Nuclear Forces and Currents in Chiral Effective Field Theory

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Polarized light ion physics with EIC
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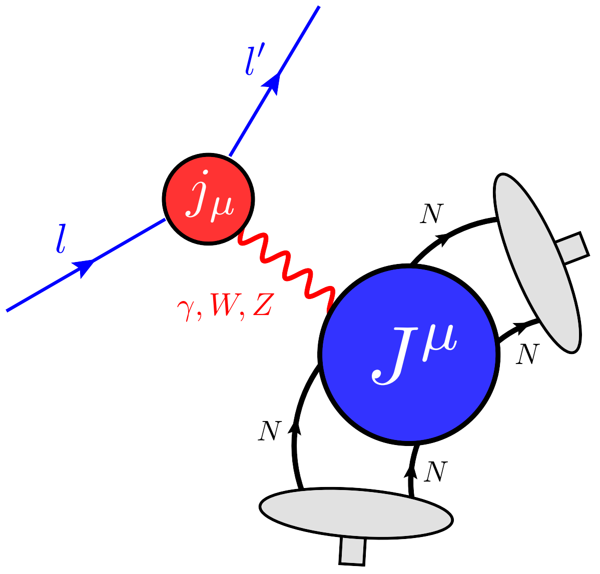
Outline

- Nuclear forces in chiral EFT
  - Introduction to chiral EFT
  - Long & short-range physics
  - Role of pion-nucleon scattering
  - Selective applications and role of 3NF

- Nuclear current in chiral EFT
  - Symmetries for currents
  - Nuclear currents up to N³LO



ChPT and low energy QCD

Spontaneous + explicit (by small quark masses) breaking of chiral symmetry in QCD

$\Downarrow$

Existence of light weakly interacting Goldstone bosons

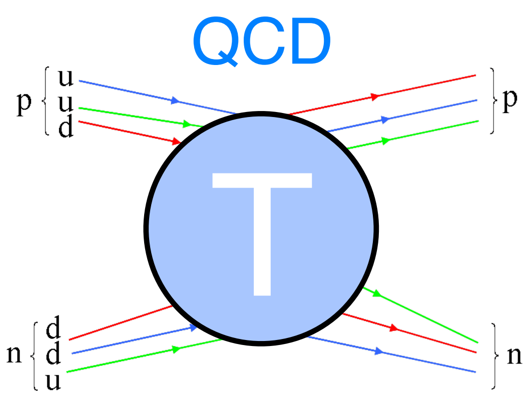
$\Downarrow$

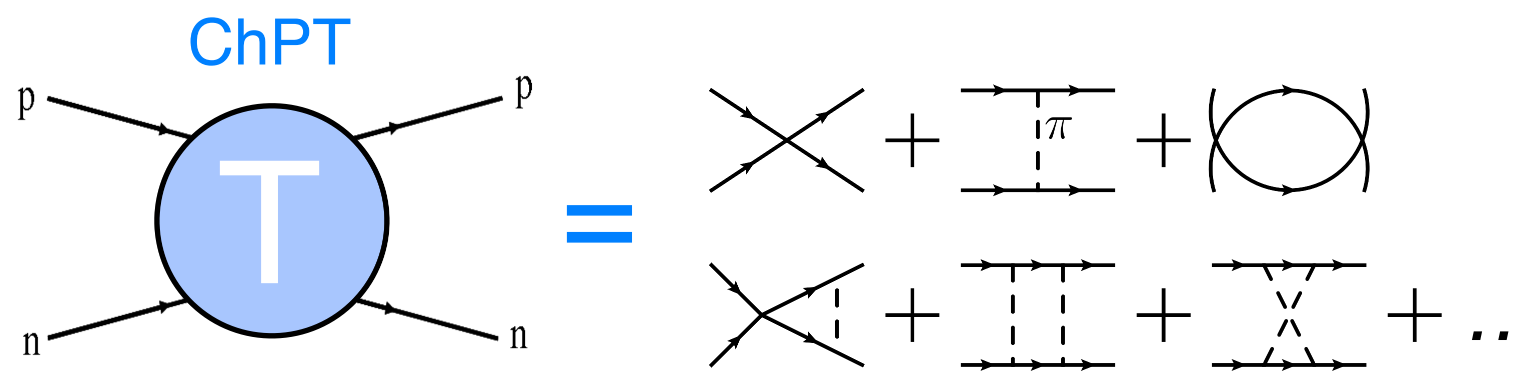
Chiral Perturbation theory (ChPT)
Expansion in small momenta and masses of Goldstone bosons

$\Downarrow$

Systematic description of QCD by ChPT in low energy sector
(low momenta and masses $q, M_\pi \ll \Lambda \simeq 1\,\mathrm{GeV}$)

From QCD to nuclear physics

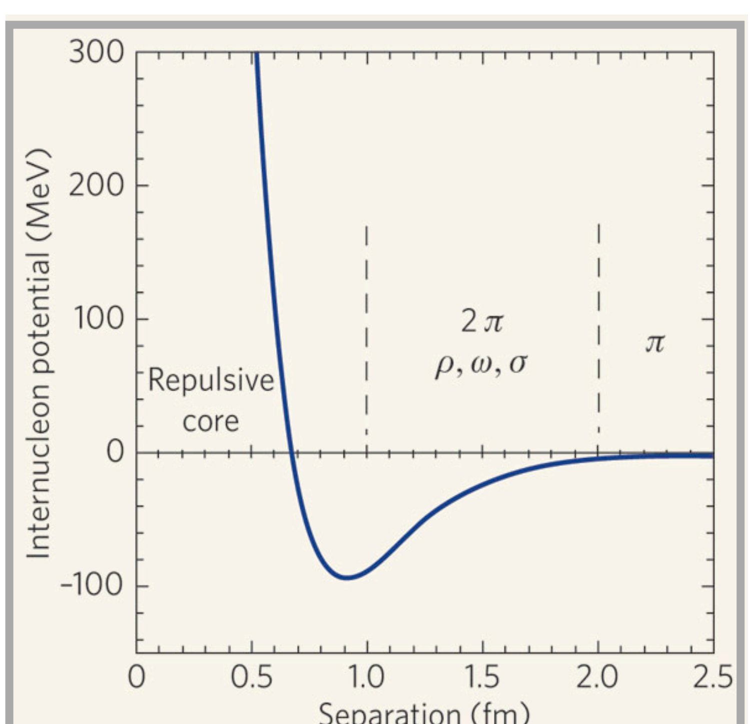


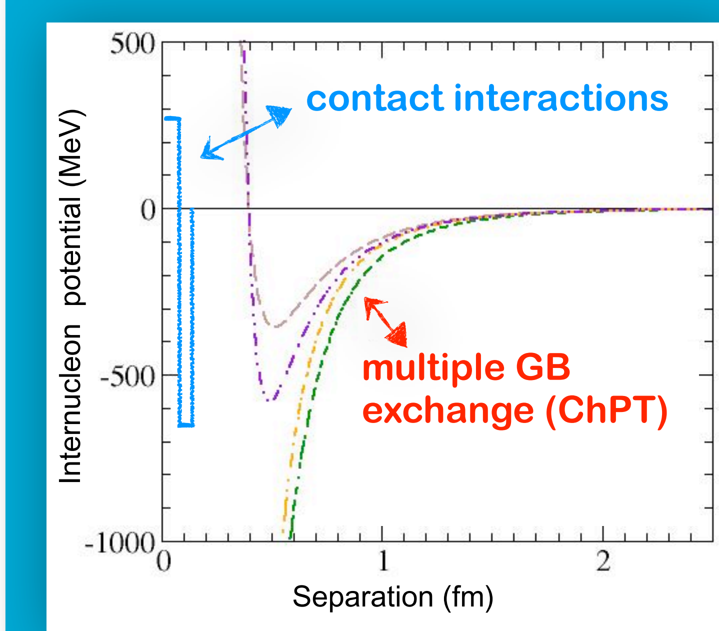


- **NN interaction is strong**: resummations/nonperturbative methods needed
- $1/m_N$ - expansion: nonrelativistic problem ($|\vec{p}_i| \sim M_\pi \ll m_N$) ⇨ the QM A-body problem

$$\left[\left(\sum_{i=1}^{A} \frac{-\vec{\nabla}_i^2}{2m_N} + \mathcal{O}(m_N^{-3})\right) + \underbrace{V_{2N} + V_{3N} + V_{4N} + \ldots}_{\textit{derived within ChPT}}\right]|\Psi\rangle = E|\Psi\rangle$$

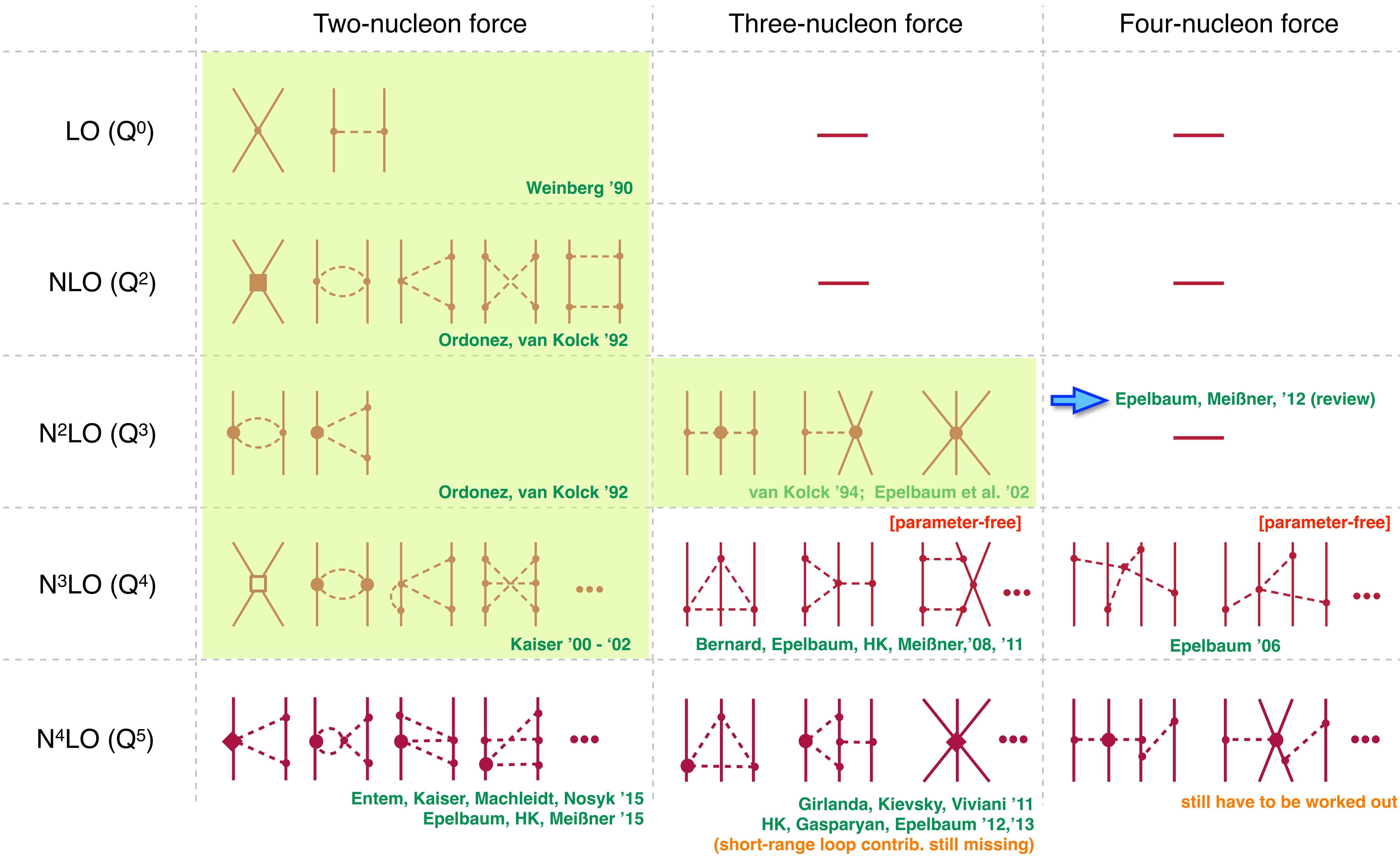
Weinberg '91





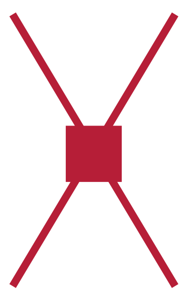
- unified description of $\pi\pi$, πN and NN
- consistent many-body forces and currents
- systematically improvable
- bridging different reactions (electroweak, π-prod., ...)
- precision physics with/from light nuclei

Chiral Expansion of the Nuclear Forces



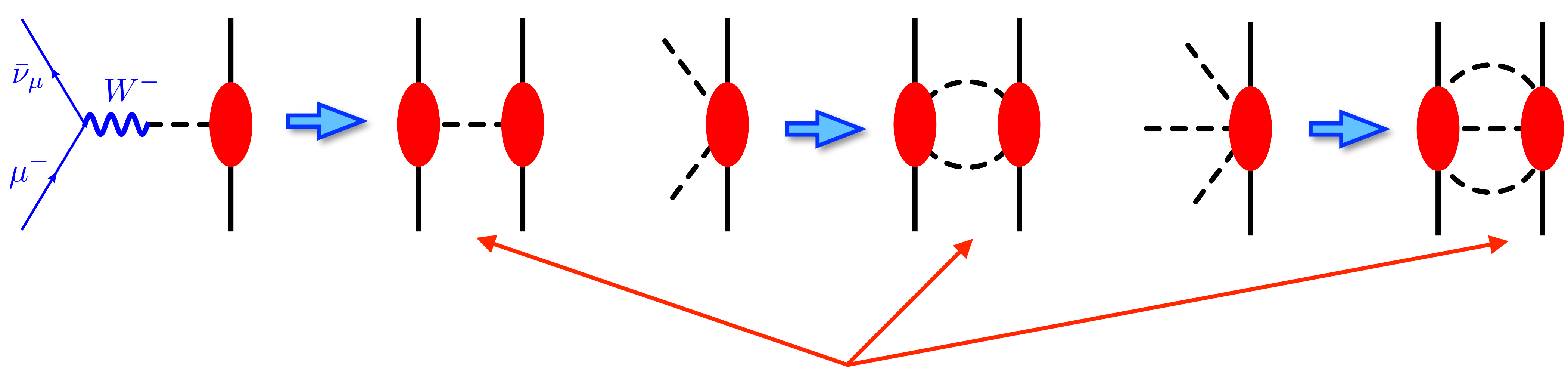
Long and Short Range Interactions

- Couplings of short-range interactions are fixed from NN - data.
  In the isospin limit we have:



LO [Q^0]: 2 operators (S-waves)
NLO [Q^2]: + 7 operators (S-, P-waves and ε_1)
N^{2}LO [Q^3]: no new terms
N^{3}LO [Q^4]: + 12 operators (S-, P-, D-waves and ε_1, ε_2)
N^{4}LO [Q^5]: no new terms

- Long range part of the nuclear forces are predictions (chiral symmetry of QCD)
  once couplings from single-nucleon subprocess are determined



NN parameter free predictions

Pion-Nucleon Scattering

- Effective chiral Lagrangian:

low-energy constants

$$\mathcal{L}_\pi = \mathcal{L}_\pi^{(2)} + \mathcal{L}_\pi^{(4)} + \ldots$$

$$\mathcal{L}_{\pi N} = \underbrace{\bar{N}\left(i\gamma^\mu D_\mu[\pi] - m + \frac{g_A}{2}\gamma^\mu\gamma_5 u_\mu[\pi]\right)N}_{\mathcal{L}_{\pi N}^{(1)}} + \underbrace{\sum_i \boldsymbol{c}_i \bar{N}\hat{O}_i^{(2)}[\pi]N}_{\mathcal{L}_{\pi N}^{(2)}} + \underbrace{\sum_i \boldsymbol{d}_i \bar{N}\hat{O}_i^{(3)}[\pi]N}_{\mathcal{L}_{\pi N}^{(3)}} + \underbrace{\sum_i \boldsymbol{e}_i \bar{N}\hat{O}_i^{(4)}[\pi]N}_{\mathcal{L}_{\pi N}^{(4)}} + \ldots$$

- Pion-nucleon scattering is calculated up to Q^4 in heavy-baryon ChPT

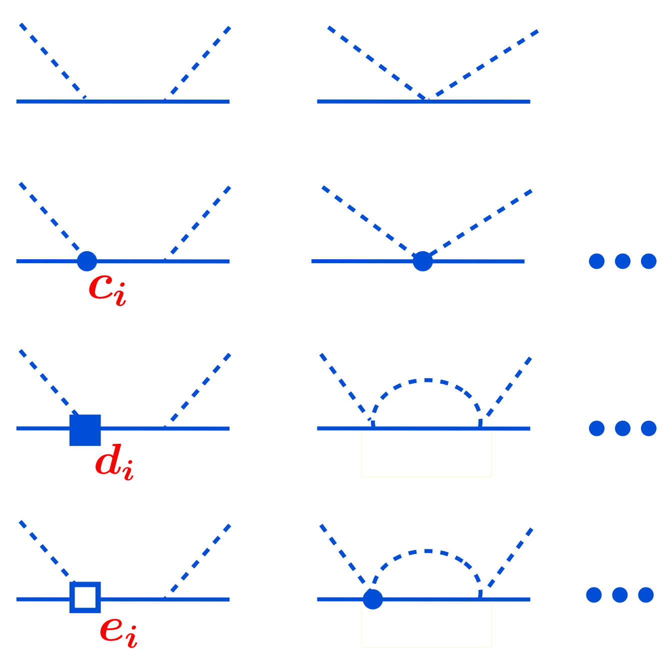
Fettes, Meißner '00; HK, Gasparyan, Epelbaum '12

Order Q:

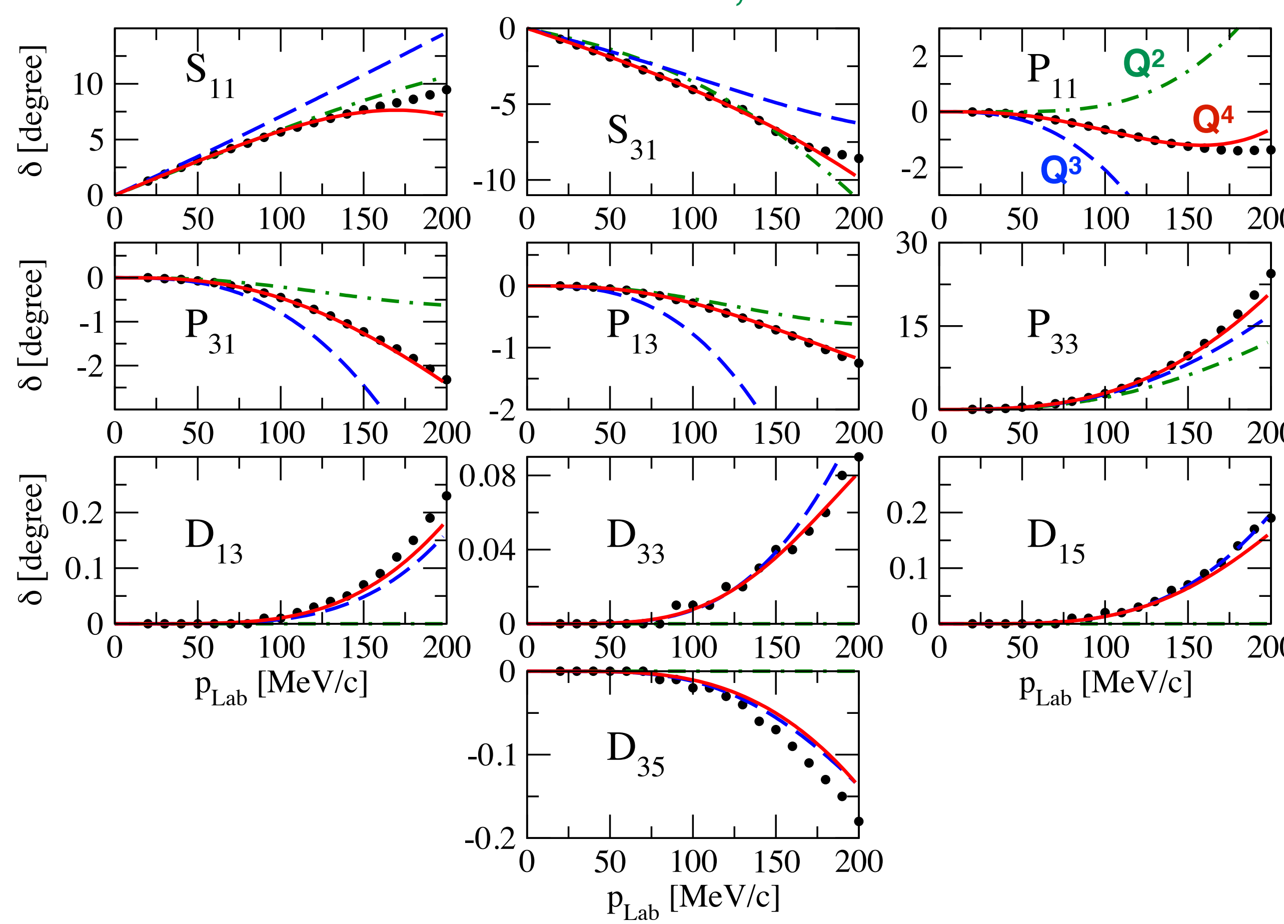
Order Q^2:

Order Q^3:

Order Q^4:



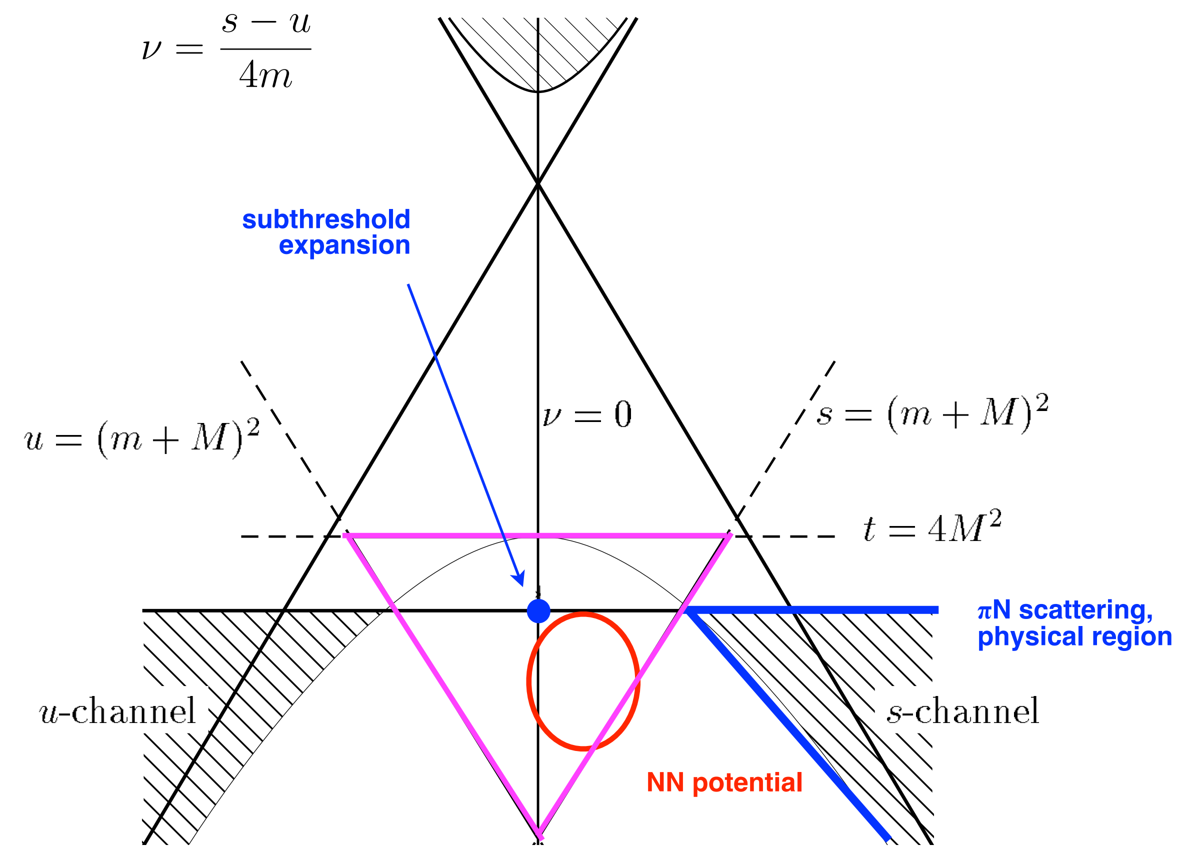
Karlsruhe-Helsinki PWA, Koch et al. '86



Dispersive analysis of πN scattering

- Roy-Steiner equations for πN scattering
  Hoferichter et al., Phys. Rept. 625 (16) 1

Partial Wave Decomposition of
Hyperbolic dispersion relations
$\pi N \to \pi N$ & $\pi\pi \to \bar{N}N$ channels



Input:
S- and P-waves above $s_{\rm m} = (1.38\,{\rm GeV})^2$
Higher partial waves for all *s*
Inelasticities for $s < s_m$ and scattering lengths

Output:
S- and P-waves with error bands, σ-term,
Subthreshold coefficients $\bar{X} = \sum_{m,n} x_{mn}\, \nu^{2m+k} t^n, \qquad X = \{A^\pm, B^\pm\}$

- c_i, d_i, e_i are fixed from subthreshold coefficients (within Mandelstam triangle where one expects best convergence of chiral expansion)

- Subthreshold point is closer to kinematical region of NN force than the physical region of πN scattering

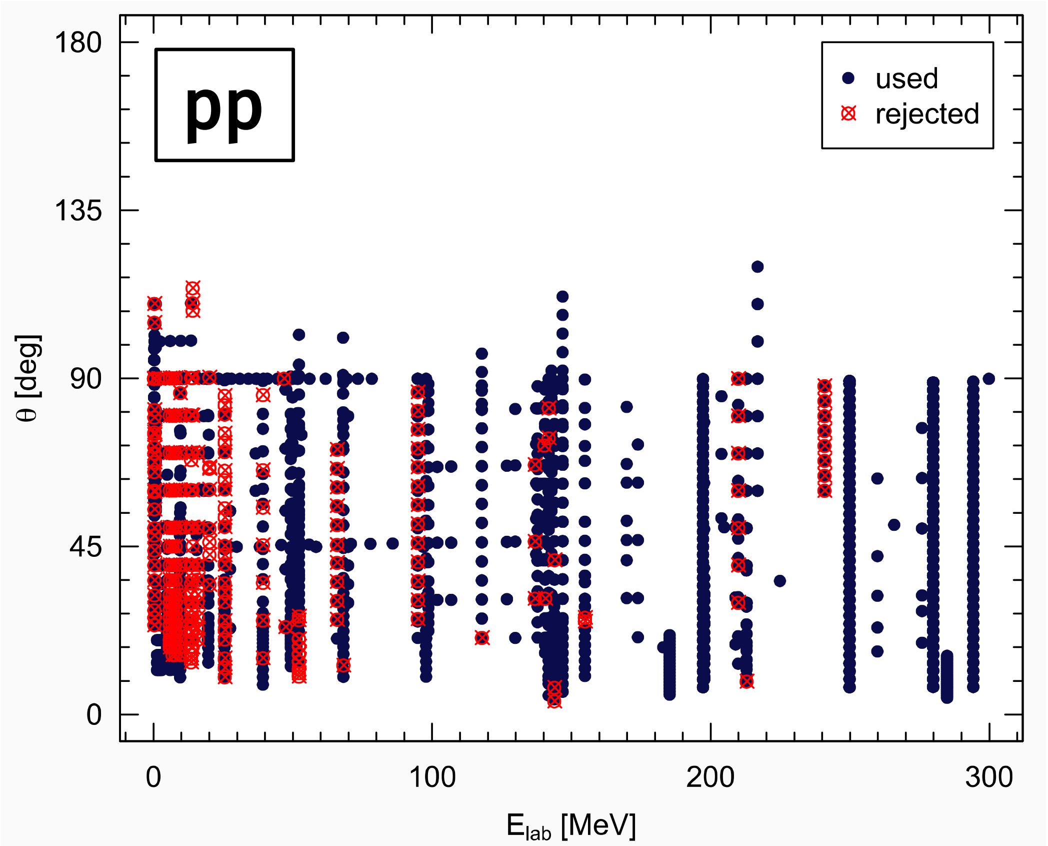
NN Data Used in the Fits

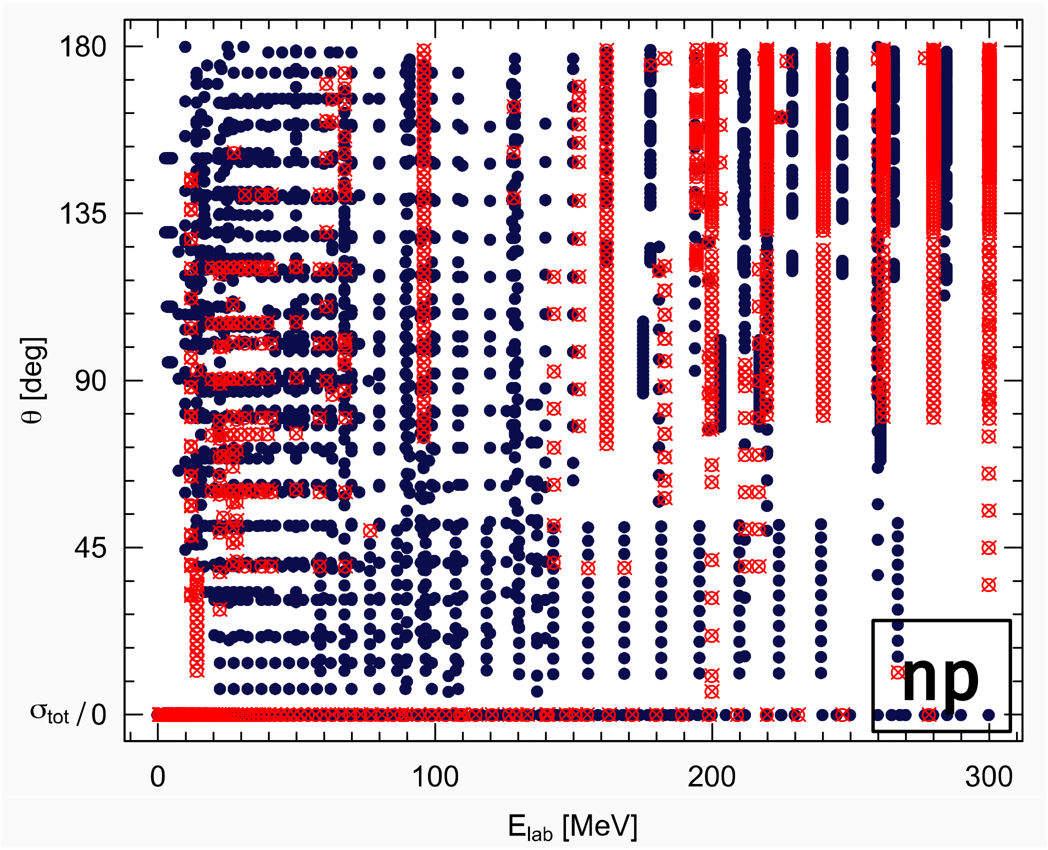
Reinert, HK, Epelbaum '17

- From 1950 on around 3000 proton-proton + 5000 neutron-proton scattering data below 350 MeV have been measured
- Not all of these data are compatible. Rejections are required to get a reasonable fit
- Granada 2013 base used: *Navarro Perez et al. '13* rejection by 3σ-criterion

  ⇨ 31% of np + 11% of pp data have been rejected

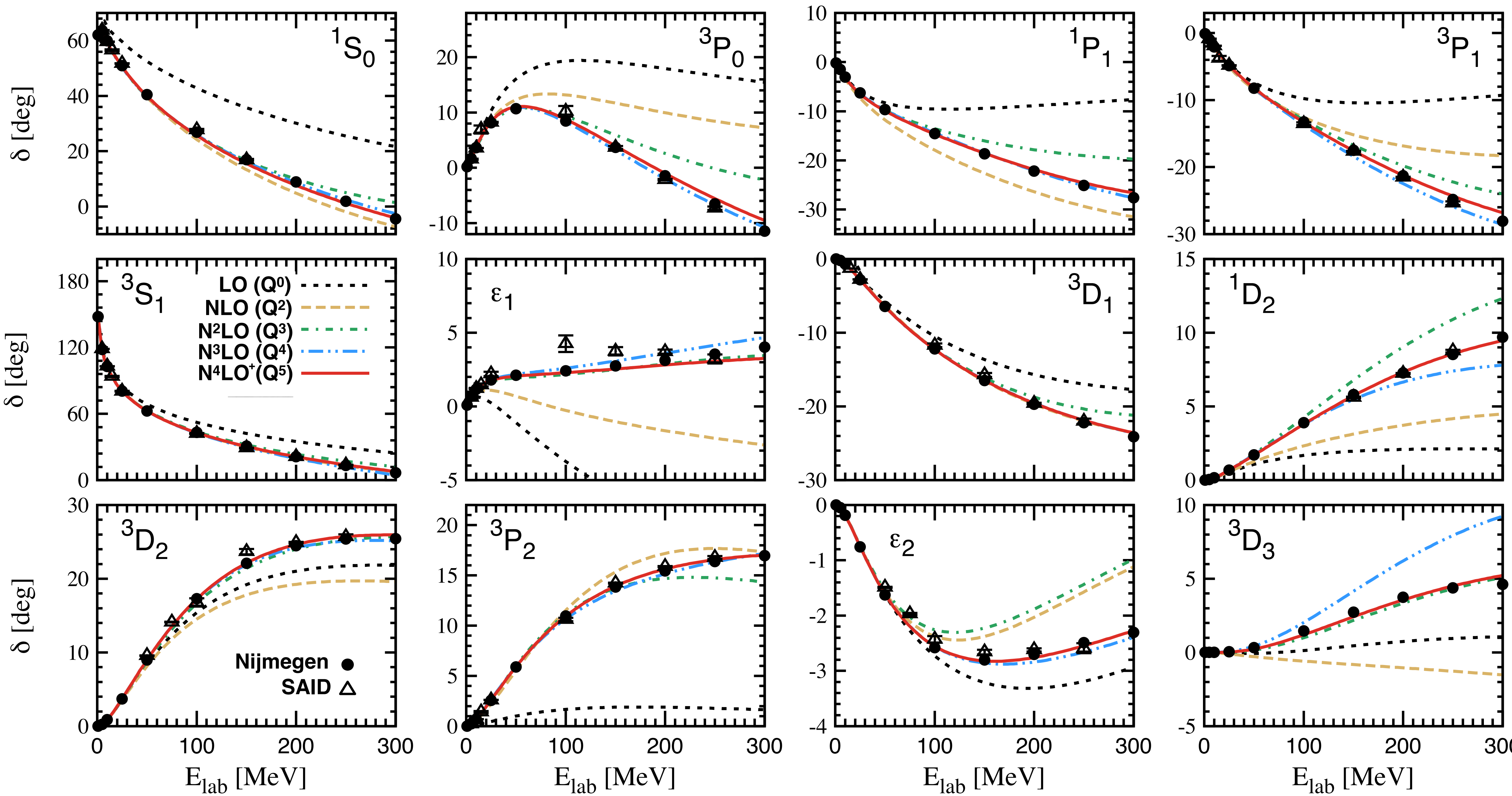
Resulting data base consists of 2697 np + 2158 pp data for E_{lab}=0-300 MeV





Chiral Expansion of np Phase Shifts

Reinert, HK, Epelbaum '17



- Good convergence of chiral expansion & excellent agreement with NPWA data
- Chiral potential match in precision phenomenological potentials (CD Bonn, Av18,…) with around 40% less parameter

Uncertainty Estimate

Epelbaum, HK, Meißner '15

- Uncertainties in the experimental data
- Uncertainties in the estimation of πN LECs
- Uncertainties in the determination of contact interaction LECs
- Uncertainties of the fits due to the choice of E_{max}
- Systematic uncertainty due to truncation of the chiral expansion at a given order

Estimate the uncertainty via expected size of higher-order corrections

For a N^{4}LO prediction of an observable $X^{\mathrm{N^4LO}}$ we get an uncertainty

$$\begin{aligned}\Delta X^{\mathrm{N^4LO}}(p) \;=\; \max\Big(& Q \times |X^{\mathrm{N^3LO}}(p) - X^{\mathrm{N^4LO}}(p)|, Q^2 \times |X^{\mathrm{N^2LO}}(p) - X^{\mathrm{N^3LO}}(p)|, \\ & Q^3 \times |X^{\mathrm{NLO}}(p) - X^{\mathrm{N^2LO}}(p)|, Q^4 \times |X^{\mathrm{LO}}(p) - X^{\mathrm{NLO}}(p)|, Q^6 \times |X^{\mathrm{LO}}(p)|\Big)\end{aligned}$$

with chiral expansion parameter $Q = \max\left(\frac{p}{\Lambda_b}, \frac{M_\pi}{\Lambda_b}\right)$

For σ_{tot} errors → 68% degree-of-belief intervals(Bayesian analysis): *Furnstahl et al. '15*

Uncertainty Quantification

Reinert, HK, Epelbaum '17

Effective range, deuteron properties and phase-shift with quantified uncertainty

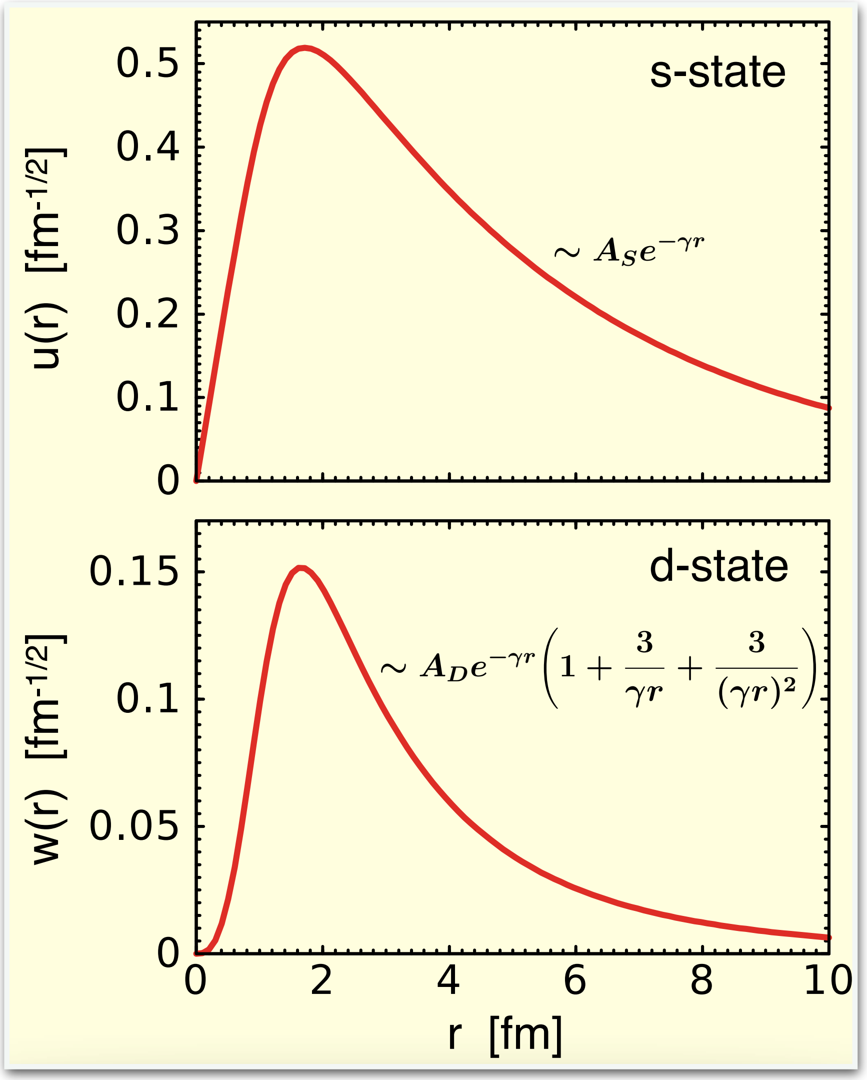
Example: deuteron asymptotic normalization

	$\Lambda = 400$ MeV	$\Lambda = 450$ MeV	$\Lambda = 500$ MeV	$\Lambda = 550$ MeV
A_S (fm$^{-1/2}$)	$0.8847^{(+3)}_{(-3)}(6)(4)(4)$	$0.8847^{(+3)}_{(-3)}(3)(5)(1)$	$0.8849^{(+3)}_{(-3)}(1)(7)(0)$	$0.8851^{(+3)}_{(-3)}(3)(8)(1)$
η	$0.0255^{(+1)}_{(-1)}(1)(3)(2)$	$0.0255^{(+1)}_{(-1)}(1)(4)(1)$	$0.0257^{(+1)}_{(-1)}(1)(5)(1)$	$0.0258^{(+1)}_{(-1)}(1)(5)(1)$

truncation error, statistical error, πN LECs, variation of E_{max}

$$A_S = 0.8847^{(+3)}_{(-3)}(3)(5)(1)\ \mathrm{fm}^{-1/2}$$

$$\eta \equiv \frac{A_D}{A_S} = 0.0255^{(+1)}_{(-1)}(1)(4)(1)$$



Exp: $A_S = 0.8781(44)\ \mathrm{fm}^{-1/2}$, $\eta = 0.0256(4)$

Borbely et al. '85 *Rodning, Knutson '90*

Nijmegen PWA [errors are „educated guesses"] *Stoks et al. '95*

$$A_S = 0.8845(8)\ \mathrm{fm}^{-1/2}, \quad \eta = 0.0256(4)$$

Granada PWA [errors purely statistical] *Navarro Perez et al. '13*

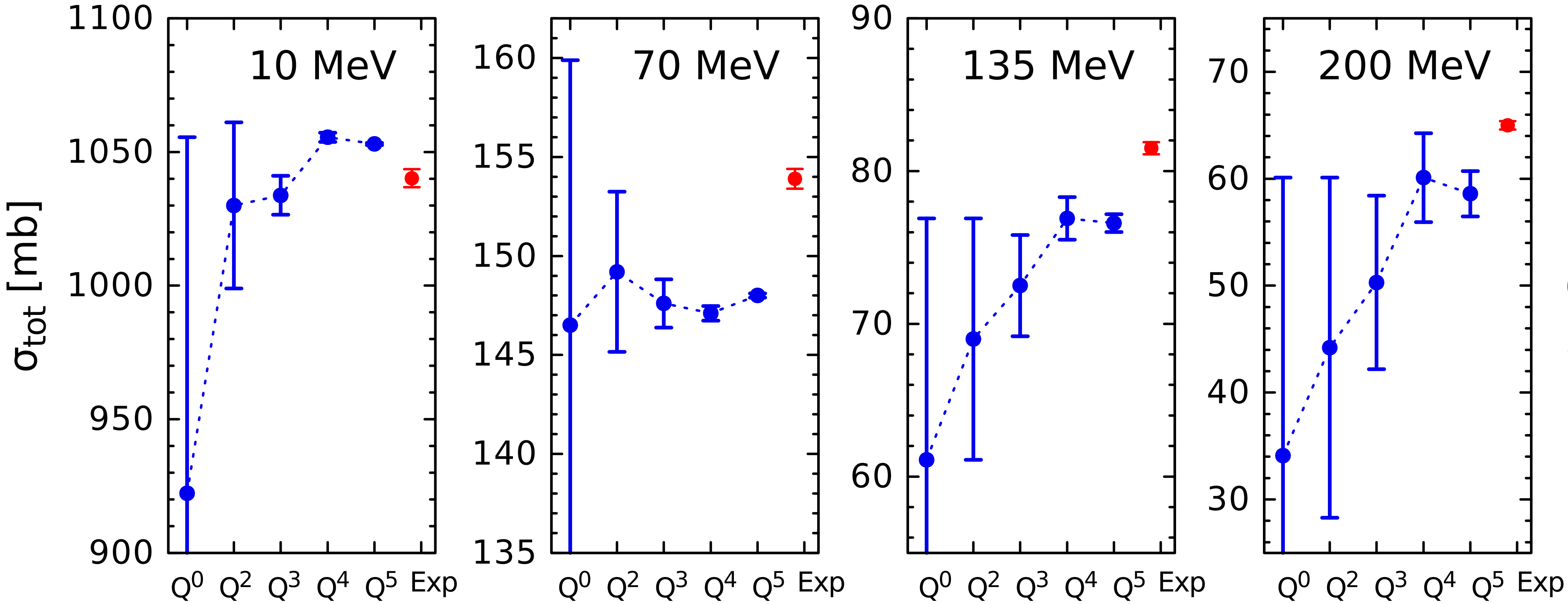
$$A_S = 0.8829(4)\ \mathrm{fm}^{-1/2}, \quad \eta = 0.0249(1)$$

Role of the 3NFs

NN-force from Epelbaum, HK, Meißner '15

Total cross section for Nd scattering: without 3NF

LENPIC collaboration, Binder et al. '15



Clear evidence of missing 3NFs at higher energy

- Significant discrepancy between experiment and theory
- The discrepancy at 10 MeV is much lower than at other energies

Cross section at low energy is governed by S-wave spin-doublet and spin-quartet Nd scattering lengths: ${}^4a \gg {}^2a$ (one order of magnitude) and ${}^4a$ is much less sensitive to 3NF (Pauli principle)

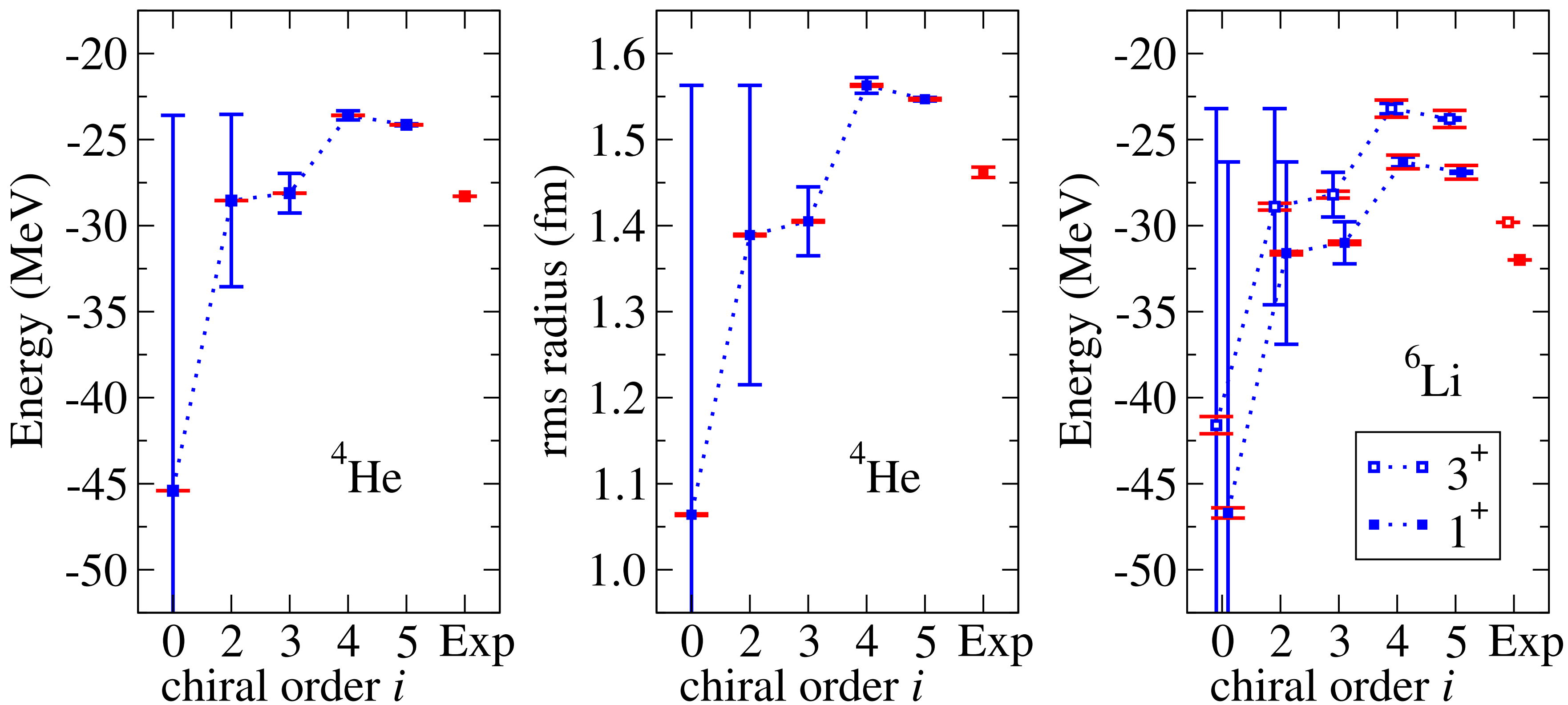
LENPIC: Low Energy Nuclear Physics International Collaboration

Role of the 3NFs for A > 3

NN-force from Epelbaum, HK, Meißner '15

Selected observables for ^{4}He & ^{6}Li

LENPIC collaboration, Binder et al. '15



Clear evidence of missing 3NFs at higher energy

- Results for ^{4}He are obtained by solving Faddeev-Yakubovski eq. and No-Core Shell Model (NCSM) which agree within estimated uncertainties
- Results for ^{6}Li are obtained by NCSM with Similarity Renormalization Group(SRG) evolution (induced 3NF's are taken into account).

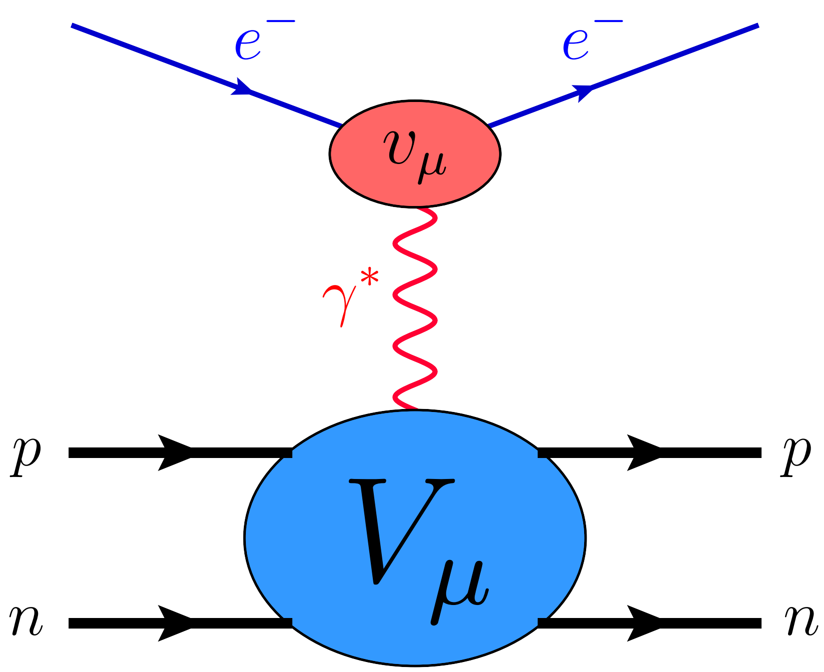
Summary

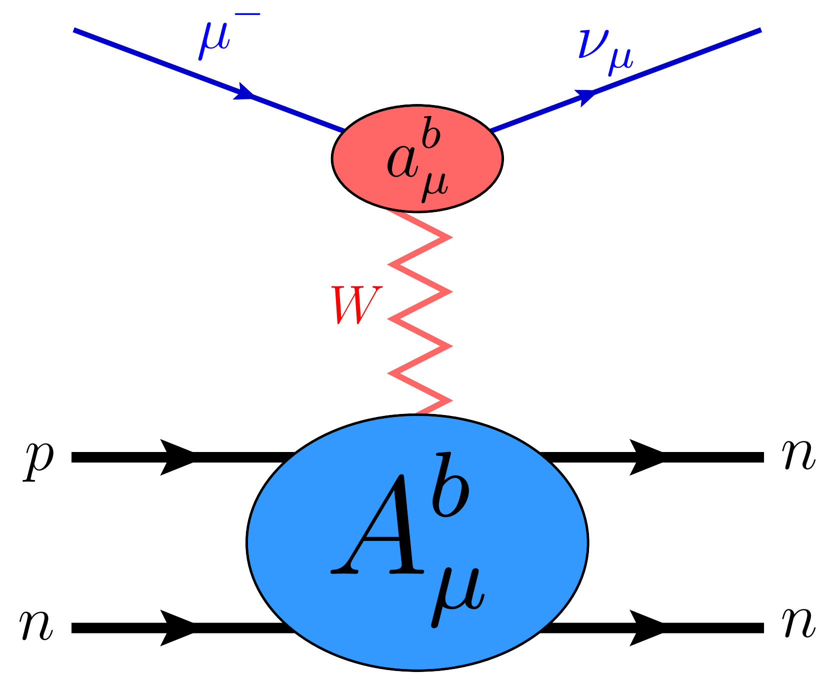
Chiral Nucleon-Nucleon Force

- Chiral nuclear NN forces are calculated up to N^4LO
- Phase-shifts, deuteron properties, … with uncertainty quantification
- Chiral NN force match in precision phenomenological potentials (CD Bonn, Av18,…) with around 40% less parameter
- Clear evidence for missing 3NF for $A > 2$

Nuclear currents in chiral EFT

Electroweak probes on nucleons and nuclei can be described by current formalism





Chiral EFT Hamiltonian depends on external sources

$$H[a, v, s, p]$$

Axial-vector source

Vector source

Scalar source

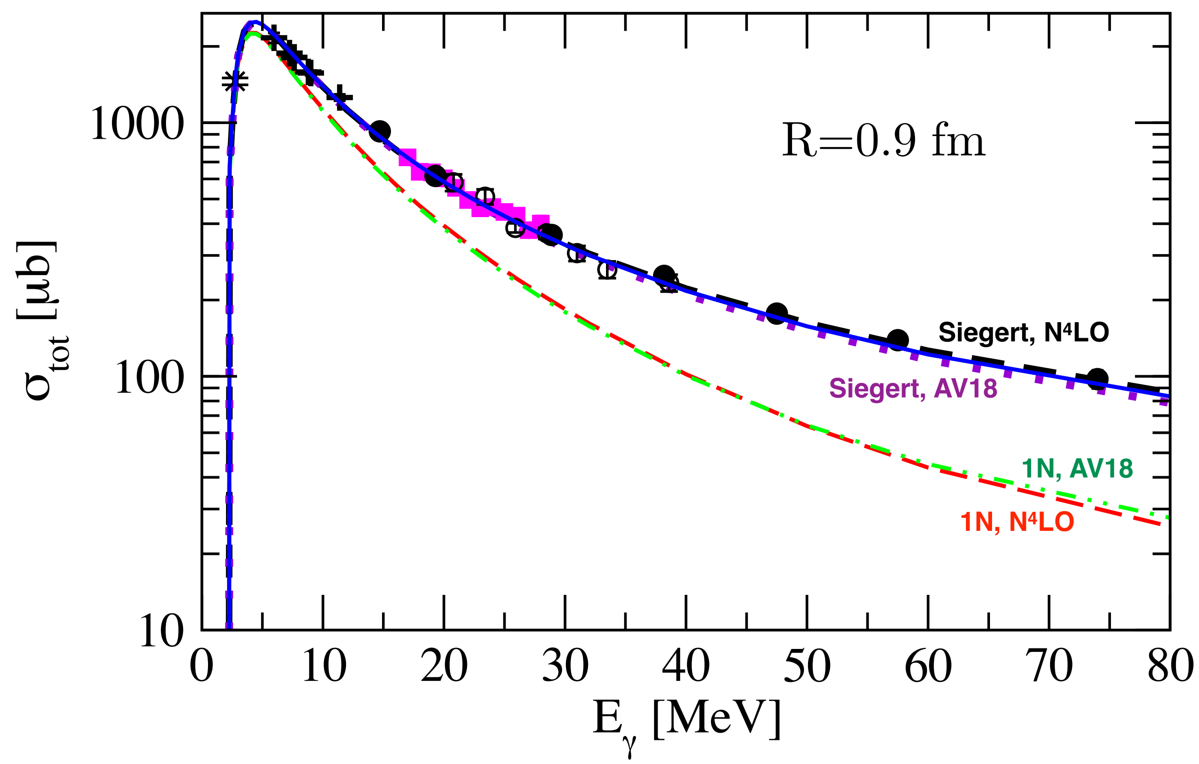
Pseudoscalar source

Siegert theorem + N4LO

Skibinski et al. PRC93 (2016) no. 6, 064002

Generate longitudinal component of NN current by continuity equation

$$\left[H_{\mathrm{strong}}, \boldsymbol{\rho}\right] = \vec{k} \cdot \vec{\boldsymbol{J}} \leftarrow$$ regularized longitudinal current (Siegert theorem)

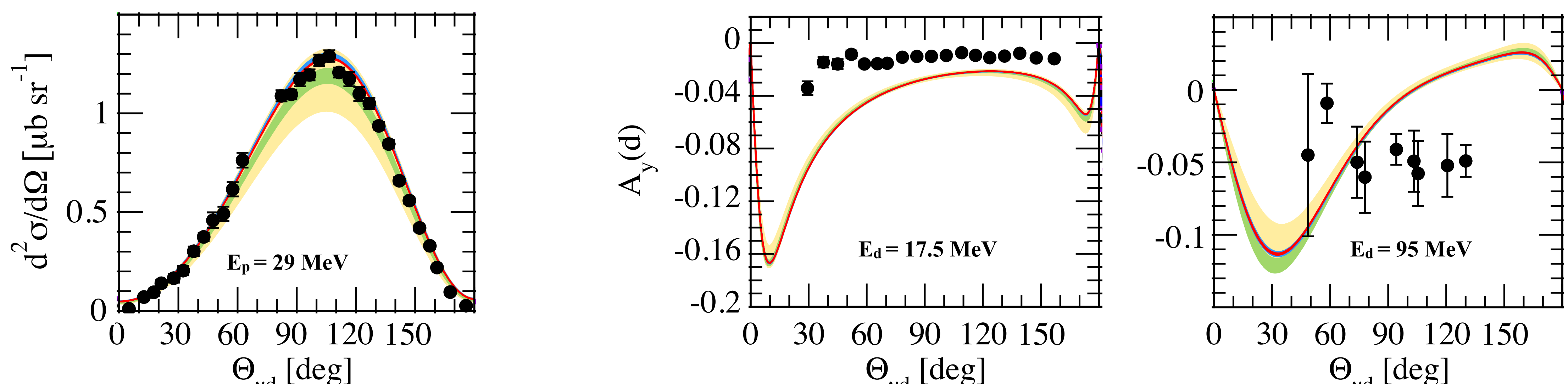


Deuteron photo-disintegration

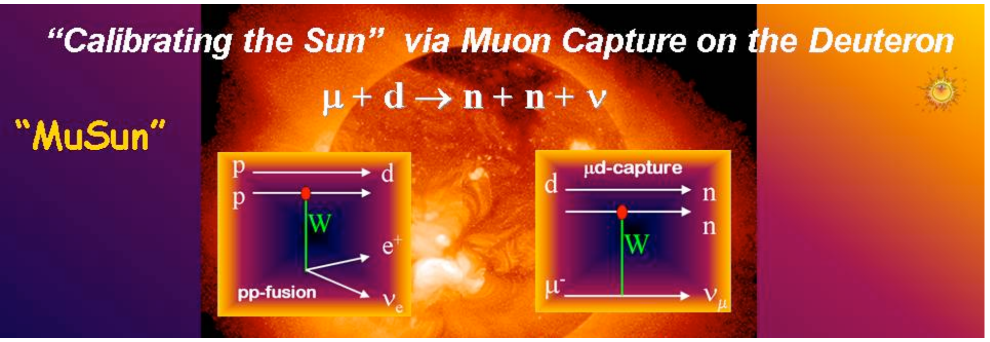
$$\gamma + d \rightarrow p + n$$

- consistent regularization via cont. eq.
- improvement by 1N+Siegert
- implementation of transverse part & exchange currents work in progress

Nucleon-deuteron radiative capture: $p(n) + d \rightarrow ^3\mathrm{H}(^3\mathrm{He}) + \gamma$

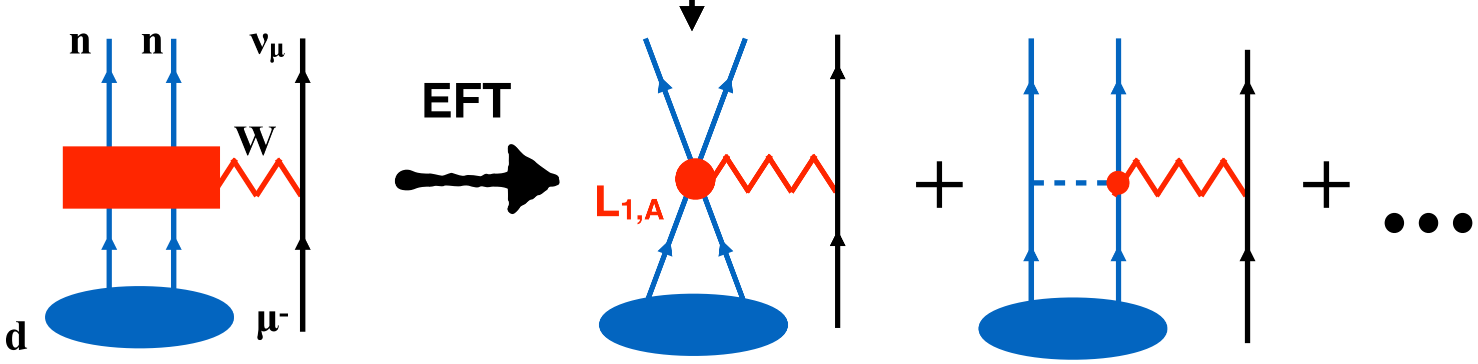


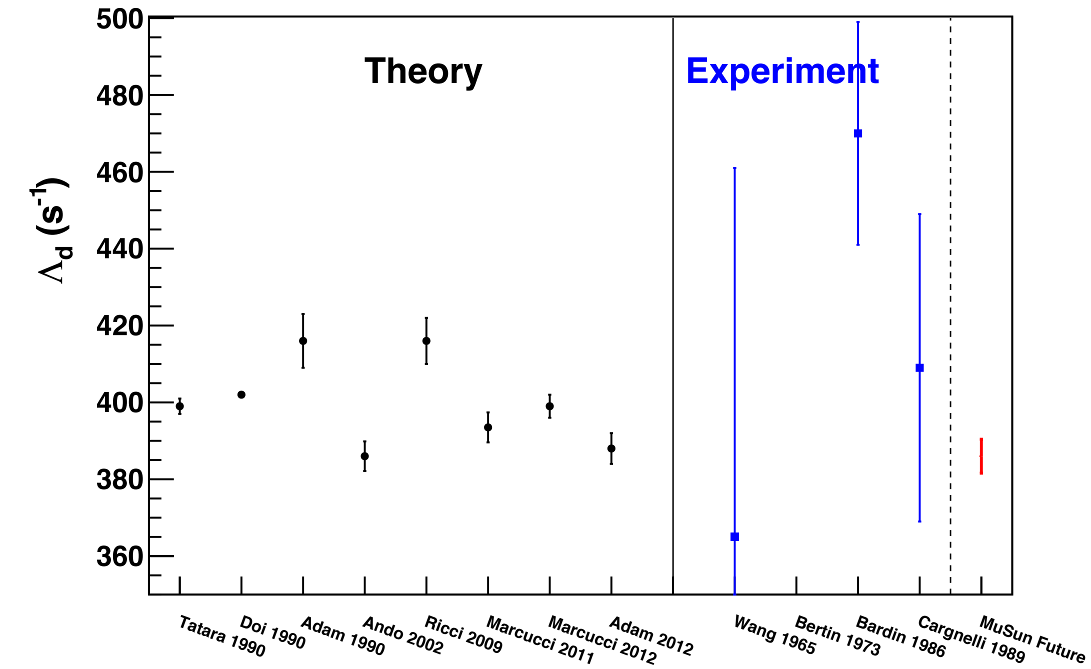
MuSun experiment at PSI



Main goal: measure the doublet capture rate Λ_d in
$\mu^- + d \to \nu_\mu + n + n$ with the accuracy of ~ 1.5%

This will strongly constrain the short-range axial current





The resulting axial exchange current can be used to make precision calculations for

- triton half life, $fT_{1/2}$ = 1129.6 ± 3.0 s, and the muon capture rate on ^{3}He, Λ_0 = 1496 ± 4 s^{-1} → precision tests of the theory
- weak reactions of astrophysical interest such as e.g. the pp chain of the solar burning:
- $L_{1,A}$ governs the leading 3NF

$$p + p \to d + e^+ + \nu_e$$
$$p + p + e^- \to d + \nu_e$$
$$p + {}^3He \to {}^4He + e^+ + \nu_e$$
$${}^7Be + e^- \to {}^7Li + \nu_e$$
$${}^8B \to {}^8Be^* + e^+ + \nu_e$$

Historical remarks

- Meson-exchange theory, Skyrme model, phenomenology, …
  Brown, Adam, Mosconi, Ricci, Truhlik, Nakamura, Sato, Ando, Kubodera, Riska, Sauer, Friar, Gari, …

- First derivation within chiral EFT to leading 1-loop order using TOPT
  Park, Min, Rho Phys. Rept. 233 (1993) 341; NPA 596 (1996) 515;
  Park et al., Phys. Rev. C67 (2003) 055206
  – only for the threshold kinematics
  – pion-pole diagrams ignored
  – box-type diagrams neglected
  – renormalization incomplete

- Leading one-loop expressions using TOPT for general kinematics (still incomplete, e.g. no 1/m corrections)
  Pastore, Girlanda, Schiavilla, Goity, Viviani, Wiringa;
  PRC78 (2008) 064002; PRC80 (2009) 034004; PRC84 (2011) 024001 ← Vector current

  Baroni, Girlanda, Pastore, Schiavilla, Viviani;
  PRC93 (2016) 015501, Erratum: PRC 93 (2016) 049902 ← Axial vector current

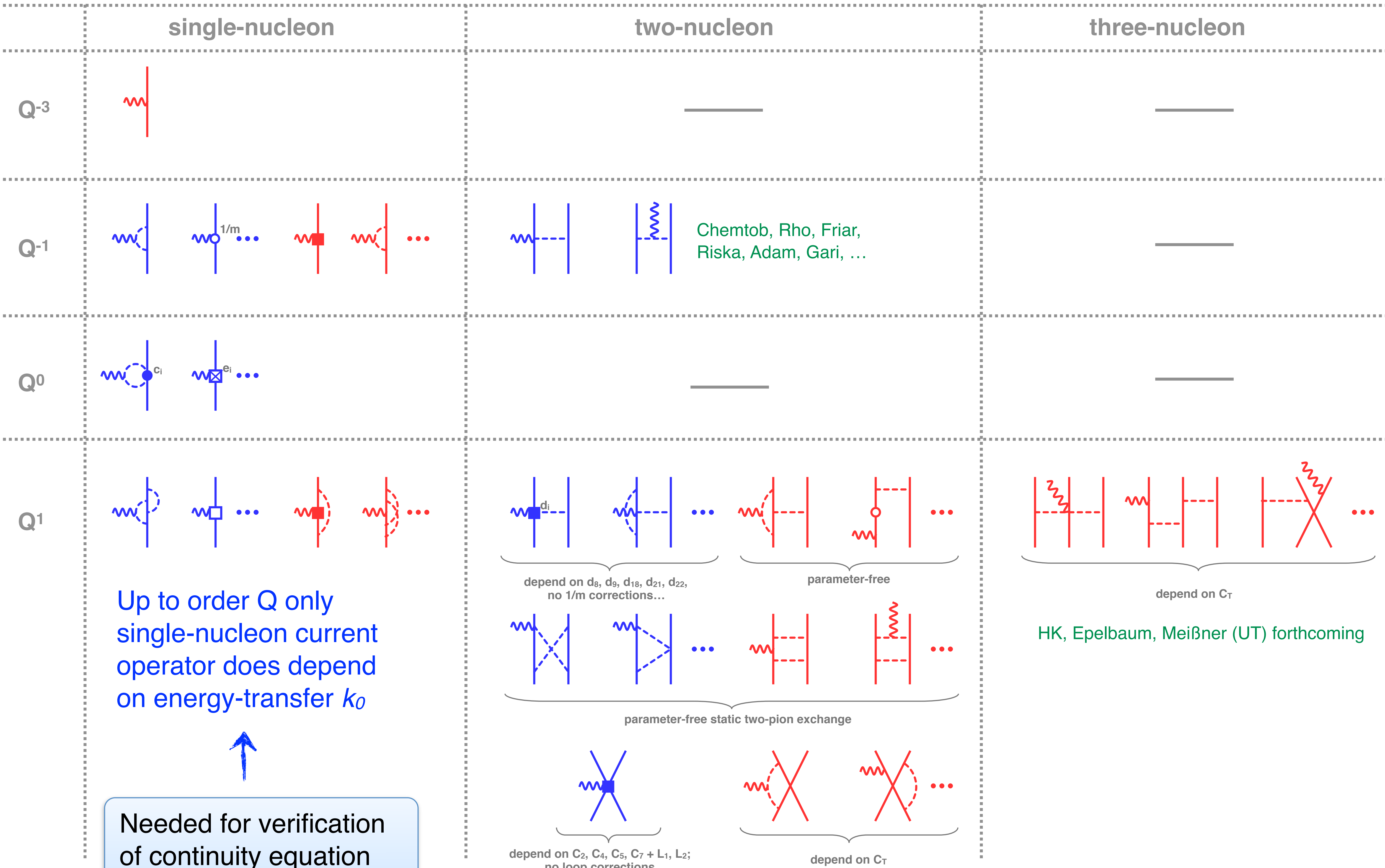
Complete derivation to leading one-loop order using the method of UT

Kölling, Epelbaum, HK, Meißner;
PRC80 (2009) 045502; PRC84 (2011) 054008 ← Vector current

HK, Epelbaum, Meißner, Ann. Phys. 378 (2017) 317 ← Axial vector current

Vector currents in chiral EFT

Chiral expansion of the electromagnetic **current** and **charge** operators



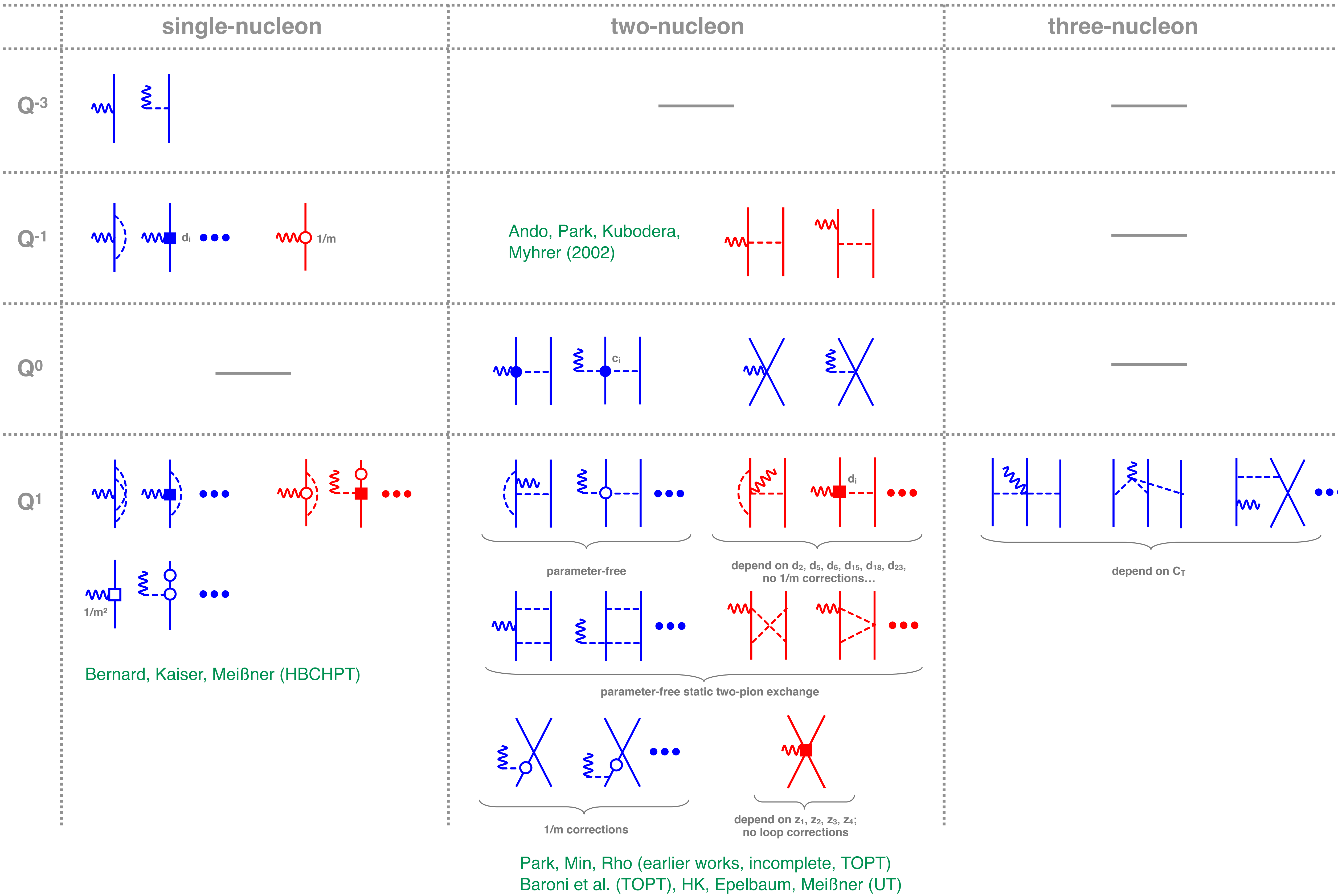
Up to order Q only single-nucleon current operator does depend on energy-transfer k_0

Needed for verification of continuity equation for OPE part

Park, Min, Rho, Kubodera, Song, Lazauskas (earlier works, incomplete, TOPT)
Pastore, Schiavilla et al. (TOPT), Kölling, Epelbaum, HK, Meißner (UT)

Axial vector operators in chiral EFT

Chiral expansion of the axial vector **current** and **charge** operators



Summary

Nuclear Currents *Forthcoming review HK, EPJA*

- Vector & axial-vector currents are calculated up to N^3LO
- Numerical implementation require symmetry-respecting regularization (work in progress)