GENERALIZED TMDS
IN HADRONIC COLLISIONS

SHOHINI BHATTACHARYA

TEMPLE UNIVERSITY

LIGHT CONE 2018

OUTLINE

➢ **Generalized TMDs (GTMDs)**

➢ **Quark GTMDs in Exclusive Double Drell-Yan Process**

(S. Bhattacharya, A. Metz, J. Zhou / arXiv: 1702.04387)

▪ **Old + New results on accessing quark GTMDs**

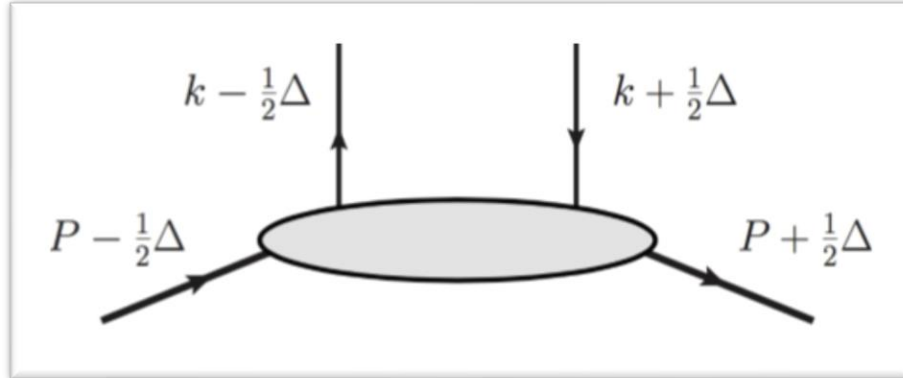➢ **Gluon GTMDs in Exclusive Double Quarkonia Production Process**

(S. Bhattacharya, A. Metz, V. K. Ojha, J. Tsai, J. Zhou / arXiv: 1802.10550)

▪ **Define the process**

▪ **Kinematics**

▪ **Scattering Amplitude**

▪ **Gluon GTMDs**

▪ **Polarization Observables**

➢ **Summary and Outlook**

Generalized Transverse Momentum Dependent distributions

GTMD matrix element- graphical depiction

$$P = \frac{(p + p')}{2} \qquad \Delta = p' - p \qquad t = \Delta^2$$
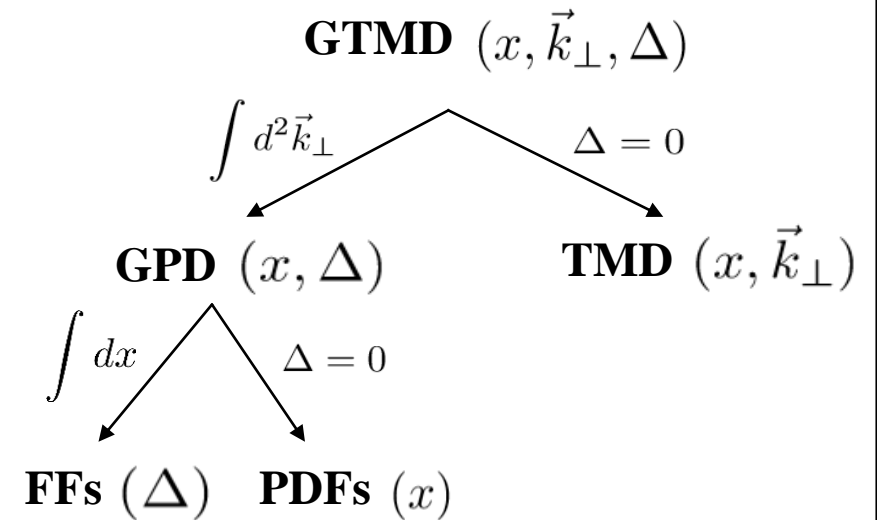
GTMD matrix element definition

$$W_{\lambda,\lambda'}^{q\,[\Gamma]}(P, \Delta, x, \vec{k}_\perp) =$$

$$\int \frac{dz^- \, d^2\vec{z}_\perp}{2(2\pi)^3} \, e^{ik\cdot z} \, \langle p', \lambda'| \, \bar{q}(-\tfrac{z}{2}) \, \Gamma \, \mathcal{W}(-\tfrac{z}{2}, \tfrac{z}{2}) \, q(\tfrac{z}{2}) \, |p, \lambda\rangle \Big|_{z^+=0}$$

Matrix elements parameterized through GTMDs

$$X(x, \xi, \vec{k}_\perp, \vec{\Delta}_\perp)$$

$$x = \frac{k^+}{P^+} \qquad\qquad \xi = \frac{p^+ - p'^+}{p^+ + p'^+} = -\frac{\Delta^+}{2P^+}$$

GTMDs as 'mother distributions'

GTMD $(x, \vec{k}_\perp, \Delta)$

$\int d^2\vec{k}_\perp \qquad\qquad \Delta = 0$

GPD (x, Δ) \qquad\qquad TMD $(x, \vec{k}_\perp)$

$\int dx \qquad\qquad \Delta = 0$
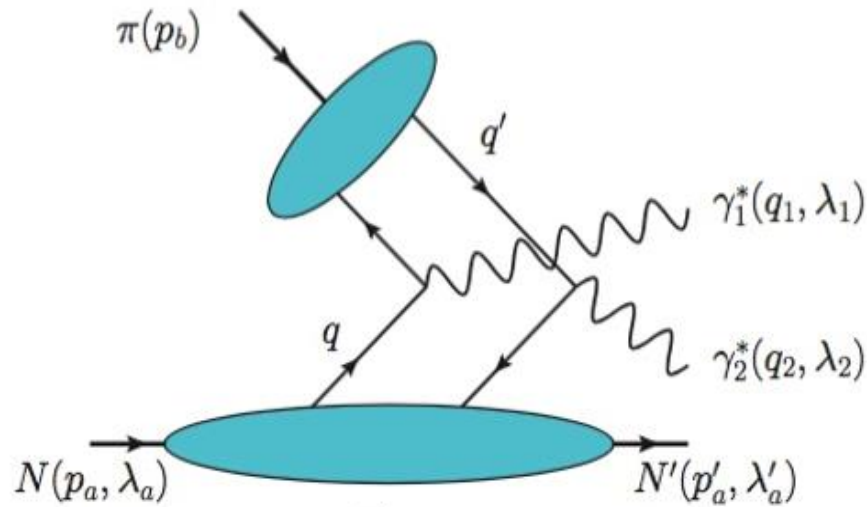
FFs (Δ) \qquad PDFs (x)

In particular, $F_{1,4}$ and $G_{1,1}$ have a direct role in understanding spin-structure of hadrons (Lorcé, Pasquini, 2011 / Hatta, 2011 / Lorcé, 2014). But, how would we measure them?
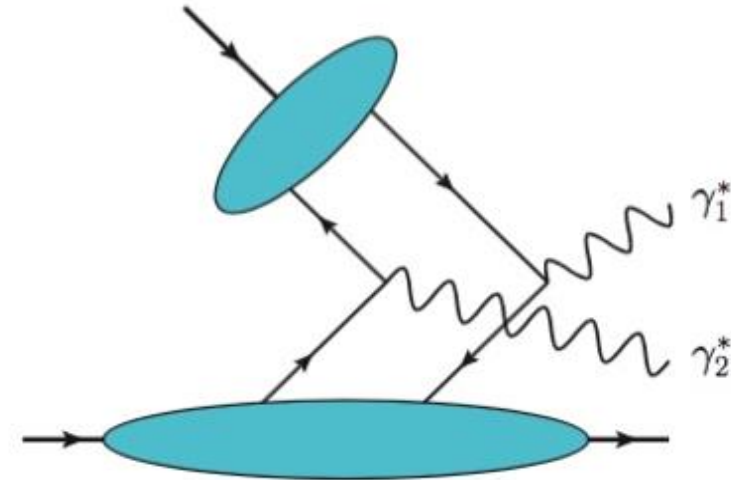
Process $\pi(p_b)\, N(p_a, \lambda_a) \to (l_1^- l_1^+)(l_2^- l_2^+)\, N'(p_a, \lambda_a)$

Leading-order diagrams

Graph (a)

Graph (b)

Kinematics (TMD type)

Kinematical variables

$s = (p_a + p_b)^2 \approx 2p_a^+ p_b^-$ large

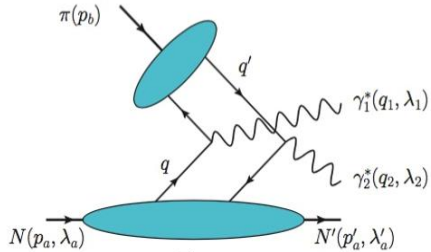$q_1^2,\ q_2^2$ large $\qquad |\vec{q}_{i\perp}^2| \ll q_i^2$

$x_a = \dfrac{k_a^+}{P_a^+} \qquad x_b = \dfrac{k_b^-}{p_b^-} \qquad \xi_a = \dfrac{(q_1^+ + q_2^+)}{2P_a^+} \qquad \vec{\Delta}_{a\perp} = -(\vec{q}_{1\perp} + \vec{q}_{2\perp})$

$$
\mathcal{T}^{\mu\nu}_{\lambda_a,\lambda'_a} = i \sum_{q,q'} e_q e'_q e^2 \frac{1}{N_c} \int d^2\vec{k}_{a\perp} \int d^2\vec{k}_{b\perp} \delta^{(2)}\left(\frac{\Delta\vec{q}_\perp}{2} - \vec{k}_{a\perp} - \vec{k}_{b\perp}\right) \Phi^{q'q}_\pi(x_b, \vec{k}^2_{b\perp})
$$

$$
\left[-i\varepsilon^{\mu\nu}_\perp \left(W^{qq'\,[\gamma^+]}_{\lambda_a,\lambda'_a}(x_a, \vec{k}_{a\perp}) - W^{qq'\,[\gamma^+]}_{\lambda_a,\lambda'_a}(-x_a, -\vec{k}_{a\perp}) \right) \right.
$$

$$
\left. - g^{\mu\nu}_\perp \left(W^{qq'\,[\gamma^+\gamma_5]}_{\lambda_a,\lambda'_a}(x_a, \vec{k}_{a\perp}) + W^{qq'\,[\gamma^+\gamma_5]}_{\lambda_a,\lambda'_a}(-x_a, -\vec{k}_{a\perp}) \right) \right]
$$

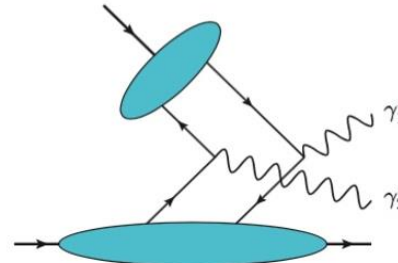|		
Graph (a) **Longitudinal parton momenta fixed as:** $$x_a = \frac{(q_1^+ - q_2^+)}{2P_a^+}$$ $$x_b = 1 - \frac{q_1^-}{p_b^-} = \frac{q_2^-}{p_b^-}$$	**Graph (b)** **Longitudinal parton momenta fixed as:** $$x_a = -\frac{(q_1^+ - q_2^+)}{2P_a^+}$$ $$x_b = \frac{q_1^-}{p_b^-}$$	**ERBL region** $$\xi_a = \frac{(q_1^+ + q_2^+)}{2P_a^+}$$ $$-\xi_a \le x_a \le \xi_a$$ **Independent transverse vectors** $$\vec{\Delta}_{a\perp} = -(\vec{q}_{1\perp} + \vec{q}_{2\perp})$$ $$\Delta\vec{q}_\perp = (\vec{q}_{1\perp} - \vec{q}_{2\perp})$$

$$W^{q\,[\gamma^+]}_{\lambda,\lambda'} \;=\; \frac{1}{2M}\,\bar{u}(p',\lambda')\left[F^q_{1,1} + \frac{i\sigma^{i+}k^i_\perp}{P^+}F^q_{1,2} + \frac{i\,\sigma^{i+}\Delta^i_\perp}{P^+}F^q_{1,3} + \frac{i\sigma^{ij}k^i_\perp\Delta^j_\perp}{M^2}F^q_{1,4}\right]u(p,\lambda)$$

$$W^{q\,[\gamma^+\gamma_5]}_{\lambda,\lambda'} \;=\; \frac{1}{2M}\,\bar{u}(p',\lambda')\left[-\frac{i\varepsilon^{ij}_\perp k^i_\perp\Delta^j_\perp}{M^2}G^q_{1,1} + \frac{i\sigma^{i+}\gamma_5 k^i_\perp}{P^+}G^q_{1,2} + \frac{i\sigma^{i+}\gamma_5\Delta^i_\perp}{P^+}G^q_{1,3} + i\sigma^{+-}\gamma_5 G^q_{1,4}\right]u(p,\lambda)$$

GTMDs have real and imaginary part

Polarization Observables

Unpolarized

$$\tau_{UU} = \frac{1}{2} \sum_{\lambda,\lambda'} |\mathcal{T}_{\lambda,\lambda'}|^2$$

SSA

$$\tau_{LU} = \frac{1}{2} \sum_{\lambda'} \left(|\mathcal{T}_{+,\lambda'}|^2 - |\mathcal{T}_{-,\lambda'}|^2 \right)$$

DSA

$$\tau_{LL} = \frac{1}{2} \left(\left(|\mathcal{T}_{+,+}|^2 - |\mathcal{T}_{+,-}|^2 \right) - \left(|\mathcal{T}_{-,+}|^2 - |\mathcal{T}_{-,-}|^2 \right) \right)$$

Key point is to realize that if we ~
Define 'Polarization Observables' in a certain fashion
Take proper linear combination of those observables

↓

'Single out' what you want!

Remarks:
- Summation over photon polarization (λ_1, λ_2) is implied
- If photon polarization (λ_1, λ_2) is summed over, there is no interference between the objects $W^{q[\gamma^+]}_{\lambda,\lambda'}$ and $W^{q[\gamma^+ \gamma^5]}_{\lambda,\lambda'}$

Simple Case: **Addressing GTMDs at a specific kinematical point** $\vec{\Delta}_{a\perp} = 0$

Surviving Candidates: $F_{1,1}$ $F_{1,2}$ $G_{1,2}$ $G_{1,4}$

Direct Access

- $\frac{1}{4}(\tau_{UU}+\tau_{LL}+\tau_{XX}+\tau_{YY}) = 2C^{(-)}\left[F_{1,1}(x_a,\xi_a,\vec{k}_{a\perp}^2)\,\phi_\pi(x_b,\vec{k}_{b\perp}^2)\right]C^{(-)}\left[F_{1,1}^*(x_a,\xi_a,\vec{p}_{a\perp}^2)\,\phi_\pi^*(x_b,\vec{p}_{b\perp}^2)\right]$

- $\frac{1}{4}(\tau_{UU}+\tau_{LL}-\tau_{XX}-\tau_{YY}) = 2C^{(+)}\left[G_{1,4}(x_a,\xi_a,\vec{k}_{a\perp}^2)\,\phi_\pi(x_b,\vec{k}_{b\perp}^2)\right]C^{(+)}\left[G_{1,4}^*(x_a,\xi_a,\vec{p}_{a\perp}^2)\,\phi_\pi^*(x_b,\vec{p}_{b\perp}^2)\right]$

- $\tau_{XY} = \tau_{YX}$

$$= -4\frac{(1-\xi_a^2)^2}{M_a^2}\Delta q_\perp^1 \Delta q_\perp^2 \left\{ C^{(+)}\left[\frac{\Delta\vec{q}_\perp.\vec{k}_{a\perp}}{\Delta\vec{q}_\perp^2}F_{1,2}(x_a,\xi_a,\vec{k}_{a\perp}^2)\,\phi_\pi(x_b,\vec{k}_{b\perp}^2)\right]C^{(+)}\left[\frac{\Delta\vec{q}_\perp.\vec{p}_{a\perp}}{\Delta\vec{q}_\perp^2}F_{1,2}^*(x_a,\xi_a,\vec{p}_{a\perp}^2)\,\phi_\pi^*(x_b,\vec{p}_{b\perp}^2)\right]\right.$$

$$\left. - C^{(-)}\left[\frac{\Delta\vec{q}_\perp.\vec{k}_{a\perp}}{\Delta\vec{q}_\perp^2}G_{1,2}(x_a,\xi_a,\vec{k}_{a\perp}^2)\,\phi_\pi(x_b,\vec{k}_{b\perp}^2)\right]C^{(-)}\left[\frac{\Delta\vec{q}_\perp.\vec{p}_{a\perp}}{\Delta\vec{q}_\perp^2}G_{1,2}^*(x_a,\xi_a,\vec{p}_{a\perp}^2)\,\phi_\pi^*(x_b,\vec{p}_{b\perp}^2)\right]\right\}$$

Convolution integral

$$C^{(\pm)}\left[w(\vec{k}_{a\perp},\vec{k}_{b\perp})\,X\,\Phi_\pi\right] = \frac{e^2}{\sqrt{1-\xi_a^2}N_c}\sum_{q,q'}e_q e_q' \int d^2\vec{k}_{a\perp}\int d^2\vec{k}_{b\perp}\,\delta^{(2)}\left(\frac{\Delta\vec{q}_\perp}{2}-\vec{k}_{a\perp}-\vec{k}_{b\perp}\right)w(\vec{k}_{a\perp},\vec{k}_{b\perp})$$

$$\times\left[X^{qq'}(x_a,\vec{k}_{a\perp}) \pm X^{qq'}(-x_a,-\vec{k}_{a\perp})\right]\Phi_\pi^{q'q}(x_b,\vec{k}_{b\perp}^2)$$

$$\varepsilon_\perp^{\mu\nu}\,\epsilon_\mu^*\,\epsilon_\nu^*\,W_{\lambda_a,\lambda_a'}^{qq'\,[\gamma^+]}(x_a,\vec{k}_{a\perp})$$

$$g_\perp^{\mu\nu}\,\epsilon_\mu^*\,\epsilon_\nu^*\,W_{\lambda_a,\lambda_a'}^{qq'\,[\gamma^+\gamma_5]}(x_a,\vec{k}_{a\perp})$$

Not discussed in arXiv: 1702.04387

- $\tau_{XU} = 4\dfrac{(1-\xi_a^2)}{M_a}\Delta q_\perp^2 \, \boxed{Im.}\Big\{ C^{(-)}\Big[F_{1,1}(x_a,\xi_a,\vec{k}_{a\perp}^{\,2})\,\phi_\pi(x_b,\vec{k}_{b\perp}^{\,2})\Big] C^{(+)}\Big[\dfrac{\Delta\vec{q}_\perp.\vec{p}_{a\perp}}{\Delta\vec{q}_\perp^2} F_{1,2}^*(x_a,\xi_a,\vec{p}_{a\perp}^{\,2})\,\phi_\pi^*(x_b,\vec{p}_{b\perp}^{\,2})\Big]$

  $\qquad - C^{(+)}\Big[G_{1,4}(x_a,\xi_a,\vec{k}_{a\perp}^{\,2})\,\phi_\pi(x_b,\vec{k}_{b\perp}^{\,2})\Big] C^{(-)}\Big[\dfrac{\Delta\vec{q}_\perp.\vec{p}_{a\perp}}{\Delta\vec{q}_\perp^2} G_{1,2}^*(x_a,\xi_a,\vec{p}_{a\perp}^{\,2})\,\phi_\pi^*(x_b,\vec{p}_{b\perp}^{\,2})\Big]\Big\}$

- $\tau_{XL} = -4\dfrac{(1-\xi_a^2)}{M_a}\Delta q_\perp^1 \, \boxed{Re.}\Big\{ C^{(-)}\Big[F_{1,1}(x_a,\xi_a,\vec{k}_{a\perp}^{\,2})\,\phi_\pi(x_b,\vec{k}_{b\perp}^{\,2})\Big] C^{(+)}\Big[\dfrac{\Delta\vec{q}_\perp.\vec{p}_{a\perp}}{\Delta\vec{q}_\perp^2} F_{1,2}^*(x_a,\xi_a,\vec{p}_{a\perp}^{\,2})\,\phi_\pi^*(x_b,\vec{p}_{b\perp}^{\,2})\Big]$

  $\qquad - C^{(+)}\Big[G_{1,4}(x_a,\xi_a,\vec{k}_{a\perp}^{\,2})\,\phi_\pi(x_b,\vec{k}_{b\perp}^{\,2})\Big] C^{(-)}\Big[\dfrac{\Delta\vec{q}_\perp.\vec{p}_{a\perp}}{\Delta\vec{q}_\perp^2} G_{1,2}^*(x_a,\xi_a,\vec{p}_{a\perp}^{\,2})\,\phi_\pi^*(x_b,\vec{p}_{b\perp}^{\,2})\Big]\Big\}$

Remark:

$Re.F_{1,1}$ and $Re.G_{1,4}$ treated as 'key' candidates to get access to real and imaginary part of other GTMDs since they are presumably large

✓ $Re.(F_{1,1})\big|_{\Delta=0} = f_1(x,\vec{k}_\perp^{\,2})$

✓ $Re.(G_{1,4})\big|_{\Delta=0} = g_1(x,\vec{k}_\perp^{\,2})$

$Im.(F_{1,2})\big|_{\Delta=0} = -f_{1T}^\perp$

$Re.(G_{1,2})\big|_{\Delta=0} = g_{1T}$

General Case: Keep $\vec{\Delta}_{a\perp}$!

Direct Access

Old piece! But now,

$$G_{1,4}(x, \xi, \vec{k}_\perp^2, \vec{\Delta}_\perp^2, \vec{k}_\perp \cdot \vec{\Delta}_\perp)$$

$$\bullet \frac{1}{4}\left(\tau_{UU} + \tau_{LL} - \tau_{XX} - \tau_{YY}\right) = 2\,C^{(+)}\Big[G_{1,4}\,\phi_\pi\Big]C^{(+)}\Big[G_{1,4}^*\,\phi_\pi^*\Big] +$$

$$\frac{2}{M_a^4}\left(\varepsilon_\perp^{ij}\Delta q_\perp^i \Delta_{a\perp}^j\right)^2 C^{(+)}\Big[\vec{\beta}_\perp \cdot \vec{k}_{a\perp}\,F_{1,4}\,\phi_\pi\Big]C^{(+)}\Big[\vec{\beta}_\perp \cdot \vec{p}_{a\perp}\,F_{1,4}^*\,\phi_\pi^*\Big]$$

New piece!

Old piece!

$$\bullet \frac{1}{4}\left(\tau_{UU} + \tau_{LL} + \tau_{XX} + \tau_{YY}\right) = 2\,C^{(-)}\Big[F_{1,1}\,\phi_\pi\Big]C^{(-)}\Big[F_{1,1}^*\,\phi_\pi^*\Big] +$$

$$\frac{2}{M_a^4}\left(\varepsilon_\perp^{ij}\Delta q_\perp^i \Delta_{a\perp}^j\right)^2 C^{(-)}\Big[\vec{\beta}_\perp \cdot \vec{k}_{a\perp}\,G_{1,1}\,\phi_\pi\Big]C^{(-)}\Big[\vec{\beta}_\perp \cdot \vec{p}_{a\perp}\,G_{1,1}^*\,\phi_\pi^*\Big]$$

New piece!

weight factor $\quad \vec{\beta}_\perp = \dfrac{\vec{\Delta}_{a\perp}^2\,\Delta\vec{q}_\perp - (\vec{\Delta}_{a\perp} \cdot \Delta\vec{q}_\perp)\,\vec{\Delta}_{a\perp}}{\vec{\Delta}_{a\perp}^2\,\Delta\vec{q}_\perp^2 - (\vec{\Delta}_{a\perp} \cdot \Delta\vec{q}_\perp)^2} = \dfrac{\vec{\Delta}_{a\perp} \times (\Delta\vec{q}_\perp \times \vec{\Delta}_{a\perp})}{\left(\varepsilon_\perp^{ij}\Delta q_\perp^i \Delta_{a\perp}^j\right)^2}$

Access through Interference $(\vec{\Delta}_{a\perp} \neq 0)$

$$\frac{1}{2}\left(\tau_{XY} - \tau_{YX}\right) = \frac{4}{M_a^2}\left(\varepsilon_\perp^{ij}\Delta q_\perp^i \Delta_{a\perp}^j\right) Re. \left\{ C^{(-)}\left[F_{1,1}\,\phi_\pi\right]C^{(+)}\left[\vec{\beta}_\perp \cdot \vec{k}_{a\perp} F_{1,4}^*\,\phi_\pi^*\right] \right.$$

$$\left. - C^{(+)}\left[G_{1,4}\,\phi_\pi\right]C^{(-)}\left[\vec{\beta}_\perp \cdot \vec{p}_{a\perp} G_{1,1}^*\,\phi_\pi^*\right]\right\}$$

Remark:
This linear combination is sensitive to $Re.F_{1,4}$ **and** $Re.G_{1,1}$ **(sensitivity to strength of spin-orbit correlation)**

What about $F_{1,2}$ $F_{1,3}$ $G_{1,2}$ $G_{1,3}$ **in the general case,** $\vec{\Delta}_{a\perp} \neq 0$ **?**

Not discussed in arXiv:
1702.04387

Example: $F_{1,2}$

Direct Access   **May not be possible in this case!**

$$\varepsilon_\perp^{\mu\nu} \epsilon_\mu^* \epsilon_\nu^* W_{\lambda_a,\lambda_a'}^{qq' [\gamma^+]}(x_a, \vec{k}_{a\perp})$$

Access through Interference   **Challenging! But possible!**

$$g_\perp^{\mu\nu} \epsilon_\mu^* \epsilon_\nu^* W_{\lambda_a,\lambda_a'}^{qq' [\gamma^+\gamma_5]}(x_a, \vec{k}_{a\perp})$$

- $$\frac{1}{4}\left[\Delta_{a\perp}^1(\tau_{XU} + \tau_{UX}) + \frac{\xi_a \vec{\Delta}_{a\perp}^2}{2M_a}(\tau_{LU} + \tau_{UL})\right]$$

$$= -2\frac{(1-\xi_a^2)}{M_a}(\varepsilon_\perp^{ij}\Delta q_\perp^i \Delta_{a\perp}^j)\boxed{Im.}\left\{C^{(-)}\Big[F_{1,1}\,\phi_\pi\Big]C^{(+)}\Big[\vec{\beta}_{a\perp}.\vec{p}_{a\perp}\,\boxed{F_{1,2}^*}(x_a, \xi_a, \vec{p}_{a\perp}^2, \vec{\Delta}_{a\perp}^2, \vec{p}_{a\perp}.\vec{\Delta}_{a\perp})\,\phi_\pi^*\Big]\right\} + \ldots$$

- $$\frac{1}{4}\left[\Delta_{a\perp}^1(\tau_{LY} - \tau_{YL}) - \frac{\xi_a \vec{\Delta}_{a\perp}^2}{2M_a}(\tau_{XY} - \tau_{YX})\right]$$

$$= -2\frac{(1-\xi_a^2)}{M_a}(\varepsilon_\perp^{ij}\Delta q_\perp^i \Delta_{a\perp}^j)\boxed{Re.}\left\{C^{(-)}\Big[F_{1,1}\,\phi_\pi\Big]C^{(+)}\Big[\vec{\beta}_{a\perp}.\vec{p}_{a\perp}\,\boxed{F_{1,2}^*}(x_a, \xi_a, \vec{p}_{a\perp}^2, \vec{\Delta}_{a\perp}^2, \vec{p}_{a\perp}.\vec{\Delta}_{a\perp})\,\phi_\pi^*\Big]\right\} + \ldots$$

Remark:
Similar consideration can be made for $F_{1,3}$ **,** $G_{1,2}$ **and** $G_{1,3}$

Exclusive Double Quarkonium Production and GTMDs of gluon
(SB, Metz, Ojha, Tsai, Zhou / arXiv: 1802.10550)

Process $\quad N_a(p_a, \lambda_a) + N_b(p_b, \lambda_b) \rightarrow \eta_Q(q_1) + \eta_Q(q_2) + N_a(p_a', \lambda_a') + N_b(p_b', \lambda_b')$

Leading-order diagrams
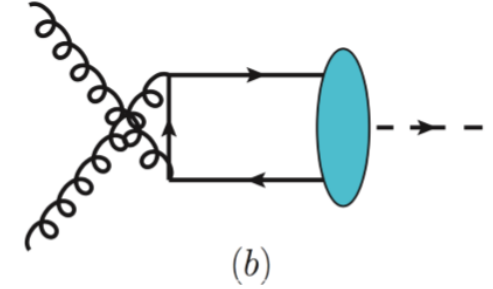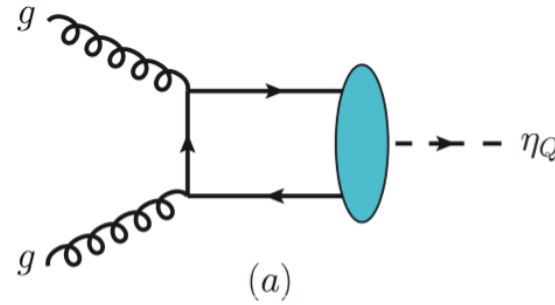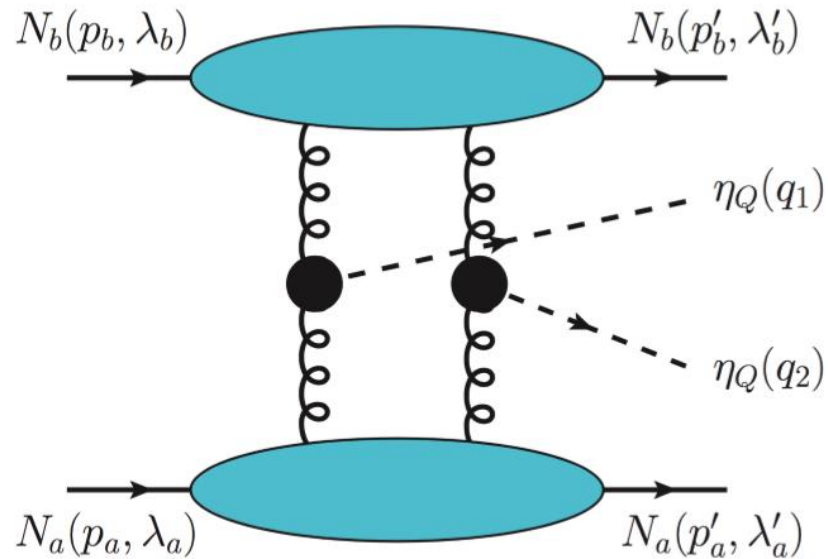
Kinematics (TMD type)

$$s = (p_a + p_b)^2 \approx 2p_a^+ p_b^- \text{ large} \qquad |\vec{q}_{i\perp}| << m_\eta \qquad q_1^2 = q_2^2 = m_\eta^2$$

$$\xi_a = \frac{(q_1^+ + q_2^+)}{2P_a^+} \qquad \xi_b = \frac{(q_1^- + q_2^-)}{2P_b^-}$$

Scattering Amplitude

$$\Delta \vec{q}_\perp = (\vec{q}_{1\perp} - \vec{q}_{2\perp})$$

$$\mathcal{T}_{\lambda_a, \lambda_a'; \lambda_b, \lambda_b'} = -iA \int d^2 \vec{k}_{a\perp} \int d^2 \vec{k}_{b\perp}\, \delta^{(2)} \left(\frac{\Delta \vec{q}_\perp}{2} - \vec{k}_{a\perp} - \vec{k}_{b\perp} \right)$$

$$\times \left[W^g_{\lambda_a, \lambda_a'}(x_a, \vec{k}_{a\perp})\, W^g_{\lambda_b, \lambda_b'}(x_b, \vec{k}_{b\perp}) + W^g_{\lambda_a, \lambda_a'}(-x_a, -\vec{k}_{a\perp})\, W^g_{\lambda_b, \lambda_b'}(-x_b, -\vec{k}_{b\perp}) \right.$$

$$\left. + \widetilde{W}^g_{\lambda_a, \lambda_a'}(x_a, \vec{k}_{a\perp})\, \widetilde{W}^g_{\lambda_b, \lambda_b'}(x_b, \vec{k}_{b\perp}) + \widetilde{W}^g_{\lambda_a, \lambda_a'}(-x_a, -\vec{k}_{a\perp})\, \widetilde{W}^g_{\lambda_b, \lambda_b'}(-x_b, -\vec{k}_{b\perp}) \right]$$

Radial wavefunction of η_Q at origin

$$A = \frac{g_s^4\, R_0^2(0)\, s}{N_c(N_c^2 - 1)\, \pi\, m_\eta^5\, (1 + \xi_a)(1 + \xi_b)}$$

Longitudinal parton momenta fixed:

$$x_a = \frac{(q_1^+ - q_2^+)}{2P_a^+} \qquad x_b = \frac{(q_1^- - q_2^-)}{2P_b^-} \qquad -\xi \le x \le \xi \quad \textbf{ERBL region}$$

Gluon GTMDs

$$W^{g\,[ij]}_{\lambda, \lambda'}(P, \Delta, x, \vec{k}_\perp) = \frac{1}{P^+} \int \frac{dz^-\, d^2 \vec{z}_\perp}{(2\pi)^3}\, e^{ik\cdot z} \langle p', \lambda' | F_a^{+i}(-\tfrac{z}{2})\, \mathcal{W}_{ab}(-\tfrac{z}{2}, \tfrac{z}{2})\, F_b^{+j}(\tfrac{z}{2}) | p, \lambda \rangle \Big|_{z^+ = 0}$$

$$W^g_{\lambda, \lambda'} = \delta_\perp^{ij} W^{g[ij]}_{\lambda, \lambda'} \to W^{q[\gamma^+]}_{\lambda, \lambda'}$$

$$\widetilde{W}^g_{\lambda, \lambda'} = -i\epsilon_\perp^{ij} W^{g[ij]}_{\lambda, \lambda'} \to W^{q[\gamma^+ \gamma^5]}_{\lambda, \lambda'}$$

Polarization Observables

$$\tau_{UU} = \frac{1}{2}\sum_{\lambda_b,\lambda_b'}\frac{1}{2}\sum_{\lambda_a,\lambda_a'}|\mathcal{T}_{\lambda_a,\lambda_a';\lambda_b,\lambda_b'}|^2$$

$$\tau_{LU} = \frac{1}{2}\sum_{\lambda_b,\lambda_b'}\frac{1}{2}\sum_{\lambda_a'}\left(|\mathcal{T}_{+,\lambda_a';\lambda_b,\lambda_b'}|^2 - |\mathcal{T}_{-,\lambda_a';\lambda_b,\lambda_b'}|^2\right)$$

$$\tau_{LL} = \frac{1}{2}\sum_{\lambda_b,\lambda_b'}\frac{1}{2}\left(\left(|\mathcal{T}_{+,+;\lambda_b,\lambda_b'}|^2 - |\mathcal{T}_{+,-;\lambda_b,\lambda_b'}|^2\right) - \left(|\mathcal{T}_{-,+;\lambda_b,\lambda_b'}|^2 - |\mathcal{T}_{-,-;\lambda_b,\lambda_b'}|^2\right)\right)$$

We sum / average over polarizations of Nucleon `b'

Direct Access

Assumed hierarchy of magnitude of GTMDs

$$F_{1,1} > G_{1,4} > \text{others}$$

$$\frac{1}{4}\left(\tau_{UU} + \tau_{LL} - \tau_{XX} - \tau_{YY}\right)$$

$$\approx \quad \frac{1}{M^4}\left(\varepsilon_\perp^{ij}\Delta q_\perp^i \Delta_{a\perp}^j\right)^2 C\left[\vec{\beta}_\perp \cdot \vec{k}_{a\perp} \, F_{1,4}(x_a, \vec{k}_{a\perp}) \, F_{1,1}(x_b, \vec{k}_{b\perp})\right] C\left[\vec{\beta}_\perp \cdot \vec{p}_{a\perp} \, F_{1,4}^*(x_a, \vec{p}_{a\perp}) \, F_{1,1}^*(x_b, \vec{p}_{b\perp})\right]$$

$$+ \quad C\left[G_{1,4}(x_a, \vec{k}_{a\perp}) \, G_{1,4}(x_b, \vec{k}_{b\perp})\right] C\left[G_{1,4}^*(x_a, \vec{p}_{a\perp}) \, G_{1,4}^*(x_b, \vec{p}_{b\perp})\right]$$

convolution integral

$$C\left[w(\vec{k}_{a\perp}, \vec{k}_{b\perp}) \, X \, Y\right] = \frac{2A}{\sqrt{1-\xi_a^2}\sqrt{1-\xi_b^2}} \int d^2\vec{k}_{a\perp} \int d^2\vec{k}_{b\perp} \, \delta^{(2)}\left(\frac{\Delta\vec{q}_\perp}{2} - \vec{k}_{a\perp} - \vec{k}_{b\perp}\right)$$

$$\times \, w(\vec{k}_{a\perp}, \vec{k}_{b\perp}) \, X(x_a, \vec{k}_{a\perp}) \, Y(x_b, \vec{k}_{b\perp})$$

weight factor $\quad \vec{\beta}_\perp = \dfrac{\vec{\Delta}_{a\perp}^2 \, \Delta\vec{q}_\perp - (\vec{\Delta}_{a\perp} \cdot \Delta\vec{q}_\perp)\,\vec{\Delta}_{a\perp}}{\vec{\Delta}_{a\perp}^2 \, \Delta\vec{q}_\perp^2 - (\vec{\Delta}_{a\perp} \cdot \Delta\vec{q}_\perp)^2} = \dfrac{\vec{\Delta}_{a\perp} \times (\Delta\vec{q}_\perp \times \vec{\Delta}_{a\perp})}{\left(\varepsilon_\perp^{ij}\Delta q_\perp^i \Delta_{a\perp}^j\right)^2}$

Accessing $G_{1,1}$ ($\vec{\Delta}_{a\perp} \neq 0, \vec{\Delta}_{b\perp} = 0$)

Direct Access

Assumed hierarchy of magnitude of GTMDs
$$F_{1,1} > G_{1,4} > \text{others}$$

$$\frac{1}{4}\left(\tau_{UU} + \tau_{LL} + \tau_{XX} + \tau_{YY}\right)$$

$$\approx \quad \frac{\varepsilon_\perp^{ij}\Delta_{a\perp}^j}{M} \frac{\varepsilon_\perp^{kl}\Delta_{a\perp}^l}{M} C\left[\frac{k_{a\perp}^i}{M}G_{1,1}(x_a,\vec{k}_{a\perp})\,G_{1,4}(x_b,\vec{k}_{b\perp})\right] C\left[\frac{p_{a\perp}^k}{M}G_{1,1}^*(x_a,\vec{p}_{a\perp})\,G_{1,4}^*(x_b,\vec{p}_{b\perp})\right]$$

$$+ \; C\left[F_{1,1}(x_a,\vec{k}_{a\perp})\,F_{1,1}(x_b,\vec{k}_{b\perp})\right] C\left[F_{1,1}^*(x_a,\vec{p}_{a\perp})\,F_{1,1}^*(x_b,\vec{p}_{b\perp})\right]$$

Takes over $\left|G_{1,1}\right|^2$ contribution. Hence, $G_{1,1}$ cannot probably be accessed directly.

Access through Interference ($\vec{\Delta}_{a\perp} \neq 0, \vec{\Delta}_{b\perp} = 0$)

Assumed hierarchy of magnitude of GTMDs
$$F_{1,1} > G_{1,4} > \text{others}$$

- $\dfrac{1}{2}\left(\tau_{UL} + \tau_{LU}\right)$

$$\approx 2\,\mathrm{Im}\left\{ -\frac{1}{M^2}\left(\varepsilon_\perp^{ij}\Delta q_\perp^i \Delta_{a\perp}^j\right) C\left[\vec{\beta}_\perp \cdot \vec{k}_{a\perp} F_{1,4}(x_a,\vec{k}_{a\perp})\,F_{1,1}(x_b,\vec{k}_{b\perp})\right] C\left[F_{1,1}^*(x_a,\vec{p}_{a\perp})\,F_{1,1}^*(x_b,\vec{p}_{b\perp})\right] \right.$$

$$+ \frac{1}{M^2}\left(\varepsilon_\perp^{ij}\Delta q_\perp^i \Delta_{a\perp}^j\right) C\left[\vec{\beta}_\perp \cdot \vec{k}_{a\perp} G_{1,1}(x_a,\vec{k}_{a\perp})\,G_{1,4}(x_b,\vec{k}_{b\perp})\right] C\left[G_{1,4}^*(x_a,\vec{p}_{a\perp})\,G_{1,4}^*(x_b,\vec{p}_{b\perp})\right] \right\}$$

$$\approx 2\,\mathrm{Im}\left\{ -\frac{1}{M^2}\left(\varepsilon_\perp^{ij}\Delta q_\perp^i \Delta_{a\perp}^j\right) C\left[\vec{\beta}_\perp \cdot \vec{k}_{a\perp}\,F_{1,4}(x_a,\vec{k}_{a\perp})\,F_{1,1}(x_b,\vec{k}_{b\perp})\right] C\left[F_{1,1}^*(x_a,\vec{p}_{a\perp})\,F_{1,1}^*(x_b,\vec{p}_{b\perp})\right] \right\}$$

$Re.G_{1,1}$ $Im.G_{1,1}$ **overwhelmed by three powers of** $F_{1,1}$

- $\dfrac{1}{2}\left(\tau_{XY} - \tau_{YX}\right)$

$$\approx 2\,\mathrm{Re}\left\{ -\frac{1}{M^2}\left(\varepsilon_\perp^{ij}\Delta q_\perp^i \Delta_{a\perp}^j\right) C\left[\vec{\beta}_\perp \cdot \vec{k}_{a\perp}\,F_{1,4}(x_a,\vec{k}_{a\perp})\,F_{1,1}(x_b,\vec{k}_{b\perp})\right] C\left[F_{1,1}^*(x_a,\vec{p}_{a\perp})\,F_{1,1}^*(x_b,\vec{p}_{b\perp})\right] \right\}$$

Remark:
The second linear combination is sensitive to gluon Orbital Angular Momentum

Summary and Outlook

➢ **G**eneralized **T**ransverse **M**omentum **D**ependent distributions are important-

• 'mother' distributions to TMDs, GPDs and other non-perturbative functions

• direct link to partonic orbital angular momentum ($F_{1,4}$ and $G_{1,1}$)

➢ **Proposal:** Processes that are sensitive to quark / gluon **GTMDs** are the **Exclusive Double Drell-Yan and Exclusive Double Quarkonia production**

   1) Quark / Gluon **GTMDs** can be accessed in the ERBL region

   2) **GTMDs** can be accessed through **P**olarization **O**bservables

➢ **What else can be done?**

• include effect of linearly polarized gluons in exclusive double quarkonia production

• search for process that is sensitive to GTMDs in DGLAP region

• numerical estimate for observables