Elastic form factors for resonances from LQCD

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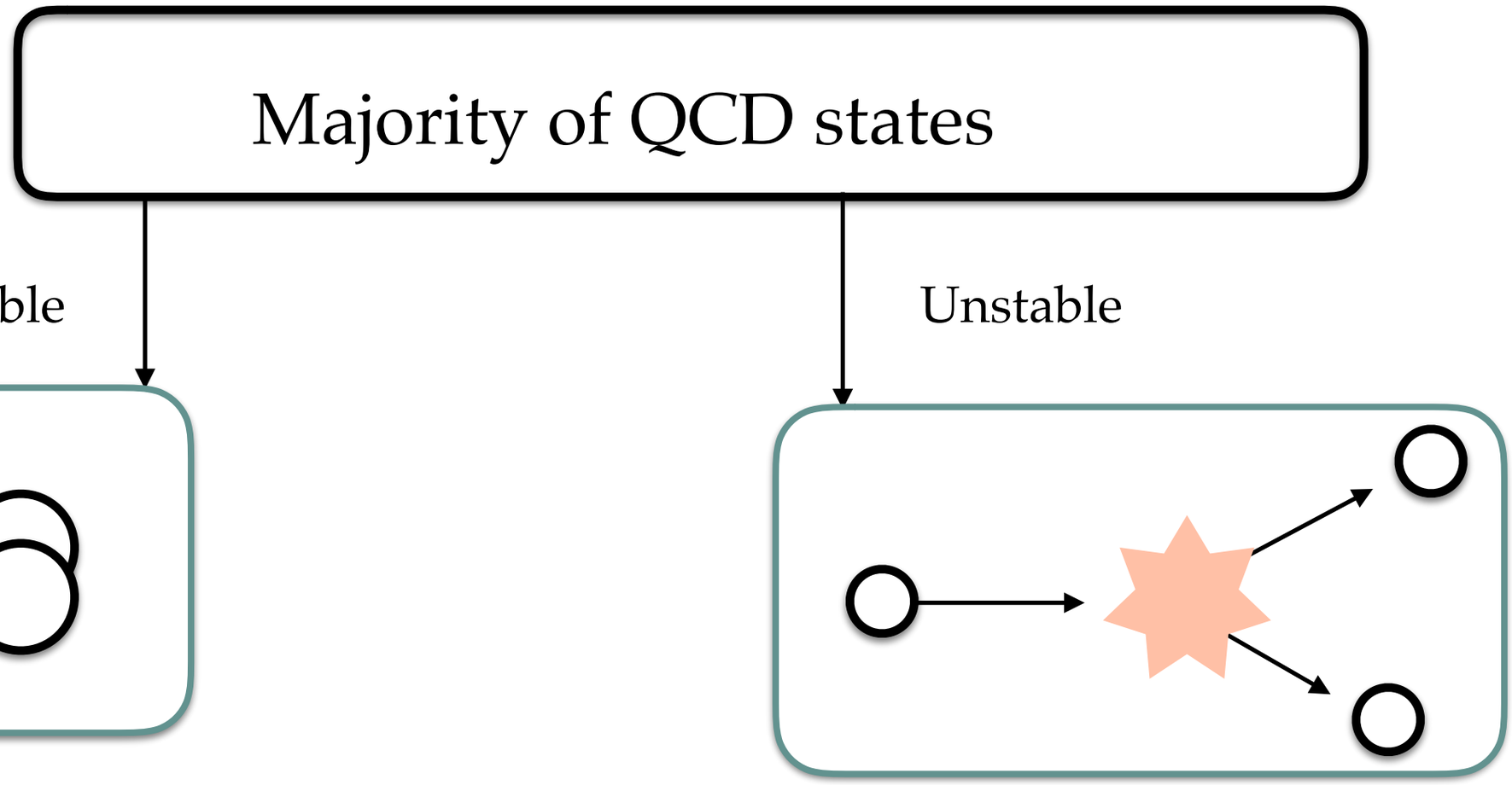
Light cone 2018
Jefferson Lab

Composite states



- Bound states
  - nuclei (deuteron …)
  - hypernuclei
  - …

- Resonances
  - ϱ → ππ
  - the Roper (Nπ, Nππ,…)
  - …

Goals

- Developing a framework for studying :
  - structure of composite states
    - structure of the deuteron
    - structure of resonances
  - Weak processes involving few-hadron systems
    - parity violation
    - p-p fusion

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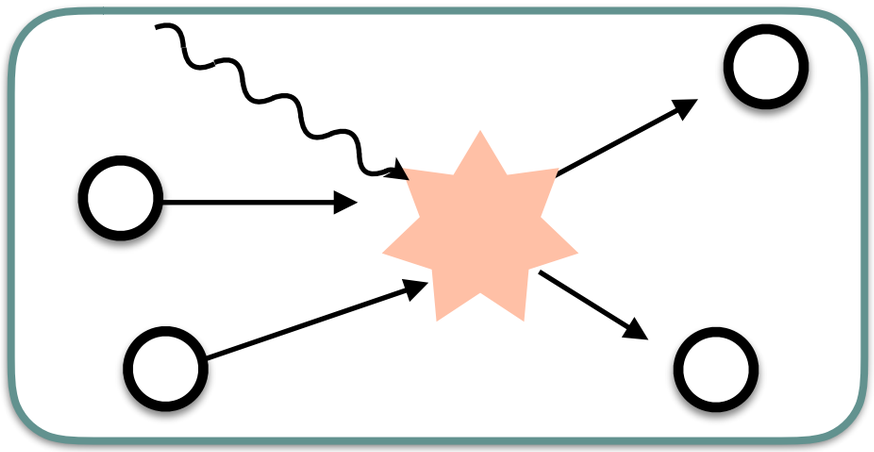
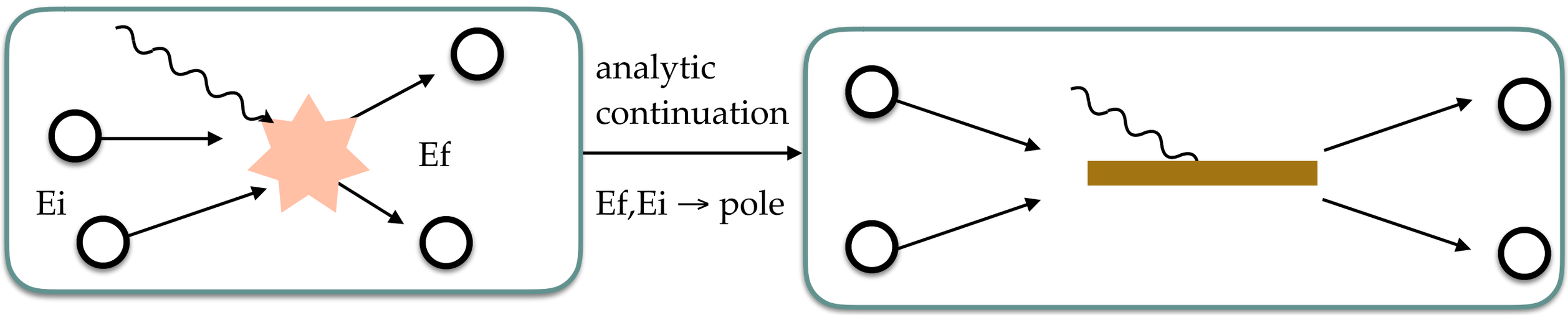
Form factor of a resonance

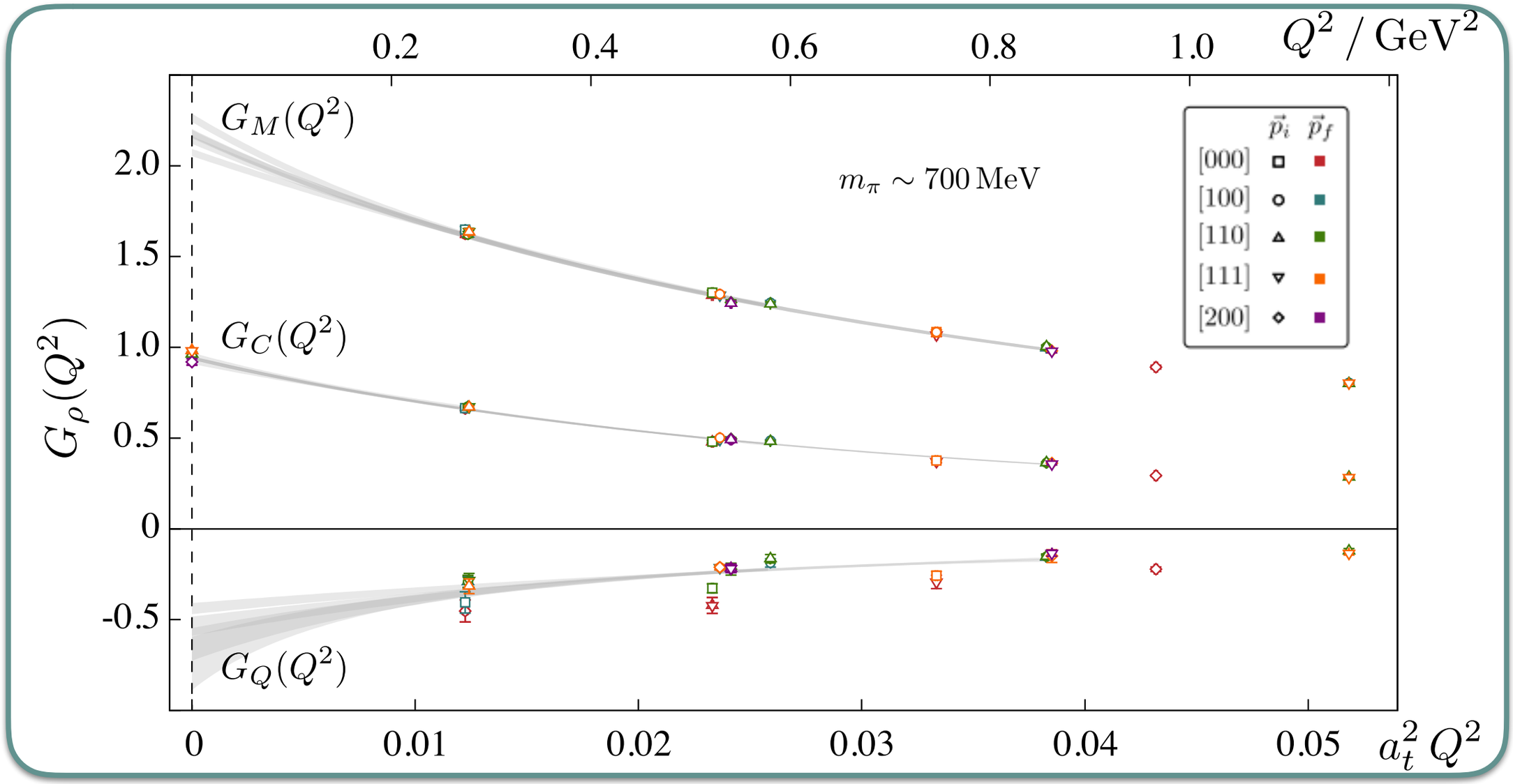
- Resonance are not asymptotic states → Form factor?



- We can get the form factors from amplitudes



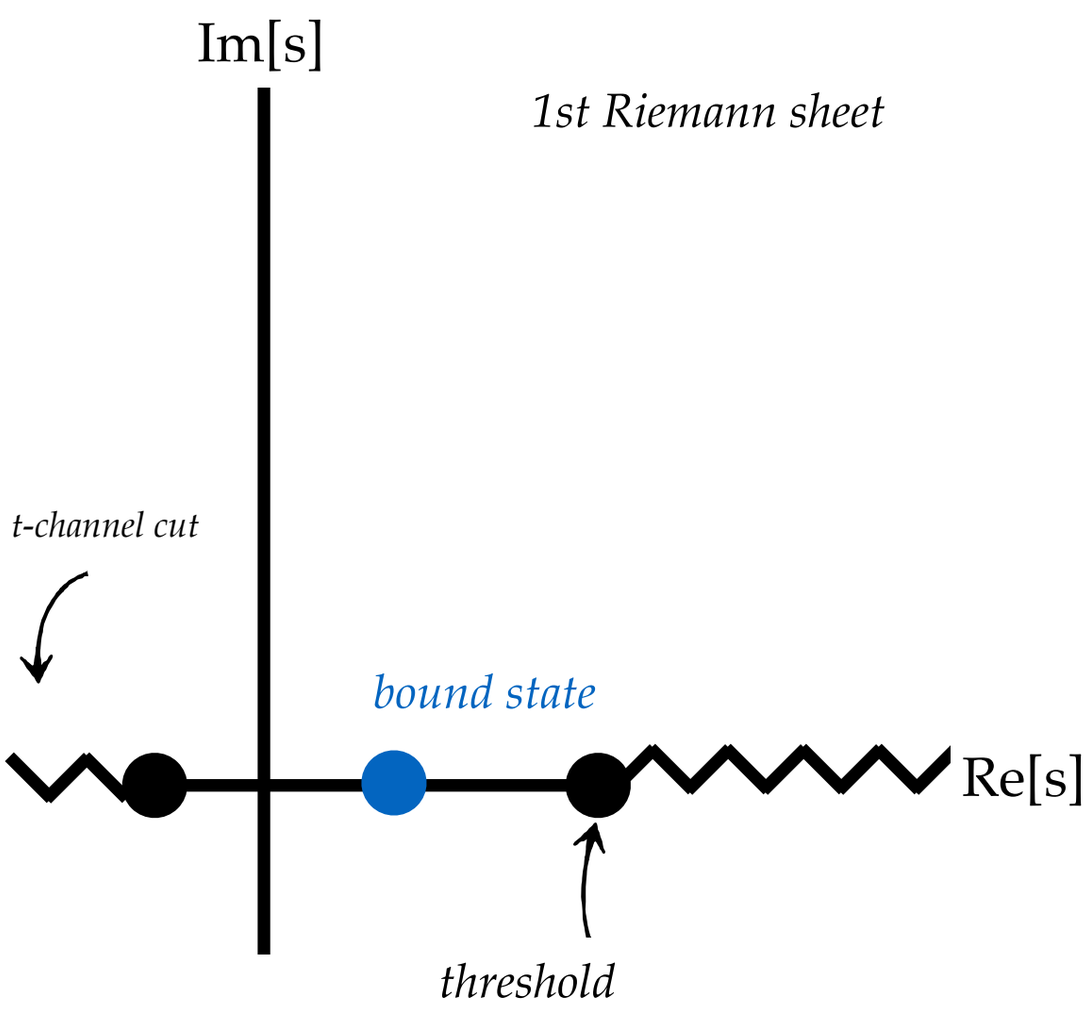
Form factor of a stable “resonance“



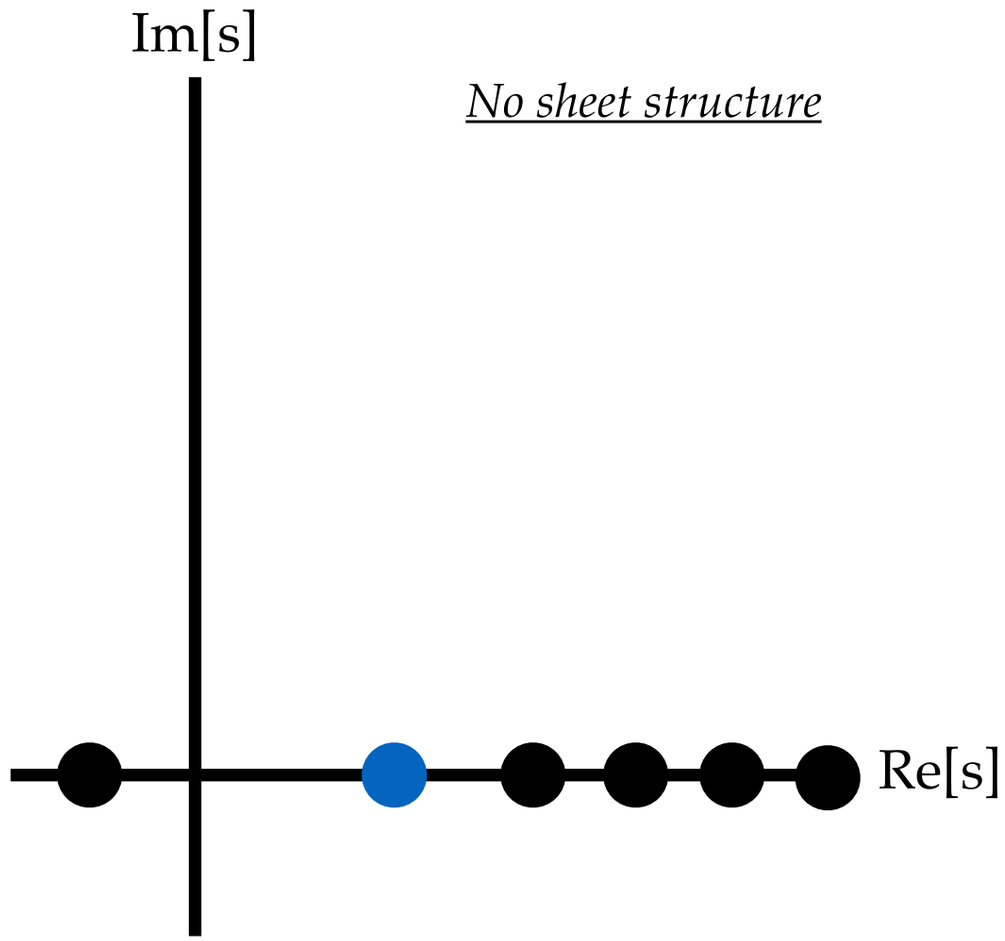
- Shultz, Dudek, and Edwards PRD (2015)

Scattering amplitude I

Infinite volume



Finite volume



$$\mathrm{s} = E_{\mathrm{cm}}^2$$

Scattering amplitude II

Infinite volume

Im[s]

2nd Riemann sheet

t-channel cut

Narrow resonance

Re[s]

$$s_R = (E_R - \frac{i}{2}\Gamma_R)^2$$

$$\mathrm{s} = E_{\mathrm{cm}}^2$$

Finite volume

Im[s]

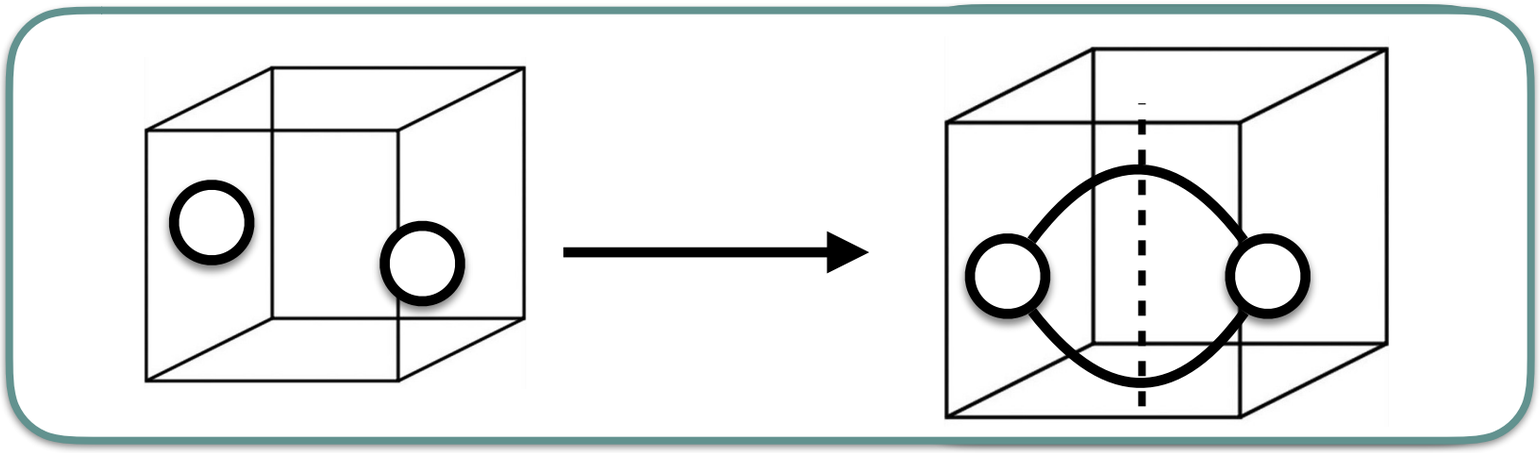
No sheet structure

No resonances

Re[s]

Finite volume effects

- Finite volume effects are complicated for matrix elements with multi-hadron states
  - On-shell intermediate states give singularities

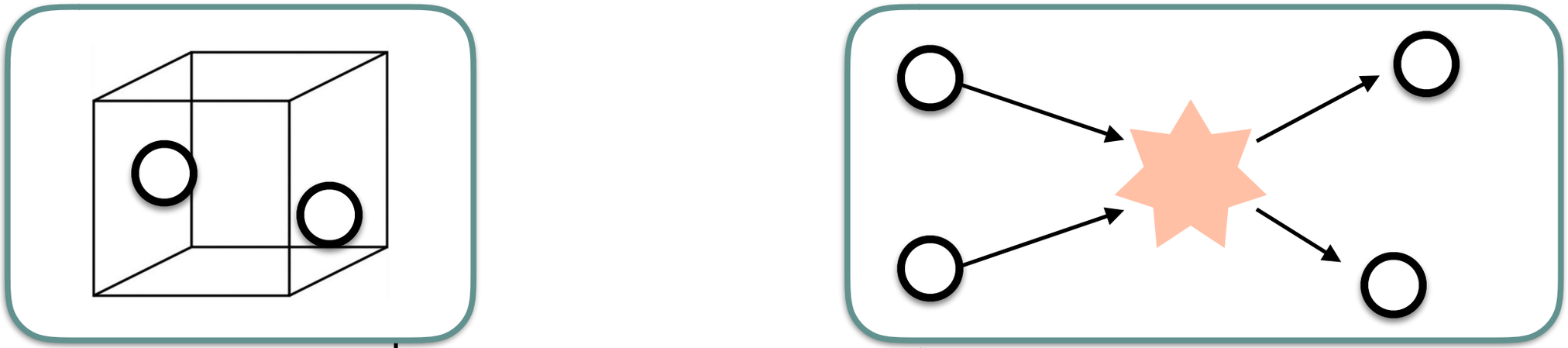


- Formalism needed to deal with these effects:

  - Briceño and Hansen (2016) - general, inelastic, relativistic
  - Rusetsky et al. (2012) - EFT dependent, NR
  - Briceño and Davoudi (2012) - EFT dependent, NR

$2 \to 2$

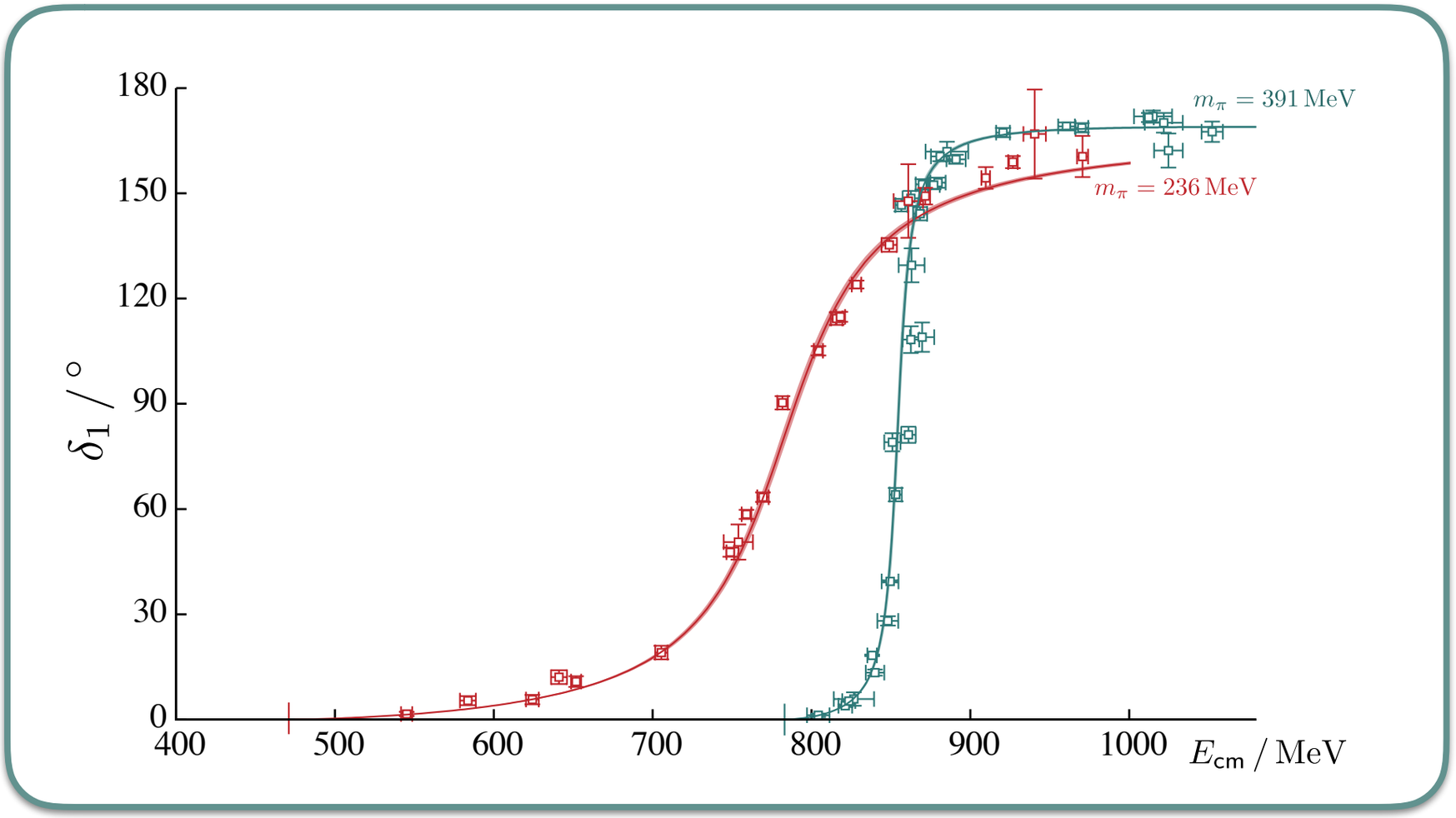
- FV spectra to infinite volume purely hadronic amplitudes
- Holds for a generic QFT with hadronic d.o.f, up to multi-particle thresholds



$$\det\left[F^{-1}(E_L, L) + M(E_L)\right] = 0$$

- Lüscher (1986, 1991) [elastic scalar bosons]
- Rummukainen & Gottlieb (1995) [moving elastic scalar bosons]
- Kim, Sachrajda, & Sharpe/Christ, Kim & Yamazaki (2005) [QFT derivation]
- Feng, Li, & Liu (2004) [inelastic scalar bosons]
- Hansen & Sharpe / Briceño & Davoudi (2012) [moving inelastic scalar bosons]
- Briceño (2014) [general 2-body result]

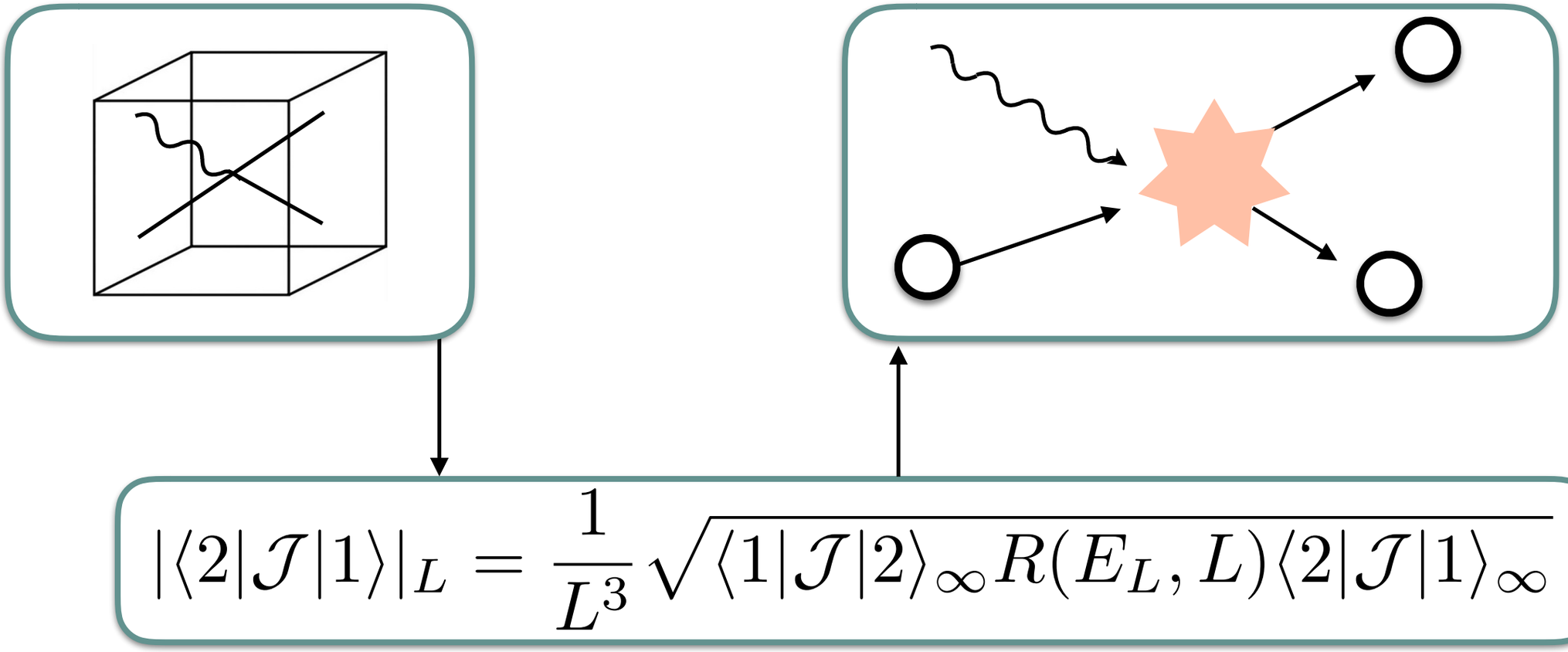
$2 \to 2$



- Wilson, Briceño, Dudek, Edwards, and Thomas PRD (2015)

$1+\mathcal{J}\to 2$

- FV matrix elements to infinite volume electroweak amplitudes

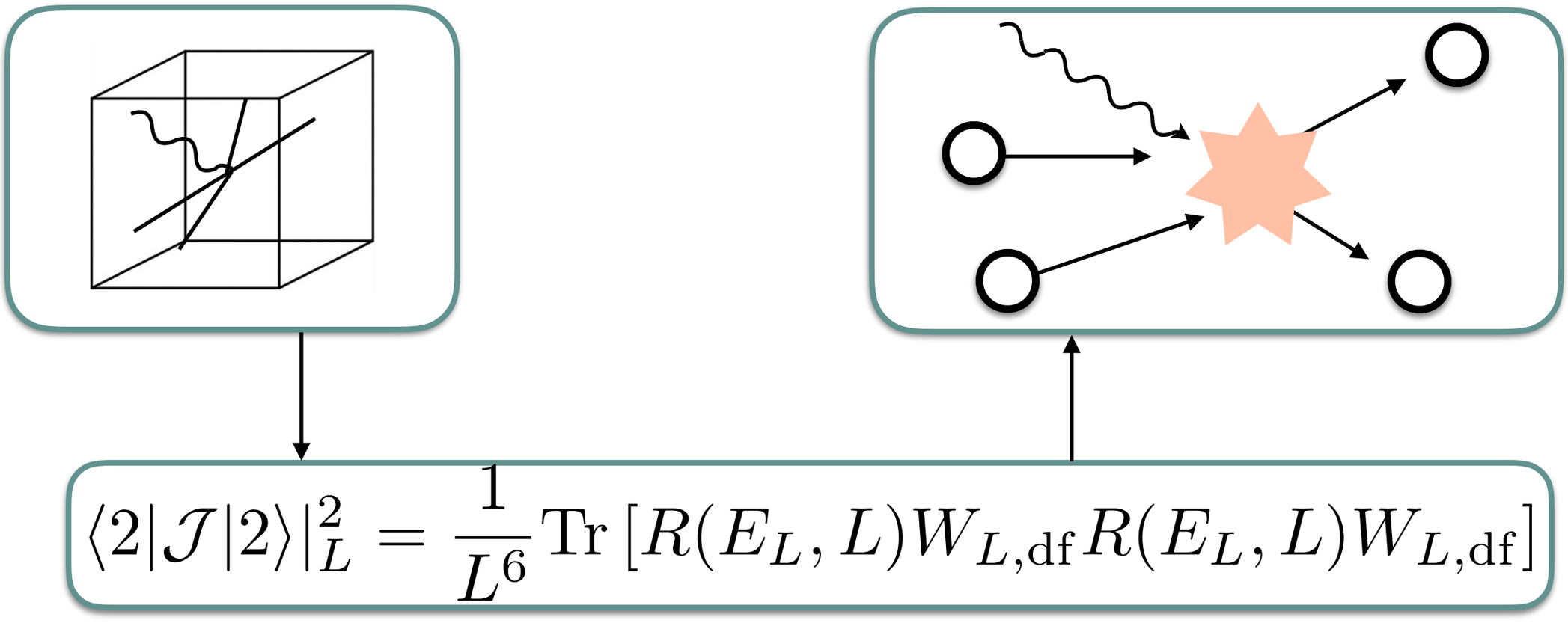


$$|\langle 2|\mathcal{J}|1\rangle|_L = \frac{1}{L^3}\sqrt{\langle 1|\mathcal{J}|2\rangle_\infty R(E_L, L)\langle 2|\mathcal{J}|1\rangle_\infty}$$

- Lellouch & Lüscher (2000) [K-to-ππ at rest]
- Kim, Sachrajda, & Sharpe/Christ, Kim & Yamazaki (2005) [moving K-to-ππ]
- Hansen & Sharpe (2012) [D-to-ππ/KK]
- Briceño, Hansen Walker-Loud /Briceño & Hansen(2014-2015)[general 1-to-2]

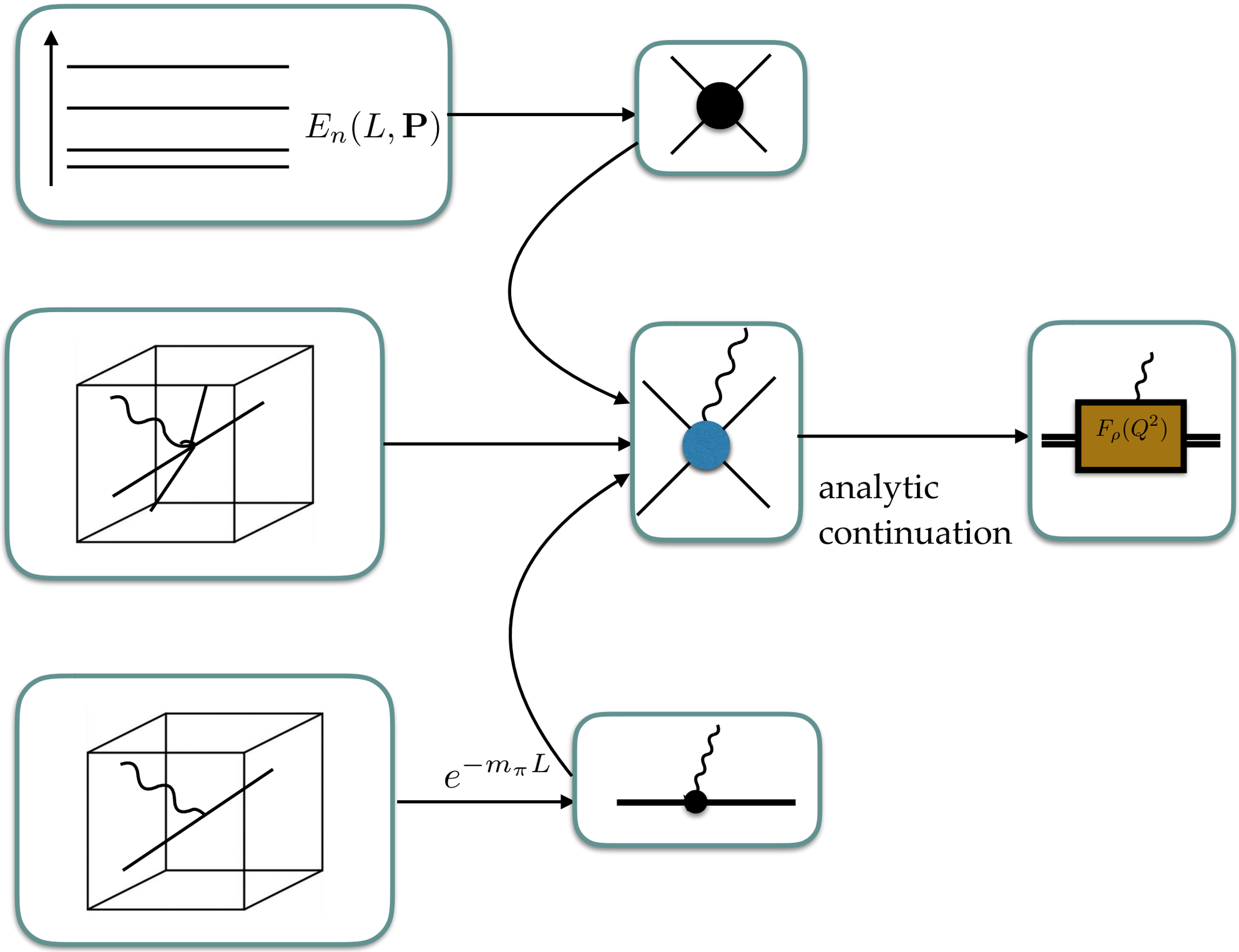
$2+\mathcal{J} \to 2$

- FV matrix elements to infinite volume electroweak amplitudes



- Briceño & Hansen (2016)

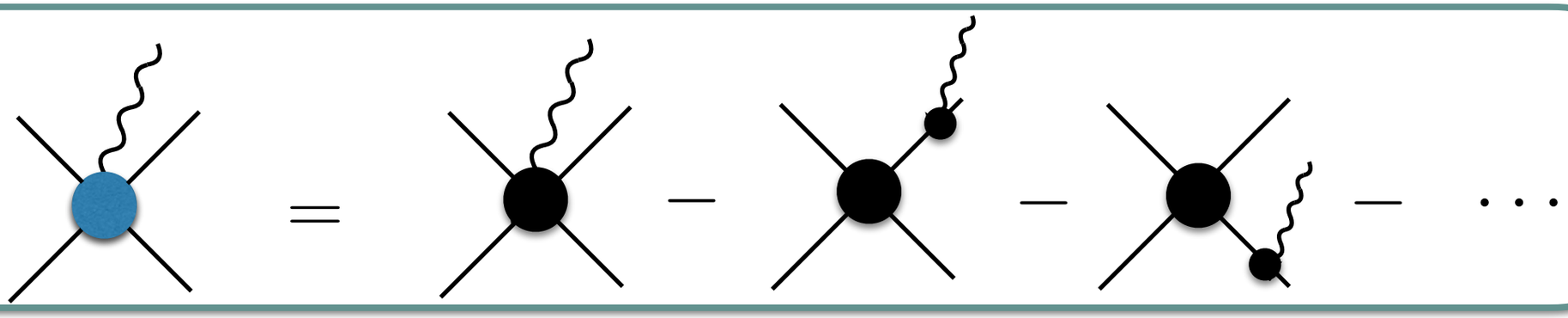
Workflow

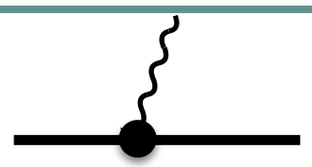


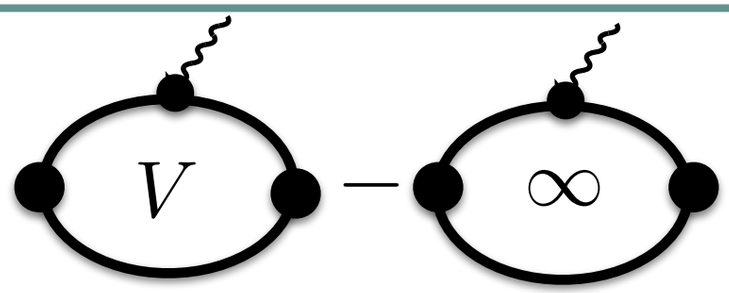
$2+\mathcal{J}\to 2$

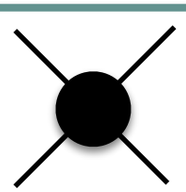
$$\langle 2|\mathcal{J}|2\rangle|^2_L = \frac{1}{L^6}\mathrm{Tr}\left[R(E_L,L)W_{L,\mathrm{df}}R(E_L,L)W_{L,\mathrm{df}}\right]$$

$$W_{L,\mathrm{df}} = W_{\mathrm{df}} + MG(L,w)M$$

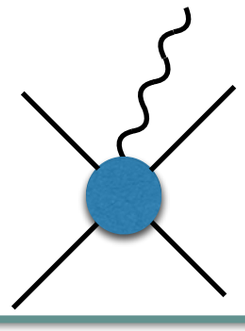
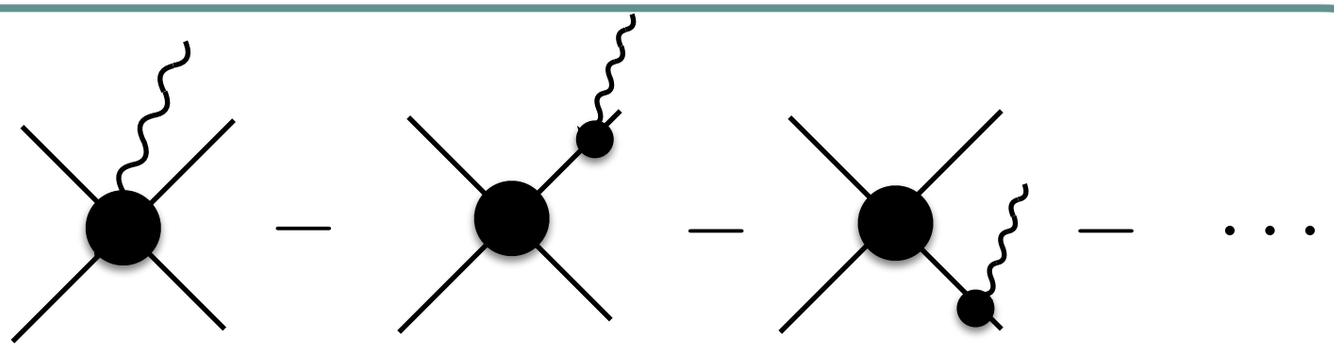
$W_{\mathrm{df}} =$  $= \cdots$

$w =$ 

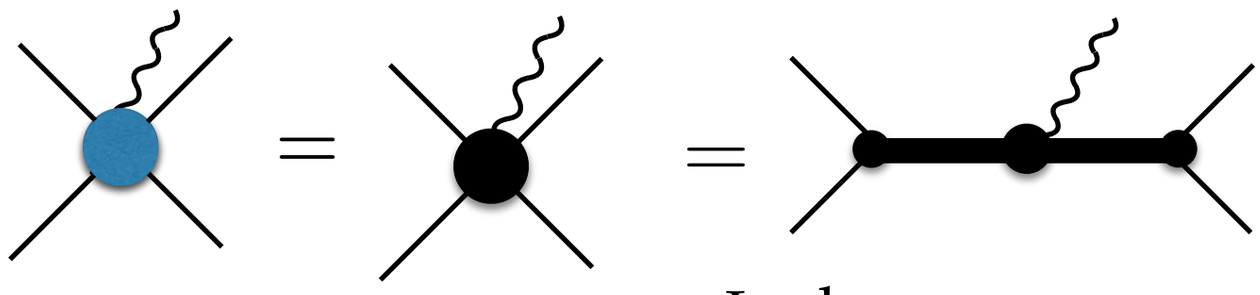
$G(L,w) =$ 

$M =$ 

A detail

$W_{\rm df} =$  $=$ 

Kinematic singularities not showing up in this limit

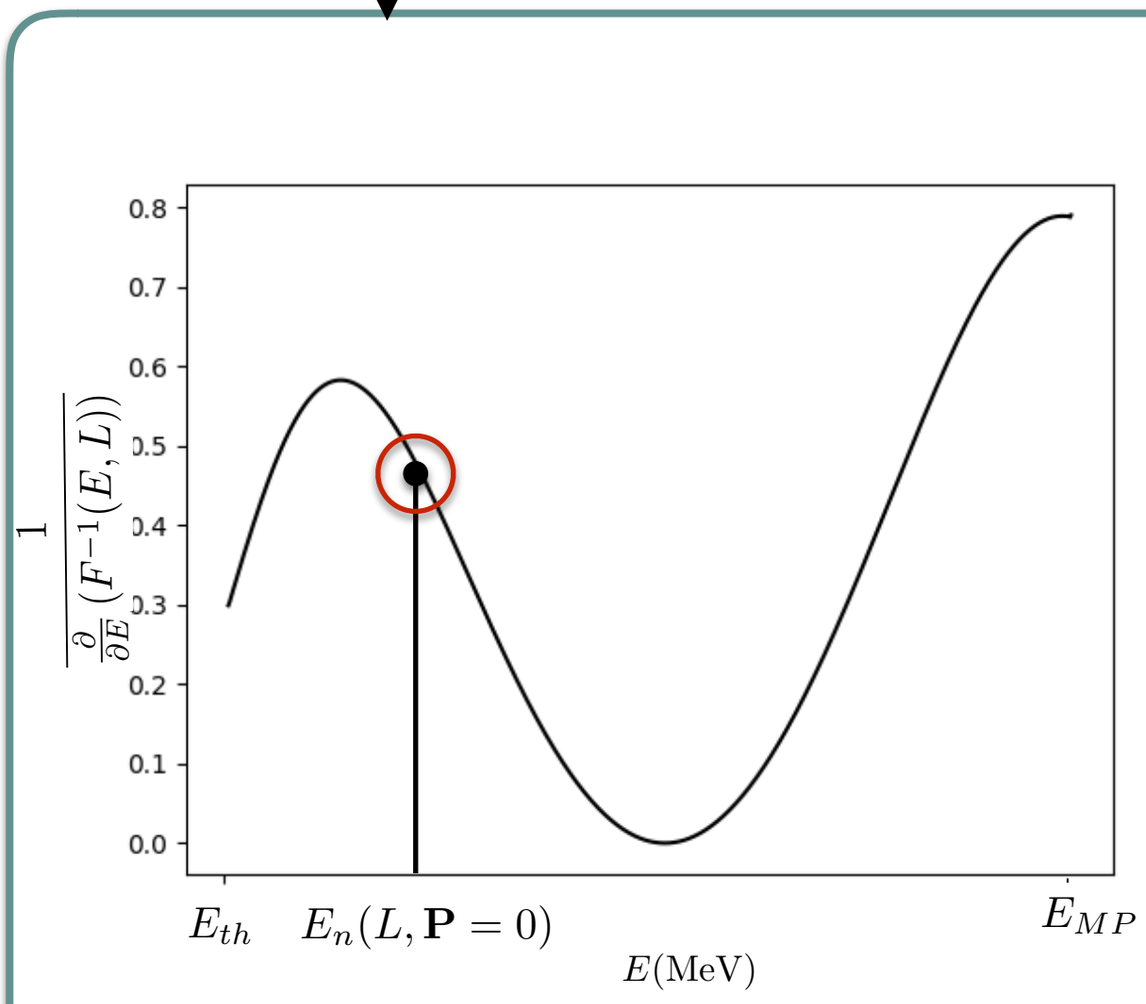
$\lim_{E_i^{\rm cm}, E_f^{\rm cm} \to E_R}$ 

In the resonance region

Kinematic functions

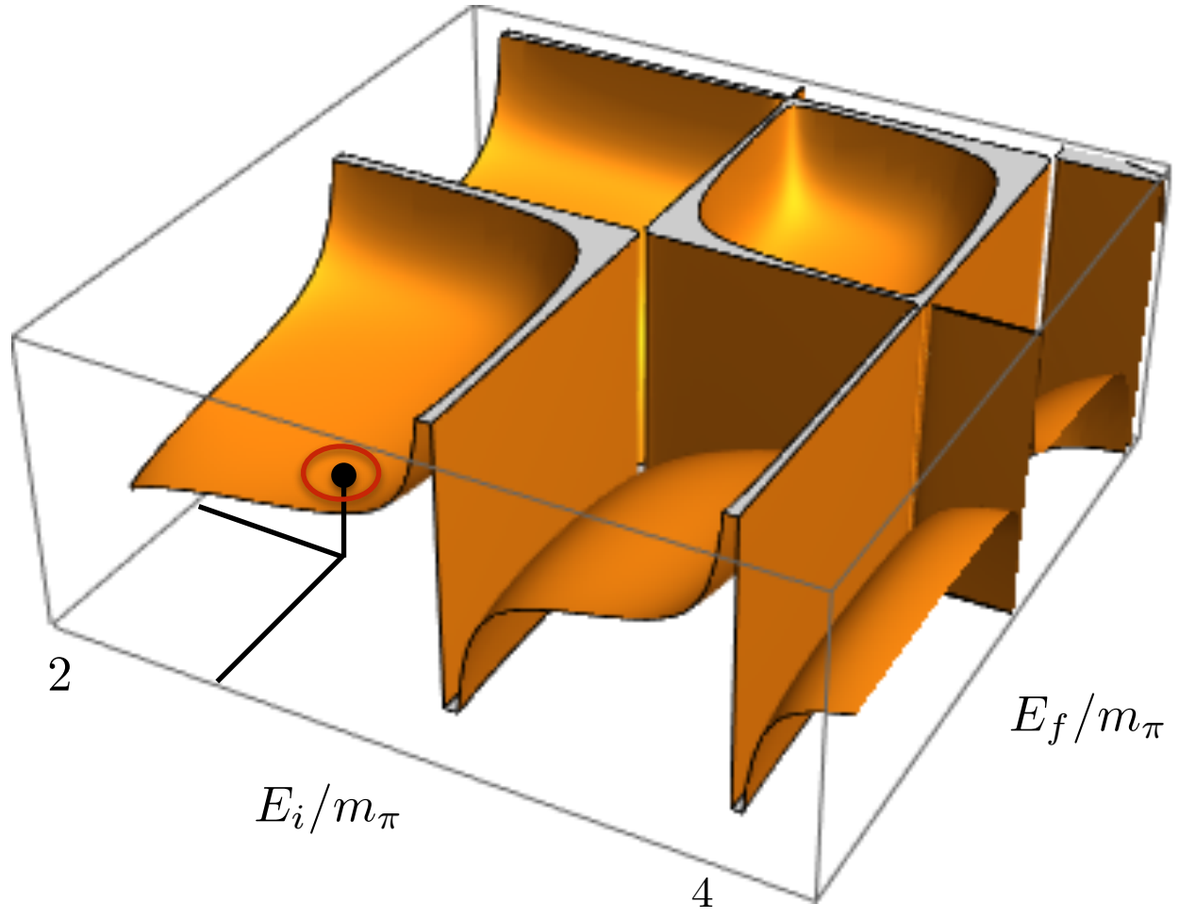
$$R(E_L, L) = \left. \frac{1}{\frac{\partial}{\partial E}\left(F^{-1}(E, L) + M(E)\right)} \right|_{E=E_L}$$

$$G(E_i, E_f, L) = \left[\frac{1}{L^3} \sum_{\mathbf{k}} - \int d\mathbf{k} \right] (\cdots\cdots)$$



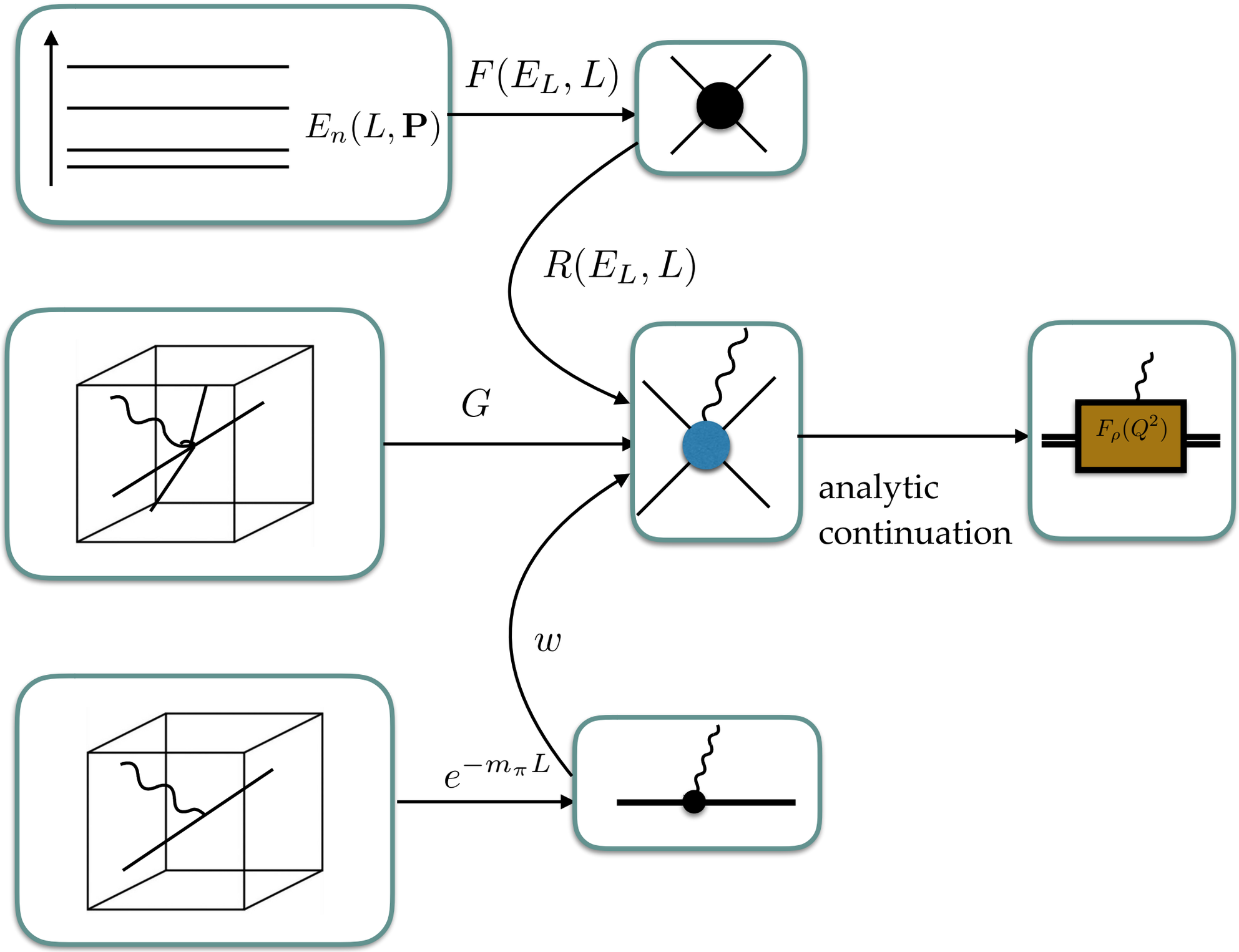
E_{th} = threshold energy

E_{MP} = multiparticle states energy



Singularities at free particle energies

Workflow


$E_n(L, \mathbf{P})$
$F(E_L, L)$
$R(E_L, L)$
G
w
$e^{-m_\pi L}$
analytic
continuation
$F_\rho(Q^2)$

Thank you!

Some challenges

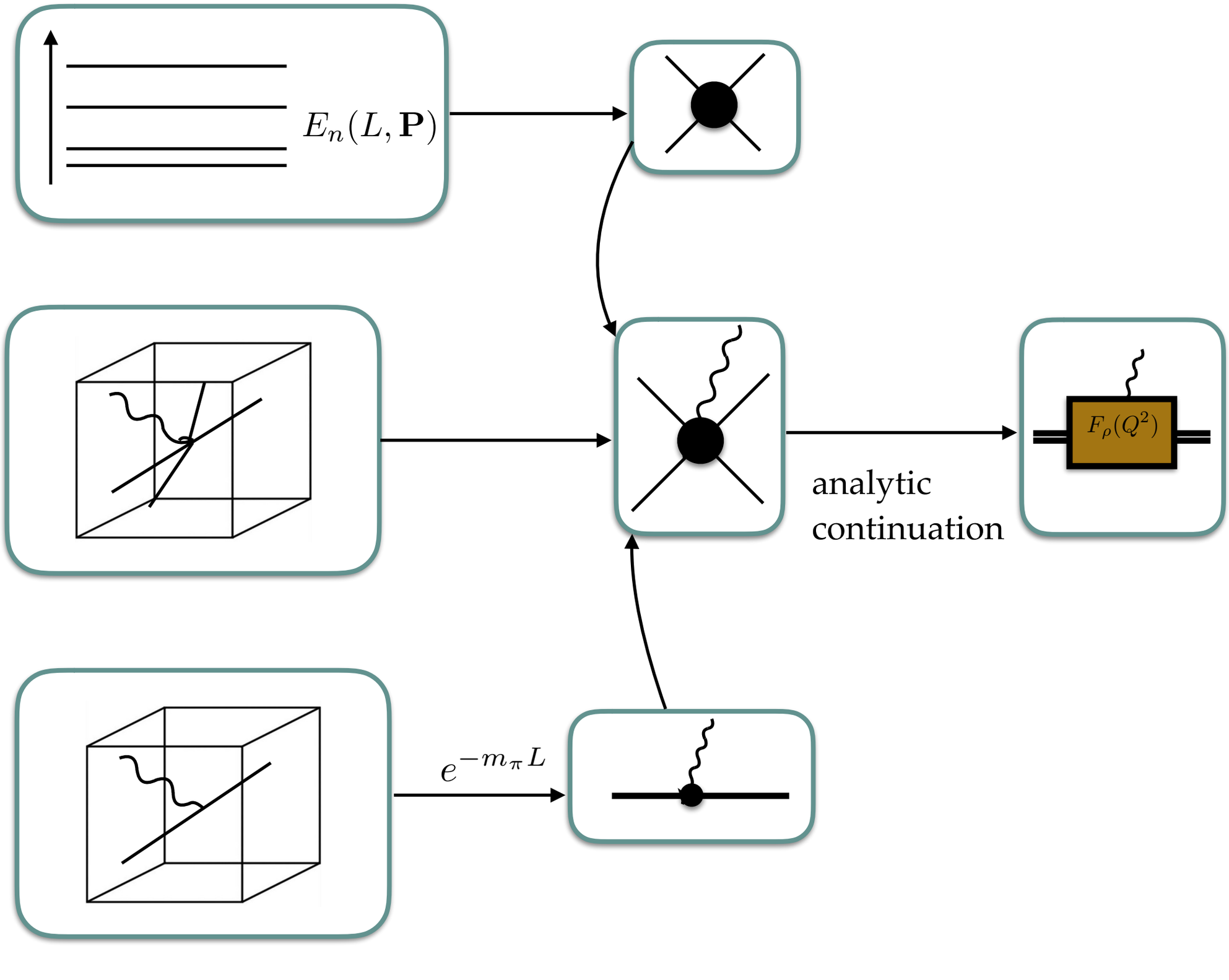
- A spin 1 particle between non-degenerate states has four form factors

- There is not a one-to-one mapping between matrix elements and amplitudes
  - Solved problem for spectrum analysis

- Analytical continuation of the amplitudes

Backup slides

Workflow 101



Steps left

$$\langle 2|\mathcal{J}|2\rangle_{\mathrm{FV}} \to \langle 2|\mathcal{J}|2\rangle_{\infty}$$

- Evaluate kinematic functions for every value of energy and momenta
- Understand how to extract the form factors, mixing of waves…..

- From $\langle 2|\mathcal{J}|2\rangle_{\infty}$ how do we get the four form factors?
  - Analytic continuation