Ab-initio methods for light nuclei from low to high resolution

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Polarized light ion physics with EIC Ghent, Belgium Feb. 5 – 9, 2018

Meeting Topics include:

* Neutron spin structure from polarized deep-inelastic scattering on light nuclei (d, 3He)

- * Nuclear fragmentation and final-state interactions in high-energy processes
- * Spin-orbit effects and azimuthal asymmetries in scattering on proton and light nuclei
- * Tensor-polarized deuteron in low- and high-energy processes
- * Theoretical methods for light nuclear structure: Few-body, Lattice, Light-front
- * Nuclear structure at variable scales: Effective degrees of freedom, EFT methods
- * Quarks and gluons in light nuclei: EMC effect, non-nucleonic degrees of freedom
- * Diffraction and nuclear shadowing in DIS on light nuclei
- * Polarized light ion beams: Sources, acceleration, polarimetry
- * Forward detection of spectators and nuclear fragments at EIC

issues in this talk



Effective Nucleon Interaction (Chiral Perturbation Theory)

Chiral perturbation theory (χ PT) allows for controlled power series expansion



No-Core Configuration Interaction calculations

Barrett, Navrátil, Vary, Ab initio no-core shell model, PPNP69, 131 (2013)

Given a Hamiltonian operator

$$\hat{\mathbf{H}} = \sum_{i < j} \frac{(\vec{p}_i - \vec{p}_j)^2}{2 \, m \, A} + \sum_{i < j} V_{ij} + \sum_{i < j < k} V_{ijk} + \dots$$

solve the eigenvalue problem for wavefunction of A nucleons

$$\mathbf{\hat{H}} \Psi(r_1, \dots, r_A) = \lambda \Psi(r_1, \dots, r_A)$$

- Expand eigenstates in basis states $|\Psi\rangle = \sum a_i |\Phi_i\rangle$
- Diagonalize Hamiltonian matrix $H_{ij} = \langle \Phi_j | \mathbf{\hat{H}} | \Phi_i \rangle$
- No Core Full Configuration (NCFC) All A nucleons treated equally
- Complete basis \longrightarrow exact result
- In practice
 - truncate basis
 - study behavior of observables as function of truncation

Basis expansion $\Psi(r_1, \ldots, r_A) = \sum a_i \Phi_i(r_1, \ldots, r_A)$

- Many-Body basis states $\Phi_i(r_1, \ldots, r_A)$ Slater Determinants
- Single-Particle basis states $\phi_{\alpha}(r_k)$ with $\alpha = (n, l, s, j, m_j)$
- Radial wavefunctions: Harmonic Oscillator (HO), natural orbitals, Woods-Saxon, Coulomb-Sturmian, Complex Scaled HO, Berggren,...
- *M*-scheme: Many-Body basis states eigenstates of \hat{J}_z

$$\hat{\mathbf{J}}_{\mathbf{z}}|\Phi_i\rangle = M|\Phi_i\rangle = \sum_{k=1}^A m_{ik}|\Phi_i\rangle$$

Nmax truncation: Many-Body basis states satisfy

$$\sum_{\alpha \text{ occ.}}^{A} (2n+l)_{\alpha} \leq N_0 + N_{\max}$$

 $N_{\rm max}$ runs from zero to computational limit. $(N_{\rm max}, \hbar\Omega)$ fix HO basis

Alternatives:

- Full Configuration Interaction (single-particle basis truncation)
- Importance Truncation
 Roth, PRC79, 064324 (2009)
- No-Core Monte-Carlo Shell Model Abe et al, PRC86, 054301 (2012)
- SU(3) Truncation Dytrych *et al*, PRL111, 252501 (2013)

Calculation of three-body forces at N³LO



Goal

Calculate matrix elements of 3NF in a partialwave decomposed form which is suitable for different few- and many-body frameworks

Challenge

Due to the large number of matrix elements, the calculation is extremely expensive.

Strategy

Develop an efficient code which allows to treat arbitrary local 3N interactions. (Krebs and Hebeler) Initial LENPIC Collaboration results: Chiral NN results for ⁶Li by Chiral order Orange: Chiral order uncertainties; Blue/Green: Many-body method uncertainties S. Binder, et al, Phys. Rev. C **93**, 044002 (2016); arXiv:1505.07218





S. Binder, et al., LENPIC Collaboration, in preparation

Preliminary LENPIC results with Chiral NN only and R = 1.0 fm, IA for operator S. Binder, et al., LENPIC Collaboration, in preparation



Dirac's forms of relativistic dynamics [Dirac, Rev. Mod. Phys. **21**, 392–1949] Instant form is the well-known form of dynamics starting with $x^0 = t = 0$ $K^i = M^{0i}$, $J^i = \frac{1}{2} \varepsilon^{ijk} M^{jk}$, $\varepsilon^{ijk} = (+1, -1, 0)$ for (cyclic, anti-cyclic, repeated) indeces Front form defines relativistic dynamics on the light front (LF): $x^+ = x^0 + x^3 = t + z = 0$

$$P^{\pm} \triangleq P^0 \pm P^3$$
, $\vec{P}^{\perp} \triangleq (P^1, P^2)$, $x^{\pm} \triangleq x^0 \pm x^3$, $\vec{x}^{\perp} \triangleq (x^1, x^2)$, $E^i = M^{+i}$, $E^+ = M^{+-}$, $F^i = M^{-i}$



Adapted from talk by Yang Li

Discretized Light Cone Quantization Pauli & Brodsky c1985

Basis Light Front Quantization*

$$\phi(\vec{x}) = \sum_{\alpha} \left[f_{\alpha}(\vec{x}) a_{\alpha}^{+} + f_{\alpha}^{*}(\vec{x}) a_{\alpha} \right]$$

Operator-valued distribution function

where $\{a_{\alpha}\}$ satisfy usual (anti-) commutation rules.

Furthermore, $f_{\alpha}(\vec{x})$ are arbitrary except for conditions:

Orthonormal: $\int f_{\alpha}(\vec{x}) f_{\alpha'}^{*}(\vec{x}) d^{3}x = \delta_{\alpha\alpha'}$ Complete: $\sum f_{\alpha}(\vec{x}) f_{\alpha}^{*}(\vec{x}') = \delta^{3}(\vec{x} - \vec{x}')$

=> Wide range of choices for $f_a(\vec{x})$ and our initial choice is

$$f_{\alpha}(\vec{x}) = Ne^{ik^{+}x^{-}}\Psi_{n,m}(\rho,\varphi) = Ne^{ik^{+}x^{-}}f_{n,m}(\rho)\chi_{m}(\varphi)$$

*J.P. Vary, H. Honkanen, J. Li, P. Maris, S.J. Brodsky, A. Harindranath, G.F. de Teramond, P. Sternberg, E.G. Ng and C. Yang, PRC 81, 035205 (2010). ArXiv:0905:1411

Set of transverse 2D HO modes for n=4



J.P. Vary, H. Honkanen, J. Li, P. Maris, S.J. Brodsky, A. Harindranath, G.F. de Teramond, P. Sternberg, E.G. Ng and C. Yang, PRC 81, 035205 (2010). ArXiv:0905:1411

Symmetries & Constraints

$$\sum_{i} b_{i} = B$$

$$\sum_{i} e_{i} = Q$$

$$\sum_{i} (m_{i} + s_{i}) = J_{z}$$

$$\sum_{i} k_{i} = K$$
Finite basis regulators
$$\sum_{i} [2n_{i} + |m_{i}| + 1] \leq N_{max}$$
Global Color Singlets (QCD)
Light Front Gauge
Optional - Fock space cutoffs
$$H \rightarrow H + \lambda H_{CM}$$

Effective Yukawa Model in BLFQ Wenyang Qian, et al. in preparation



LF treatment:

Approximate the contact term by heavy scalar boson exchange + effective one pion exchange

Basis Light-Front Quantization(BLFQ) Approach:

Hamiltonian formalism Relativistic theory Light-front wave functions provides direct access to all physical observables

* R. Machleidt, D.R. Entem, Phys.Rept.503:1-75 (2011)



Light Front (LF) Hamiltonian Defined by its Elementary Vertices in LF Gauge



Light-Front Regularization and Renormalization Schemes

- 1. Regulators in BLFQ (Ω , N_{max}, K)
- 2. Additional Fock space truncations (if any)
- 3. Counterterms identified/tested*
- 4. Sector-dependent renormalization**
- 5. SRG & OLS in NCSM*** adapted to BLFQ (future)

*D. Chakrabarti, A. Harindranath and J.P. Vary, Phys. Rev. D **69**, 034502 (2004) *P. Wiecki, Y. Li, X. Zhao, P. Maris and J.P. Vary, Phys. Rev. D **91**, 105009 (2015)

**V. A. Karmanov, J.-F. Mathiot, and A. V. Smirnov,
Phys. Rev. D 77, 085028 (2008); Phys. Rev. D 86, 085006 (2012)
**Y. Li, V.A. Karmanov, P. Maris and J.P. Vary,
Phys. Letts. B. 748, 278 (2015); arXiv: 1504.05233

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***B.R. Barrett, P. Navratil and J.P. Vary,
Prog. Part. Nucl. Phys. 69, 131 (2013)
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Light-Front Schrödinger equation for quark – antiquark systems

$$V = \kappa^{2} \zeta_{\perp}^{2} - \frac{\kappa^{4}}{4m_{q}^{2}} \partial_{x} (x(1-x)\partial_{x}) - \frac{C_{F}4\pi\alpha_{s}(Q^{2})}{Q^{2}} \overline{u}_{s}(k)\gamma^{\mu}u_{s}'(k')\overline{v_{s'}}(\overline{k}')\gamma^{v}v_{\overline{s}}(\overline{k})d_{\mu\nu}$$

$$Krautgartner-Pauli-Wolz one-gluon exchange with running coupling$$

- ► Implement quark masses with longitudinal degree of freedom: identical to LFH in both chiral limit and NR limit; X(x) ~ x^a(1 − x)^b
- Introduce short-distance physics with spinor structure [cf. Głazek '17]
- Self-energy and dynamical chiral symmetry breaking to be implemented
- Basis light-front quantization

[Vary et al, PRC 2010]

Spectroscopy

[Li, Maris & Vary, PRD '17; Tang, Li, Maris & Vary, in preparation]





Energy Physics 2015, 14 (2015). C. Hughes et al., Phys. Rev. D 92, 094501 (2015).

Anji Yu, et al., in preparation Baryons $\begin{array}{ccc} (1-\xi)(1-x) & \vec{k}_{\perp} \equiv \vec{k}_{1\perp} = \frac{x_1 \vec{p}_{2\perp} - x_2 \vec{p}_{1\perp}}{x_1 + x_2} & \vec{b}_{\perp} \equiv \vec{b}_{1\perp} = \vec{r}_{2\perp} - \vec{r}_{1\perp} \\ \hline \vec{b}_{\perp} & 1-x \\ \hline \vec{r}_{\perp} & \xi(1-x) & \vec{p}_{\perp} \equiv \vec{k}_{2\perp} = \vec{p}_{3\perp} - x_3 \vec{P}_{\perp} & \vec{r}_{\perp} \equiv \vec{b}_{2\perp} = \frac{\vec{r}_{3\perp} - \vec{R}_{\perp}}{1-x_3} \end{array}$ P^+ $egin{array}{ccc} \overline{x} & ec{P}_{ot} = \sum^3 ec{p}_{aot} & ec{R}_{ot} = \sum^3 x_a ec{r}_{aot} \end{array}$ **Effective Hamiltonian** Kinetic energy Harmonic oscillator confining potential $H_{\rm eff} = \frac{\vec{p}_{\perp}^2 + m_3^2}{x} + \frac{1}{1-x} \left[\vec{p}_{\perp}^2 + \frac{\vec{k}_{\perp}^2 + m_2^2}{\xi} + \frac{\vec{k}_{\perp}^2 + m_1^2}{1-\xi} \right] + \kappa^4 x (1-x) \vec{r}_{\perp}^2 + \kappa^4 (1-x) \xi (1-\xi) \vec{b}_{\perp}^2$ $-\frac{\kappa^4}{(m_1+m_2+m_3)^2} \left[\partial_x \left(x(1-x)\partial_x\right) + \frac{1}{1-x}\partial_\xi \left(\xi(1-\xi)\partial_\xi\right)\right] + \text{const.}$

Longitudinal confining potential

Mass eig

Jenvalue

$$\begin{bmatrix} M_{\rm L}^2 = (m_1 + m_2)^2 + \frac{m_1 + m_2}{m_1 + m_2 + m_3} \kappa^2 (2{\rm L} + 1) + \frac{\kappa^4}{(m_1 + m_2 + m_3)^2} {\rm L}({\rm L} + 1) \end{bmatrix}$$

$$E_{{\rm n}_1,{\rm m}_1,{\rm n}_2,{\rm m}_2,{\rm L},{\rm I}} = (m_3 + M_{\rm L})^2 + 2\kappa^2 (2{\rm n}_1 + |{\rm m}_1| + 2{\rm n}_2 + |{\rm m}_2| + 2)$$

$$+ \frac{M_{\rm L} + m_3}{m_1 + m_2 + m_3} \kappa^2 (2{\rm I} + 1) + \frac{\kappa^4}{(m_1 + m_2 + m_3)^2} {\rm I}({\rm I} + 1) + \text{const.}$$

Eigenfunction

$$\Phi_{\mathtt{n}_1 \mathtt{m}_1 \mathtt{n}_2 \mathtt{m}_2 \mathtt{L1}}(ec{p}_{\perp}, x, ec{k}_{\perp}, \xi) = \phi_{\mathtt{n}_1 \mathtt{m}_1}(ec{q}_{1\perp}) \chi_{\mathtt{L}}^{(A,B)}(\xi) \phi_{\mathtt{n}_2 \mathtt{m}_2}(ec{q}_{2\perp}) \chi_{1}^{(lpha_{\mathtt{L}},eta)}(x).
onumber \ A = 2m_1(m_1 + m_2 + m_3)/\kappa^2, B = 2m_2(m_1 + m_2 + m_3)/\kappa^2, \ lpha_{\mathtt{L}} = 2M_L(m_1 + m_2 + m_3)/\kappa^2, eta = 2m_3(m_1 + m_2 + m_3)/\kappa^2 \quad ec{q}_{1\perp} \equiv rac{ec{k}_{\perp}}{\sqrt{(1 - x)\xi(1 - \xi)}} \quad ec{q}_{2\perp} \equiv rac{ec{p}_{\perp}}{\sqrt{x(1 - x)}}$$



Brodsky, S. J., de Téramond, G. F., Dosch, H. G., & Erlich, J., "Light-front holographic QCD and emerging confinement", *Physics Reports*, 584, 1 (2015)



Measuring VM LFWF

Diffractive VM production $\sigma_{\rm tot} \sim \Psi_{photon} \circledast \sigma_{\rm dipole} \circledast \Psi_{VM}$ □Photon LFWF can be calculated from first principles. **Dipole cross section can be** γ^* 20000 obtained by measuring the inclusive DIS cross section. Provide measurements of VM A. Mueller, '90 N. Nikolaev, '91 LFWF and gluon distribution. K. Golec-Biernat et al., '99

Confront existing data

• In agreement with HERA, RHIC and LHC data.



Chen, Li, Maris, Tuchin and Vary, PLB 769, 477, 2017

Prediction for future experiment

Guangyao Chen, et al., in preparation

Electron Ion Collider--high luminosity, wide kinematic range.

Enable precision measurement of VM LFWF,

especially the higher excited states. $y^*_{p \rightarrow \Psi(2s)p}$



Chen, Li, Maris, Tuchin and Vary, PLB 769, 477, 2017

Looking ahead: under what conditions do we require a quark-based description of nuclear structure? "Quark Percolation in Cold and Hot Nuclei"



Sketch: hierarchy of strong interaction scales

Effective Field Theory	Scale	Range of Q	Phenomena
Pionless	Chiral symmetry breaking ~Λ _{QCD} ~ m _N	Q < m _π ~ k _F Q ~ 0.2 k _F	Scattering lengths Stellar burning Halo nuclei Clustering,
Pionfull, Deltafull	Chiral symmetry breaking ~ Λ _{QCD} ~ m _N	Q < m _N Q ~ m _π	Low Energy Nuclear Structure & Reactions ¹⁴ C anomalous lifetime Tetraneutron,
Quark Clusters	Chiral symmetry crossover transition $\sim (1 - 4) \Lambda_{QCD}$ $\sim (1 - 4) m_N$	Q < (1 - 4) m _N Q ~ m _N	X > 1 staircase EMC effect Quark percolation Color conducting drops Deconfining fluctuations,
QCD	Chiral symmetry restoration	Q < m _{Planck}	Asymptotic freedom pQCD domain sQCD-Quark-Gluon Plasma Color glass condensate Hadron tomography,

Conclusions and Outlook

- Chiral EFT is making rapid progress for nuclear structure at low Q
- BLFQ/tBLFQ are practical approaches to light-front QFT
- Provide a pathway to understand nuclei at high resolution
- Next goal: two-baryon systems with effective LF Hamiltonians from chiral EFT to quark-gluon systems
- Next goal: mesons and baryons with one dynamical gluon
- Future: EFT at the quark-percolation scale