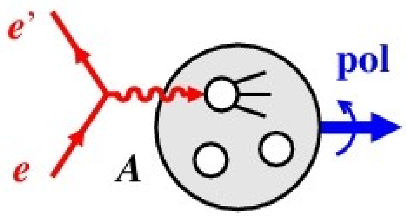

Polarized light ion physics with EIC

OAM in QCD

Based on [C.L., Mantovani, Pasquini (2018)]

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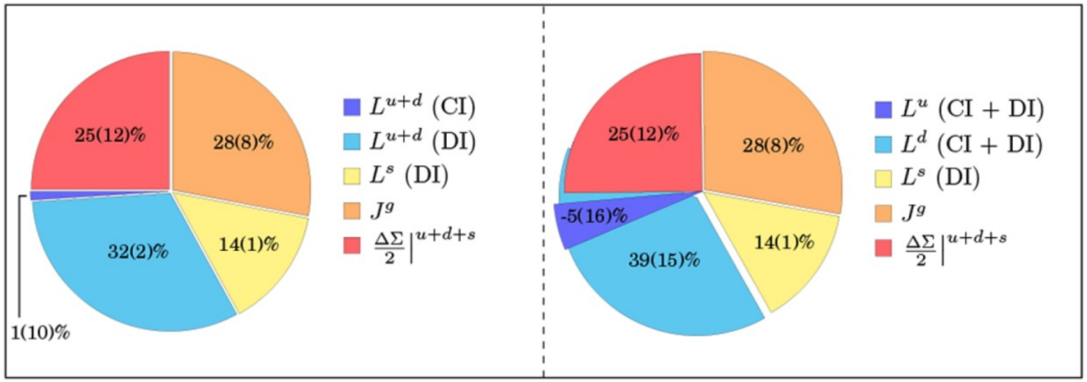
Outline

1. **Angular momentum in QFT**
2. **Energy-momentum tensor**
3. **3D distributions in Breit frame**
4. **2D distributions in elastic frame**
5. **Results in scalar diquark model**

Orbital angular momentum

Large contribution to nucleon spin ⟶ **relativistic nature of the system**



χQCD collaboration [Deka et al. (2015)]

Responsible for most of spin asymmetries ⟶ **fine structure and spin-orbit correlation**

Quark polarization

Nucleon polarization

ρ_X	U	L	T_x	T_y
U	● $\langle 1 \rangle$	$\langle S^q_L \ell^q_L \rangle$	$\langle S^q_x \ell^q_x \rangle$	$\langle S^q_y \ell^q_y \rangle$
L	$\langle S_L \ell^q_L \rangle$	● $\langle S_L S^q_L \rangle$	$\langle S_L \ell^q_L S^q_x \ell^q_x \rangle$	$\langle S_L \ell^q_L S^q_y \ell^q_y \rangle$
T_x	$\langle S_x \ell^q_x \rangle$	$\langle S_x \ell^q_x S^q_L \ell^q_L \rangle$	● $\langle S_x S^q_x \rangle$	$\langle S_x \ell^q_x S^q_y \ell^q_y \rangle$
T_y	$\langle S_y \ell^q_y \rangle$	$\langle S_y \ell^q_y S^q_L \ell^q_L \rangle$	$\langle S_y \ell^q_y S^q_x \ell^q_x \rangle$	● $\langle S_y S^q_y \rangle$

FFs, GPDs, TMDs, GTMDs, ...

PDFs

[C.L., Pasquini (2016)]

Angular momentum

Quantum mechanics

$$\vec{J} = \vec{L} + \vec{S} \qquad \vec{L} = \vec{r} \times \vec{p}$$

Quantum field theory

$$J^{\mu\alpha\beta}(x) = L^{\mu\alpha\beta}(x) + S^{\mu\alpha\beta}(x) \qquad L^{\mu\alpha\beta}(x) = x^\alpha T^{\mu\beta}(x) - x^\beta T^{\mu\alpha}(x)$$

Poincaré covariance

$$\partial_\mu J^{\mu\alpha\beta}(x) = 0, \quad \partial_\mu T^{\mu\nu}(x) = 0 \quad \Longrightarrow \quad \boxed{T^{[\alpha\beta]}(x) = -\partial_\mu S^{\mu\alpha\beta}(x)}$$

Absent in GR

Canonical, kinetic or Belinfante?

Canonical (Jaffe-Manohar)

$$T^{\mu\nu}_{\text{can},q}(x) = \overline{\psi}(x)\gamma^\mu \tfrac{i}{2}\overleftrightarrow{\partial}^\nu \psi(x)$$

Kinetic (Ji)

$$T^{\mu\nu}_{\text{kin},q}(x) = \overline{\psi}(x)\gamma^\mu \tfrac{i}{2}\overleftrightarrow{D}^\nu \psi(x)$$

Belinfante

$$T^{\mu\nu}_{\text{Bel}}(x) = T^{\mu\nu}(x) + \partial_\lambda G^{\lambda\mu\nu}(x), \qquad G^{[\lambda\mu]\nu}(x) = 0$$

superpotential

$$\begin{aligned} J^{\mu\alpha\beta}_{\text{Bel}}(x) &= J^{\mu\alpha\beta}(x) + \partial_\lambda[x^\alpha G^{\lambda\mu\beta} - x^\beta G^{\lambda\mu\alpha}] \\ &= x^\alpha T^{\mu\beta}_{\text{Bel}} - x^\beta T^{\mu\alpha}_{\text{Bel}} \end{aligned} \quad \Longrightarrow \quad T^{[\alpha\beta]}_{\text{Bel}}(x) = 0$$

Energy-momentum tensor

Matrix elements

$$P = \frac{p' + p}{2}, \qquad \Delta = p' - p, \qquad t = \Delta^2$$

$$\langle p', s'|T^{\mu\nu}(0)|p, s\rangle =$$

$$\overline{u}(p', s')\left[\frac{P^\mu P^\nu}{M}A(t) + \frac{P^{\{\mu}i\sigma^{\nu\}\lambda}\Delta_\lambda}{4M}(A+B)(t) + \frac{\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2}{M}C(t)\right.$$

$$\left. + M g^{\mu\nu}\bar{C}(t) + \frac{P^{[\mu}i\sigma^{\nu]\lambda}\Delta_\lambda}{4M}D(t)\right]u(p, s)$$

Non-conservation **Intrinsic spin**

Mass-shell conditions

$$p'^2 = p^2 = M^2 \quad \Longrightarrow \quad P^2 = M^2 - \frac{t}{4}, \qquad P \cdot \Delta = 0$$

Timelike (↓ P) ⟹ **« frame »**

Spacelike (↑ Δ) ⟹ **« position »**

Energy-momentum form factors

Mellin moment of twist-2 vector GPDs $\langle p', s'|T^{++}(0)|p, s\rangle$

$$\int \mathrm{d}x\, x\, H(x,\xi,t) = A(t) + 4\xi^2 C(t)$$

$$\int \mathrm{d}x\, x\, E(x,\xi,t) = B(t) - 4\xi^2 C(t)$$

[Ji (1996)]

Poincaré covariance $\langle p', s'|T_q^{[\alpha\beta]}(0)|p, s\rangle = -i\Delta_\mu \langle p', s'|S_q^{\mu\alpha\beta}(0)|p, s\rangle$

$$D_q(t) = -G_A^q(t)$$

[C.L., Mantovani, Pasquini (2018)]

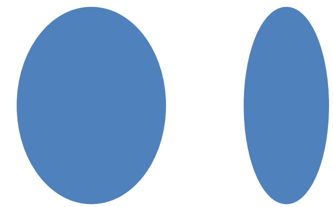
No contribution to AM from $\bar{C}(t)$

3D distribution in Breit frame

Lorentz factors

$$p^0 = \gamma M, \qquad p'^0 = \gamma' M$$

Initial ψ **Final** ψ'^*



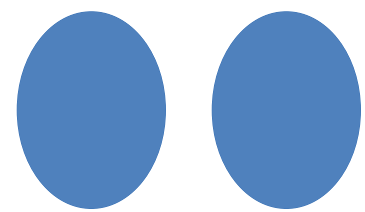
$$\Delta^0 \neq 0$$

Breit frame

$\vec{p}\,'$ $\rightarrow$

$\leftarrow$ $\vec{p}$

$$\vec{P} = \vec{0} \quad \Longrightarrow \quad \Delta^0 = \frac{\vec{P} \cdot \vec{\Delta}}{P^0} = 0$$



$$\Delta^0 = 0$$

$$\rho \sim \psi^* \psi$$

3D distribution of OAM

$$\langle L^i \rangle(\vec{x}) = -i\epsilon^{ijk} \int \frac{\mathrm{d}^3\Delta}{(2\pi)^3}\, e^{-i\vec{\Delta}\cdot\vec{x}}\, \frac{\partial \langle T^{0k} \rangle_{\mathrm{BF}}}{\partial \Delta^j}$$

$$\langle T^{\mu\nu} \rangle_{\mathrm{BF}} \equiv \frac{\langle \frac{\vec{\Delta}}{2}, \vec{s} | T^{\mu\nu}(0) | -\frac{\vec{\Delta}}{2}, \vec{s} \rangle}{2P^0}$$

3D distribution in Breit frame

Kinetic OAM

$$\langle L^i\rangle(\vec{x}) = \int \frac{\mathrm{d}^3\Delta}{(2\pi)^3}\, e^{-i\vec{\Delta}\cdot\vec{x}} \left[s^i L(t) + [(\vec{\Delta}\cdot\vec{s})\Delta^i - \vec{\Delta}^2 s^i]\,\frac{\mathrm{d}L(t)}{\mathrm{d}t}\right]_{t=-\vec{\Delta}^2}, \qquad L(t) \equiv \frac{A(t)+B(t)+D(t)}{2}$$

Spin

$$\langle S^i\rangle(\vec{x}) = \int \frac{\mathrm{d}^3\Delta}{(2\pi)^3}\, e^{-i\vec{\Delta}\cdot\vec{x}} \left[\frac{s^i}{2} G_A(t) - \frac{(\vec{\Delta}\cdot\vec{s})\Delta^i}{4}\,\frac{\mathrm{d}G(t)}{\mathrm{d}t}\right]_{t=-\vec{\Delta}^2}, \qquad \frac{\mathrm{d}G(t)}{\mathrm{d}t} \equiv \frac{1}{2P^0}\left[\frac{G_A(t)}{P^0+M} + \frac{G_P(t)}{M}\right]$$

Belinfante total AM

$$\langle J^i_{\mathrm{Bel}}\rangle(\vec{x}) = \int \frac{\mathrm{d}^3\Delta}{(2\pi)^3}\, e^{-i\vec{\Delta}\cdot\vec{x}} \left[s^i J(t) + [(\vec{\Delta}\cdot\vec{s})\Delta^i - \vec{\Delta}^2 s^i]\,\frac{\mathrm{d}J(t)}{\mathrm{d}t}\right]_{t=-\vec{\Delta}^2}, \qquad J(t) \equiv \frac{A(t)+B(t)}{2}$$

$$\boxed{\int \mathrm{d}^3x\, [\langle L^i\rangle(\vec{x}) + \langle S^i\rangle(\vec{x})] = \int \mathrm{d}^3x\, \langle J^i_{\mathrm{Bel}}\rangle(\vec{x})}$$

$$\int \mathrm{d}^3x \leftrightarrow \vec{\Delta} = \vec{0}$$

But $\langle L^i\rangle(\vec{x}) + \langle S^i\rangle(\vec{x}) \neq \langle J^i_{\mathrm{Bel}}\rangle(\vec{x})$ **!!!!**

3D distribution in Breit frame

Kinetic OAM

$$\langle L^i\rangle(\vec{x}) = \int \frac{\mathrm{d}^3\Delta}{(2\pi)^3}\, e^{-i\vec{\Delta}\cdot\vec{x}} \left[s^i L(t) + [(\vec{\Delta}\cdot\vec{s})\Delta^i - \vec{\Delta}^2 s^i]\,\frac{\mathrm{d}L(t)}{\mathrm{d}t}\right]_{t=-\vec{\Delta}^2}, \qquad L(t) \equiv \frac{A(t)+B(t)+D(t)}{2}$$

Spin

$$\langle S^i\rangle(\vec{x}) = \int \frac{\mathrm{d}^3\Delta}{(2\pi)^3}\, e^{-i\vec{\Delta}\cdot\vec{x}} \left[\frac{s^i}{2}\, G_A(t) - \frac{(\vec{\Delta}\cdot\vec{s})\Delta^i}{4}\,\frac{\mathrm{d}G(t)}{\mathrm{d}t}\right]_{t=-\vec{\Delta}^2}, \qquad \frac{\mathrm{d}G(t)}{\mathrm{d}t} \equiv \frac{1}{2P^0}\left[\frac{G_A(t)}{P^0+M} + \frac{G_P(t)}{M}\right]$$

Belinfante total AM

$$\langle J^i_{\mathrm{Bel}}\rangle(\vec{x}) = \int \frac{\mathrm{d}^3\Delta}{(2\pi)^3}\, e^{-i\vec{\Delta}\cdot\vec{x}} \left[s^i J(t) + [(\vec{\Delta}\cdot\vec{s})\Delta^i - \vec{\Delta}^2 s^i]\,\frac{\mathrm{d}J(t)}{\mathrm{d}t}\right]_{t=-\vec{\Delta}^2}, \qquad J(t) \equiv \frac{A(t)+B(t)}{2}$$

Superpotential

$$\langle M^i\rangle(\vec{x}) = -\int \frac{\mathrm{d}^3\Delta}{(2\pi)^3}\, e^{-i\vec{\Delta}\cdot\vec{x}} \left[\frac{(\vec{\Delta}\cdot\vec{s})\Delta^i - \vec{\Delta}^2 s^i}{2}\,\frac{\mathrm{d}G_A(t)}{\mathrm{d}t} + \frac{(\vec{\Delta}\cdot\vec{s})\Delta^i}{4}\,\frac{\mathrm{d}G(t)}{\mathrm{d}t}\right]_{t=-\vec{\Delta}^2}$$

$$\boxed{\langle L^i\rangle(\vec{x}) + \langle S^i\rangle(\vec{x}) = \langle J^i_{\mathrm{Bel}}\rangle(\vec{x}) + \langle M^i\rangle(\vec{x})} \qquad \int \mathrm{d}^3x\, \langle M^i\rangle(\vec{x}) = 0$$

Link with other 3D distributions

Naive distribution

$$\langle J^i\rangle(\vec{x}) =$$

$$\underbrace{s^i \int \frac{\mathrm{d}^3\Delta}{(2\pi)^3}\, e^{-i\vec{\Delta}\cdot\vec{x}}\, J(-\vec{\Delta}^2)}_{\langle J^i\rangle_{\text{naive}}(\vec{x})} + \underbrace{\int \frac{\mathrm{d}^3\Delta}{(2\pi)^3}\, e^{-i\vec{\Delta}\cdot\vec{x}} \left[[(\vec{\Delta}\cdot\vec{s})\Delta^i - \vec{\Delta}^2 s^i]\, \frac{\mathrm{d}L(t)}{\mathrm{d}t} - \frac{(\vec{\Delta}\cdot\vec{s})\Delta^i}{4}\, \frac{\mathrm{d}G(t)}{\mathrm{d}t} \right]_{t=-\vec{\Delta}^2}}_{\langle J^i\rangle_{\text{corr}}(\vec{x})}$$

Polyakov's distribution

[Polyakov (2003)]
[Goeke *et al.* (2007)]

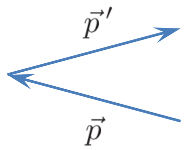
$$\langle J^i_{\text{Bel}}\rangle(\vec{x}) =$$

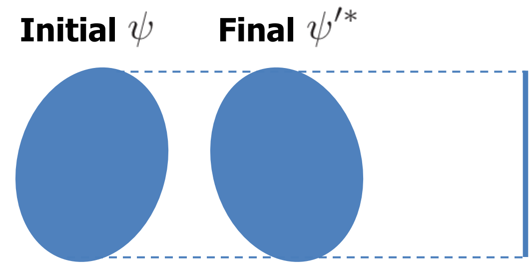
$$\underbrace{s^i \int \frac{\mathrm{d}^3\Delta}{(2\pi)^3}\, e^{-i\vec{\Delta}\cdot\vec{x}} \left[J(t) + \frac{2t}{3}\, \frac{\mathrm{d}J(t)}{\mathrm{d}t} \right]_{t=-\vec{\Delta}^2}}_{\langle J^i_{\text{Bel}}\rangle_{\text{mono}}(\vec{x})} + \underbrace{s^j \int \frac{\mathrm{d}^3\Delta}{(2\pi)^3}\, e^{-i\vec{\Delta}\cdot\vec{x}} \left[\left(\Delta^j\Delta^i - \tfrac{1}{3}\,\delta^{ji}\vec{\Delta}^2\right) \frac{\mathrm{d}J(t)}{\mathrm{d}t} \right]_{t=-\vec{\Delta}^2}}_{\langle J^i_{\text{Bel}}\rangle_{\text{quad}}(\vec{x})}$$

Discarded without justification

2D distribution in elastic frame

Elastic frame





$$\rho_\perp \sim \int \mathrm{d}z\, \psi^* \psi$$

$$\vec{P} \neq \vec{0}, \quad \Delta^0 = 0 \quad \Longrightarrow \quad \vec{P} \cdot \vec{\Delta} = 0$$

$$\vec{P} = (\vec{0}_\perp, P), \qquad \vec{\Delta} = (\vec{\Delta}_\perp, 0) \qquad \int \mathrm{d}z \leftrightarrow \Delta^z = 0$$

2D distribution of longitudinal OAM

$$\langle L^z \rangle(\vec{b}_\perp) = -i\epsilon^{3jk} \int \frac{\mathrm{d}^2\Delta_\perp}{(2\pi)^2}\, e^{-i\vec{\Delta}_\perp \cdot \vec{b}_\perp}\, \frac{\partial \langle T^{0k} \rangle_{\mathrm{EF}}}{\partial \Delta^j_\perp}$$

2D distribution in elastic frame

Kinetic OAM

$$\langle L^z\rangle(\vec{b}_\perp) = s^z \int \frac{\mathrm{d}^2\Delta_\perp}{(2\pi)^2}\, e^{-i\vec{\Delta}_\perp\cdot\vec{b}_\perp} \left[L(t) + t\,\frac{\mathrm{d}L(t)}{\mathrm{d}t}\right]_{t=-\vec{\Delta}^2_\perp}$$

Spin

$$\langle S^z\rangle(\vec{b}_\perp) = \frac{s^z}{2} \int \frac{\mathrm{d}^2\Delta_\perp}{(2\pi)^2}\, e^{-i\vec{\Delta}_\perp\cdot\vec{b}_\perp}\, G_A(-\vec{\Delta}^2_\perp)$$

Belinfante total AM

$$\langle J^z_{\mathrm{Bel}}\rangle(\vec{b}_\perp) = s^z \int \frac{\mathrm{d}^2\Delta_\perp}{(2\pi)^2}\, e^{-i\vec{\Delta}_\perp\cdot\vec{b}_\perp} \left[J(t) + t\,\frac{\mathrm{d}J(t)}{\mathrm{d}t}\right]_{t=-\vec{\Delta}^2_\perp}$$

Superpotential

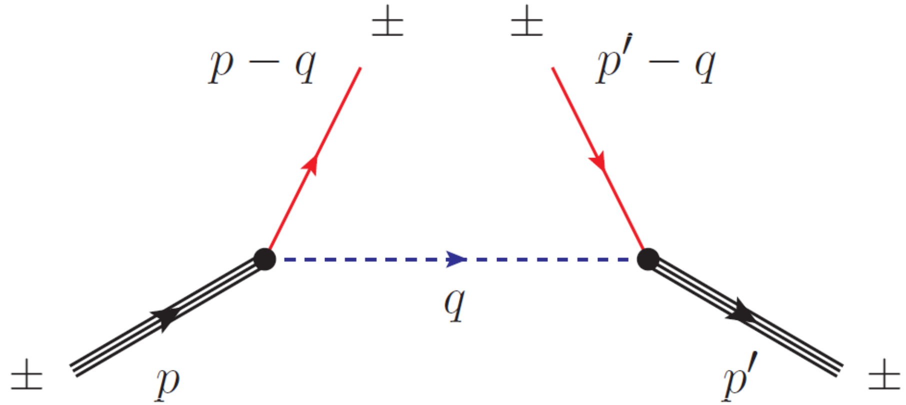
$$\langle M^z\rangle(\vec{b}_\perp) = -\frac{s^z}{2} \int \frac{\mathrm{d}^2\Delta_\perp}{(2\pi)^2}\, e^{-i\vec{\Delta}_\perp\cdot\vec{b}_\perp} \left[t\,\frac{\mathrm{d}G_A(t)}{\mathrm{d}t}\right]_{t=-\vec{\Delta}^2_\perp}$$

P **-independent !**

$P = 0$ → $P \to \infty$

Breit frame → **Infinite momentum frame** ⇒ **Light-front distribution**

Scalar diquark model



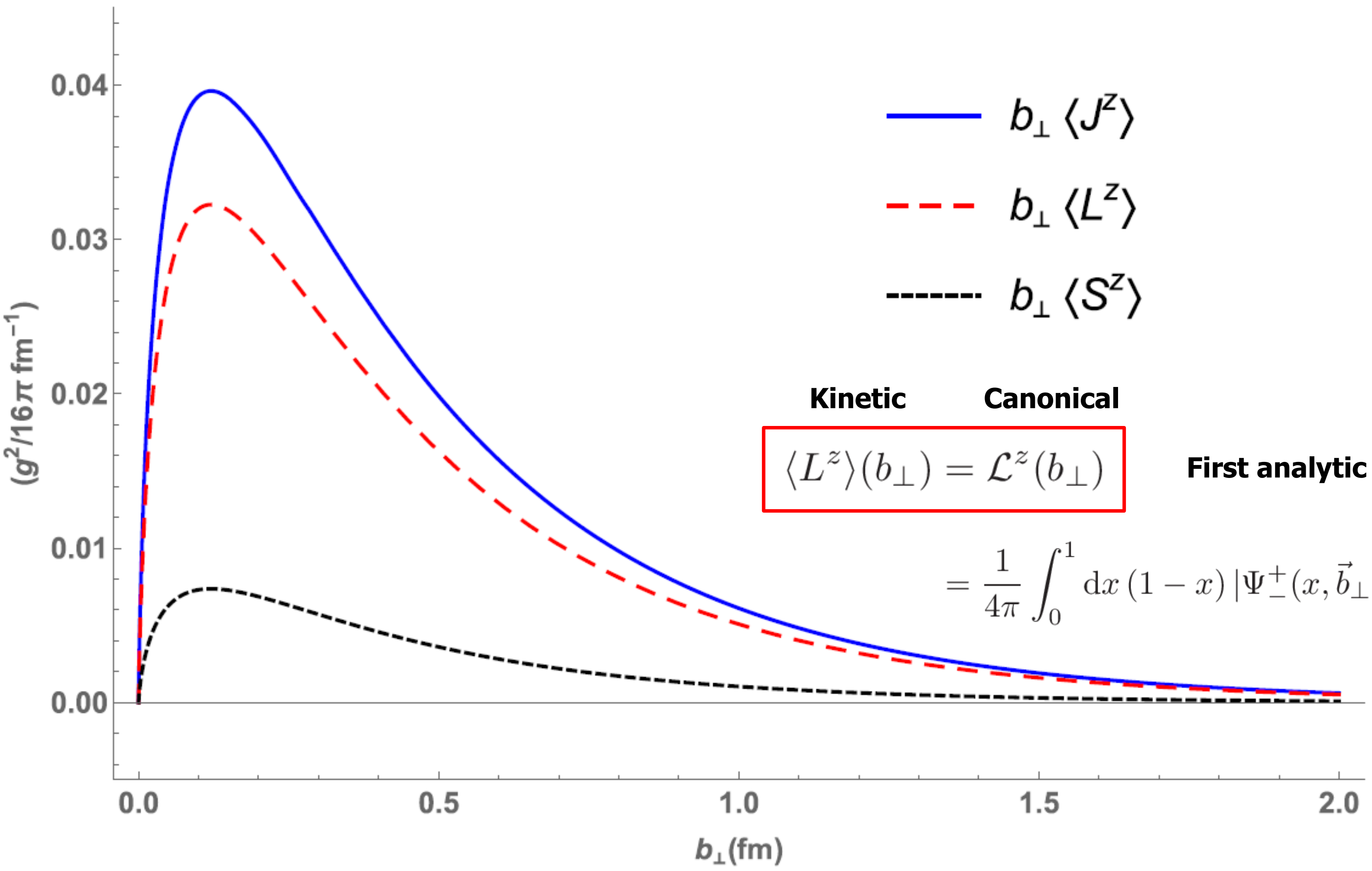
Light-front wave functions

$$\psi_+^+(x, \vec{k}_\perp) = \psi_-^-(x, \vec{k}_\perp) = (xM + m)\,\phi(x, \vec{k}_\perp^2)$$
$$\psi_-^+(x, \vec{k}_\perp) = -[\psi_+^-(x, \vec{k}_\perp)]^* = -(k^x + ik^y)\,\phi(x, \vec{k}_\perp^2)$$

$$\phi(x, \vec{k}_\perp^2) = -\frac{g\sqrt{1-x}}{\vec{k}_\perp^2 - \Lambda^2}, \qquad \Lambda^2 = x m_D^2 + (1-x)m^2 - x(1-x)M^2$$

Diquark **Quark** **Nucleon**

Scalar diquark model



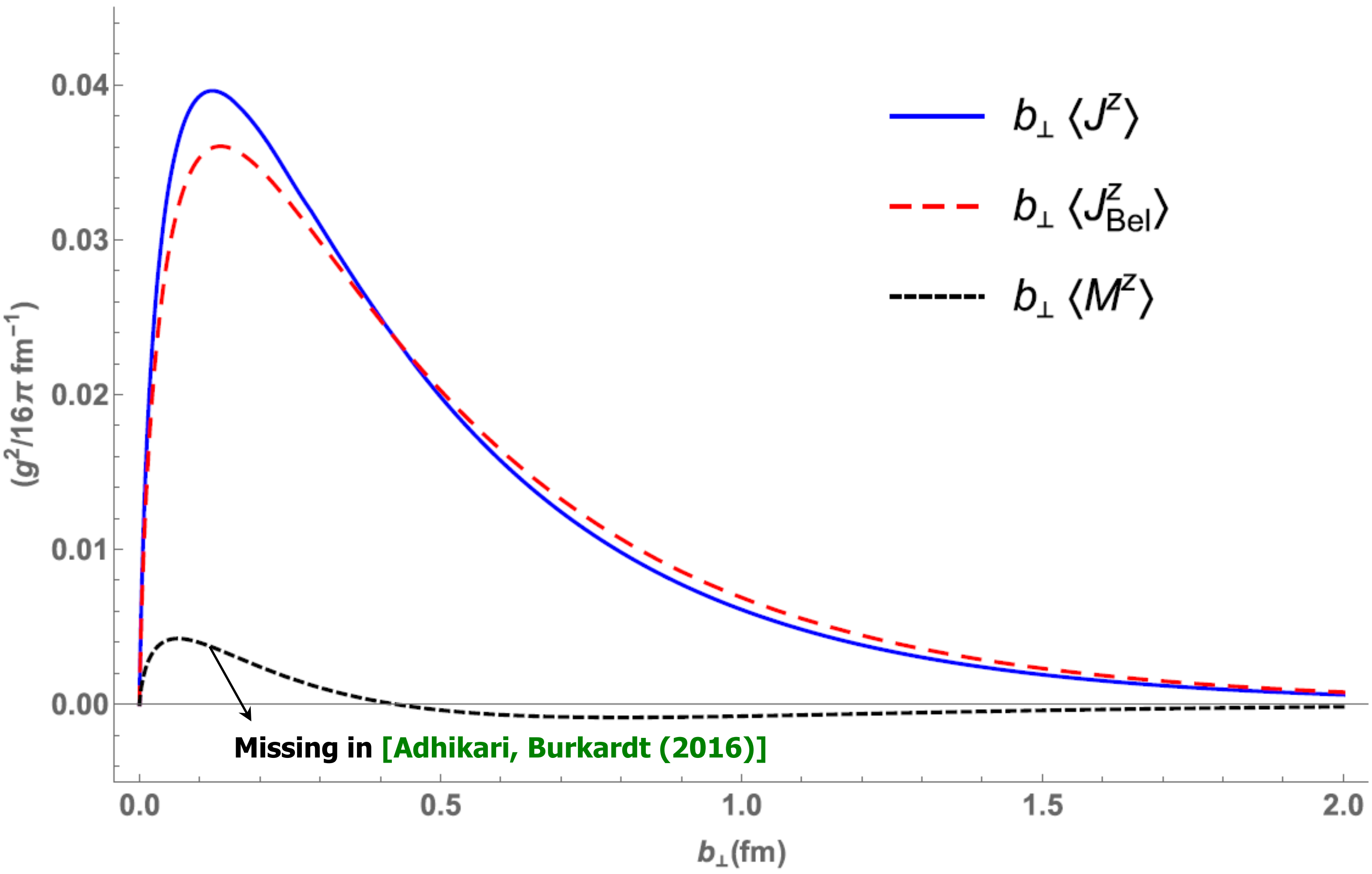
Kinetic **Canonical**

$$\langle L^z\rangle(b_\perp) = \mathcal{L}^z(b_\perp)$$

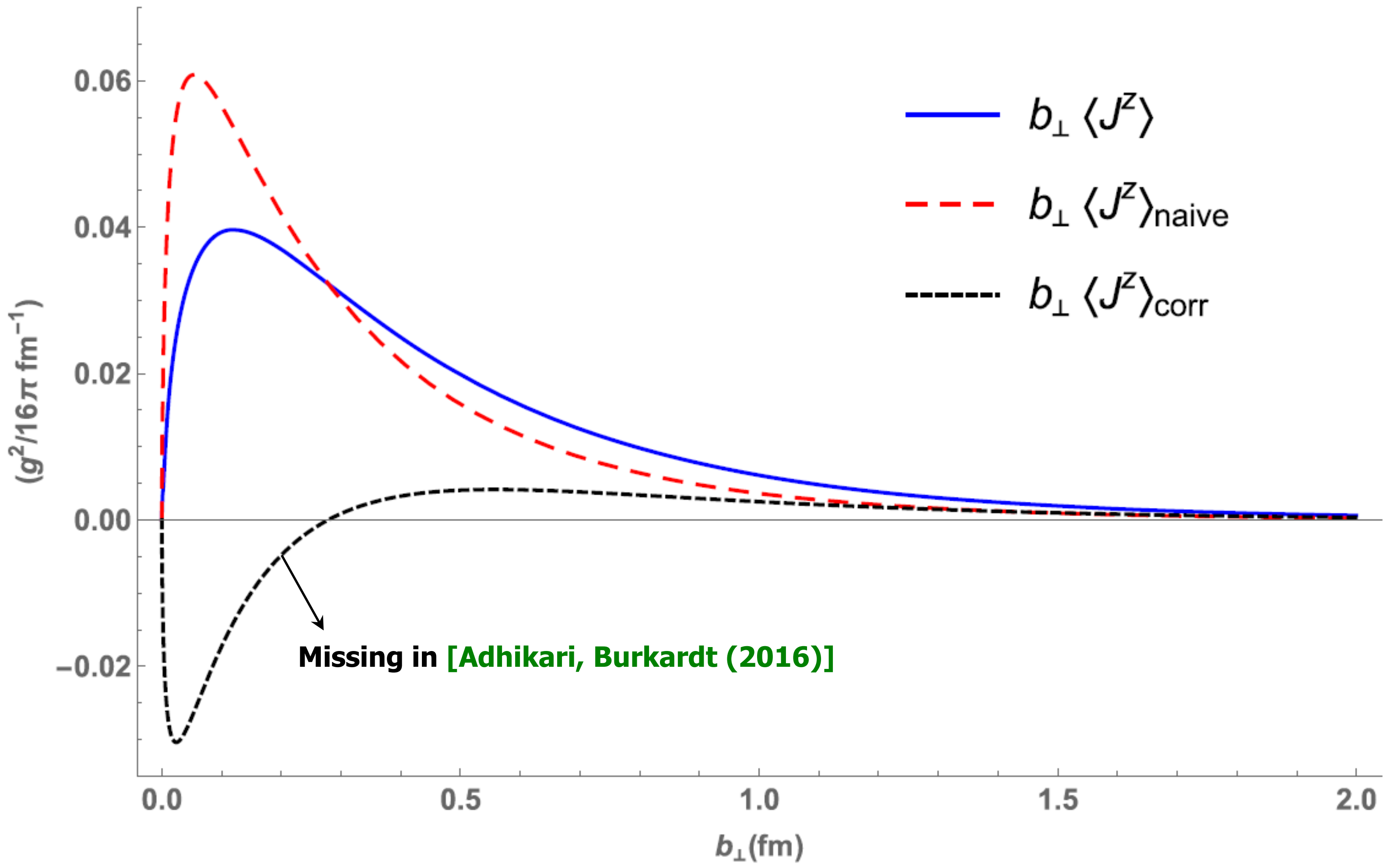
First analytic proof!

$$= \frac{1}{4\pi}\int_0^1 \mathrm{d}x\,(1-x)\,|\Psi^+_-(x,\vec{b}_\perp)|^2$$

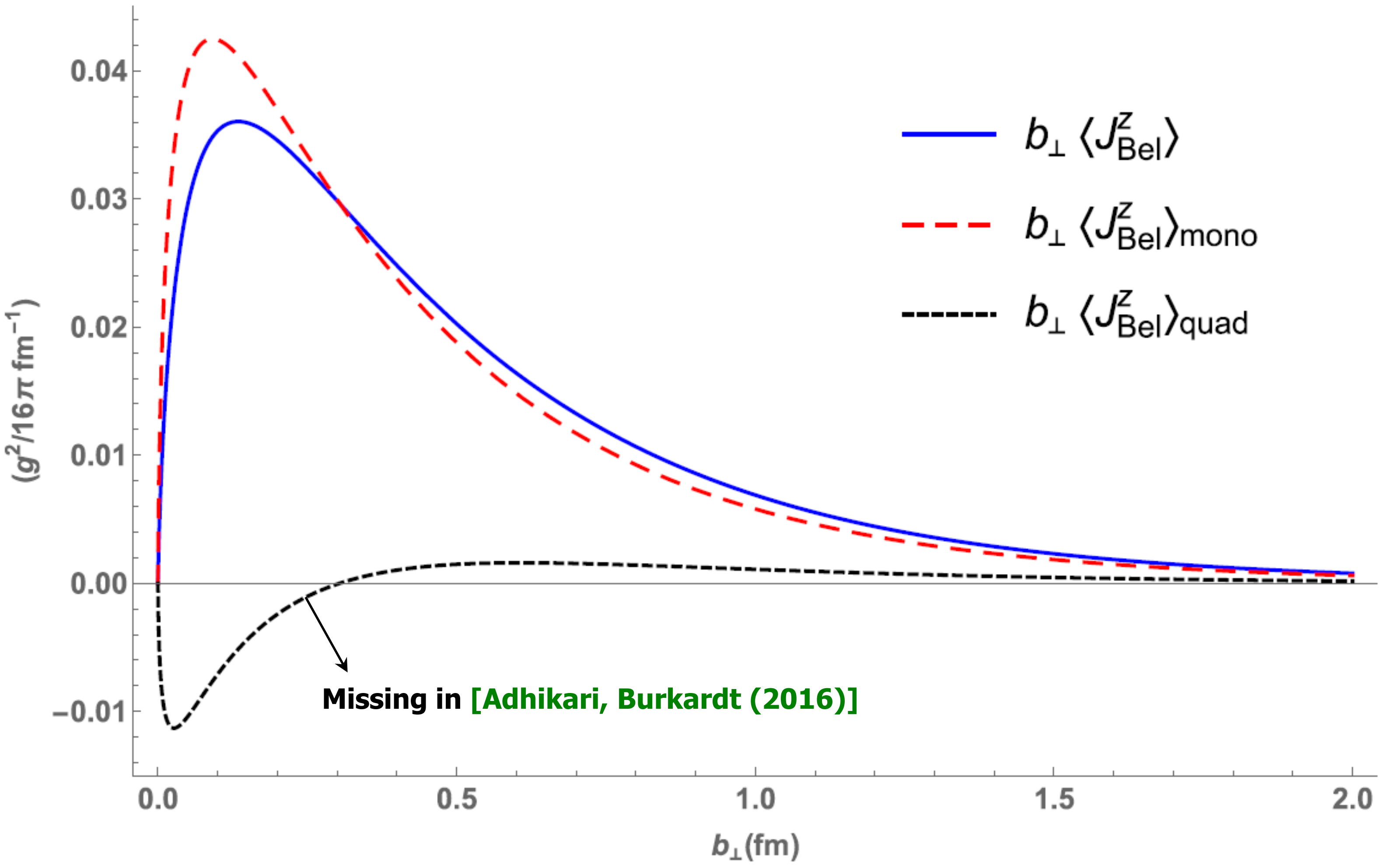
Scalar diquark model



Scalar diquark model



Scalar diquark model



Polarized light ions

Our results are valid for any spin-1/2 target !

Opportunities with light ions:

- **Flavor separation**
- **Nuclear OAM vs hadronic OAM**
- **Controlled OAM state ?**
- **Nuclear medium modification**
- **Higher spin**

Summary

- **Because of spin, the EMT is asymmetric**
- **AM can be defined at the distribution level**
- **Pay attention to contributions with vanishing integral**
- **Longitudinal AM does not depend on target momentum**
- **Scalar diquark model shows large OAM contribution**
- **Polarized light ions offer many interesting opportunities!**