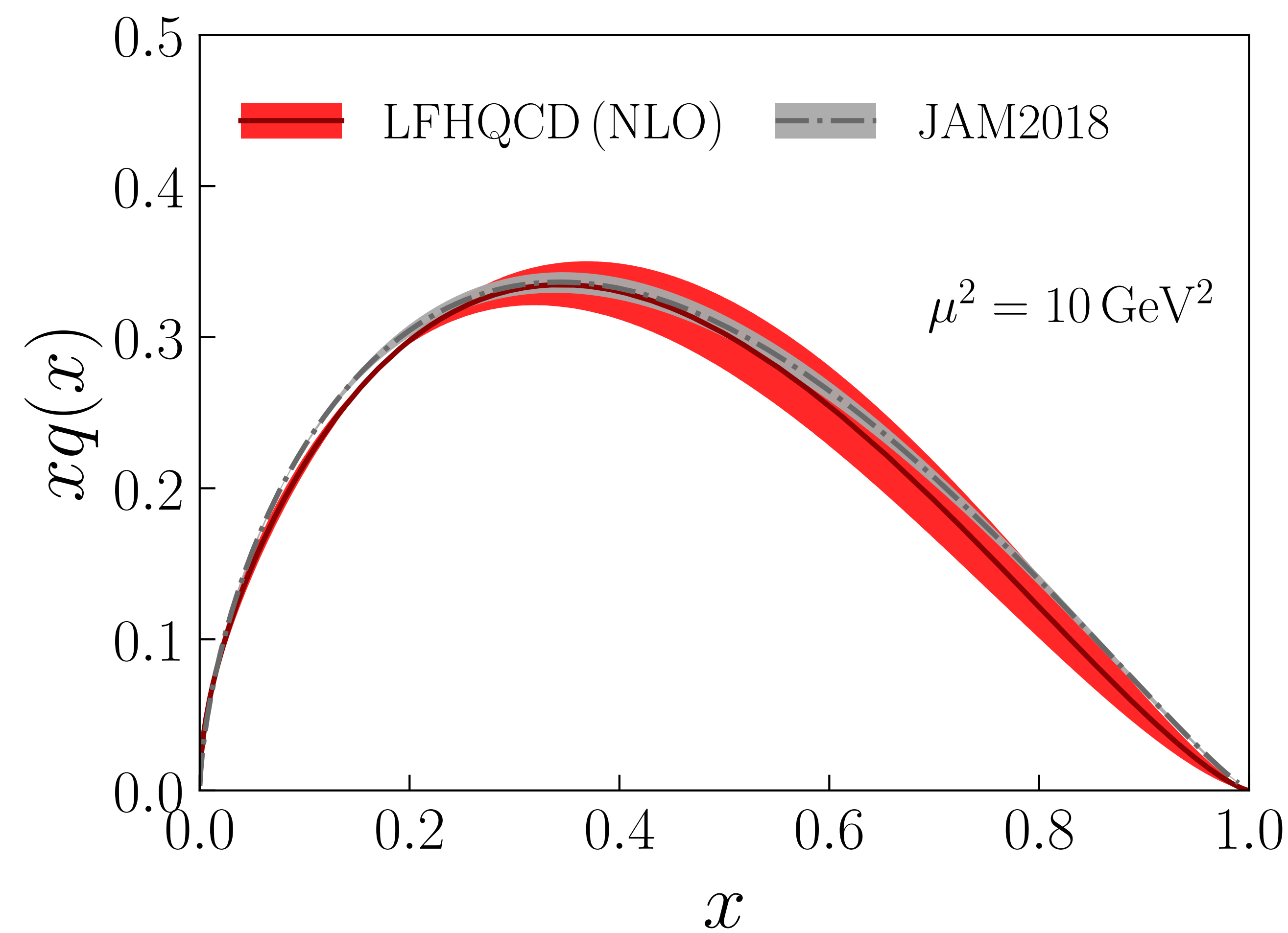
Pion PDF

Prediction for pion PDF with the same $w(x)$

$$w(x) = x^{1-x} e^{-a(1-x)^2}$$



Evolved from the matching scale
1.1 ± 0.2 GeV (NLO)
1.06 ± 0.15 GeV (NNLO)

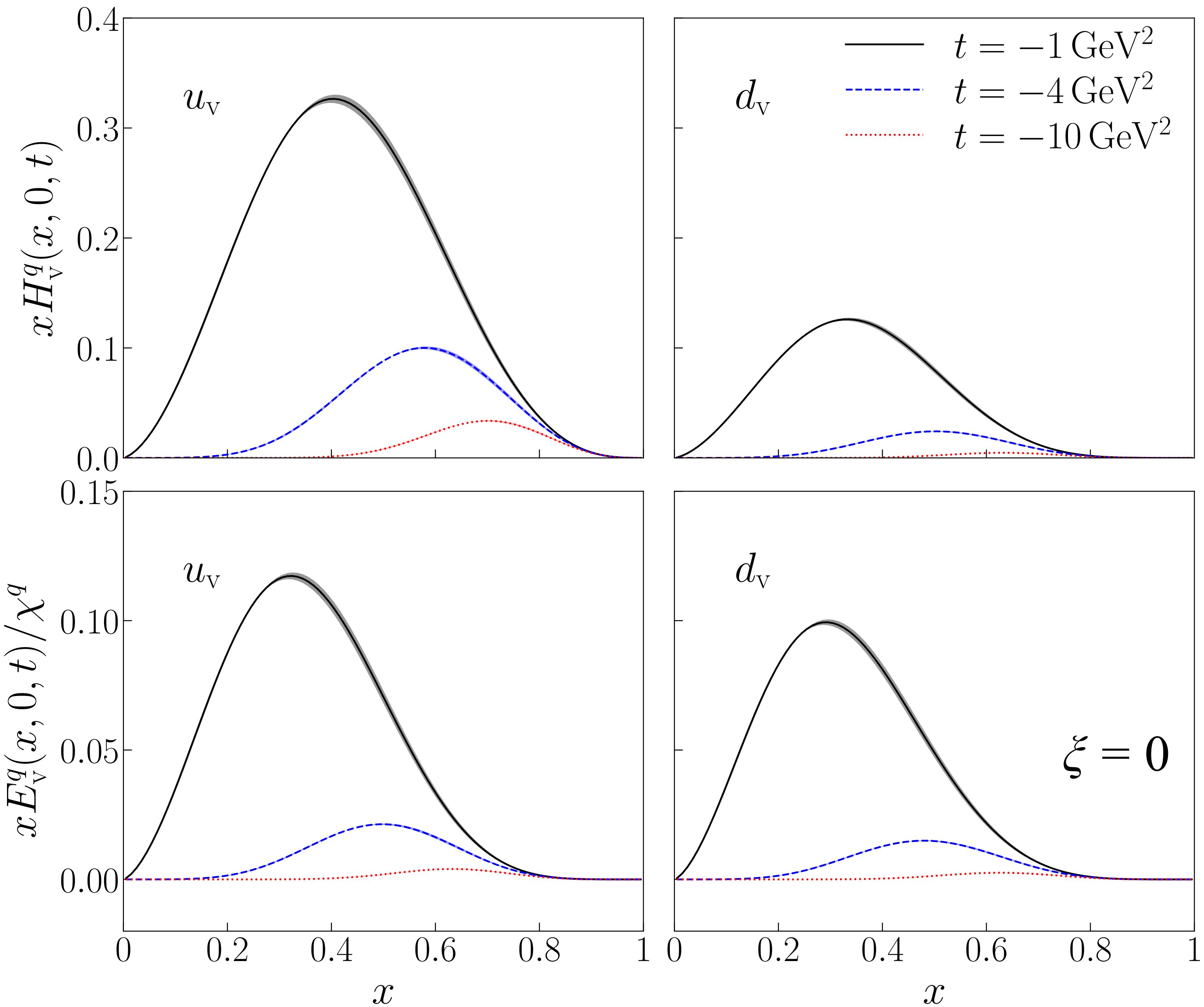


Comparison with a new global fit by JAM [arXiv:1804.01965]

see P. Barry's talk

de Téramond, TL, Sufian, Dosch, Brodsky, Deur, Phys. Rev. Lett. 120, 182001 (2018).

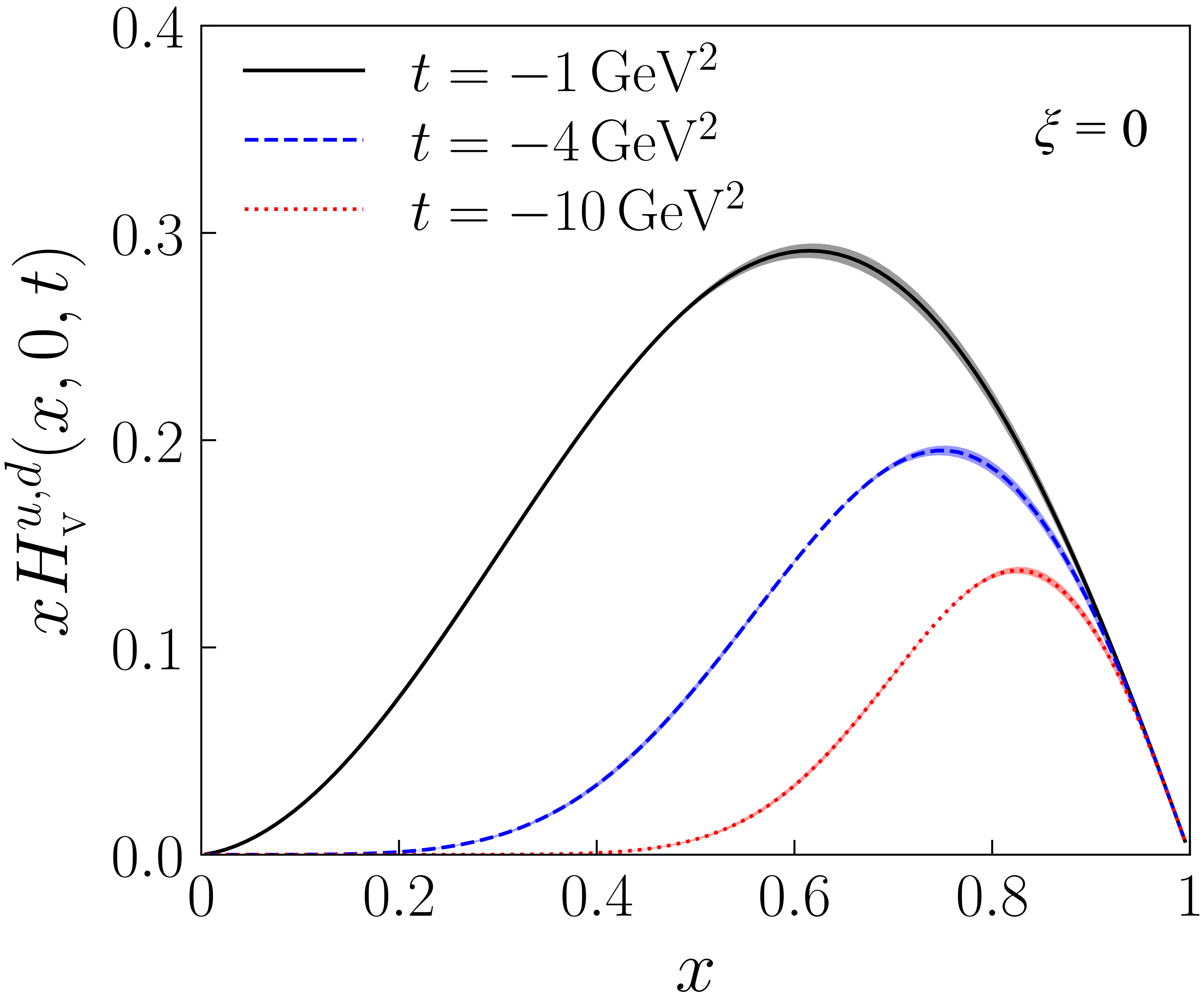
Nucleon GPDs



At the matching scale
1.06 ± 0.15 GeV

G.F. de Téramond, TL, R.S. Sufian, H.G. Dosch, S.J. Brodsky, A. Deur, Phys. Rev. Lett. 120, 182001 (2018).

Pion GPD

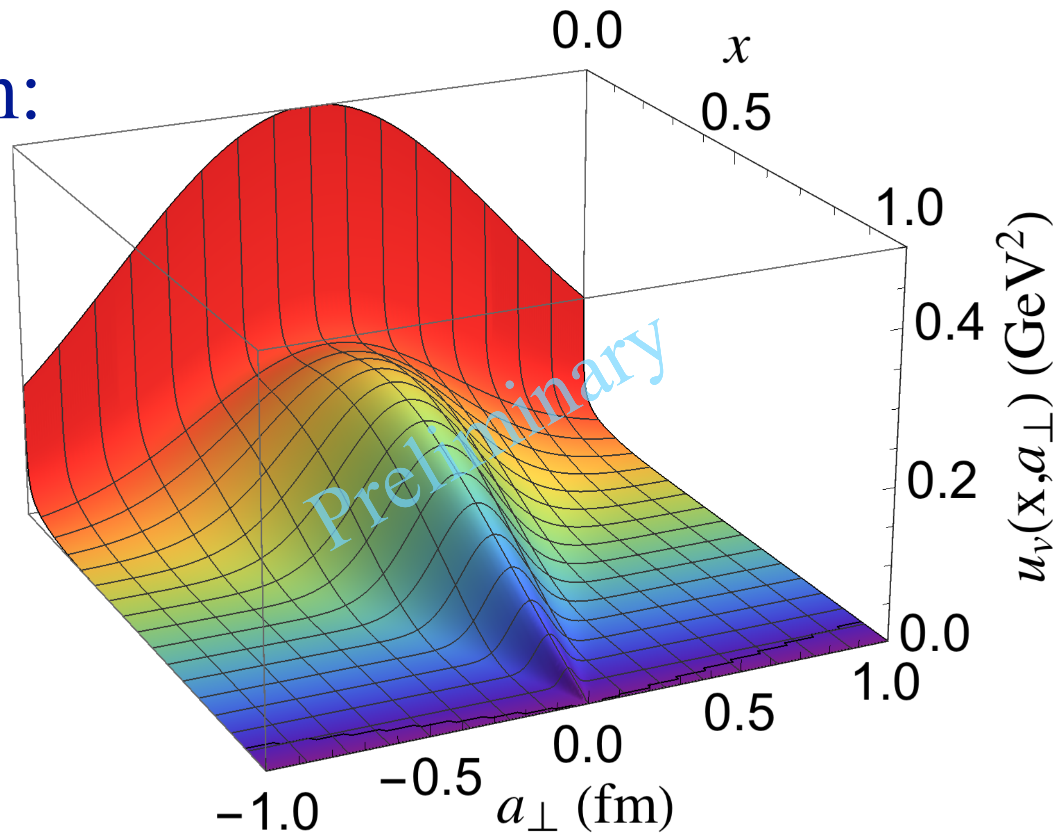


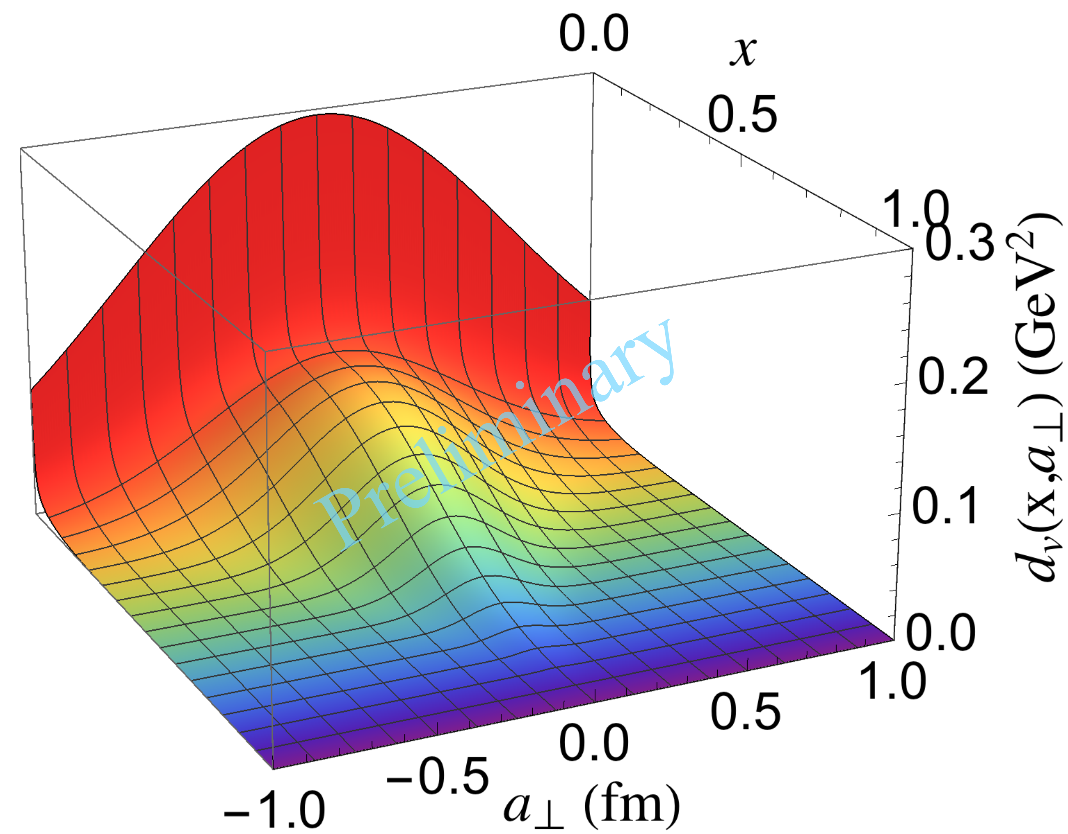
At the matching scale
1.1 ± 0.2 GeV

G.F. de Téramond, TL, R.S. Sufian, H.G. Dosch, S.J. Brodsky, A. Deur, Phys. Rev. Lett. 120, 182001 (2018).

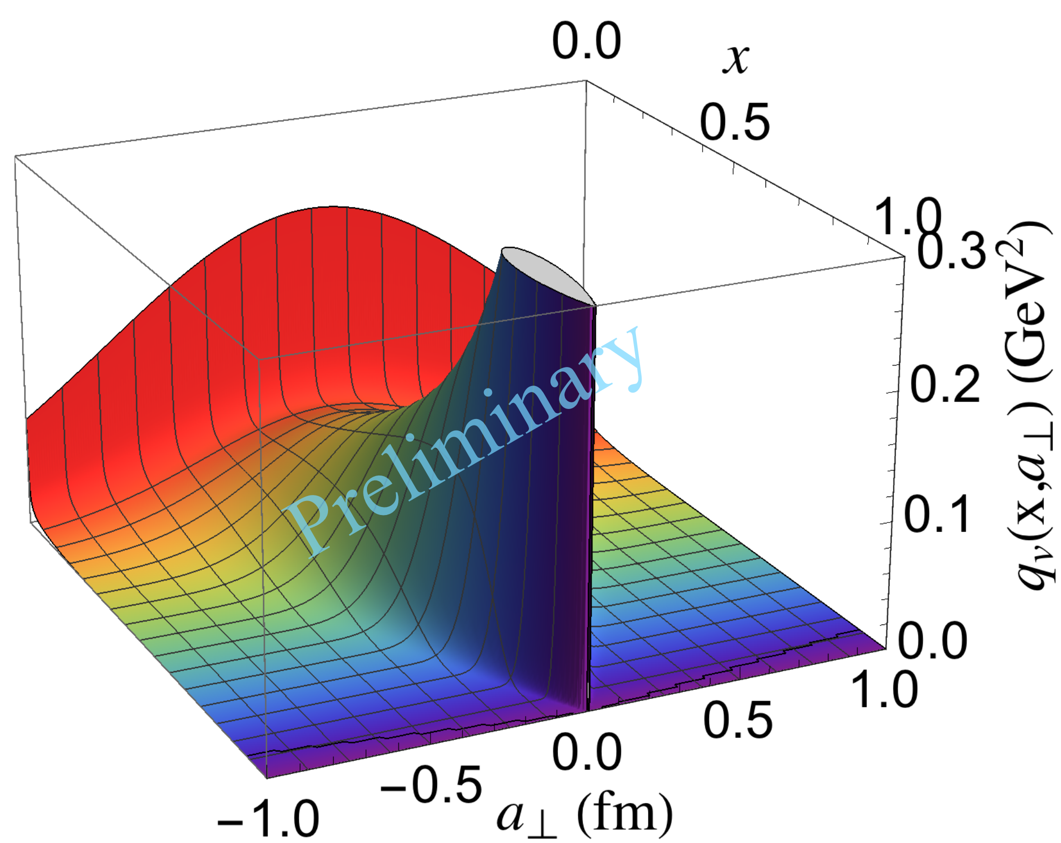
Impact Parameter Distributions

Nucleon:





Pion:



Summary

The structure of GPDs is determined from LF holographic QCD up to a universal reparametrization function w(x).

Imposing constraints on w(x) to incorporate Regge behavior of small-x and inclusive counting rules at $x \to 1$. A simple w(x) ansatz results in precise descriptions of parton distributions for both nucleon and pion.

The LF holographic QCD provides a unified framework to describe hadron spectrum and structure.

Thanks!

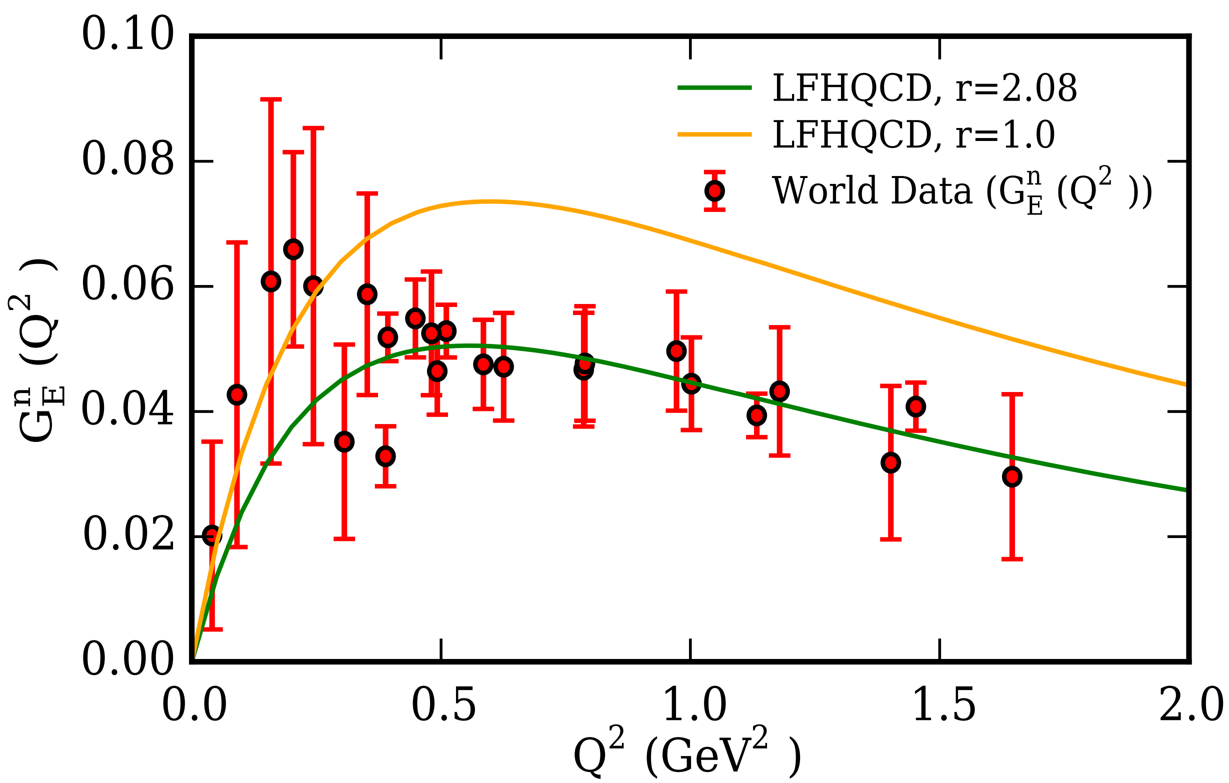
Backup

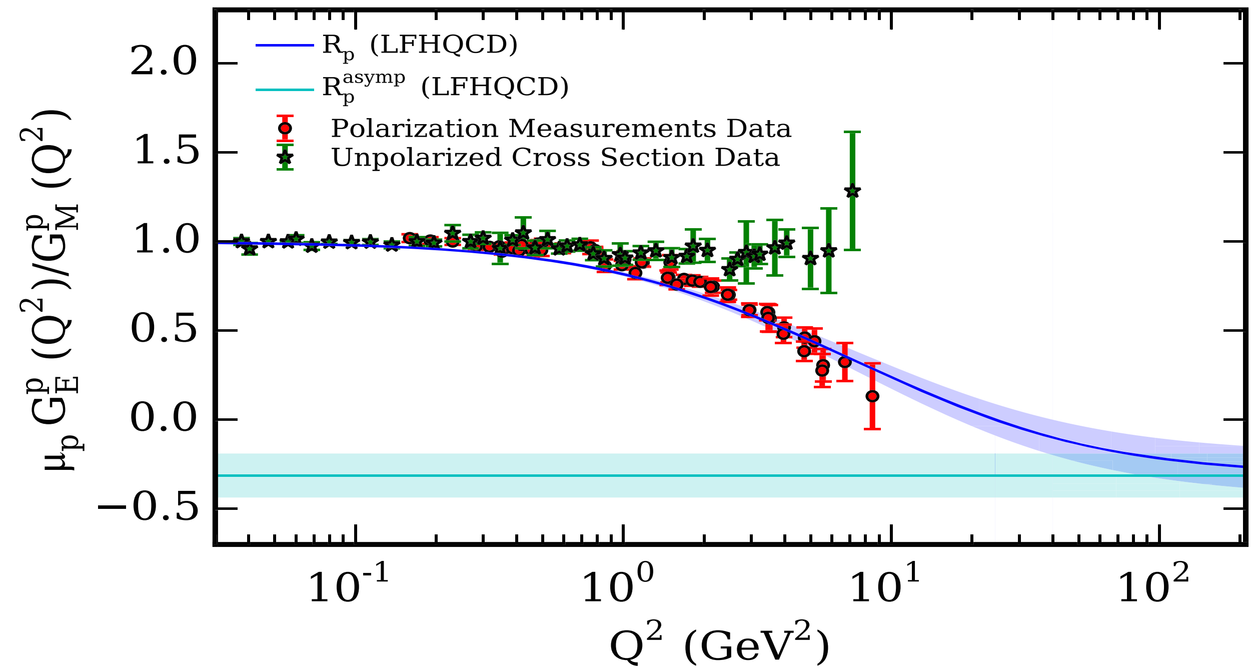
Nucleon Form Factors

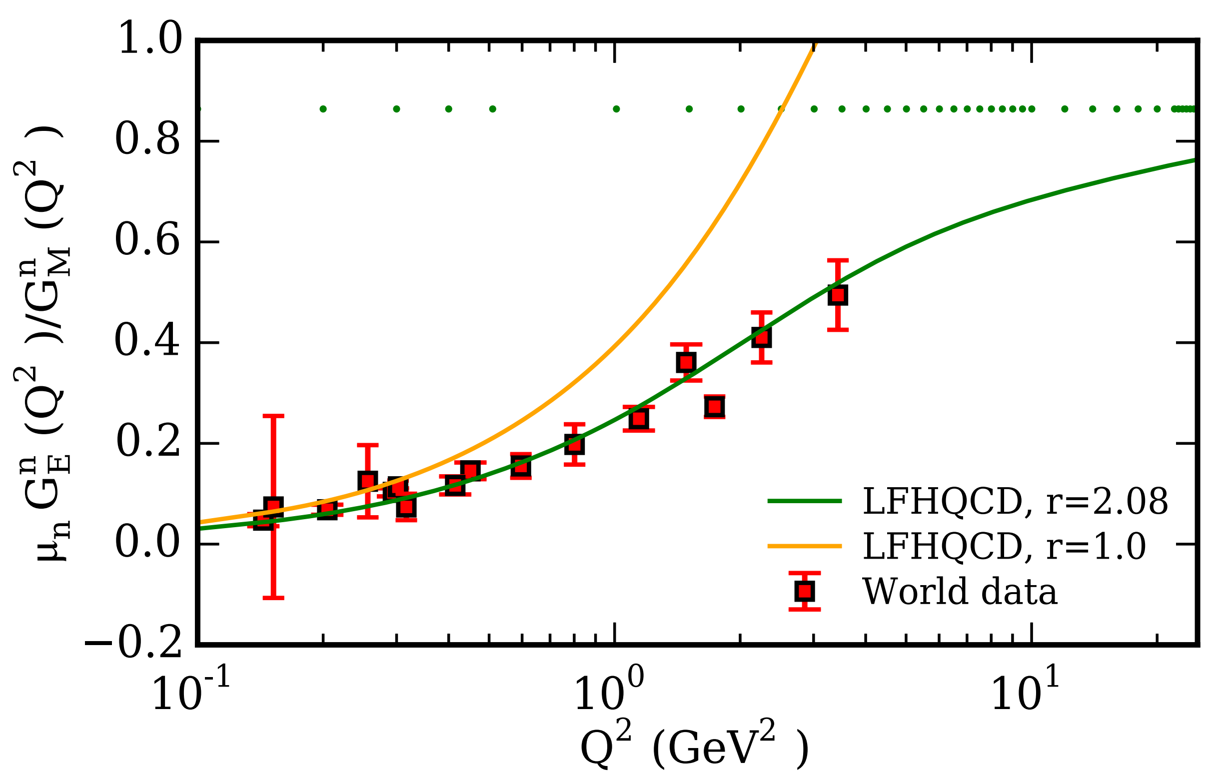
Sachs form factors

$$G_E^N(Q^2) = F_1^N(Q^2) - \frac{Q^2}{4m_N^2} F_2^N(Q^2)$$

$$G_M^N(Q^2) = F_1^N(Q^2) + F_2^N(Q^2).$$



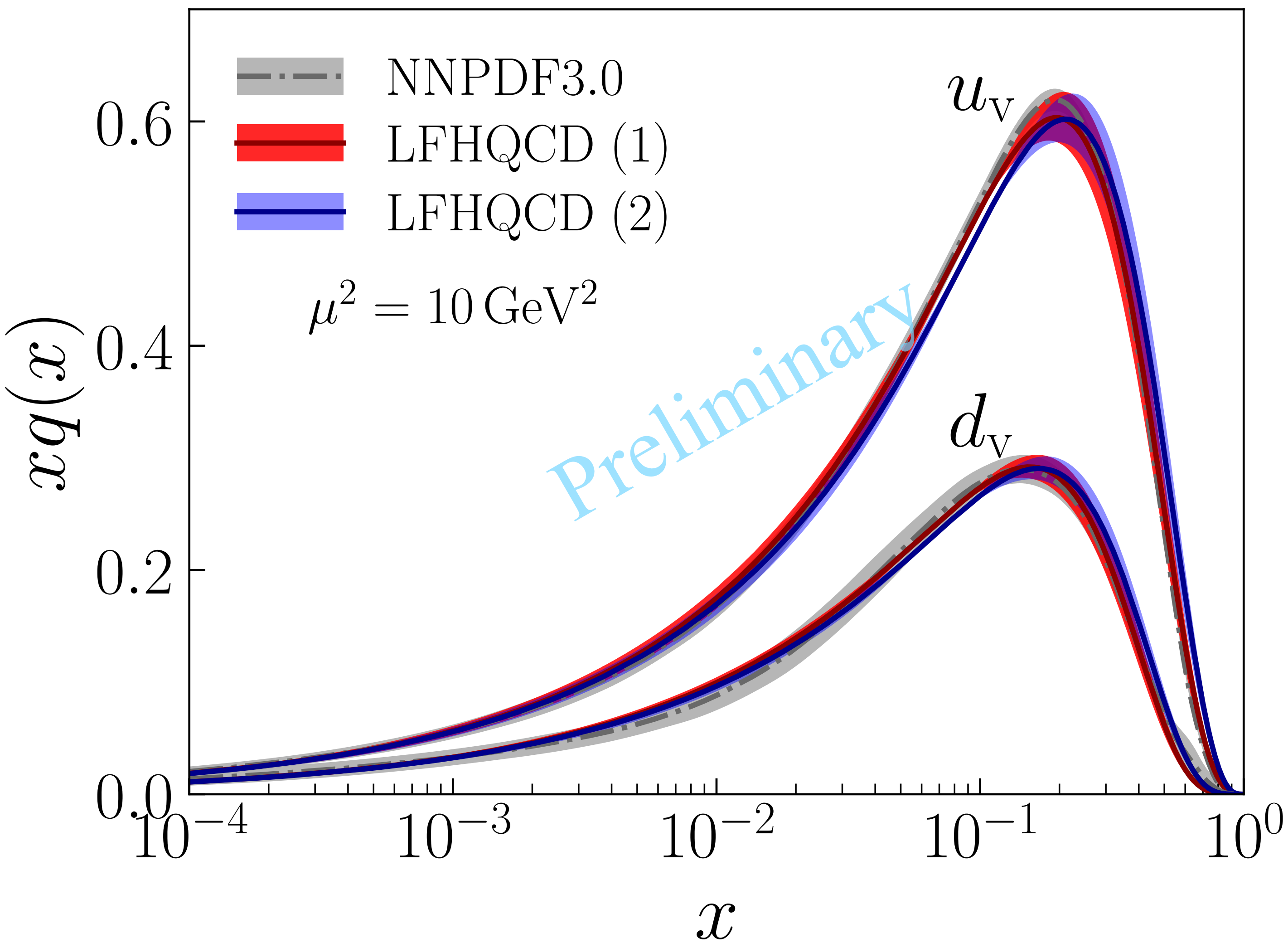




R.S. Sufian, G.F. de Téramond, S.J. Brodsky, A. Deur, H.G. Dosch, Phys. Rev. D 95 014011 (2017).

Different w(x) ansatz

Nucleon PDFs with different $w(x)$ forms



(1) $w(x) = x^{1-x} e^{-a(1-x)^2}$

(2) $w(x) = Ax + Bx^2 + Cx^3$

$$B = 3 - 2A, \quad C = A - 2$$

Effective LF Wave Functions

$\mathbf{b}_\perp$-space:

$$\psi^\tau_{\text{eff}}(x,\mathbf{b}_\perp) = \frac{1}{2\sqrt{\pi}}\sqrt{\frac{q_\tau(x)}{f(x)}}(1-x)\exp\left(-\frac{(1-x)^2}{8f(x)}\mathbf{b}_\perp^2\right)$$

normalization: $$\int_0^1 dx \int d^2\mathbf{b}_\perp |\psi_{\text{eff}}(x,\mathbf{b}_\perp)|^2 = 1$$

$\mathbf{k}_\perp$-space:

$$\psi^\tau_{\text{eff}}(x,\mathbf{k}_\perp) = 8\pi\frac{\sqrt{q_\tau(x)f(x)}}{1-x}\exp\left(-\frac{2f(x)}{(1-x)^2}\mathbf{k}_\perp^2\right)$$

normalization: $$\int_0^1 dx \int \frac{d^2\mathbf{k}_\perp}{16\pi^3} |\psi_{\text{eff}}(x,\mathbf{k}_\perp)|^2 = 1$$