The Similarity Renormalization Group and Deuteron Disintegration

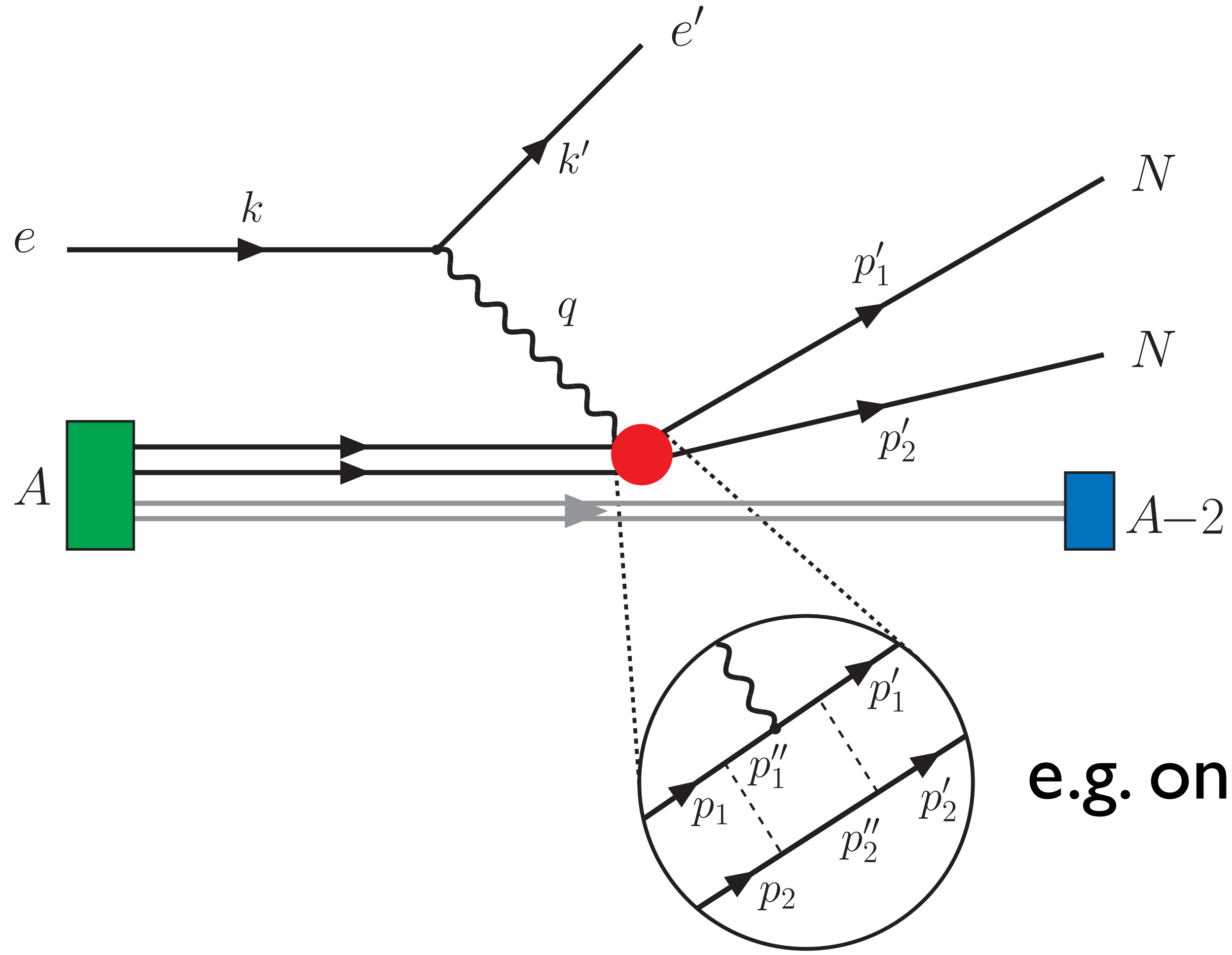
Polarized light ion physics with EIC

Kai Hebeler

Ghent, Feb. 6, 2018

with Sushant More, Dick Furnstahl and Sebastian König

Reaction processes within ab initio frameworks



e.g. one-body interactions

Calculations generally involve:

- **nuclear structure** part for description of initial/final state wave functions
- **reaction part** describes interaction with external probes

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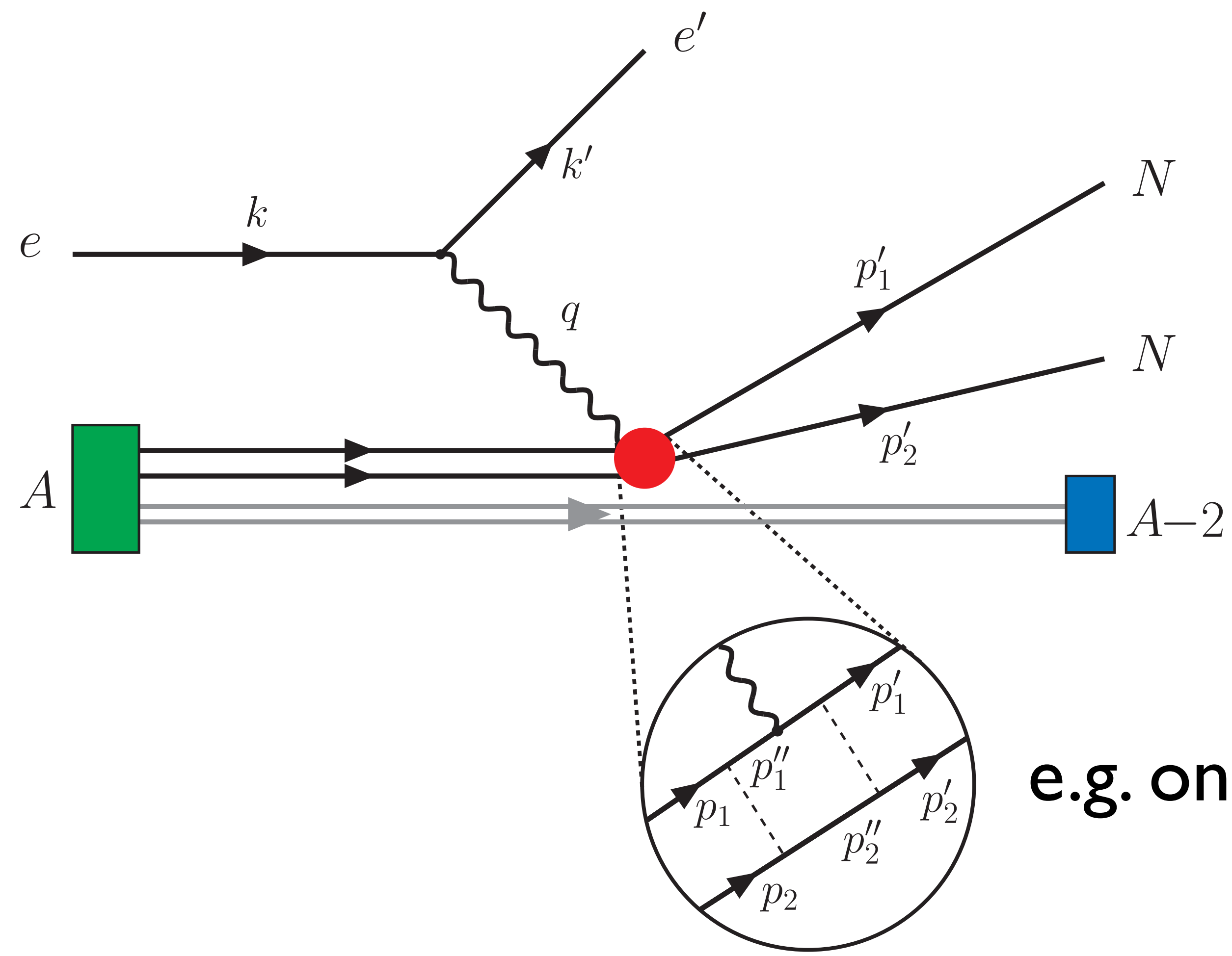

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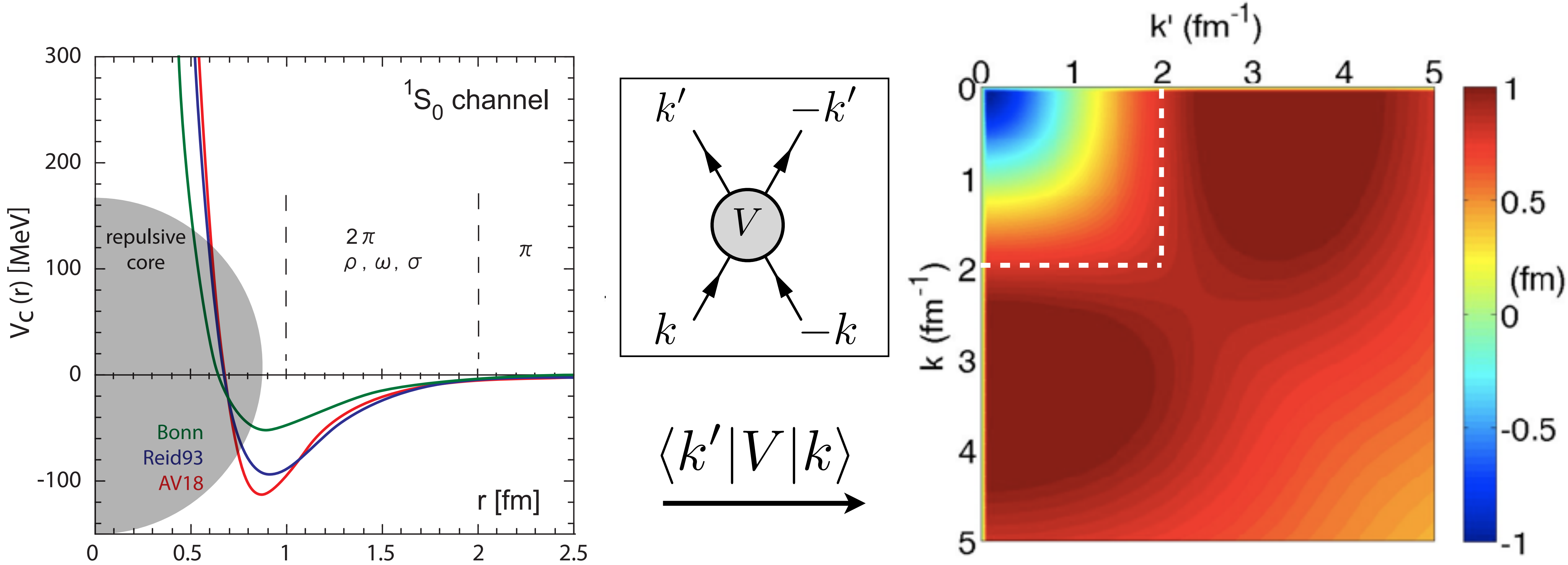
For calculations a factorization of structure and reaction parts is required, consistency between Hamiltonian and currents (see talk by Hermann Krebs):

$$\sigma \sim \langle \psi_F(\lambda) | O(\lambda) | \psi_A(\lambda) \rangle$$

whereas λ is a chosen resolution scale (and scheme)

“Traditional” NN interactions



- constructed to fit scattering data (long-wavelength information)
- long-range part dominated by one pion exchange interaction
- short range part strongly model dependent!
- traditional NN interactions contain strongly repulsive core at small distance
  - strong coupling between low and high-momenta
  - many-body problem hard to solve using basis expansion!

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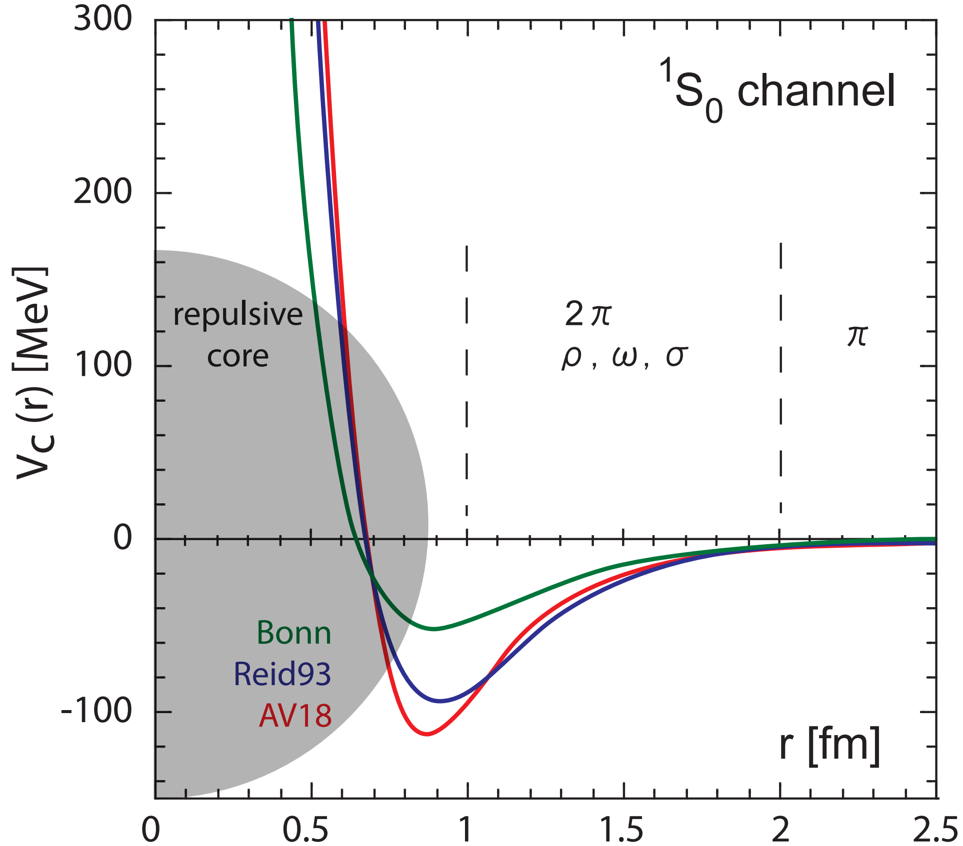
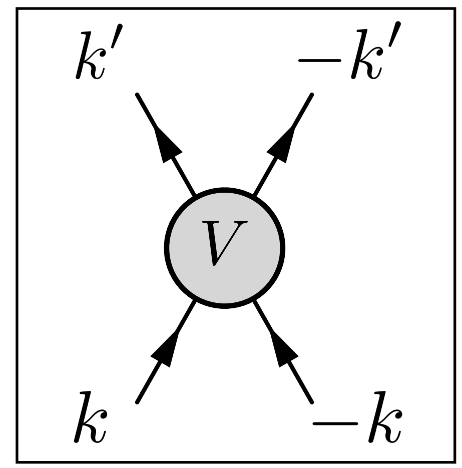
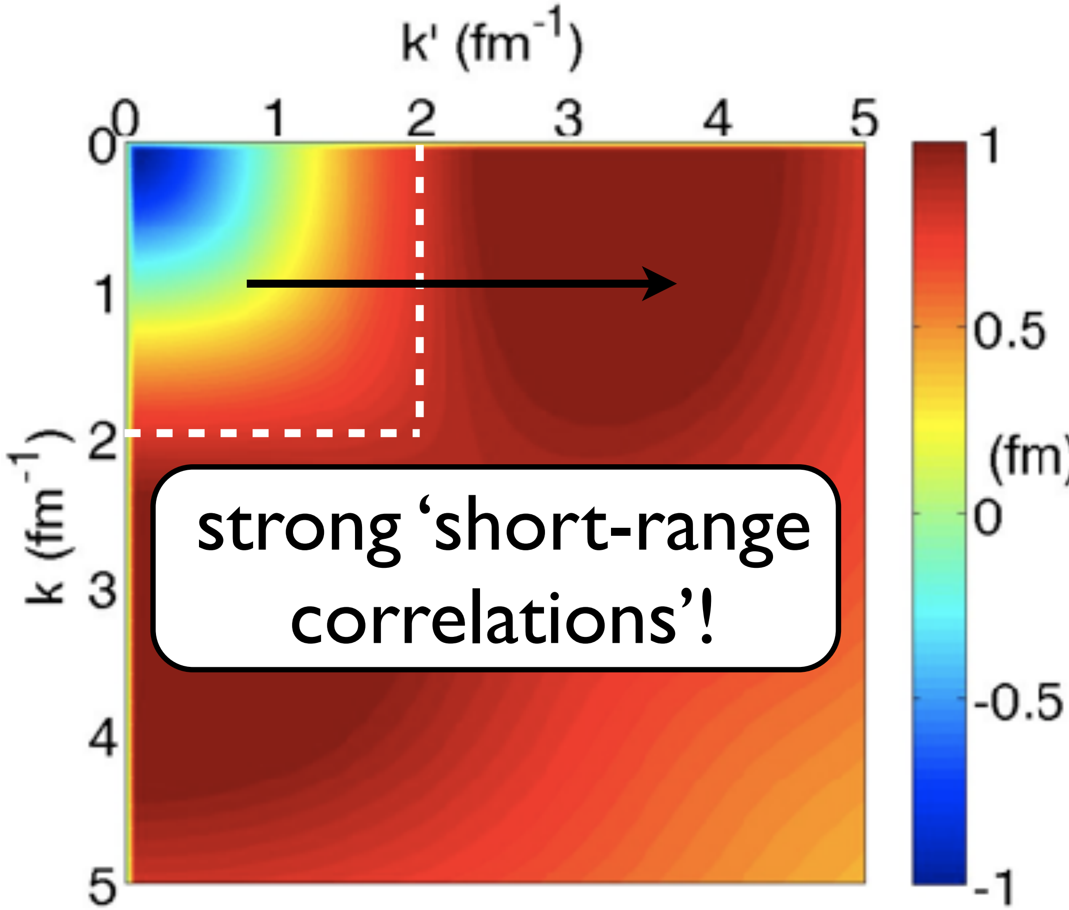




$$\langle k' | V | k \rangle$$



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Changing the resolution scale

$$\langle\psi_F(\lambda_0)|O(\lambda_0)|\psi_A(\lambda_0)\rangle = \langle\psi_F(\lambda_0)|U^\dagger U(\lambda) O(\lambda_0) U^\dagger U(\lambda)|\psi_A(\lambda)\rangle$$
$$= \langle\psi_F(\lambda)|O(\lambda)|\psi_A(\lambda)\rangle$$

with

$$|\psi(\lambda)\rangle = U(\lambda)\,|\psi(\lambda_0)\rangle \qquad O(\lambda) = U^\dagger(\lambda) O(\lambda_0) U(\lambda)$$

$$U(\lambda)U^\dagger(\lambda) = 1$$

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Convenient to choose resolution scale λ such that

- wave functions include only momentum scales that are constrained by scattering data (reduction of scheme dependence)
- nuclear structure calculations are simplified
- hierarchy of many-body forces is preserved (more later)

One solution: the Similarity Renormalization Group

- generate unitary transformation which decouples low- and high momenta:

$$H_\lambda = U_\lambda H U_\lambda^\dagger$$ with the resolution parameter λ

- change resolution systematically in small steps: $$\frac{dH_\lambda}{d\lambda} = [\eta_\lambda, H_\lambda]$$

- generator η_λ can be chosen and tailored to different applications

- observables are preserved due to unitarity of transformation

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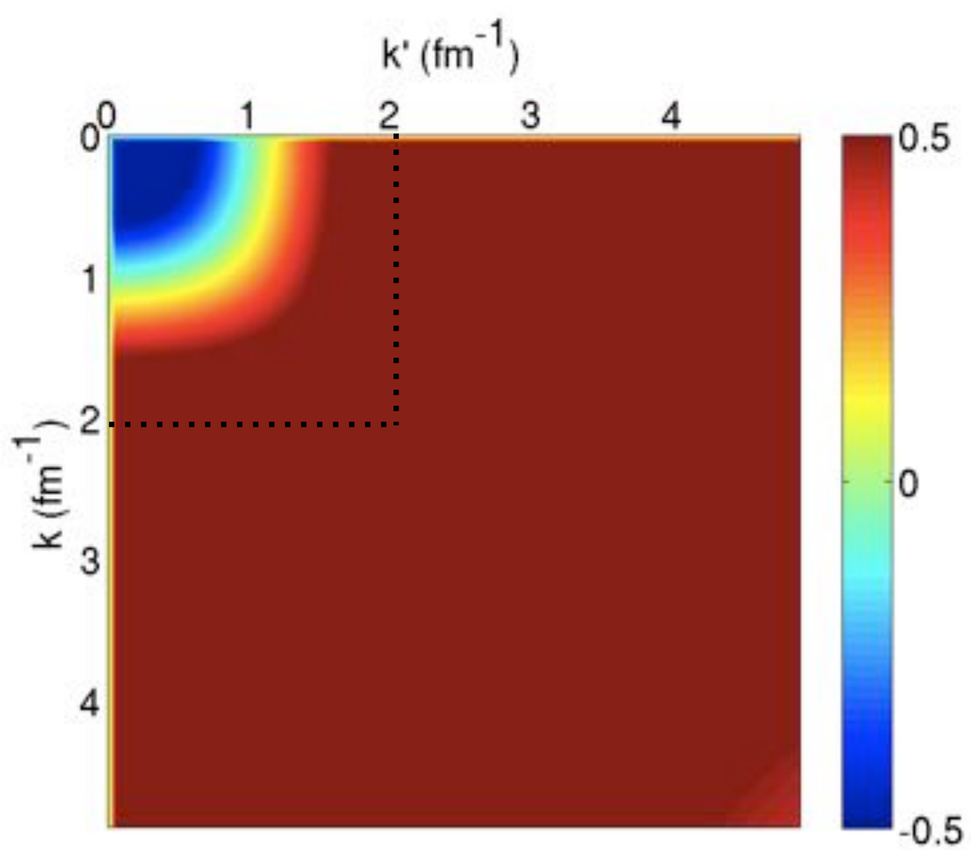

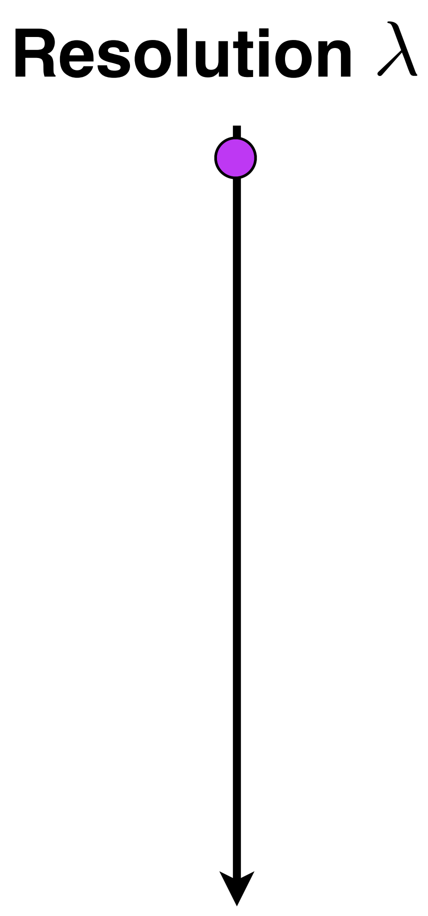

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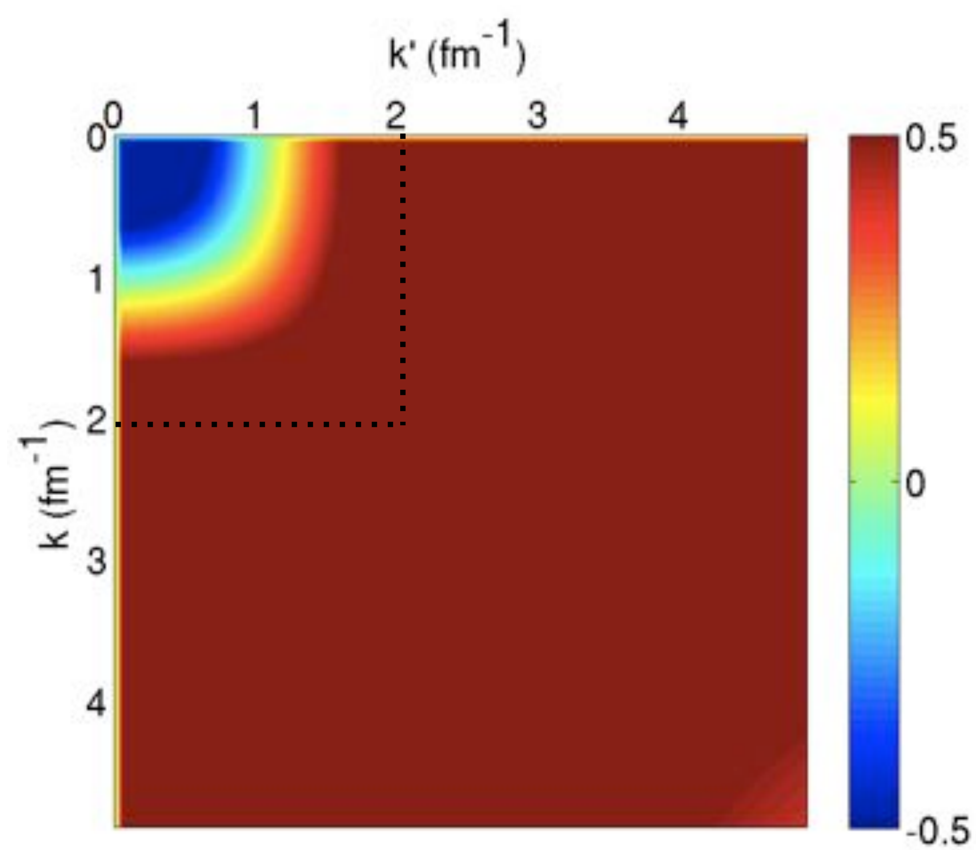

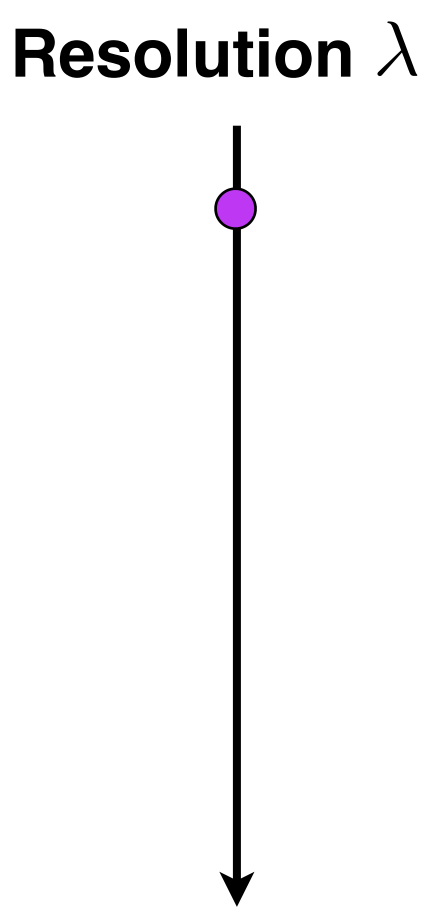

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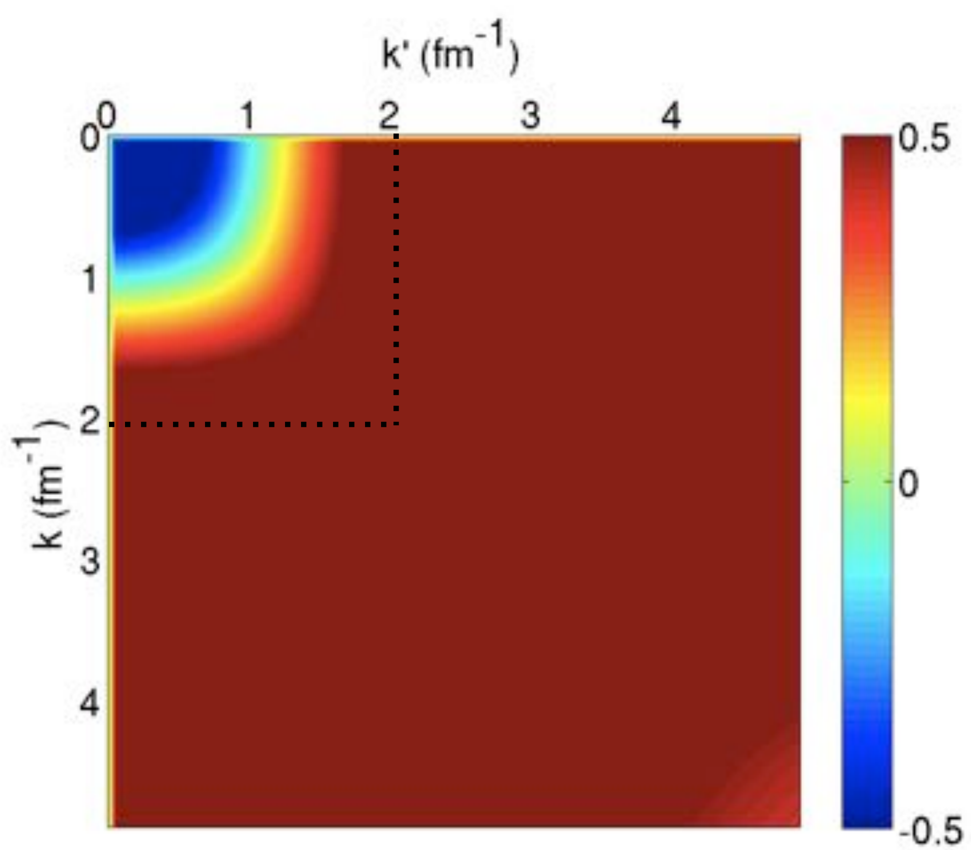

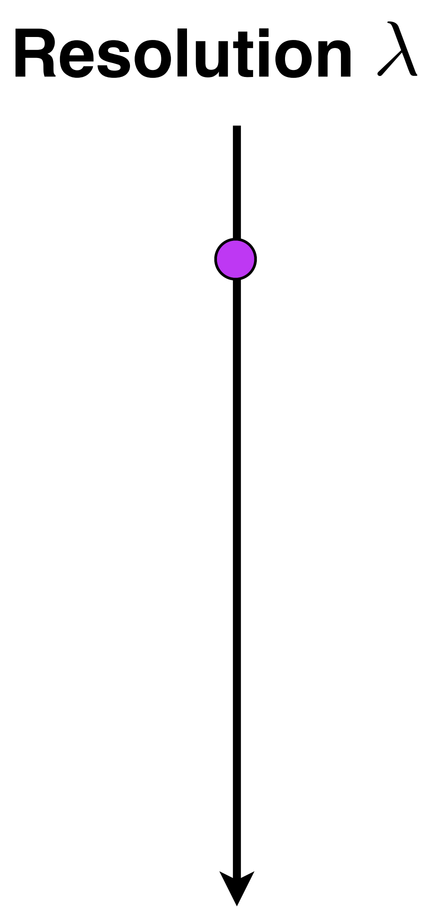

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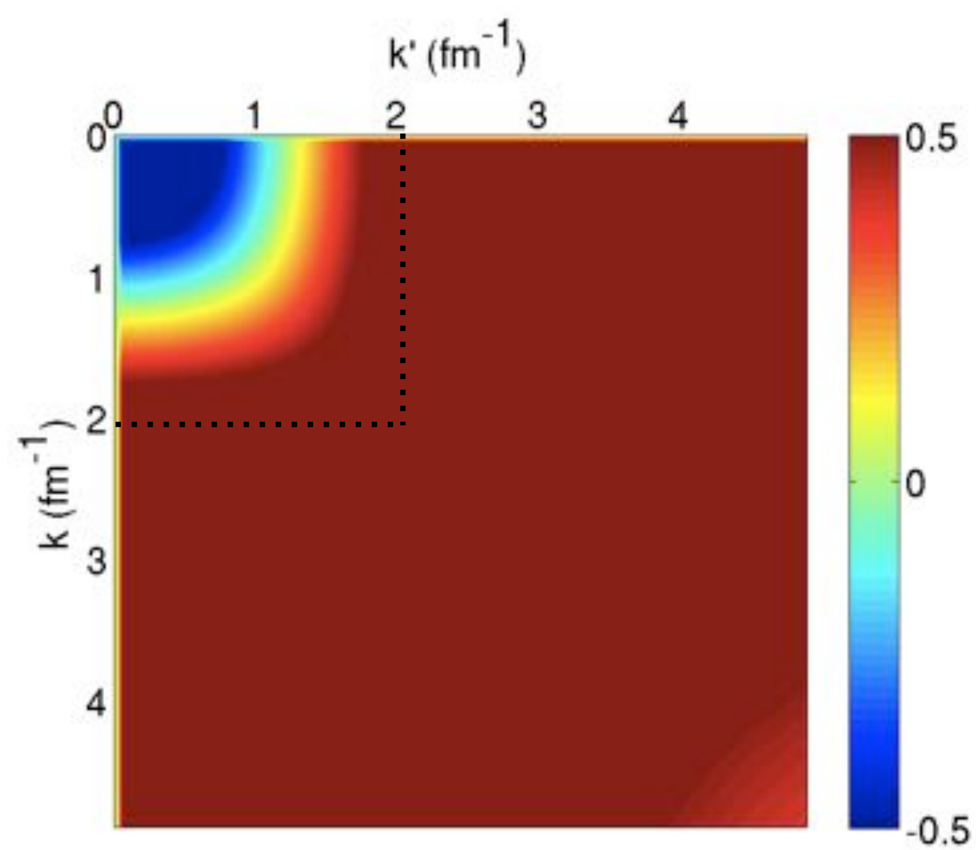

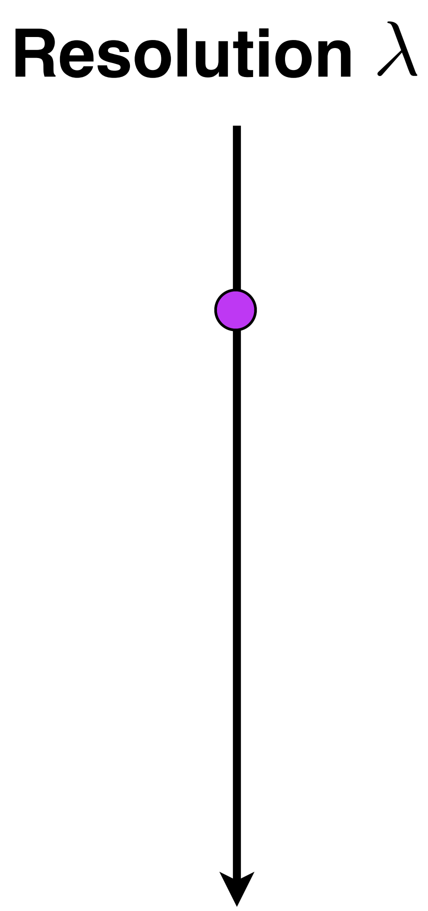

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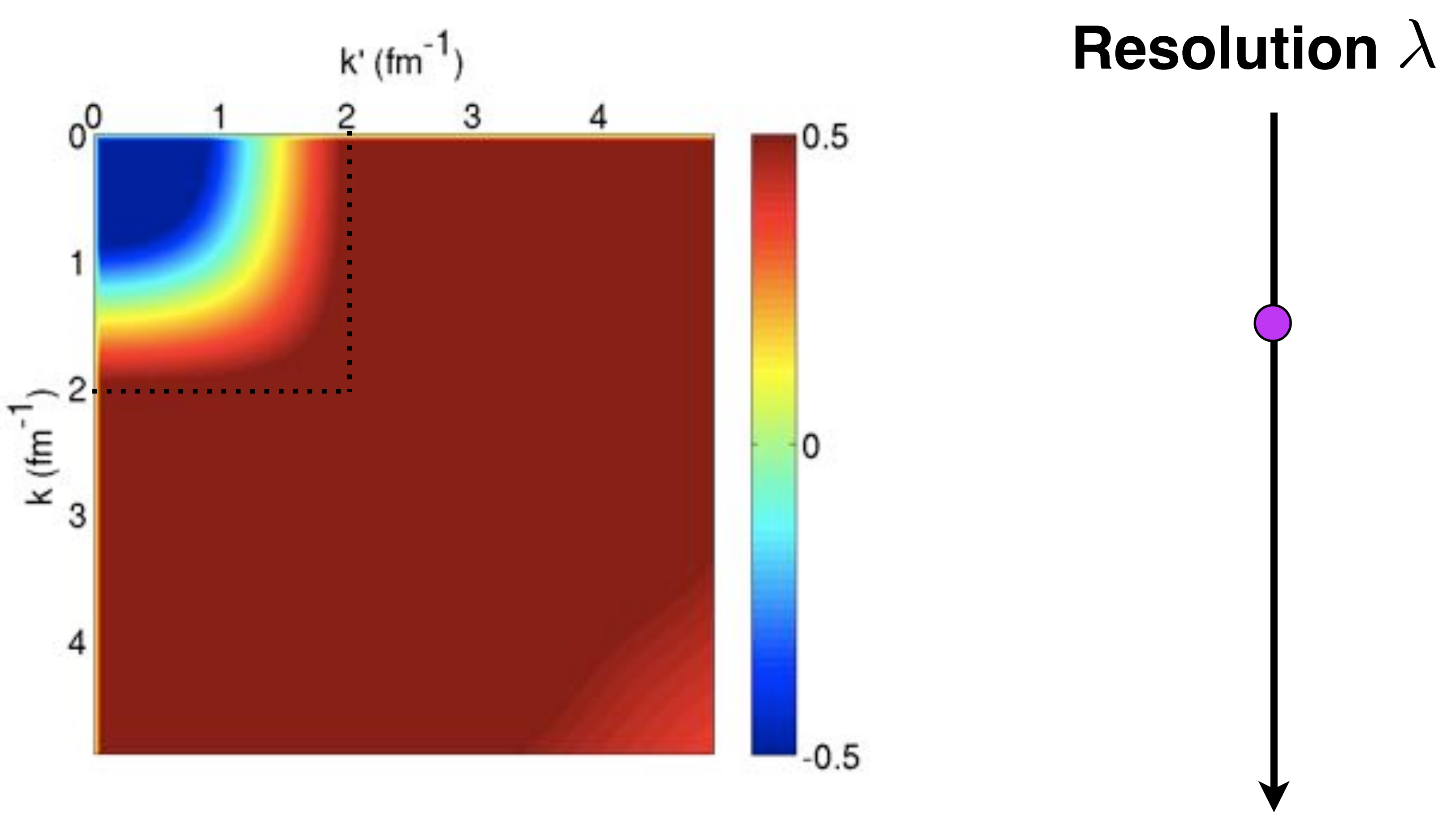

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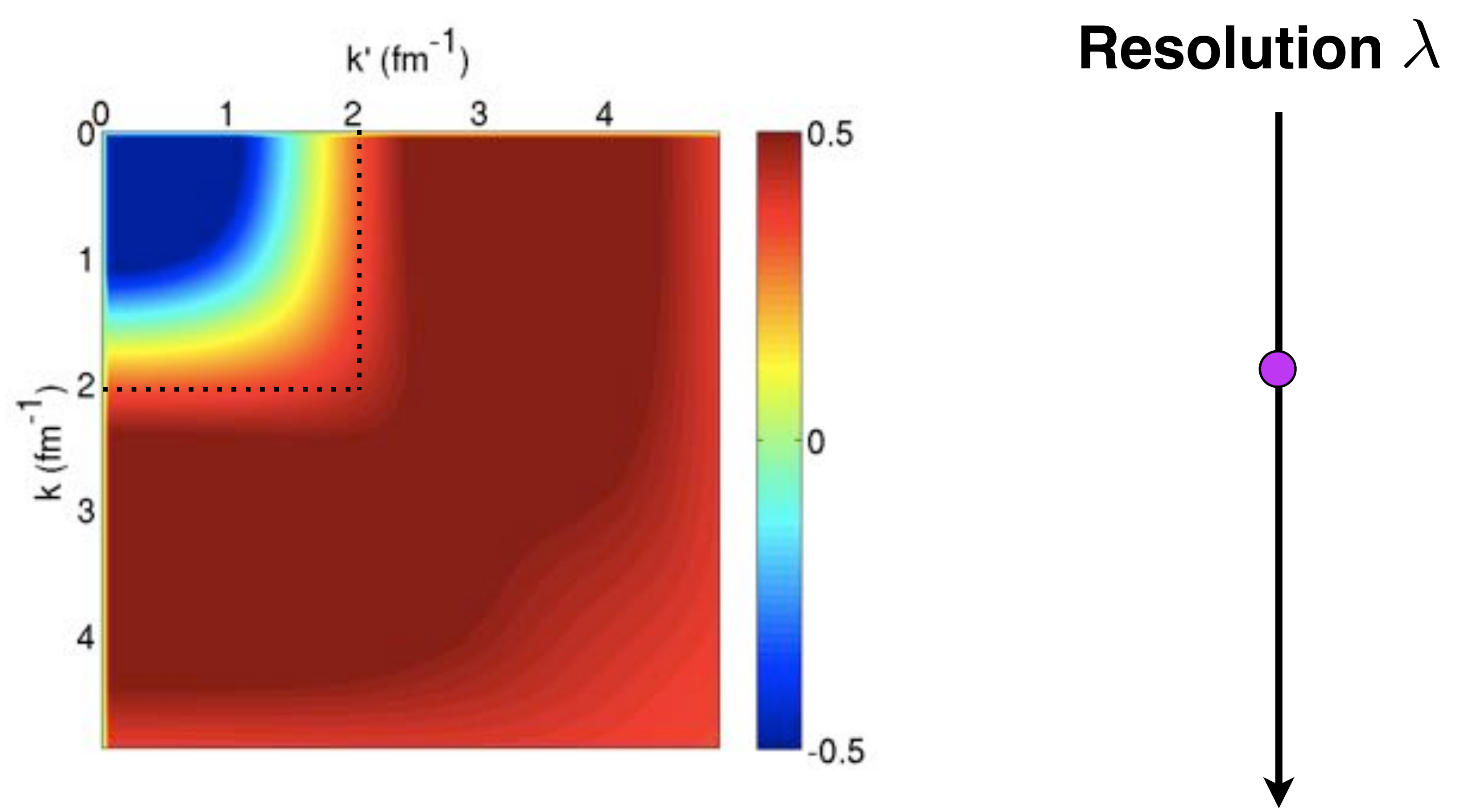

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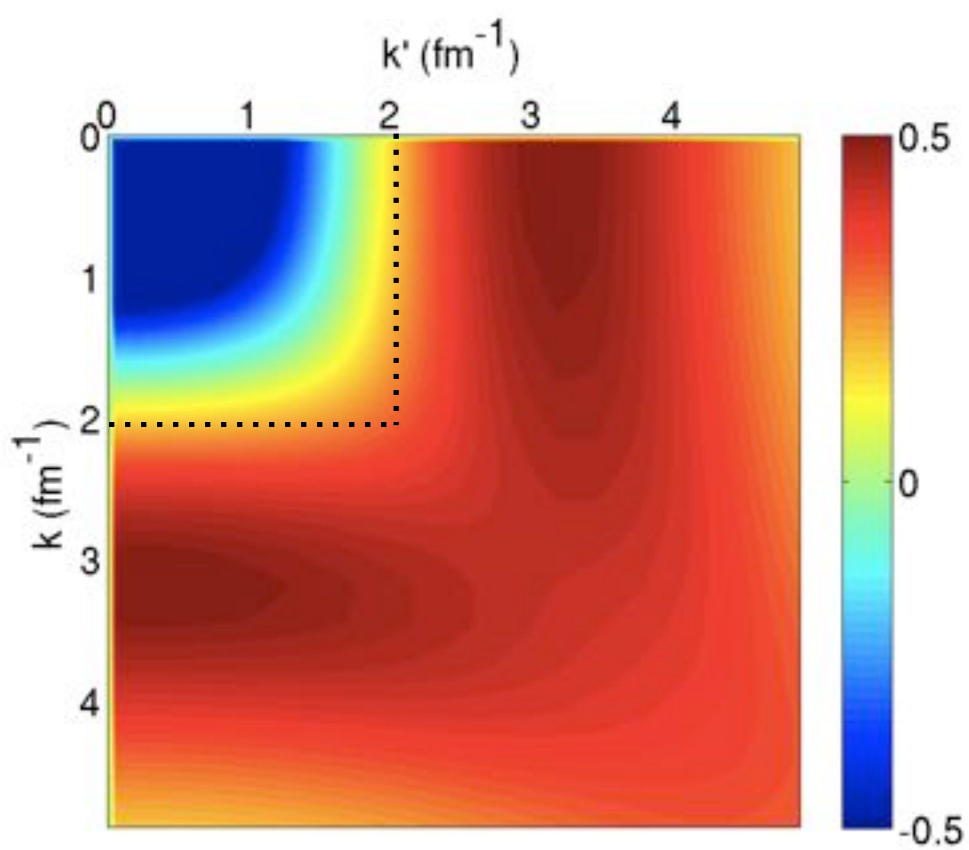
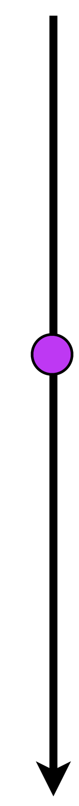


Resolution λ



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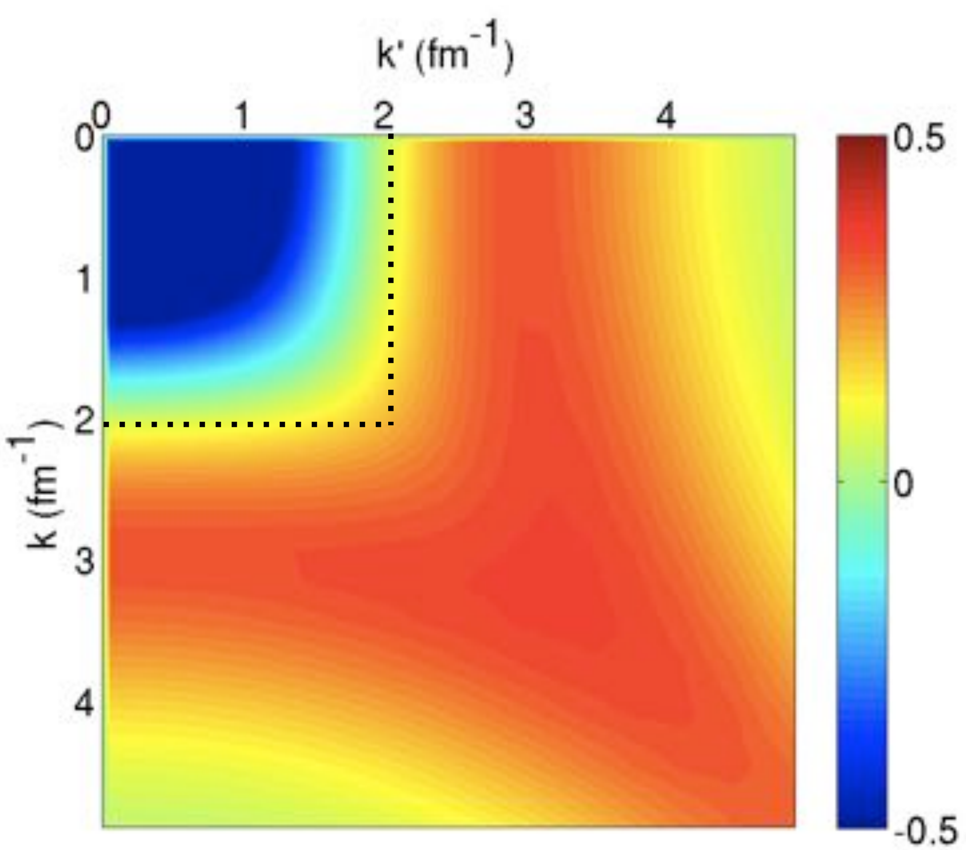

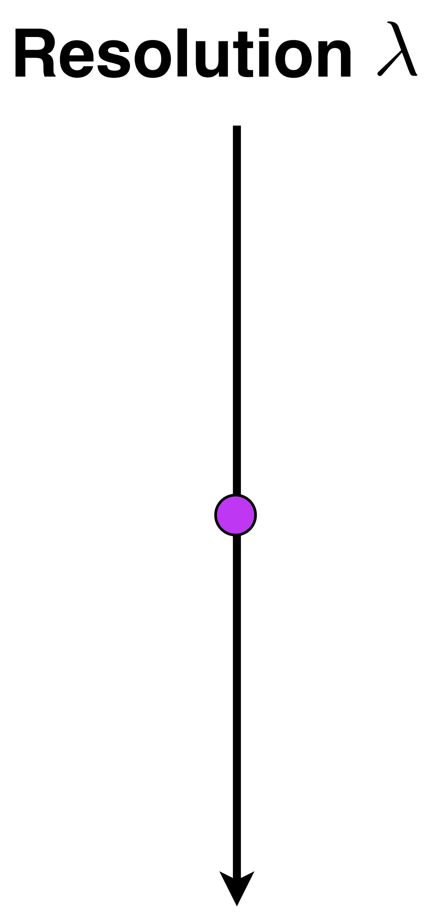

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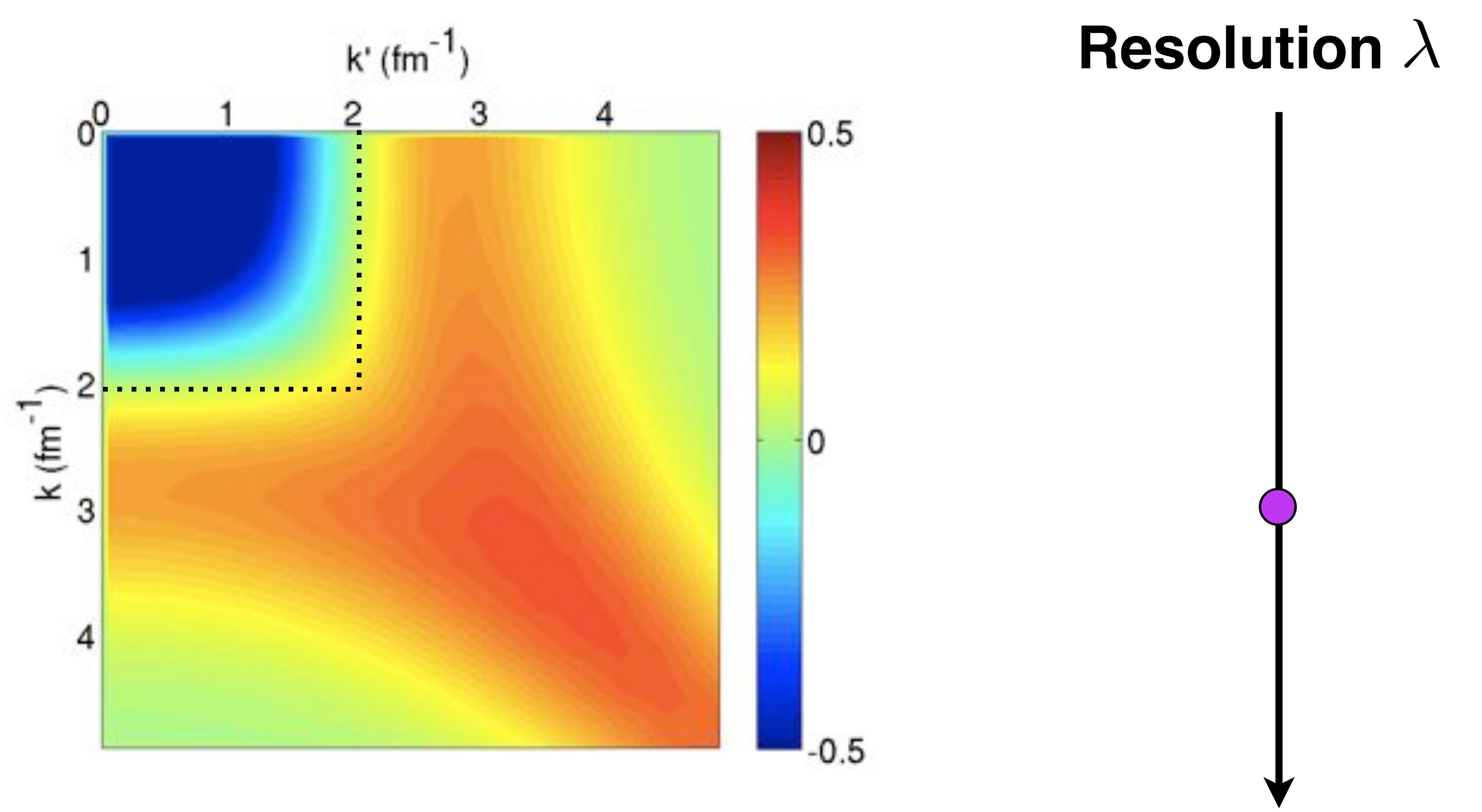

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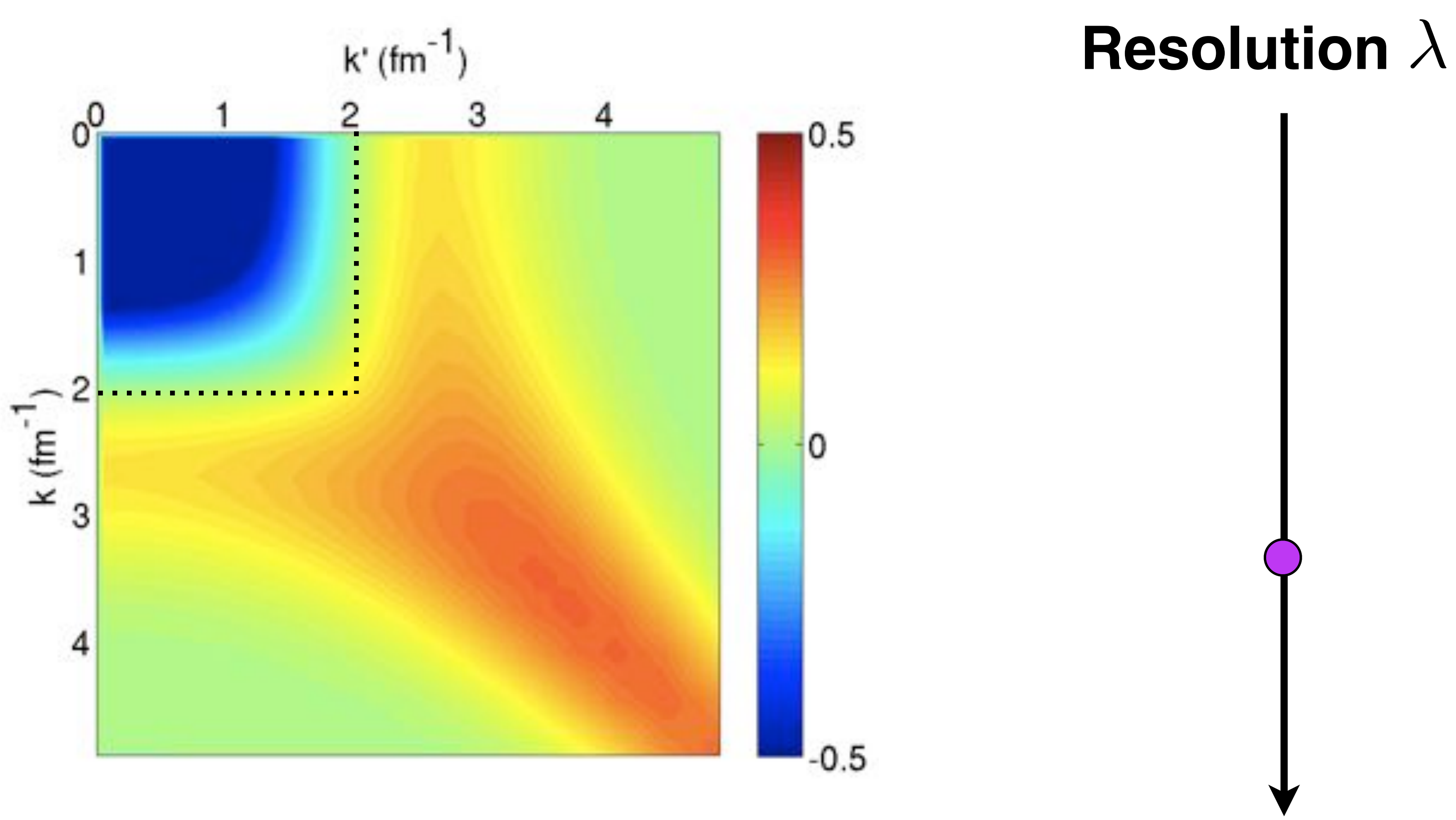

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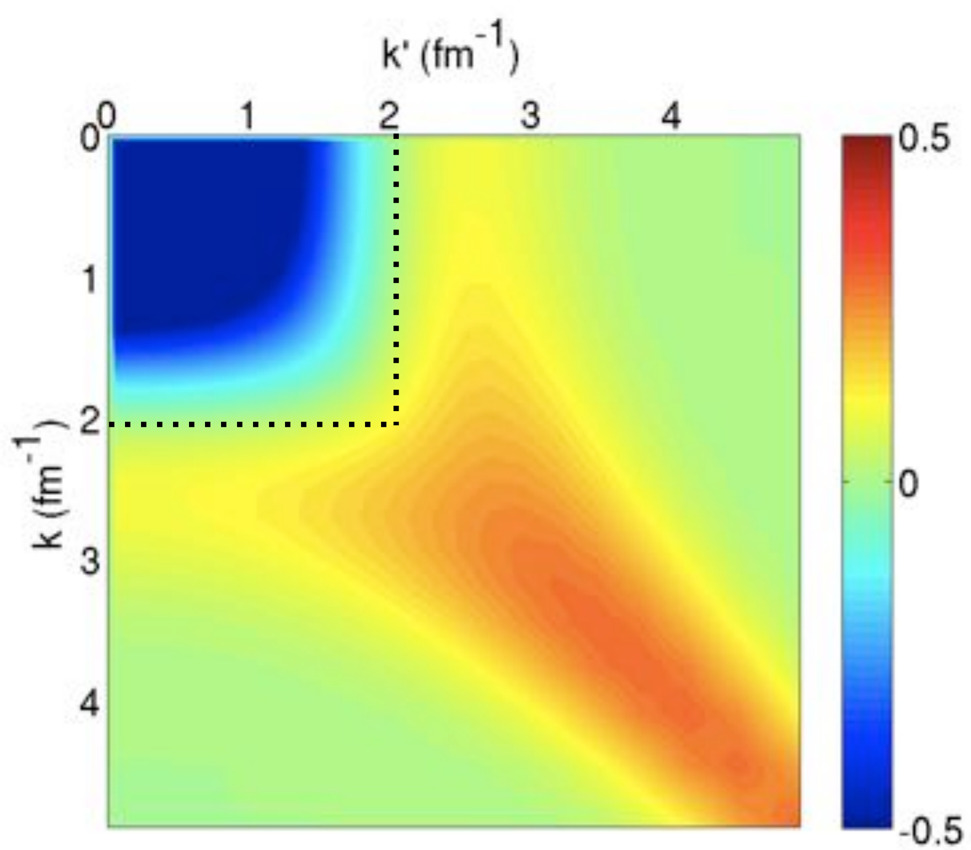

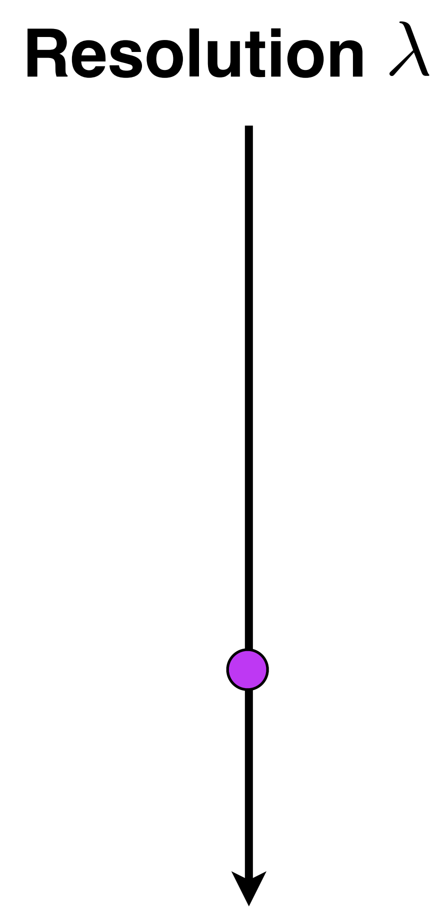

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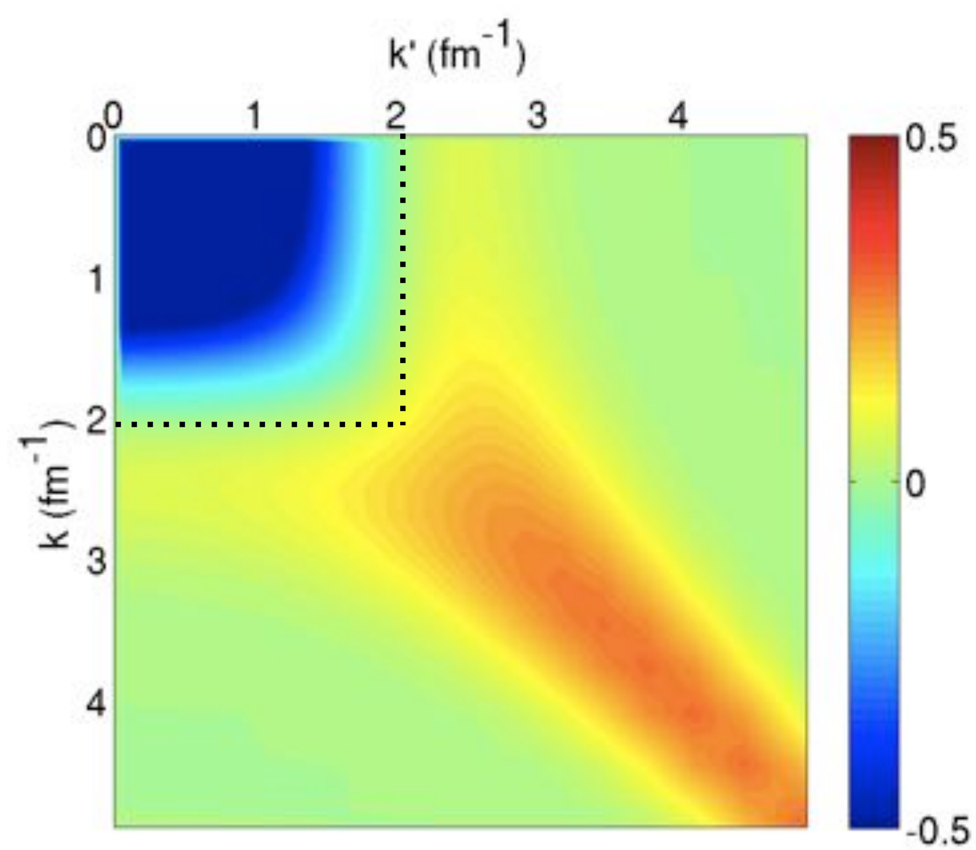

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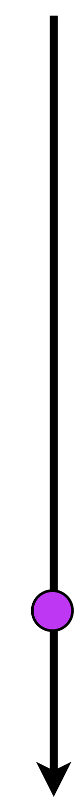

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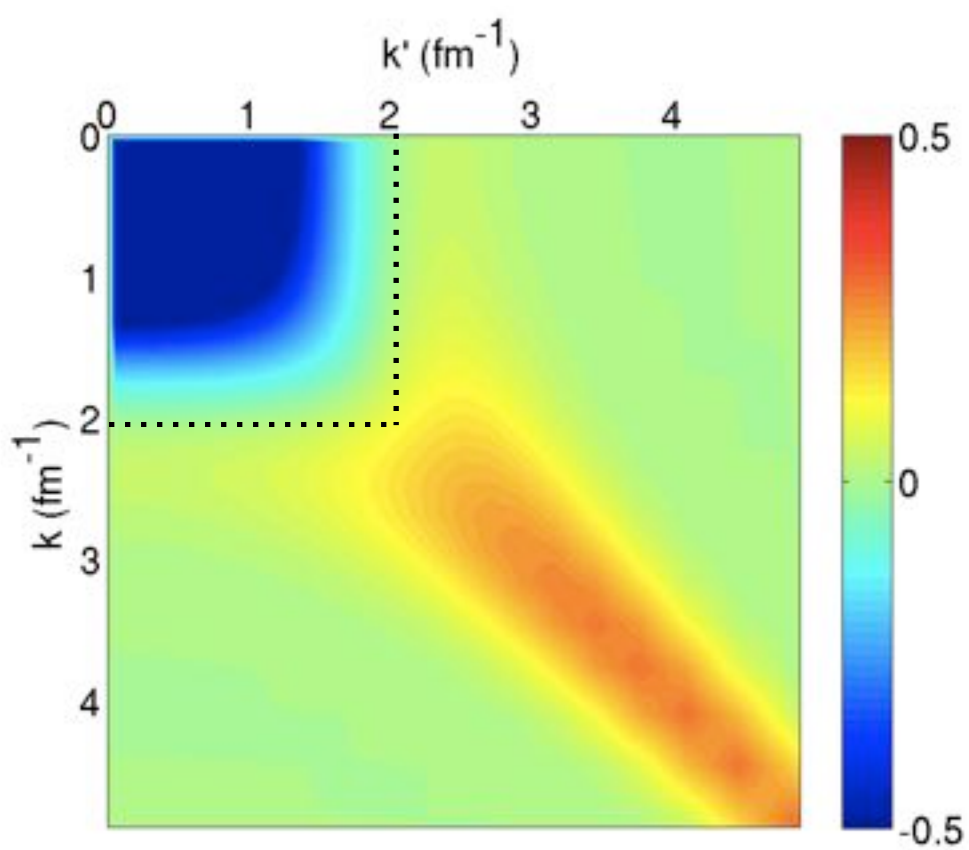

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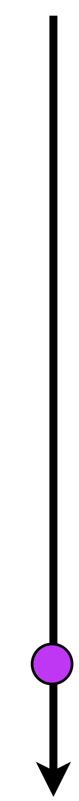

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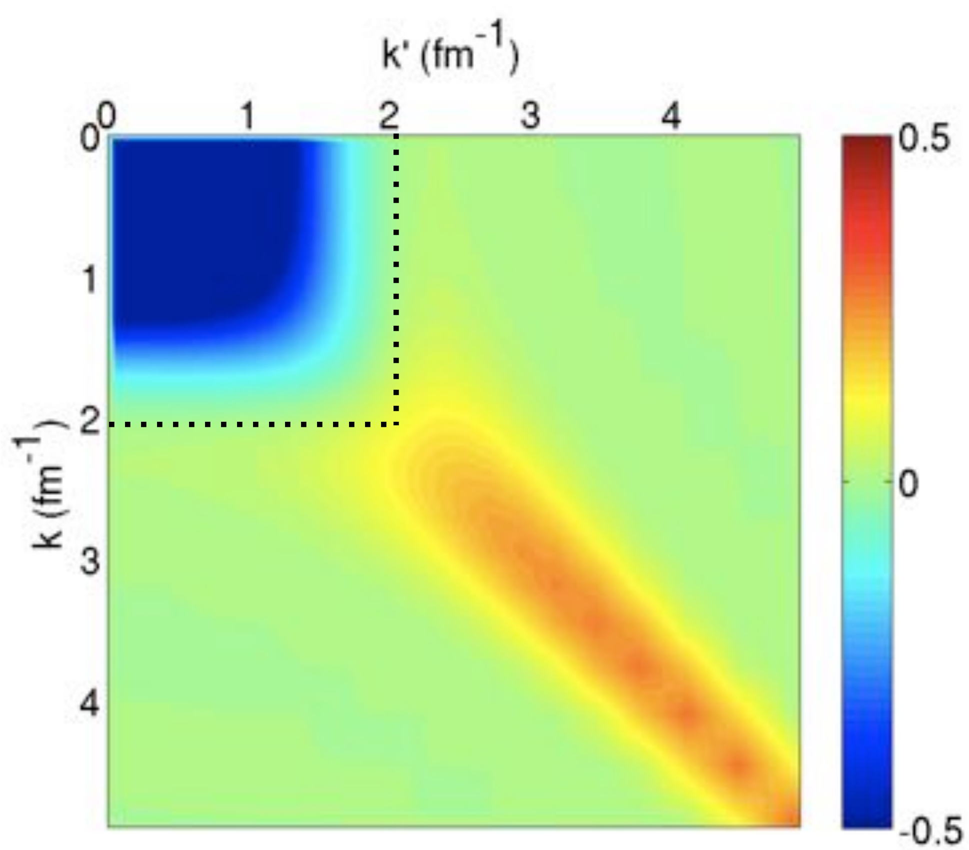

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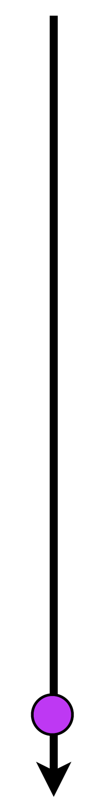
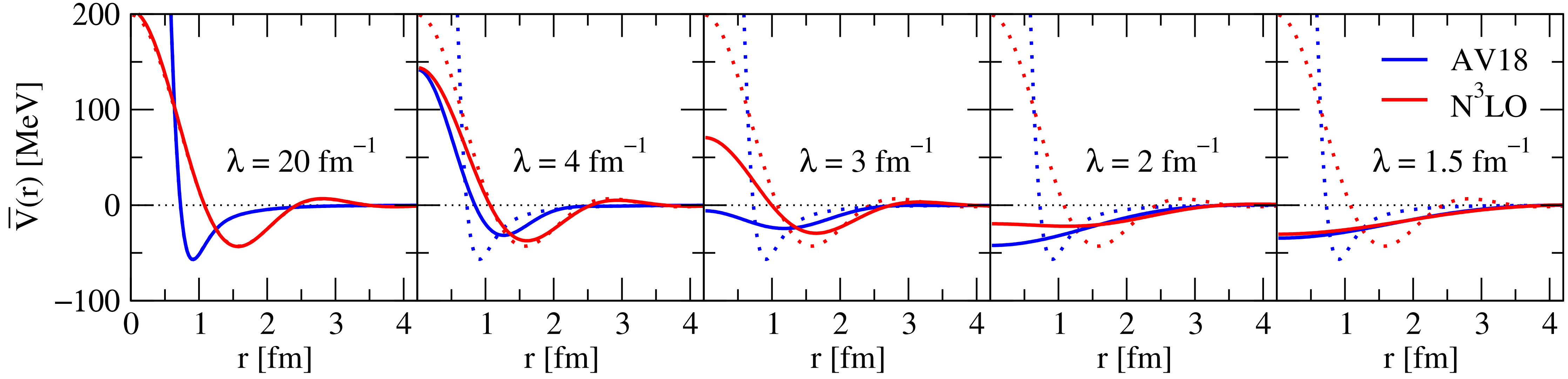
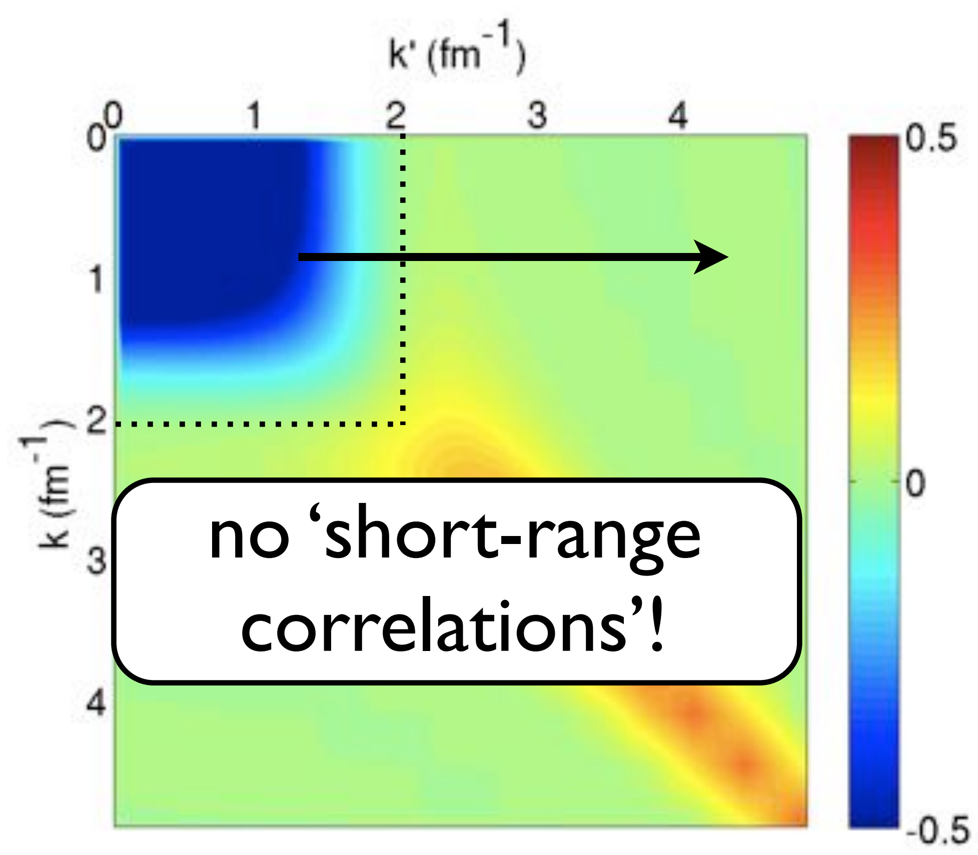


One solution: the Similarity Renormalization Group

$$\overline{V}_\lambda(r) = \int dr' r'^2 V_\lambda(r,r')$$





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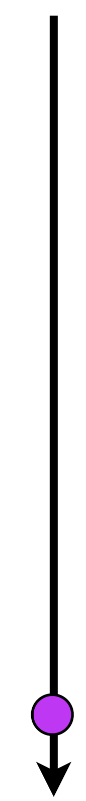
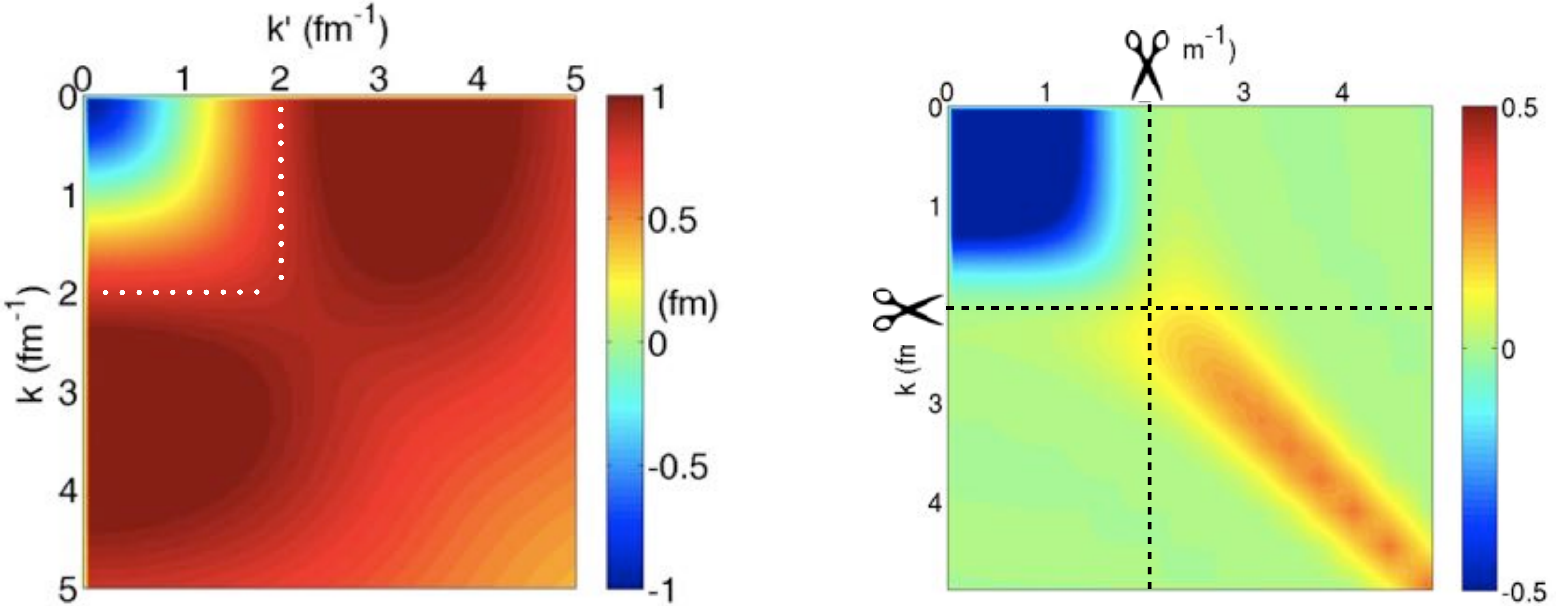


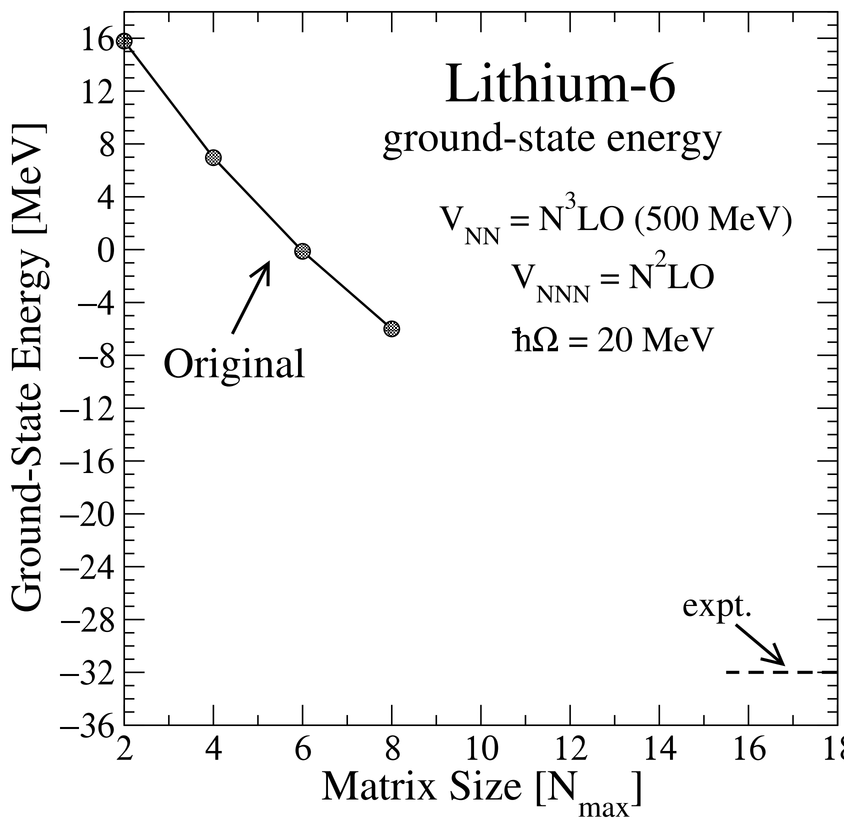
Systematic decoupling of high-momentum physics: the Similarity Renormalization Group

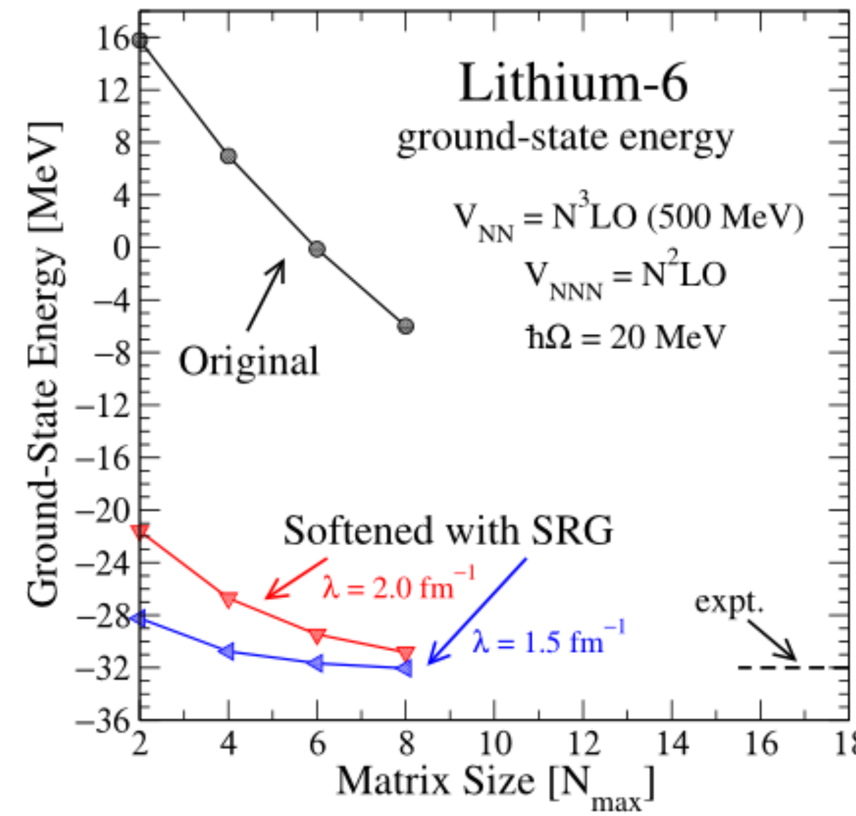


- elimination of coupling between low- and high momentum components,
  ⟶ simplified many-body calculations!
- observables unaffected by resolution change (for exact calculations)
- residual resolution dependences can be used as tool to test calculations

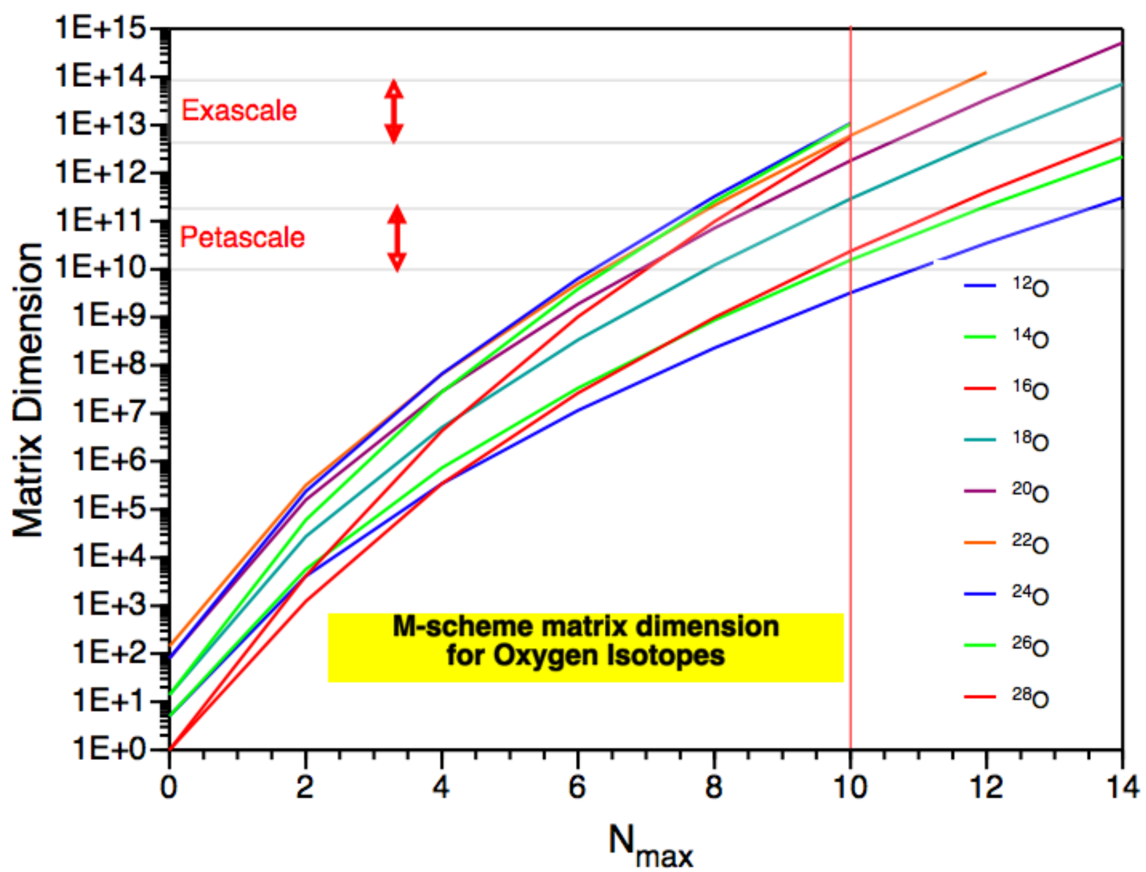
Not the full story:
RG transformations also change three-body (and higher-body) interactions and ALL other operators!

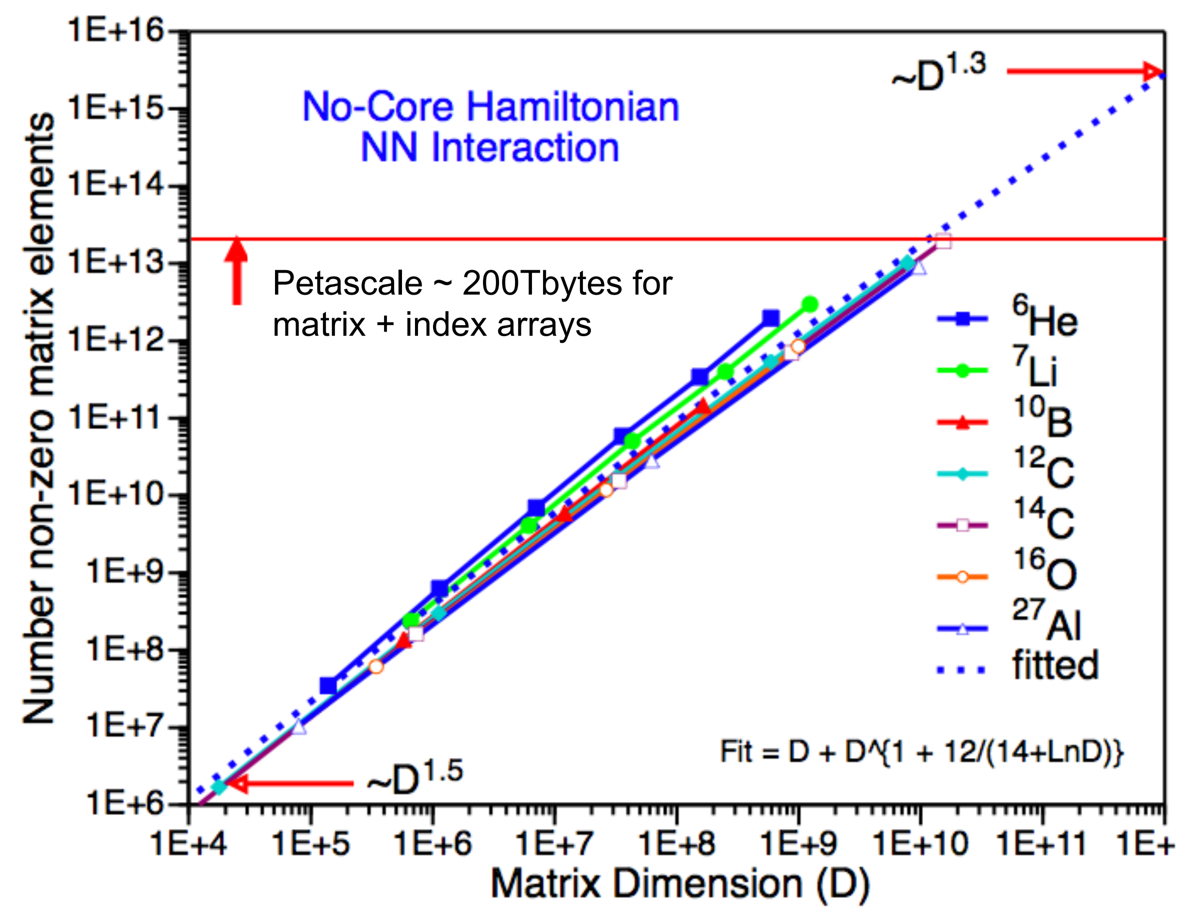
Basis size and matrix dimensions





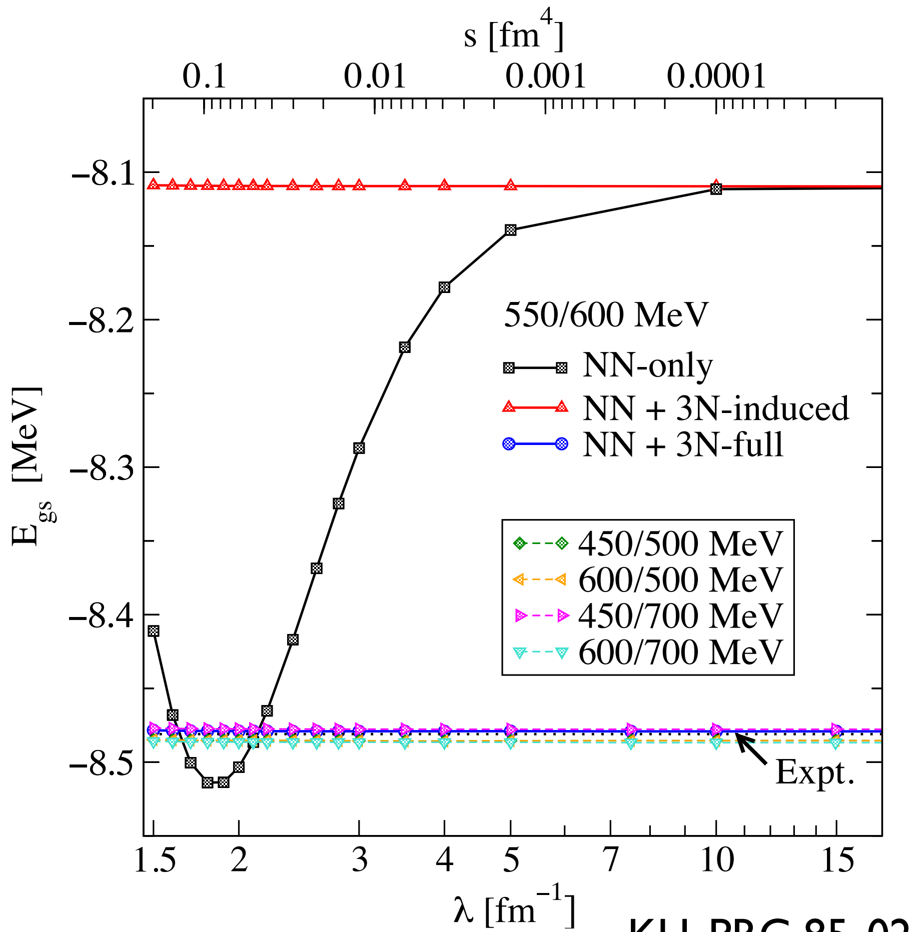
Bogner et al.,
PPNP 65, 95 (2010)





Barrett, Navratil,Vary, PPNP 69, 131 (2013)

Application of SRG-evolved NN+3N forces to nuclei



KH, PRC 85, 021002 (2012)

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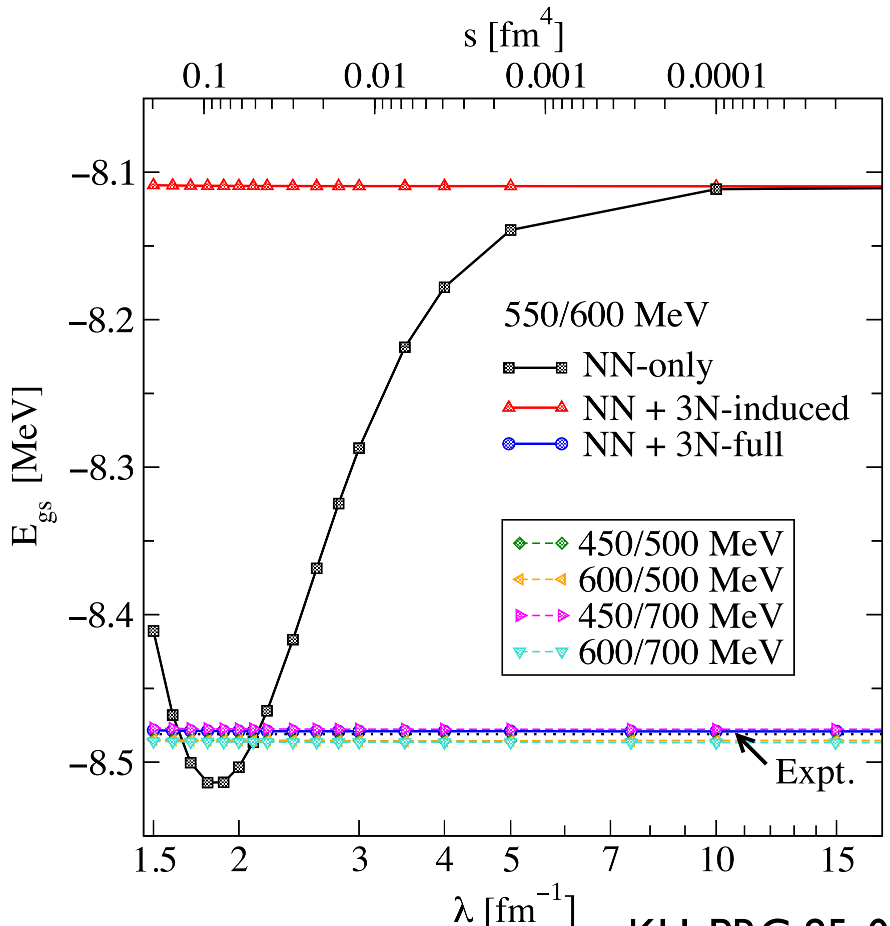

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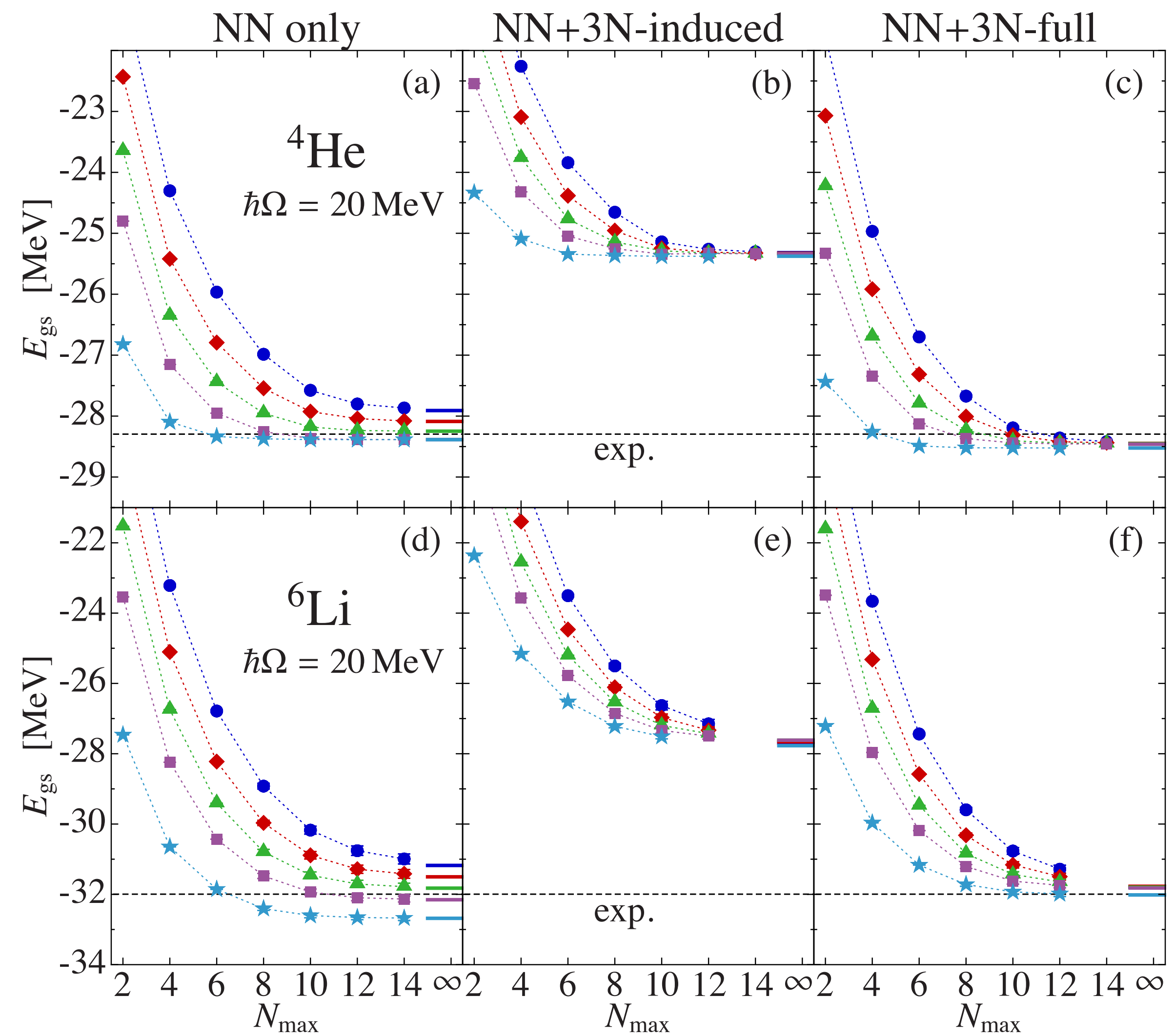

Roth et al. PRL 107, 072501 (2011)

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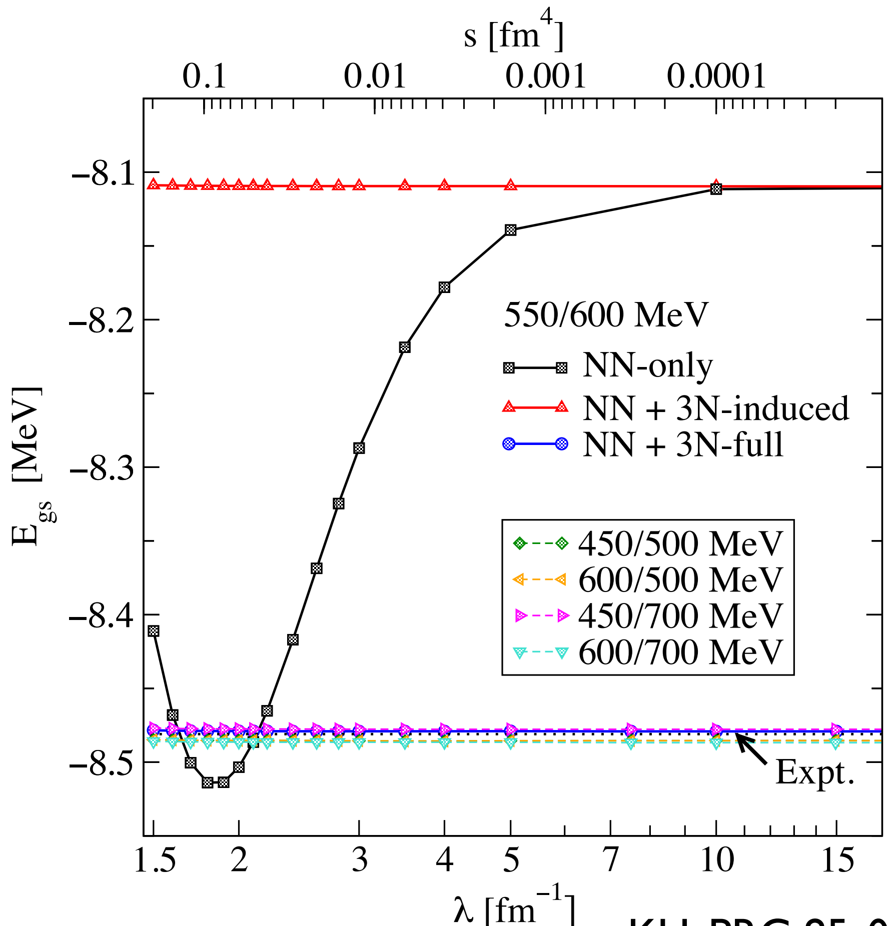

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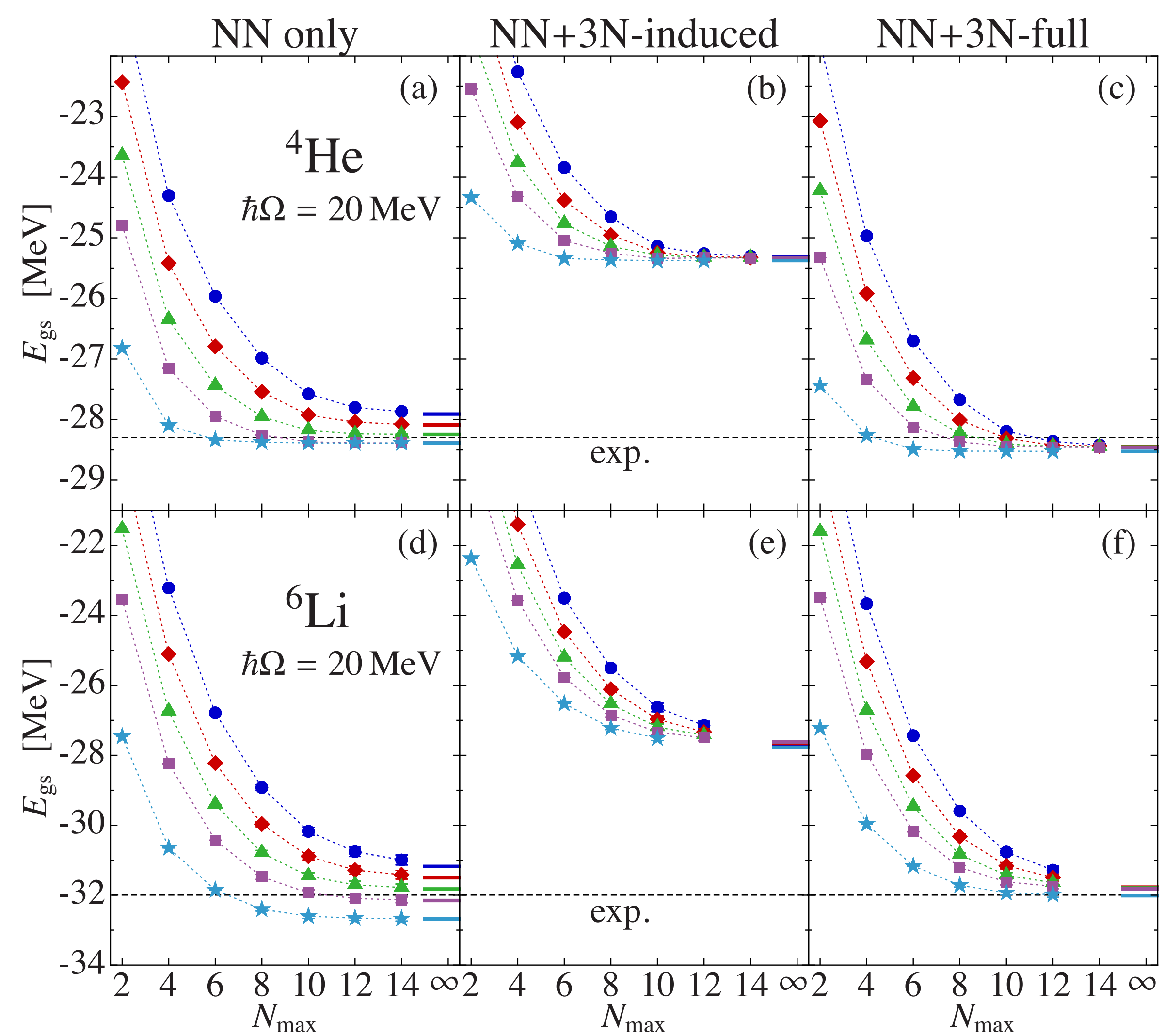

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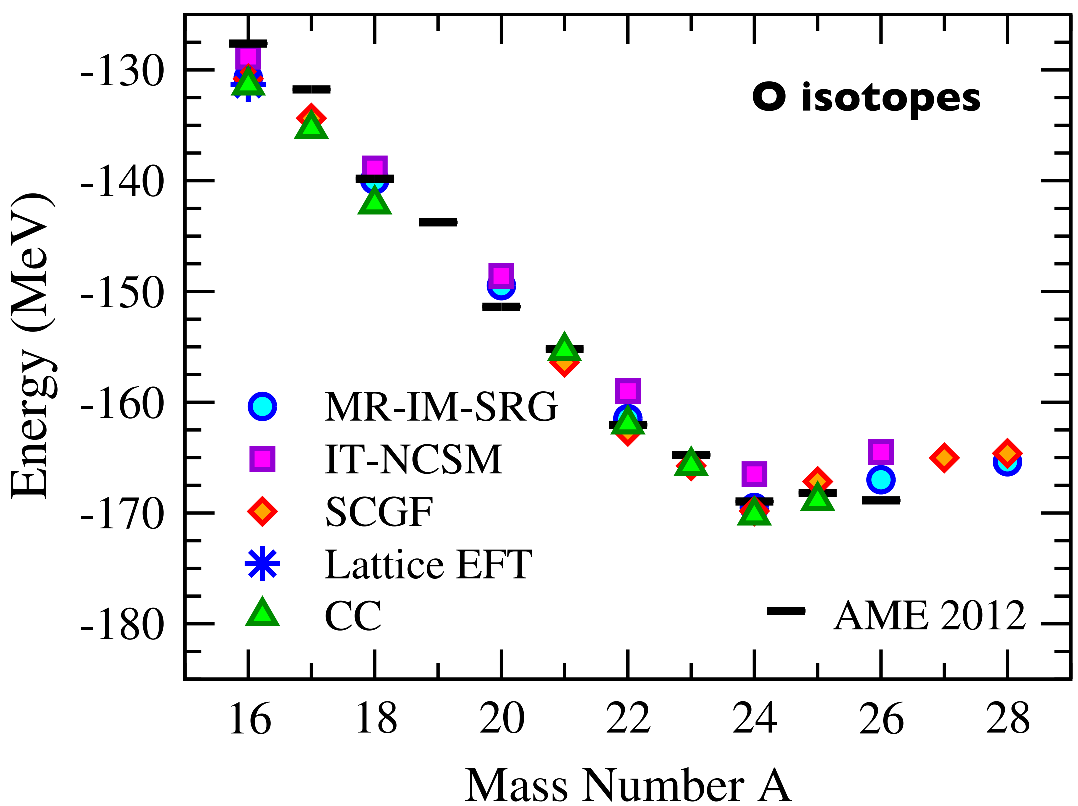

KH et al., Ann. Rev. Nucl. Part. Sci.165, 457 (2015)

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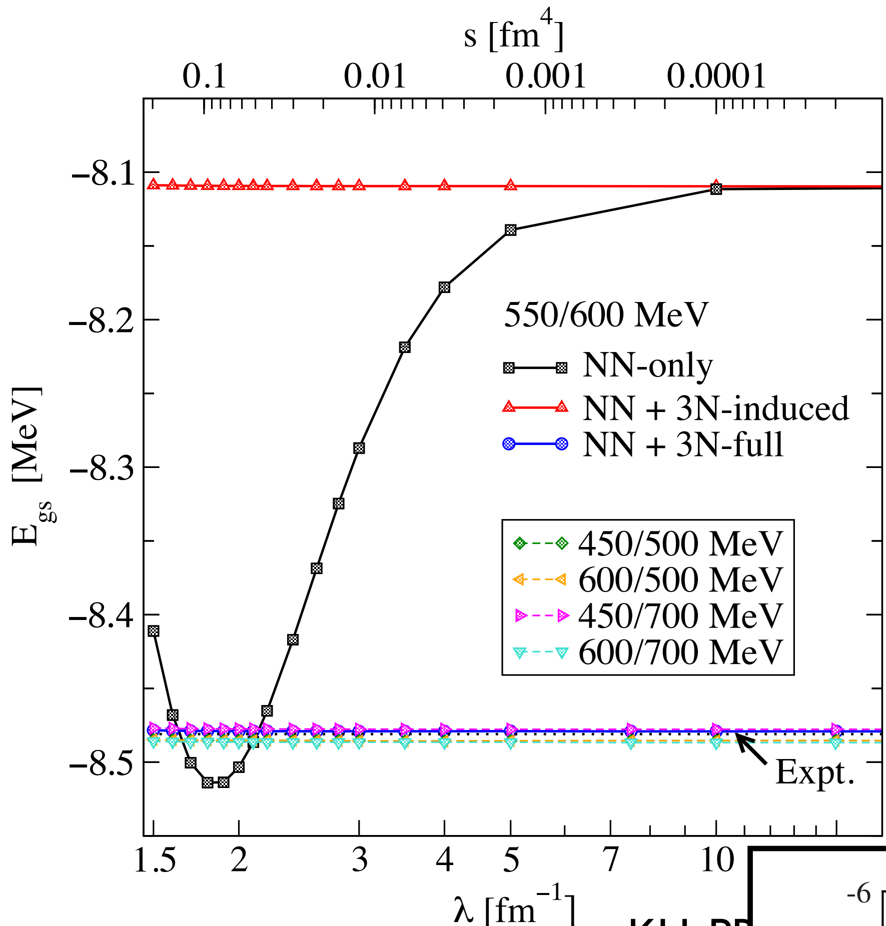
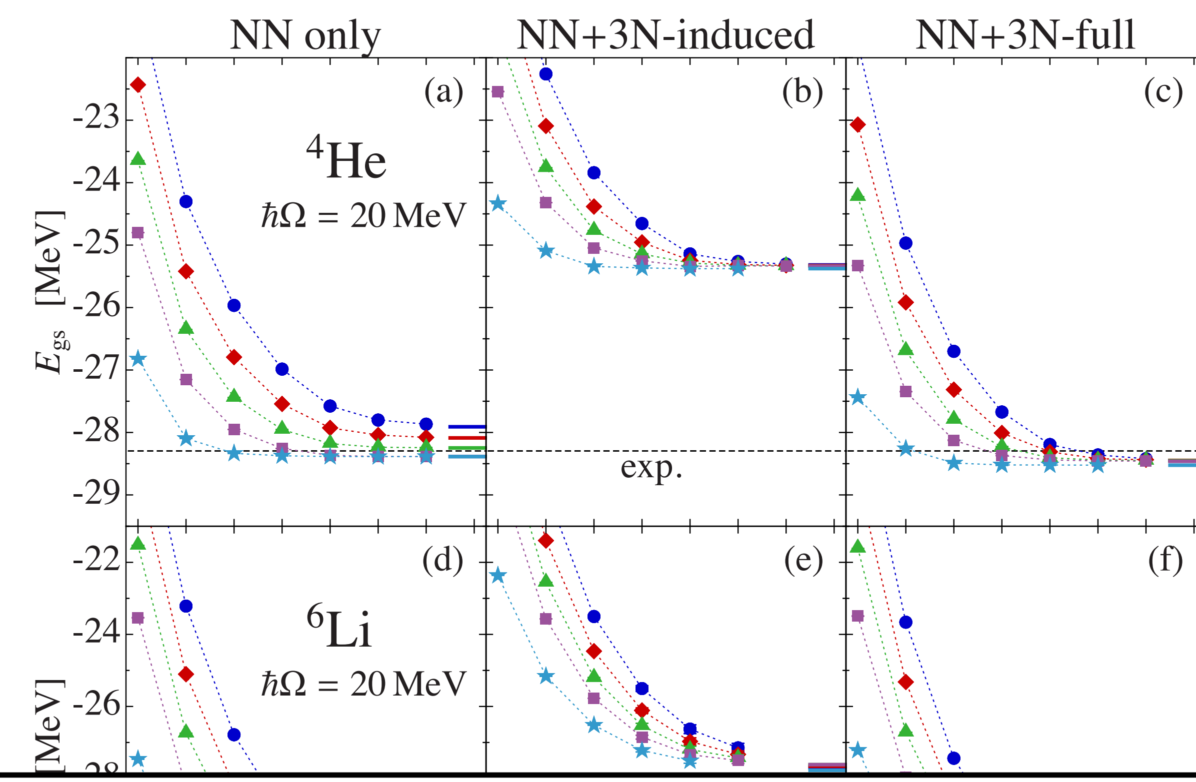
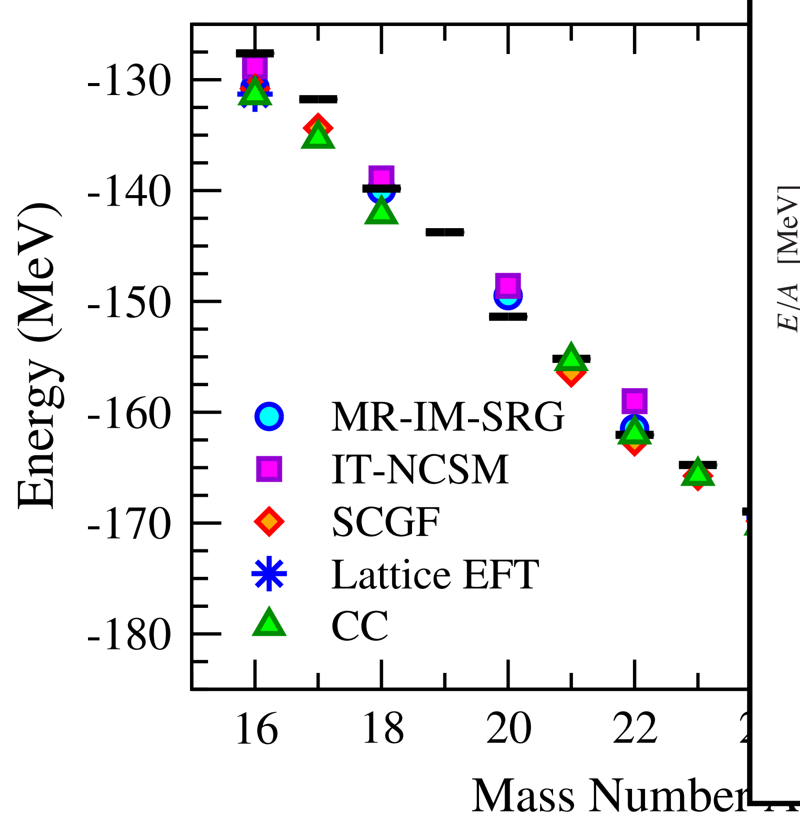
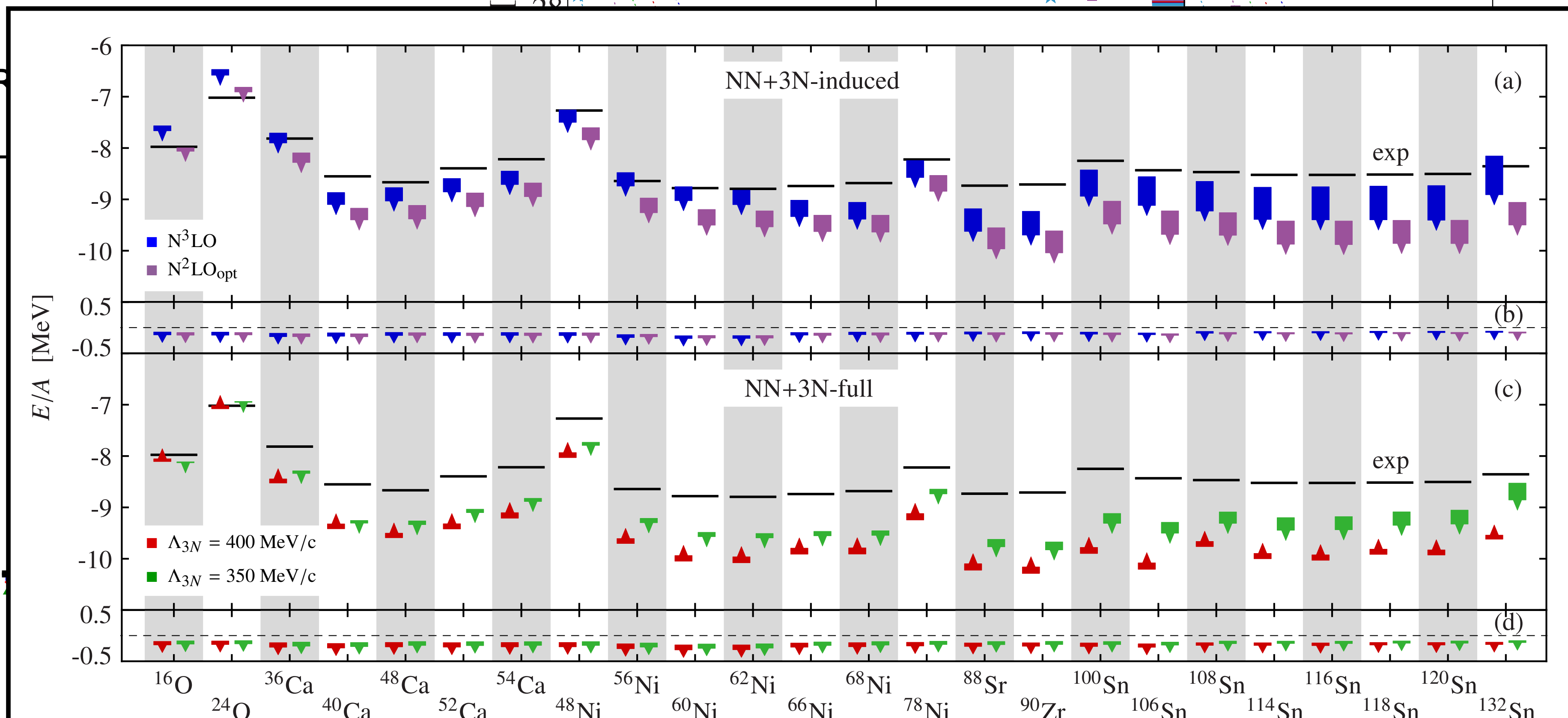


KH, PR

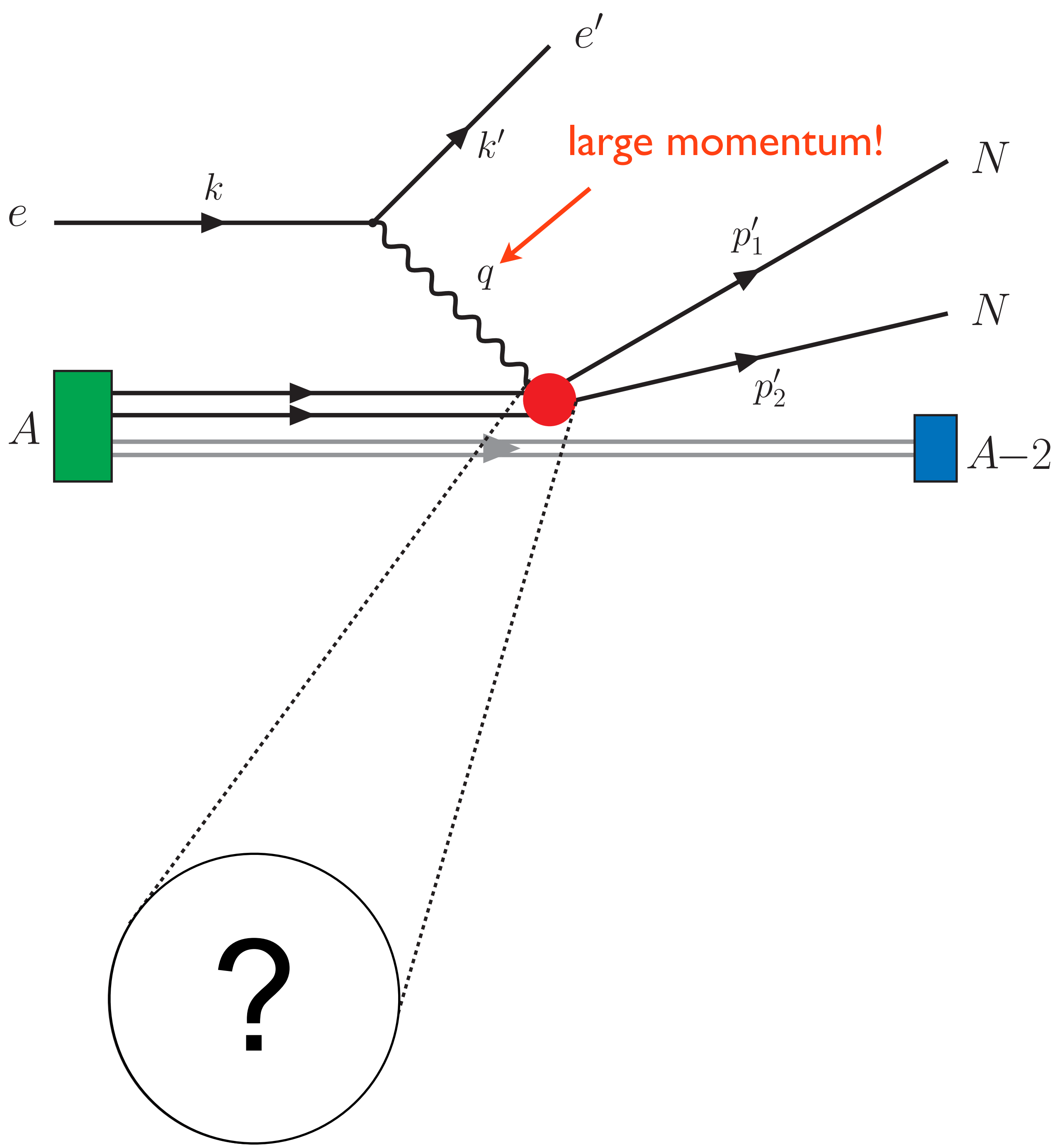


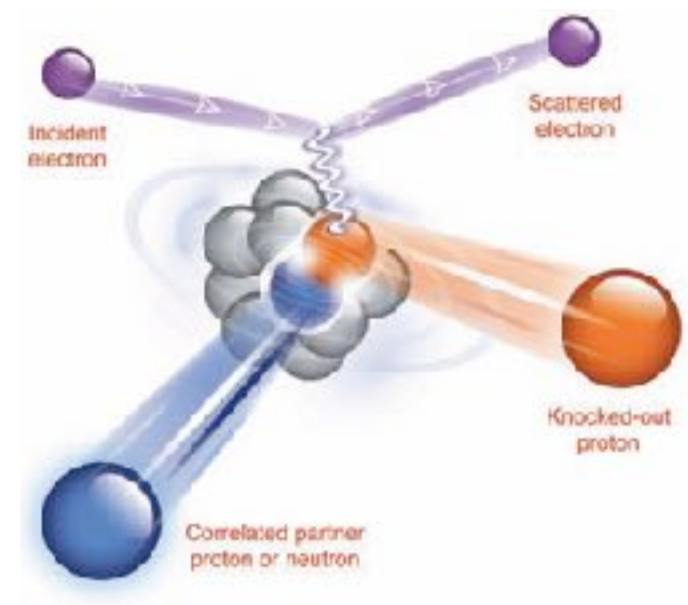




Binder et al., Phys. Lett B 736, 119 (2014)

Application to deep-inelastic knock-out reactions

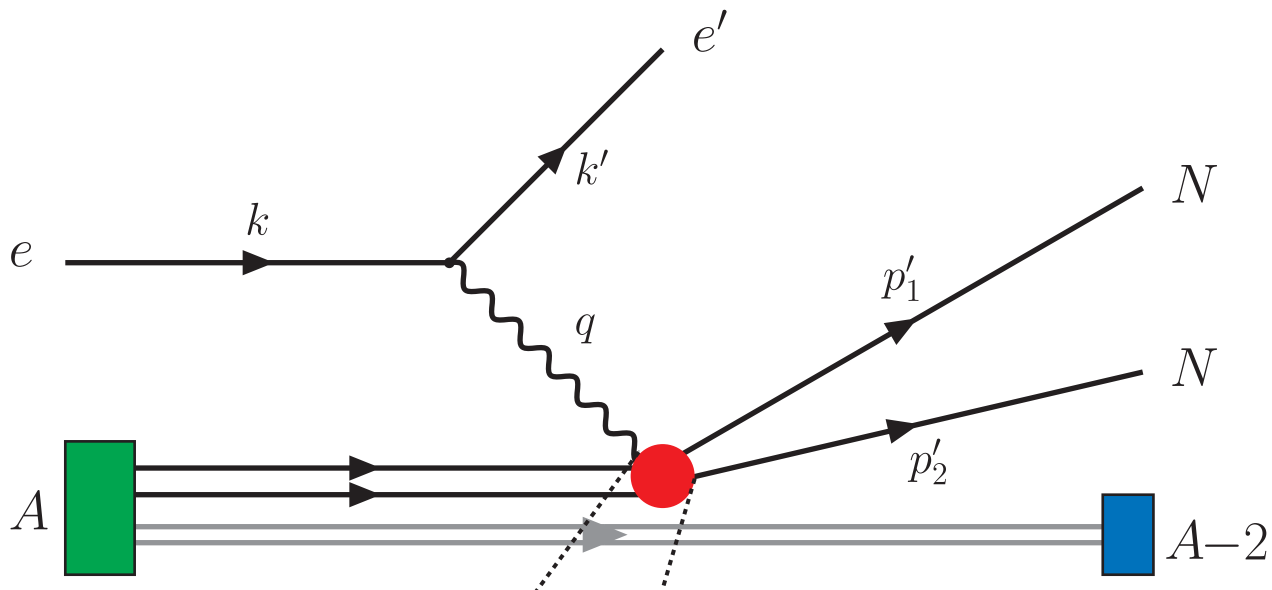


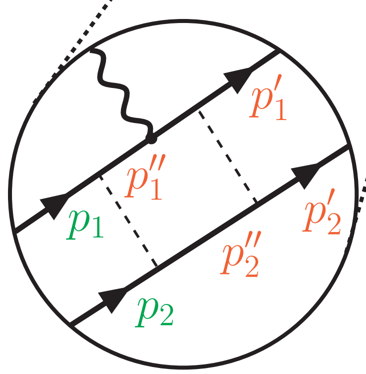


- detection of knocked out pairs with large relative momenta
- excess of np pairs over pp pairs

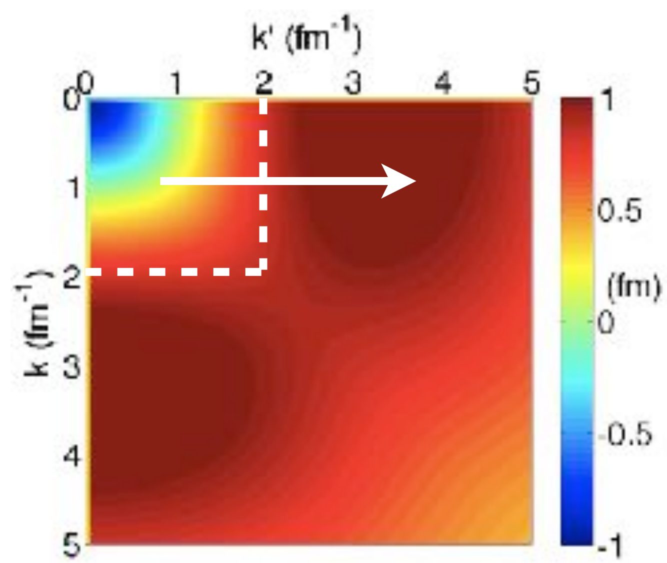
Subedi et al., Science 320, 1476 (2008)

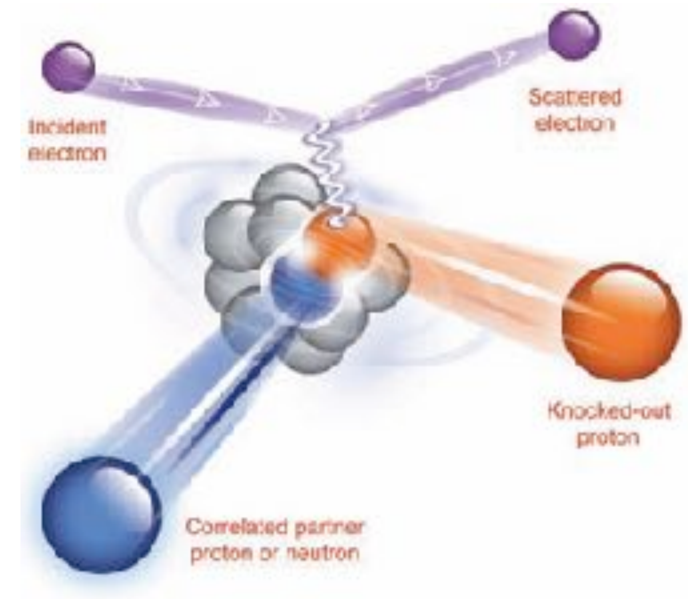
Application: deep-inelastic knock-out reactions





Short-range-correlation interpretation (SRC):

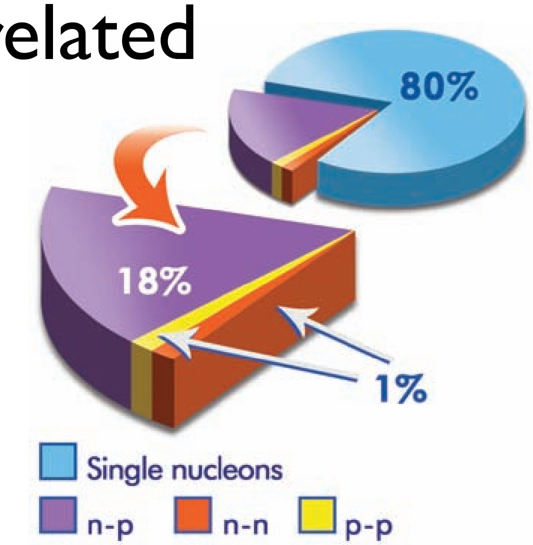




- detection of knocked out pairs with large relative momenta
- excess of np pairs over pp pairs

Subedi et al., Science 320, 1476 (2008)

'measurement' of number of short range-correlated pairs in ^{12}C



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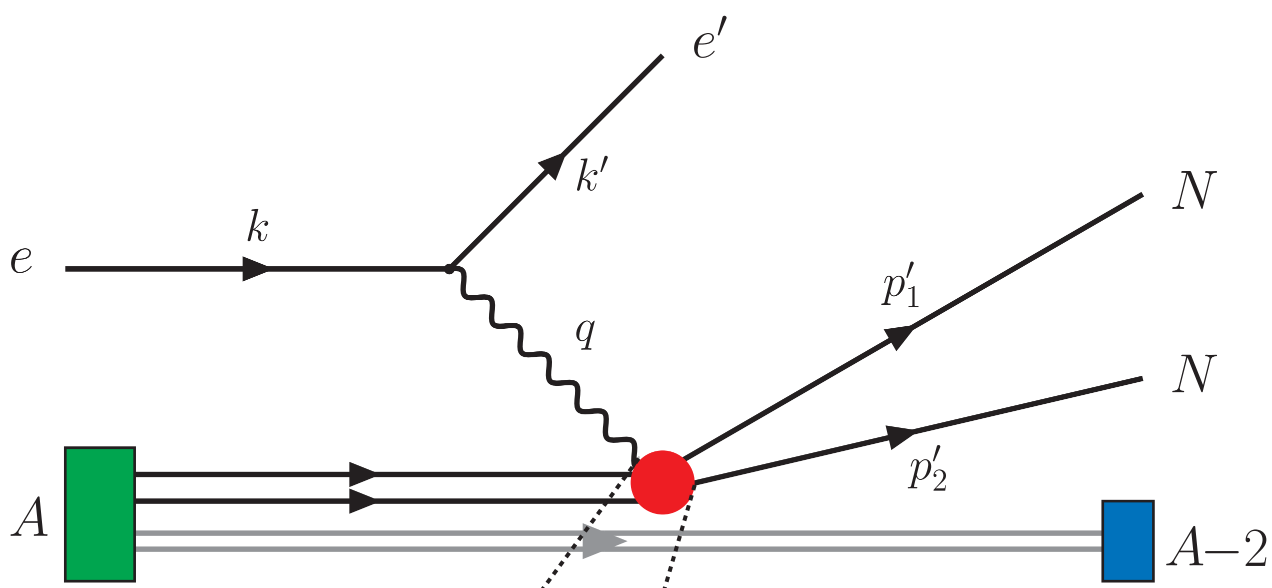

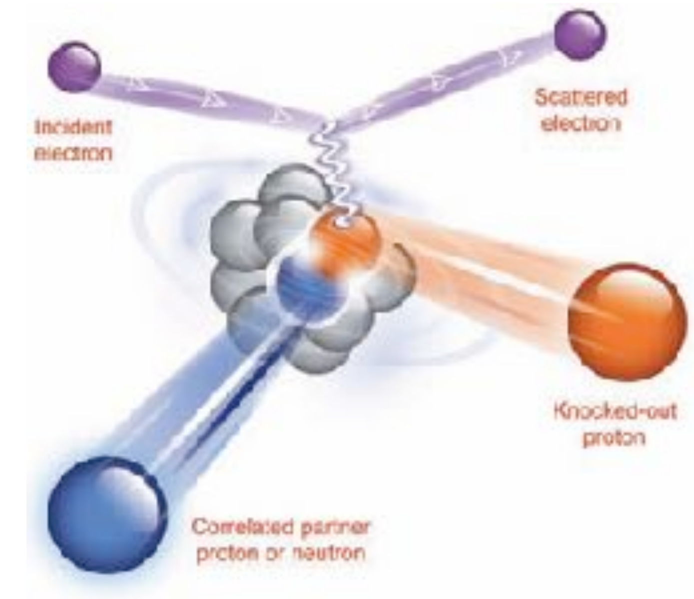

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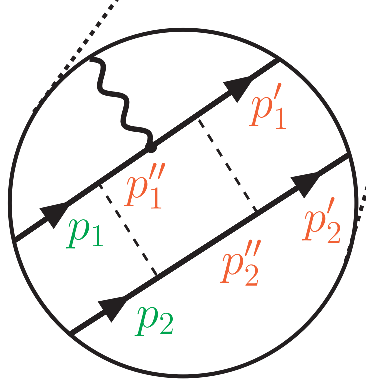
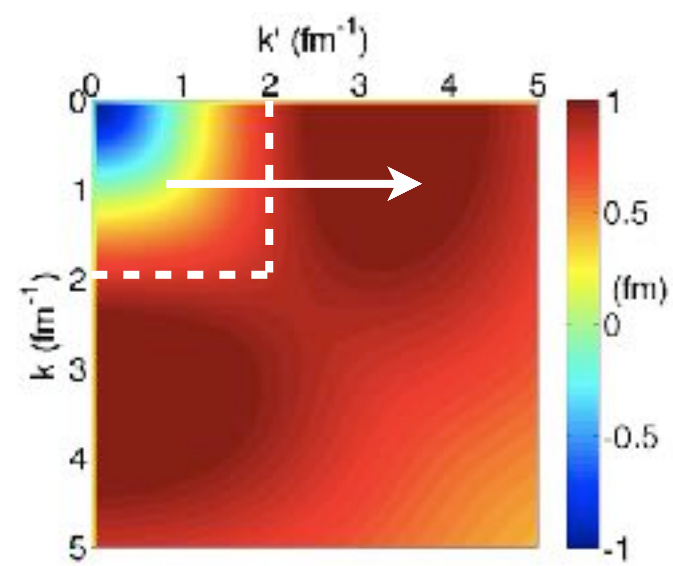
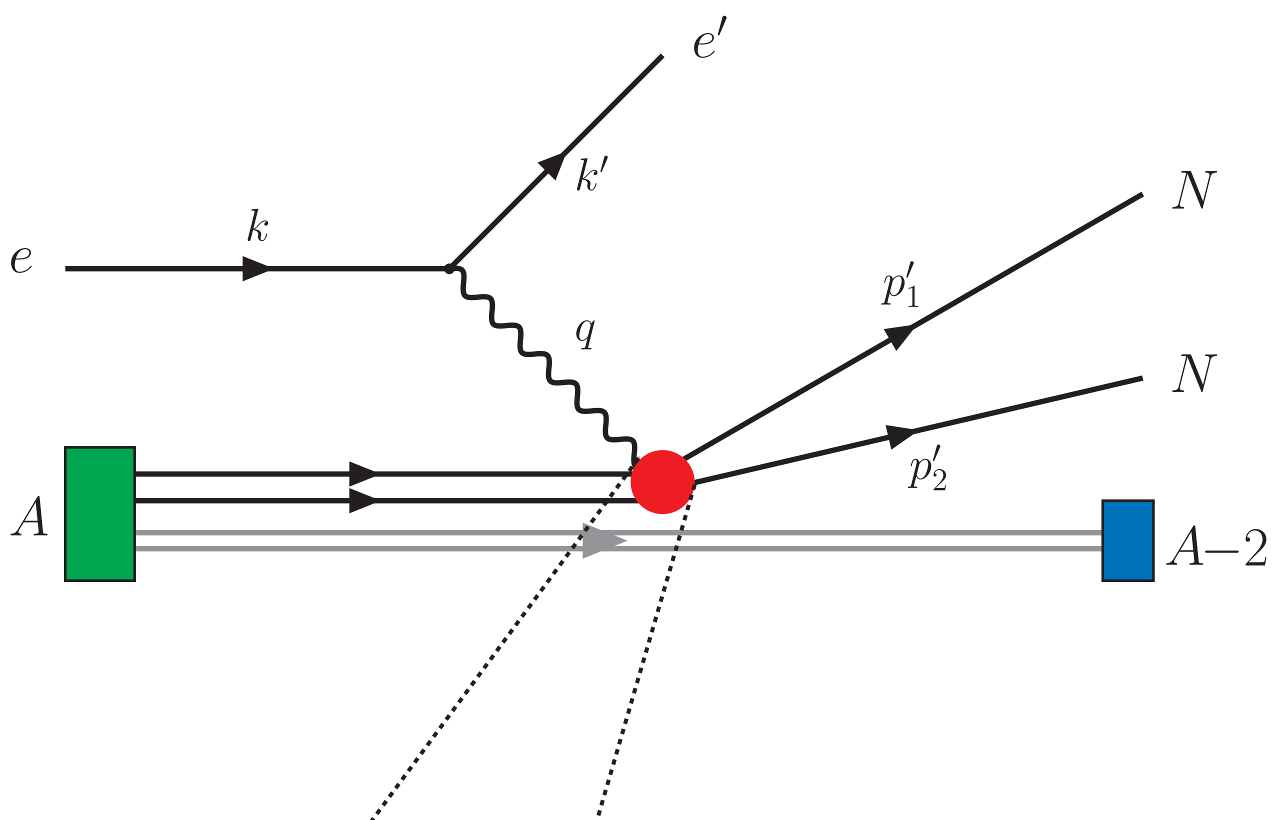
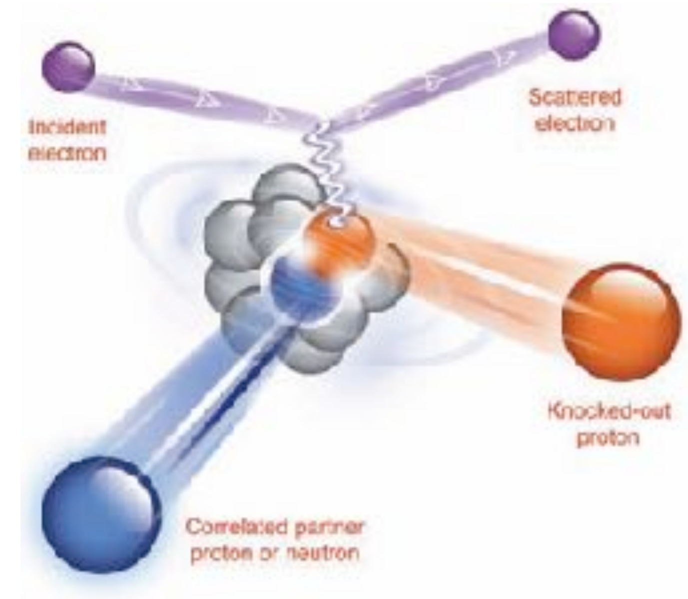




Explanation in terms
of low-momentum interactions?

Application: deep-inelastic knock-out reactions





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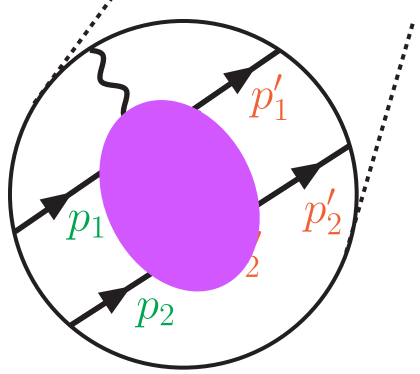
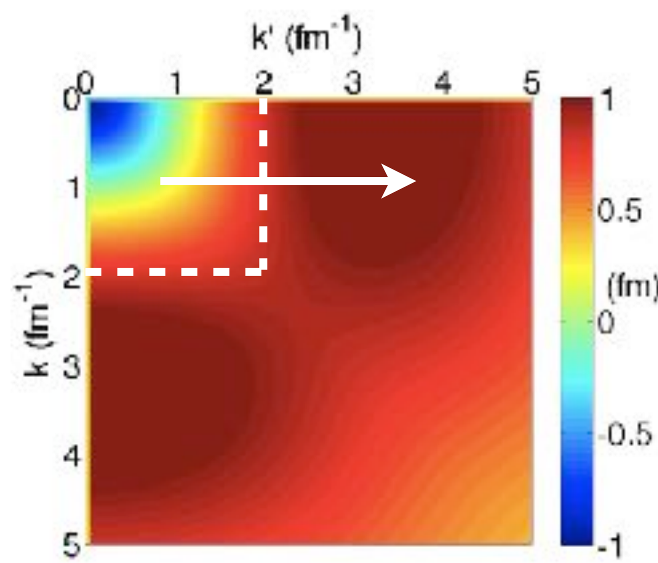




Explanation in terms of low-momentum interactions?

Vertex depends on the resolution!
One-body current and SRC changes to two-body current and simple wave function.

Furnstahl, KH, Rep. Prog. Phys. 76, 126301 (2013)

RG evolution of operators

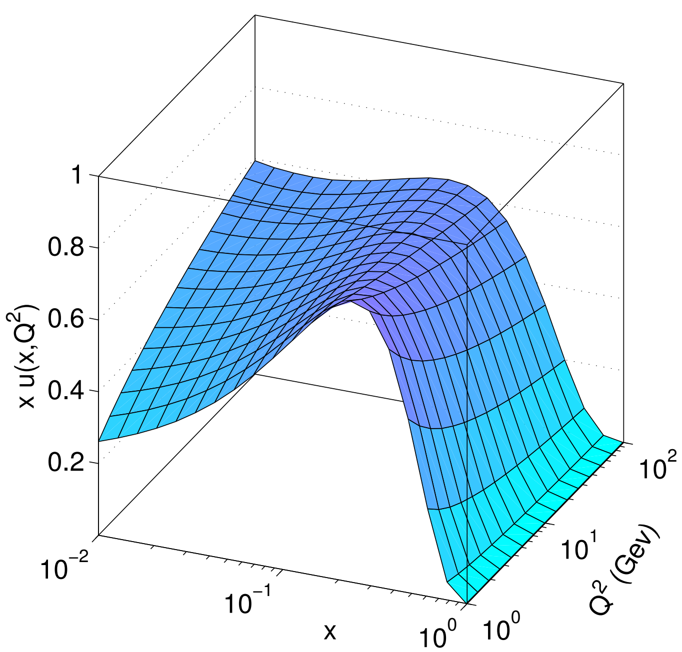
one unitary SRG transformation renormalizes all operators

$$O_\lambda = U_\lambda O U_\lambda^\dagger, \quad \frac{dO_\lambda}{d\lambda} = [\eta_\lambda, O_\lambda]$$

$\Rightarrow$ quantities like momentum distributions are generally scale dependent:

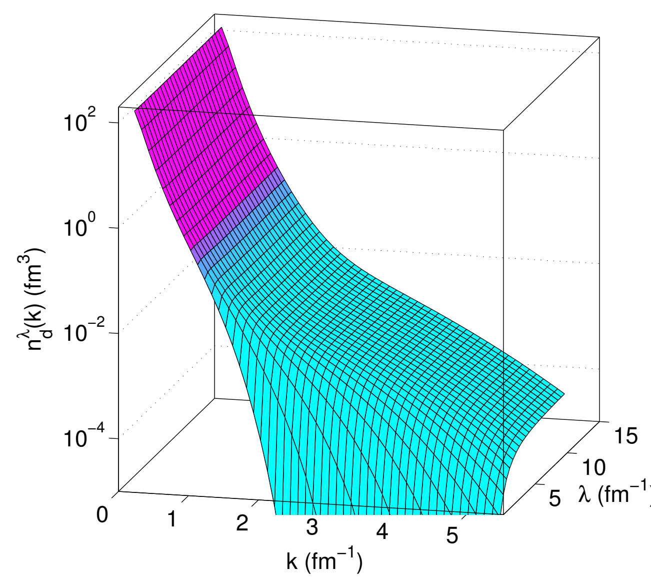
quark parton distribution

$xq(x, Q^2)$



deuteron momentum distribution

$n(k, \lambda)$

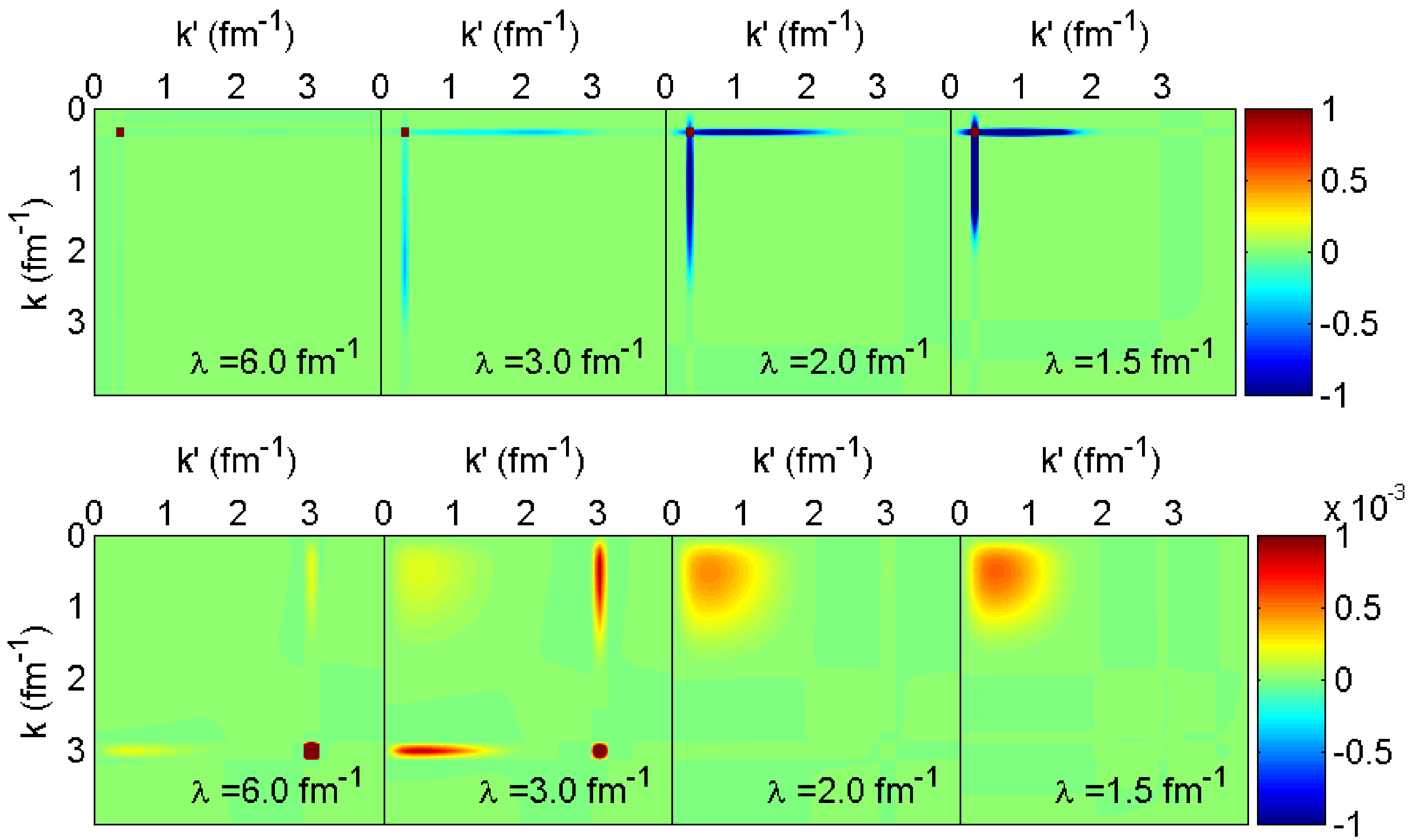


Furnstahl, KH, Rep. Prog. Phys. 76, 126301 (2013)

- applies generally to all quantities like spectroscopic factors, short-range corr.,...
- consistency requires consistent RG evolution of reaction and structure parts
- key for all momenta involving high-momentum components

Evolved density operator in the deuteron

investigate $\langle \psi_D(\lambda) | U(\lambda) a_q^\dagger a_q U^\dagger(\lambda) | \psi_D(\lambda) \rangle$



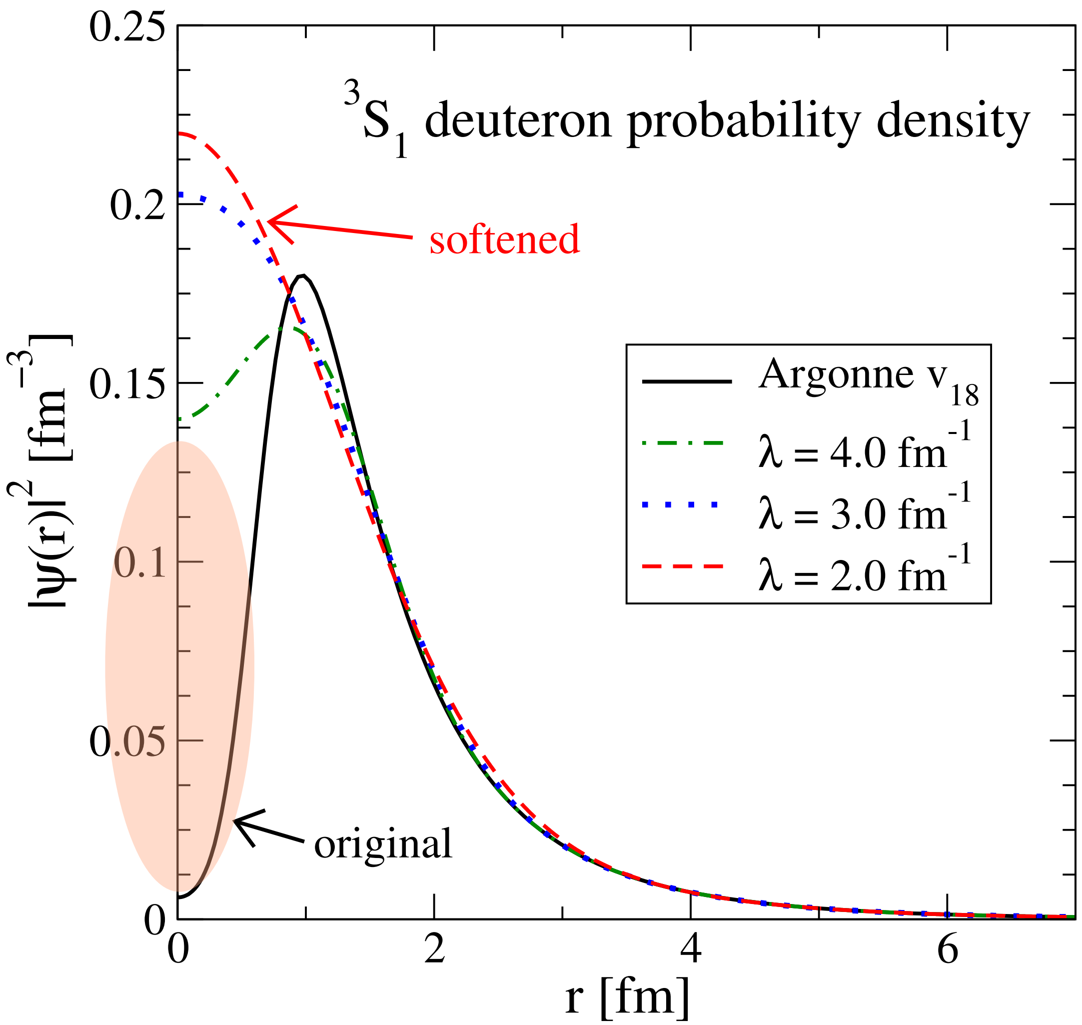
$q = 0.32 \text{ fm}^{-1}$

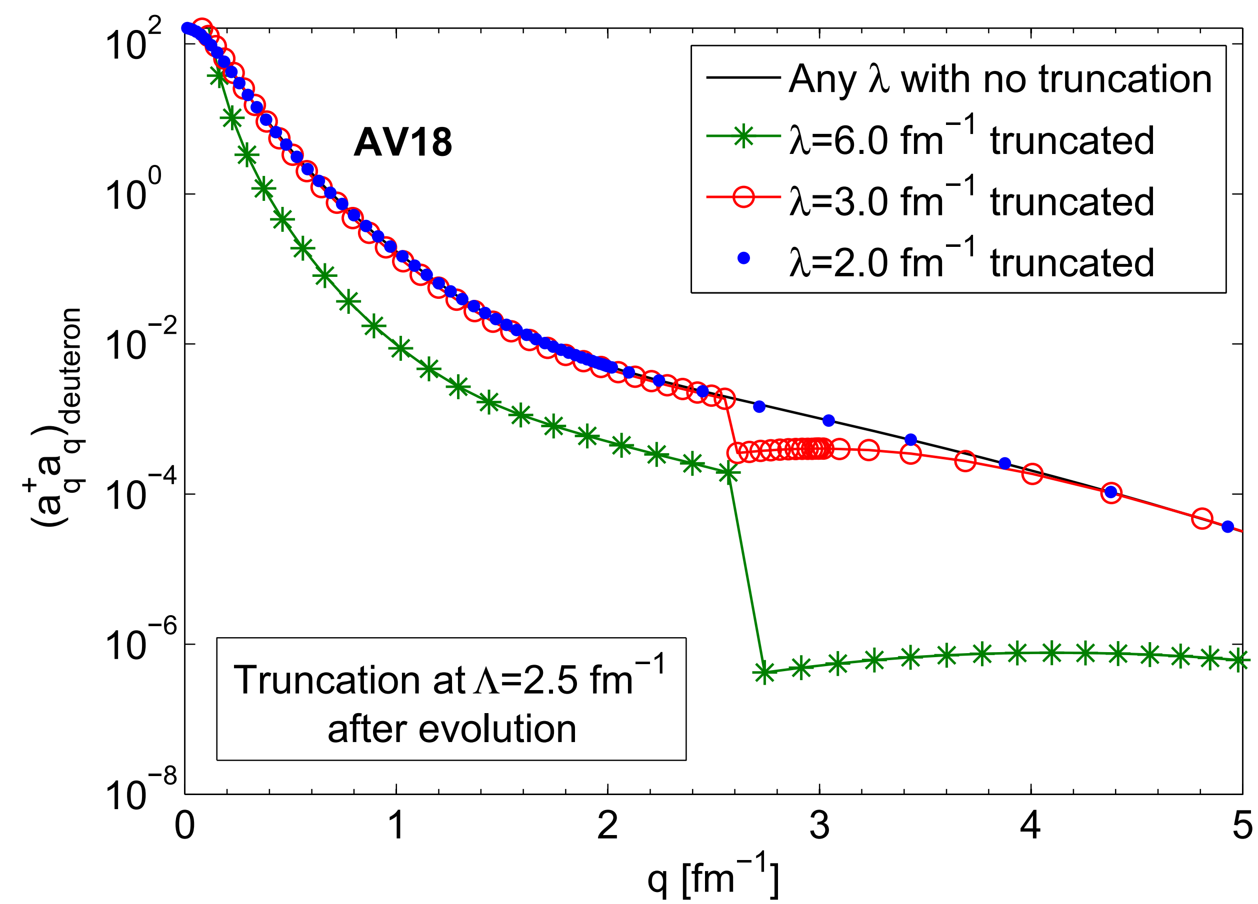
$q = 3.0 \text{ fm}^{-1}$

Anderson, Bogner, Furnstahl, Perry, PRC 82, 054001 (2010)

- for low-momentum operators RG evolution provides only small corrections
- for high-momentum operators induced two-body contributions at small momenta completely dominate contribution at small resolution scales

Evolved density operator in the deuteron

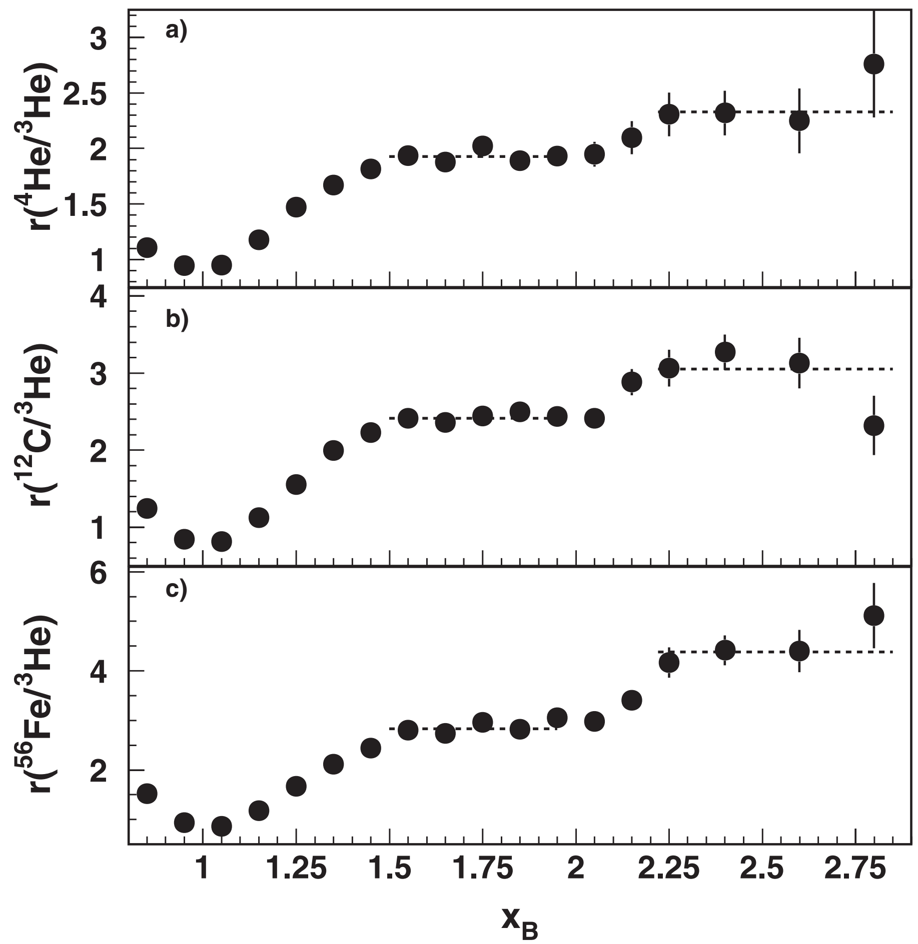




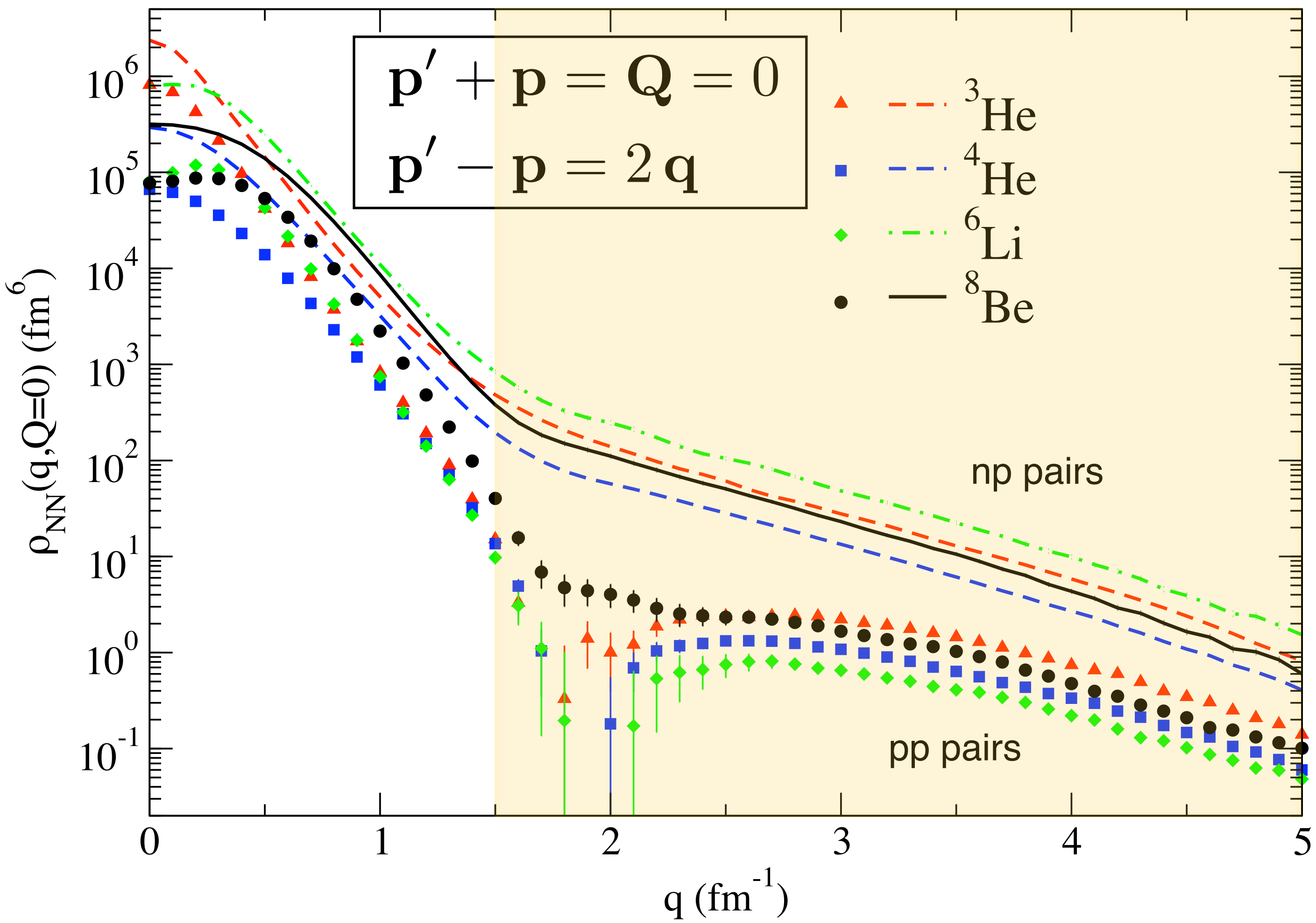
Anderson, Bogner, Furnstahl, Perry, PRC 82, 054001 (2010)

- short-distance correlations in wave function are resolution dependent!
- perfect invariance of momentum distribution function with consistently evolved density operator
- $U_\lambda(k,q)$ factorizes for $k < \lambda$ and $q \gg \lambda$: $U_\lambda(k,q) \approx K_\lambda(k) Q_\lambda(q)$

Scaling in nuclear systems



Egiyan et al. PRL 96, 1082501 (2006)



Schiavilla et al., PRL 98, 132501 (2007)

- scaling behavior of momentum distribution function:

$$\rho_{\rm NN}(q, Q=0) \approx C_A \times \rho_{\rm NN,Deuteron}(q, Q=0) \quad \text{at large} \quad q$$

- dominance of np pairs over pp pairs
- “hard” (high resolution) interaction used
- dominance explained by short-range tensor forces

Nuclear scaling at low resolution

$\langle\psi_\lambda|O_\lambda|\psi_\lambda\rangle$ **factorizes** into a low-momentum structure and a **universal** high momentum part if the initial operator only weakly couples low and high momenta $\longrightarrow$ explains scaling

key: $U_\lambda(k,q) \approx K(k)Q(q)$ for $k < \lambda$ and $q \gg \lambda$

That leads to:

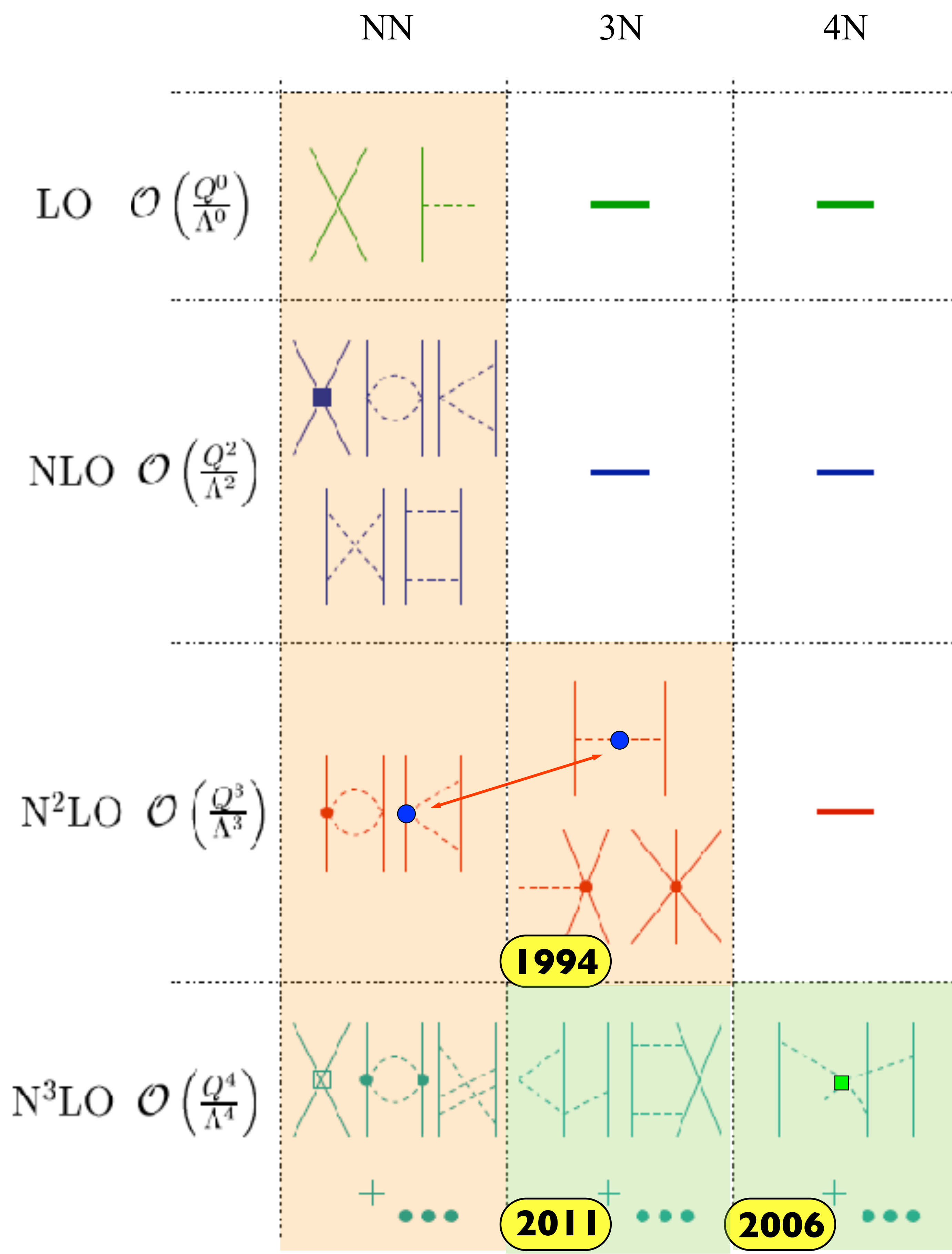
$$\begin{aligned}\langle\psi_\lambda|\,O_\lambda\,|\psi_\lambda\rangle &= \int_0^\lambda dk\,dk' \int_0^\infty dq\,dq'\,\psi^\dagger(k)U_\lambda(k,q)O(q,q')U_\lambda(q',k')\,\psi_\lambda(k') \\ &\approx \int_0^\lambda dk\,dk'\,\psi_\lambda^\dagger(k)\left[\int_0^\lambda dq\,dq'\,K(k)K(q)O(q,q')K(q')K(k') + I_{QOQ}K(k)K(k')\right]\psi_\lambda(k')\end{aligned}$$

with the **universal** quantity:

$$I_{QOQ} = \int_\lambda^\infty dq\,dq'\,Q(q)O(q,q')Q(q')$$

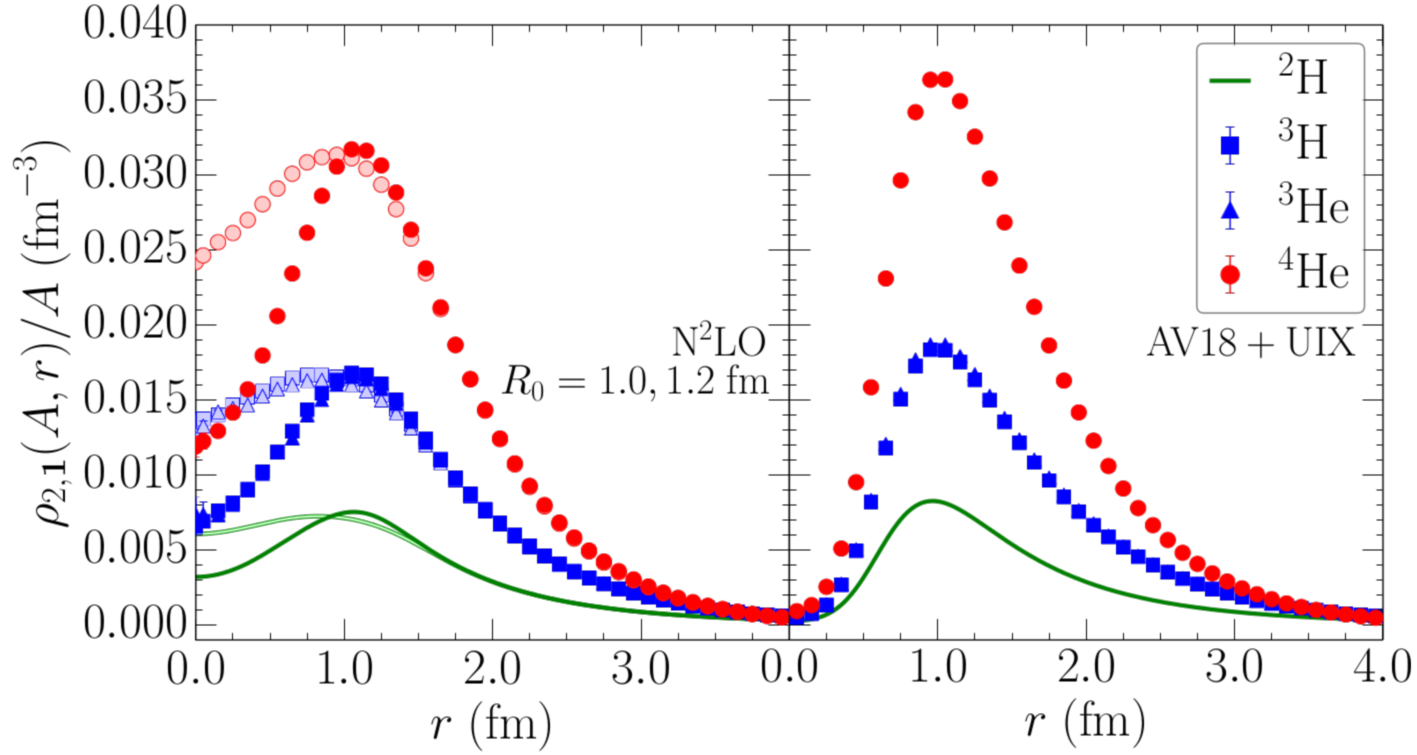
Chiral EFT for nuclear forces and currents

- choose relevant degrees of freedom: here nucleons and pions
- operators constrained by symmetries of QCD
- short-range physics captured in few short-range couplings
- separation of scales: $Q << \Lambda_b$, breakdown scale Λ_b~500 MeV
- power-counting: expand in powers Q/Λ_b
- systematic: work to desired accuracy, obtain error estimates



Nuclear scaling within chiral EFT

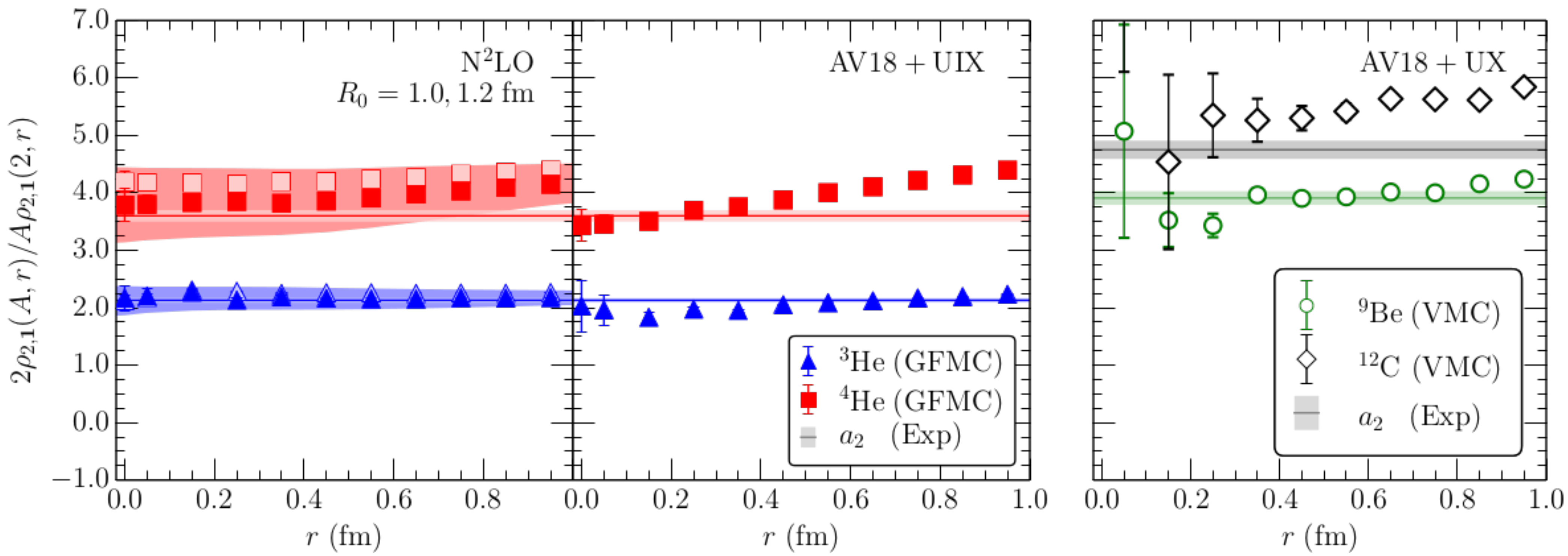
$$P \gg \Lambda_b \gg m_\pi \Rightarrow F_2^A(x, Q^2) \sim g_2(A, \Lambda) f_2(x, Q^2, \Lambda)$$



two-body distribution functions scheme and scale dependent!

Chen, Detmold,
PLB 625, 165 (2005)

Chen, Detmold, Lynn, Schwenk,
PRL 119, 262502 (2017)



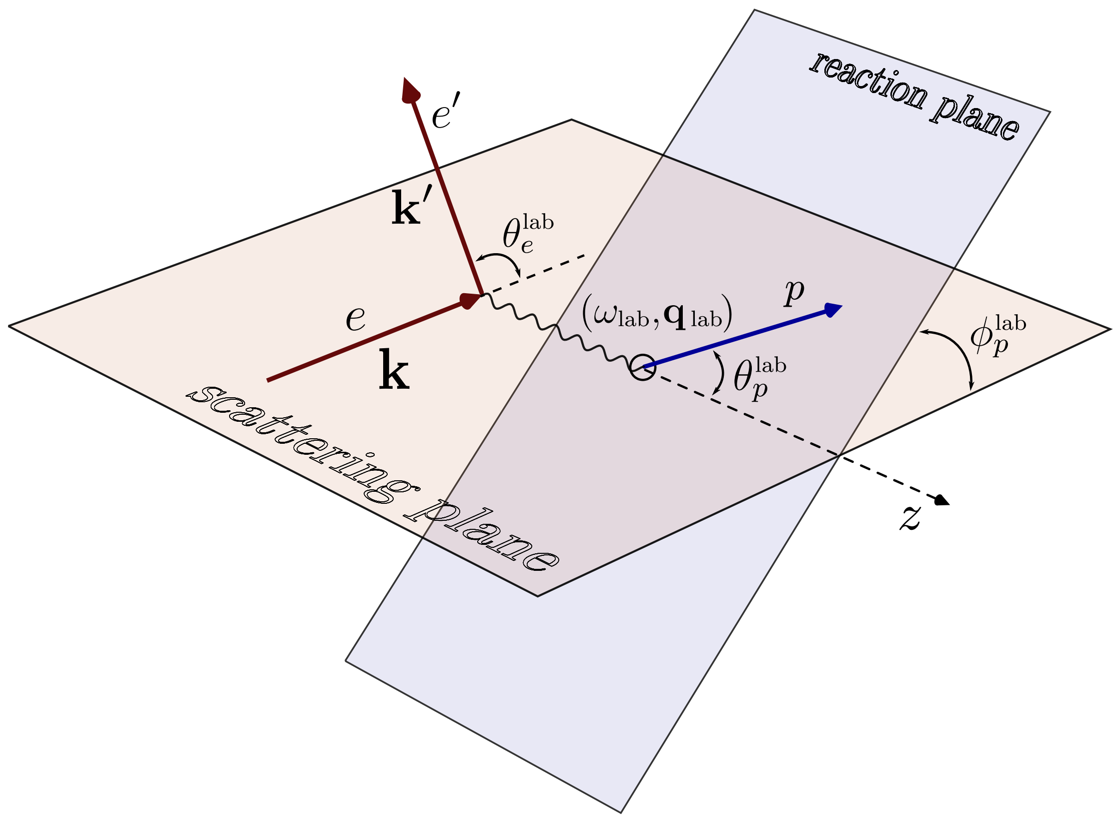
...ratios are not!

SRC Correlation Factors

Detailed comparison of experiment and theory

	N^2LO (R_0 = 1.0 – 1.2 fm)	AV18+UIX	Exp
^3H	2.1(2) – 2.3(3)	2.0(4)	
^3He	2.1(2) – 2.1(3)	2.0(4)	2.13(4)
^4He	3.8(7) – 4.2(8)	3.4(3)	3.60(10)

Deuteron disintegration at low resolution scales



More, König, Furnstahl, KH,
PRC 92, 064002 (2015)

decomposition of differential cross section:

$$\frac{d^3\sigma}{dk'^{\text{lab}} d\Omega_e^{\text{lab}} d\Omega_p^{\text{lab}}} \sim v_L f_L + v_T f_T + v_{TT} f_{TT} \cos 2\phi_p^{\text{lab}} + v_{LT} f_{LT} \cos \phi_p^{\text{lab}}$$

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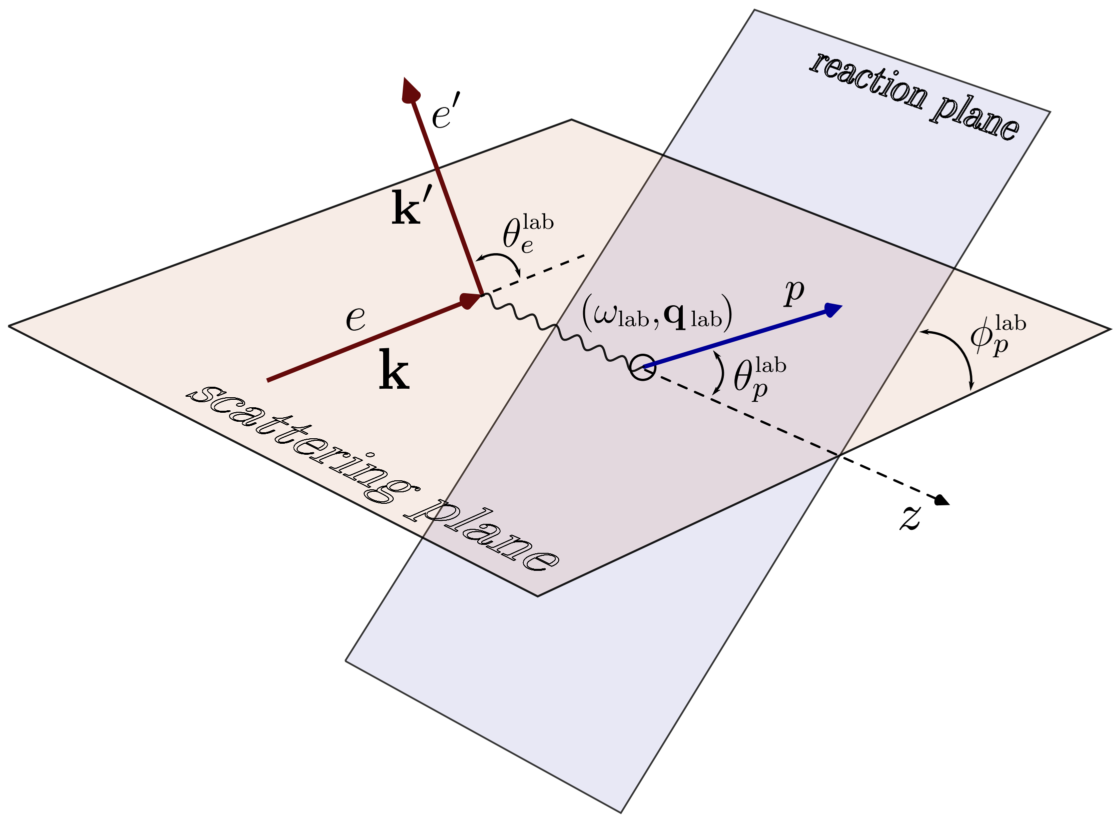

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kinematic factors

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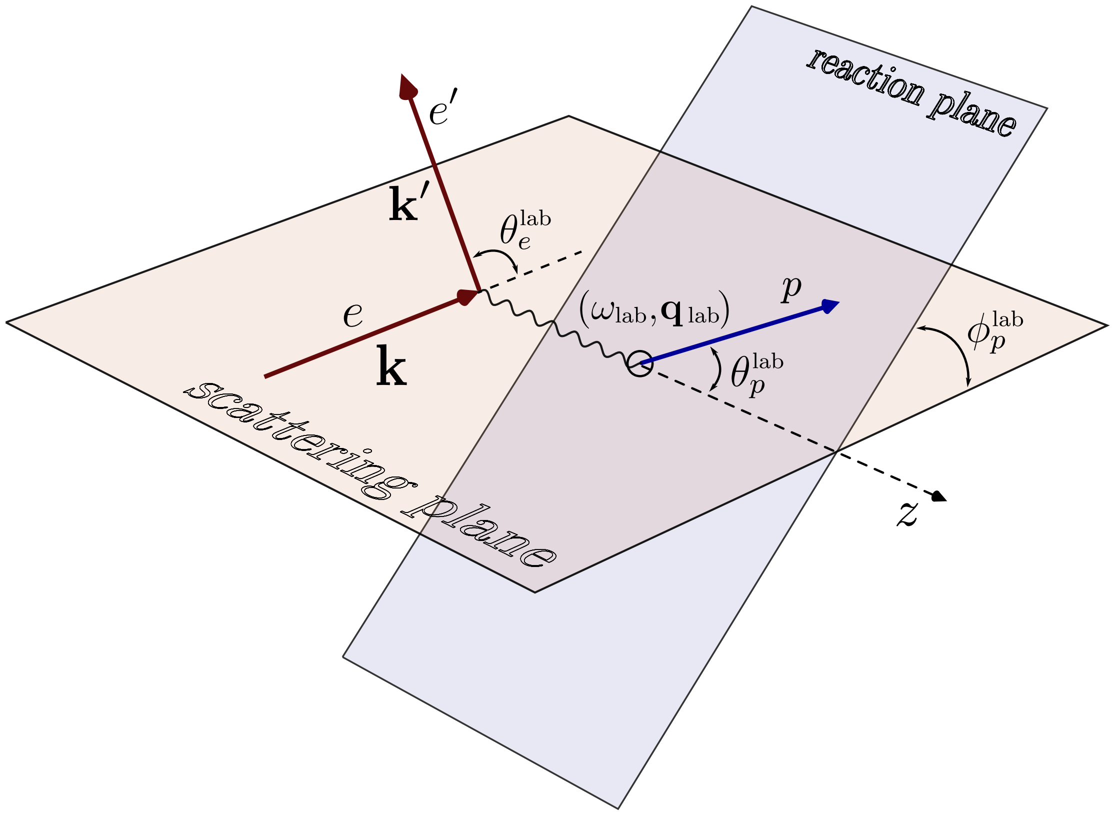

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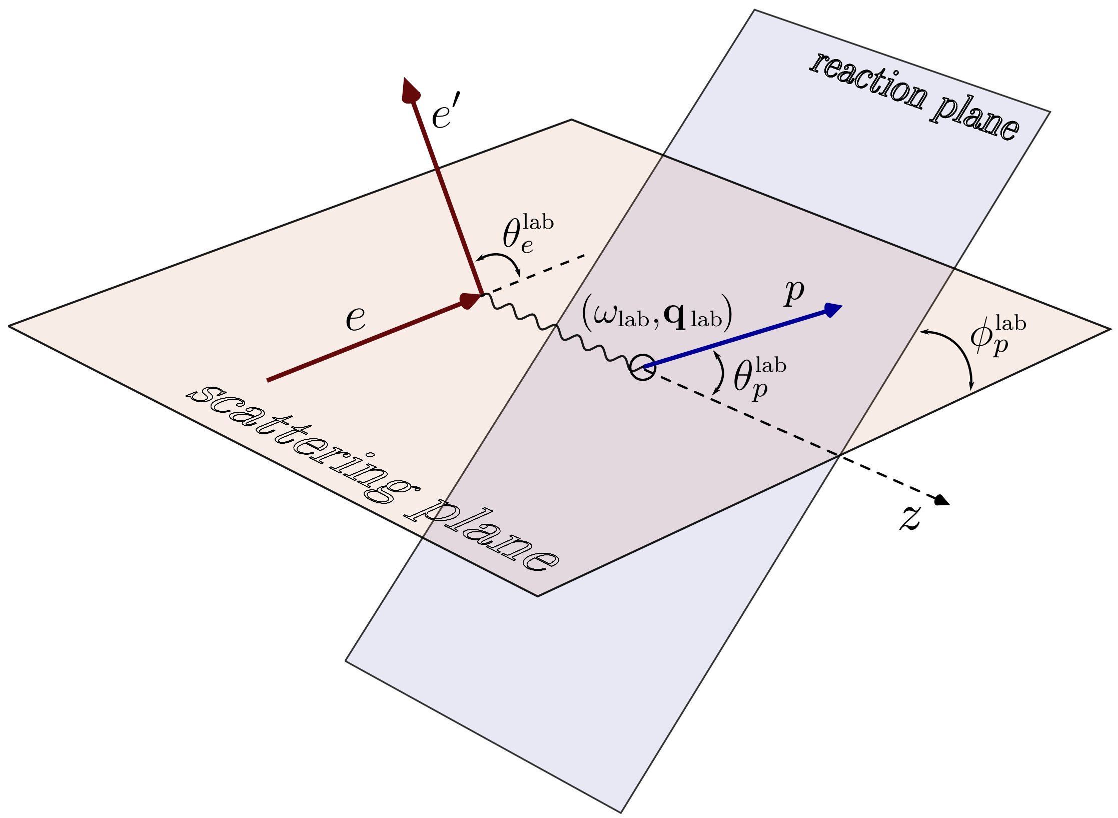
kinematic factors

structure functions: observables (cross sections
up to kinematical factors)

Deuteron disintegration at low resolution scales

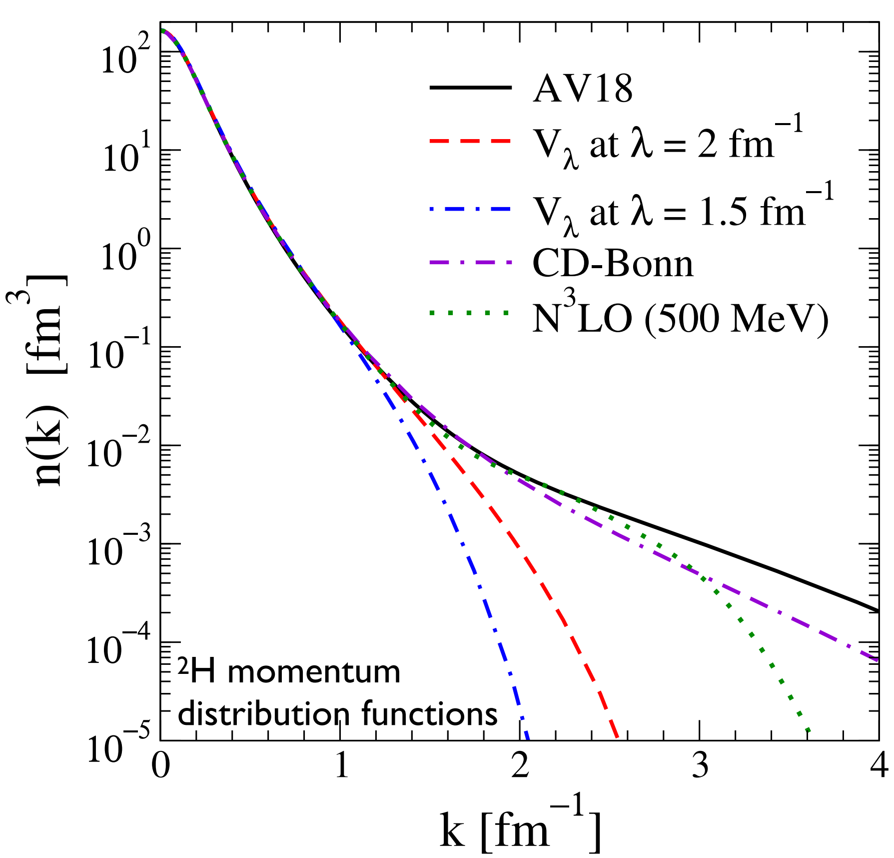


More, König, Furnstahl, KH,
PRC 92, 064002 (2015)

consider initial one-body current:

$$\langle \mathbf{k}_1\, T_1 |\, J_0(\mathbf{q})\, | \,\mathbf{k}_2\, T = 0\rangle = \frac{1}{2}\left(G_E^p + (-1)^{T_1} G_E^n\right)\delta(\mathbf{k}_1 - \mathbf{k}_2 - \mathbf{q}/2) + \frac{1}{2}\left((-1)^{T_1} G_E^p + G_E^n\right)\delta(\mathbf{k}_1 - \mathbf{k}_2 + \mathbf{q}/2)$$

$$\langle \psi_f |\, J_0\, | \psi_i\rangle = \underbrace{\langle \phi |\, J_0\, | \psi_i\rangle}_{\text{IA}} + \underbrace{\langle \phi | t^\dagger\, G_0^\dagger\, J_0\, | \psi_i\rangle}_{\text{FSI}} = \left\langle \psi_f | U^\dagger U J_0 U^\dagger U | \psi_i \right\rangle = \left\langle \psi_f^\lambda | J_0^\lambda | \psi_i^\lambda \right\rangle$$



study longitudinal structure function

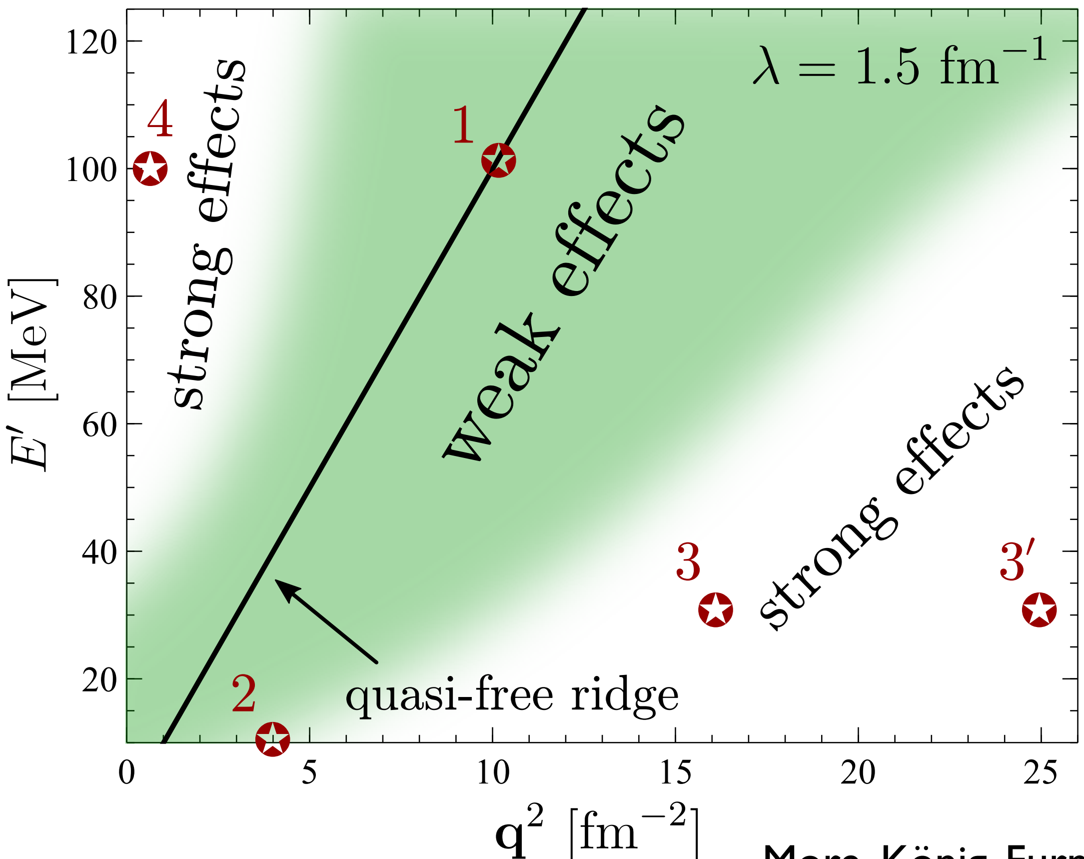
$$f_L(E', \mathbf{q}'^2; \cos\theta') \sim \left|\left\langle \psi_v^\lambda \right|\, J_0^\lambda\, \left| \psi_i^\lambda \right\rangle\right|^2$$

E' : energy of outgoing nucleons (in CM frame)

$\mathbf{q}'$: momentum transfer (in CM frame)

θ' : angle of outgoing nucleons (CM frame)

Deuteron disintegration at low resolution scales



More, König, Furnstahl, KH, PRC 92, 064002 (2015)

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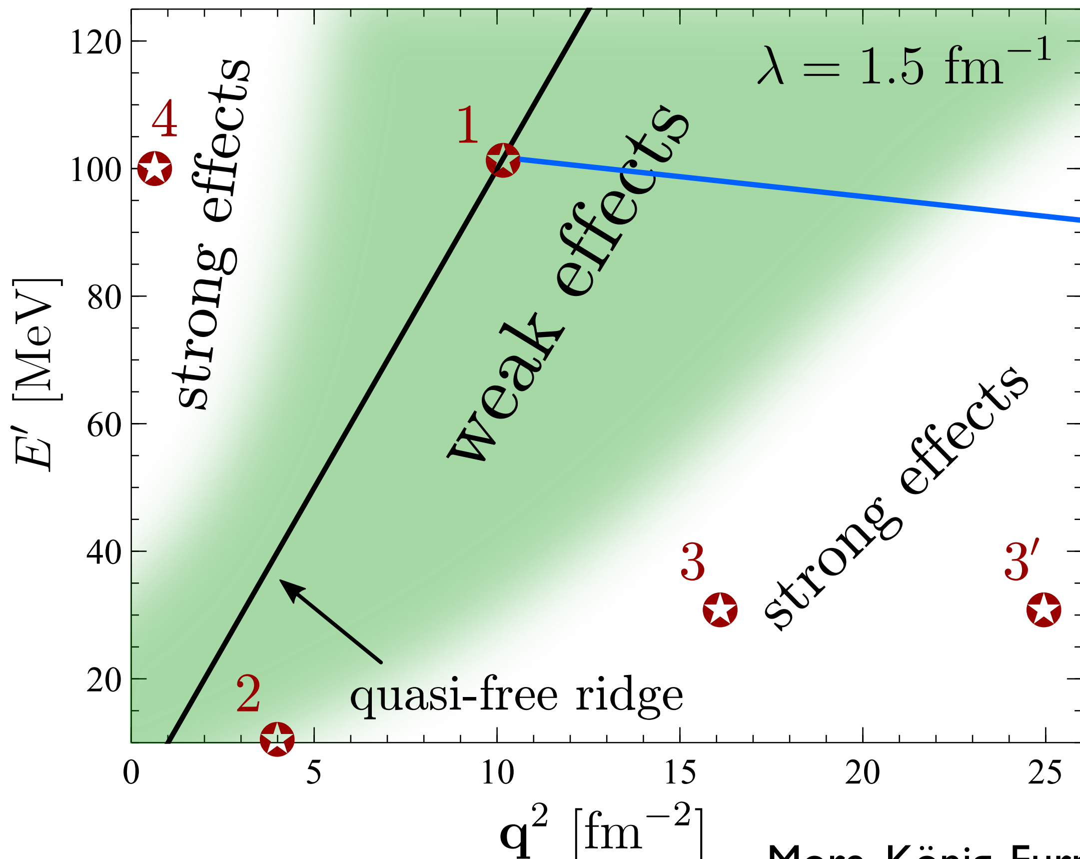

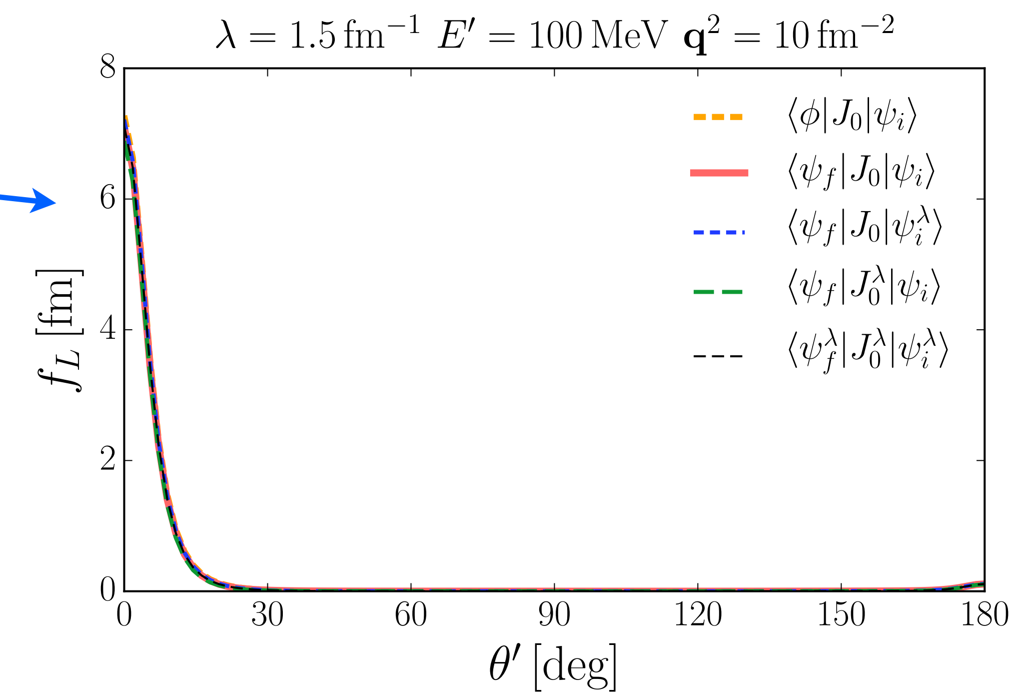

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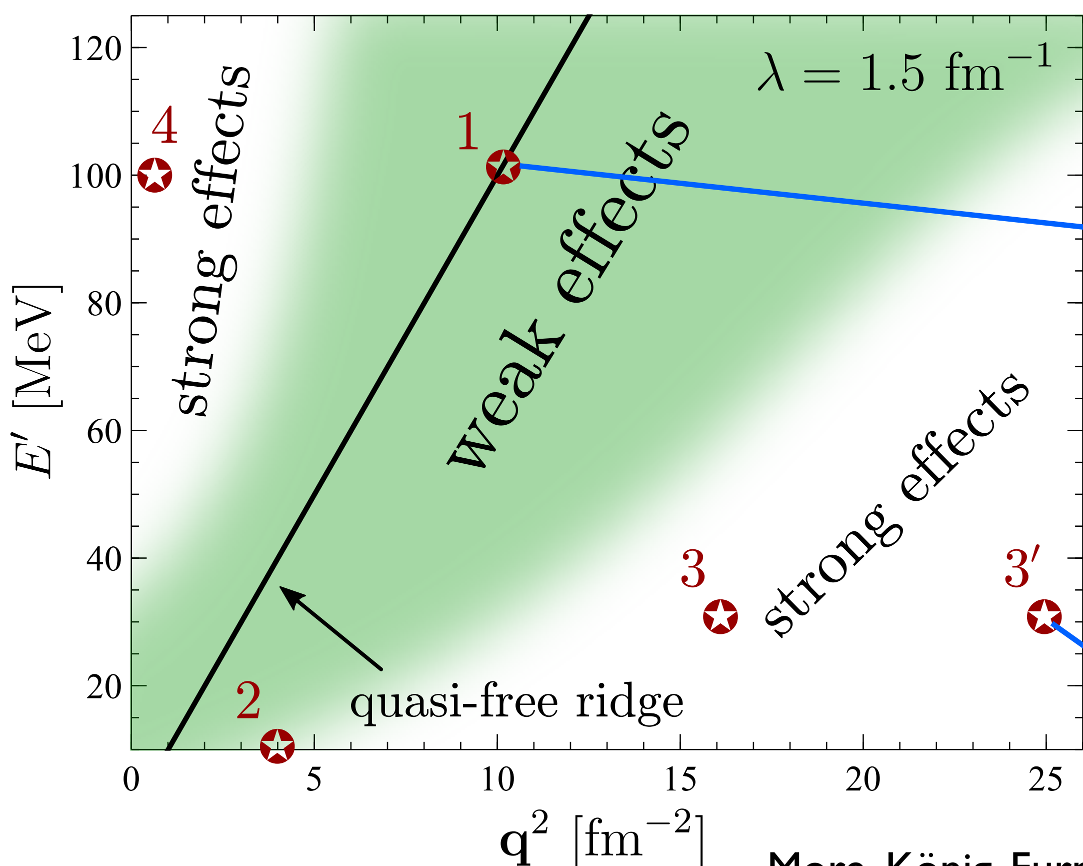

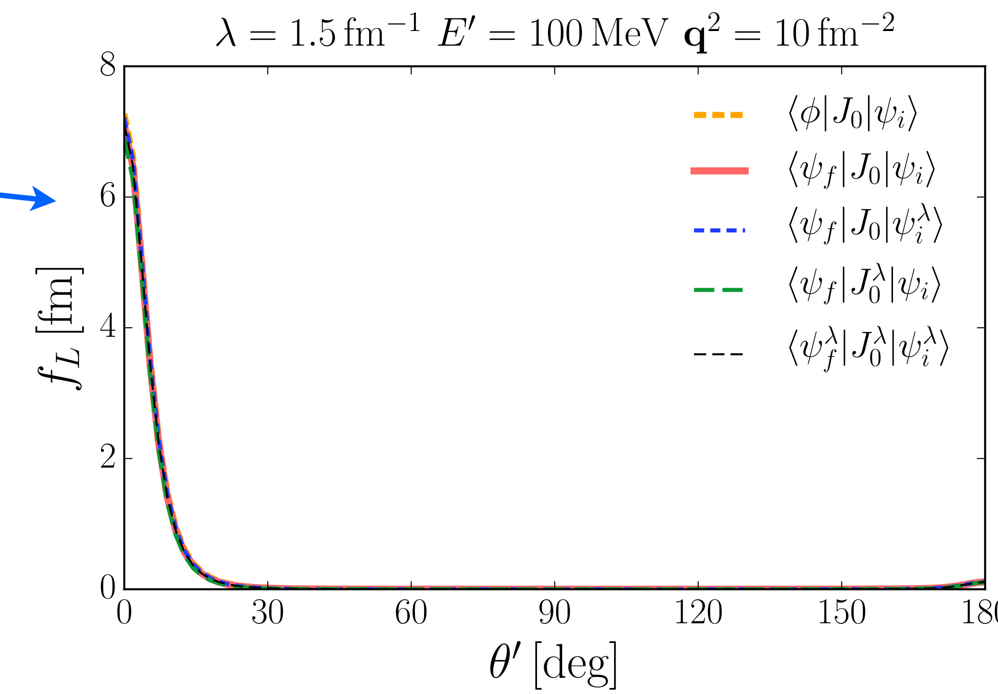

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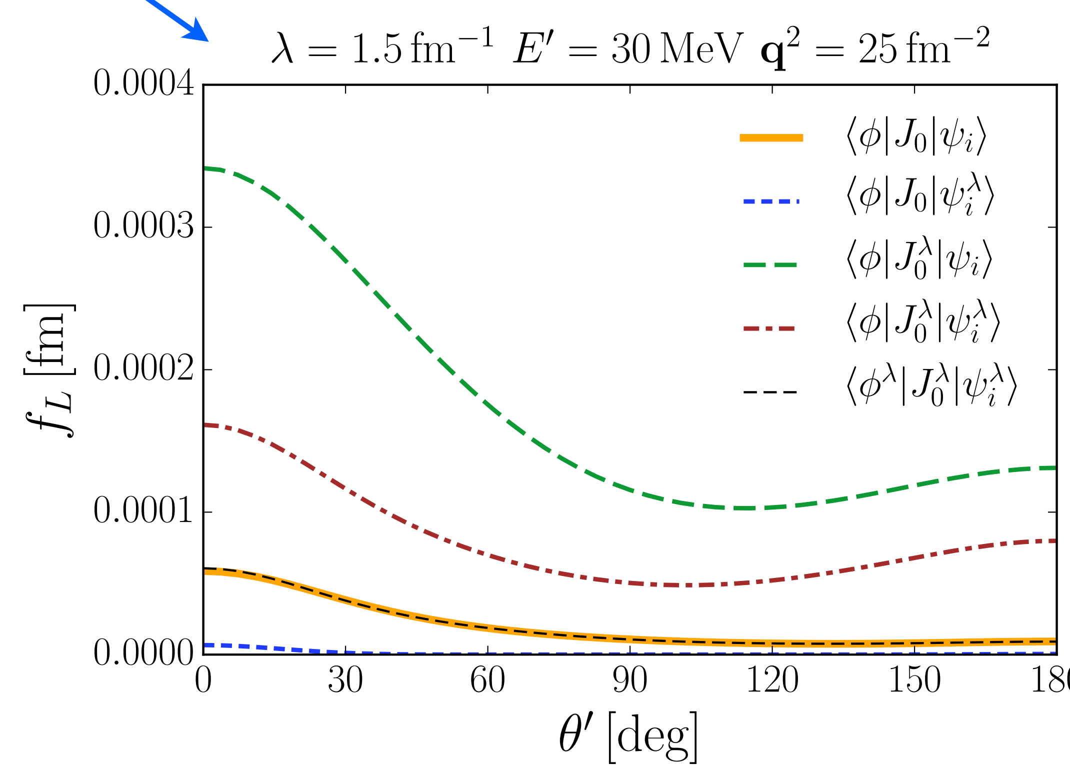

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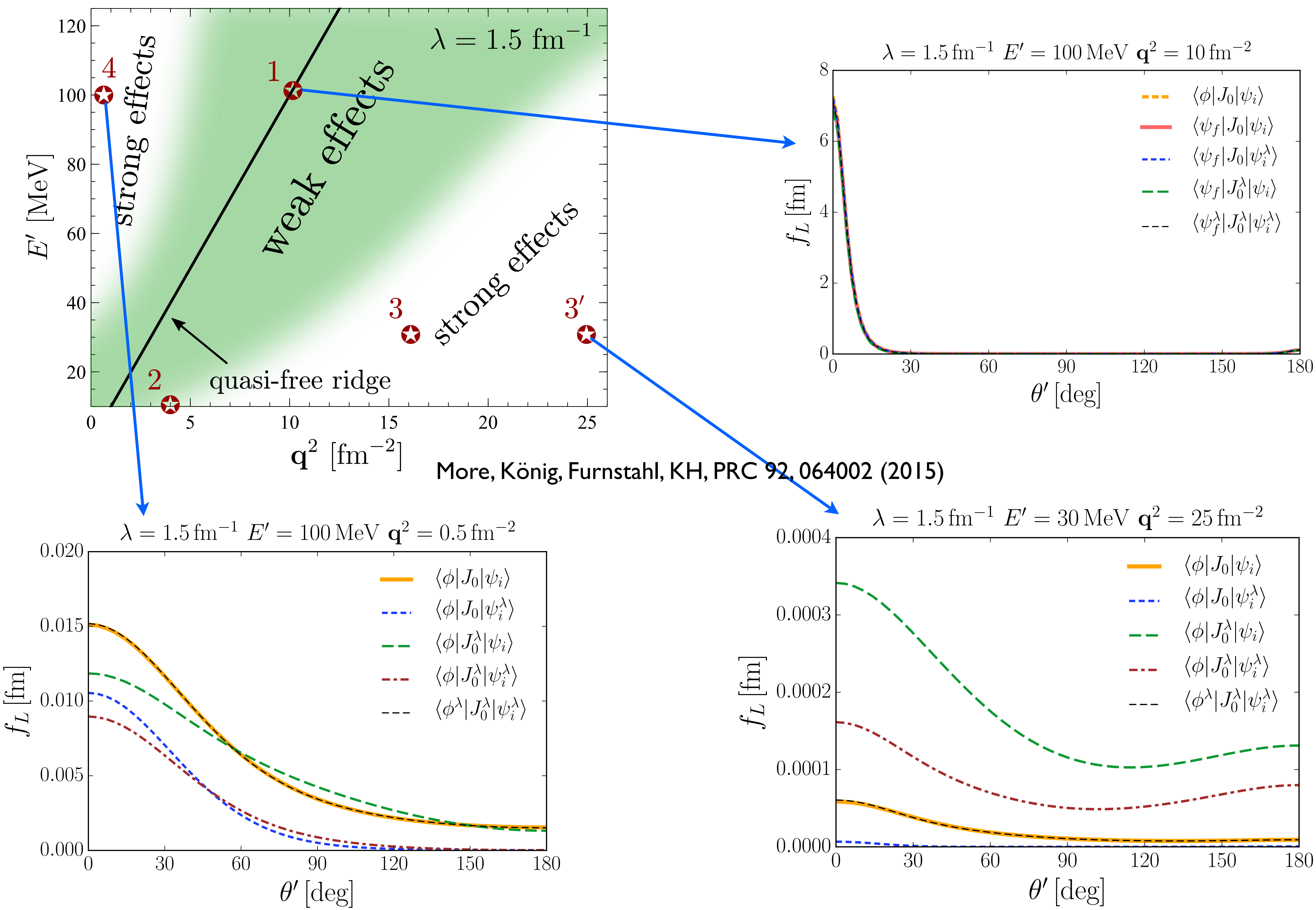

More, König, Furnstahl, KH, PRC 92, 064002 (2015)

Summary

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• theoretical interpretation sensitively depends on the resolution scale

• resolution scale change shifts contributions between structure and reaction

• deep inelastic cross sections usually explained in terms of short-range correlations, scheme dependent, observables can also be explained by separation of scales and factorization

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- ✦ found perfect RG invariance of longitudinal structure function
- ✦ impact of RG evolution strongly depends on kinematics

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Thank you!