Electron scattering on ^{3}He and ^{3}H with Faddeev-type methods

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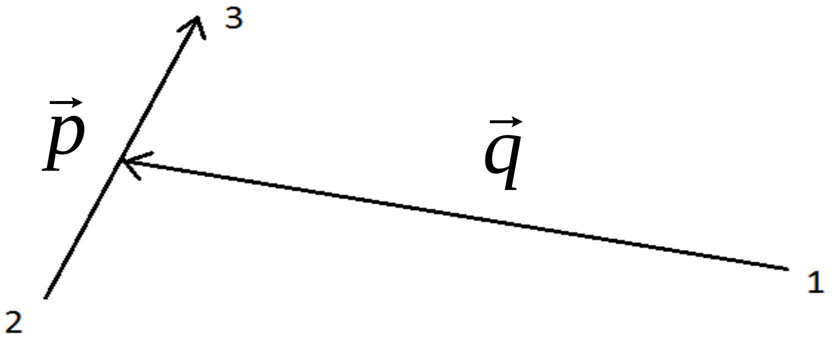
Outline

1. Theoretical framework and technicalities
2. 3N bound state
3. Inclusive processes
4. pd break-up of ^{3}He
5. ppn break-up of ^{3}He
6. Conclusions and outlook

3N basis states in momentum space

In the 3N system we need two relative (Jacobi) momenta



and use states $\mid \vec{p}\, \vec{q}\, \vec{P}\, m_1 \nu_1\ m_2 \nu_2\ m_3 \nu_3 \rangle$

with individual spin (m_i) and isospin (ν_i) magnetic quantum numbers

$$\vec{p} = \frac{1}{2}(\vec{p}_2 - \vec{p}_3),\ \ \vec{q} = \frac{2}{3}\left(\vec{p}_1 - \frac{1}{2}(\vec{p}_2 + \vec{p}_3)\right),\ \ \vec{P} = \vec{p}_1 + \vec{p}_2 + \vec{p}_3$$

Instead of $| \vec{p}\, \vec{q}\, \vec{P}\; m_1 \nu_1\; m_2 \nu_2\; m_3 \nu_3 \rangle$ one can use states of the total 3N spin and isospin:

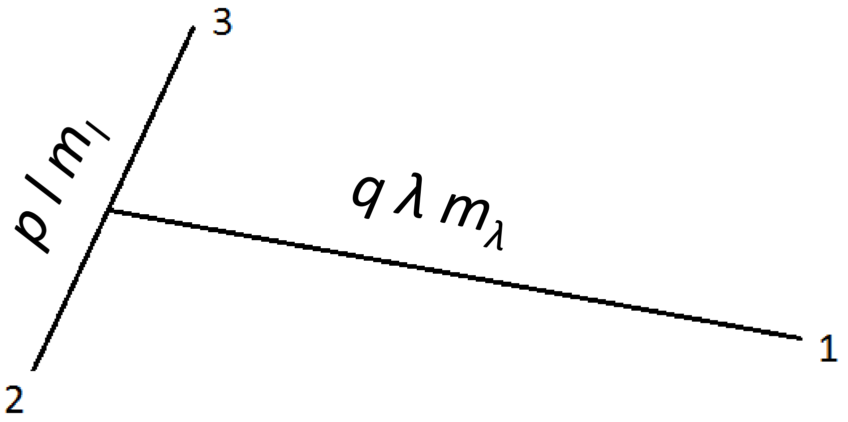
$$\left| \vec{p}\, \vec{q}\, \vec{P} \left(s \frac{1}{2} \right) S\, M_S \right\rangle \left| \left(t \frac{1}{2} \right) T\, M_T \right\rangle ,$$

where the 2N subsystem (iso)spin is coupled with the (iso)spin of the third nucleon)

$$\left| \left(s \frac{1}{2} \right) S\, M_S \right\rangle = \sum_{m_s} c\left(s, \frac{1}{2}, S; m_s, M_S - m_s, M_S \right) \left| s\, m_s \right\rangle \left| \frac{1}{2} M_S - m_s \right\rangle$$

$$\left| \left(t \frac{1}{2} \right) T\, M_T \right\rangle = \sum_{m_t} c\left(t, \frac{1}{2}, T; m_t, M_T - m_t, M_T \right) \left| t\, m_t \right\rangle \left| \frac{1}{2} M_T - m_t \right\rangle$$

We introduce *partial waves*, eigenstates of the two (relative) orbital angular momentum operators



and build the states of the TOTAL angular momentum of the 3N system:

$$\left| pq\,(l\lambda)L\,M_L \right\rangle = \sum_{m_l} c\left(l,\lambda,L;m_l,M_L-m_l,M_L\right)\left| p l m_l \right\rangle \left| q\lambda M_L - m_l \right\rangle$$

They are normalized as

$$\left\langle \vec{p}'\,\vec{q}'\,\middle|\,pq\,(l\lambda)L\,M_L\right\rangle = \frac{\delta\left(p-\left|\vec{p}'\right|\right)}{p^2}\,\frac{\delta\left(q-\left|\vec{q}'\right|\right)}{q^2}\,\mathrm{Y}_{l\lambda}^{LM_L}\left(\hat{p}',\hat{q}'\right),$$

where

$$\mathrm{Y}_{l\lambda}^{LM_L}\left(\hat{p}',\hat{q}'\right)=\sum_{m_l} c\left(l,\lambda,L;m_l,M_L-m_l,M_L\right)Y_{l\,m_l}\left(\hat{p}'\right)Y_{\lambda M_L-m_l}\left(\hat{q}'\right)$$

Together wih $\left\langle \vec{p}'\middle|\,\vec{p}\right\rangle=\delta^3\left(\vec{p}'-\vec{p}\right)$ and $\left\langle \vec{q}'\middle|\,\vec{q}\right\rangle=\delta^3\left(\vec{q}'-\vec{q}\right)$

we get also

$$\left\langle p'q'\left(l'\lambda'\right)L'\,M_L'\,\middle|\,pq\,(l\lambda)L\,M_L\right\rangle =$$

$$\frac{\delta\left(p-p'\right)}{p^2}\,\frac{\delta\left(q-q'\right)}{q^2}\,\delta_{ll'}\,\delta_{\lambda\lambda'}\,\delta_{LL'}\,\delta_{M_LM_L'}$$

and

$$\sum_{l=0}^{\infty}\sum_{\lambda=0}^{\infty}\sum_{L=|l-\lambda|}^{l+\lambda}\sum_{M_L=-L}^{L}\int_0^{\infty}dp\,p^2\int_0^{\infty}dq\,q^2\left|pq(l\lambda)Lm_L\right\rangle\left\langle pq(l\lambda)m_L\right|=1.$$

Next from the product states $\left| pq\,(l\lambda)L\,M_L \right\rangle \left| \left(s\frac{1}{2}\right)S\,M_S \right\rangle$

we build eigenstates of the TOTAL 3N angular momentum operator *J:*

$$\left| pq(l\,\lambda)L\left(s\frac{1}{2}\right)S\,(LS)\,JM \right\rangle \equiv \sum_{M_L} c(L,S,J;M_L,M-M_L,M) \left| pq\,(l\lambda)L\,M_L \right\rangle \left| \left(s\frac{1}{2}\right)S\,M-M_L \right\rangle$$

Multiplying these states with the isospin states $\left| \left(t\frac{1}{2}\right)T\,M_T \right\rangle$,

we get the final form of basis states in the LS-coupling:

$$\left| pq\beta \right\rangle \equiv \left| pq(l\,\lambda)L\left(s\frac{1}{2}\right)S\,(LS)\,JM \right\rangle \left| \left(t\frac{1}{2}\right)T\,M_T \right\rangle$$

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The 3N partial waves $\left| pq(l\lambda)L\left(s\frac{1}{2}\right)S(LS)JM\right\rangle \left|\left(t\frac{1}{2}\right)T M_T\right\rangle,$

have a definite parity: $\pi = (-1)^{l+\lambda}$

Condition $(-1)^{l+s+t} = -1,$

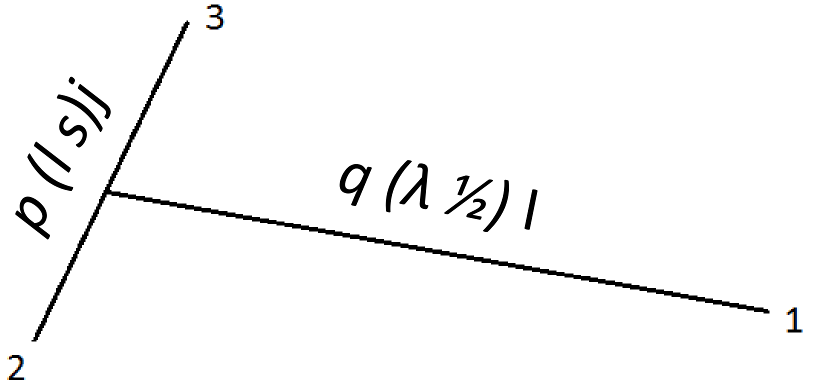
guarantees that the states are antisymmetric with respect to the exchange of nucleons 2 and 3

Rotational invariance and parity conservation allows us to solve the Faddeev equations SEPARATELY for each total 3N angular momentum *J* and its projection *M* and given parity π !

We have two continuous variables (*p* and *q*) and about 100 combinations of discrete quantum numbers

States in the so-called *jI* coupling $\quad \left| pq\alpha \right\rangle \equiv \left| pq(l\,s)j\left(\lambda\frac{1}{2}\right)I\,(jI)\,JM \right\rangle \left| \left(t\frac{1}{2}\right)T\,M_T \right\rangle,$

formed from the eigenstates of the total angular momentum of subsystem (23) and nucleon 1



9j recoupling coefficient

The relation between the two types of 3N partial waves is following:

$$\left| pq(l\,s)j\left(\lambda\frac{1}{2}\right)I\,(jI)\,JM \right\rangle = \sum_{L,S}\sqrt{(2L+1)(2S+1)(2j+1)(2I+1)}\begin{Bmatrix} l & s & j \\ \lambda & 1/2 & I \\ L & S & J \end{Bmatrix}\left| pq(l\,\lambda)L\left(s\frac{1}{2}\right)S\,(LS)\,JM \right\rangle$$

3N bound state

The Faddeev equation for the 3N bound state:

$$|\psi\rangle = G_0 \left(V + V^{(1)}\right)(1 + P)|\psi\rangle$$

Faddeev component

free 3N propagator

2N potential

part of 3N potential symmetric under the exchange (2↔3)

$$P_{12}P_{23} + P_{13}P_{23}$$

Another form of the Faddeev equation:

$$|\psi\rangle = G_0\, t\, P|\psi\rangle + \left(1 + G_0 t\right) G_0\, V^{(1)} \left(1 + P\right) |\psi\rangle$$

$$t = V + V\, G_0^{2N}\, t \longleftarrow$$ Lippmann-Schwinger equation for t-matrix

The full 3N wave function $$\left|\Psi\right\rangle = \left(1+P\right)\left|\psi\right\rangle$$

is expanded as $$\left|\Psi\right\rangle = \sum_{\beta}\int dp\, p^2 \int dq\, q^2\, \psi_{\beta}(p,q)\left|pq\beta\right\rangle$$

or $$\left|\Psi\right\rangle = \sum_{\alpha}\int dp\, p^2 \int dq\, q^2\, \psi_{\alpha}(p,q)\left|pq\alpha\right\rangle$$

Components
of the wave function $$\left|\Psi\right\rangle = \left|\Psi\right\rangle_{L=0} + \left|\Psi\right\rangle_{L=1} + \left|\Psi\right\rangle_{L=2}$$

S-wave
(dominant)

P-wave
(tiny)

D-wave)

Principal S-state of ^{3}He

$$\left|\Psi m\right\rangle^{PS} \equiv \left|\phi_S\right\rangle \left|\xi_a m\right\rangle$$

totally antisymmetric spin-isospin part

totally symmetric
momentum part

$$\left|\xi_a m\right\rangle \equiv \frac{1}{\sqrt{2}}\left(\left|\left(t=0,\frac{1}{2}\right)T=\frac{1}{2}\right\rangle\left|\left(s=1,\frac{1}{2}\right)S=\frac{1}{2}\right\rangle - \left|\left(t=1,\frac{1}{2}\right)T=\frac{1}{2}\right\rangle\left|\left(s=0,\frac{1}{2}\right)S=\frac{1}{2}\right\rangle\right)$$

The principal S-state is a part of $\left|\Psi m\right\rangle_{L=0}$ and $\left|\phi_S\right\rangle \equiv \left\langle \xi_a m \middle| \Psi m\right\rangle_{L=0}$

In LS-coupling

$$\left|\phi_S\right\rangle = \left\langle \xi_a m \middle| \Psi m \right\rangle_{L=0} = \sum_{\beta} \int dp\, p^2 \int dq\, q^2 \left\langle \xi_a m \middle| pq\beta \right\rangle_{L=0} \psi_\beta(p,q)$$

$$\left\langle \xi_a m \middle| pq\beta \right\rangle_{L=0} = \left| pq(l\,l)00 \right\rangle \frac{1}{\sqrt{2}} \left(\delta_{s1}\delta_{t0} - \delta_{s0}\delta_{t1} \right)$$

$$\left|\phi_S\right\rangle = \sum_{l=0,2,4,\ldots} \int dp\, p^2 \int dq\, q^2 \left| pq(l\,l)00 \right\rangle \frac{1}{\sqrt{2}} \left(\psi_{(ll)0\left(1\frac{1}{2}\right)\frac{1}{2}\left(0\frac{1}{2}\right)\frac{1}{2}}(p,q) - \psi_{(ll)0\left(0\frac{1}{2}\right)\frac{1}{2}\left(1\frac{1}{2}\right)\frac{1}{2}}(p,q) \right)$$

Expansion amplitudes for the principal S-state

$$\psi_\beta^{PS}(p,q) \equiv \langle pq\beta | \Psi m \rangle^{PS} = \langle pq\beta | \phi_S \rangle | \xi_a m \rangle = \delta_{L0}\ \delta_{\lambda l}\ \phi_l(p,q) \frac{1}{\sqrt{2}} (\delta_{s1}\delta_{t0} - \delta_{s0}\delta_{t1})$$

for $l = 0, 2, 4, \ldots$

$$\phi_l(p,q) \equiv \frac{1}{\sqrt{2}} \left(\psi_{(ll)0\left(1\frac{1}{2}\right)\frac{1}{2}\left(0\frac{1}{2}\right)\frac{1}{2}}(p,q) - \psi_{(ll)0\left(0\frac{1}{2}\right)\frac{1}{2}\left(1\frac{1}{2}\right)\frac{1}{2}}(p,q) \right)$$

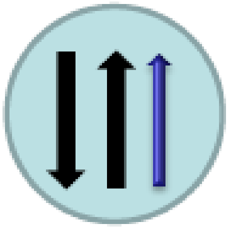
L	symmetry	Ps [%] Bonn B	Ps [%] AV18	Ps [%] AV18 + Urbana IX
0	totally symmetric	91.6	89.9	89.3
0	totally antisymmetric	0	0	0
0	mixed	1.18	1.53	1.24
1		0.05	0.07	0.14
2		7.17	8.40	9.19

$$\left|\xi_a M_S=\frac{1}{2}\right\rangle \equiv \frac{1}{\sqrt{2}}\left(\left|\left(t=0,\frac{1}{2}\right)T=\frac{1}{2},M_T=\frac{1}{2}\right\rangle\left|\left(s=1,\frac{1}{2}\right)S=\frac{1}{2},M_S=\frac{1}{2}\right\rangle\right.$$

$$\left.-\left|\left(t=1,\frac{1}{2}\right)T=\frac{1}{2},M_T=\frac{1}{2}\right\rangle\left|\left(s=0,\frac{1}{2}\right)S=\frac{1}{2},M_S=\frac{1}{2}\right\rangle\right)$$

$$=\frac{1}{\sqrt{3}}\left(|n,+\rangle_2\,|p\rangle_3\,|p\rangle_1\left|\left(\frac{1}{2},\frac{1}{2}\right)00\right\rangle_{31}+|n,+\rangle_1\,|p\rangle_2\,|p\rangle_3\left|\left(\frac{1}{2},\frac{1}{2}\right)00\right\rangle_{23}\right.$$

$$\left.+|n,+\rangle_3\,|p\rangle_1\,|p\rangle_2\left|\left(\frac{1}{2},\frac{1}{2}\right)00\right\rangle_{12}\right)$$



$$\overrightarrow{{}^3\mathrm{He}}\approx\vec{\mathrm{n}}\ \ (90\ \%)$$

In the principal S-state spins of two protons couple to zero !
The total angular momentum of ^{3}He is carried by the neutron !

Can we use that fact ? YES !

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3N bound state in three dimensions

Our starting point is the operator form of ^{3}H
(see I. Fachruddin *et al*., Phys. Rev. C 69, 064002 (2004)),
although other possibilities exist
(see S. Bayegan *et al*., Phys. Rev. C77, 064005 (2008))

The Faddeev equation for the 3N bound state reads:

$$\psi = G_0\, t\, P \psi + \left(1 + G_0 t\right) G_0 V^{(1)} \left(1 + P\right) \psi \ ,$$

where
Ψ – Faddeev component,
t – 2N t-matix,
G_0 – free 3N propagator,
$P = P_{12} P_{23} + P_{13} P_{23}$ (permutation operator),
$V^{(1)}$ - part of 3N force symmetric under the exchange (2↔3)

It is possible to work with the 2N force *V* and not to use *t* !

$$\psi = G_0 V(1+P)\psi + G_0 V^{(1)}(1+P)\psi \, .$$

The full 3N wave function is given as

$$\Psi = (1+P)\psi \, .$$

Three possible isospin states in ^{3}H

$$|\gamma_1\rangle = \left|\left(0\frac{1}{2}\right)\frac{1}{2}\right\rangle, \; |\gamma_2\rangle = \left|\left(1\frac{1}{2}\right)\frac{1}{2}\right\rangle, \; |\gamma_3\rangle = \left|\left(1\frac{1}{2}\right)\frac{3}{2}\right\rangle$$

are used to expand the Faddeev component

$$\psi = \sum_i \psi_{\gamma_i} |\gamma_i\rangle \equiv \sum_{t,T} \psi_{tT} \left|\left(t\frac{1}{2}\right)T\right\rangle$$

We need two (relative) momenta to represent the Faddeev component:

$$\vec{p} = \frac{1}{2}\left(\vec{p}_2 - \vec{p}_3\right), \quad \vec{q} = \frac{2}{3}\left(\vec{p}_1 - \frac{1}{2}\left(\vec{p}_2 + \vec{p}_3\right)\right)$$

The operator form for the Faddeev component:

$$\left\langle \vec{p},\vec{q} \middle| \psi_{tT} = \sum_{i=1}^{8} \phi_{tT}^{i}\left(\vec{p},\vec{q}\right) O^{i}\left(\vec{\sigma}_1,\vec{\sigma}_2,\vec{\sigma}_3,\vec{p},\vec{q}\right) \middle| \left(0\frac{1}{2}\right)\frac{1}{2}m \right\rangle,$$

with

$$O^{1}\left(\vec{\sigma}_1,\vec{\sigma}_2,\vec{\sigma}_3,\vec{p},\vec{q}\right) = 1,$$

$$O^{2}\left(\vec{\sigma}_1,\vec{\sigma}_2,\vec{\sigma}_3,\vec{p},\vec{q}\right) = \frac{1}{\sqrt{3}}\vec{\sigma}_{23}\cdot\vec{\sigma}_1, \qquad \vec{\sigma}_{23} \equiv \frac{1}{2}\left(\vec{\sigma}_2 - \vec{\sigma}_3\right)$$

$$O^{3}\left(\vec{\sigma}_1,\vec{\sigma}_2,\vec{\sigma}_3,\vec{p},\vec{q}\right) = \sqrt{\frac{3}{2}}\frac{1}{i}\vec{\sigma}_1\cdot\left(\vec{p}\times\vec{q}\right),$$

$$O^{4}\left(\vec{\sigma}_1,\vec{\sigma}_2,\vec{\sigma}_3,\vec{p},\vec{q}\right) = \frac{1}{\sqrt{2}}\left(i\vec{\sigma}_{23}\cdot\left(\vec{p}\times\vec{q}\right) - \left(\vec{\sigma}_1\times\vec{\sigma}_{23}\right)\cdot\left(\vec{p}\times\vec{q}\right)\right), \ldots$$

Note that ϕ^{i}_{tT} are scalar functions !

$$\phi^{i}_{tT}\left(\vec{p},\vec{q}\right)=\phi^{i}_{tT}\left(p,q,x\equiv\hat{p}\cdot\hat{q}\right)$$

Using the steps outlined in W. Glöckle *et al*., Few-Body Systems 47, 25 (2010), we can write schematically the Faddeev equation as

$$\phi=\underbrace{C^{-1}\left(D+D'\right)\phi}_{\text{term with P}}\quad+\quad\underbrace{C^{-1}L\,\phi}_{\text{term without P}}$$

A huge eigenvalue problem after discretization (p_i,q_j,x_k):

ϕ has $3 \times 8 \times N_p \times N_q \times N_x$ components !

At present we keep only the total isospin $T=1/2$,
so ϕ has only $2 \times 8 \times N_p \times N_q \times N_x$ components.

$16 \times 40^3 =$ 1024000

We solve this equation by iterations, employing the Lanczos method.

Like in the deuteron case, also for ^{3}H we need several sets of scalar coefficients

$$C_{ki}(\vec{p},\vec{q};\vec{p},\vec{q}) = \sum_m \left\langle \chi^m \middle| O^k(\vec{\sigma}_1,\vec{\sigma}_2,\vec{\sigma}_3,\vec{p},\vec{q})\, O^i(\vec{\sigma}_1,\vec{\sigma}_2,\vec{\sigma}_3,\vec{p},\vec{q}) \middle| \chi^m \right\rangle,$$

$$D_{kjk'}(\vec{p},\vec{q};\vec{p}\,',\vec{q}\,';\vec{p}\,'',\vec{q}\,'') = -\frac{1}{2}\sum_m \left\langle \chi^m \middle| O^k(\vec{\sigma}_1,\vec{\sigma}_2,\vec{\sigma}_3,\vec{p},\vec{q})\, w_j(\vec{\sigma}_2,\vec{\sigma}_3,\vec{p}\,',\vec{q}\,') \right.$$
$$\left. O^{k'}(\vec{\sigma}_2,\vec{\sigma}_3,\vec{\sigma}_1,\vec{p}\,'',\vec{q}\,'')(1+\vec{\sigma}_{23}\cdot\vec{\sigma}_1) \middle| \chi^m \right\rangle,$$

$$D'_{kjk'}(\vec{p},\vec{q};\vec{p}\,',\vec{q}\,';\vec{p}\,'',\vec{q}\,'') = \frac{1}{2}\sum_m \left\langle \chi^m \middle| O^k(\vec{\sigma}_1,\vec{\sigma}_2,\vec{\sigma}_3,\vec{p},\vec{q})\, w_j(\vec{\sigma}_2,\vec{\sigma}_3,\vec{p}\,',\vec{q}\,') \right.$$
$$\left. O^{k'}(\vec{\sigma}_3,\vec{\sigma}_2,\vec{\sigma}_1,\vec{p}\,'',\vec{q}\,'')(1-\vec{\sigma}_{23}\cdot\vec{\sigma}_1) \middle| \chi^m \right\rangle$$

and more if we include the 3N force.

Of course all done (easily !) with *Mathematica* ®

The full wave function $\Psi = (1+P)\psi$.

We have to consider the isospin, spin and momentum spaces:

$$\psi = \sum_\gamma |\gamma\rangle \psi_\gamma \equiv \sum_{tT} |\left(t\frac{1}{2}\right) T\rangle \psi_{tT}$$

$$\langle\gamma|P|\gamma'\rangle = \delta_{TT'} P_{tt'T} = \delta_{TT'} F_{tt'T} (P_{12}^{sm} P_{23}^{sm} + (-)^{t+t'} P_{13}^{sm} P_{23}^{sm}) \ ,$$

$$\langle\gamma|P\psi = \sum_{t'} F_{tt'T} \left(P_{12}^{sm} P_{23}^{sm} + (-)^{t+t'} P_{13}^{sm} P_{23}^{sm}\right) \psi_{t'T}$$

$$\begin{aligned} \langle \vec{p}\,\vec{q}\,|\, P_{12}^{sm} P_{23}^{sm} \,|\, \vec{p}\,'\,\vec{q}\,'\rangle &= \delta(\vec{p}\,' - \vec{P}_1(\vec{p},\vec{q}))\delta(\vec{q}\,' - \vec{Q}_1(\vec{p},\vec{q}))P_{12}^s P_{23}^s \,, \\ \langle \vec{p}\,\vec{q}\,|\, P_{13}^{sm} P_{23}^{sm} \,|\, \vec{p}\,'\,\vec{q}\,'\rangle &= \delta(\vec{p}\,' - \vec{P}_2(\vec{p},\vec{q}))\delta(\vec{q}\,' - \vec{Q}_2(\vec{p},\vec{q}))\underbrace{P_{13}^s P_{23}^s}_{\text{spin space operators}} \end{aligned}$$

$$\vec{P}_1(\vec{p},\vec{q}) = \frac{1}{2}\vec{p} + \frac{3}{4}\vec{q}, \qquad \vec{P}_2(\vec{p},\vec{q}) = \frac{1}{2}\vec{p} - \frac{3}{4}\vec{q},$$

$$\vec{Q}_1(\vec{p},\vec{q}) = \vec{p} - \frac{1}{2}\vec{q}, \qquad \vec{Q}_2(\vec{p},\vec{q}) = -\vec{p} - \frac{1}{2}\vec{q}.$$

$$\psi_{tT}(\vec{p},\vec{q}) \equiv \langle \vec{p}\,\vec{q}\,|\langle\gamma|\psi \equiv \langle\vec{p}\vec{q}|\psi_{tT}\rangle = \sum_{i=1}^{8} \phi_{tT}^{(i)}(\vec{p},\vec{q})\; O_i|\chi^m\rangle\;,$$

$$\langle\,\vec{p}\,\vec{q}\,|\langle\gamma|P\psi_{\cdot} = \sum_{i=1}^{8} \alpha_{tT}^{(i)}(\vec{p},\vec{q})\; O_i|\chi^m\rangle\;,$$

expansion
coefficients

$$\alpha_{tT}^{(m)}(\vec{p},\vec{q}) \;=\; \sum_{t'} F_{tt'T}\; \sum_{k=1}^{8} C_{mk}^{-1}\,(\vec{p}\vec{q},\vec{p}\vec{q})\; \sum_{l=1}^{8} \Big\{ W_{kl}\left(\vec{p}\vec{q},\vec{P}_1\vec{Q}_1\right)\; \phi_{t'T}^{(l)}(\vec{P}_1,\vec{Q}_1\,) + (-)^{t+t'}\; Z_{kl}\left(\vec{p}\vec{q},\vec{P}_2\vec{Q}_2\right)\; \phi_{t'T}^{(l)}(\vec{P}_2,\vec{Q}_2\,)\Big\}\;.$$

$$C_{ki}\,(\vec{p}\vec{q},\vec{p}\vec{q}) = \sum_m \langle\chi^m|O_k(\vec{\sigma}_{(1)},\vec{\sigma}_{(2)},\vec{\sigma}_{(3)},\vec{p},\vec{q})O_i(\vec{\sigma}_{(1)},\vec{\sigma}_{(2)},\vec{\sigma}_{(3)},\vec{p},\vec{q})|\chi^m\rangle\;,$$

$$W_{ki}\left(\vec{p}\vec{q},\vec{P}_1\vec{Q}_1\,\right) = -\frac{1}{2}\sum_m \langle\chi^m|O_k(\vec{\sigma}_{(1)},\vec{\sigma}_{(2)},\vec{\sigma}_{(3)},\vec{p},\vec{q})\; O_i(\vec{\sigma}_{(2)},\vec{\sigma}_{(3)},\vec{\sigma}_{(1)},\vec{P}_1,\vec{Q}_1\,)\,(1+\vec{\sigma}(23)\cdot\vec{\sigma}_{(1)})|\chi^m\rangle\;,$$

$$Z_{ki}\left(\vec{p}\vec{q},\vec{P}_2\vec{Q}_2\right) = -\frac{1}{2}\sum_m \langle\chi^m|O_k(\vec{\sigma}_{(1)},\vec{\sigma}_{(2)},\vec{\sigma}_{(3)},\vec{p},\vec{q})$$

$$O_i(\vec{\sigma}_{(3)},\vec{\sigma}_{(1)},\vec{\sigma}_{(2)},\vec{P}_2,\vec{Q}_2)\,(1-\vec{\sigma}(23)\cdot\vec{\sigma}_{(1)})|\chi^m\rangle\,.$$

Finally the expansion coefficients for the full wave function are:

$$\beta_\gamma^{(i)}(\vec{p},\vec{q}) \;=\; \phi_\gamma^{(i)}(\vec{p},\vec{q}) \;+\; \alpha_\gamma^{(i)}(\vec{p},\vec{q})$$

and we write

$$\Psi = \sum_\gamma \int d^3p \int d^3q \,|\,\vec{p}\vec{q}\rangle\,|\,\gamma\rangle \sum_{i=1}^{8} \beta_\gamma^{(i)}(\vec{p},\vec{q})\;O_i\left(\vec{\sigma}_{(1)},\vec{\sigma}_{(2)},\vec{\sigma}_{(3)},\vec{p},\vec{q}\right)\,|\chi^m\rangle$$

Normalisation condition:

$$1 = \langle \Psi \mid \Psi \rangle = \langle \psi(1+P) \mid (1+P)\psi \rangle = 3 \langle \psi \mid \Psi \rangle$$

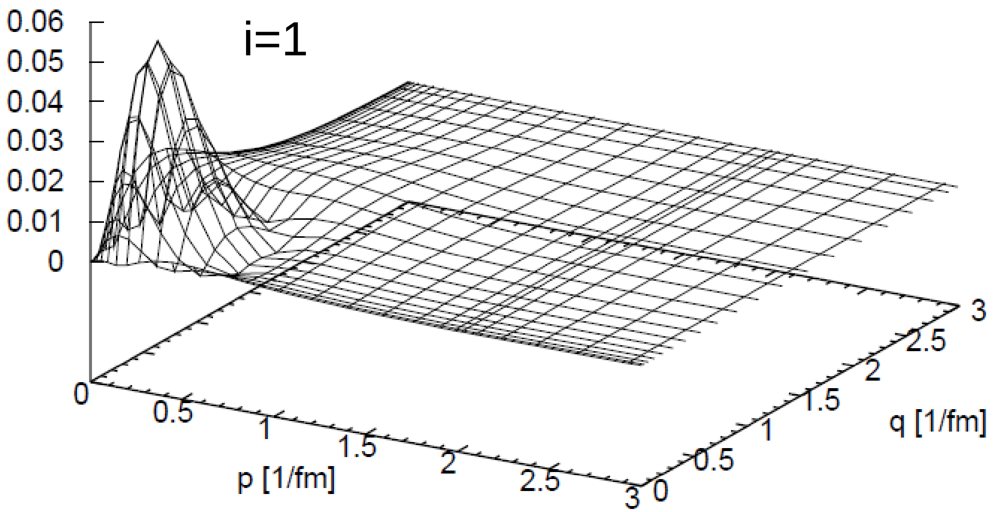
requires that

$$2 = 3 \sum_{\gamma} \int d^3p \int d^3q \sum_{i,j=1}^{8} \phi_{\gamma}^{(i)}(\vec{p},\vec{q})\ \beta_{\gamma}^{(j)}(\vec{p},\vec{q})\ R_{ij}(\vec{p}\vec{q},\vec{p}\vec{q})\ ,$$

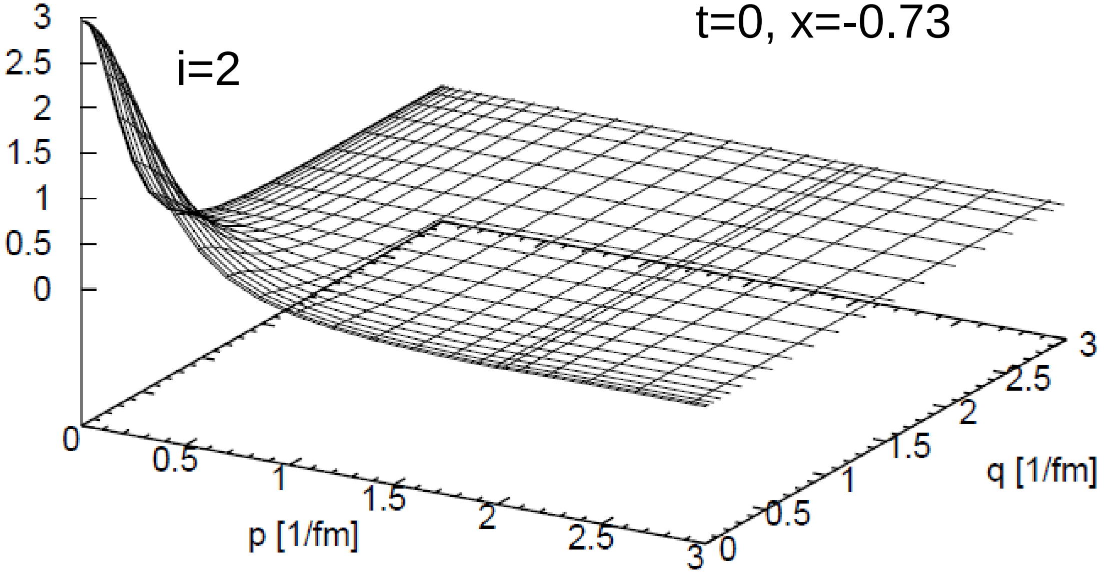
where

$$R_{ij}(\vec{p}\vec{q},\vec{p}\vec{q}) = \sum_{m} \langle \chi^m | O_i^{\dagger}\left(\vec{\sigma}_{(1)},\vec{\sigma}_{(2)},\vec{\sigma}_{(3)},\vec{p},\vec{q}\right) O_j\left(\vec{\sigma}_{(1)},\vec{\sigma}_{(2)},\vec{\sigma}_{(3)},\vec{p},\vec{q}\right) |\chi^m\rangle$$

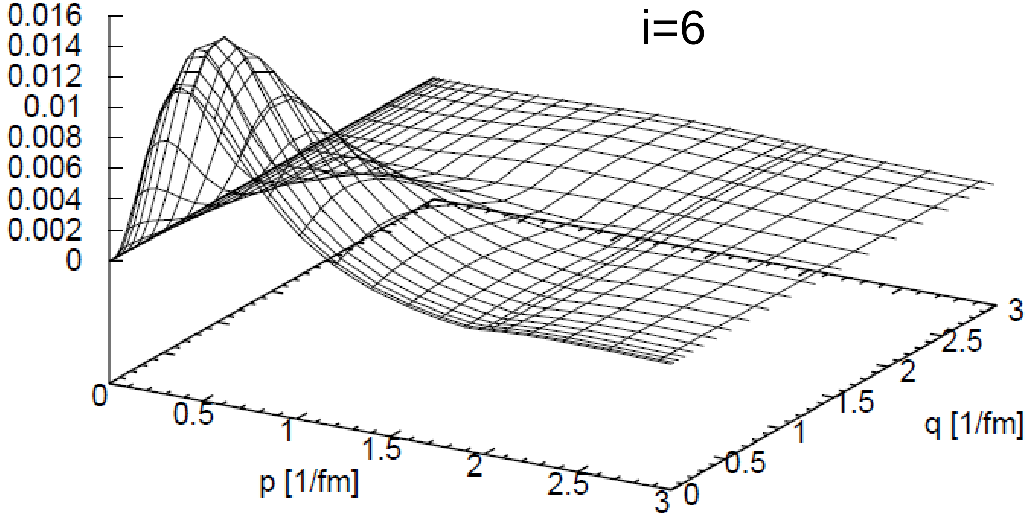
$$\beta_{\gamma}^{(i)}(\vec{p},\vec{q})$$



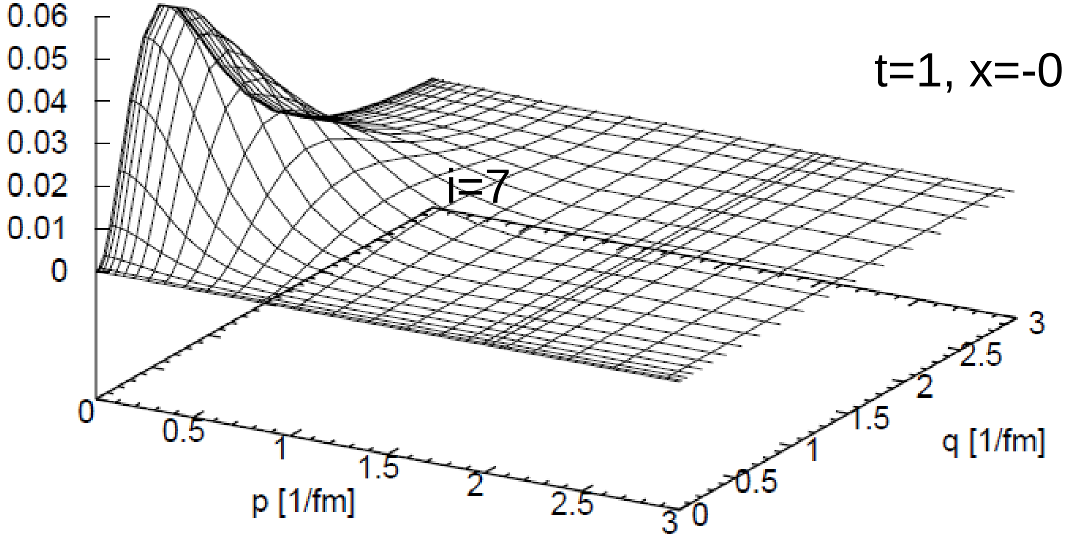
t=0, x=-0.73



$$\beta_\gamma^{(i)}(\vec{p},\vec{q})$$



t=1, x=-0.73



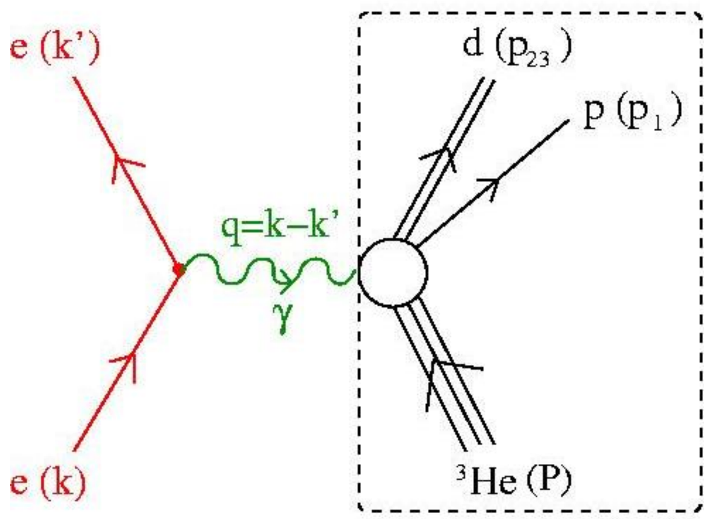
Several tests passed:

(1) Eigenvalue 1 for the binding energy *E*

(2) expectation values of the kinetic and potential energy operators sum up to *E*

(3) two formulations give equivalent results

Factorization of the total transition amplitude under the one-photon approximation



$e + {}^3He \rightarrow e + {}^3He$

$e + {}^3He \rightarrow e + p + d$

$e + {}^3He \rightarrow e + p + p + n$

$$\mathbf{A} \sim L^{\mu} \frac{1}{q^2} \mathbf{N}_{\mu}$$

QED (known analytically)

The crucial (nuclear) matrix element

$$N_\mu = < \Psi_f \,|\, j_\mu \,|\, \Psi_i >$$

Final 3N state. Can be bound (elastic case) or scattering (describes either two-body or three-body break-up of ^{3}He)

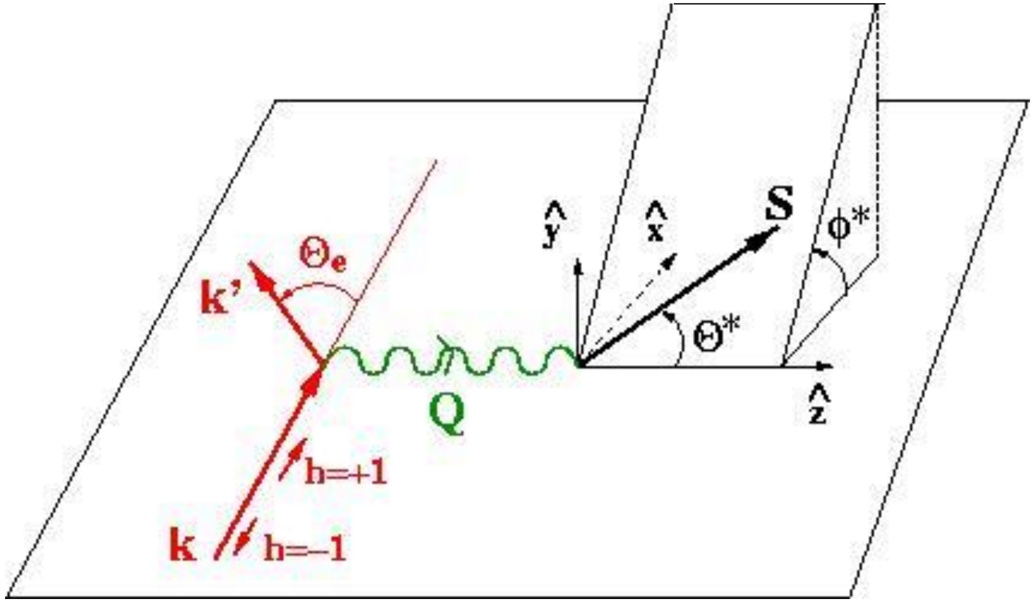
Initial 3N bound state

3N electromagnetic current operator

Reference frame given by the electron arm

$\vec{k}$ and $\vec{k}'$ span the x-z plane !

$$\hat{y} = \frac{\vec{k}\times\vec{k}'}{\left|\vec{k}\times\vec{k}'\right|}, \quad \vec{Q} = \vec{k} - \vec{k}' \,||\, \hat{z}$$ (quantization axis)



Θ* and Φ* define
the direction
of the initial ^{3}He spin

General formula for the (exclusive) cross section

Based on three assumptions:
(1) one-photon approximation
(2) nuclear current conservation
(3) final electron helicity is not measured

$$\omega N^0 = \vec{Q} \cdot \vec{N} \xrightarrow{\vec{Q} \| \hat{z}} N_z = \frac{\omega}{|\vec{Q}|} N^0$$

$$\sigma = \Sigma + \Delta h$$

the only dependence on the initial electron helicity !

$$\Sigma = \sigma_{Mott} \left(v_L R_L + v_T R_T + v_{TT} R_{TT} + v_{TL} R_{TL} \right) \rho \,,$$

$$\Delta = \sigma_{Mott} \left(v_{T'} R_{T'} + v_{TL'} R_{TL'} \right) \rho$$

$$\sigma_{Mott} = \frac{\alpha^2 \cos^2(\frac{\theta_e}{2})}{4E^2 \sin^4(\frac{\theta_e}{2})}$$

Mott cross section

phase space factor (kinematical factor)

Kinematical factors v_i

$$
\begin{aligned}
v_L &= \frac{(\mathbf{q}^2)^2}{Q^4}, \\
v_T &= -\frac{1}{2}\frac{\mathbf{q}^2}{Q^2} + \tan^2\frac{\theta_e}{2}, \\
v_{TT} &= \frac{1}{2}\frac{\mathbf{q}^2}{Q^2}, \\
v_{TL} &= \frac{1}{\sqrt{2}}\frac{\mathbf{q}^2}{Q^2}\left[-\frac{\mathbf{q}^2}{Q^2} + \tan^2\frac{\theta_e}{2}\right]^{1/2}, \\
v_{T'} &= \tan\frac{\theta_e}{2}\left[-\frac{\mathbf{q}^2}{Q^2} + \tan^2\frac{\theta_e}{2}\right]^{1/2}, \\
v_{TL'} &= \frac{1}{\sqrt{2}}\frac{\mathbf{q}^2}{Q^2}\tan\frac{\theta_e}{2},
\end{aligned}
$$

Dynamical quantities (response functions) *Ri*

They still carry information about the polarizations of the nuclear fragments in the final state !

$$
\begin{aligned}
R_L &= |N^0|^2 \,, \\
R_T &= |N_{+1}|^2 + |N_{-1}|^2 \,, \\
R_{TT} &= 2\mathrm{Re}(N_{+1} N^*_{-1}) \,, \\
R_{TL} &= -2\mathrm{Re}(N^0 (N_{+1} - N_{-1})^*) \,, \\
R_{T'} &= |N_{+1}|^2 - |N_{-1}|^2 \,, \\
R_{TL'} &= -2\mathrm{Re}(N^0 (N_{+1} + N_{-1})^*) \,,
\end{aligned}
$$

$$N_{+1} = -\frac{1}{\sqrt{2}}\left(N_x + i\,N_y\right), \qquad N_{-1} = \frac{1}{\sqrt{2}}\left(N_x - i\,N_y\right)$$

spherical components

All we need is … $N_\mu = < \Psi_f \mid j_\mu \mid \Psi_i >$!

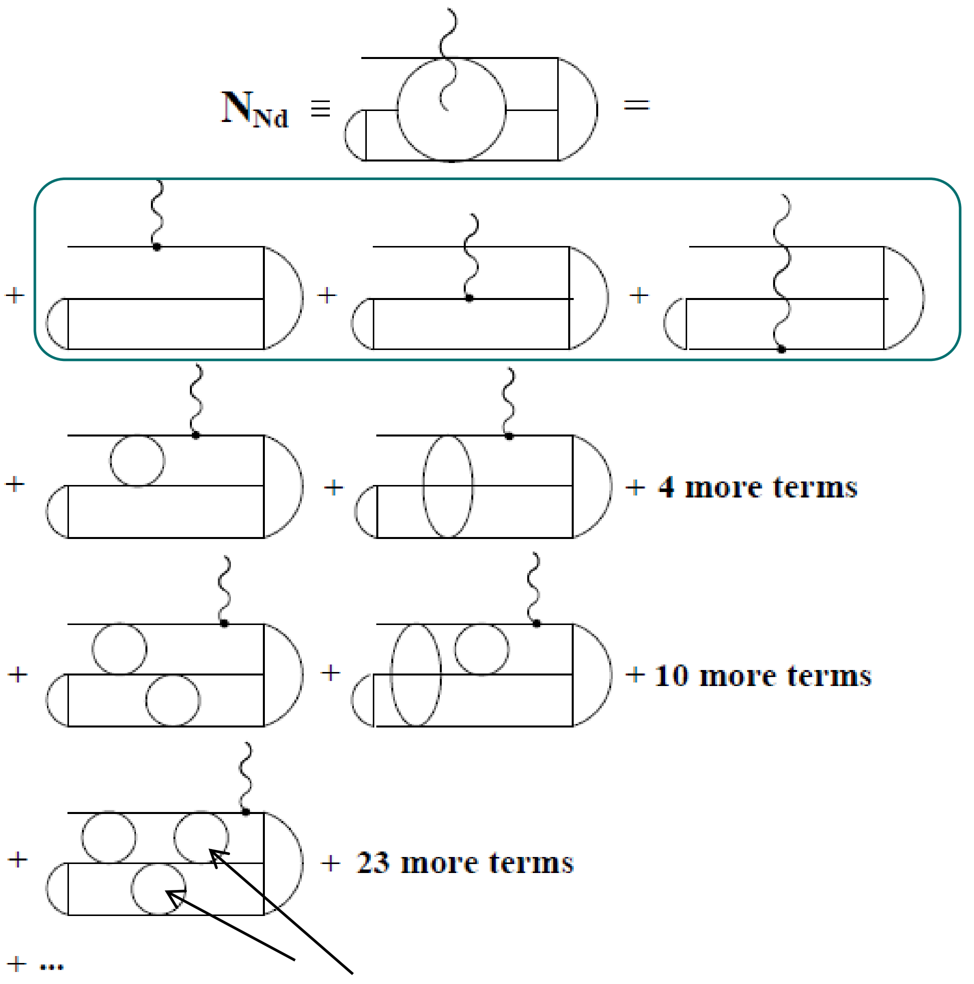
Also for photodisintegration processes !

General strategy in the few-nucleon physics:

Ab initio calculations that

- use the basic dynamical ingredients (2N and 3N potentials, current operators)
- solve the dynamical equations (Schrödinger equation, Lippmann-Schwinger equation, Faddeev equations)
- give properties of the bound states and reaction observables

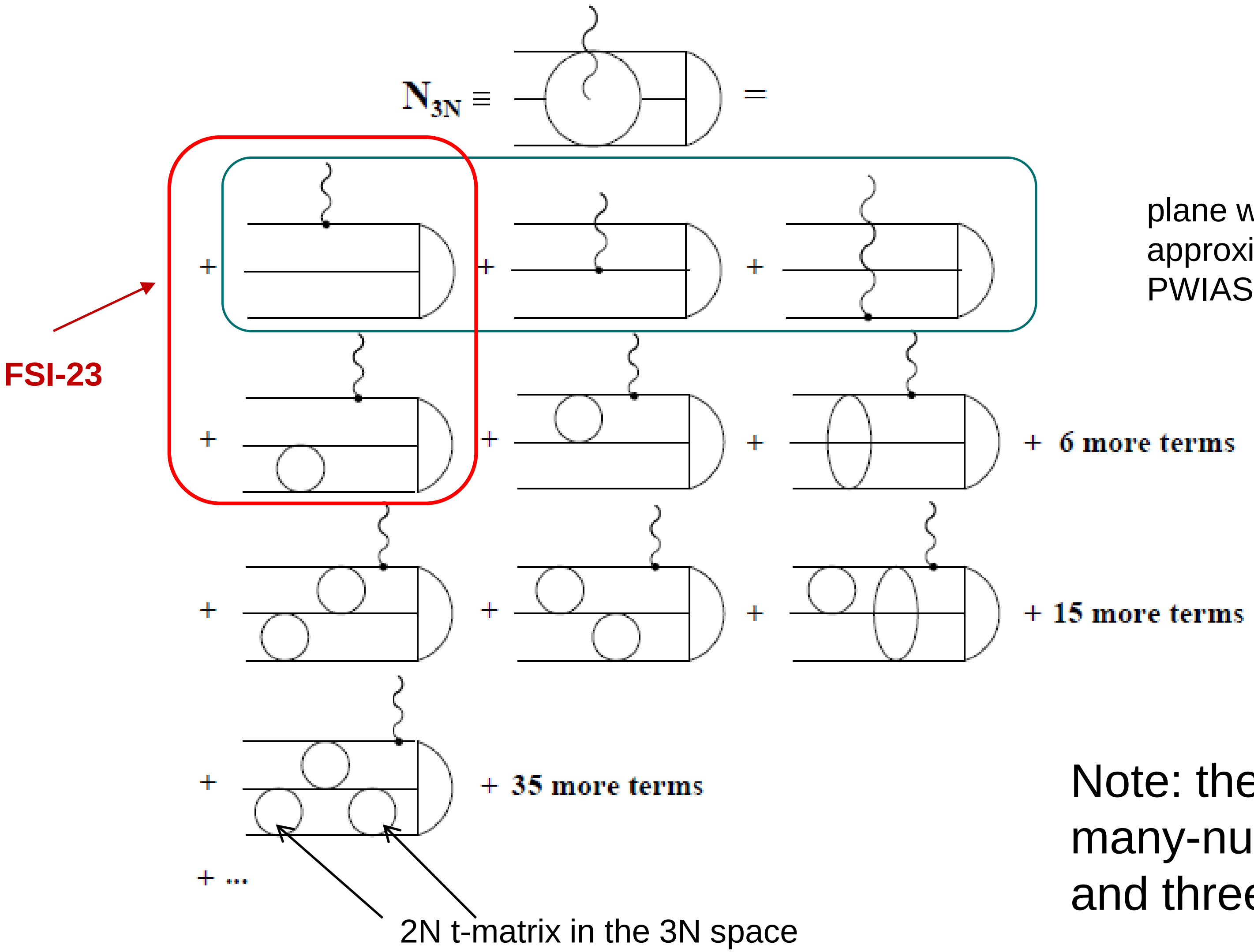
Two-body break-up of ^{3}He: e + ^{3}He $\rightarrow$ e + p + d



2N t-matrix in the 3N space

plane wave impulse
approximation (symmetrized)
PWIAS

Three-body break-up of ^{3}He: $e + {}^3He \rightarrow e + p + p + n$



plane wave impulse
approximation (symmetrized)
PWIAS

Note: the diagrams neglect
many-nucleon currents
and three-nucleon forces !

How to get N_μ ?

$$N_\mu = N_\mu^{\mathrm{RESCATT}} + N_\mu^{\mathrm{PWIAS}}$$

calculated directly

calculated in two steps

(1) Solve an auxiliary equation for the $| U_\mu >$ state

$$\left| U_\mu \right\rangle = \left[t\,G_0 + \frac{1}{2}(1+P)V^{(1)}G_0(1+tG_0) \right](1+P)j_\mu \left| \Psi_i \right\rangle$$

$$+ \left[t\,G_0 P + \frac{1}{2}(1+P)V^{(1)}G_0(1+tG_0)P \right] \left| U_\mu \right\rangle$$

(2) Get N_μ^{RESCATT} by quadratures

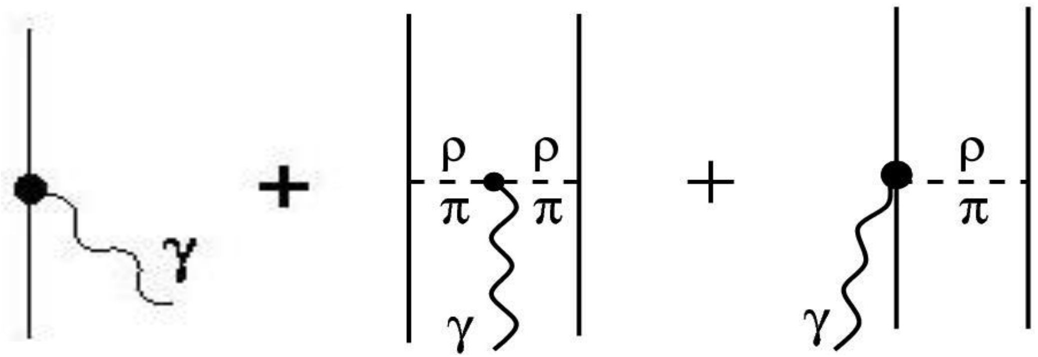
$$N_\mu^{RESCATT} = \begin{cases} \left\langle \Phi_{Nd} \right| P \left| U_\mu \right\rangle, & 2BB \\ \left\langle \Phi_{3N} \right| tG_0(1+P)j_\mu \left| \Psi_i \right\rangle + \left\langle \Phi_{3N} \right| P \left| U_\mu \right\rangle + \left\langle \Phi_{3N} \right| tG_0 P \left| U_\mu \right\rangle, & 3BB \end{cases}$$

Very efficient approach !

(1) $|U_\mu\rangle = |U_\mu(\omega,Q)\rangle$ *for given* j_μ
(2) The same equation is solved for 2BB and 3BB

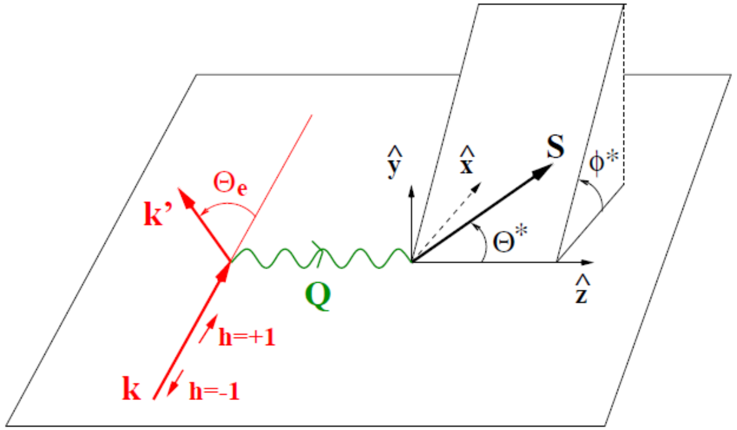
Basis states $|\,pq\alpha\,\rangle$ are used to solve the equation on $|U_\mu\rangle$

The bulk of predictions obtained with the AV18 2N force, the UrbanaIX 3N force,
the single nucleon current supplemented with π- and ρ-like 2N currents linked to AV18 (no 3N currents)



What we calculate (keep in mind: $\sigma\left(h,\vec{S},\{m_f\}\right)=\Sigma\left(\vec{S},\{m_f\}\right)+h\,\Delta\left(\vec{S},\{m_f\}\right)$

set of spin magnetic quantum numbers in the final state



(1) Unpolarized cross sections

$$\sigma_0=\frac{1}{2}\sum_{\{m_f\}}\left(\Sigma\left(\vec{S},\{m_f\}\right)+\Sigma\left(-\vec{S},\{m_f\}\right)\right)$$

(2) Target analyzing power

$$A_y=\frac{\sum_{\{m_f\}}\left(\Sigma\left(\hat{y},\{m_f\}\right)-\Sigma\left(-\hat{y},\{m_f\}\right)\right)}{\sum_{\{m_f\}}\left(\Sigma\left(\hat{y},\{m_f\}\right)+\Sigma\left(-\hat{y},\{m_f\}\right)\right)}$$

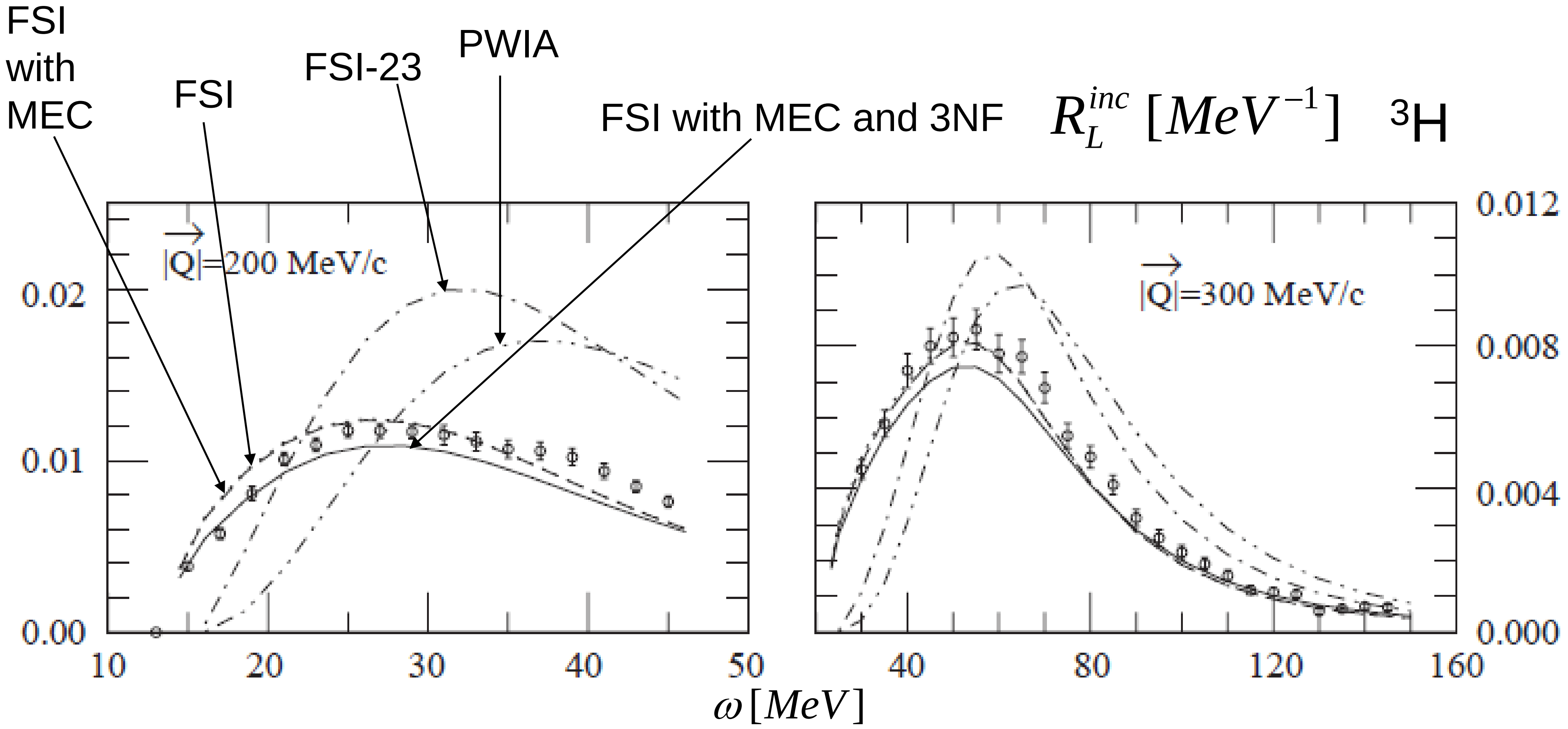
(3) Spin dependent helicity asymmetries

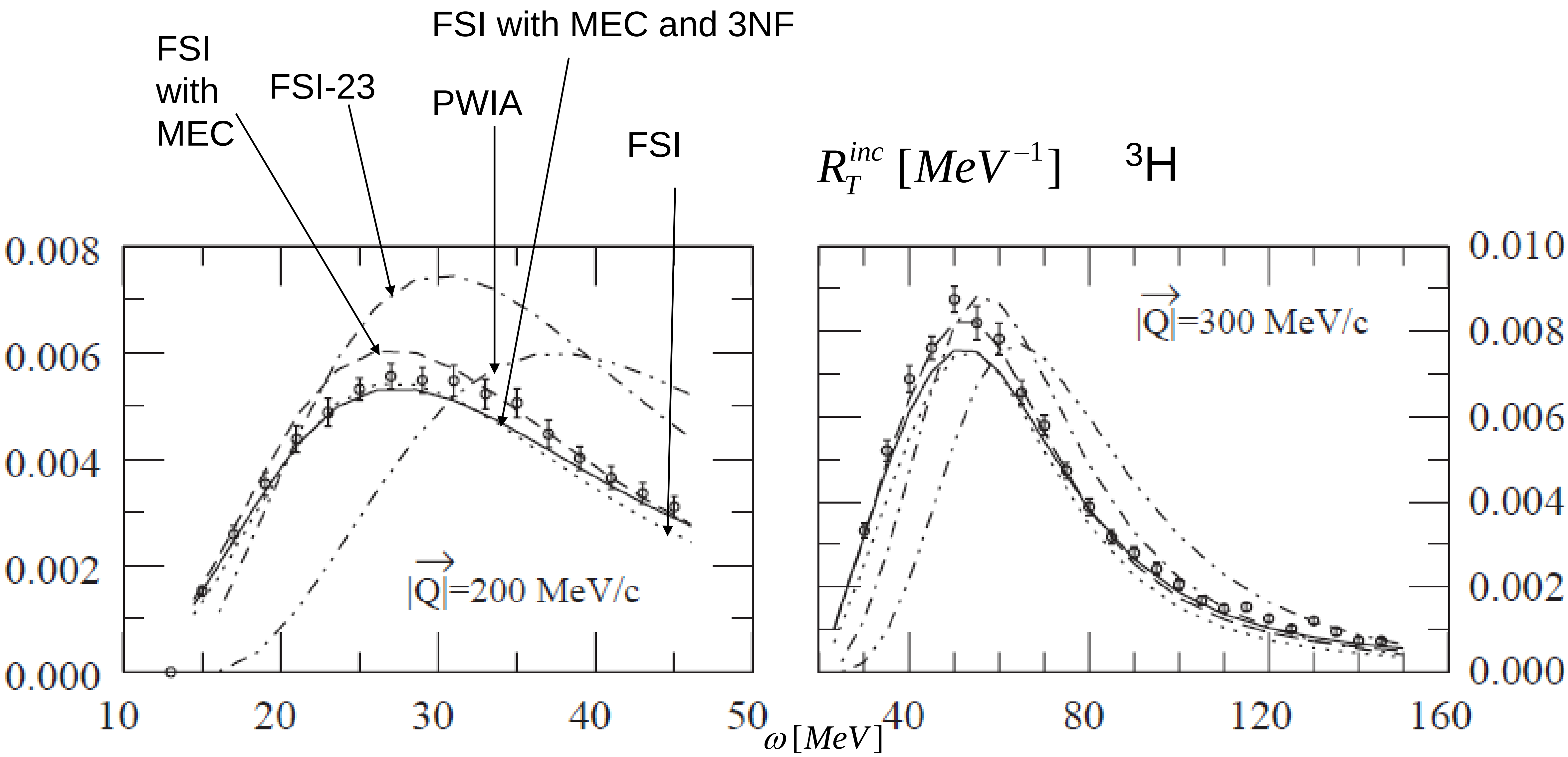
$$A\left(\vec{S}\right)\equiv\frac{\sum_{\{m_f\}}\left(\sigma\left(h=1,\vec{S},\{m_f\}\right)-\sigma\left(h=-1,\vec{S},\{m_f\}\right)\right)}{\sum_{\{m_f\}}\left(\sigma\left(h=1,\vec{S},\{m_f\}\right)+\sigma\left(h=-1,\vec{S},\{m_f\}\right)\right)}$$

Inclusive electron scattering

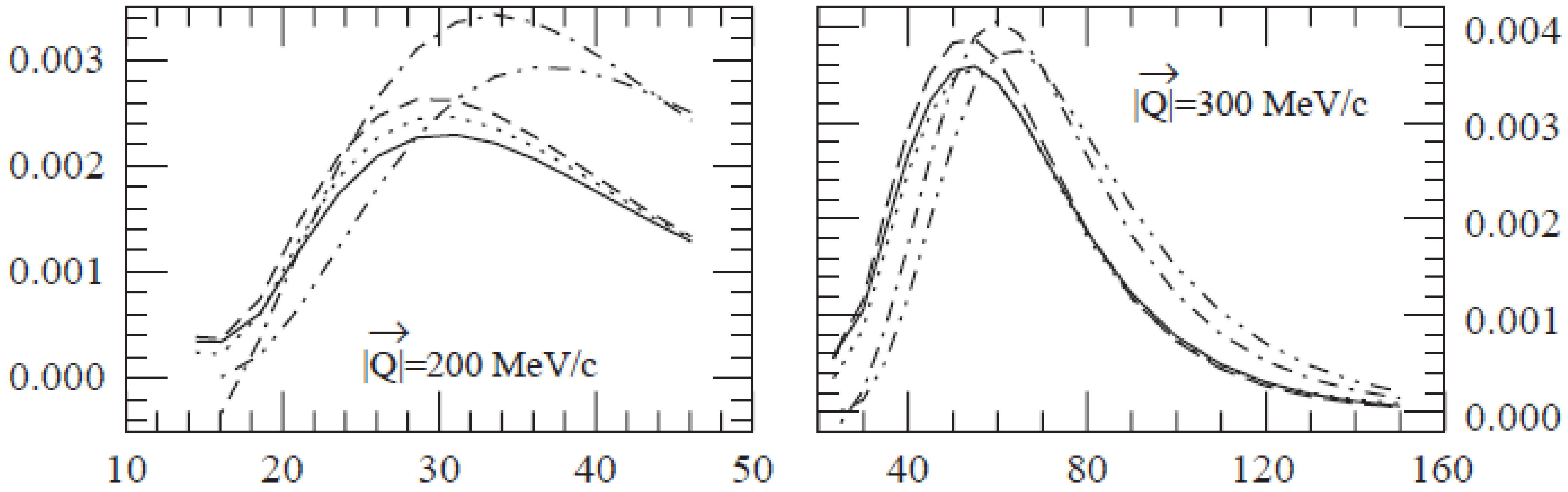
J. Golak et al. / Physics Reports 415 (2005) 89–205

$$\sigma^{inc} = \sigma_{Mott}\left(v_L R_L^{inc} + v_T R_T^{inc} - h\left(v_{T'} R_{T'}^{inc} \cos\theta^* + 2 v_{TL'} R_{TL'}^{inc} \sin\theta^* \cos\phi^*\right)\right) \qquad R_i^{inc} \equiv R_i^{inc}\left(\omega, \left|\vec{Q}\right|\right)$$



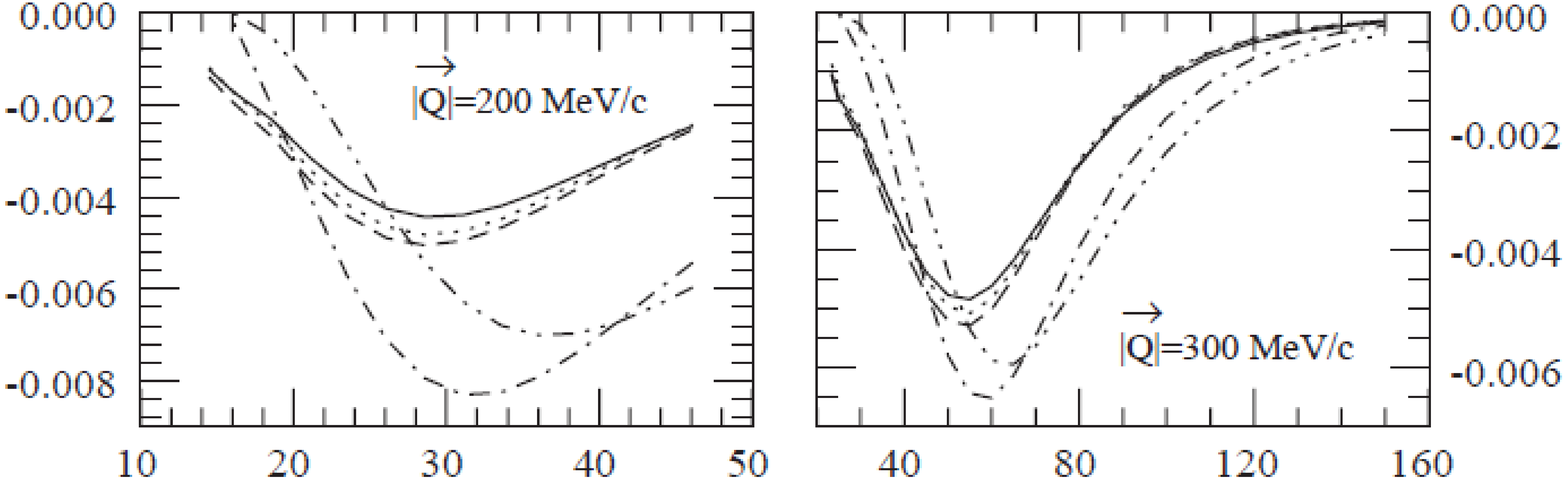

FSI with MEC and 3NF
FSI with MEC
FSI-23
PWIA
FSI
$R_T^{inc}\ [MeV^{-1}]$
^{3}H
0.008
0.006
0.004
0.002
0.000
10
20
30
40
50
|Q|=200 MeV/c
0.010
0.008
0.006
0.004
0.002
0.000
40
80
120
160
|Q|=300 MeV/c
$\omega\,[MeV]$

$R_{T'}^{inc}\ [MeV^{-1}]$ ^{3}H

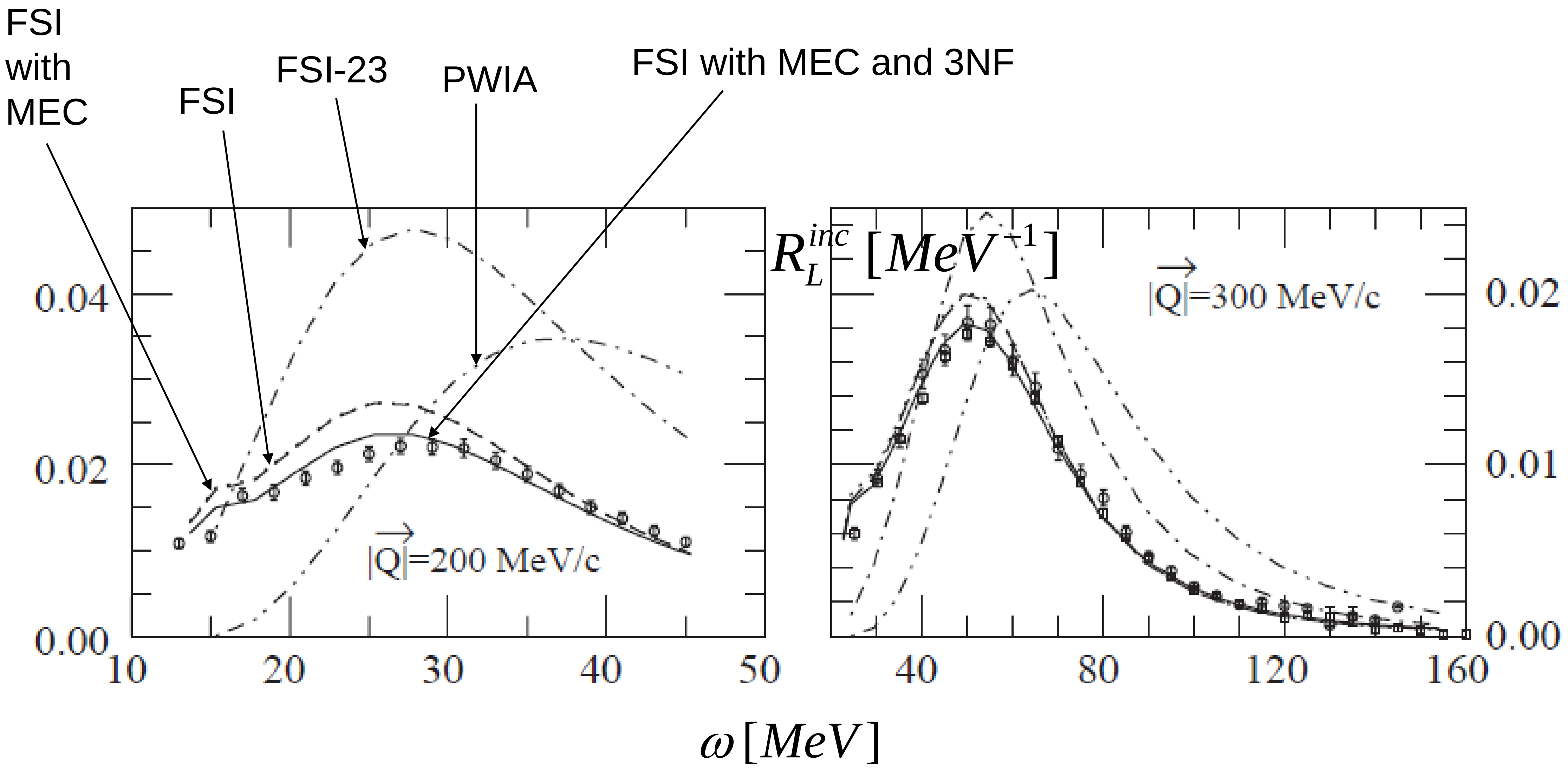


$\omega\,[MeV]$

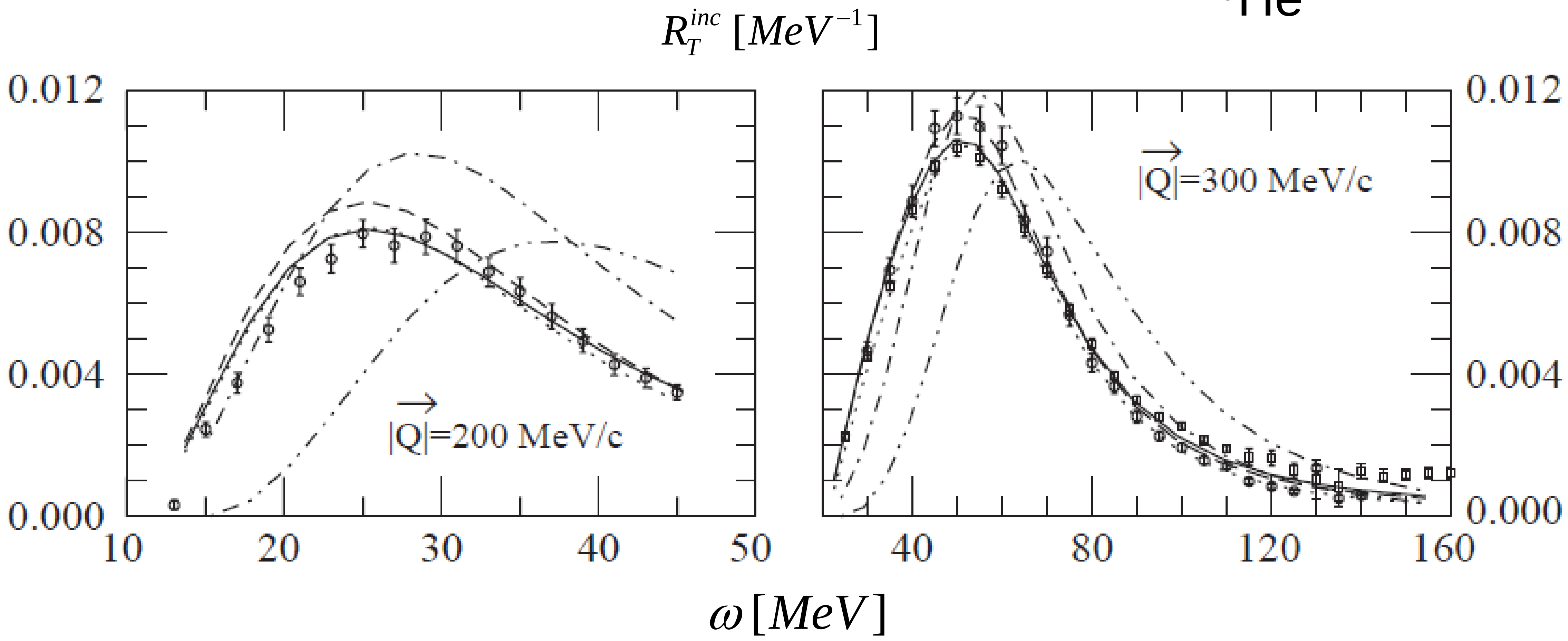
$R_{TL'}^{inc}\ [MeV^{-1}]$ ^3H



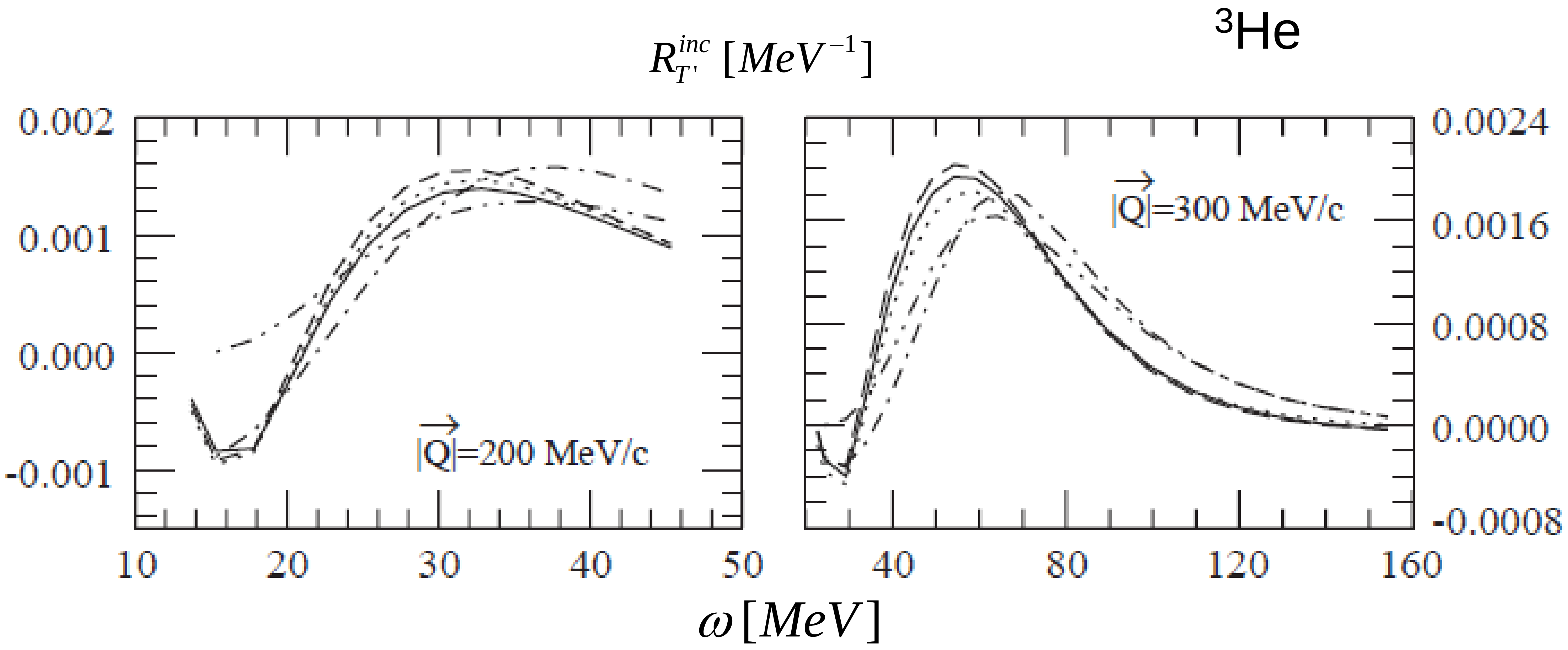
$\omega\,[MeV]$


FSI with MEC
FSI
FSI-23
PWIA
FSI with MEC and 3NF
$R_L^{inc}\,[MeV^{-1}]$
$|\vec{Q}|$=200 MeV/c
$|\vec{Q}|$=300 MeV/c
0.00
0.02
0.04
0.00
0.01
0.02
10
20
30
40
50
40
80
120
160
$\omega\,[MeV]$

^{3}He



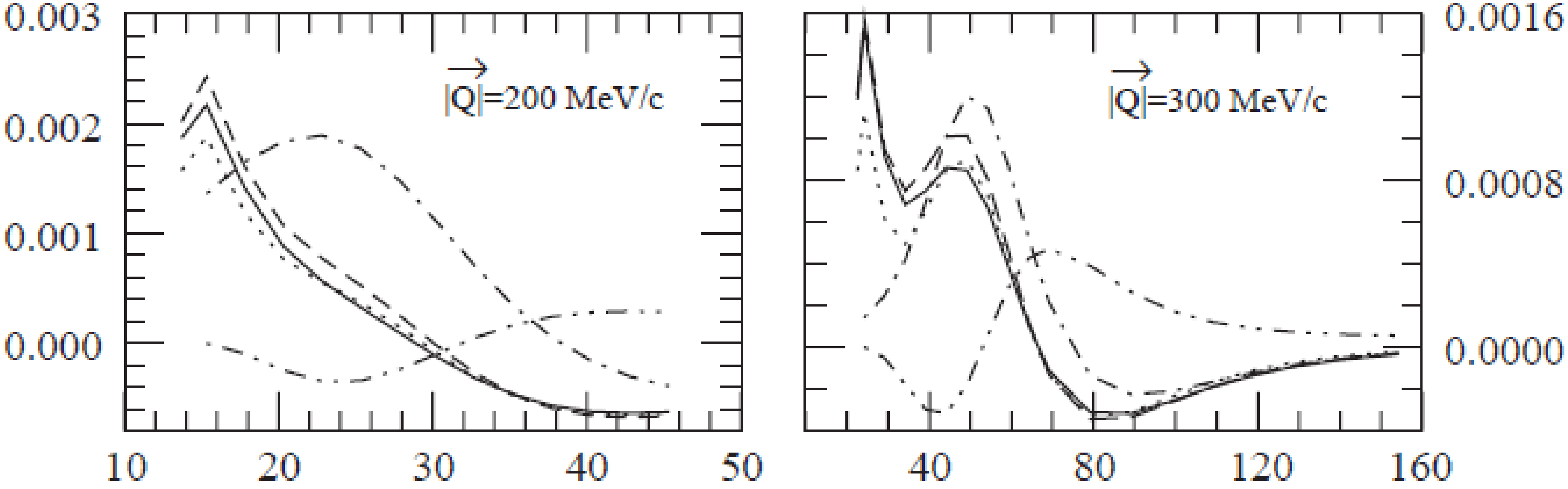
$R_T^{inc}\ [MeV^{-1}]$
0.012
0.008
0.004
0.000
$\overrightarrow{|Q|}$=200 MeV/c
$\overrightarrow{|Q|}$=300 MeV/c
10
20
30
40
50
40
80
120
160
$\omega\,[MeV]$



^{3}He
$R_{T'}^{inc}$ [MeV^{-1}]
0.002
0.001
0.000
-0.001
$|\vec{Q}|$=200 MeV/c
10
20
30
40
50
$|\vec{Q}|$=300 MeV/c
0.0024
0.0016
0.0008
0.0000
-0.0008
40
80
120
160
ω [MeV]

$R^{inc}_{TL'}\ [MeV^{-1}]$

^{3}He



$\omega\,[MeV]$

Spin dependent
helicity asymmetries

$$A(\vec{S}) \equiv \frac{\sum_{\{m_f\}} \left(\sigma(h=1,\vec{S},\{m_f\}) - \sigma(h=-1,\vec{S},\{m_f\})\right)}{\sum_{\{m_f\}} \left(\sigma(h=1,\vec{S},\{m_f\}) + \sigma(h=-1,\vec{S},\{m_f\})\right)}$$

Under PWIA
approximation

$$A_{\parallel} \equiv A(\theta^* = 0^o, \phi^* = 0^o) \equiv A_{T'} \propto (G_M^n)^2$$

$$A_{\perp} \equiv A(\theta^* = 90^o, \phi^* = 0^o) \equiv A_{TL'} \propto G_M^n \, G_E^n$$

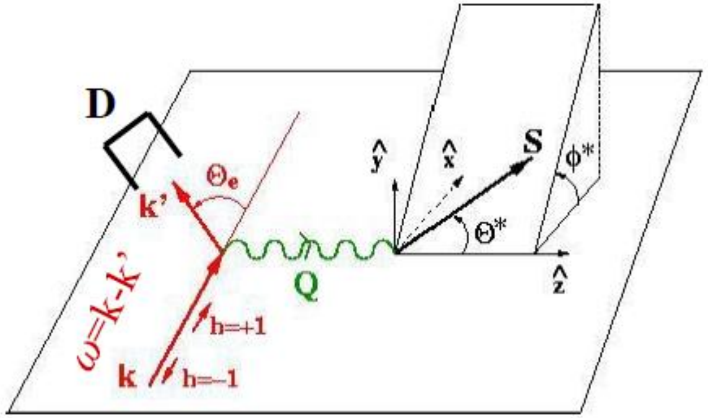
magnetic
neutron
formfactor

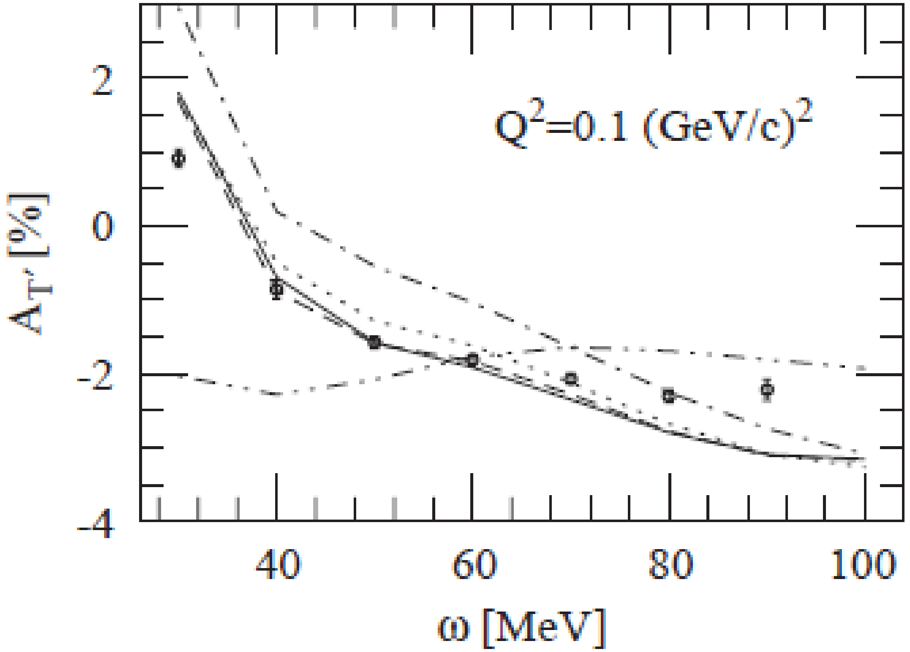
electric
neutron
formfactor

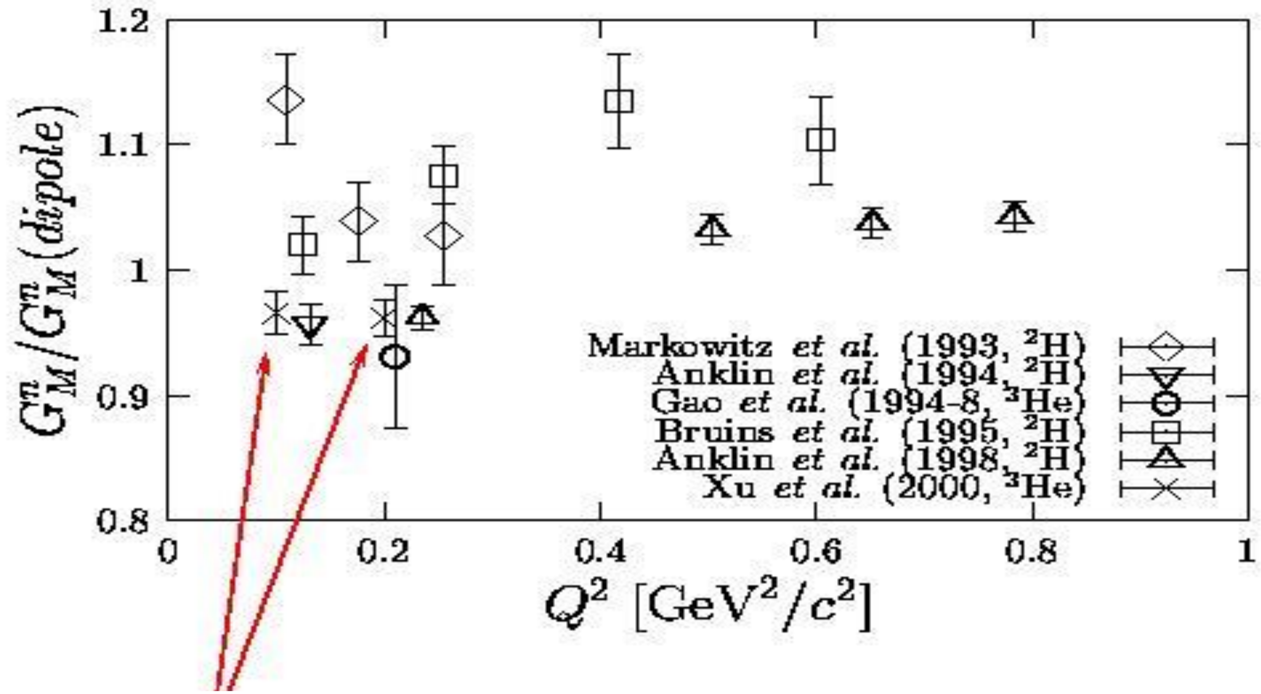
Question: Can we use inclusive asymmetries to obtain information about G_M^n and G_E^n ?

Answer: YES for G_M^n NO for G_E^n !!!

W. Xu *et al.*, Phys. Rev. Lett. **85**, 2900 (2000)

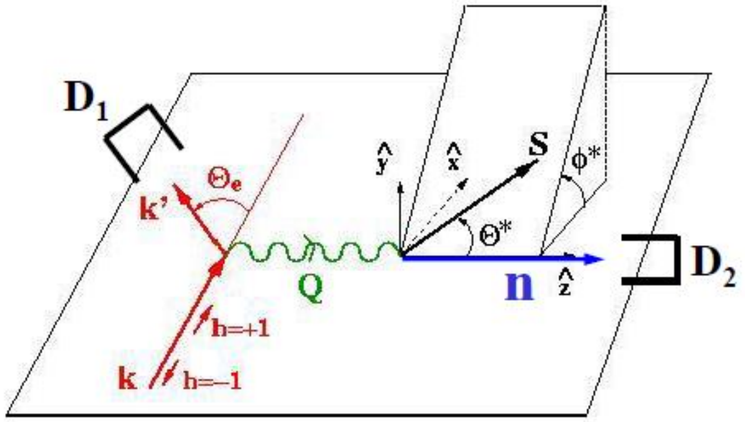


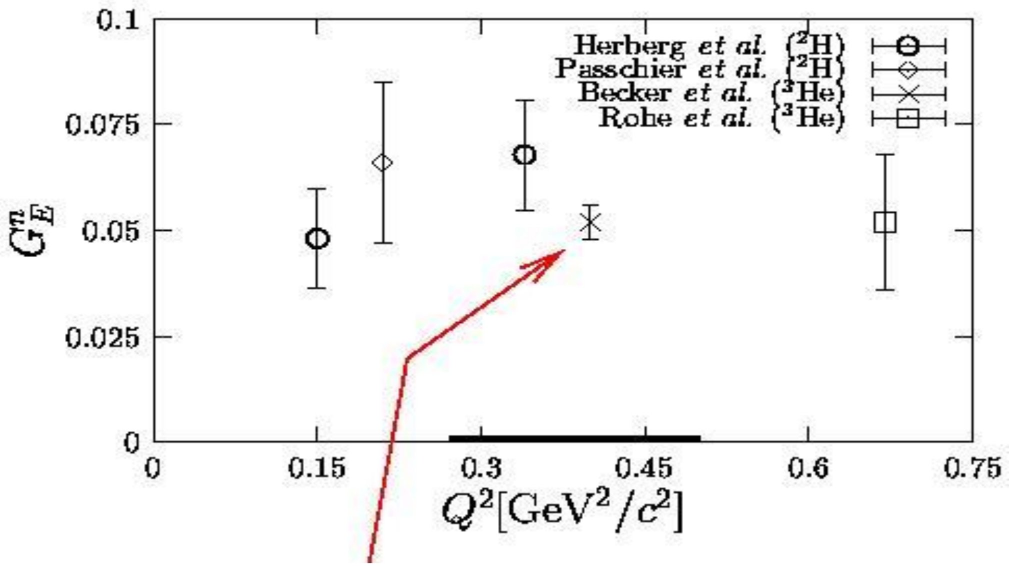




our results

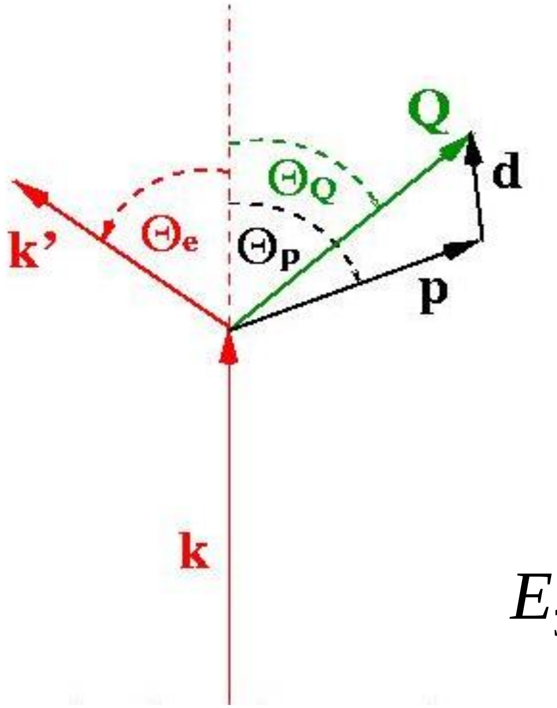
Analysis of a Mainz experiment,
where the ejected neutron was measured
in coincidence with the outgoing electron





our result

Exclusive e + ^{3}He → e + p + d reaction



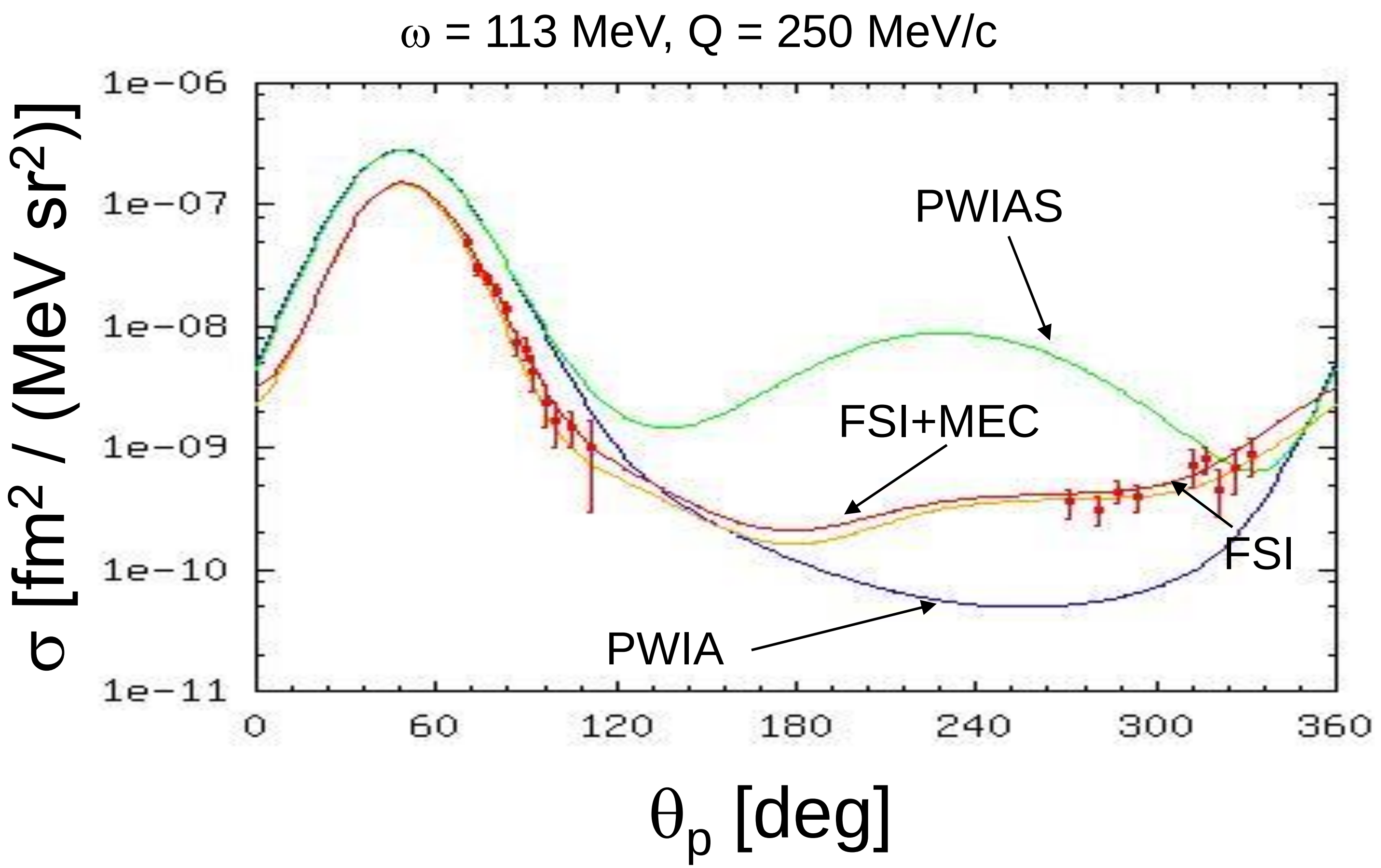
Usually electron and hadronic
planes coincide !

Two-nucleon kinematics,
provided the „electron arm" is fixed

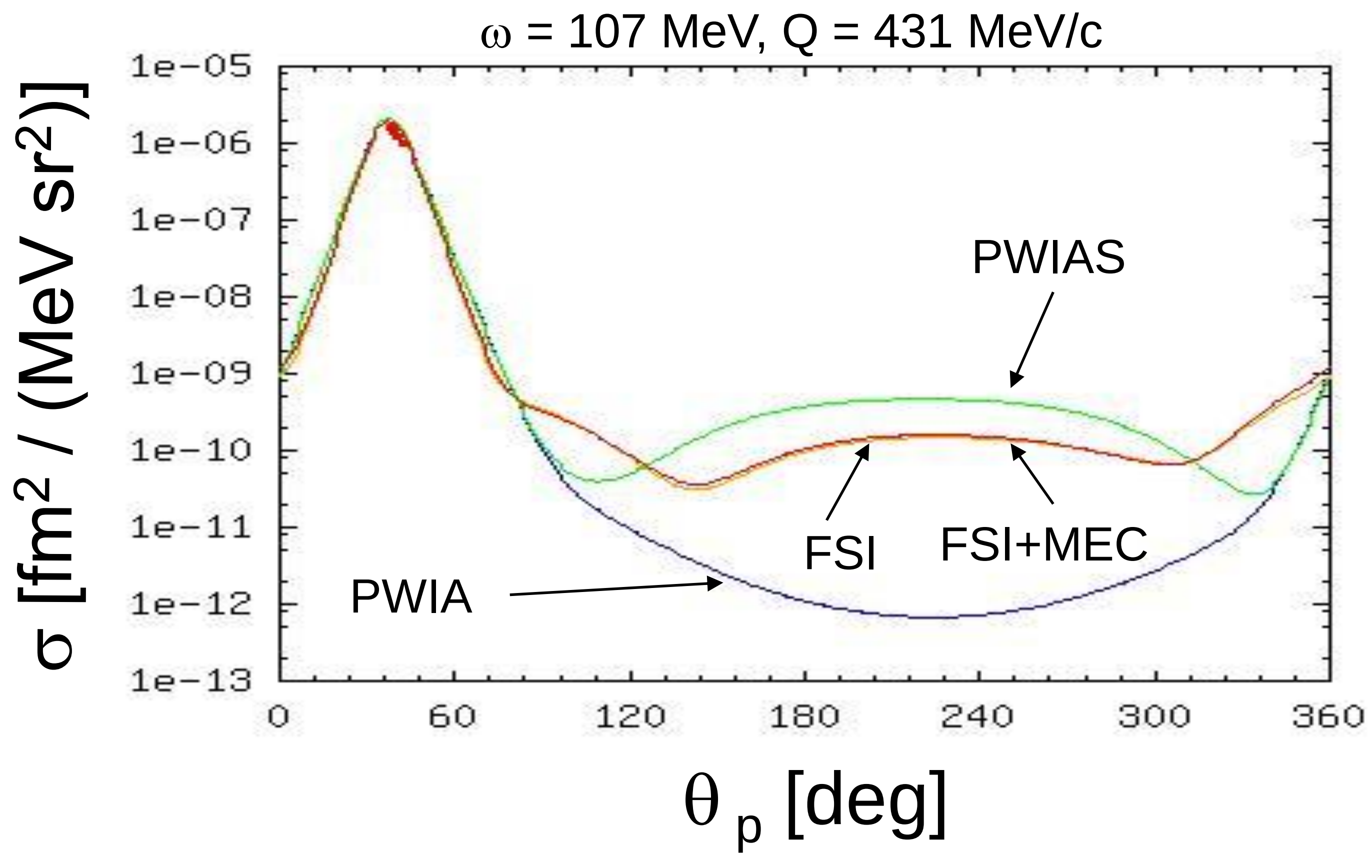
$$E_{3N} \approx \omega - \left|E^b_{3He}\right| + \left|E^b_{2H}\right| - \frac{\vec{Q}^2}{6m}$$

internal energy
of the 3N system

J. Golak et al. / Physics Reports 415 (2005) 89–205

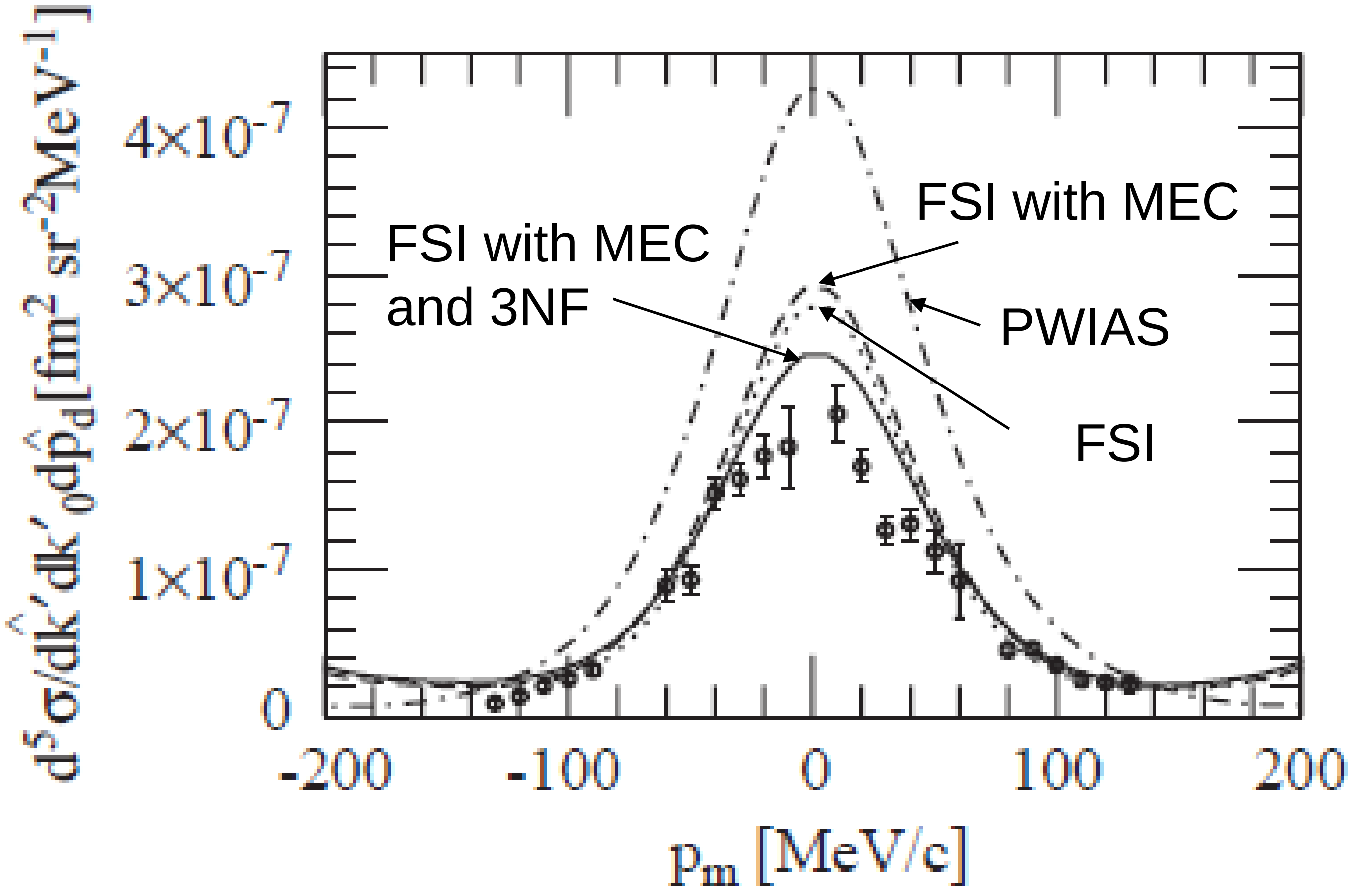


FSI is important for all proton angles !



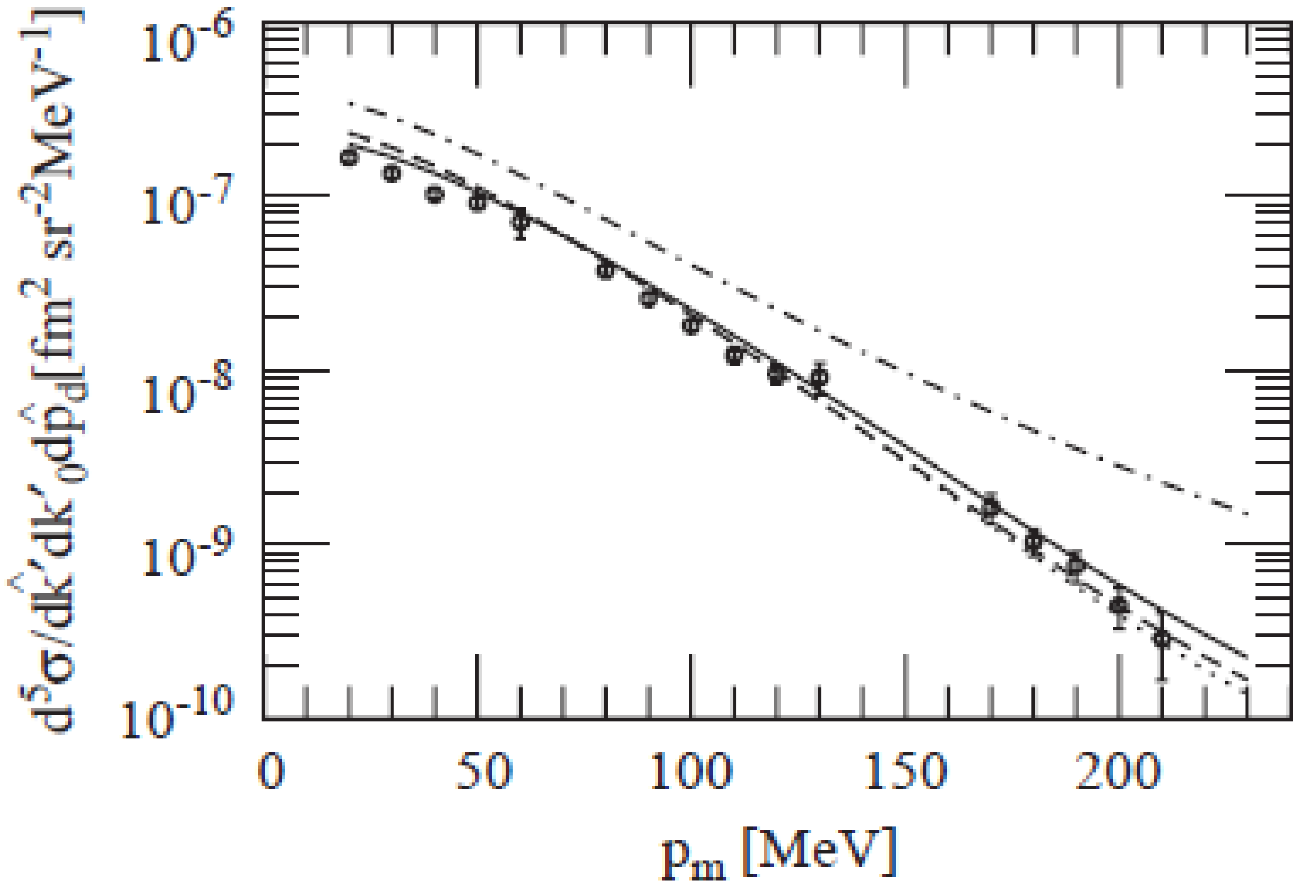
FSI less visible in the proton knockout kinematics,
MEC do not play any important role

Deuteron knock-out cross section as a function of the missing (proton) momentum



E= 370 MeV
ω= 50 MeV
Q=412 MeV/c

Deuteron knock-out cross section for the „paralel kinematics"
as a function of the missing (proton) momentum



E= 370 MeV
Q=412 MeV/c

Semi-exclusive ^{3}He(e,e'p)pn and ^{3}He(e,e'n)pp reactions

Target analyzing power

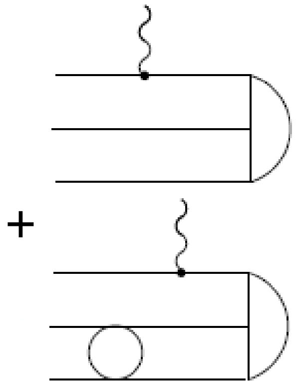
J. Bermuth et al. / Physics Letters B 564 (2003) 199–204

Table 1

Results of A_y^0 for the $^3\overrightarrow{\mathrm{He}}(\mathrm{e},\mathrm{e}'\mathrm{n})$ and $^3\overrightarrow{\mathrm{He}}(\mathrm{e},\mathrm{e}'\mathrm{p})$ reactions. The experimental data at $Q^2 = 0.37\ (\mathrm{GeV}/c)^2$ are compared to results of a complete Faddeev calculation. For $(\mathrm{e},\mathrm{e}'\mathrm{n})$ the effects of dropping different contributions in the calculation are also shown

$Q^2\ (\mathrm{GeV}/c)^2$	0.37	0.67
$^3\overrightarrow{\mathrm{He}}(\mathrm{e},\mathrm{e}'\mathrm{n})$:		
Experiment	0.144 ± 0.034	0.028 ± 0.010
Theory	0.178	
Theory without MEC	0.186	
Theory with $G_{\mathrm{ep}} = G_{\mathrm{mp}} = 0$	0.004	
$^3\overrightarrow{\mathrm{He}}(\mathrm{e},\mathrm{e}'\mathrm{p})$:		
Experiment	-0.025 ± 0.005	-0.016 ± 0.005
Theory	-0.017	

Eur. Phys. J. A **21**, 335–348 (2004)



Spectral function
in the FS23
approximation can be obtained
both from R_L and R_T

$$S(E,k) = \frac{1}{2}\, m\, p_{23}\, \frac{1}{(G_E)^2} \int d\hat{p}_{23} R_L(\mathrm{FSI23})$$
$$= \frac{1}{2}\, m\, p_{23}\, \frac{2m^2}{Q^2 (G_M)^2} \int d\hat{p}_{23} R_T(\mathrm{FSI23})$$

nucleon mass

relative momentum of nucleons 2 and 3

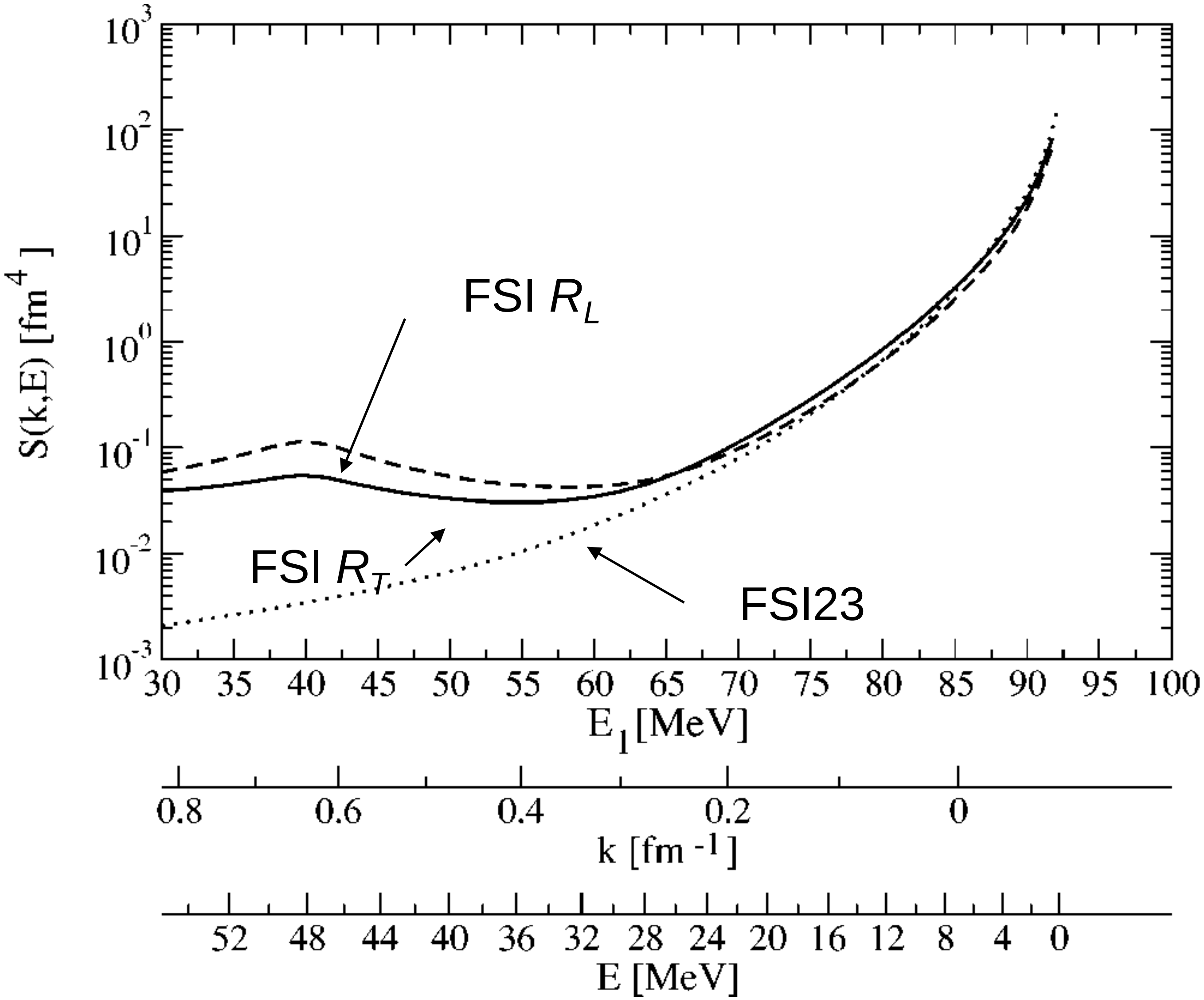
$$E \equiv \frac{\vec{p}_{23}^{\,2}}{m}, \quad \vec{k} \equiv \vec{p}_1 - \vec{Q}$$

momentum of nucleon 1

What do we get for other dynamical pictures ?

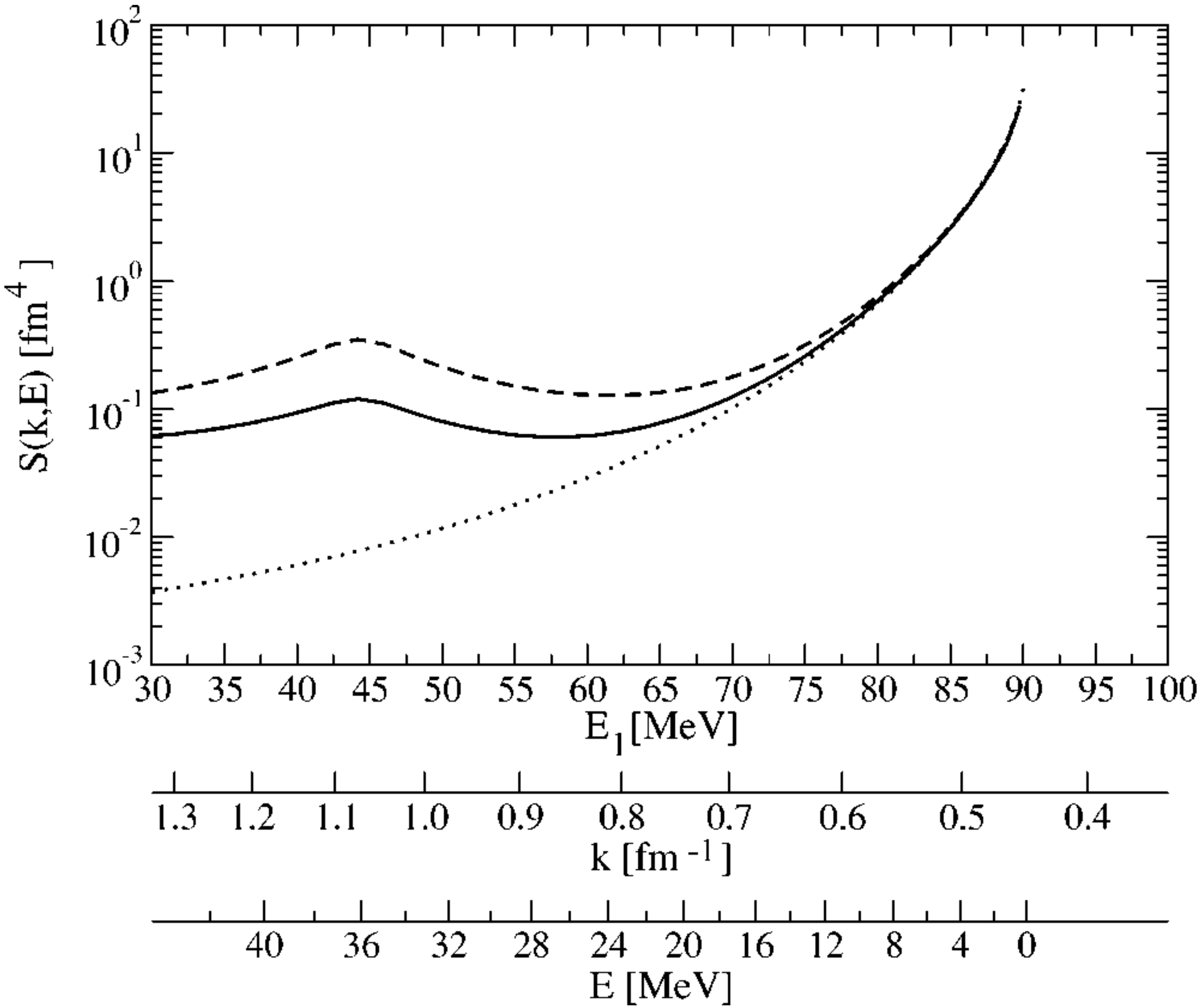
$$S(E,k)^{\mathrm{FSI}}_L = \frac{1}{2}\, m\, p_{23}\, \frac{1}{(G_E)^2} \int d\hat{p}_{23} R_L(\mathrm{FSI}),$$
$$S(E,k)^{\mathrm{FSI}}_T = \frac{1}{2}\, m\, p_{23}\, \frac{2m^2}{Q^2 (G_M)^2} \int d\hat{p}_{23} R_T(\mathrm{FSI})$$

Proton knock-out



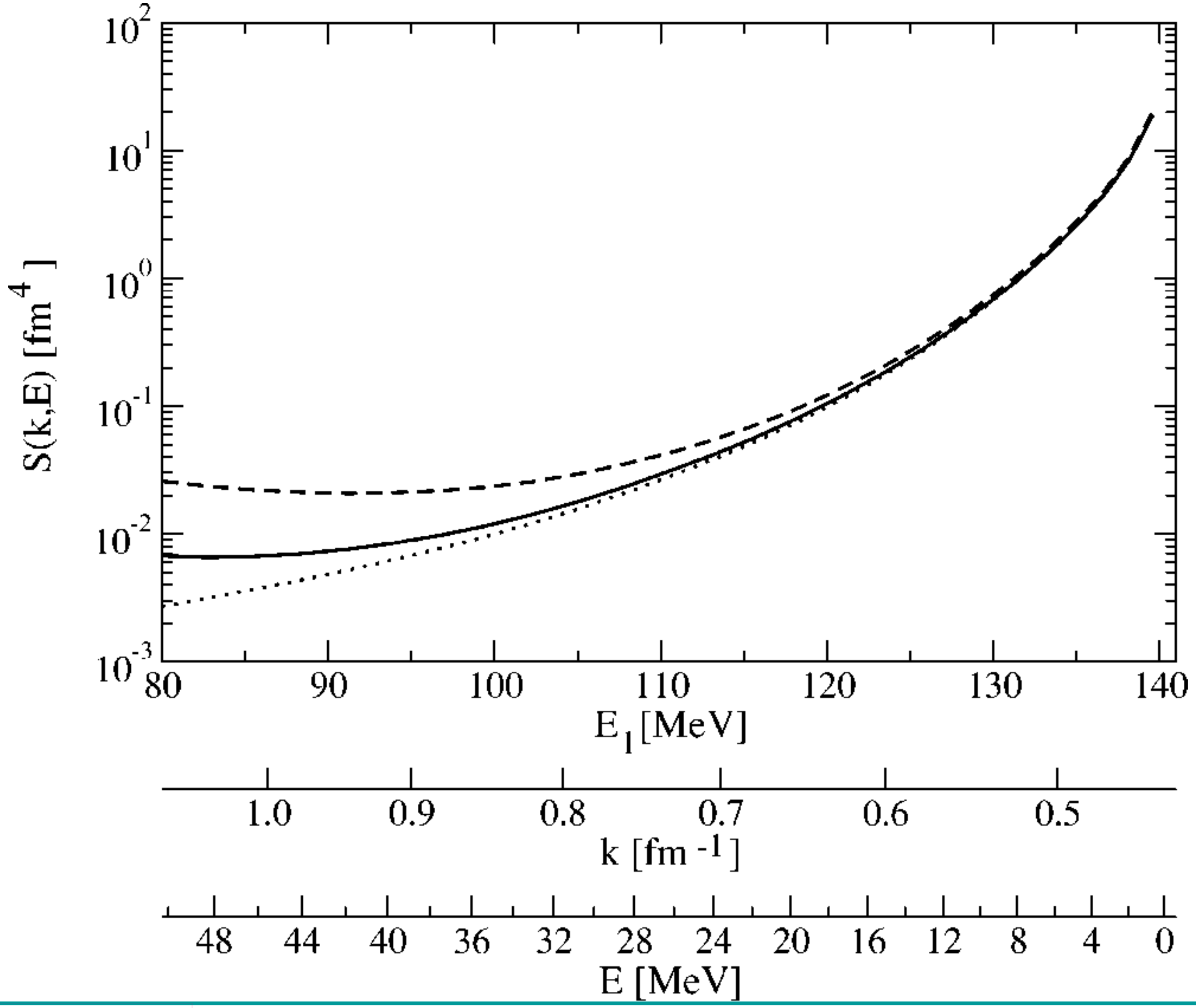
ω= 100 MeV
Q= 400 MeV/c

Proton knock-out



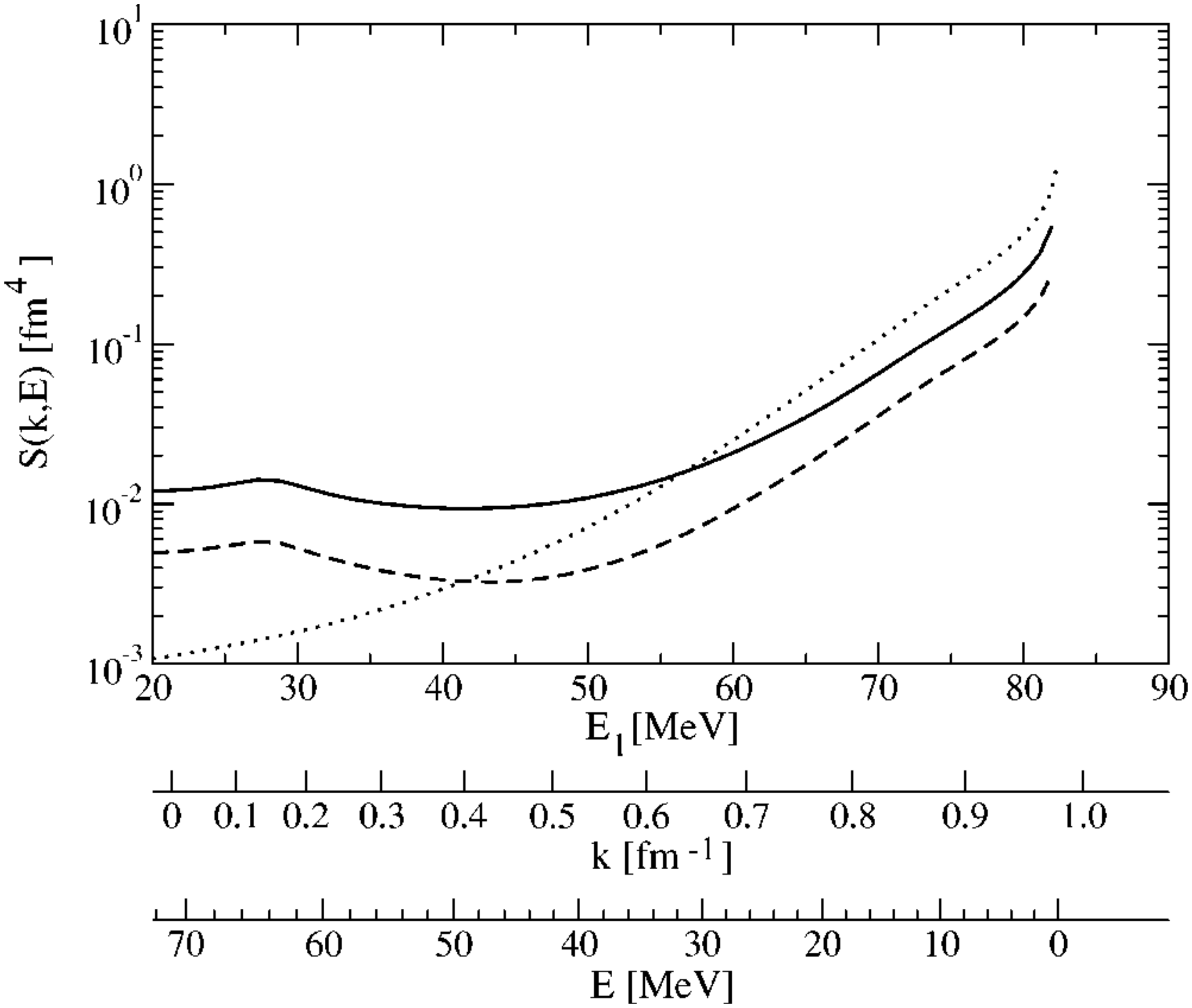
ω= 100 MeV
Q= 500 MeV/c

Proton knock-out



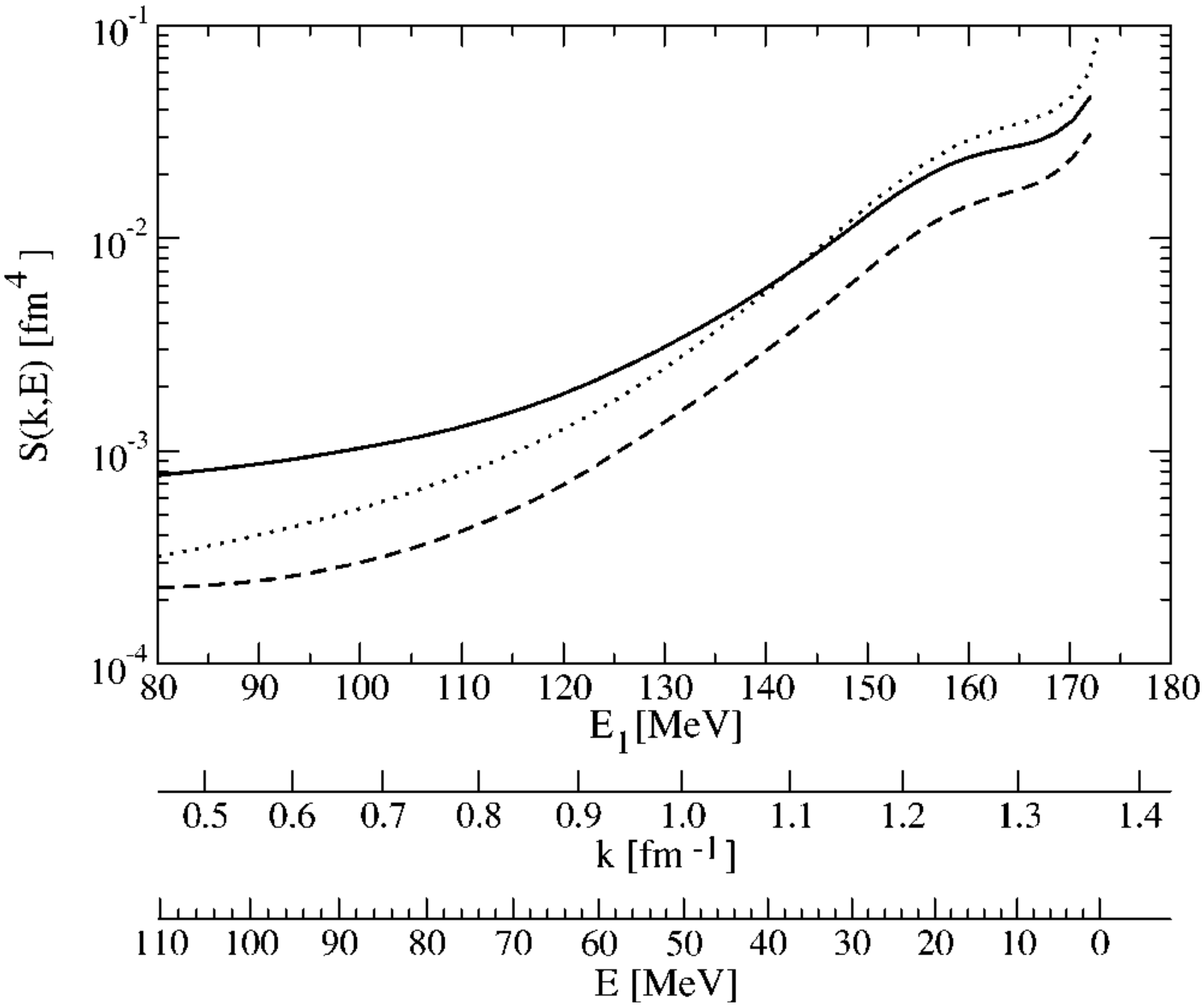
ω= 150 MeV
Q= 600 MeV/c

Proton knock-out



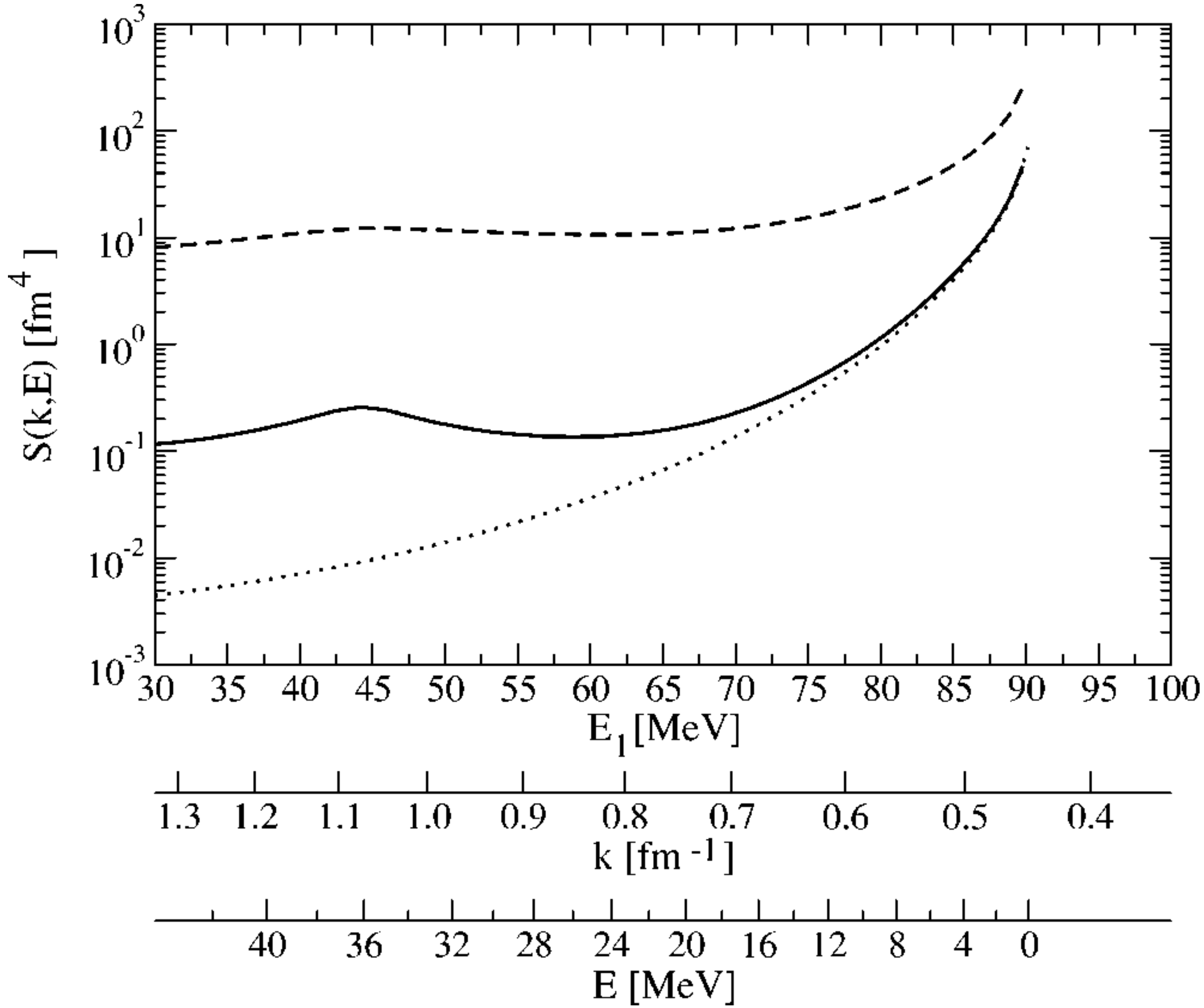
ω= 100 MeV
Q= 200 MeV/c

Proton knock-out



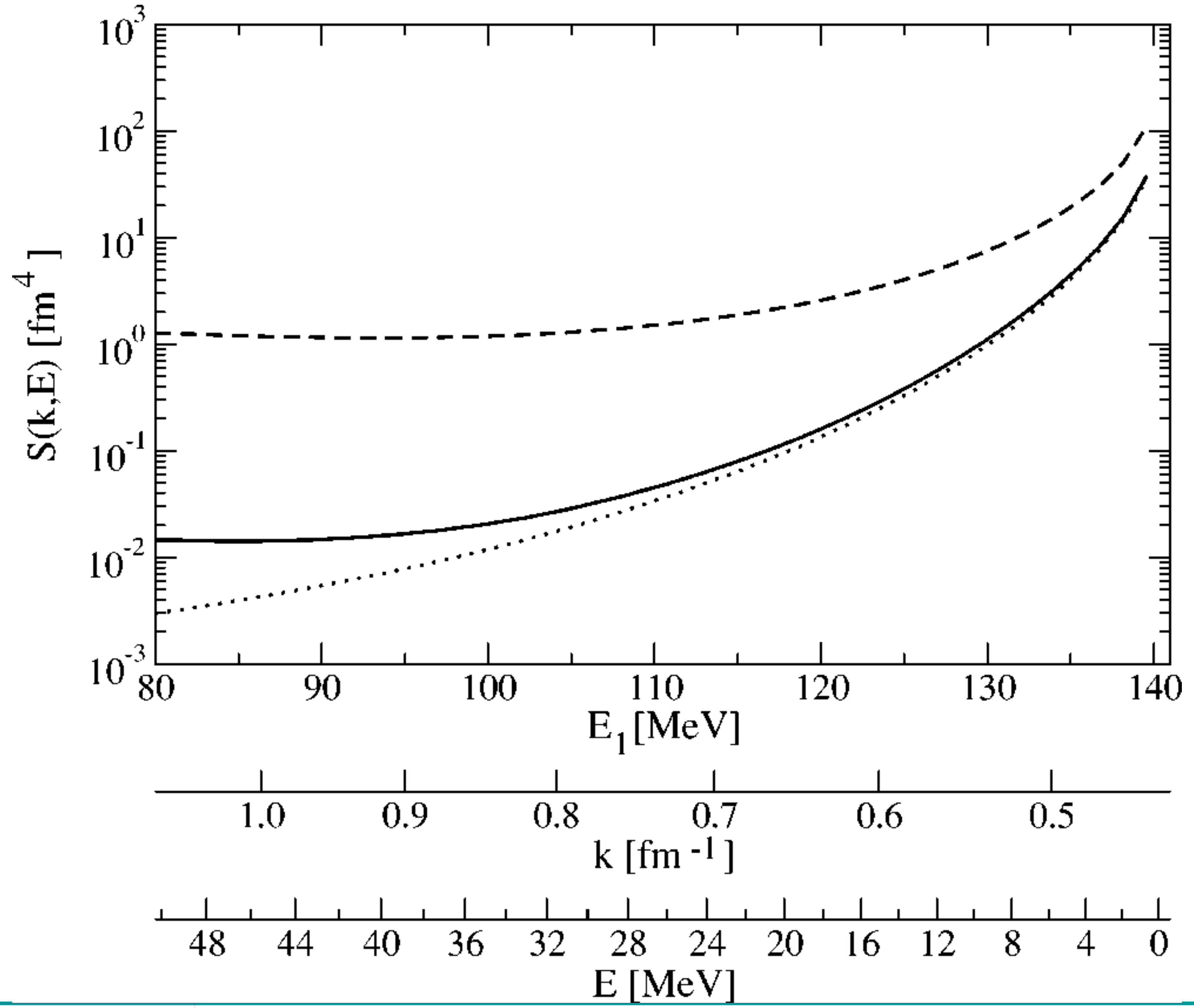
ω= 200 MeV
Q= 300 MeV/c

Neutron knock-out



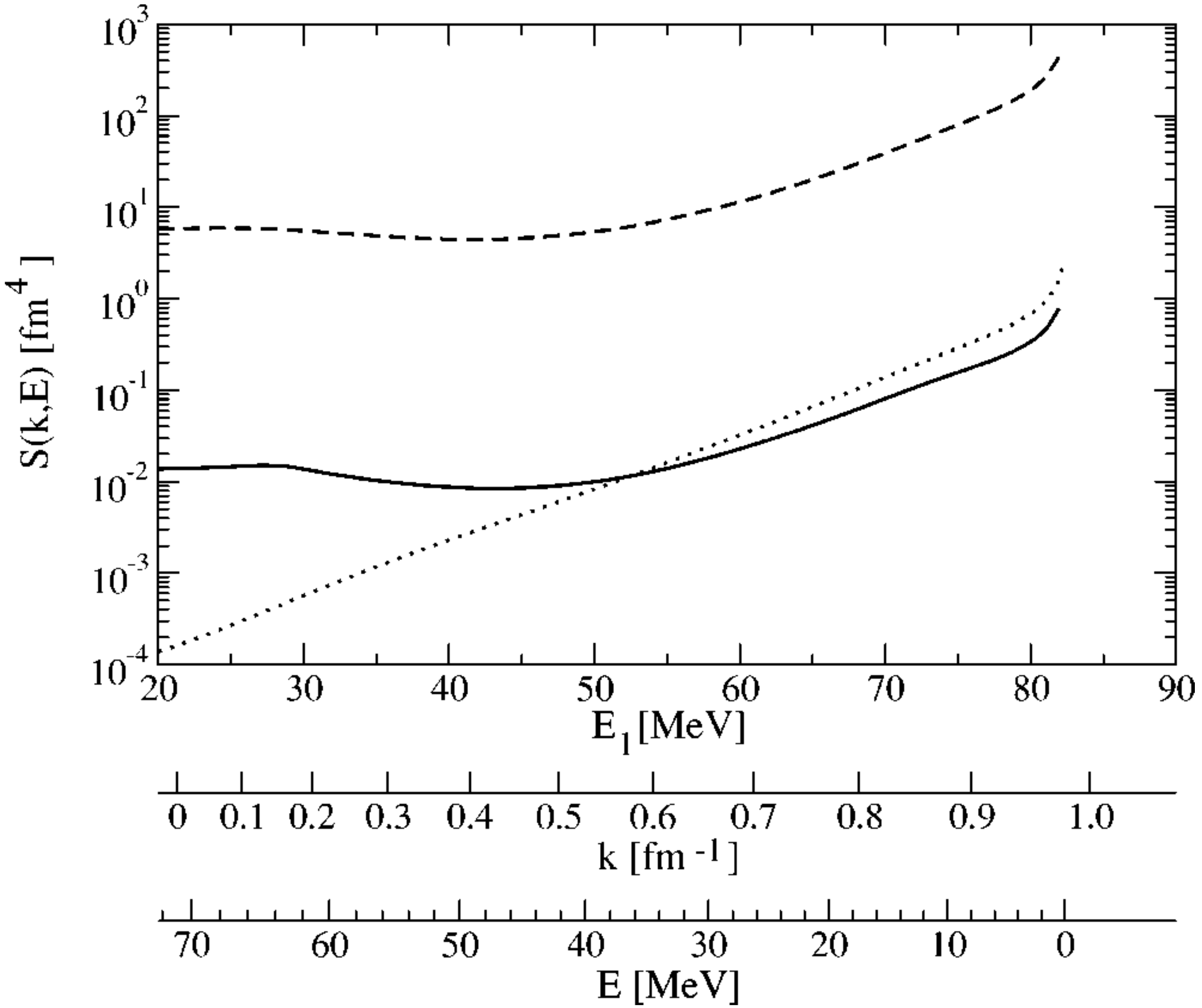
ω= 100 MeV
Q= 500 MeV/c

Neutron knock-out



ω= 150 MeV
Q= 600 MeV/c

Neutron knock-out



ω= 100 MeV
Q= 200 MeV/c

Some remarks

For a realistic situation we deal with some acceptances, which means that we need to integrate over ***exclusive*** cross section Σ_f.

$$\overline{\Sigma} \equiv \int dE \int d\theta_e' \int dE' \int df' \, \frac{1}{2} \sum_{m_i} \sum_{\{m_f\}} \Sigma_f \left(\theta^* \left(E, \theta_e', E' \right), \phi^* \left(E, \theta_e', E' \right) \right)$$

Note that we deal here with two types of integrations. First three integrations are for the „electron arm”. Their ranges decide how many equations for $| U_\mu >$ have to be solved !

The integral marked as $\int df'$ is meant for the „hadronic arm”. Here we may deal with two- and three-body break-up of ^{3}He, different angles and energies of the nuclear fragments but for fixed „electron arm”. All vectors and angles in the „hadronic arm” have to calculated with respect to the system of reference given by the „electron arm”.

A lot of „$\int df'$” can obscure physics !

Summary and outlook

1. We have at our disposal a universal nonrelativistic framework to investigate several electromagnetic (electroweak) processes on ^{2}H and ^{3}He below the pion production threshold
2. We work with AV18, UrbanaIX and related current operators
3. We can learn about basic reaction mechanisms and verify popular approximations
4. Observables provide information about EM properties of the nucleon, momentum distributions in ^{3}He, nucleon-nucleon correlations, …
5. The same framework can be applied also to other processes with 3N (muon capture on ^{3}He, neutrino scattering on A=2,3 nuclei, non-mesonic and mesonic weak decays of the hypertriton, non-radiative and radiative pion capture,   …)

Some references:
Phys. Rept. 415, 89 (2005),
Eur. Phys. J. A25, 177 (2005),
Phys. Rev. C 72, 054005 (2005),
Phys. Rev. Lett. 101, 022303 (2008),
Phys. Rev. Lett. 103, 152501 (2009)

Summary and outlook

1. We have at our disposal a universal nonrelativistic framework to investigate several electromagnetic (electroweak) processes on ^{2}H and ^{3}He below the pion production threshold (Limitations: nonrelativistic character and lack of Coulomb force in the 3N continuum)
2. We can learn about basic reaction mechanisms and verify popular approximations
3. Observables provide information about EM properties of the nucleon, momentum distributions in ^{3}He, nucleon-nucleon correlations, …
4. The same framework can be applied also to other processes with 3N (muon capture on ^{3}He, neutrino scattering on A=2,3 nuclei, non-mesonic and mesonic weak decays of the hypertriton, non-radiative and radiative pion capture, …)

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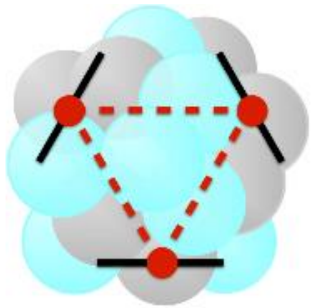
Phys. Rev. Lett. 103, 152501 (2009)

Summary and outlook (*cont.*)

6. To make better predictions we need improved models of the nuclear forces and current operators (special role played by the Chiral Effective Field Theory).
7. We need very good tools to deal with many spin-isospin structures that appear in the 2N forces, 3N forces (even 4N forces) and in the current operators.
8. New („3D”) approach - expansion in independent operators to work with scalar functions (already done for the deuteron, NN scattering, electroweak processes in the 2N system, 3N bound state). Work in progress for Nd scattering states as formulated in Eur. Phys. J A43, 339 (2010).
9. LENPIC (Low Energy Nuclear Physics International Collaboration) to coordinate few-nucleon and many-nucleon calculations

http://www.lenpic.org



``to understand nuclear structure and reactions with chiral forces”

LENPIC

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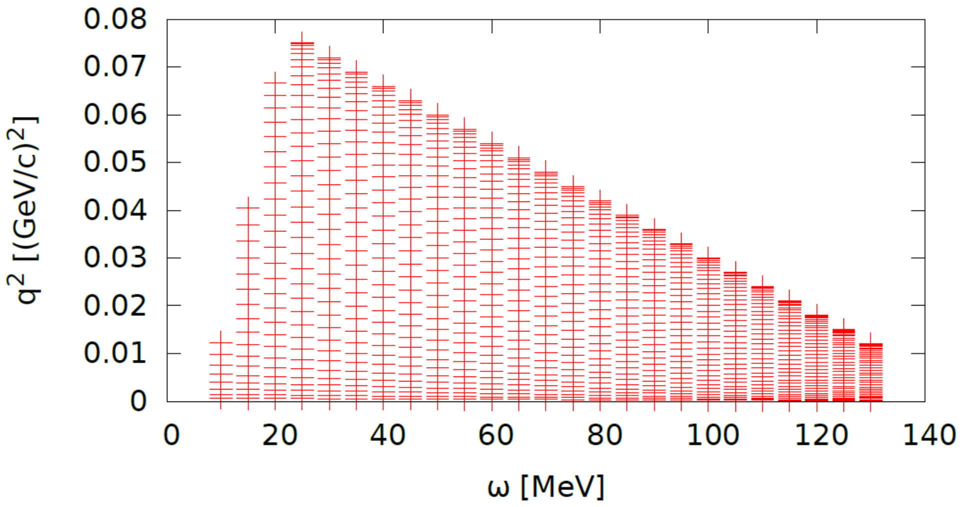
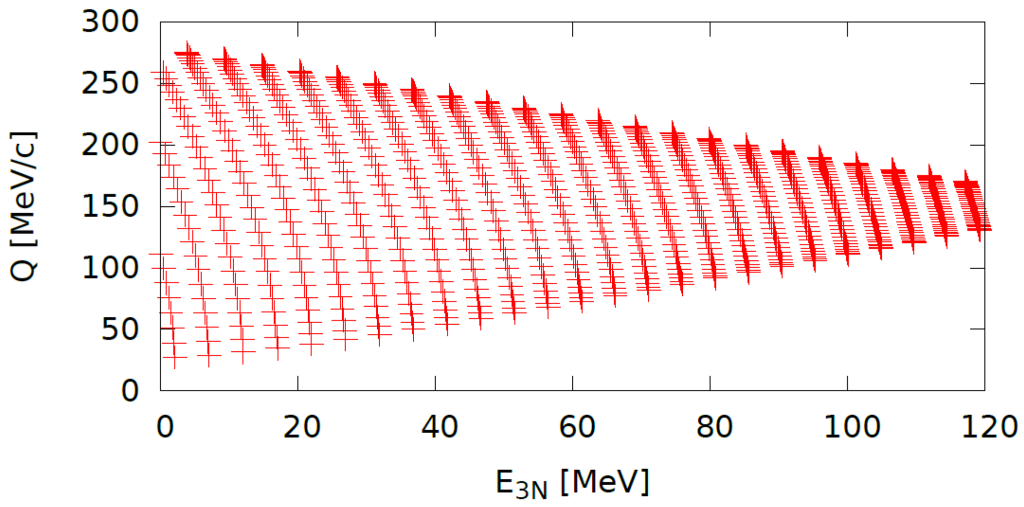
Summary and outlook *(cont.)*

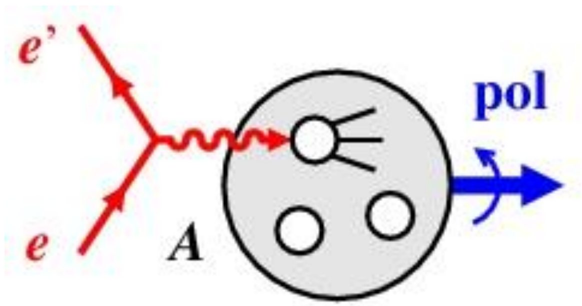
10. precision measurements are badly needed for various reactions in the kinematical regimes, where chiral predictions are applicable (MESA – a new linear accelerator in Mainz can be very important)





Expected MESA parameters (E= 150 MeV, E' > 20 MeV, Θ_e > 10 deg)
ideal to study few-nucleon dynamics within the nonrelativistic framework with the input from ChEFT !

From Abhay Desphande’s lectures at National Nuclear Physics Summer School held at MIT in July 2016



„What are you doing here ?”

The EIC Machine parameters:

For e-p/n collisions:

- Polarized e, p, deteron or ^{3}He beams
- Electron beam energy ~ 5-20 GeV
- Proton beam energy up to ~50 - 250 GeV (RHIC exists!)
- *Luminosity* L_{ep}~ $10^{33\text{-}34}$ $cm^{-2}sec^{-1}$
- Center of mass energy ~ sqrt(4 x E_p x E_e) ~ 30 – 140 GeV

For e-A collisions: (use the same collider ring…)

- Wide range in Nuclei (proton-to-Uranium)
- Luminosity per nucleon (scaled) by the one for e-p
- Variable CM energy (scaled by A)

Thank you for your attention !