SRCs with similarity transforms and Wigner function

Thomas Neff
GSI Helmholtzzentrum für Schwerionenforschung, Darmstadt

Polarized light ion physics with EIC
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Overview

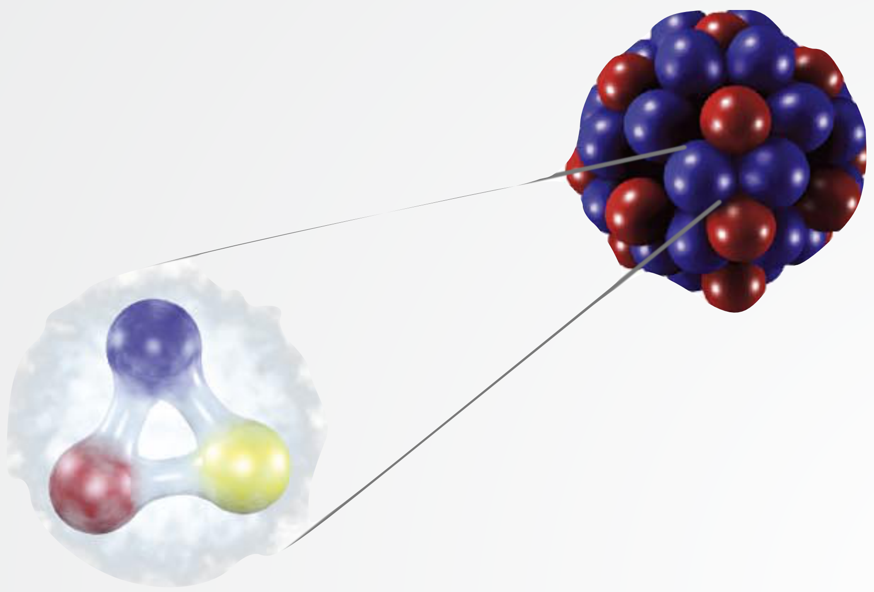
Similarity Transformations

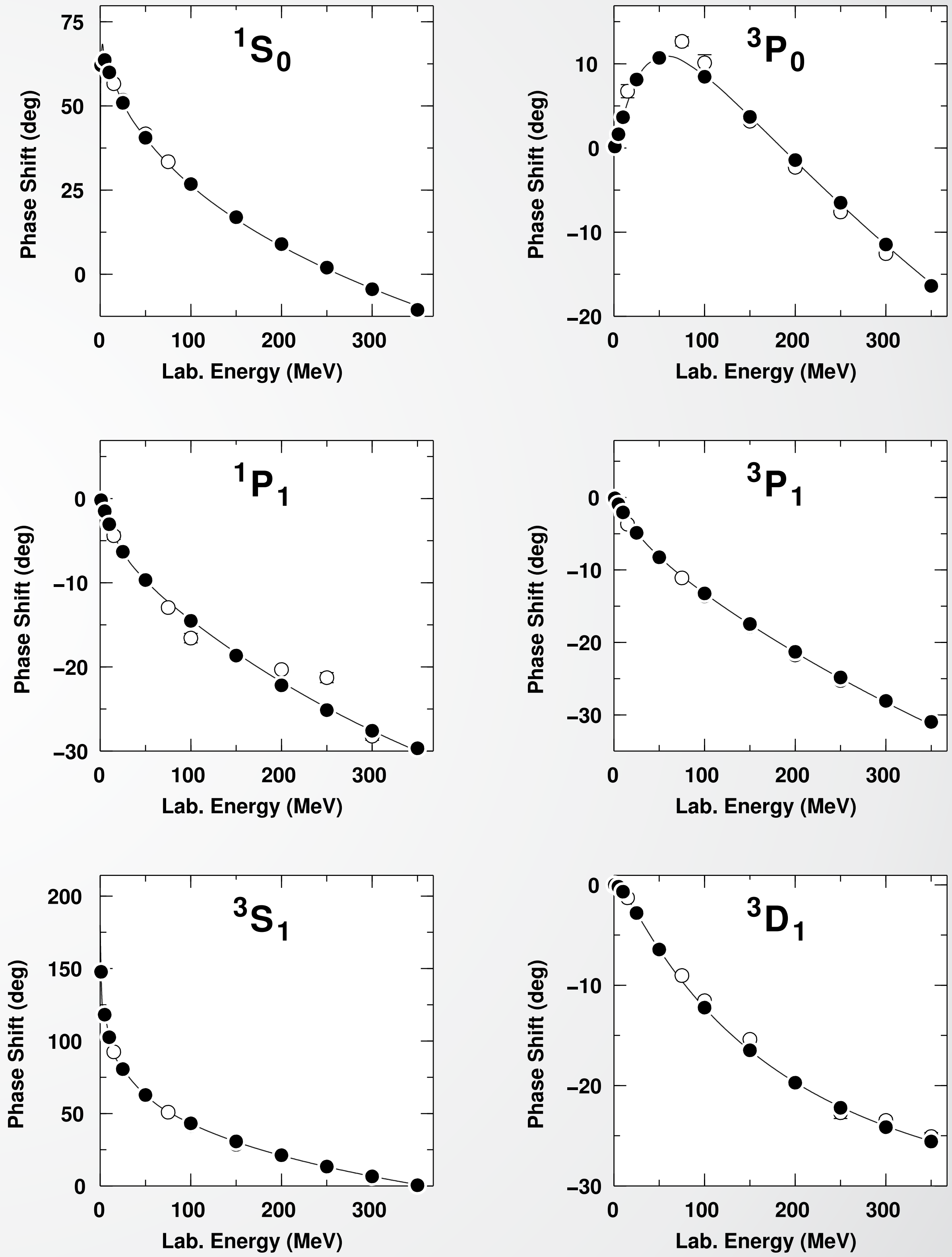
- Soften the NN interaction (AV18/Chiral EFT) with SRG
- Solve many-body problem with NCSM
- Recover short-range physics with SRG transformed operators
- SRG transformation in two-body approximation — comparing two-body densities in ^{4}He obtained with bare and transformed interactions/operators allows to disentangle two- and many-body correlations
- Two-body densities as a function of distance of two nucleons $\rho^{rel}(r)$, relative momentum $n^{rel}(k)$ and as a function of pair and relative momentum n(K,k)
- Dominant role of deuteron-like S=1,T=0 pairs and tensor correlations at high relative momenta (dominance of pn over pp pairs)
- NCSM with SRG transformed operators allows to study SRC in heavier (light) nuclei

Wigner function of the Deuteron

- Wigner function contains the complete information about the system
- Obtain the quantum-mechanical analogue to a phase-space picture
- How are short-distance and high-momentum components connected?

Nucleon-Nucleon Interactions





- Nucleons are not point-like, complicated quark and gluon sub-structure
- Nucleon-nucleon (NN) interaction: residual interaction
- Calculation within QCD not possible yet — construct **realistic NN potentials** …
- describe two-nucleon properties (scattering, Deuteron) with high accuracy
- **high-momentum and off-shell behavior not constrained by scattering data**

Nucleon-Nucleon Interactions

N3LO

- potential derived using chiral EFT
- includes full π dynamics
- power counting
- short-range behavior given by contact-terms
- regulated by non-local cut-off (500 MeV)

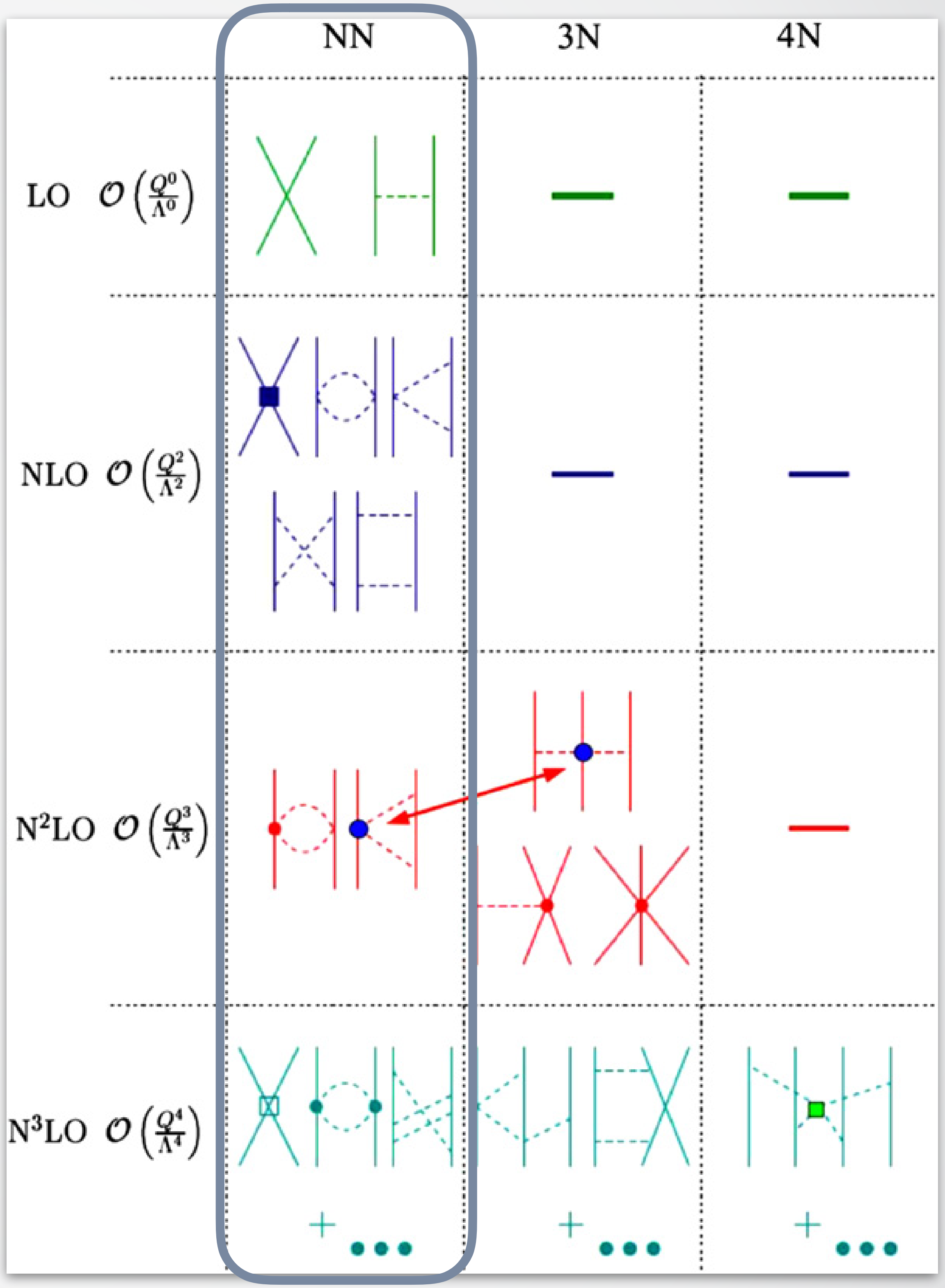
Entem, Machleidt, Phys. Rev. C **68**, 041001 (2003)

new
developments in chiral EFT
➜ talk by Hermann Krebs

Argonne V18/V8'

- π-exchange, phenomenological short-range
- "as local as possible"
- fitted to phase shifts up to 350 MeV, but describes elastic phase shifts up to 1 GeV

Wiringa, Stoks, Schiavilla, Phys. Rev. C **51**, 38 (1995)

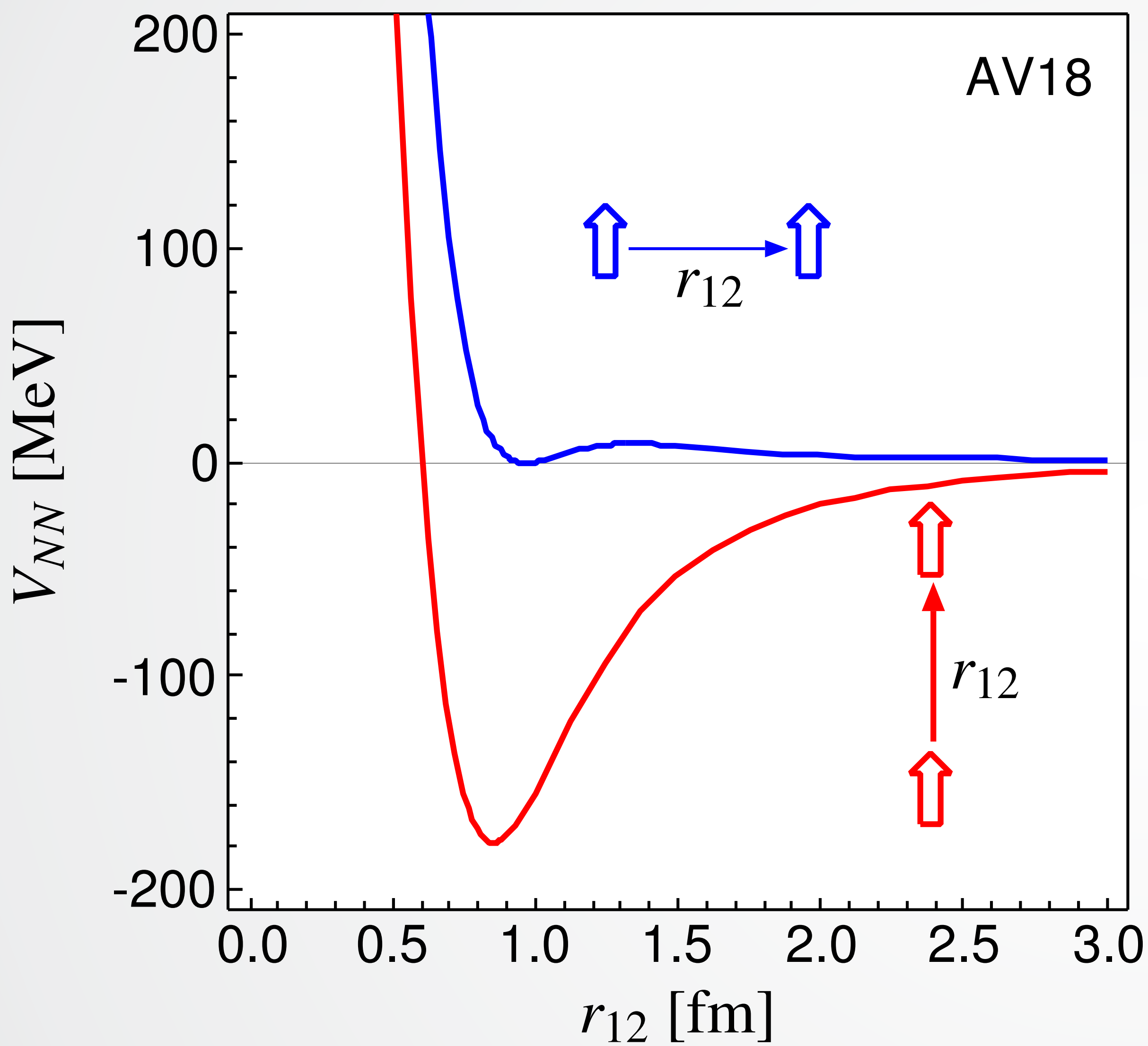


Bogner, Furnstahl, Schwenk, Prog. Part. Nucl. Phys. **65**, 94 (2010)

Nucleon-Nucleon Interaction

S=1, T=0

$$S_{ij}(\hat{\mathbf{r}},\hat{\mathbf{r}}) = 3(\boldsymbol{\sigma}_i\cdot\hat{\mathbf{r}})(\boldsymbol{\sigma}_j\cdot\hat{\mathbf{r}}) - (\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j)$$



- **repulsive core**: nucleons can not get closer than ≈ 0.5 fm
- strong dependence on the orientation of the spins due to the **tensor force** (mainly from π-exchange)
- the nuclear force will induce strong short-range correlations in the nuclear wave function

π-Exchange

$$V_\pi(\mathbf{q}) = -\frac{f_{\pi NN}^2}{m_\pi^2}(\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j)\frac{(\boldsymbol{\sigma}_i\cdot\mathbf{q})(\boldsymbol{\sigma}_j\cdot\mathbf{q})}{m_\pi^2+q^2}$$

$$V_\pi(\mathbf{r}) = \frac{1}{3}m_\pi\frac{f_{\pi NN}^2}{4\pi}(\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j)\left[(\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j) + S_{ij}(\hat{\mathbf{r}},\hat{\mathbf{r}})\left(1+\frac{3}{m_\pi r}+\frac{3}{(m_\pi r)^2}\right)\right]\frac{e^{-m_\pi r}}{m_\pi r}$$

Short-range correlations in nuclei using No-Core Shell Model and SRG

Short-range correlations with “soft” interactions

Neff, Feldmeier, Horiuchi, Phys. Rev. C **92**, 024003 (2015)

Unitary Transformations

- Many-body problem very hard to solve for bare interaction
- Universality of short-range correlations motivates the use of **unitary transformations** to obtain a "soft" realistic interaction ($V_{\text{low-k}}$, UCOM, SRG, ...)

$$\hat{H}_{eff} = \hat{U}^{\dagger}\hat{H}\hat{U}$$

- The **transformation is done in *N*-body approximation**

$$\hat{H}_{eff} = \hat{T} + \hat{V}^{[2]}_{eff} + \ldots \hat{V}^{[N]}_{eff}$$

  and is therefore unitary only up to the *N*-body level
- Deuteron binding energy and *NN* phase shifts are conserved
- **Not only the Hamiltonian, all operators, including density operators, have to be transformed**

$$\hat{B}_{eff} = \hat{U}^{\dagger}\hat{B}\hat{U} = \hat{B}^{[1]} + \hat{B}^{[2]}_{eff} + \ldots \hat{B}^{[N]}_{eff}$$

- SRG operator evolution studied for Deuteron

Anderson, Bogner, Furnstahl, Perry, Phys. Rev. C **82**, 054001 (2010)

- SRG operator evolution for radius and Gaussian two-body operator on 3-body level

Schuster, Quaglioni, Johnson, Jurgenson, Navrátil, Phys. Rev. C **90**, 011301 (2014)

Similarity Renormalization Group

- SRG provides a family of similarity transformations depending on a flow parameter α
- Evolve Hamiltonian and unitary transformation matrix (momentum space)

$$\frac{d\hat{H}_\alpha}{d\alpha} = \left[\hat{\eta}_\alpha, \hat{H}_\alpha\right]_- , \qquad \frac{d\hat{U}_\alpha}{d\alpha} = -\hat{U}_\alpha \hat{\eta}_\alpha$$

- Intrinsic kinetic energy as metagenerator

$$\hat{\eta}_\alpha = (2\mu)^2 \left[\hat{T}_{\text{int}}, \hat{H}_\alpha\right]_-$$

- Evolution is done here on the **2-body level** – α-dependence can be used to investigate the role of missing higher-order contributions
- Hamiltonian evolution can nowadays be done on the 3-body level

(Jurgenson, Roth, Hebeler, . . .)

Bogner, Furnstahl, Perry, Phys. Rev. C, **75**, 061001 (2007)

Roth, Neff, Feldmeier, Prog. Part. Nucl. Phys. **65**, 50 (2010)

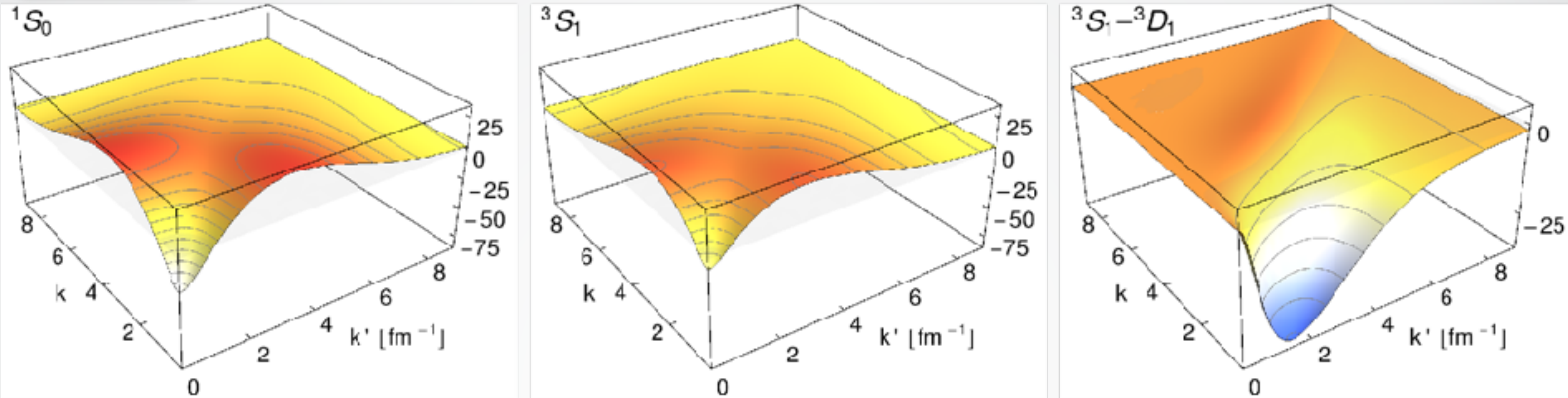
α=0:
bare interaction
fully correlated wave function

↓

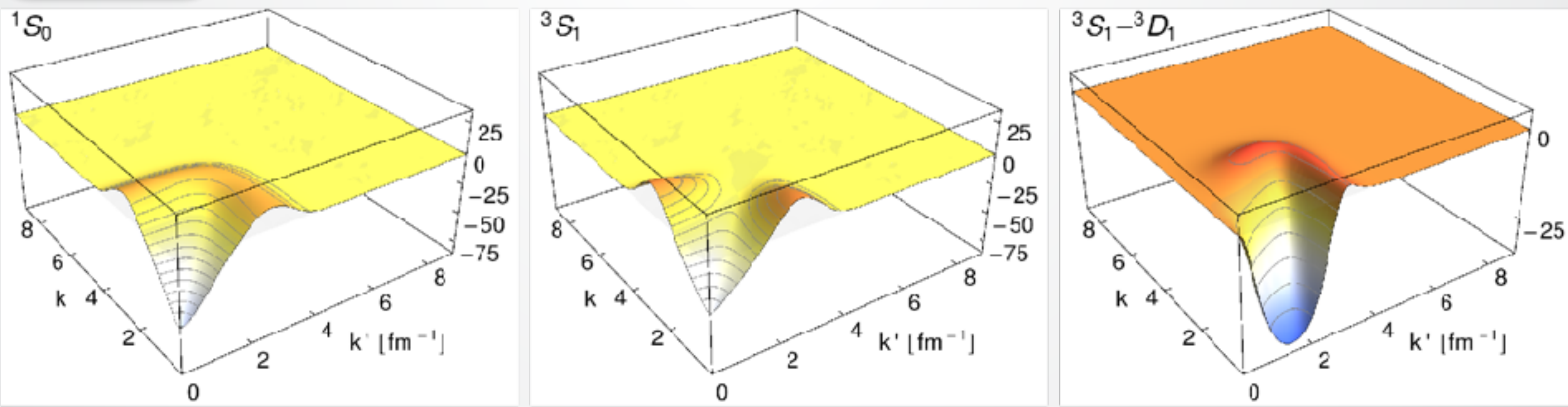
α large:
mean-field like wave function
with **pairwise correlations**

Similarity Renormalization Group

AV8′



N3LO

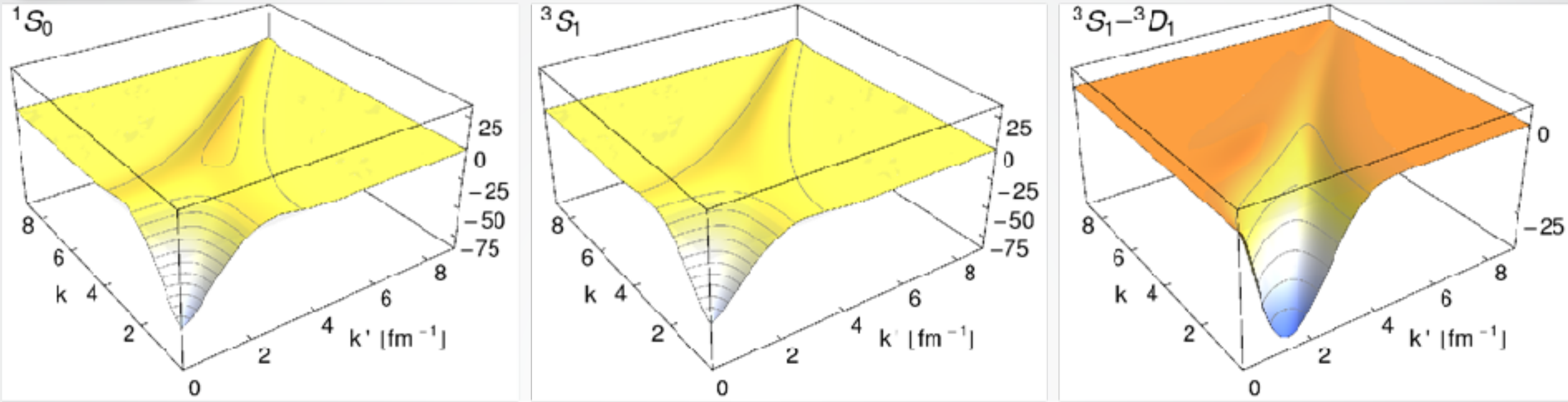


α=0.00 fm⁴

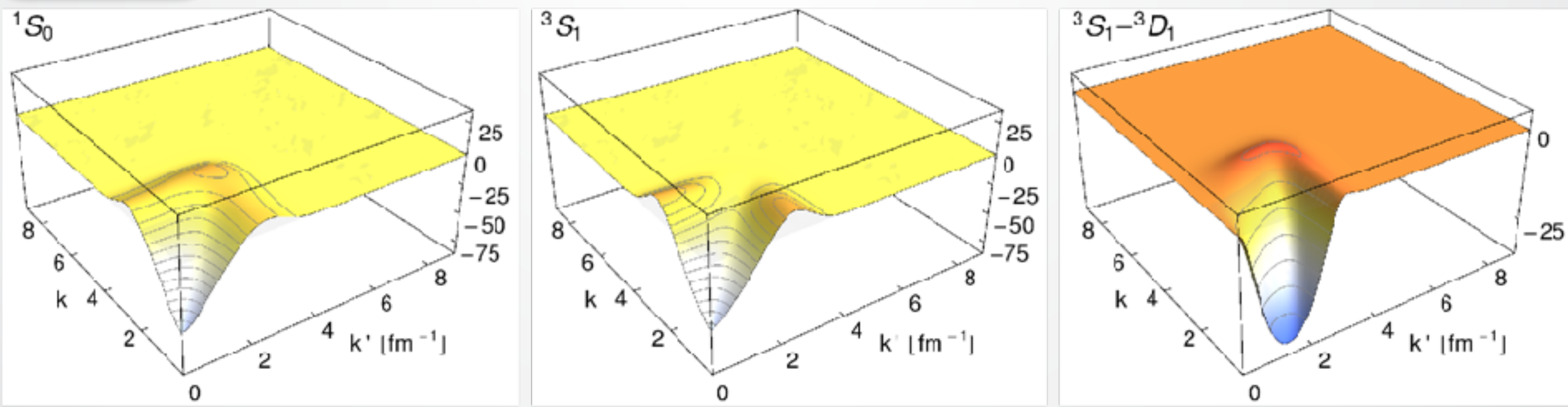
$$V_{(LL'S)J}(k,k') = \left\langle k(LS)J \middle| \hat{V} \middle| k'(L'S)J \right\rangle$$

Similarity Renormalization Group

AV8′



N3LO

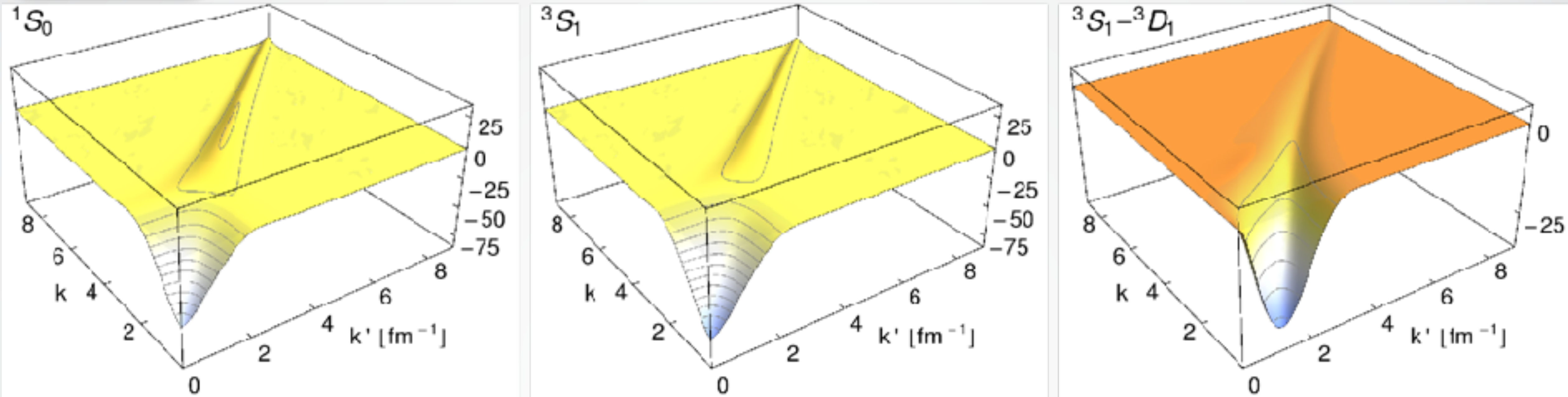


$$V_{(LL'S)J}(k,k') = \left\langle k(LS)J \middle| \hat{V} \middle| k'(L'S)J \right\rangle$$

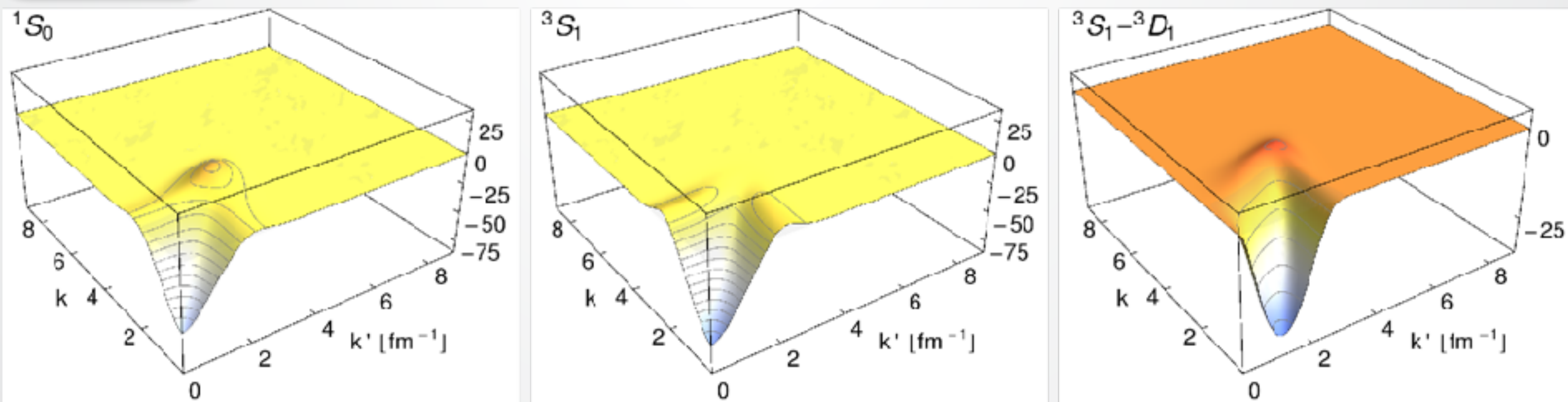
$\alpha = 0.01\ \text{fm}^4$

Similarity Renormalization Group

AV8′



N3LO

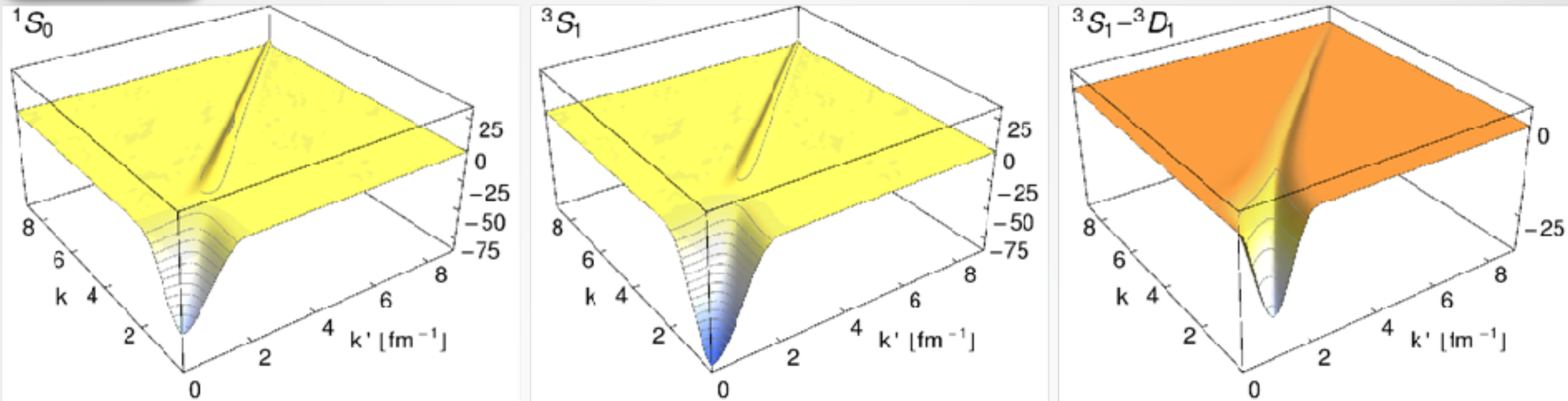


$$V_{(LL'S)J}(k,k') = \langle k(LS)J | \hat{V} | k'(L'S)J \rangle$$

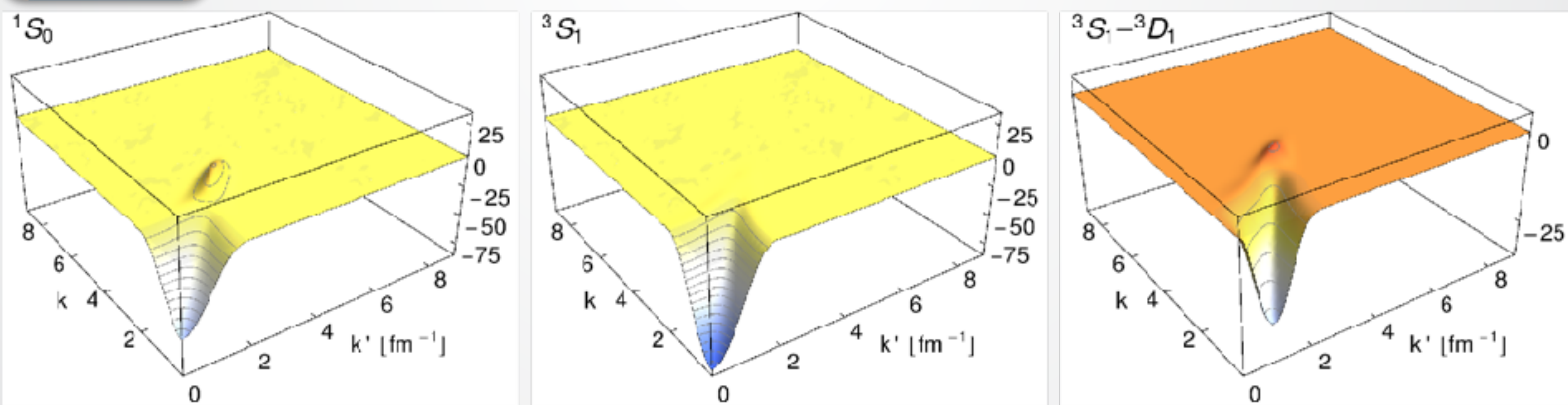
$\alpha = 0.04\ \mathrm{fm}^4$

Similarity Renormalization Group

AV8′



N3LO



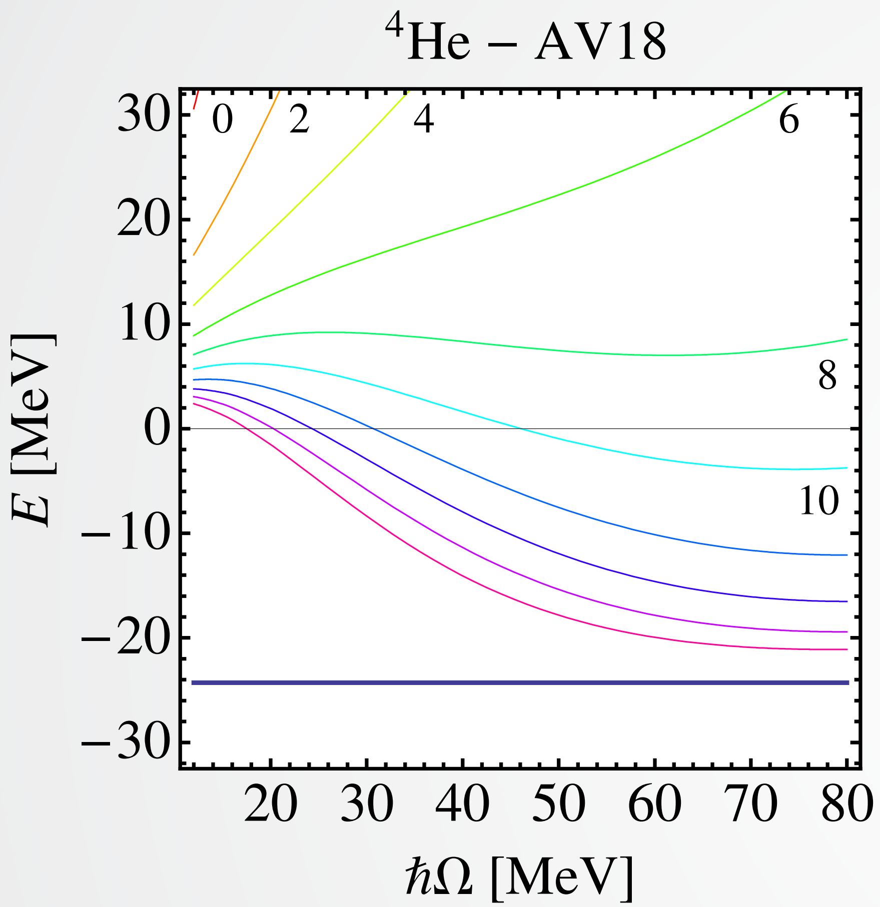
$$V_{(LL'S)J}(k,k') = \langle k(LS)J | \hat{V} | k'(L'S)J \rangle$$

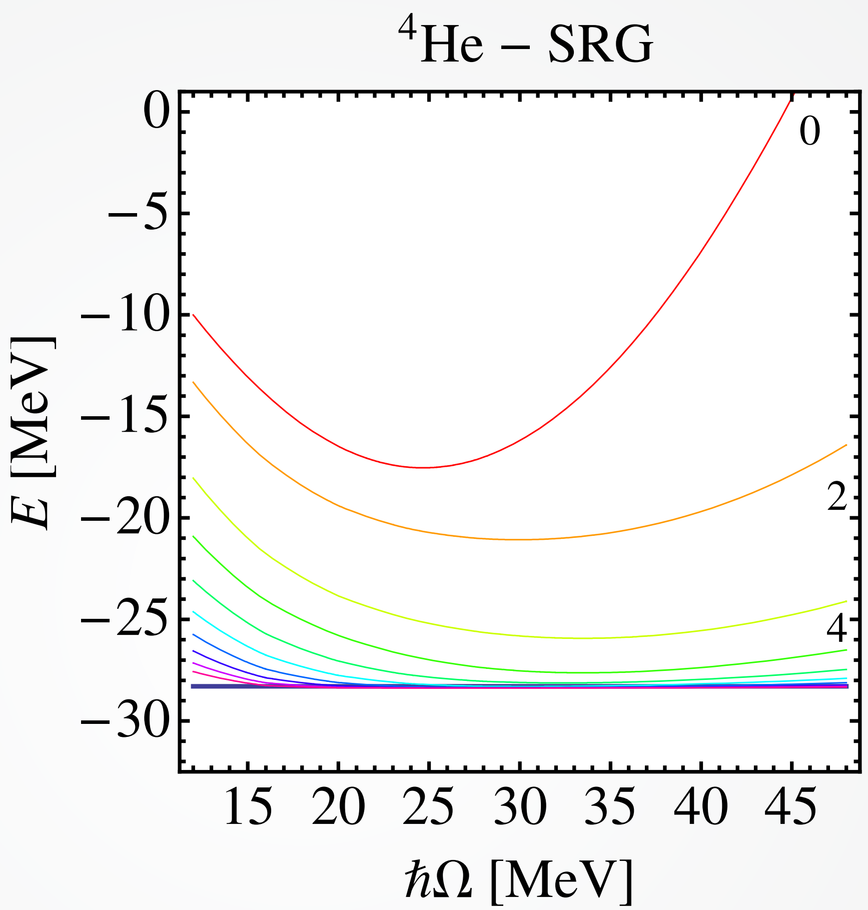
α=0.20 fm^4

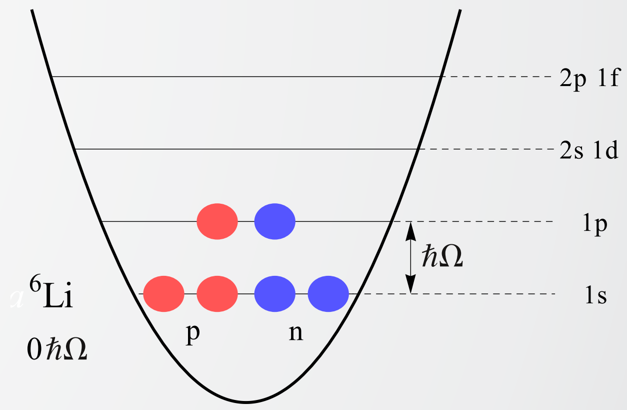
Convergence in No-Core Shell Model

bare interaction

SRG (α=0.03 fm^4)

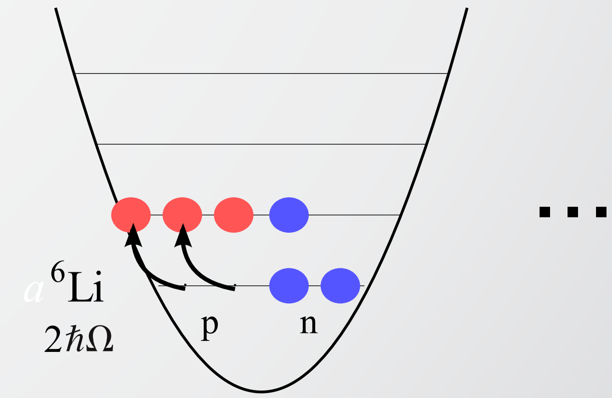






No-Core Shell Model (NCSM)

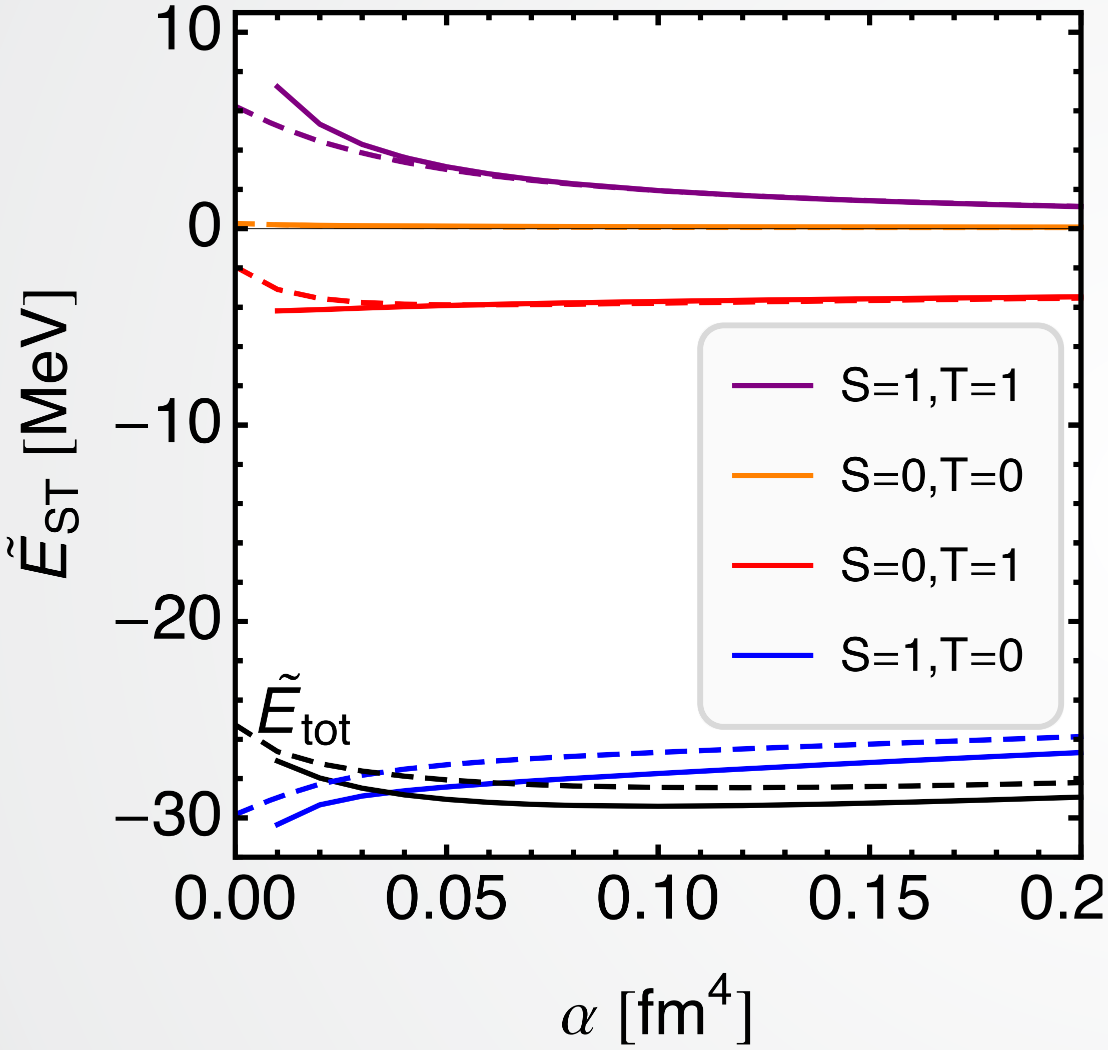
- Diagonalization of Hamiltonian in harmonic oscillator basis
- $N\ \hbar\Omega$ configuration: N oscillator quanta above 0 $\hbar\Omega$ configuration
- Model space sizes grow rapidly with A and N_{max}



Roth, Neff, Feldmeier, Prog. Part. Nucl. Phys. **65**, 50 (2010)

Contributions to the binding energy

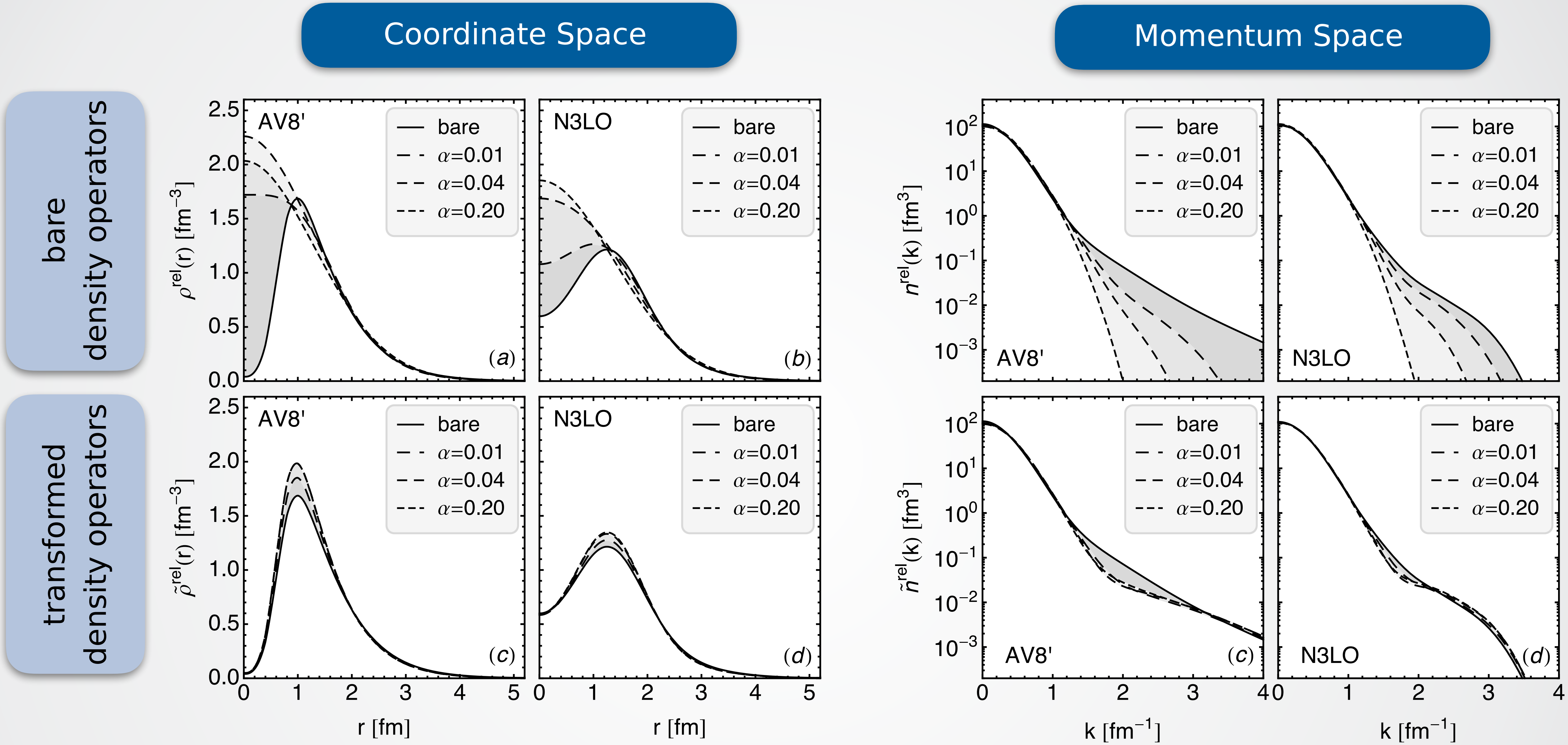
solid: AV8', dashed: N3LO



- Energy depends slightly on flow parameter — indicates missing three-body terms in effective Hamiltonian
- Binding energy dominated by (ST)=(10) channel, contribution from tensor part of effective Hamiltonian decreases with flow parameter
- Sizeable repulsive contribution from odd (ST)=(11) channel related to many-body correlations — decreases with flow parameter

Neff, Feldmeier, Horiuchi, Phys. Rev. C **92**, 024003 (2015)

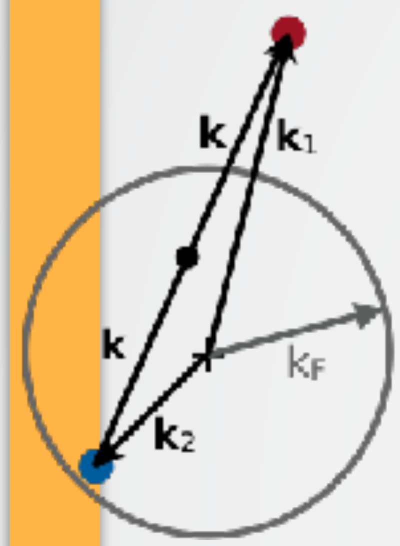
^{4}He: $\rho^{rel}(r)$ and $n^{rel}(k)$

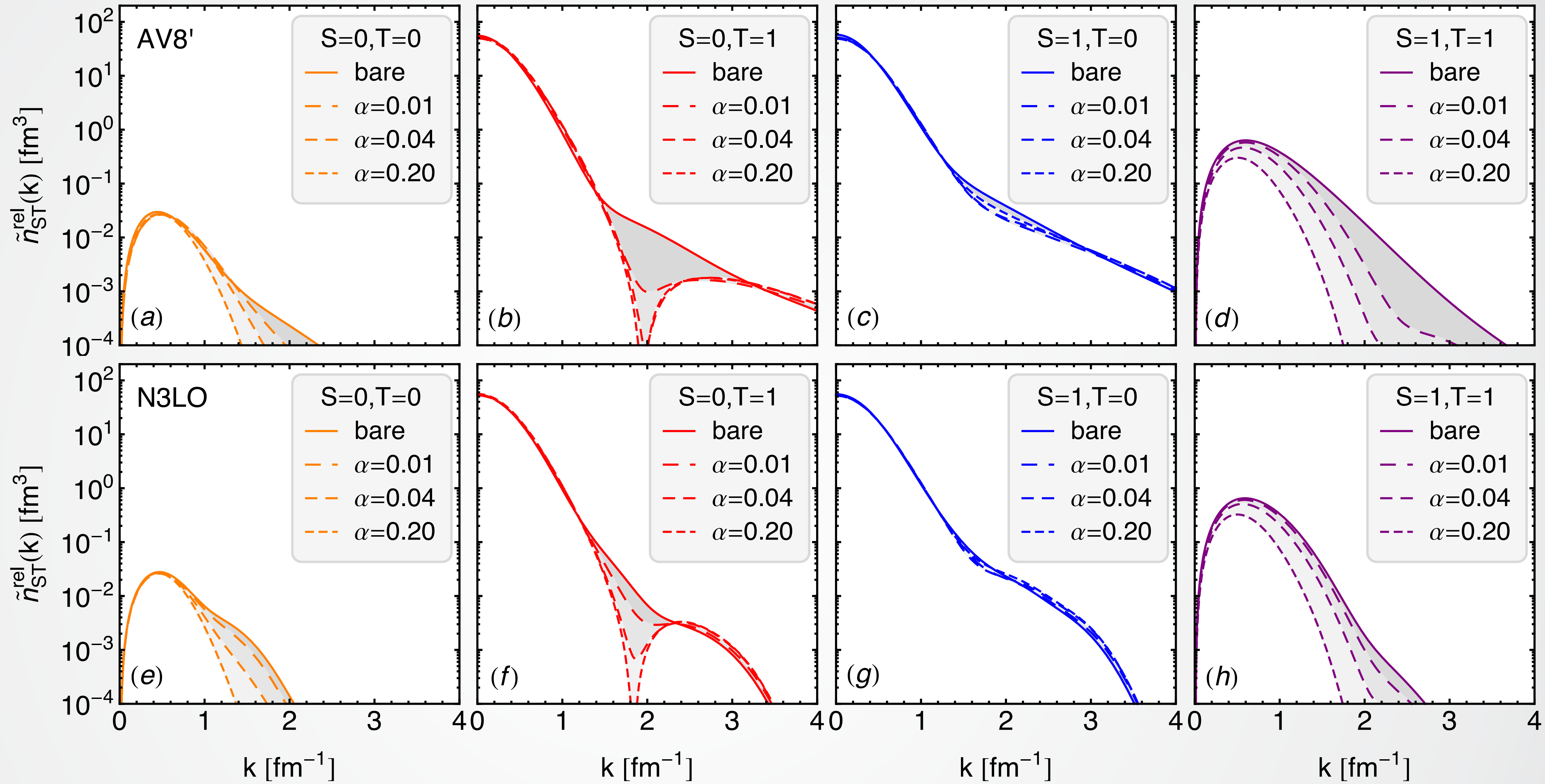


- SRG softens interaction - suppression at short distances and high-momentum components removed in wave function
- these features are recovered with SRG transformed density operators
- small but noticeable dependence on flow parameter α

Neff, Feldmeier, Horiuchi, Phys. Rev. C **92**, 024003 (2015)

^{4}He: $n^{rel}_{ST}(k)$

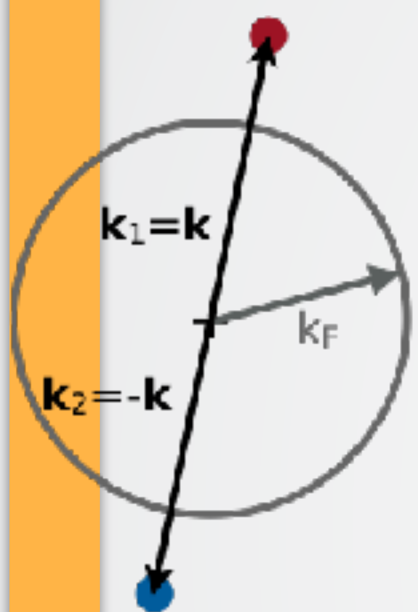


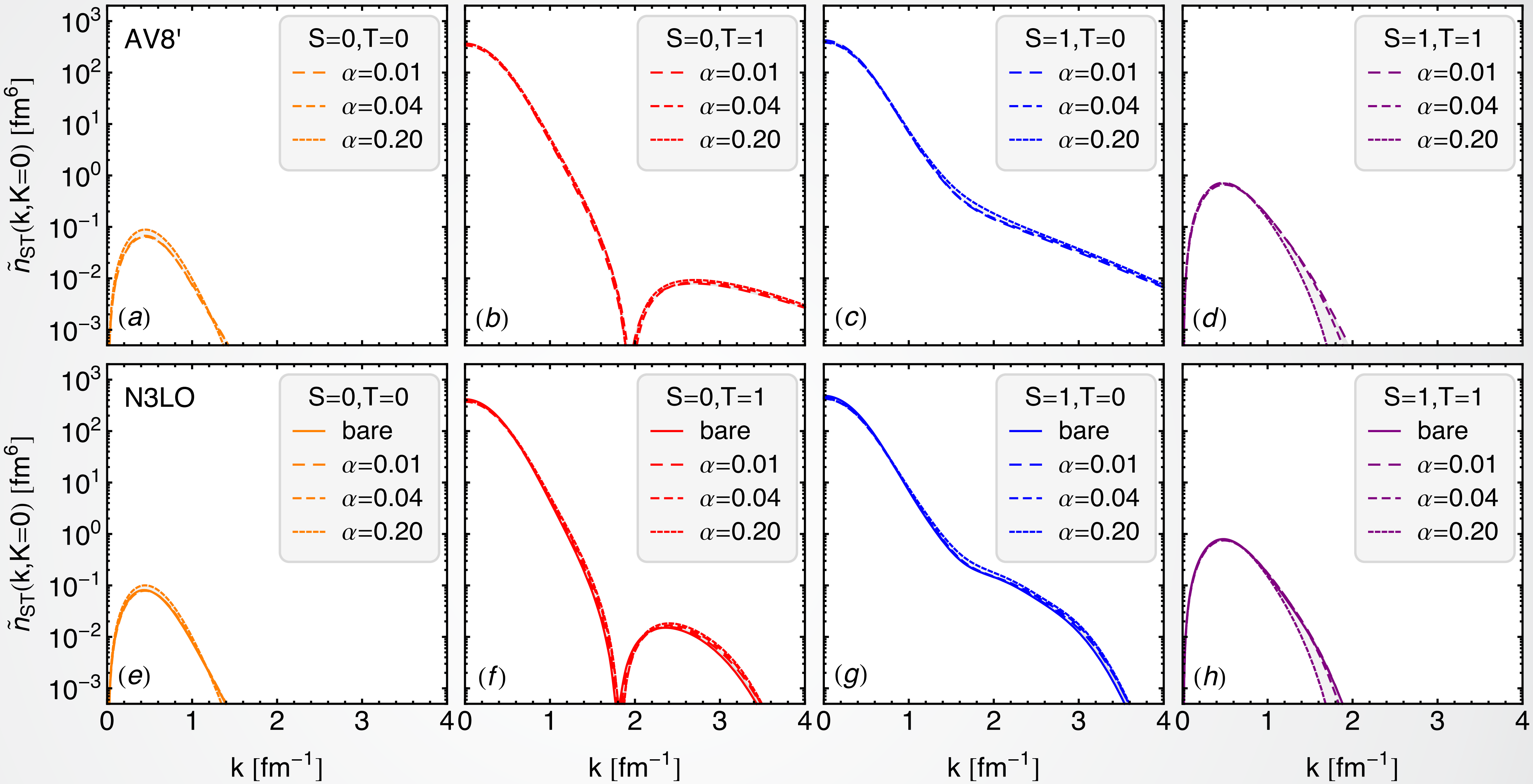


- high-momentum components much stronger in (ST)=(10) channel
- flow dependence is weak in (ST)=(10) channel
- flow dependence is strong in (ST)=(01) and (11) channels, especially for momenta above Fermi momentum — signal of many-body correlations

Neff, Feldmeier, Horiuchi, Phys. Rev. C **92**, 024003 (2015)

^{4}He: n_{ST}(k, K=0)





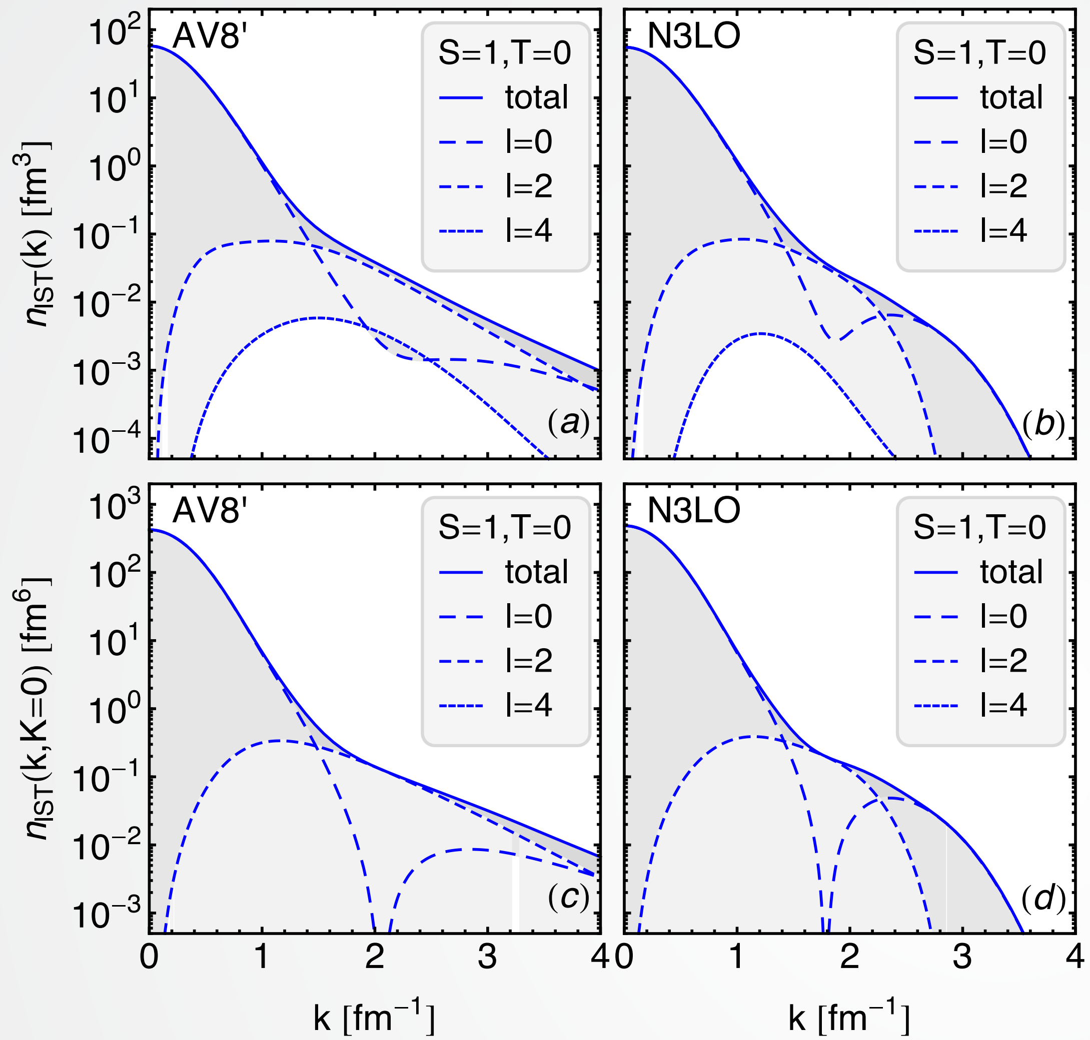
- Relative momentum distributions for K=0 pairs show a very weak dependence on flow parameter and therefore on many-body correlations — ideal to study two-body correlations
- Momentum distribution vanishes for relative momenta around 1.8 fm^{-1} in the (ST)=(01) channel

Neff, Feldmeier, Horiuchi, Phys. Rev. C **92**, 024003 (2015)

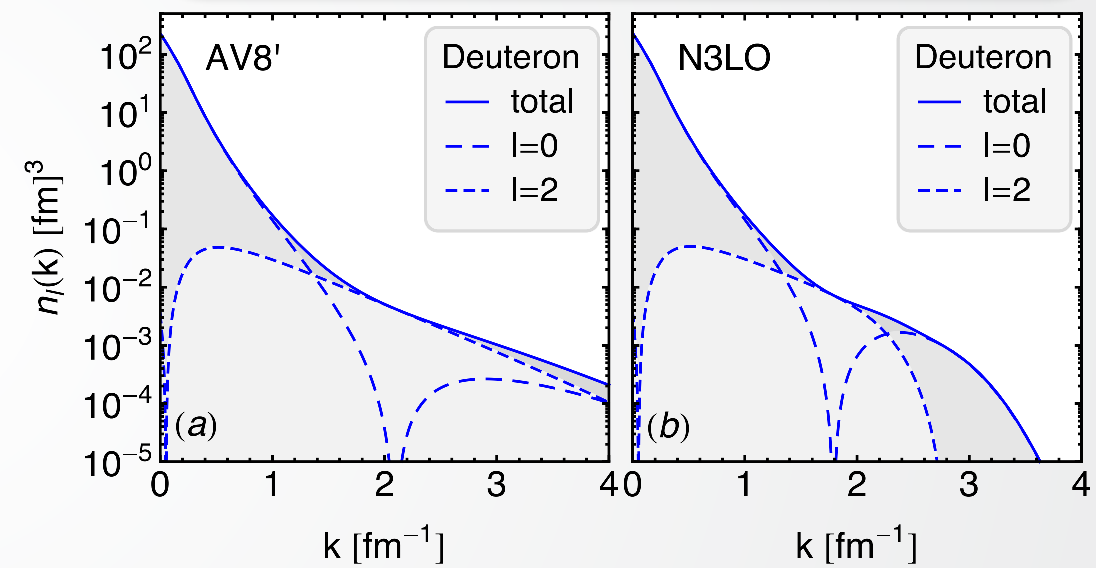
^{4}He: Tensor Correlations

all pairs

K=0 pairs



Deuteron



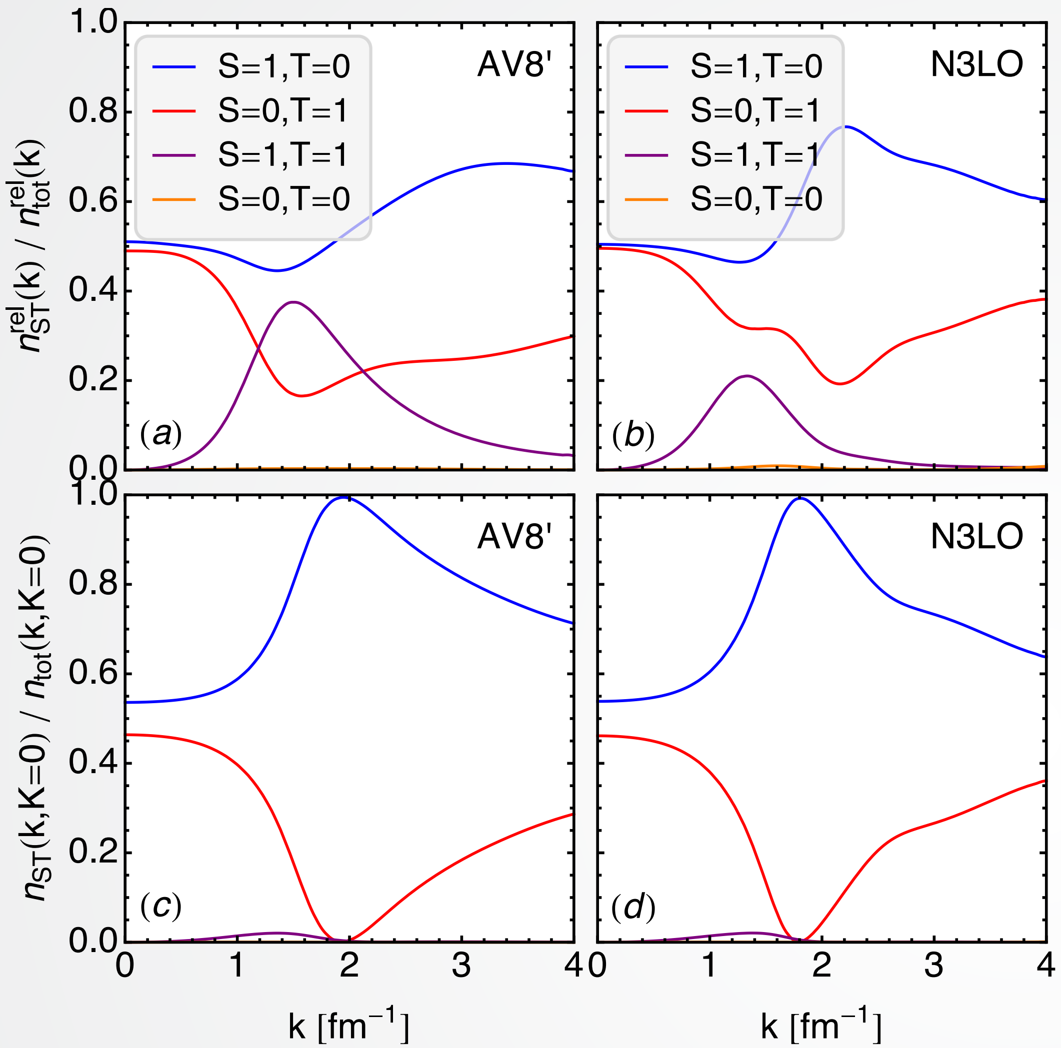
- In (ST)=(10) channel momentum distributions above Fermi momentum dominated by pairs with orbital angular momentum L=2
- For K=0 pairs only L=0,2 relevant, for all pairs also higher orbital angular momenta contribute
- The ^{4}He K=0 momentum distributions in (ST)=(10) channel above 1.5 fm^{-1} look like Deuteron momentum distributions

Neff, Feldmeier, Horiuchi, Phys. Rev. C **92**, 024003 (2015)

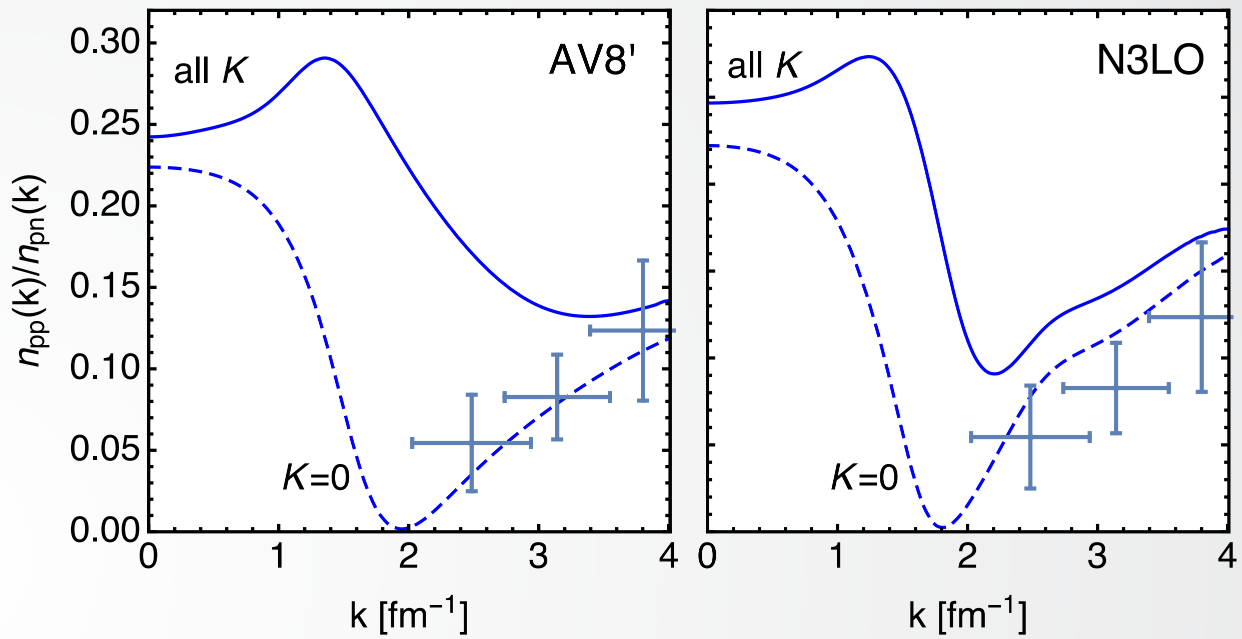
^{4}He: Relative Probabilities

all pairs

K=0 pairs



as ratio of **pp/pn** pairs



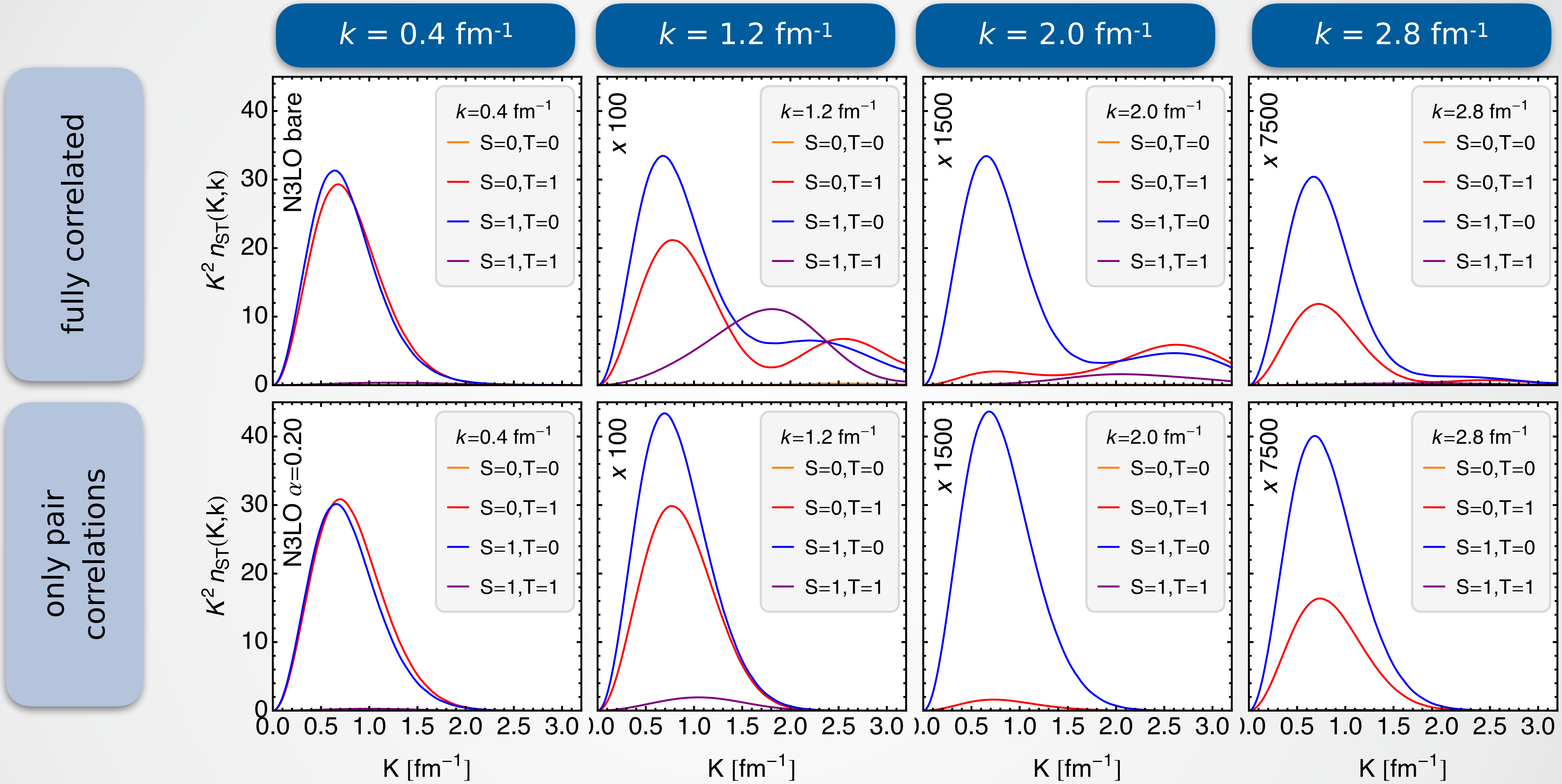
JLab Hall A data

Korover *et al.*, Phys. Rev. Lett. **113**, 022501 (2014)

- Relative probabilities for *K*=0 pairs similar for AV8’ and N3LO interactions, AV8’ in good agreement with JLab data
- For *K*=0 pairs ratio of pp/pn pairs goes to zero for relative momenta of about 1.8 fm^{-1}
- This is not the case if we look at all pairs, here many-body correlations generate many pairs in the (ST)=(11) channel

Neff, Feldmeier, Horiuchi, Phys. Rev. C **92**, 024003 (2015)

^{4}He: Which K contribute to $n_{ST}(k)$?

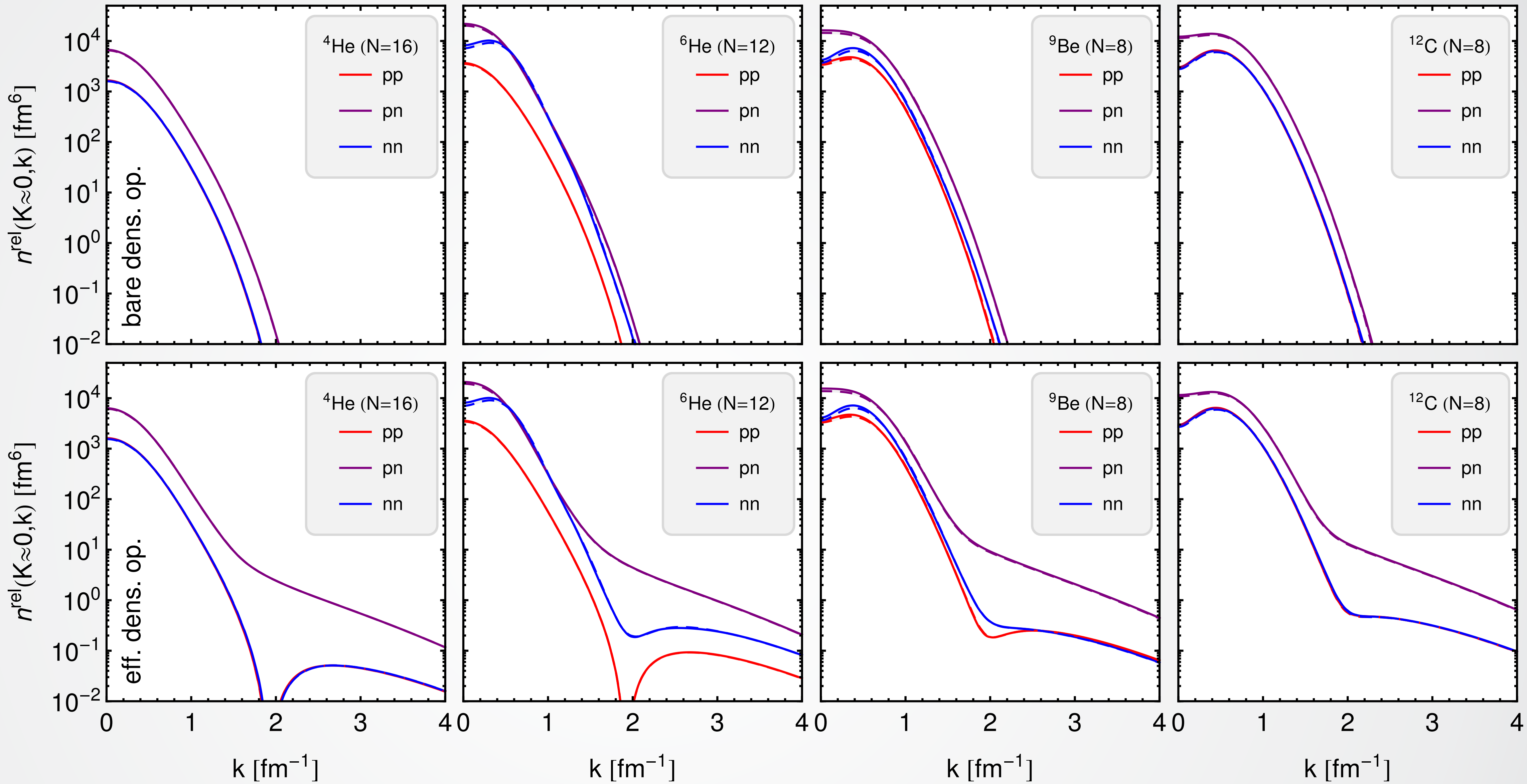


- many-body correlations responsible for pairs with pair momenta $K \gtrsim 2.0\ \mathrm{fm}^{-1}$
- these play a significant role for relative momenta $1.0\ \mathrm{fm}^{-1} \lesssim k \lesssim 2.5\ \mathrm{fm}^{-1}$
- pairs with high relative momenta are only mildly affected

^{4}He, ^{6}He, ^{9}Be, ^{12}C: *n*(k, K=0)

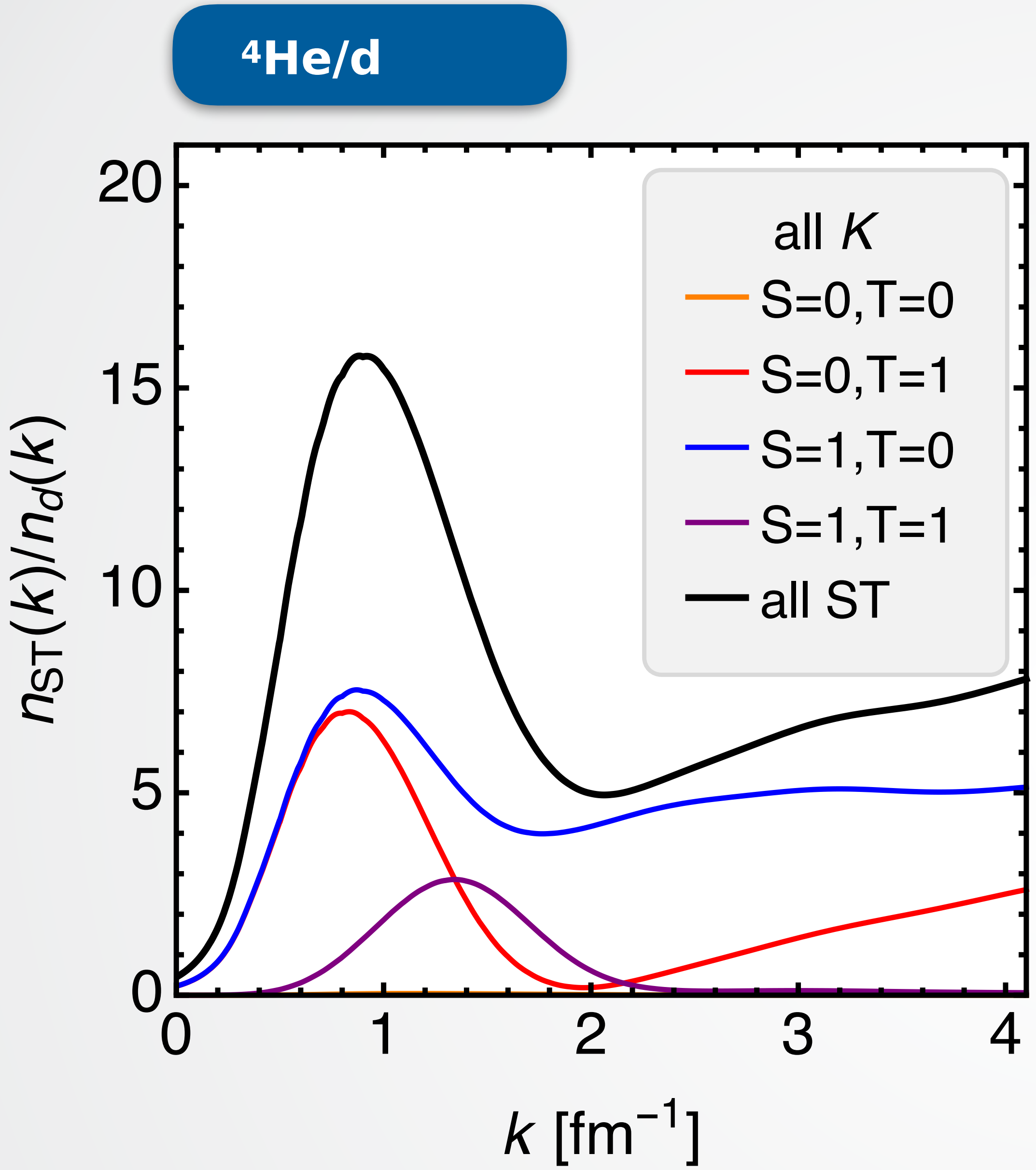
bare density operators

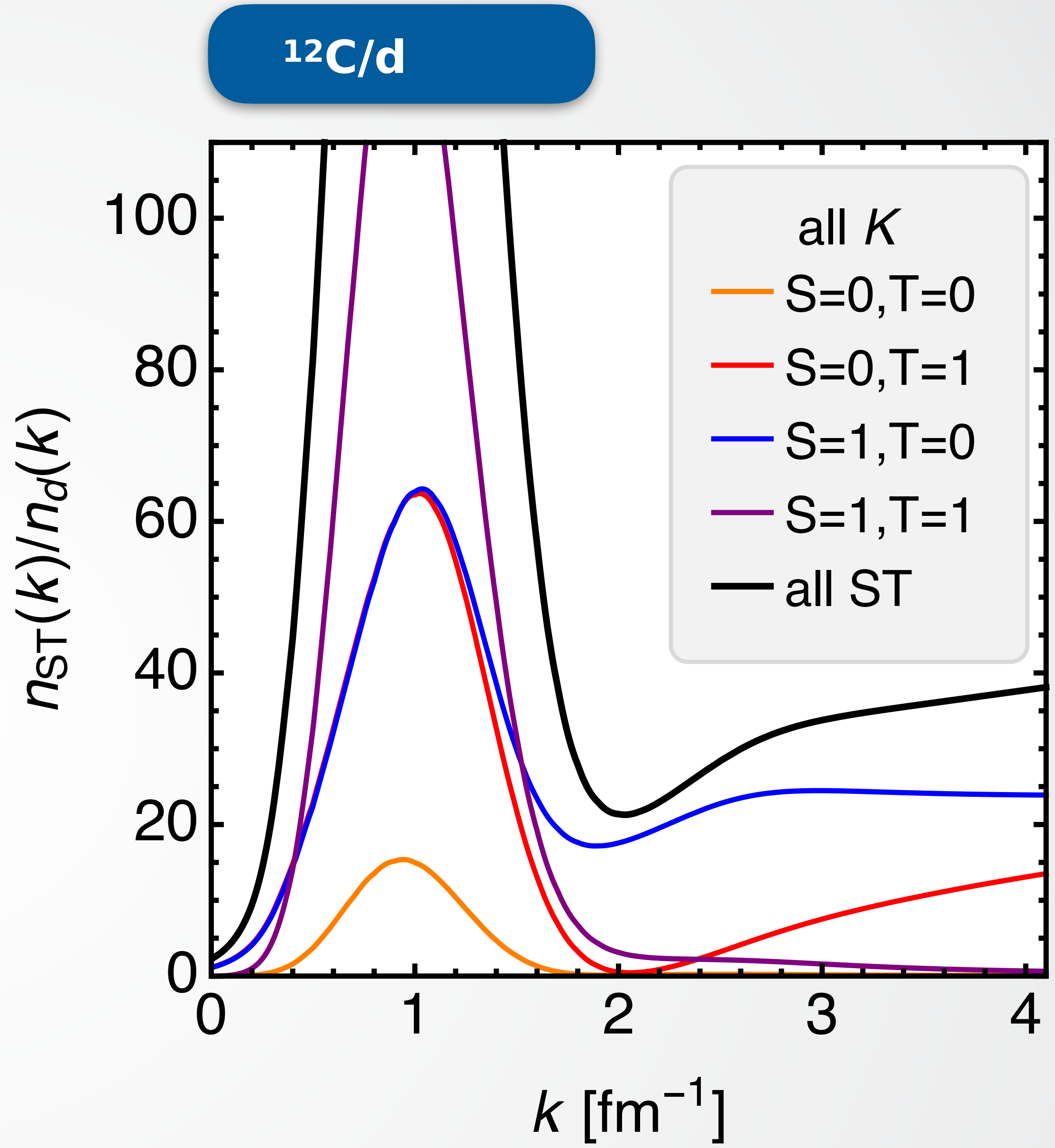
transformed density operators



- Momentum distributions obtained in NCSM are well converged for larger flow parameters
- high-momentum *pn* (and total) momentum distributions very similar for all nuclei
- *p*-shell nucleons fill up the node around 1.8 fm^{-1} for pp/nn pairs

Scaling with $n_d(k)$?





- S=1, T=0 channel: $n^{rel}(k)$ proportional to Deuteron momentum distribution for $k \gtrsim 2.5$ fm^{-1}
- contribution of S=0, T=1 channel grows with relative momentum

The Wigner Function of the Deuteron

A phase-space picture of short-range correlations

Neff, Feldmeier, arXiv:1610.04066

$$W_{M_S,M_S'}(\mathbf{r},\mathbf{p}) = \frac{1}{(2\pi)^3}\int d^3s \left\langle \mathbf{r}+\tfrac{1}{2}\mathbf{s}; SM_S \middle| \hat{\rho} \middle| \mathbf{r}-\tfrac{1}{2}\mathbf{s}; SM_S' \right\rangle e^{-i\mathbf{p}\cdot\mathbf{s}}$$

$$\rho_{M_S}(\mathbf{r}) = \left\langle \mathbf{r}; SM_S \middle| \hat{\rho} \middle| \mathbf{r}; SM_S \right\rangle = \int d^3p\, W_{M_S,M_S}(\mathbf{r},\mathbf{p})$$

$$\hat{\rho} = \frac{1}{3}\sum_M \left|\Psi; 1M\right\rangle\left\langle\Psi; 1M\right|$$

$$n_{M_S}(\mathbf{p}) = \left\langle \mathbf{p}; SM_S \middle| \hat{\rho} \middle| \mathbf{p}; SM_S \right\rangle = \int d^3r\, W_{M_S,M_S}(\mathbf{r},\mathbf{p})$$

Wigner Function of the Deuteron

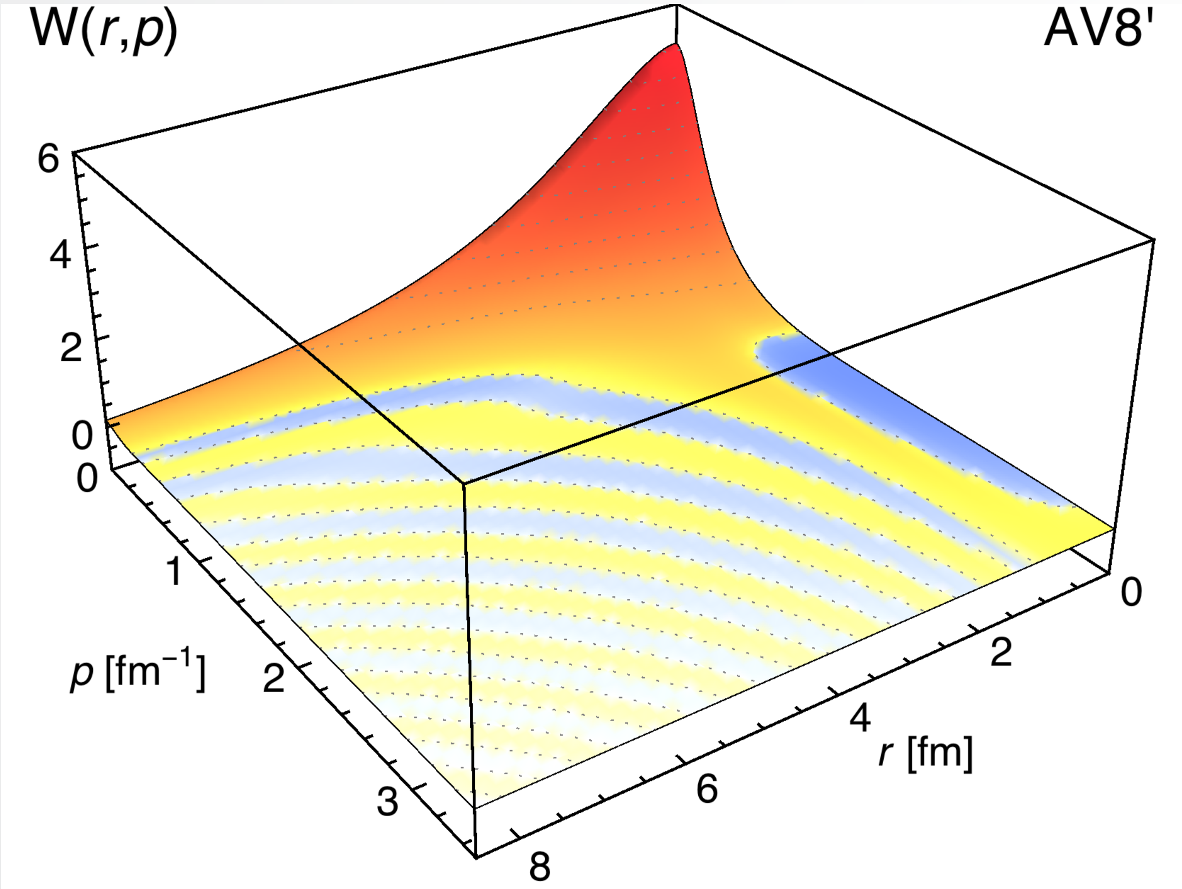
$$W(\mathbf{r},\mathbf{p}) = \frac{1}{(2\pi)^3}\int d^3s \left\langle \mathbf{r}+\tfrac{1}{2}\mathbf{s}\middle|\hat{\rho}\middle|\mathbf{r}-\tfrac{1}{2}\mathbf{s}\right\rangle e^{-i\mathbf{p}\cdot\mathbf{s}}$$
$$= \frac{1}{(2\pi)^3}\int d^3s\, \Psi(\mathbf{r}+\tfrac{1}{2}\mathbf{s})\Psi(\mathbf{r}-\tfrac{1}{2}\mathbf{s})^\star e^{-i\mathbf{p}\cdot\mathbf{s}}$$

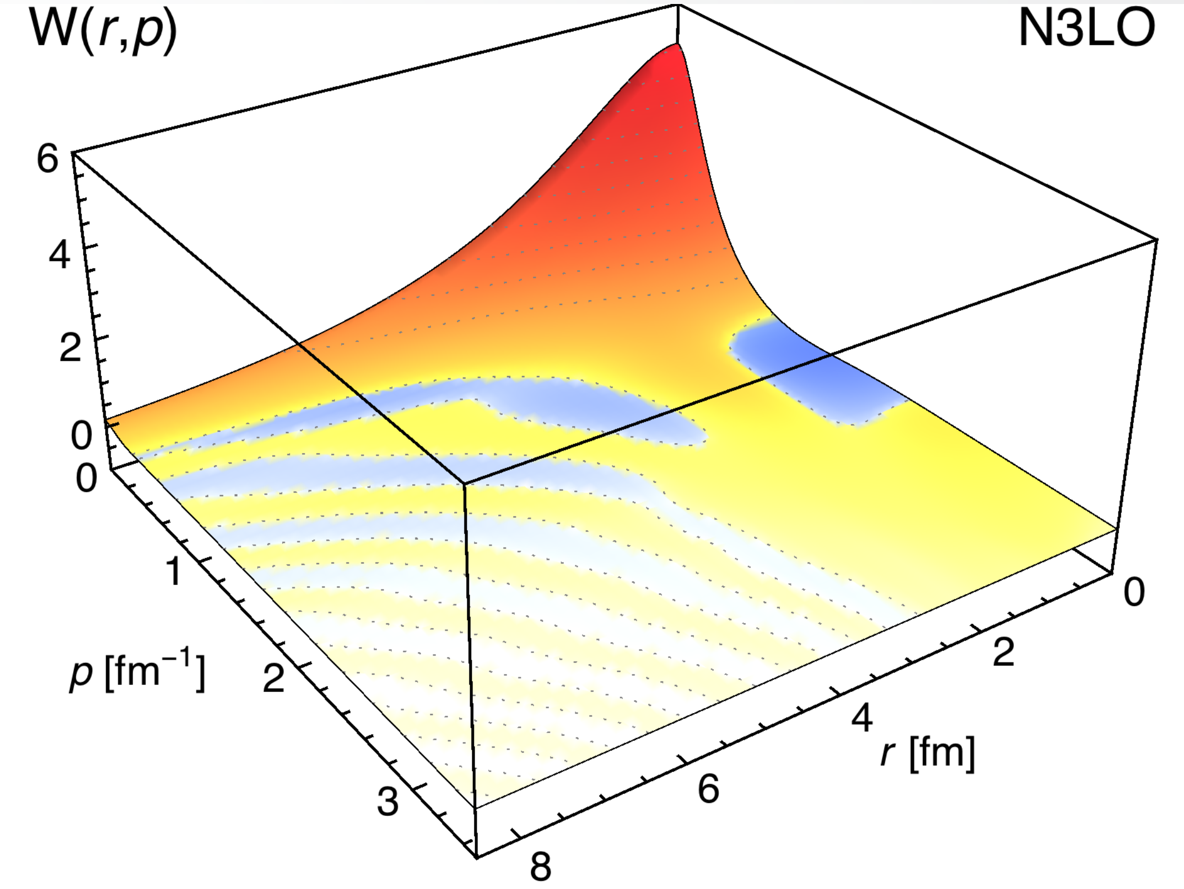
- Integrate over angles

$$W(r,p) = \int d\Omega_r \int d\Omega_p\, W(\mathbf{r},\mathbf{p})$$

AV8′

N3LO





- Wigner function not suppressed at small distances r
- short-range physics is encoded in high-momentum region

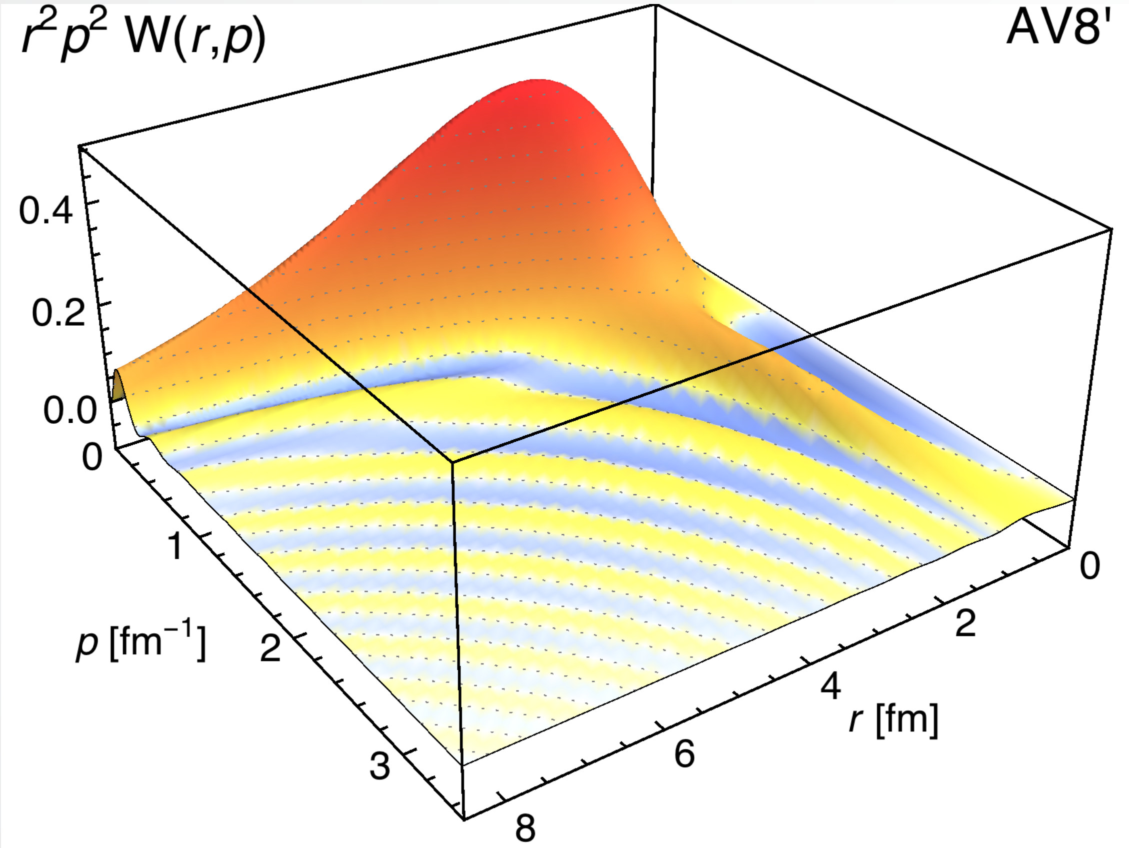
Wigner Function of the Deuteron

$$W(\mathbf{r},\mathbf{p}) = \frac{1}{(2\pi)^3}\int d^3s \left\langle \mathbf{r}+\tfrac{1}{2}\mathbf{s}\middle|\hat{\rho}\middle|\mathbf{r}-\tfrac{1}{2}\mathbf{s}\right\rangle e^{-i\mathbf{p}\cdot\mathbf{s}}$$
$$= \frac{1}{(2\pi)^3}\int d^3s\, \Psi(\mathbf{r}+\tfrac{1}{2}\mathbf{s})\Psi(\mathbf{r}-\tfrac{1}{2}\mathbf{s})^\star e^{-i\mathbf{p}\cdot\mathbf{s}}$$

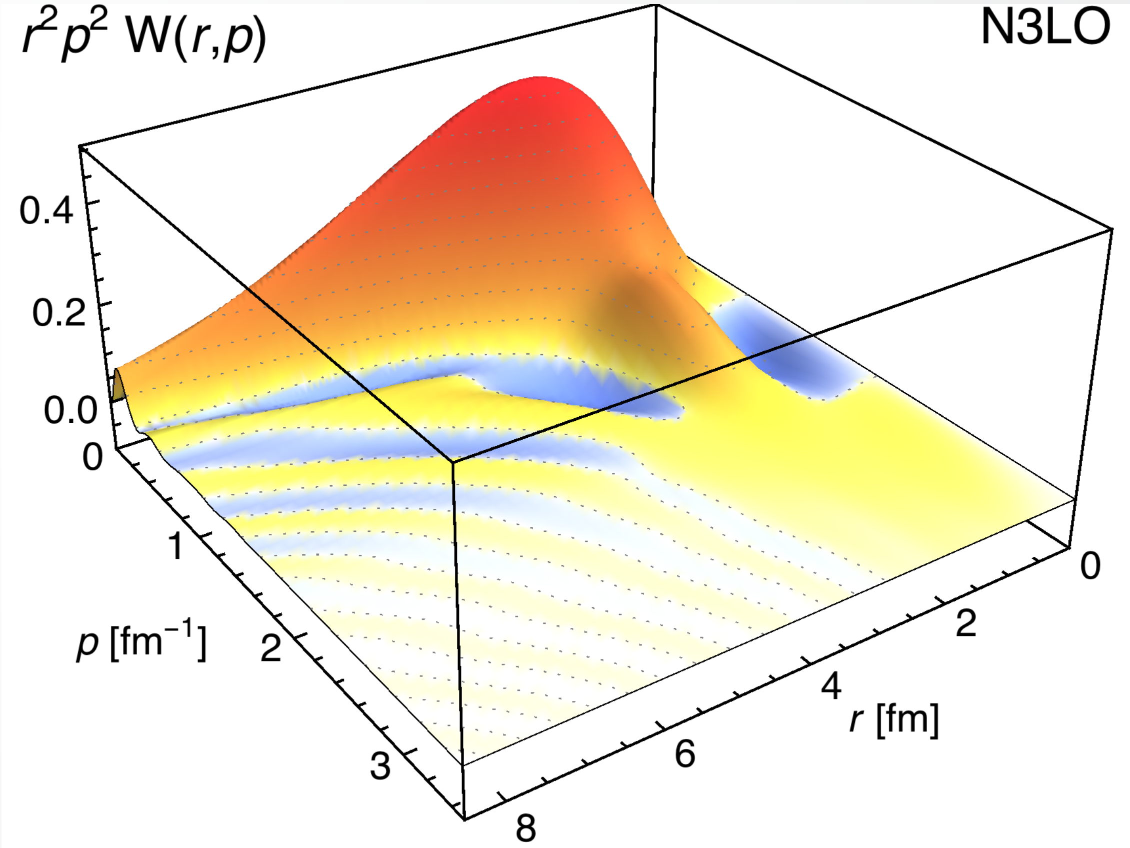
- Integrate over angles

$$W(r,p) = \int d\Omega_r \int d\Omega_p\, W(\mathbf{r},\mathbf{p})$$

AV8′

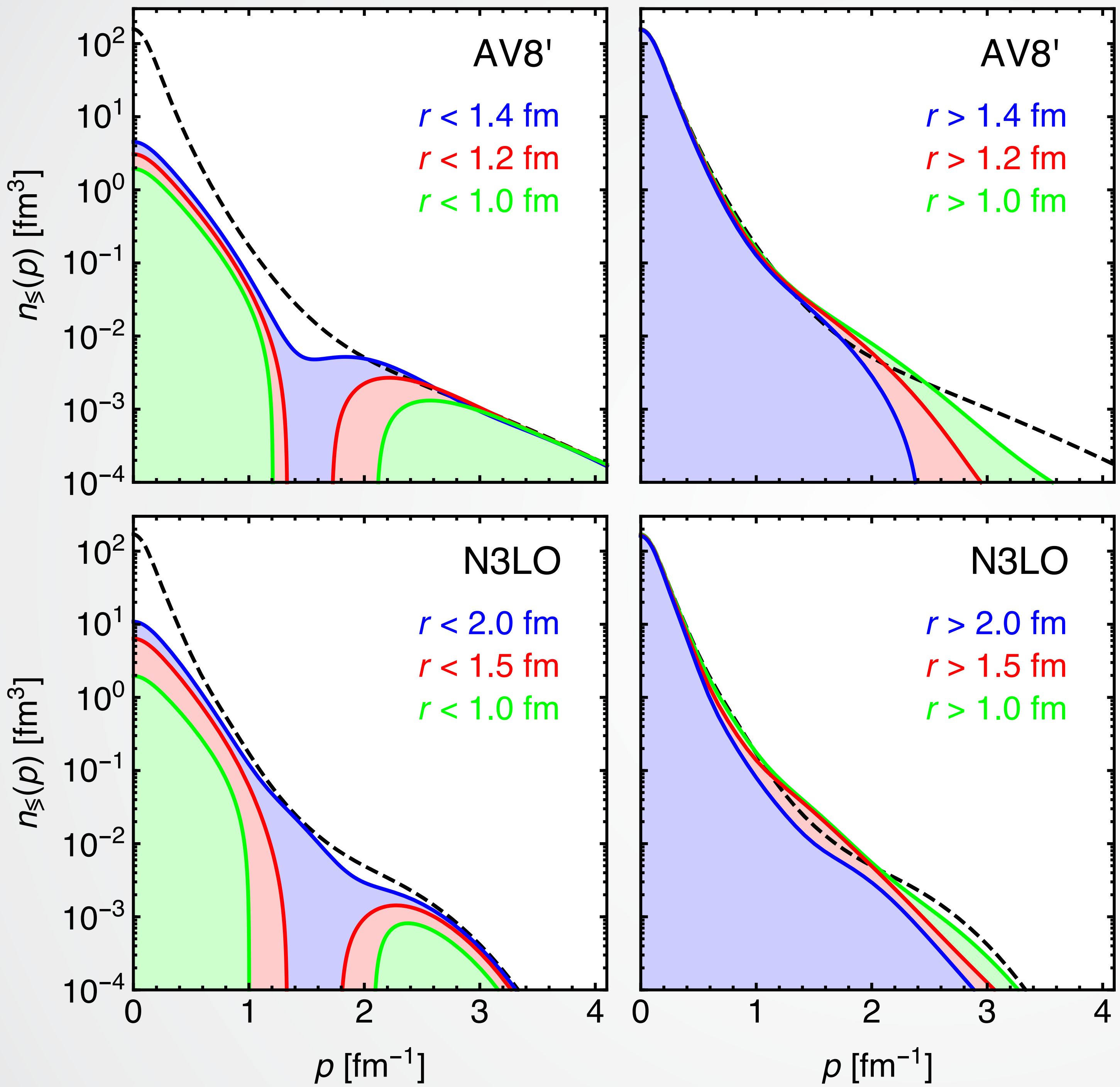


N3LO



- “quasi-probability” to find pair at distance r and relative momentum p
- high-momentum components are seen as a shoulder at small distances
- Oscillations due to quantum effects (interference of low- and high-momentum components)

(Partial) Momentum Distributions

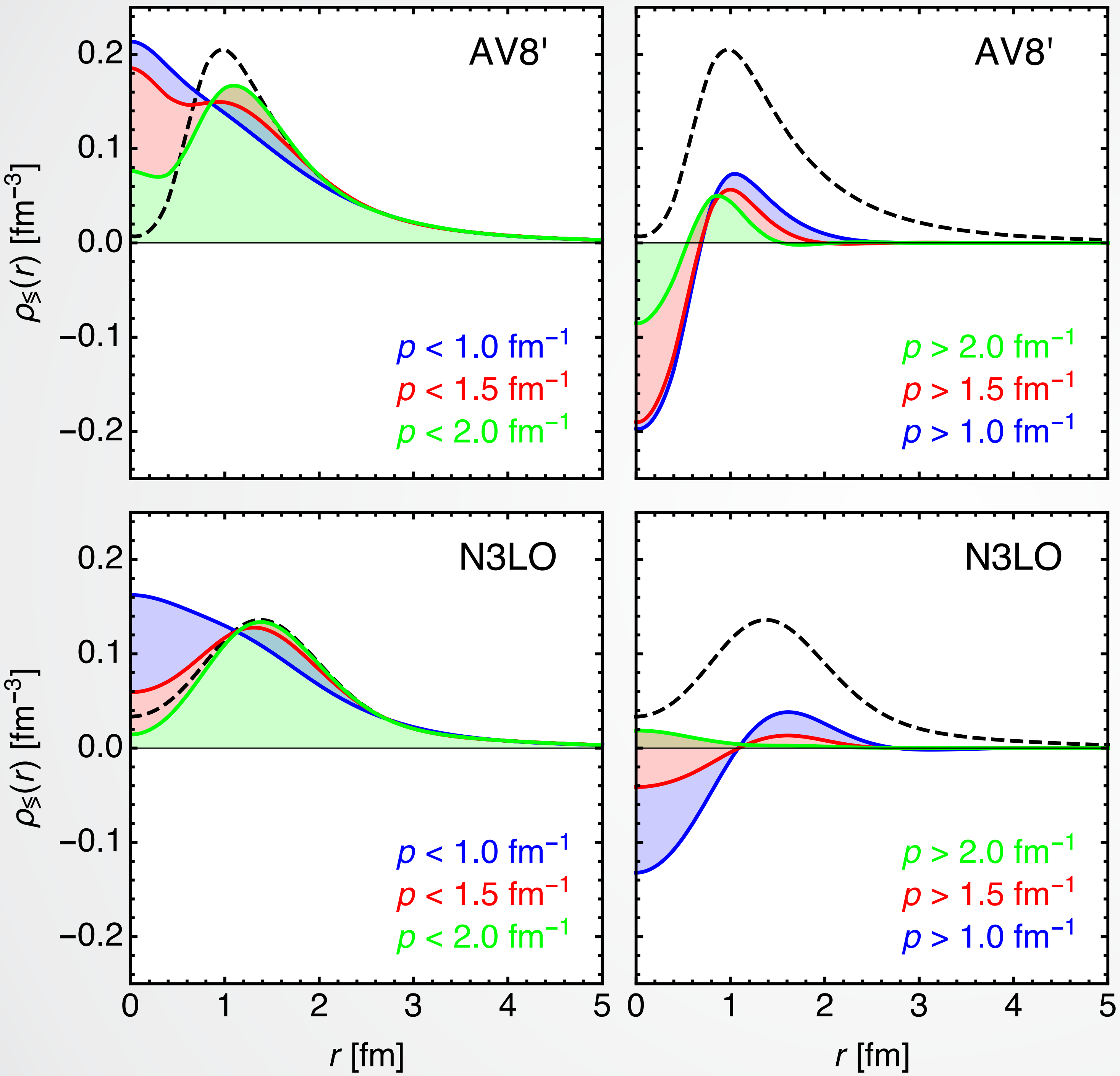


$$n_{\lessgtr}(p) = \int_{r \lessgtr r_{sep}} dr\, r^2 W(r,p)$$

- Integrate Wigner function over small or large distance regions
- not an observable but provides intuition

- small distance pairs determine high momentum part of momentum distribution
- large distance pairs give momentum distributions in low momentum region

(Partial) Coordinate Space Distributions



$$\rho_{\lessgtr}(r) = \int_{p \lessgtr p_{sep}} dp\, p^2 W(r,p)$$

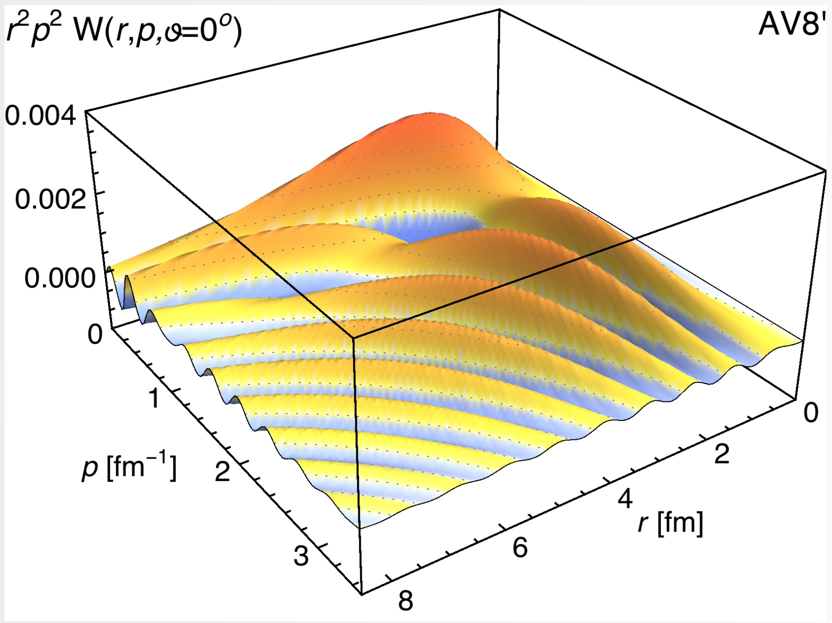
- Integrate Wigner function over regions of low and high momenta

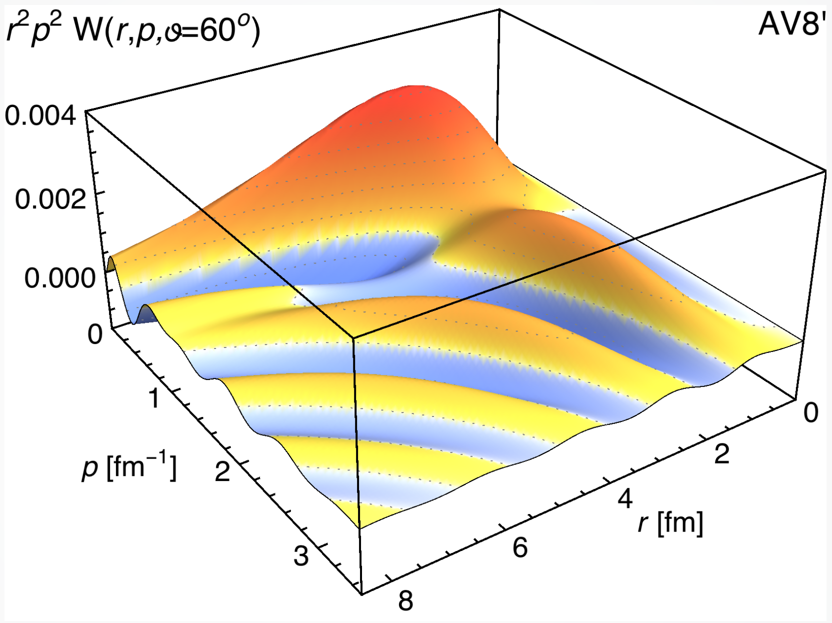
- density at large distances given by low-momentum pairs
- correlation hole at small distances is created by interference of low- and high-momentum pairs

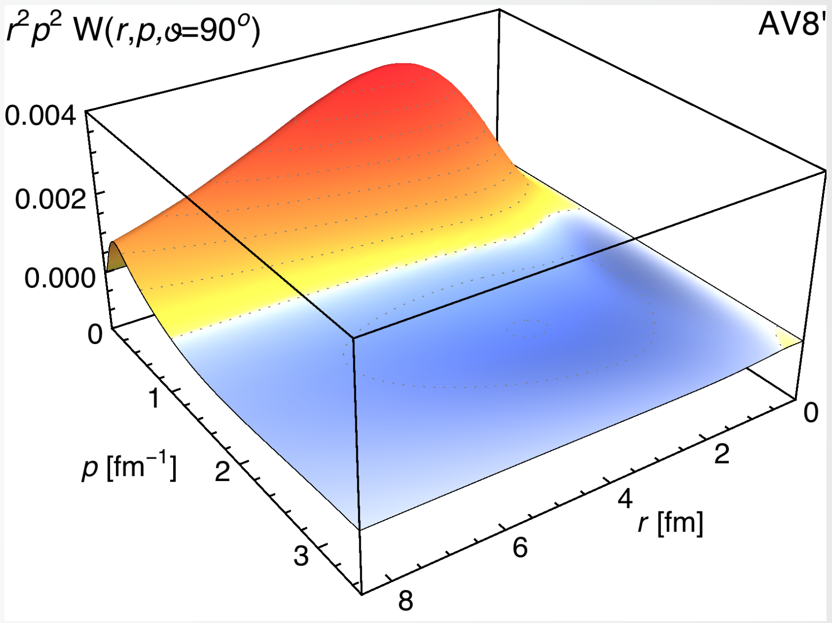
Orientation dependence

$$W(\mathbf{r}, \mathbf{p}) = W(r, p, \cos\vartheta)$$

AV8′







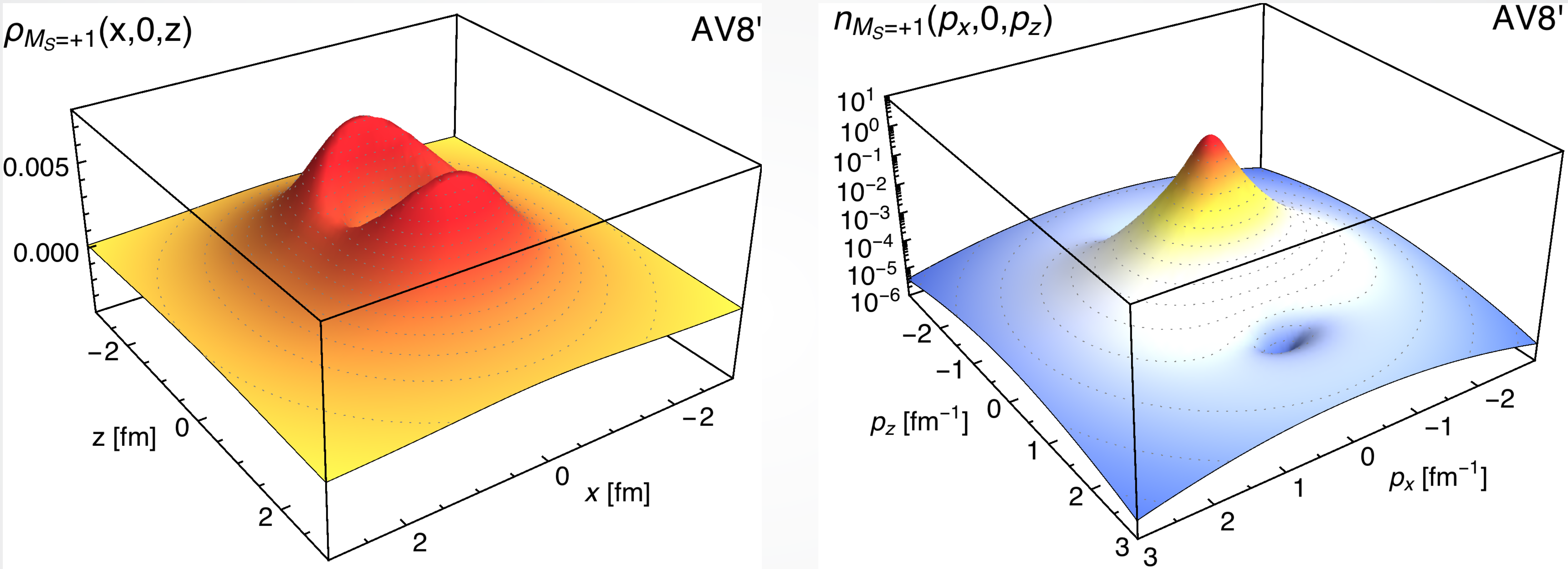
r and **p** parallel

r and **p** perpendicular

- oscillations reflect uncertainty principle for non-commuting observables
- three-dimensional problem, small angles correspond to small impact parameters, angles around 90° to circular motion around the core
- highest probability for angles around 90°

Spin dependence

$$\rho_{M_S}(\mathbf{r}) = \langle \mathbf{r}; SM_S | \hat{\rho} | \mathbf{r}; SM_S \rangle = \int d^3p\, W_{M_S,M_S}(\mathbf{r},\mathbf{p}) \qquad n_{M_S}(\mathbf{p}) = \langle \mathbf{p}; SM_S | \hat{\rho} | \mathbf{p}; SM_S \rangle = \int d^3r\, W_{M_S,M_S}(\mathbf{r},\mathbf{p})$$



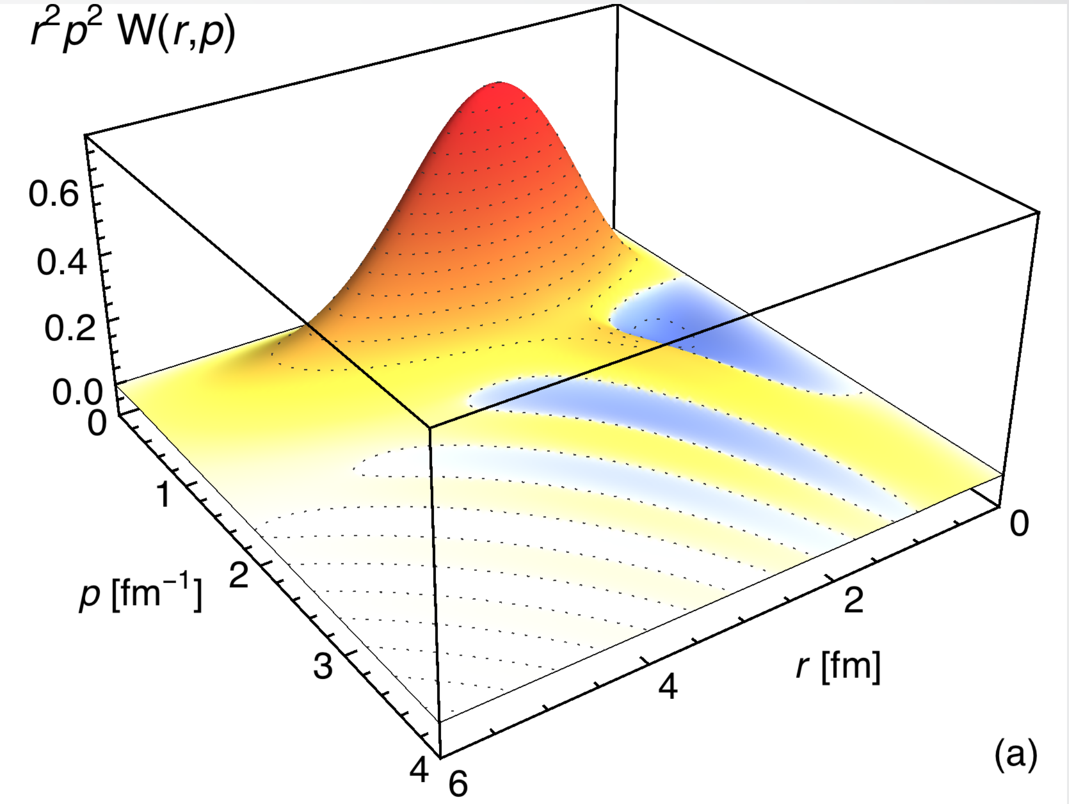
- density and momentum distributions depend on orientation of the spin due to tensor force
- dumbbell (M_S=±1) and donut (M_S=0) shapes in coordinate space
- dip in momentum distribution for momenta parallel to spin orientation
- tensor correlations strongest in mid-momentum region ($1.5\ \text{fm}^{-1} \lesssim p \lesssim 2.5\ \text{fm}^{-1}$)

Wigner function of two-Gaussian toy model

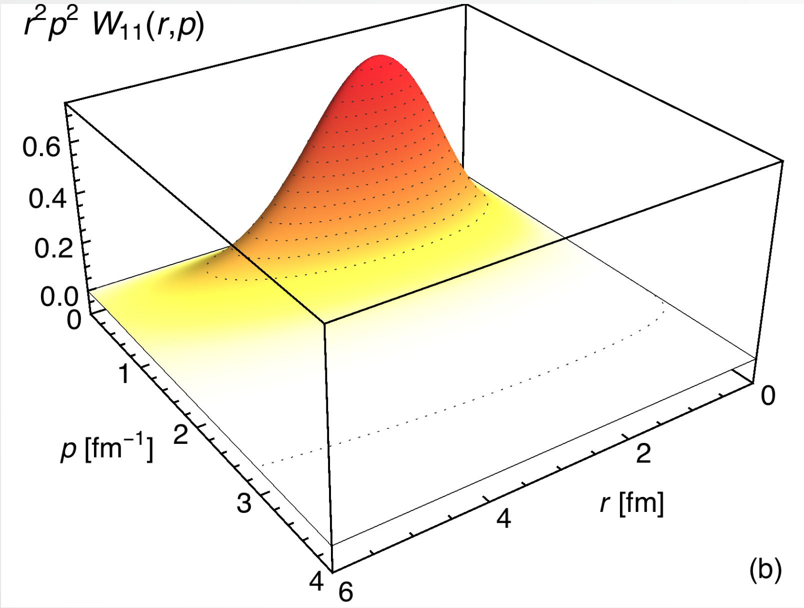
$$\psi(\mathbf{r}) = \alpha_1 \psi_1(\mathbf{r}) + \alpha_2 \psi_2(\mathbf{r})$$

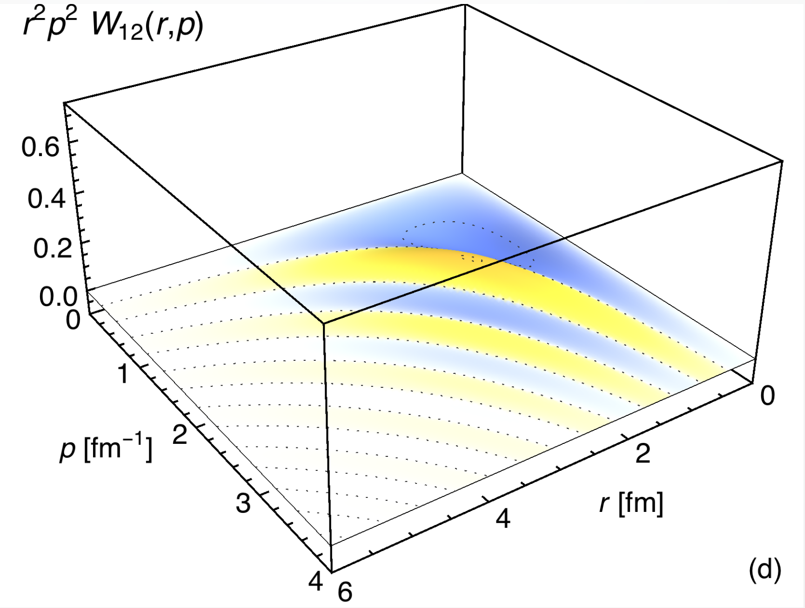
$$\psi_i(\mathbf{r}) = \frac{1}{(\pi a_i)^{3/4}} \exp\left\{-\frac{\mathbf{r}^2}{2a_i}\right\}$$

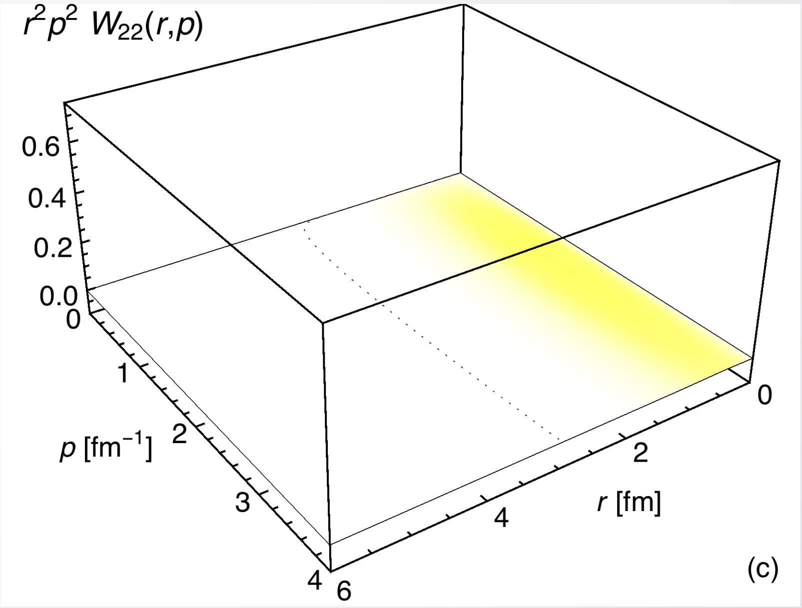
$$W(\mathbf{r},\mathbf{p}) = W_{11}(\mathbf{r},\mathbf{p}) + W_{12}(\mathbf{r},\mathbf{p}) + W_{22}(\mathbf{r},\mathbf{p})$$



interference term







Summary

Two-body Densities with NCSM and SRG transformed Operators

- SRG eliminates high-momentum components from wave function
- high-momentum information can be recovered with SRG transformed operators
- use flow dependence to study many-body correlations and learn about induced three-body forces
- back-to-back pairs only weakly, pairs with large pair momentum strongly affected by many-body correlations
- high-momentum components for AV18 and N3LO interactions similar for momenta up to 2.5 fm^{-1}, dominated by tensor correlations, show dominance of *pn* over *pp* pairs
- observed *pp*/*pn* ratios in ^4He in good agreement with calculated ratios of $K=0$ pairs
- at high momenta calculated total two-nucleon momentum distributions of heavier nuclei scale with ^3He/^4He but not with Deuteron
- two-nucleon distributions as a function of pair momentum $n(k, K)$ or pair position $n(k, X)$

- densities are not observables, how big is the role of two-body currents (MEC) and FSI ?

Wigner Function of the Deuteron

- Shows how short-distance and high-momentum physics is connected
- Oscillations reflect quantum nature and can be understood as interference between low- and high-momentum components