# Vacuum and 1-particle states. Equal-time vs. Light-front

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# Motivations

- **what is the light-front vacuum state?**
- **how the information about mass appears at the light-front hypersurface**
- **how to deal with the light-front infra-red singularities for $k^+ \to 0$?**
- **discussion for free field massive scalar field**

# Introductory notes

- **quantum fields are operator-valued distributions**
- **smeared quantum fields - local field operators - act in the Hilbert space**
- **the vacuum sector of Hilbert space - $\mathcal{H}_0$ - consists of all states that can be created from the vacuum by local field operators**
- **the purpose of smearing for states in $\mathcal{H}_0$ is to make sure that these states have finite norm and thus really are Hilbert space states**
- **for local field operators no canonical quantization procedure**
- **canonical smearing for equal-time quantization procedure**
- **canonical smearing for light-front quantization procedure**

# Notation for equal-time quantization $\hbar = c = 2\pi = 1$

- canonically smeared scalar fields

$$\hat{\phi}[t,f] = \int_{\mathbb{R}^3} \mathrm{d}^3\boldsymbol{x}\, f(\boldsymbol{x})\, \hat{\phi}(t,\boldsymbol{x}), \qquad \boldsymbol{x} = (x,y,z)$$

- real-valued test function $f(\boldsymbol{x}) \in \mathscr{S}(\mathbb{R}^3)$ - **Schwartz class**

- Fourier transform of test function

$$\tilde{f}(\boldsymbol{k}) = \int_{\mathbb{R}^3} \mathrm{d}^3\boldsymbol{x}\, f(\boldsymbol{x})\, \mathrm{e}^{-\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{x}}, \qquad \tilde{f} \in \mathscr{S}(\mathbb{R}^3)$$

- $\omega^a f : \mathbb{R}^3 \to \mathbb{R}$, with $a \in \mathbb{Z}$, $\omega(\boldsymbol{k}) = \sqrt{m^2 + \boldsymbol{k}^2}$

$$(\omega^a f)(\boldsymbol{x}) = \int_{\mathbb{R}^3} \mathrm{d}^3\boldsymbol{k}\, \mathrm{e}^{-\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{x}}\, (m^2 + \boldsymbol{k}^2)^{a/2}\, \tilde{f}(\boldsymbol{k}), \qquad \omega^a f \in \mathscr{S}'(\mathbb{R}^3)$$

- inner product for test functions $(f,g) \in \mathscr{S}(\mathbb{R}^3)$

$$(f,g) = \int_{\mathbb{R}^3} \mathrm{d}^3\boldsymbol{x}\, f(\boldsymbol{x})\, g(\boldsymbol{x}) = \int_{\mathbb{R}^3} \mathrm{d}^3\boldsymbol{k}\, \tilde{f}^*(\boldsymbol{k})\, \tilde{g}(\boldsymbol{k}) < \infty$$

# Equal-time canonical quantization

- canonical commutators at $t = 0$ we denote $\hat{\phi}[0,f] = \hat{\phi}[f]$

$$\left[\hat{\phi}[f], \hat{\pi}[g]\right] = \mathrm{i}\,(f,g)\,\mathbf{1}_{op}, \quad \left[\hat{\phi}[f], \hat{\phi}[g]\right] = 0, \quad [\hat{\pi}[f], \hat{\pi}[g]] = 0.$$

- Heisenberg time evolution equations

$$\mathrm{d}_t\hat{\phi}[f] = \frac{1}{\mathrm{i}}\left[\hat{\phi}[f], \hat{H}\right] = \hat{\pi}[f], \quad \mathrm{d}_t\hat{\pi}[f] = \frac{1}{\mathrm{i}}\left[\hat{\pi}[f], \hat{H}\right] = -\hat{\phi}[\omega^2 f].$$

- **canonically smeared annihilation and creation operators:**

$$\hat{a}[f] := \hat{\phi}[\omega f] + \mathrm{i}\hat{\pi}[f], \quad \hat{a}^\dagger[f] := \hat{\phi}[\omega f] - \mathrm{i}\hat{\pi}[f], \quad (\hat{a}[f])^\dagger = \hat{a}^\dagger[f],$$

- satisfy Heisenberg equations

$$[\hat{H}, \hat{a}[f]] = -\hat{a}[\omega f], \quad \left[\hat{H}, \hat{a}^\dagger[f]\right] = \hat{a}^\dagger[\omega f],$$

- have canonical commutators

$$\left[\hat{a}[f], \hat{a}^\dagger[g]\right] = 2\,(f|\omega g)\,\mathbf{1}_{op}, \quad [\hat{a}[f], \hat{a}[g]] = \left[\hat{a}^\dagger[f], \hat{a}^\dagger[g]\right] = 0,$$

# Equal-time canonical quantization, cont'd

- **equivalence for canonically smeared operators**

$$\hat{\phi}[f] = \frac{1}{2}\left(\hat{a}[\omega^{-1}f] + \hat{a}^\dagger[\omega^{-1}f]\right), \qquad \hat{\pi}[f] = \frac{\mathrm{i}}{2}\left(\hat{a}^\dagger[f] - \hat{a}[f]\right).$$

- smeared operators lead to operator-valued distributions

$$\begin{aligned}
\hat{a}[f] &= \int_{\mathbb{R}^3} \mathrm{d}^3\boldsymbol{x}\, f(\boldsymbol{x})\, \hat{a}(\boldsymbol{x}) = \int_{\mathbb{R}^3} \mathrm{d}^3\boldsymbol{k}\, \tilde{f}^*(\boldsymbol{k})\, \hat{a}(\boldsymbol{k}),\\
\hat{a}^\dagger[f] &= \int_{\mathbb{R}^3} \mathrm{d}^3\boldsymbol{x}\, f(\boldsymbol{x})\, \hat{a}^\dagger(\boldsymbol{x}) = \int_{\mathbb{R}^3} \mathrm{d}^3\boldsymbol{k}\, \tilde{f}(\boldsymbol{k})\, \hat{a}^\dagger(\boldsymbol{k}).
\end{aligned}$$

- commutators for operator-valued distributions

$$\left[\hat{a}(\boldsymbol{k}), \hat{a}^\dagger(\boldsymbol{p})\right] = 2\omega(\boldsymbol{k})(2\pi)^3\delta^3(\boldsymbol{k}-\boldsymbol{p})\,\mathbf{1}_{op}, \qquad [\hat{a}(\boldsymbol{k}), \hat{a}(\boldsymbol{p})] = \left[\hat{a}^\dagger(\boldsymbol{k}), \hat{a}^\dagger(\boldsymbol{p})\right] = 0$$

# Equal-time vacuum and diagonalization of $\hat{H}$

- vacuum state $|0\rangle$: unique, normalized, Poincaré invariant

$$\langle 0|0\rangle = 1, \quad \hat{H}|0\rangle = 0, \quad \hat{\boldsymbol{P}}|0\rangle = 0, \quad \hat{M}_{\mu\nu}|0\rangle = 0,$$

- Heisenberg equations lead to

$$\hat{H}\,\hat{a}[f]|0\rangle = -\hat{a}[\omega f]|0\rangle, \quad \hat{H}\,\hat{a}^\dagger[f]|0\rangle = \hat{a}^\dagger[\omega f]|0\rangle.$$

- there are eigenvectors of $\hat{H}$ with **positive and negative eigenvalues**

$$\hat{H}\,\hat{a}(\boldsymbol{k})|0\rangle = -\omega(\boldsymbol{k})\,\hat{a}(\boldsymbol{k})|0\rangle, \qquad \hat{H}\,\hat{a}^\dagger(\boldsymbol{k})|0\rangle = \omega(\boldsymbol{k})\hat{a}^\dagger(\boldsymbol{k})|0\rangle.$$

- **vacuum state removes** the negative eigenvalue states from $\mathcal{H}_0$

$$\hat{a}(\boldsymbol{k})|0\rangle = 0 \quad \Longrightarrow \quad \hat{a}[f]|0\rangle = 0 \quad \Longrightarrow \quad \hat{\pi}[f]|0\rangle = \mathrm{i}\hat{\phi}[\omega f]|0\rangle,$$

# Equal-time 1-particle states

- canonically smeared 1-particle state:

$$|f\rangle := \hat{a}^\dagger[f]|0\rangle = 2\hat{\phi}[\omega f]|0\rangle,$$

- decomposition of the canonical commutators

$$\langle 0|\left[\hat{\phi}[f], \hat{\pi}[g]\right]|0\rangle = \mathrm{i}\,(f,g) \implies \langle 0|\hat{\phi}[f]\,\hat{\phi}[g]|0\rangle = \frac{1}{2}\left(f, \omega^{-1} g\right) < \infty$$

- inner product for canonically smeared 1-particle states

$$\langle f|g\rangle = 4\langle 0|\hat{\phi}[\omega f]\,\hat{\phi}[\omega g]|0\rangle = 2\,(f, \omega g) < \infty,$$

- canonically smeared 1-particle states are in $\mathcal{H}_0$

# Distribution-valued states - momentum dependent kets

- canonically smeared 1-particle states

$$|f\rangle = \int_{\mathbb{R}^3} \mathrm{d}^3\boldsymbol{k}\,\tilde{f}(\boldsymbol{k})\,|\boldsymbol{k}\rangle \qquad \Longrightarrow \qquad |\boldsymbol{k}\rangle = \hat{a}^\dagger(\boldsymbol{k})|0\rangle$$

- **momentum dependent** kets $|\boldsymbol{k}\rangle$ have inner product

$$\langle \boldsymbol{p}|\boldsymbol{k}\rangle = 2\omega(\boldsymbol{k})\,\delta^3(\boldsymbol{k}-\boldsymbol{p}),$$

- they are eigenvectors for translation generators operators

$$\hat{H}\,|\boldsymbol{k}\rangle = \omega(\boldsymbol{k})|\boldsymbol{k}\rangle, \qquad \hat{P}_j\,|\boldsymbol{k}\rangle = -k^j\,|\boldsymbol{k}\rangle = k_j\,|\boldsymbol{k}\rangle$$

- distribution-valued states are not states in $\mathcal{H}_0$

# 2-particle states

- canonically smeared 2-particle states:

$$|f,g\rangle := \hat{a}^\dagger[f]\,\hat{a}^\dagger[g]|0\rangle,$$

- are orthogonal to vacuum and 1-particle states

$$\langle 0|f,g\rangle = 0 \qquad \langle h|f,g\rangle = 0$$

- have finite norm

$$\langle f,g|f,g\rangle = 4(f,\omega f)(g,\omega g) + 4(f,\omega g)(g,\omega f)$$

- accordingly

$$|f,g\rangle \in \mathcal{H}_0$$

- **canonically smeared field operator changes number of particles by $\pm 1$**

$$\hat{\phi}[f]\,|g\rangle \;=\; \hat{\phi}[f]\,\hat{a}^\dagger[g]|0\rangle = \frac{1}{2}\,|g,\omega^{-1}f\rangle + (f,g)\,|0\rangle .$$

# Notation for light-front quantization $\hbar = c = 2\pi = 1$

- canonically smeared scalar fields

$$\hat{\phi}[x^+, \underline{f}] = \int_{\mathbb{R}^3} \mathrm{d}^3\bar{\boldsymbol{x}}\, \underline{f}(\bar{\boldsymbol{x}})\, \hat{\phi}(x^+, \bar{\boldsymbol{x}}), \qquad x^\pm = \frac{x^0 \pm x^3}{\sqrt{2}}, \quad \bar{\boldsymbol{x}} = (x^-, \boldsymbol{x}_\perp)$$

- real-valued test function $\underline{f}(\bar{\boldsymbol{x}}) \in \mathscr{S}(\mathbb{R}^3)$
- Fourier transform of test function $\tilde{\underline{f}} \in \mathscr{S}(\mathbb{R}^3)$

$$\tilde{\underline{f}}(\bar{\boldsymbol{k}}) = \int_{\mathbb{R}^3} \mathrm{d}^3\bar{\boldsymbol{x}}\, \underline{f}(\bar{\boldsymbol{x}})\, \mathrm{e}^{\mathrm{i}\bar{\boldsymbol{k}}\cdot\bar{\boldsymbol{x}}}, \qquad \bar{\boldsymbol{k}} = (k^+, \boldsymbol{k}_\perp), \qquad \bar{\boldsymbol{k}}\cdot\bar{\boldsymbol{x}} = -x^- k^+ + \boldsymbol{k}_\perp \cdot \boldsymbol{x}_\perp$$

- new test function $\omega_{LF}^a \underline{f} : \mathbb{R}^3 \to \mathbb{R}$, with $a \in \mathbb{Z}$, $\omega_{LF}(\boldsymbol{k}_\perp) = \sqrt{m^2 + \boldsymbol{k}_\perp^2}$

$$\left(\omega_{LF}^a\, \underline{f}\right)(\bar{\boldsymbol{x}}) = \int_{\mathbb{R}^3} \mathrm{d}^3\bar{\boldsymbol{k}}\, \mathrm{e}^{\mathrm{i}\bar{\boldsymbol{k}}\cdot\bar{\boldsymbol{x}}}\, (m^2 + \boldsymbol{k}_\perp^2)^{a/2}\, \tilde{\underline{f}}(\bar{\boldsymbol{k}}), \qquad \omega_{LF}^a \underline{f} \in \mathscr{S}(\mathbb{R}^3)$$

- inner product for test functions $(\underline{f}, \underline{g}) \in \mathscr{S}(\mathbb{R}^3)$

$$(\underline{f}, \underline{g}) = \int_{\mathbb{R}^3} \mathrm{d}^3\bar{\boldsymbol{x}}\, \underline{f}(\boldsymbol{x})\, g(\boldsymbol{x}) = \int_{\mathbb{R}^3} \mathrm{d}^3\bar{\boldsymbol{k}}\, \tilde{\underline{f}}^*(\bar{\boldsymbol{k}})\, \tilde{\underline{g}}(\bar{\boldsymbol{k}}) < \infty$$

# Light-Front canonical quantization

- LF canonical commutators at $x^+ = 0$ we denote $\hat{\phi}[0, \underline{f}] = \hat{\phi}[\underline{f}]$

$$\left[\hat{\phi}[\underline{f}], \hat{\phi}[\partial_- \underline{g}]\right] = -\frac{\mathrm{i}}{2}\left(\underline{f},\ \underline{g}\right)\ \mathbf{1}_{op},$$

- Heisenberg equation for temporal evolution

$$\mathrm{d}_+\hat{\phi}[\partial_- f] = -\mathrm{i}\left[\hat{\phi}[\partial_- f], \hat{P}^-\right] = \frac{1}{2}\hat{\phi}[\omega_{LF}^2 \underline{f}],$$

- kinematical infinitesimal translations

$$\left[\hat{\phi}[\underline{f}], \hat{P}^+\right] = -\mathrm{i}\hat{\phi}[\partial_- \underline{f}], \quad \left[\hat{\phi}[\underline{f}], \hat{P}_i\right] = -\mathrm{i}\hat{\phi}[\partial_i \underline{f}].$$

- **preliminary 1-particle states**

$$|\underline{f}\rangle := \hat{\phi}[\underline{f}]|0\rangle = \int_{\mathbb{R}^3}\frac{\mathrm{d}^3\bar{\boldsymbol{k}}}{2k^+}\tilde{\underline{f}}(\bar{\boldsymbol{k}})\,|\bar{\boldsymbol{k}}\rangle, \quad \Longrightarrow \quad |\bar{\boldsymbol{k}}\rangle = 2k^+\int_{\mathbb{R}^3}\mathrm{d}^3\bar{\boldsymbol{x}}\,\mathrm{e}^{\mathrm{i}\bar{\boldsymbol{k}}\cdot\bar{\boldsymbol{x}}}\,\hat{\phi}(\bar{\boldsymbol{x}})\,|0\rangle$$

# Light-front vacuum and diagonalization of $\hat{P}^+$

- LF vacuum state $|0\rangle$: normalized, unique and Poincaré invariant

$$\langle 0|0\rangle = 1, \quad \hat{\boldsymbol{M}}|0\rangle = 0, \quad \hat{\boldsymbol{M}} = \{\hat{P}^{\pm}, \hat{\boldsymbol{P}}_{\perp}, \hat{M}_{\mu\nu}\}$$

- **diagonalization of $\hat{P}^+$**

$$\hat{P}^+ \, \hat{\phi}[\underline{f}]|0\rangle = \mathrm{i}\hat{\phi}[\partial_- \underline{f}]|0\rangle, \qquad \hat{P}^+|\bar{\boldsymbol{k}}\rangle = k^+ \, |\bar{\boldsymbol{k}}\rangle,$$

- **nonnegative spectrum of $\hat{P}^+$ leads to the condition for $\mathbf{k^+ < 0}$**

$$|\bar{\boldsymbol{k}}\rangle = 0 \quad \Longrightarrow \quad 2k^+ \int_{\mathbb{R}^3} \mathrm{d}^3\bar{\boldsymbol{x}} \, \mathrm{e}^{\mathrm{i}\bar{\boldsymbol{k}}\cdot\bar{\boldsymbol{x}}} \, \hat{\phi}(\boldsymbol{x}) \, |0\rangle = 0$$

- **canonically smeared 1-particle states**

$$|\underline{f}\rangle = \int_{\mathbb{R}^3} \frac{\mathrm{d}^3\bar{\boldsymbol{k}}}{2k^+} \, \Theta(k^+) \, \tilde{\underline{f}}(\bar{\boldsymbol{k}}) \, |\bar{\boldsymbol{k}}\rangle = \hat{\phi}[\underline{f}]|0\rangle.$$

# Momentum dependent 1-particle states

- decomposition of LF canonical commutator

$$\langle 0|\hat{\phi}[\underline{f}]\,\hat{\phi}[\partial_-\,\underline{g}]|0\rangle - \langle 0|\hat{\phi}[\partial_-\,\underline{g}]\,\hat{\phi}[\underline{f}]|0\rangle = -\,\frac{\mathrm{i}}{2}\,\left(\underline{f}\,,\,\underline{g}\right)\,,$$

- **inner product for momentum dependent kets**

$$\langle \bar{k}|\bar{p}\rangle = 2k^+\delta^3(\bar{p}-\bar{k})$$

- **inner product for canonically smeared 1-particle states is ill defined** in $\mathscr{S}(\mathbb{R}^3)$

$$\langle \underline{f}|\,\underline{g}\rangle = \int_{\mathbb{R}^3}\frac{\mathrm{d}^3\bar{\boldsymbol{k}}}{2k^+}\,\Theta(k^+)\,\tilde{\underline{f}}^*(\bar{\boldsymbol{k}})\,\tilde{\underline{g}}(\boldsymbol{k}),$$

- for Schlieder-Seiler test functions

$$\hat{\mathscr{S}}(\mathbb{R}^3) := \left\{\,\underline{g}\in\mathscr{S}(\mathbb{R}^3);\quad \tilde{\underline{g}}(k^+,\boldsymbol{k}_\perp) = 0,\quad \text{for}\quad k^+ = 0\right\}$$

inner product is finite $\langle \underline{f}|\,\underline{g}\rangle < \infty$

# Light-front canonically smeared 1-particle states

- the Heisenberg equation for the canonically smeared operators

$$\left[\hat{\phi}[\partial_- \underline{f}], \hat{H}\right] = \frac{\mathrm{i}}{2}\hat{\phi}[\omega_{LF}^2 \underline{f}], \implies \hat{H}|\partial_- \underline{f}\rangle = -\frac{\mathrm{i}}{2}|\omega_{LF}^2 \underline{f}\rangle, \qquad \hat{H} = \hat{P^-}$$

- LF Schrödinger equations for momentum dependent 1-particle kets (for $k^+ > 0$)

$$\hat{H}|\bar{\boldsymbol{k}}\rangle = \frac{m^2 + \boldsymbol{k}_\perp^2}{2k^+}\,|\bar{\boldsymbol{k}}\rangle.$$

- the expectation value of the Hamiltonian operator is ill defined for $\underline{f} \in \hat{\mathscr{S}}(\mathbb{R}^3)$ quad (Schlieder-Seiler test functions)

$$\langle \underline{f}|\hat{H}|\underline{f}\rangle = \int_{\mathbb{R}^3} \frac{\mathrm{d}^3\bar{\boldsymbol{k}}}{2k^+}\,\Theta(k^+)\,\left|\tilde{\underline{f}}(\bar{\boldsymbol{k}})\right|^2 \frac{m^2 + \boldsymbol{k}_\perp^2}{2k^+}$$

- **problem for the quantum mechanical interpretation of 1-particle states**
- **manifestation of light-front infra-red problem**

# Smearing in light-front time $x^+$

- LF time dependent momentum 1-particle kets, valid for $k^+ > 0$

$$|x^+, \bar{\boldsymbol{k}}\rangle := \mathrm{e}^{\mathrm{i}x^+\hat{H}}|\bar{\boldsymbol{k}}\rangle = \mathrm{e}^{\mathrm{i}x^+k^-}|\bar{\boldsymbol{k}}\rangle, \qquad k^- = \frac{m^2 + \boldsymbol{k}_\perp^2}{2k^+}$$

- **LF time smeared 1-particle momentum dependent kets**

$$|f_+, \bar{k}\rangle := \int_{\mathbb{R}} \mathrm{d}x^+ f_+(x^+)\, |x^+, \bar{\boldsymbol{k}}\rangle = \tilde{f}_+^*\left(k^-\right) |\bar{\boldsymbol{k}}\rangle, \quad f_+(x) \in \mathscr{S}(\mathbb{R})$$

- **totally smeared 1-particle states**

$$|f_+, \underline{f}\rangle := \int_{\mathbb{R}^3} \frac{\mathrm{d}^3\bar{\boldsymbol{k}}}{2k^+}\, \Theta(k^+)\, \tilde{\underline{f}}(\bar{\boldsymbol{k}})\, |f_+, \bar{\boldsymbol{k}}\rangle$$

- the LF temporal evolution of the quantum field operator

$$\hat{\phi}[x^+, \underline{f}] = \mathrm{e}^{\mathrm{i}\hat{H}x^+}\, \hat{\phi}[\underline{f}]\, \mathrm{e}^{-\mathrm{i}\hat{H}x^+},$$

- **totally smeared field operator - local field operator**

$$\hat{\phi}[f_+, \underline{f}] := \int_{\mathbb{R}} \mathrm{d}x^+ f_+(x^+)\, \hat{\phi}[x^+, \underline{f}]$$

# Totally smeared fields - local field operators

- **totally smeared 1-particle states**

$$\boxed{\hat{\phi}[f_+, \underline{f}]|0\rangle = |f_+, \underline{f}\rangle,}$$

- **have finite norm**

$$\langle f_+, \underline{f}|f_+, \underline{f}\rangle = \int_{\mathbb{R}^3} \frac{d^3\bar{\boldsymbol{k}}}{2k^+} \Theta(k^+) \left|\tilde{\underline{f}}(\bar{\boldsymbol{k}})\right|^2 \left|f_+(k^-)\right|^2 < \infty, \qquad k^- = \frac{m^2 + \boldsymbol{k}_\perp^2}{2k^+}$$

- **finite expectation value for the Hamiltonian operator**

$$\langle f_+, \underline{f}|\hat{H}|f_+, \underline{f}\rangle = \int_{\mathbb{R}^3} \frac{d^3\bar{\boldsymbol{k}}}{(2\pi)^3} \frac{\Theta(k^+)}{2k^+} \left|\tilde{\underline{f}}(\bar{\boldsymbol{k}})\right|^2 \left|f_+(k^-)\right|^2 \frac{m^2 + \boldsymbol{k}_\perp^2}{2k^+} < \infty,$$

- **thus** $|f_+, \underline{f}\rangle \in \mathcal{H}_0$

# Conclusions

## ♦ vacuum state

Equal-time
- - diagonalization of $\hat{H}$
- - depends on interaction (mass)

Light-front
- - diagonalization of $\hat{P}^+$
- - kinematical (mass indep't)

## ♦ 1-particle state

- momentum dependent kets $\notin \mathcal{H}_0$
- canonically smeared kets

Equal-time
- - $|f\rangle \in \mathcal{H}_0$
- - depend on interaction (mass)

Light-front
- - $|\underline{f}\rangle \notin \mathcal{H}_0$
- - kinematical (mass indep't)

- totally smeared LF kets

  - $|f_+, \underline{f}\rangle \in \mathcal{H}_0$

  - depend on interaction (mass)

  - no LF infra-red problem

# Prospects

**smearing with test function is a promising tool for LF QFT**

- **LF perturbation theory**
  - propagators
  - closed loop diagrams
- **LF non-perturbative theory**
  - **? ? ?**

# Prospects

**smearing with test function is a promising tool for LF QFT**

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*Thank you for your attention*