Target fragmentation in polarized nucleon

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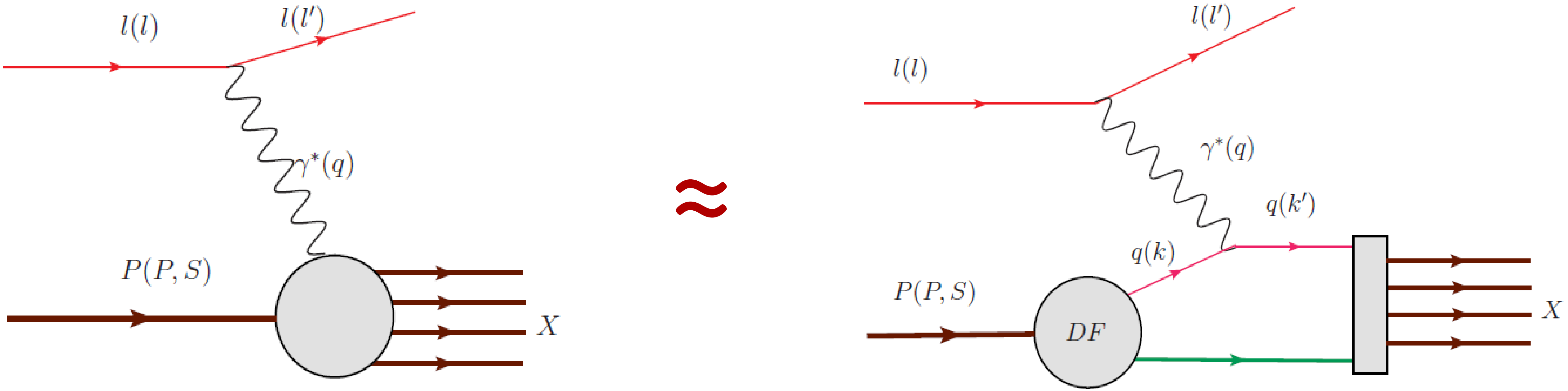
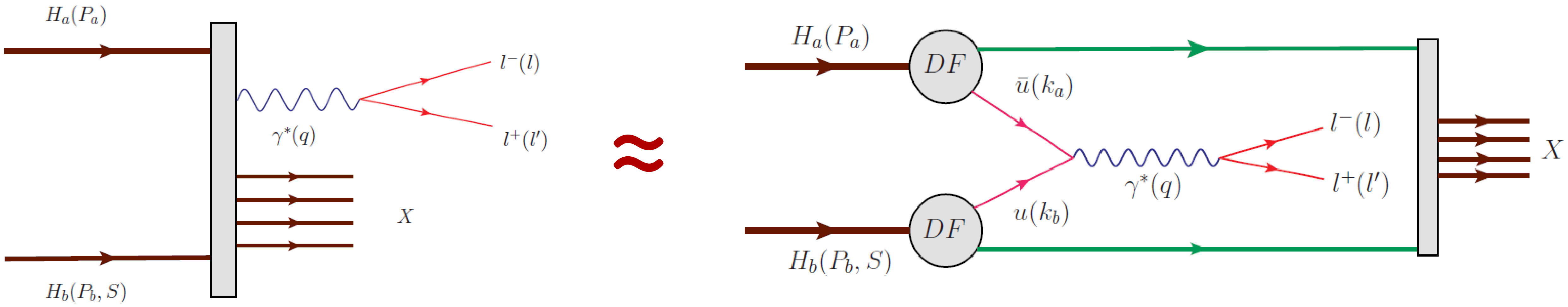
- **Only lightest ion -- nucleon**
- **Processes to access the Spin and Transverse Momentum Dependent non-perturbative inputs (STMDs)**
  - Parton Distribution Functions: STMD PDF
    - DIS, DY, SIDIS, high p_T hadron production in pp collisions
  - Parton Fragmentation Functions: STMD FF
    - Hadron production in e^+e^- annihilation: SIA, SIDIS, high p_T hadron production in pp collisions
  - STMD Fracture Functions
    - SIDIS
  - String Fragmentation
    - LEPTO, PYTHIA

QCD factorization: DIS



Access to nucleon $f_1^{q+\bar{q}}(x)$ and $g_1^{q+\bar{q}}(x)$ leading twist PDFs

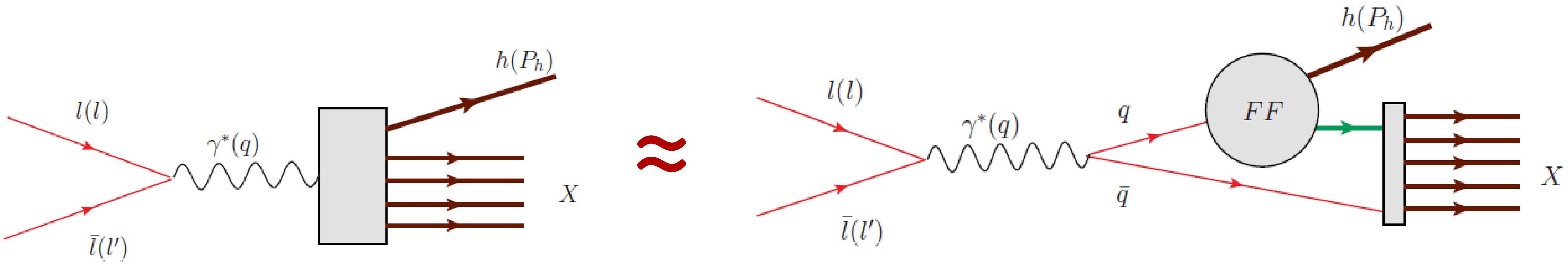
QCD TMD factorization: DY processes



Access to nucleon, pion and kaon $f_1(x,k_T^2)$, $g_1(x), h_1(x,k_T^2)$

and $h_1^{\perp}(x,k_T^2)$ leading twist PDFs

QCD TMD factorization: SIA



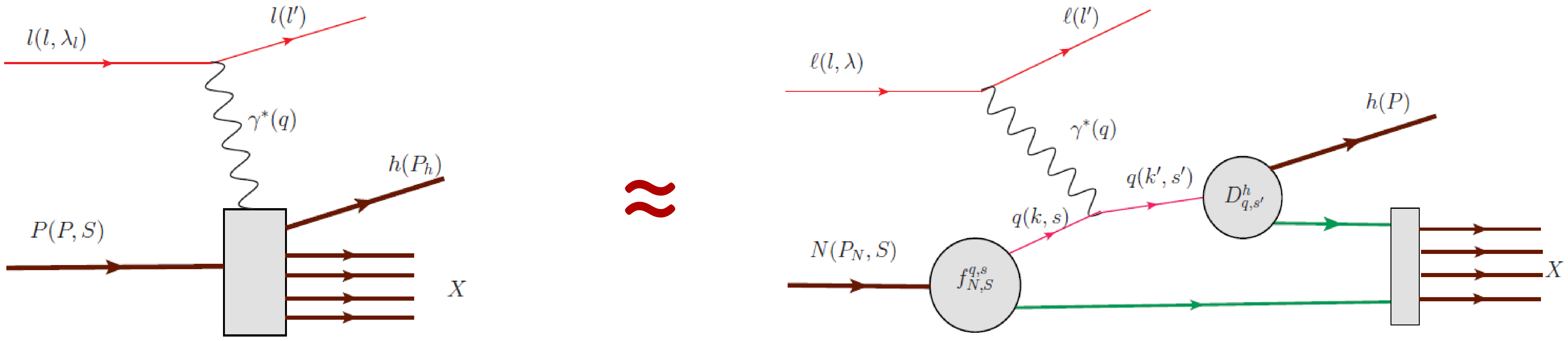
Access to $q+\bar{q}$ fragmentation finctions $D^h_{q+\bar{q}}(\mathrm{z},\mathrm{p}_\perp^2)$

Two hadron production in opposite hemispheres: acces to Collins FF $H^h_{1q}(\mathrm{z},\mathrm{p}_\perp^2)$

Two di-hadron production in opposite hemispheres:

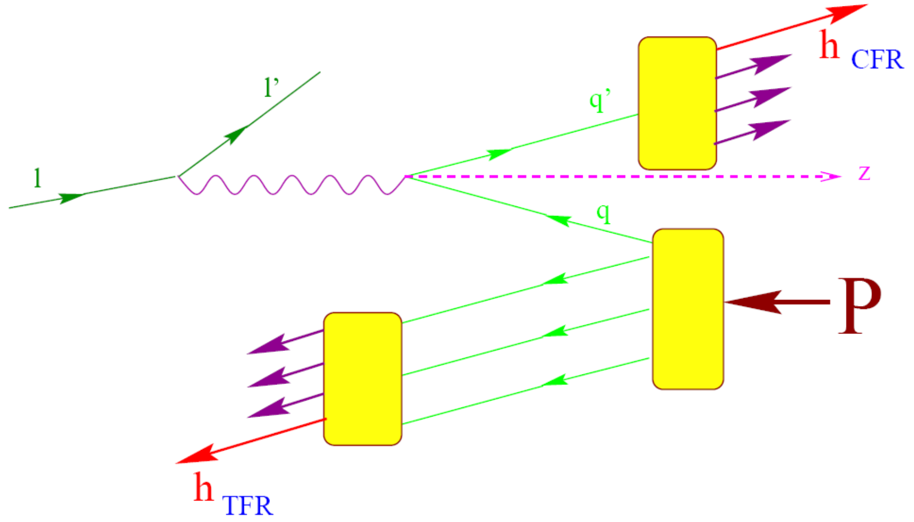
acces to $H_q^{\measuredangle}(\mathrm{z})$, $H_q^{\perp}(\mathrm{z})$ and $\mathrm{G}_q^{\perp}(\mathrm{z})$

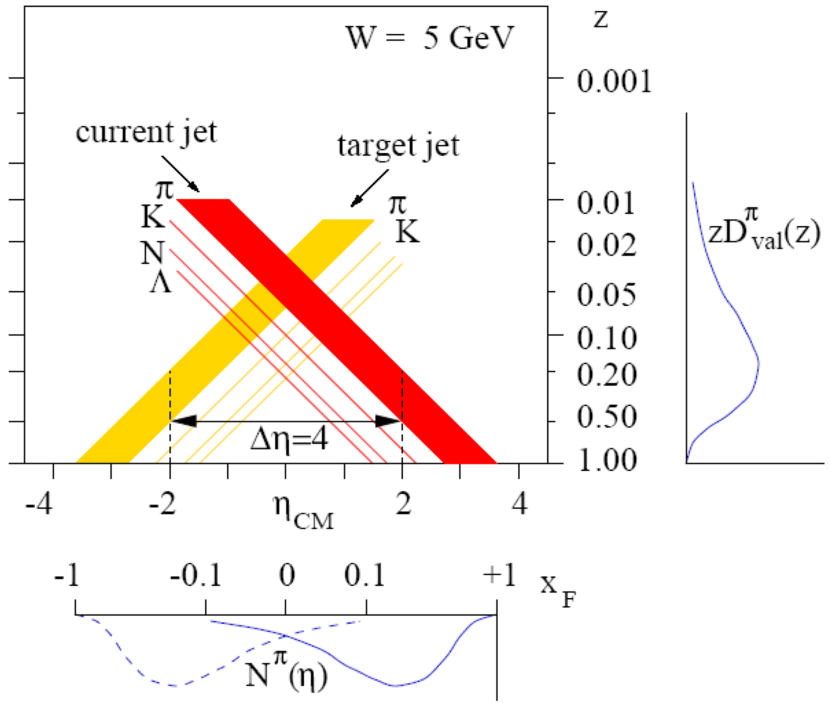
QCD TMD factorization: SIDIS in CFR

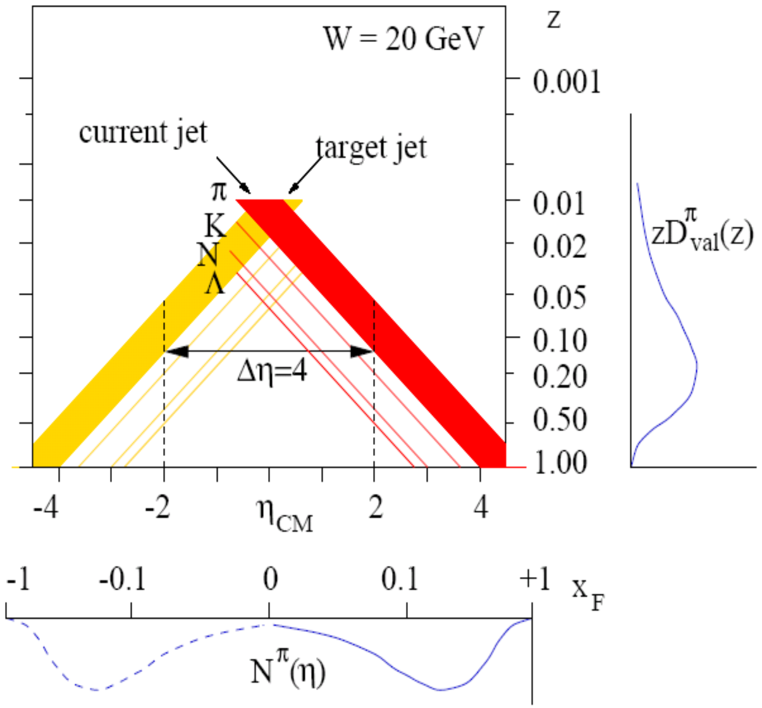


Access to nucleon $f_1^q(x, k_T^2)$, $g_1^q(x, k_T^2)$ and $h_1^q(x, k_T^2)$,… leading twist PDFs and Collins FF $H_1(z, p_T^2)$

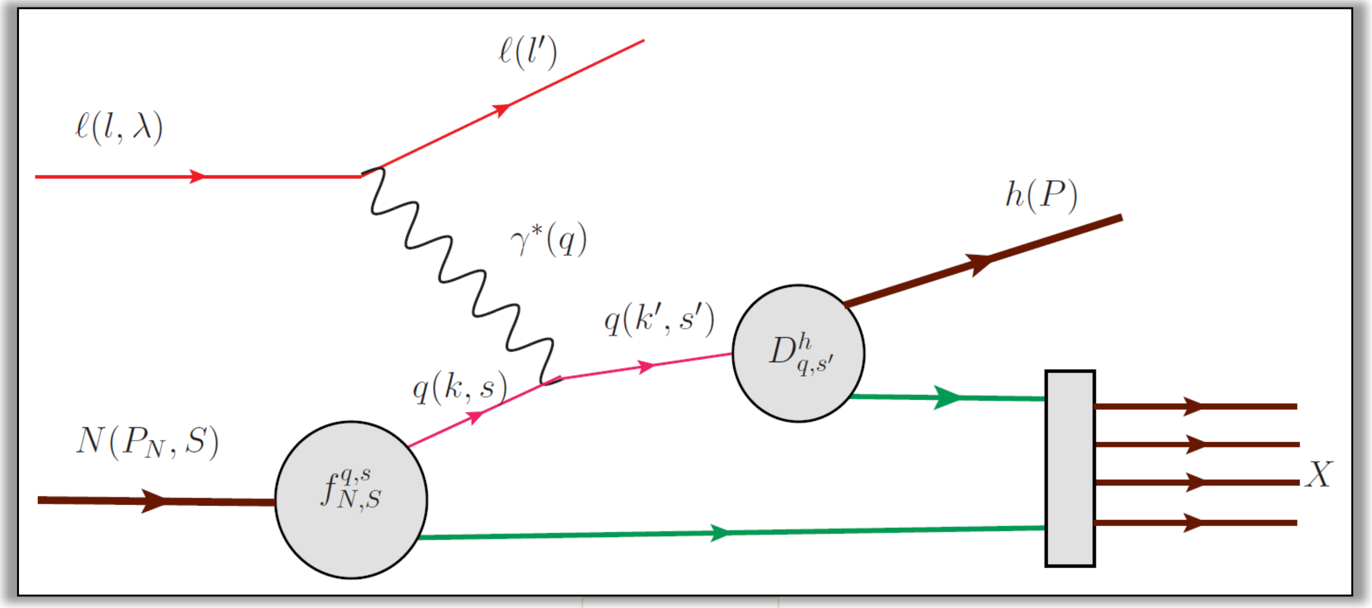
SIDIS: CFR and TFR separation







SIDIS: CFR



$$x_F > 0$$

$$\frac{d\sigma^{\ell(l,\lambda)+N(P_N,S)\to\ell(l')+h(P)+X}}{dx\,dQ^2\,d\phi_S\,dz\,d^2P_T} = f_{q,s/N,S} \otimes \frac{d\sigma^{\ell(l,\lambda)+q(k,s)\to\ell(l')+q(k',s')}}{dQ^2} \otimes D^{h_1}_{q,s'}$$

$$D^{h_1}_{q,s'}(z,\mathbf{p}_T) = D_1(z,p_T^2) + \frac{\mathbf{p}_T \times \mathbf{s}'_T}{m_h} H_1(z,p_T^2)$$

Measured by BABAR and BELLE In e^+e^- semi inclusive annihilation (SIA) to 2 back-to-back jets $e^+e^- \to h_1h_2 + X$

Twist-2 STMD qDFs

Nucleon Polarization	Quark polarization: U	Quark polarization: L	Quark polarization: T
U	$f_1^q(x, k_T^2)$		$\frac{\epsilon_T^{ij} k_T^{\ j}}{M} h_1^{\perp q}(x, k_T^2)$
L		$S_L g_{1L}^q(x, k_T^2)$	$S_L \frac{\mathbf{k}_T}{M} h_{1L}^{\perp q}(x, k_T^2)$
T	$\frac{\mathbf{k}_T \times \mathbf{S}_T}{M} f_{1T}^{\perp q}(x, k^2)$	$\frac{\mathbf{k}_T \cdot \mathbf{S}_T}{M} g_{1T}^{\perp q}(x, k_T^2)$	$\mathbf{S}_T h_{1T}^q(x, k_T^2) + \frac{\mathbf{k}_T}{M} \frac{(\mathbf{k}_T \cdot \mathbf{S}_T)}{M} h_{1T}^{\perp q}(x, k_T^2)$

All azimuthal dependences are in prefactors. TMDs do not depend on them

LO cross section in SIDIS CFR

$$\frac{d\sigma^{\ell(l,\lambda)+N(P_N,S)\to\ell(l')+h(P)+X}(x_F>0)}{dx\,dQ^2\,d\phi_S\,dz\,d^2P_T}=\frac{\alpha^2 x}{yQ^2}\left(1+(1-y)^2\right)\times$$

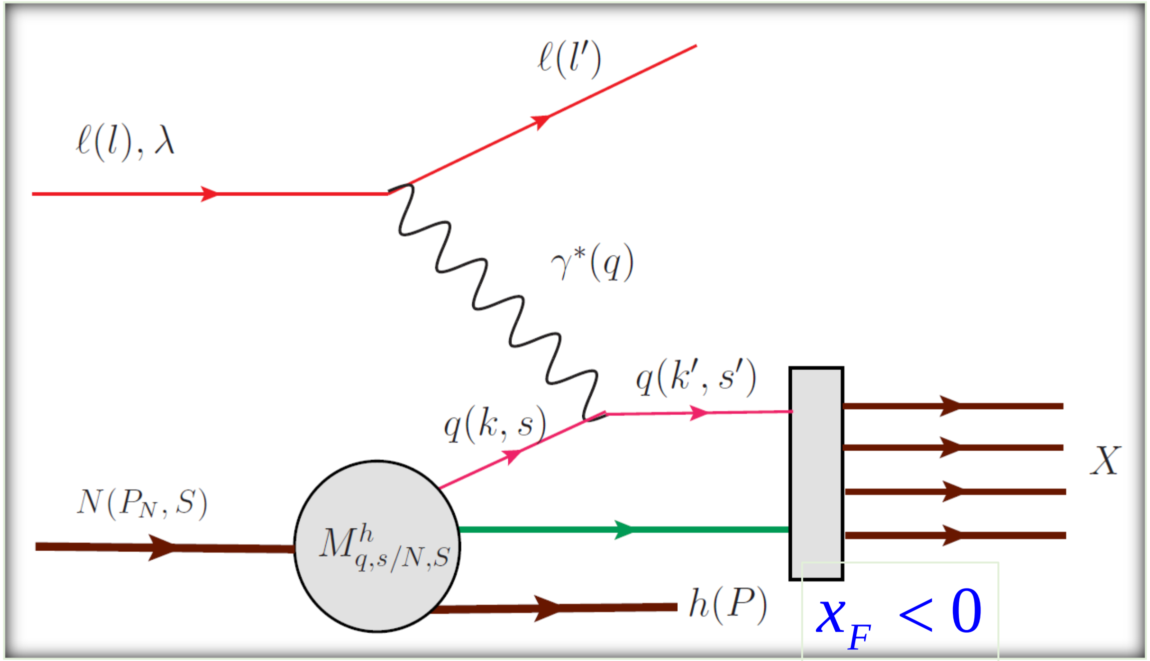
$$\times\left[\begin{array}{l} F_{UU,T}+D_{nn}(y)F_{UU}^{\cos 2\phi_h}\cos(2\phi_h)+ \\ S_L D_{nn}(y)F_{UL}^{\sin 2\phi_h}\sin(2\phi_h)+\lambda S_L D_{ll}(y)F_{LL}+ \\ S_T\left(F_{UT,T}^{\sin(\phi_h-\phi_S)}\boxed{\sin(\phi_h-\phi_S)}+D_{nn}(y)\left(\begin{array}{l}F_{UT}^{\sin(\phi_h+\phi_S)}\boxed{\sin(\phi_h+\phi_S)}+ \\ F_{UT}^{\sin(3\phi_h-\phi_S)}\sin(3\phi_h-\phi_S)\end{array}\right)\right)+ \\ \lambda S_T D_{ll}(y)F_{LT}^{\cos(\phi_h-\phi_S)}\cos(\phi_h-\phi_S)\end{array}\right]$$

$$D_{ll}(y)=\frac{y(2-y)}{1+(1-y)^2},\qquad D_{nn}(y)=\frac{2(1-y)}{1+(1-y)^2}$$

8 terms out of 18 Structure Functions, 6 azimuthal modulations

4 terms are generated by Collins effect in fragmentation

SIDIS: TFR



Trentadue, Veneziano 1994
Graudenz 1994
Collins 1998, 2000, 2002
de Florian, Sassot 1997, 1998
Grazzini, Trentadue, Veneziano 1998
Ceccopieri, Trentadue 2006, 2007, 2008
Sivers 2009
Ceccopieri , Mancusi 2013
Ceccopieri 2013
……..

$$\frac{d\sigma^{\ell(l)+N(P_N)\to\ell(l')+h(P)+X}}{dx\,dQ^2\,d\zeta} = M^h_{q/N}\left(x, Q^2, \zeta\right) \otimes \frac{d\sigma^{\ell(l)+q(k)\to\ell(l')+q(k')}}{dQ^2}$$

$$\zeta = \frac{P^-}{P_N^-} \approx x_F\,(1-x)$$

Fracture function *M* is a Conditional Probability Distribution Function (CPDF)
to observe the hadron h produced in nucleon flight direction
when hard probe interacts with parton carrying fraction x of nucleon momentum.

Collinear Frac.Func.: application to HERA data, 1

D. de Glorian, R. Sassot, Leading Proton Structure Function. PRD 58, 054003 (1998)

$$\frac{d^3\sigma^{p}_{target}}{d\beta dQ^2 dx_{\mathrm{P}}} = \frac{4\pi\alpha^2}{\beta Q^4}\left(1-y+\frac{y^2}{2}\right) M^{h}_{p}\left(\beta,Q^2,x_{\mathrm{P}}\right),\quad \beta=\frac{x}{1-\zeta},\quad \zeta=\frac{p^+_h}{p^+_N} \qquad x_{\mathrm{P}}=\zeta$$

$$xM_q^{p/p}(\beta,Q_0^2,x_{\mathrm{P}})=N_s\beta^{a_s}(1-\beta)^{b_s}\{C_{\mathrm{P}}\beta x_{\mathrm{P}}^{\alpha_{\mathrm{P}}}+C_{\mathrm{LP}}(1-\beta)^{\gamma_{\mathrm{LP}}}[1+a_{\mathrm{LP}}(1-x_{\mathrm{P}})^{\beta_{\mathrm{LP}}}]\}$$

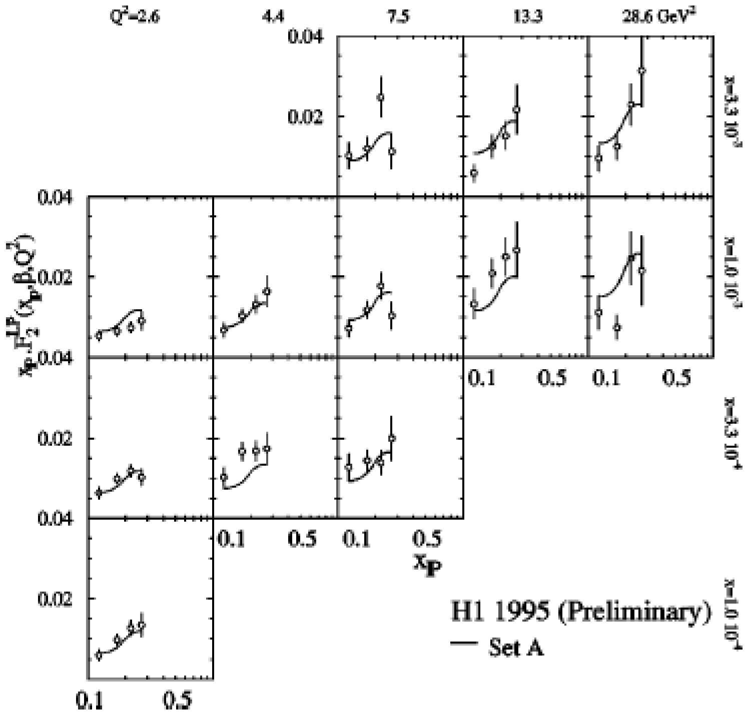


FIG. 2. H1 leading-proton data against the outcome of the fracture function parametrization (solid lines).

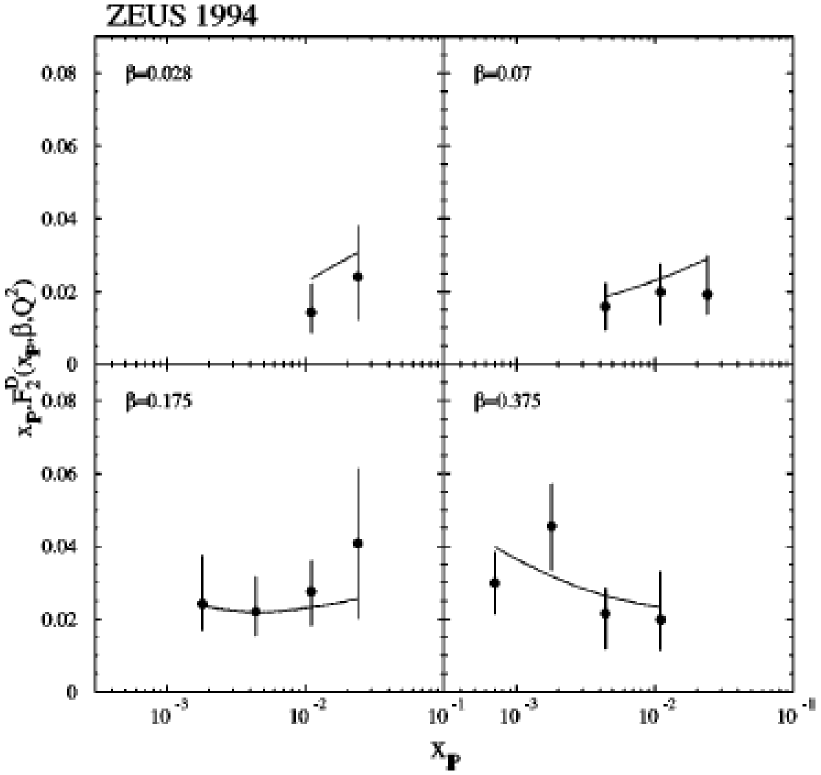
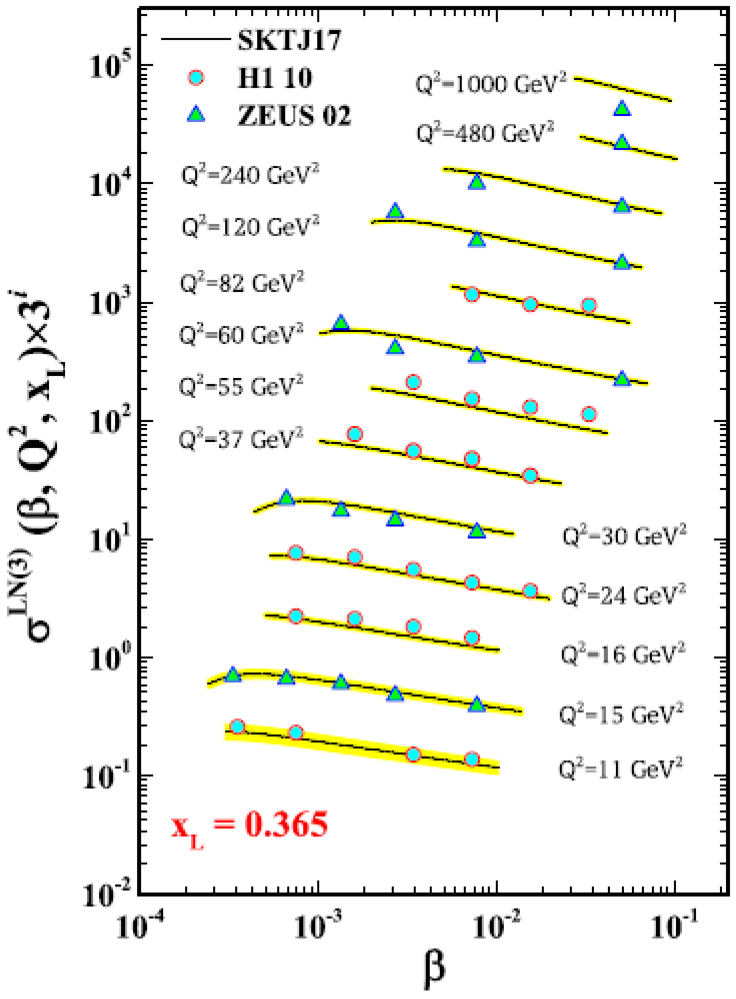
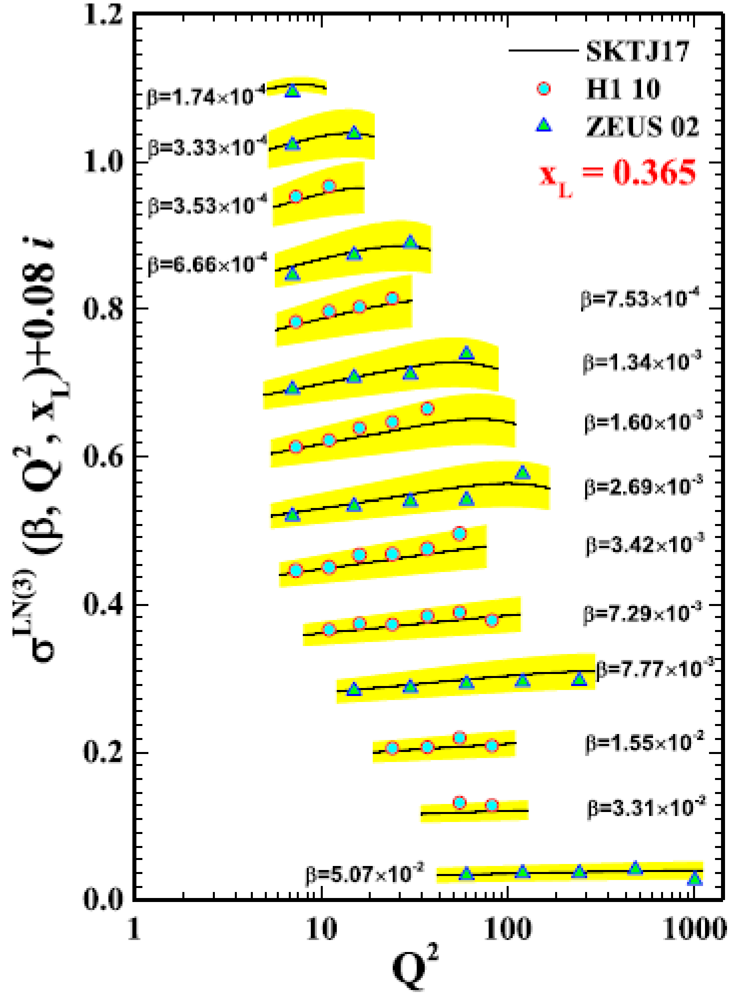


FIG. 8. ZEUS diffractive data, against the expectation coming from the fracture function parametrization (fit A).

Collinear Frac.Func.: application to HERA data, 2

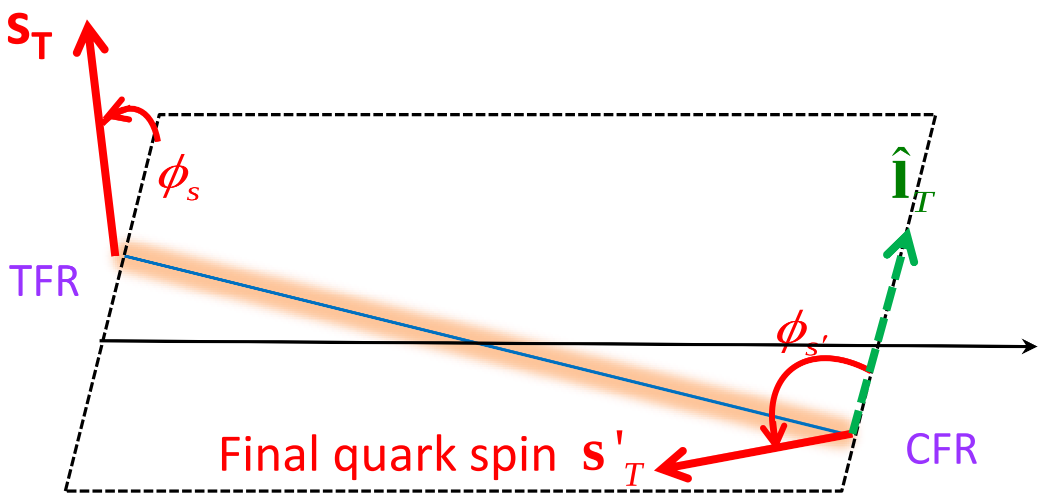
Shoeibi *et al*, Neutron fracture functions. PRD 95, 074011 (2017)





Quark transverse spin in hard *l-q* scattering

AK, Transversity workshop, Yerevan, 2009



$$\text{QED: } lq \to l'q' \;\Rightarrow\; s'_T = D_{nn}(y)s_T, \quad D_{nn}(y) = \frac{2(1-y)}{1+(1-y)^2}, \quad \phi_{s'} = \pi - \phi_s$$

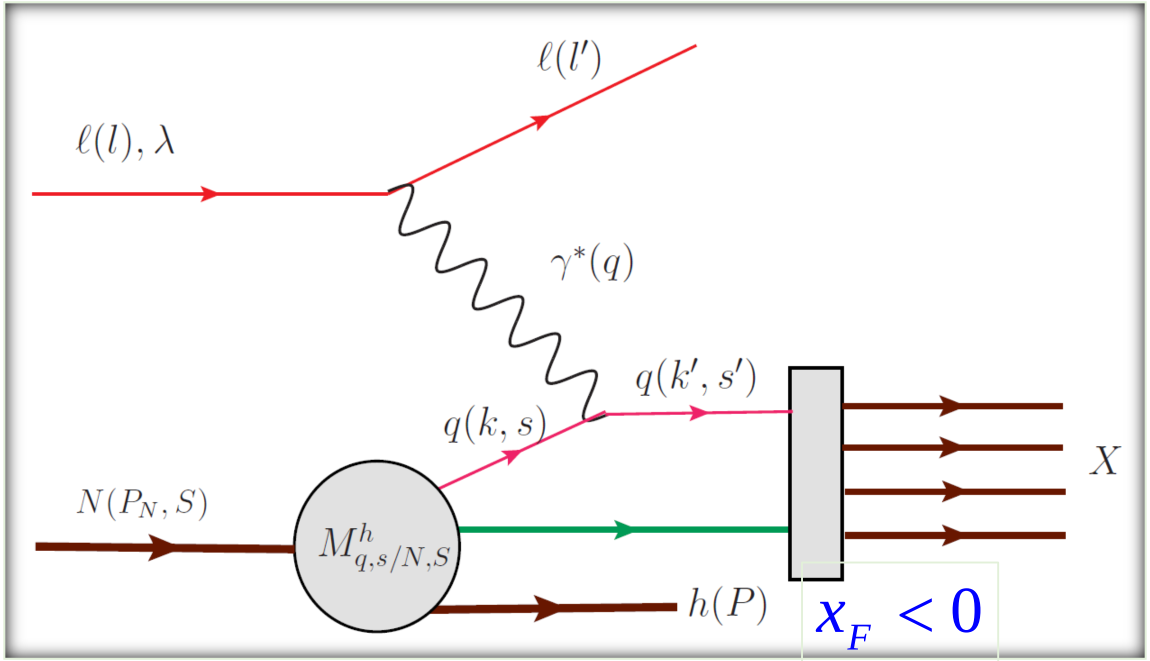
$$[\mathbf{s}_T \times \mathbf{p}_T] \propto \sin(\phi_h - \phi_{s'}) = -\sin(\phi_h + \phi_s)$$

If only one hadron in TFR of SIDIS is detected there is no final quark polarimetry.

→ No access to quark transverse polarization dependent fracture functions.

No Collins like modulation.

SIDIS TFR. Spin & TMD Fracture Functions



Anselmino, Barone and AK, **PL B 699** (2011)108**; 706** (2011)46**; 713** (2012)317

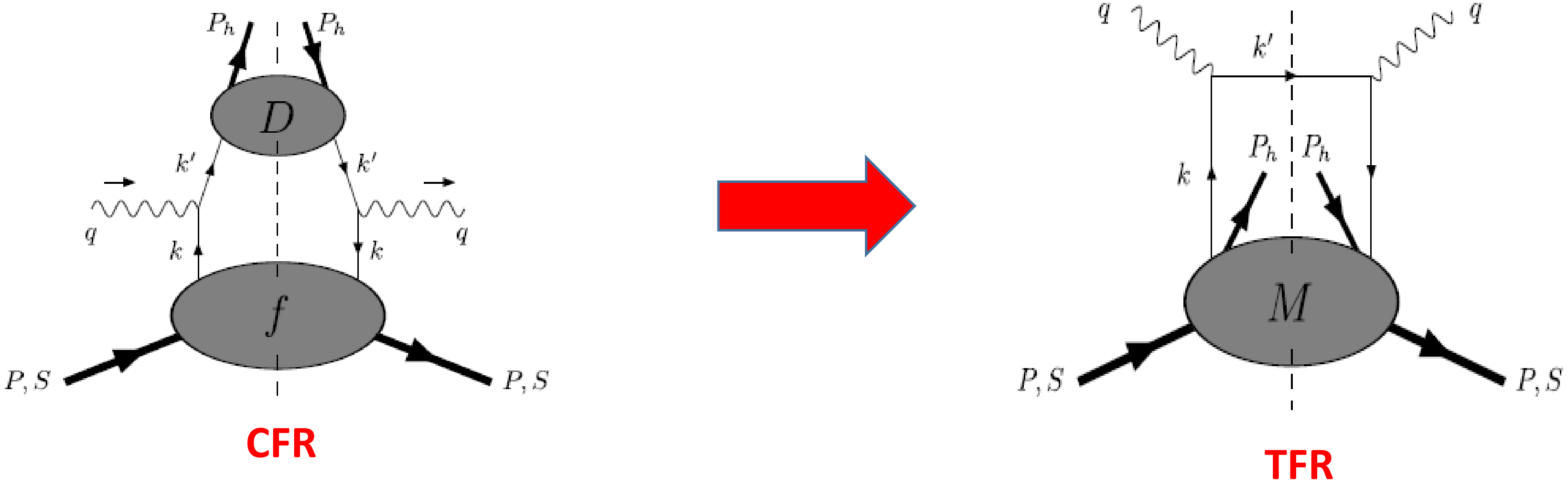
Nucleon and quark polarization are included, produced hadron and quark transverse momentum are not integrated over. Classification of twist-two Fracture Functions and cross sections expressions.

$$\frac{d\sigma^{\ell(l,\lambda)+N(P_N,S)\to\ell(l')+h(P)+X}}{dx\,dQ^2\,d\phi_S\,d\zeta\,d^2P_T} = M^h_{q,s/N,S} \otimes \frac{d\sigma^{\ell(l,\lambda)+q(k,s)\to\ell(l')+q(k',s')}}{dQ^2}$$

$$\zeta = \frac{P^-}{P_N^-} \approx x_F(1-x)$$

Quark correlator

SIDIS



$$\mathcal{M}^{[\Gamma]}(x_B,\vec{k}_\perp,\zeta,\vec{P}_{h\perp}) = \frac{1}{4\zeta}\int\frac{d\xi^+ d^2\xi_\perp}{(2\pi)^6} e^{\mathrm{i}(x_B P^-\xi^+ - \vec{k}_\perp\cdot\vec{\xi}_\perp)} \sum_X \int \frac{d^3 P_X}{(2\pi)^3 2E_X} \times$$

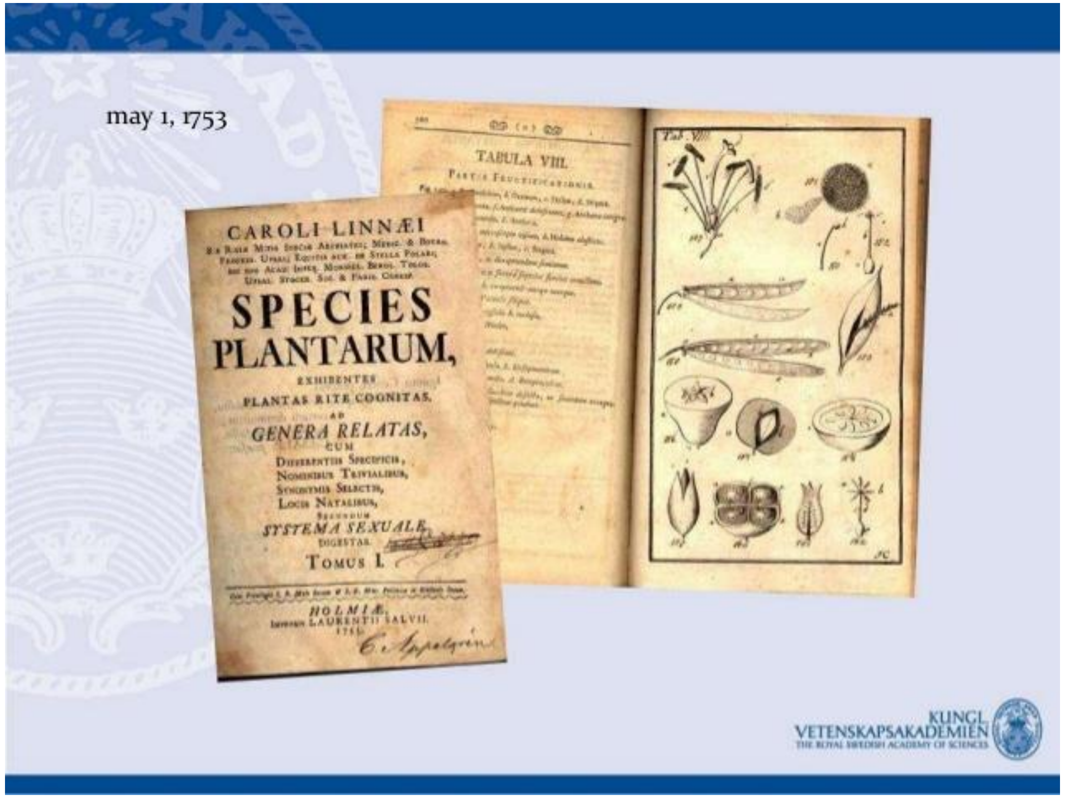
$$\times\langle P,S\,|\,\bar{\psi}(0)\Gamma\,|\,P_h,S_h;X\rangle\langle P_h,S_h;X\,|\,\psi(\xi^+,0,\vec{\xi}_\perp)\,|\,P,S\rangle$$

$$\Gamma = \gamma^-,\quad \gamma^-\gamma_5,\quad i\sigma^{i-}\gamma_5$$

At LO 16 independent STMD fracture functions. Probabilistic interpretation at LO:
the conditional probabilities to find an unpolarized ($\Gamma = \gamma -$), a longitudinally polarized ($\Gamma = \gamma -\gamma 5$) or a transversely polarized ($\Gamma = i\sigma\ i-\gamma 5$) quark with longitudinal momentum fraction x_B and transverse momentum $\boldsymbol{k}_\perp$ inside a nucleon fragmenting into a hadron carrying a fraction ζ of the nucleon longitudinal momentum and a transverse momentum $\boldsymbol{P}_{h\perp}$.

Karl Linney: plants classification

Plants were divided by it into 24 classes and 116 groups on the basis of features of a structure of their reproductive organs.



[9]

Plate IV. *Simple Leaves.*

Fig. 1. Orbicular.
2. Roundiſh.
3. Ovate.
4. Oval.
5. Oblong.
6. Lanceolate.
7. Linear.
8. Subulate or awl-ſhaped.
9. Reniform or kidney ſhaped.
10. Cordate or heart-ſhaped.
11. Lunate or creſcent-ſhaped.
12. Triangular.
13. Sagittate or arrow-ſhaped.
14. Heart-arrow ſhaped.
15. Haſtate or halbert-ſhaped.
16. Obcordate or inverſely heart-ſhaped.
17. 3-lobed.
18. Premorſe or as if bitten.
19. Lobed.
20. 5-angled.
21. Eroded or gnawed.
22. Palmate.
23. Pinnatifid or wing-cleft.
24. Laciniate or jagged.
25. Sinuate or indented.
26. Tooth-ſinuate.
27. Runcinate or barbed.
28. Parted or divided.
29. Repand or ſerpentine.
30. Toothed.
31. Serrate.
32. Doubly ſerrate.
33. D ubly crenate or ſcalloped.
34. Cartilaginous.

Fig. 35. Acutely crenate or ſcalloped.
36. Obtuſely crenate.
37. Plaited.
38. Panduræform or fiddle-ſhaped.
39. Spatulate or ſhaped like a battledore.
40. Obtuſe.
41. Acute.
42 Acuminate or pointed.
43. Obtuſe with a point.
44. Acutely emarginate or notched.
45. Cuneiform or wedge-ſhaped.
46. Retuſe.
47. Hairy.
48. Downy.
49. Hiſpid or covered with ſtiffiſh briſtles.
50. Ciliate or fringed.
51. Rhombic.
52. Veined.
53. Nerved.
54. Papillous or pimpled.
55. Parabolic.
56. Acinaciform or ſcymetar-ſhaped.
57. Dolabriform or hatched-ſhaped.
58. Deltoid.
59. Triangular.
60. Channeled.
61. Furrowed or grooved.
62. Cylindrical or without angles.

VOL. V.—B Plate

For STMD Fracture Functions I was expecting
32 (Trentadue) independent structures.
Fortunately we end up with only 16 of them at twist-two

STMD Fracture Functions for spinless hadron production

Nucleon Polarization	Quark polarization: U	L	T
U	$\hat{u}_1$	$\frac{\mathbf{k}_T \times \mathbf{P}_T}{m_N m_h} \hat{l}_1^{\perp h}$	$\frac{\epsilon_T^{ij} P_T^j}{m_h} \hat{t}_1^h + \frac{\epsilon_T^{ij} k_T^j}{m_N} \hat{t}_1^\perp$
L	$\frac{S_L(\mathbf{k}_T \times \mathbf{P}_T)}{m_N m_h} \hat{u}_{1L}^{\perp h}$	$S_L \hat{l}_{1L}$	$\frac{S_L \mathbf{P}_T}{m_h} \hat{t}_{1L}^h + \frac{S_L \mathbf{k}_T}{m_N} \hat{t}_{1L}^\perp$
T	$\frac{\mathbf{P}_T \times \mathbf{S}_T}{m_h} \hat{u}_{1T}^h + \frac{\mathbf{k}_T \times \mathbf{S}_T}{m_N} \hat{u}_{1T}^\perp$	$\frac{\mathbf{P}_T \cdot \mathbf{S}_T}{m_h} \hat{l}_{1T}^h + \frac{\mathbf{k}_T \cdot \mathbf{S}_T}{m_N} \hat{l}_{1T}^\perp$	$\mathbf{S}_T \hat{t}_{1T} + \frac{\mathbf{P}_T(\mathbf{P}_T \cdot \mathbf{S}_T)}{m_h^2} \hat{t}_{1T}^{hh} + \frac{\mathbf{k}_T(\mathbf{k}_T \cdot \mathbf{S}_T)}{m_N^2} \hat{t}_{1T}^{\perp\perp} + \frac{\mathbf{P}_T(\mathbf{k}_T \cdot \mathbf{S}_T) - \mathbf{k}_T \cdot (\mathbf{P}_T \cdot \mathbf{S}_T)}{m_N m_h} \hat{t}_{1T}^{\perp h}$

STMD fracture functions

depend on

$x, k_T^2, \zeta, P_T^2, \mathbf{k}_T \cdot \mathbf{P}_T$

$\mathbf{k}_T \cdot \mathbf{P}_T = k_T P_T \cos(\phi_h - \phi_q)$

azimuthal dependence

in fracture functions

TMD Sum Rules

$$\sum_h \int \zeta d\zeta \int d^2 P_T \, \hat{u}_1 = (1-x) \, f_1(x, k_T^2)$$

$$\sum_h \int \zeta d\zeta \int d^2 P_T \left(\hat{u}_{1T}^{\perp} + \frac{m_N}{m_h} \frac{\mathbf{k}_T \cdot \mathbf{P}}{k_T^2} \hat{u}_{1T}^{h} \right) = -(1-x) \, f_{1T}^{\perp}(x, k_T^2)$$

$$\sum_h \int \zeta d\zeta \int d^2 P_T \, \hat{l}_{1L} = (1-x) g_{1L}(x, k_T^2)$$

$$\sum_h \int \zeta d\zeta \int d^2 P_T \left(\hat{l}_{1T}^{\perp} + \frac{m_N}{m_h} \frac{\mathbf{k}_T \cdot \mathbf{P}}{k_T^2} \hat{l}_{1T}^{h} \right) = (1-x) g_{1T}(x, k_T^2)$$

$$\sum_h \int \zeta d\zeta \int d^2 P_T \left(\hat{t}_{1L}^{\perp} + \frac{m_N}{m_h} \frac{\mathbf{k}_T \cdot \mathbf{P}}{k_T^2} \hat{t}_{1L}^{h} \right) = (1-x) h_{1L}^{\perp}(x, k_T^2)$$

$$\sum_h \int \zeta d\zeta \int d^2 P_T \left(\hat{t}_{1}^{\perp} + \frac{m_N}{m_h} \frac{\mathbf{k}_T \cdot \mathbf{P}}{k_T^2} \hat{t}_{1}^{h} \right) = -(1-x) h_{1}^{\perp}(x, k_T^2)$$

$$\sum_h \int \zeta d\zeta \int d^2 P_T \left(\hat{t}_{1T}^{\perp\perp} + \frac{m_N^2}{m_h^2} \frac{2\left(\mathbf{k}_T \cdot \mathbf{P}\right)^2 - k_T^2 P_T^2}{k_T^4} \hat{t}_{1T}^{hh} \right) = (1-x) h_{1T}^{\perp}(x, k_T^2)$$

$$\sum_h \int \zeta d\zeta \int d^2 P_T \left(\hat{t}_{1T} + \frac{k_T^2}{2m_N^2} \hat{t}_{1T}^{\perp\perp} + \frac{P_T^2}{2m_h^2} \hat{t}_{1T}^{hh} \right) = (1-x) h_1(x, k_T^2)$$

Nonzero fracture functions u,l,t. Useful for modeling.

LO cross-section in TFR

$$\frac{d\sigma^{\ell(l,\lambda)+N(P_N,S)\to\ell(l')+h(P)+X}(x_F<0)}{dx\,dQ^2\,d\phi_S\,d\zeta\,d^2P_T}=\frac{\alpha^2 x}{yQ^4}\left(1+(1-y)^2\right)\sum_q e_q^2\times$$

$$\times\left[\begin{array}{l}\tilde{u}_1\left(x,\zeta,P_T^2\right)-S_T\frac{P_T}{m_h}\tilde{u}_{1T}^h\left(x,\zeta,P_T^2\right)\sin(\phi_h-\phi_S)+\\ \lambda y(2-y)\left(S_L\tilde{l}_{1L}\left(x,\zeta,P_T^2\right)+S_T\frac{P_T}{m_h}\tilde{l}_{1T}^h\left(x,\zeta,P_T^2\right)\cos(\phi_h-\phi_S)\right)\end{array}\right]$$

$$\tilde{u}_1(x_B,\zeta_2,P_{T2}^2)=\int d^2k_T\,\hat{u}_1$$

$$\tilde{u}_{1T}^h(x_B,\zeta_2,P_{T2}^2)=\int d^2k_T\left\{\hat{u}_{1T}^h+\frac{m_2}{m_N}\frac{\mathbf{k}_T\cdot\mathbf{P}_{T2}}{P_{T2}^2}\hat{u}_{1T}^\perp\right\}$$

$$\tilde{l}_{1L}(x_B,\zeta_2,P_{T2}^2)=\int d^2k_T\,\hat{l}_{1L}$$

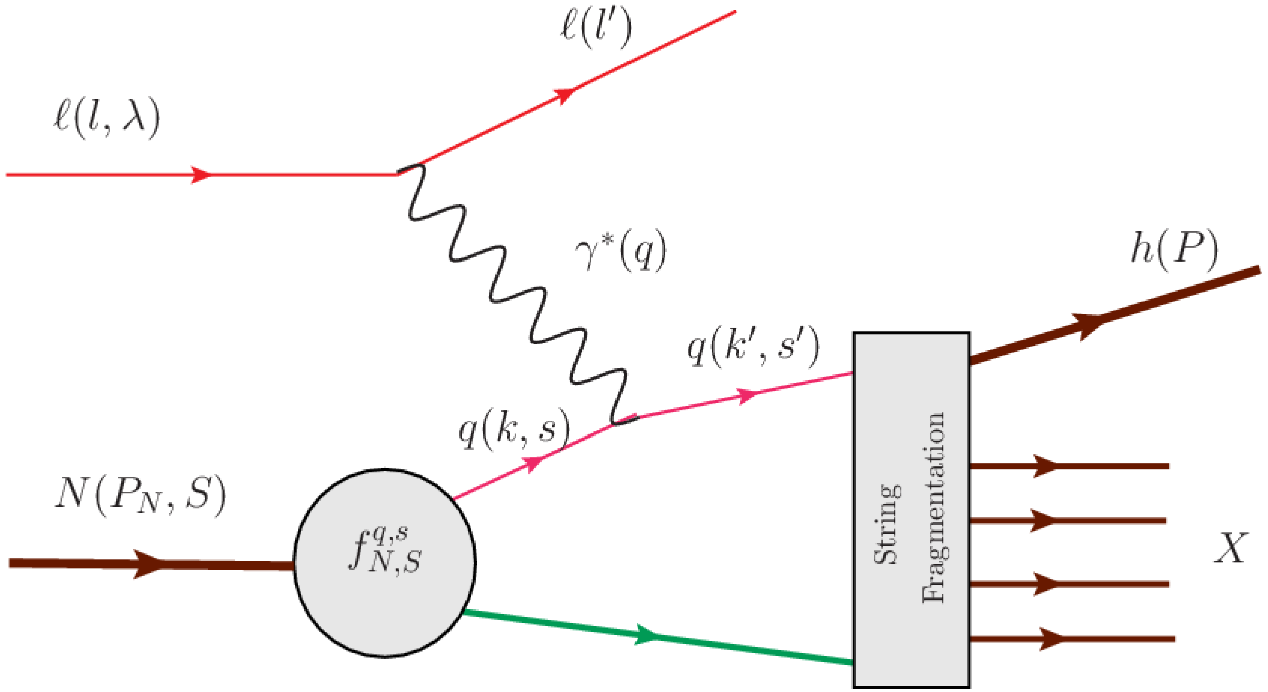
$$\tilde{l}_{1T}^h(x_B,\zeta_2,P_{T2}^2)=\int d^2k_T\left\{\hat{l}_{1T}^h+\frac{m_2}{m_N}\frac{\mathbf{k}_T\cdot\mathbf{P}_{T2}}{P_{T2}^2}\hat{l}_{1T}^\perp\right\}$$

At LO only 4 terms out of 18 Structure Functions,
Only 2 azimuthal modulations

No Collins-like $\sin(\phi_h+\phi_S)$ modulation

No access to quark transverse polarization

MC event generators (LEPTO, PYTHIA): Hadronization Function

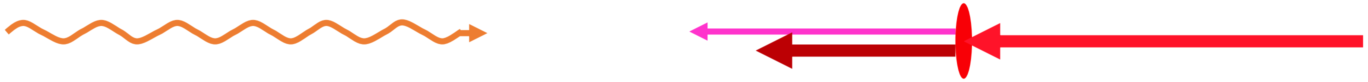


$$d\sigma^{lN\to lhX} = \sum_q f_q\left(x,\mathbf{k}_T^2\right)\otimes d\sigma^{lq\to lq}\otimes H^{q}_{h/N}\left(x,\mathbf{k}_T;x_F,\mathbf{p}_T^h\right)$$

Hadronization Function modeled by Lund String Fragmentation

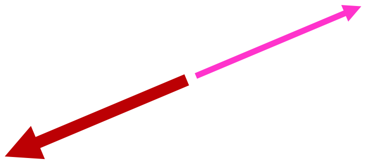
Hadronization in MC even generators

- Before



- After hard scattering

- Include k_T with isotropic azimuth



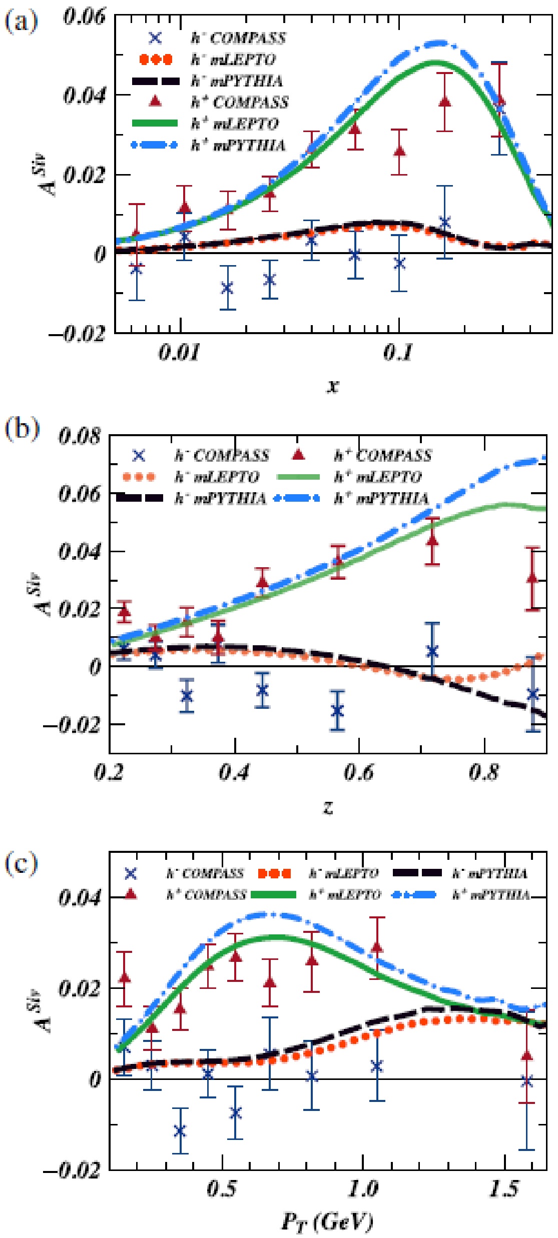
Modified, mLEPTO and mPYTHIA

Include k_T with anisotropic azimuthal modulation according Sivers function

$$d\sigma^{lN\to lhX} = \sum_q \left(f_q(x, k_T^2) + \frac{\mathbf{k_T} \times \mathbf{S_T}}{M} f_{qT}^{\perp}(x, k_T^2) \right) \otimes d\sigma^{lq\to lq} \otimes H_{h/N}^{q}(x, \mathbf{k_T}; x_F, \mathbf{p}_\mathbf{T}^{h})$$

Sivers effect in the event generators

Matevosyan, AK, Aschenauer, Avakian, Thomas, PRD 92, 054028 (2015)



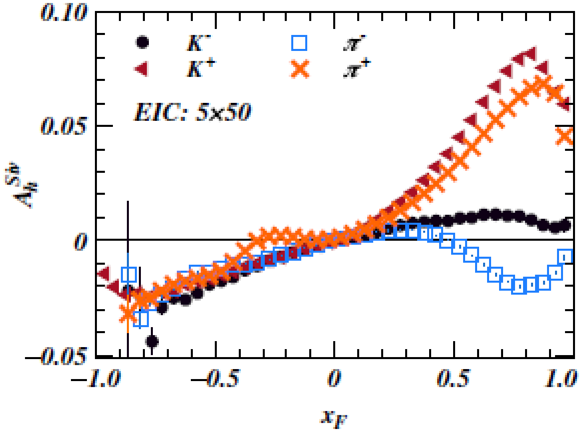


FIG. 13 (color online). EIC model SSAs for 5 × 50 SIDIS kinematics for charged pions and kaons versus x_F. The Sivers asymmetry is present both in the current and target fragmentation regions.

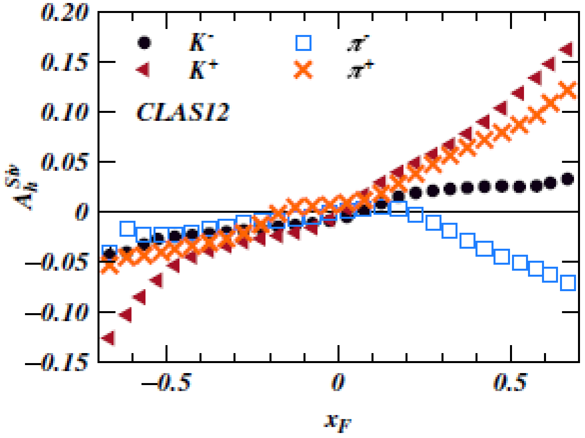
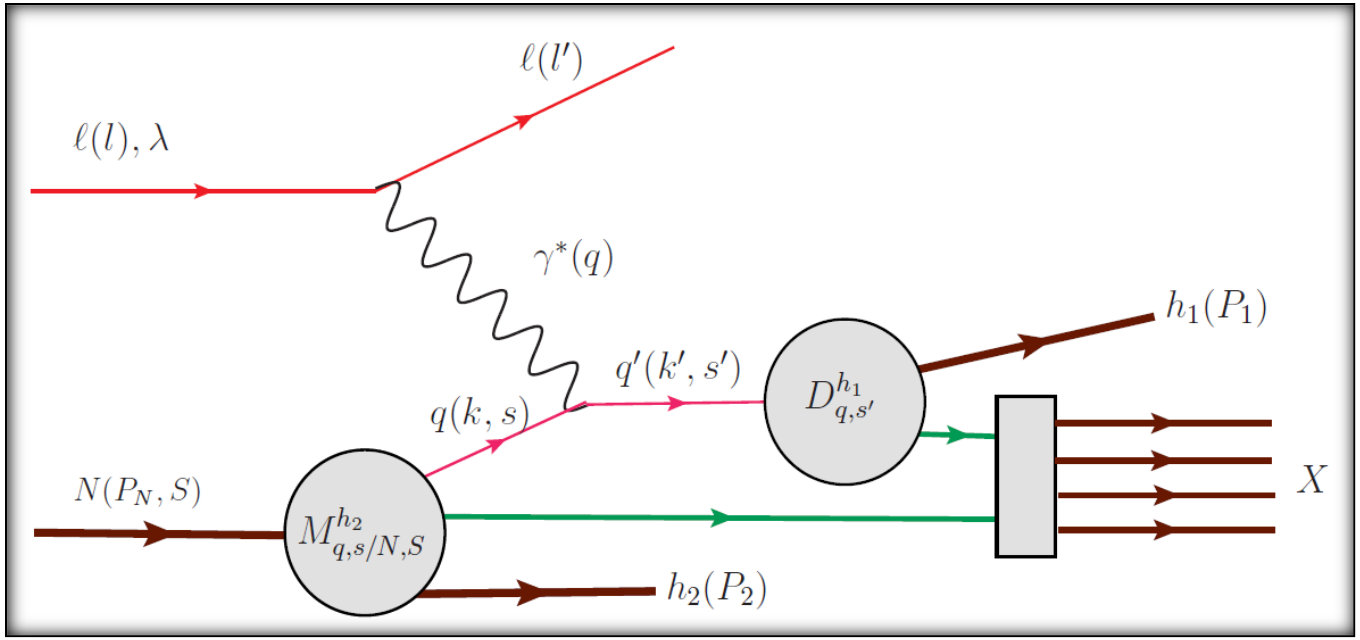


FIG. 17 (color online). Predictions for SSAs for charged pions and kaons versus x_F at CLAS12. The Sivers asymmetry is present both in the current and target fragmentation regions.

Only correlation of target $\mathbf{S_T}$ and struck quark $\mathbf{k_T}$ is explicitly parametrized using Sivers PDFs. Then this correlation is transferred to produced hadrons via unpolarized string fragmentation .

$$\tilde{u}^h_{1T}(x_B,\zeta_2,P_{T2}^2)=\int d^2k_T\left\{\hat{u}^h_{1T}+\frac{m_2}{m_N}\frac{\mathbf{k}_T\cdot\mathbf{P}_{T2}}{P_{T2}^2}\hat{u}^{\perp}_{1T}\right\}$$

Double hadron production in DIS (DSIDIS): TFR & CFR



$$x_{F2} < 0, \quad x_{F1} > 0$$

$$\frac{d\sigma^{\ell(l,\lambda)+N(P_N,S)\to\ell(l')+h_1(P_1)+h_2(P_2)+X}}{dx\,dQ^2\,d\phi_S\,dz\,d^2P_{T1}\,d\zeta\,d^2P_{T2}} = M^{h_2}_{q,s/N,S} \otimes \frac{d\sigma^{\ell(l,\lambda)+q(k,s)\to\ell(l')+q(k',s')}}{dQ^2} \otimes D^{h_1}_{q,s'}$$

$$D^{h_1}_{q,s'}(z,\mathbf{p}_T) = D_1(z,p_T^2) + \frac{\mathbf{p}_T \times \mathbf{s}'_T}{m_h} H_1(z,p_T^2)$$

Unintegrated DSIDIS cross-section: accessing quark polarization

$$\frac{d\sigma^{\ell(l,\lambda)+N(P_N,S)\to\ell(l')+h_1(P_1)+h_2(P_2)+X}}{dx\,dQ^2\,d\phi_S\,dz\,d^2P_{T1}\,d\zeta\,d^2P_{T2}} =$$

$$= \frac{\alpha^2 x}{Q^4 y}\left(1+(1-y)^2\right)\left(\hat{u}^{h_2}\otimes D_1^{h_1} + \lambda D_{ll}(y)\hat{l}^{h_2}\otimes D_1^{h_1} + \hat{t}^{h_2}\otimes\frac{\mathbf{p}_T\times\mathbf{s}'_T}{m_{h_1}}H_1^{h_1}\right)$$

$$= \frac{\alpha^2 x}{Q^4 y}\left(1+(1-y)^2\right)\left(\sigma_{UU} + S_L\sigma_{UL} + S_T\sigma_{UT} + \lambda D_{ll}\left(\sigma_{LU} + S_L\sigma_{LL} + S_T\sigma_{LT}\right)\right)$$

DSIDIS cross section is a sum of polarization independent, single and double spin dependent terms, similarly to 1h SIDIS cross section.

DSIDIS azimuthal modulations

AK @ DIS2011

$$\sigma_{UU} = F_0^{\hat{u}\cdot D_1} - D_{nn}\left(\frac{P_{T1}^2}{m_1 m_N} F_{kp1}^{\hat{t}_1^{\perp}\cdot H_1} \cos(2\phi_1) + \frac{P_{T1}P_{T2}}{m_1 m_2} F_{p1}^{\hat{t}_1^{h}\cdot H_1} \cos(\phi_1+\phi_2) + \left(\frac{P_{T2}^2}{m_1 m_N} F_{kp2}^{\hat{t}_1^{\perp}\cdot H_1} + \frac{P_{T2}^2}{m_1 m_2} F_{p2}^{\hat{t}_1^{h}\cdot H_1} \right) \cos(2\phi_2) \right)$$

$$D_{nn}(y) = \frac{2(1-y)}{1+(1-y)^2}$$

$$F_{k1}^{\hat{M}\cdot D} = C\left[\hat{M}\cdot D \frac{(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2})(\mathbf{P}_{T2}\cdot\mathbf{k}) - (\mathbf{P}_{T1}\cdot\mathbf{k})\mathbf{P}_{T2}^2}{(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2})^2 - \mathbf{P}_{T1}^2\mathbf{P}_{T2}^2} \right]$$

$$C[\hat{M}\cdot Dw] = \sum_a e_a^2 \int d^2k_T\, d^2p_T\, \delta^{(2)}(z\mathbf{k}_T + \mathbf{p}_T - \mathbf{P}_{T1}) \hat{M}_a(x,\zeta,k_T^2,P_{T2}^2,\mathbf{k}_T\cdot\mathbf{P}_{T2}) D_a(z,p_T^2) w$$

Structure functions $F_{...}^{\hat{u}\cdot D}$ depend on $x, z, \zeta, P_{T1}^2, P_{T2}^2$ and $(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2})$

$$\mathbf{P}_{T1}\cdot\mathbf{P}_{T2} = P_{T1}P_{T2}\cos(\Delta\phi), \text{ with } \Delta\phi = \phi_1 - \phi_2$$

σ_{UL}

$$\sigma_{UL} = -\frac{P_{T1}P_{T2}}{m_2 m_N} F_{k1}^{\hat{u}_{1L}^{\perp h}\cdot D_1} \sin(\phi_1 - \phi_2)$$
$$+D_{nn}\left(\frac{P_{T1}^2}{m_1 m_N} F_{kp1}^{\hat{t}_{1L}^{\perp}\cdot H_1} \sin(2\phi_1) + \frac{P_{T1}P_{T2}}{m_1 m_2} F_{p1}^{\hat{t}_{1L}^{h}\cdot H_1} \sin(\phi_1 + \phi_2) + \left(\frac{P_{T2}^2}{m_1 m_N} F_{kp2}^{\hat{t}_{1L}^{\perp}\cdot H_1} + \frac{P_{T2}^2}{m_1 m_2} F_{p2}^{\hat{t}_{1L}^{h}\cdot H_1} \right) \sin(2\phi_2) \right)$$

σ_{UT}

$$
\begin{aligned}
\sigma_{UT} = & -\frac{P_{T1}}{m_N} F_{k1}^{\hat{u}_{1T}^{\perp}\cdot D_1} \sin(\phi_1-\phi_S) \\
& -\left(\frac{P_{T2}}{m_2} F_0^{\hat{u}_{1T}^{h}\cdot D_1} + \frac{P_{T2}}{m_N} F_{k2}^{\hat{u}_{1T}^{\perp}\cdot D_1}\right)\sin(\phi_2-\phi_S) \\
& + D_{nn}(y)\Bigg[\left(\frac{P_{T1}}{m_1} F_{p1}^{\hat{t}_{1T}\cdot H_1} + \frac{P_{T1}P_{T2}^2}{2m_1m_2^2} F_{p1}^{\hat{t}_{1T}^{hh}\cdot H_1} - \frac{P_{T1}P_{T2}^2}{2m_1m_2m_N} F_{kp3}^{\hat{t}_{1T}^{\perp h}\cdot H_1} \right. \\
& \qquad \left. + \frac{P_{T1}^3}{2m_1m_N^2} F_{kkp1}^{\hat{t}_{1T}^{\perp\perp}\cdot H_1} + \frac{P_{T1}P_{T2}^2}{2m_1m_N^2} F_{kkp4}^{\hat{t}_{1T}^{\perp\perp}\cdot H_1} + \frac{P_{T1}}{m_1m_N^2} F_{kkp5}^{\hat{t}_{1T}^{\perp\perp}\cdot H_1}\right)\sin(\phi_1+\phi_S) \\
& \quad + \left(\frac{P_{T2}}{m_1} F_{p2}^{\hat{t}_{1T}\cdot H_1} + \frac{P_{T2}^3}{2m_1m_2^2} F_{p2}^{\hat{t}_{1T}^{hh}\cdot H_1} + \frac{P_{T1}^2P_{T2}}{2m_1m_2m_N} F_{kp1}^{\hat{t}_{1T}^{\perp h}\cdot H_1} + \frac{P_{T2}}{m_1m_2m_N} F_{kp4}^{\hat{t}_{1T}^{\perp h}\cdot H_1} \right. \\
& \qquad \left. + \frac{P_{T1}^2P_{T2}}{2m_1m_N^2} F_{kkp2}^{\hat{t}_{1T}^{\perp\perp}\cdot H_1} + \frac{P_{T2}^3}{2m_1m_N^2} F_{kkp3}^{\hat{t}_{1T}^{\perp\perp}\cdot H_1} + \frac{P_{T2}}{m_1m_N^2} F_{kkp6}^{\hat{t}_{1T}^{\perp\perp}\cdot H_1}\right)\sin(\phi_2+\phi_S) \\
& \quad + \frac{P_{T1}^3}{2m_1m_N^2} F_{kkp1}^{\hat{t}_{1T}^{\perp\perp}\cdot H_1}\sin(3\phi_1-\phi_S) \\
& \quad + \left(\frac{P_{T2}^3}{2m_1m_2^2} F_{p2}^{\hat{t}_{1T}^{hh}\cdot H_1} + \frac{P_{T2}^3}{2m_1m_N^2} F_{kkp3}^{\hat{t}_{1T}^{\perp\perp}\cdot H_1}\right)\sin(3\phi_2-\phi_S) \\
& \quad + \left(\frac{P_{T1}P_{T2}^2}{2m_1m_2^2} F_{p1}^{\hat{t}_{1T}^{hh}\cdot H_1} + \frac{P_{T1}P_{T2}^2}{2m_1m_N^2} F_{kkp4}^{\hat{t}_{1T}^{\perp\perp}\cdot H_1}\right)\sin(\phi_1+2\phi_2-\phi_S) \\
& \quad - \frac{P_{T1}^2P_{T2}}{2m_1m_2m_N} F_{kp1}^{\hat{t}_{1T}^{\perp h}\cdot H_1}\sin(2\phi_1-\phi_2+\phi_S) \\
& \quad - \frac{P_{T1}P_{T2}^2}{2m_1m_2m_N} F_{kp3}^{\hat{t}_{1T}^{\perp h}\cdot H_1}\sin(\phi_1-2\phi_2-\phi_S) \\
& \quad + \frac{P_{T1}^2P_{T2}}{2m_1m_N^2} F_{kkp2}^{\hat{t}_{1T}^{\perp\perp}\cdot H_1}\sin(2\phi_1+\phi_2-\phi_S)\Bigg]
\end{aligned}
$$

σ_{LU}, σ_{LL}, σ_{LT}

$$\sigma_{LU} = -\frac{P_{T1}P_{T2}}{m_2 m_N} F_{k1}^{\hat{l}_1^{\perp h}\cdot D_1} \sin(\phi_1 - \phi_2)$$

$$\sigma_{LL} = F_0^{\hat{l}_1\cdot D_1}$$

$$\sigma_{LT} = \frac{P_{T1}}{m_N} F_{k1}^{\hat{l}_{1T}^{\perp}\cdot D_1} \cos(\phi_1 - \phi_S) + \left(\frac{P_{T2}}{m_2} F_0^{\hat{l}_{1T}^{h}\cdot D_1} + \frac{P_{T2}}{m_N} F_{k2}^{\hat{l}_{1T}^{\perp}\cdot D_1} \right) \cos(\phi_2 - \phi_S)$$

A_{LU} asymmetry

Anselmino, Barone and AK, PLB **713** (2012) 317

$F_{\dots}^{\hat{u}\cdot D}$ depend on $x, z, \zeta, P_{T1}^2, P_{T2}^2$ and $(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2})$

$\mathbf{P}_{T1}\cdot\mathbf{P}_{T2} = P_{T1}P_{T2}\cos(\Delta\phi)$, with $\Delta\phi = \phi_1 - \phi_2$

One can choose as independent angles $\Delta\phi$ and ϕ_2 $(\phi_1 = \Delta\phi + \phi_2)$

Integrating σ_{UU} and σ_{lU} over ϕ_2 we obtain

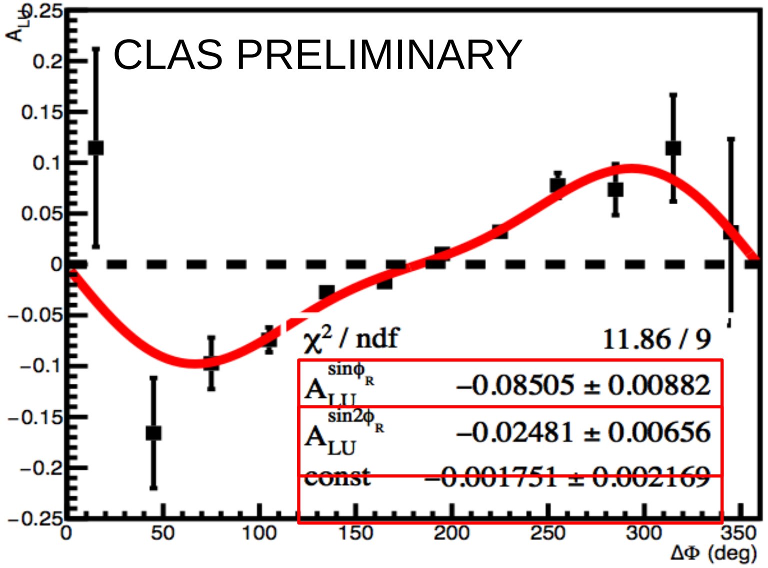
$$A_{LU} = \frac{\int d\phi_2 \sigma_{LU}}{\int d\phi_2 \sigma_{UU}} = \frac{-\frac{P_{T1}P_{T2}}{m_2 m_N} F_{k1}^{\hat{l}_1^{\perp h}\cdot D_1}\left(x, z, \zeta, P_{T1}^2, P_{T2}^2, \cos(\Delta\phi)\right)\sin(\Delta\phi)}{F_0^{\hat{u}\cdot D_1}\left(x, z, \zeta, P_{T1}^2, P_{T2}^2, \cos(\Delta\phi)\right)}$$

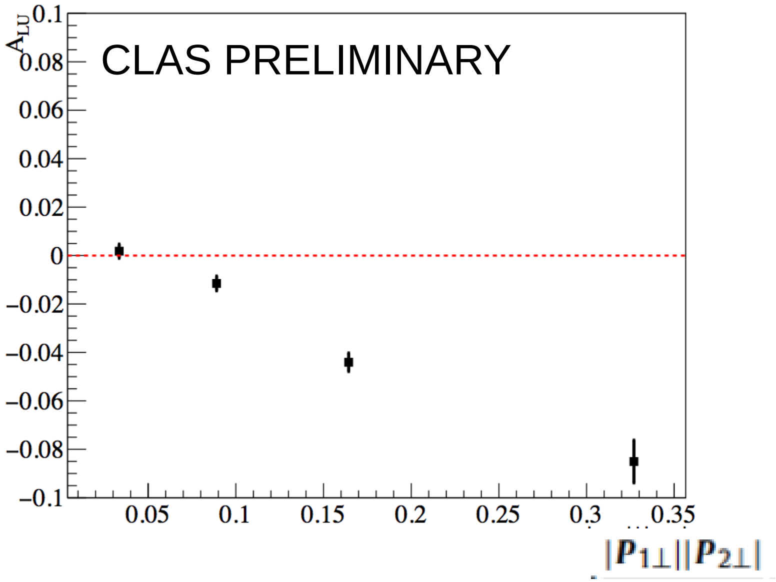
A_{LU} @ CLAS

$$A_{LU} = \frac{\sigma_{LU}(x,z,\zeta,P_{T1}^2,P_{T2}^2)\left(1+a_{LU1}\cos(\Delta\phi)+a_{LU2}\cos(2\Delta\phi)+\cdots\right)\sin(\Delta\phi)}{\sigma_{UU}(x,z,\zeta,P_{T1}^2,P_{T2}^2)\left(1+a_{UU1}\cos(\Delta\phi)+a_{UU2}\cos(2\Delta\phi)+\cdots\right)} \approx$$

$$\approx p_1\sin(\Delta\phi)+p_2\sin(2\Delta\phi)+\cdots$$

Courtesy of S.Pisano & H.Avakian





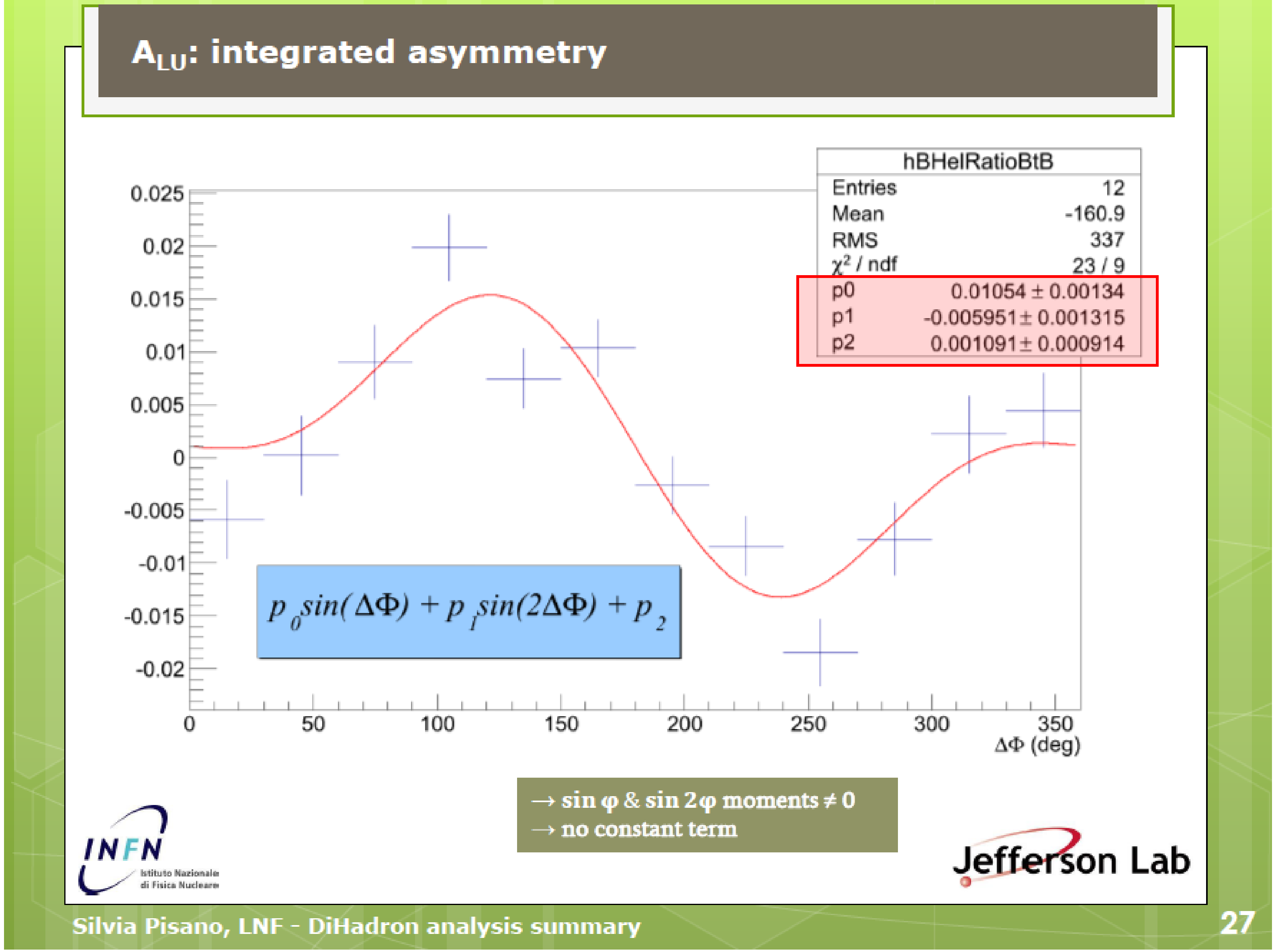
Presence of higher harmonics indicate that $\sigma_{LU}(\Delta\Phi) \neq \sigma_{UU}(\Delta\Phi)$

Conclusions

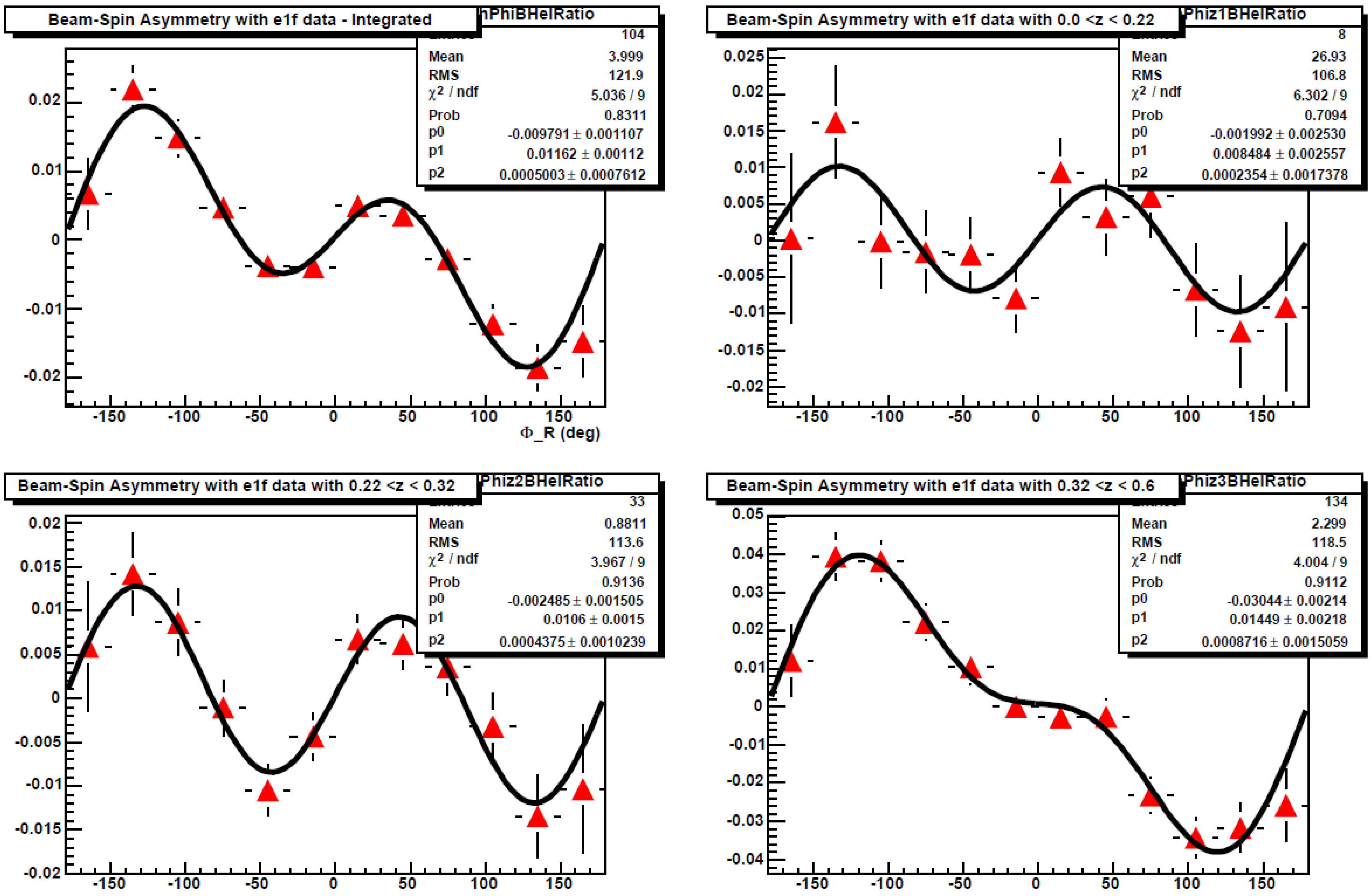
- New members of the polarized TMDs family -- 16 LO STMD fracture functions
- For hadron produced in the TFR of SIDIS, only 4 k_T-integrated fracture functions of unpolarized and longitudinally polarized quarks are probed at twist-two
  - SSA contains only a Sivers-type modulation $\sin(\phi_h - \phi_S)$ but no Collins-type $\sin(\phi_h + \phi_S)$ or $\sin(3\phi_h - \phi_S)$. The eventual observation of Collins-type asymmetry will indicate that LO factorized approach fails and long range correlations between the struck quark polarization and P_T of produced in TFR hadron might be important.
- DSIDIS cross section at LO contains 2 azimuthal independent and 20 azimuthally modulated terms.
  - Polarized SIDY cross section ($p + p \rightarrow l^+ l^- + h + X$) at LO contains 2 azimuthal independent, 20 lepton azimuth independent and 52 lepton azimuth dependent terms
- The ideal place to test the fracture functions factorization and measure these new nonperturbative objects are JLab12 and EIC facilities
- Nuclear modification of Fracture Functions???

JLab CLAS, preliminary (**courtesy of Silvia Pisano**)

π^+ in CFR, π^- in TFR,



JLab Very Preliminary



Presence of higher harmonics indicate that $\sigma_{LU}(\Delta\Phi) \neq \sigma_{UU}(\Delta\Phi)$

5.3. Lepto-production of a polarized hadron integrated over transverse momenta

We have assumed so far that the final hadron is spinless or unpolarized. Relaxing this condition, the panorama of unintegrated fracture functions becomes extremely complicated. However, the collinear case, that is, the lepto-production of a polarized hadron integrated over all transverse momenta, is still manageable. Considering only the leading terms in the expansion of the traced fracture matrix we have (the superscript denotes the polarization state of the final hadron)

$$\int \mathrm{d}^2 \boldsymbol{P}_{h\perp} \int \mathrm{d}^2 \boldsymbol{k}_\perp \mathcal{M}^{[\gamma^-]} = M(x_B,\zeta) + S_\parallel S_{h\parallel} M_L^L(x_B,\zeta) + (\boldsymbol{S}_\perp \cdot \boldsymbol{S}_{h\perp}) M_T^T(x_B,\zeta), \tag{57}$$

$$\int \mathrm{d}^2 \boldsymbol{P}_{h\perp} \int \mathrm{d}^2 \boldsymbol{k}_\perp \mathcal{M}^{[\gamma^-\gamma_5]} = S_\parallel \Delta M_L(x_B,\zeta) + S_{h\parallel} \Delta M^L(x_B,\zeta) + (\boldsymbol{S}_\perp \times \boldsymbol{S}_{h\perp}) \Delta M_T^T(x_B,\zeta). \tag{58}$$

We see that an unpolarized target can emit a longitudinally polarized quark and a longitudinally polarized hadron, via ΔM^L, a longitudinally polarized target can emit an unpolarized quark and a longitudinally polarized hadron, via M_L^L, and a transversely polarized target can produce a transversely polarized hadron in two different ways: via a correlation of the type $\boldsymbol{S}_\perp \cdot \boldsymbol{S}_{h\perp}$ by emitting an unpolarized quark, or via a correlation of the type $\boldsymbol{S}_\perp \times \boldsymbol{S}_{h\perp}$ by emitting a longitudinally polarized quark.

From Eqs. (57), (58) and (49), (50) we derive the cross section for the process $lN \to l'h^\uparrow X$:

$$\frac{\mathrm{d}\sigma^{\mathrm{TFR}}}{\mathrm{d}x_B\,\mathrm{d}y\,\mathrm{d}\zeta\,\mathrm{d}\phi_S\,\mathrm{d}\phi_{S_h}} = \frac{\alpha_{\mathrm{em}}^2}{\pi Q^2 y} \Bigg\{ \left(1 - y + \frac{y^2}{2}\right) \sum_a e_a^2 \left[M(x_B,\zeta) + S_\parallel S_{h\parallel} M_L^L(x_B,\zeta) + |\boldsymbol{S}_\perp||\boldsymbol{S}_{h\perp}| M_T^T(x_B,\zeta)\cos(\phi_{S_h} - \phi_S) \right]$$

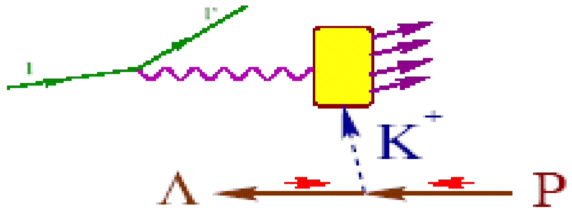
$$+ \lambda_l y \left(1 - \frac{y}{2}\right) \sum_a e_a^2 \left[S_\parallel \Delta M_L(x_B,\zeta) + S_{h\parallel} \Delta M^L(x_B,\zeta) \right.$$

$$+ |\boldsymbol{S}_\perp||\boldsymbol{S}_{h} \ldots \tag{59}$$

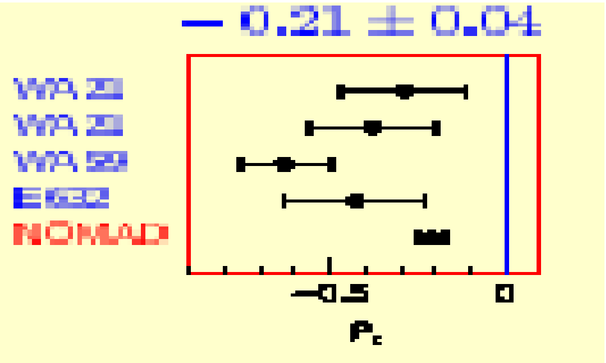
The transverse polarization terms contain modulations of the type $\cos(\phi_{S_h} - \phi_S)$ and $\sin(\phi_{S_h} - \phi_S)$, the former involving unpolarized

Λ longitudinal polarization in TFR

- Melnitchouk & Thomas, Meson Cloud Model



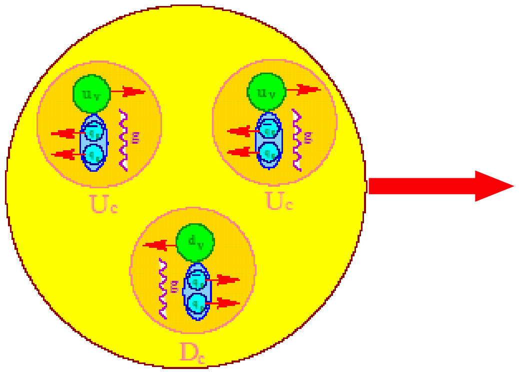
- 100 % anticorrelated with target polarization
  - contradiction with neutrino data for unpolarized target



Karliner, Kharzeev , Sapozhnikov, Alberg, Ellis, Naumov, AK: Intrinsic Strangeness Model

Nucleon wave function contains an admixture with hidden strangeness component:

$$|p\rangle = a\sum_{X=0}^{\infty}|uudX\rangle + b\sum_{X=0}^{\infty}|uud\bar{s}sX\rangle + \dots$$

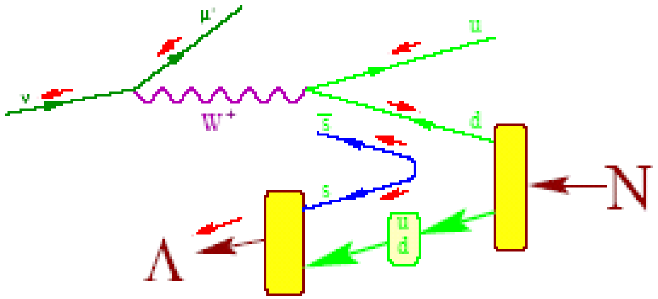


J.Ellis, A.K. & D.Naumov (2002)

$$P_\Lambda^{lN}(B) = \frac{\sum_M P_s(B(J,M)) \mid \langle B(J,M) \mid \text{diquark-quark remnant} + \text{s quark} \rangle \mid^2}{\sum_M \mid \langle B(J,M) \mid \text{diquark-quark remnant} + \text{s quark} \rangle \mid^2}$$

— $P_s(B(J,M))$ is the polarization of the strange quark in the baryon B with the spin state $\mid B(J,M)\rangle$,

— $\mid$ diquark-quark remnant + s quark $\rangle$ is the product of the wave function of the remnant diquark and the wave function of polarized s quark.



The remnant diquark-quark wave functions are:

$$|p \ominus d^\uparrow\rangle = \frac{1}{\sqrt{36}}[-\sqrt{2}(uu)_{1,0} + 2(uu)_{1,-1}]$$

$$|n \ominus d^\uparrow\rangle = \frac{1}{\sqrt{36}}[3(ud)_{0,0} + (ud)_{1,0} - \sqrt{2}(ud)_{1,-1}]$$

The wave function of polarized s quark is:

$$|s\rangle_{pol} = \frac{1}{\sqrt{2}} \mid \sqrt{(1+C_{sq})}s^\uparrow + \sqrt{(1-C_{sq})}s^\downarrow \rangle$$

Finally, the Λ^0 polarization in lN DIS is:

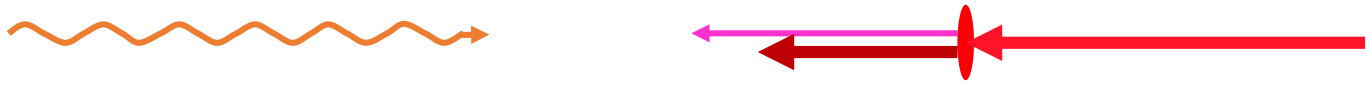
$P_\Lambda^{lN} = \sum_B \xi_B P_\Lambda^{lN}(B)$, where ξ_B is the fraction of Λ^0 produced via B

Hadronization in MC even generators

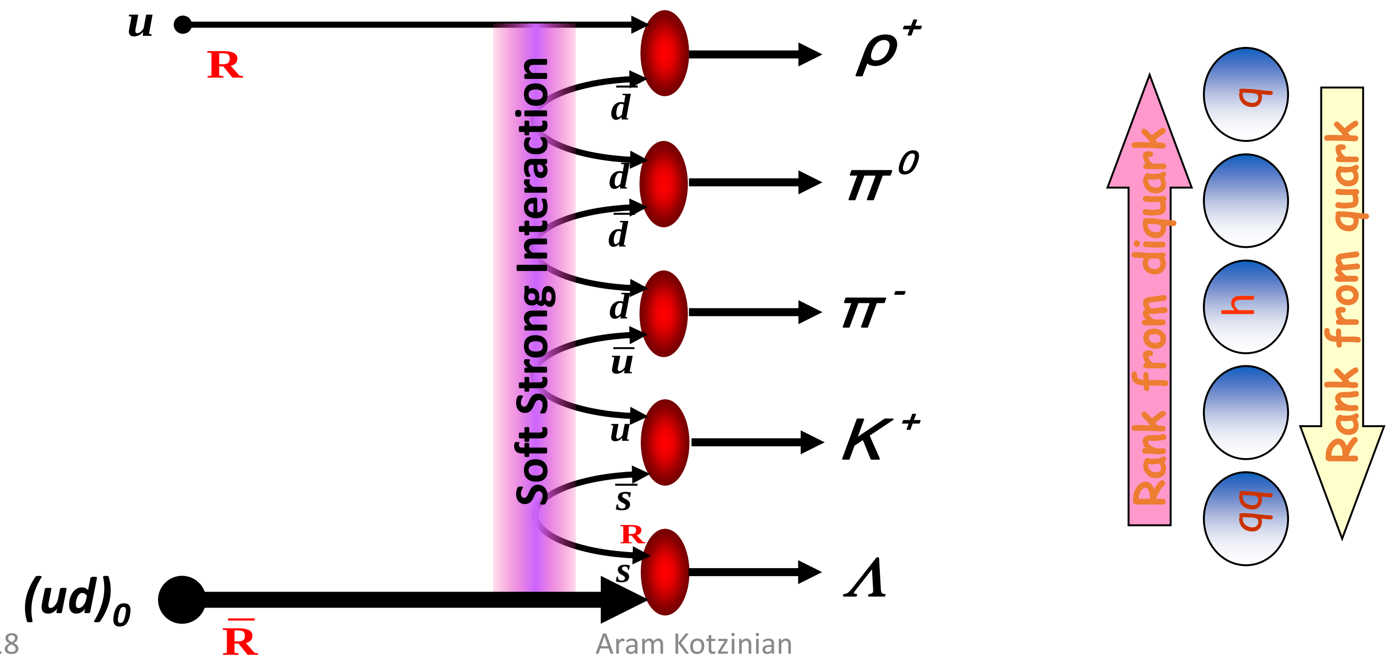
PDF , hard scattering and hadronization are factorized :

$$d\sigma^{lN\to lhX} = \sum_q f_q(x,\mathbf{k_T},\mathbf{s}_q;\mathbf{S}_N)\otimes d\sigma^{lq\to lq}\otimes H^q_{h/N}(x,\mathbf{k_T},\mathbf{s}_q;x_F,\mathbf{p}^h_\mathbf{T};\mathbf{S}_N)$$

- Before



- After hard scattering



Results and predictions from J. Ellis, A.K., D.V. Naumov, Eur. Phys. J. C 25, 603 (2002)

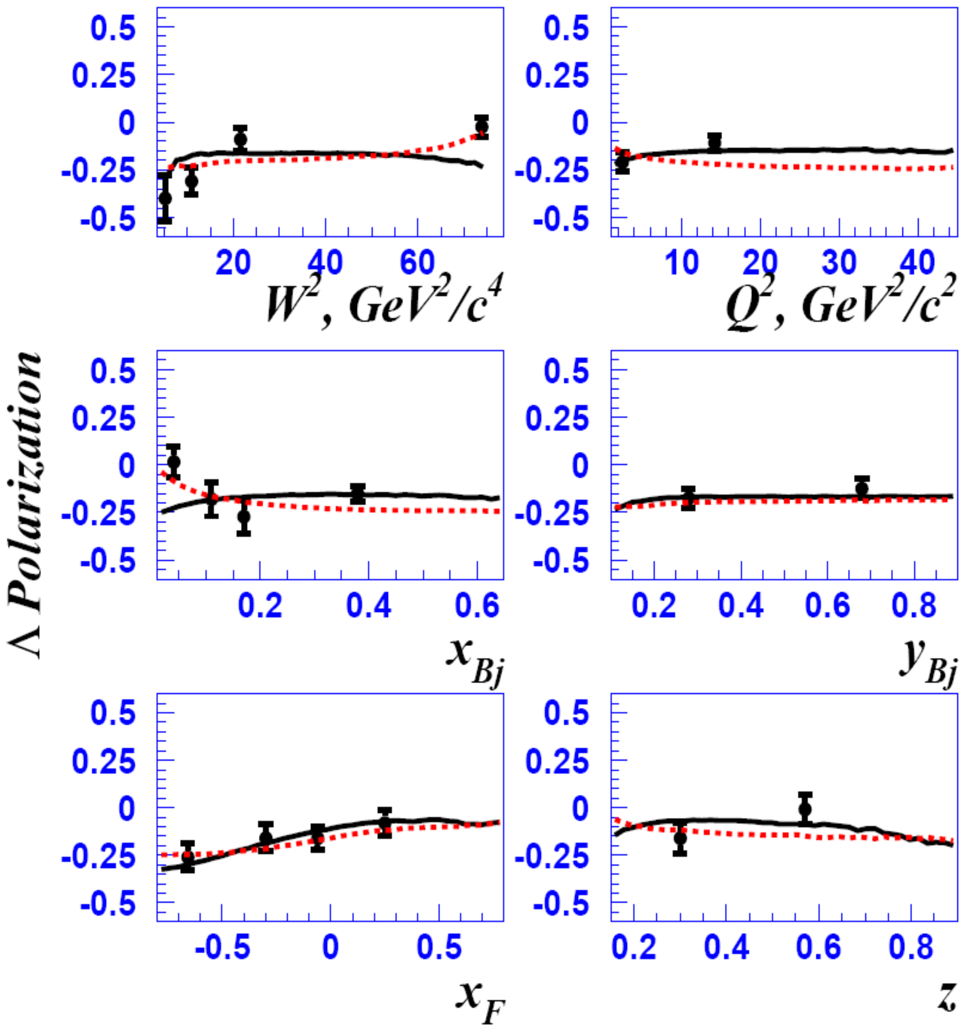
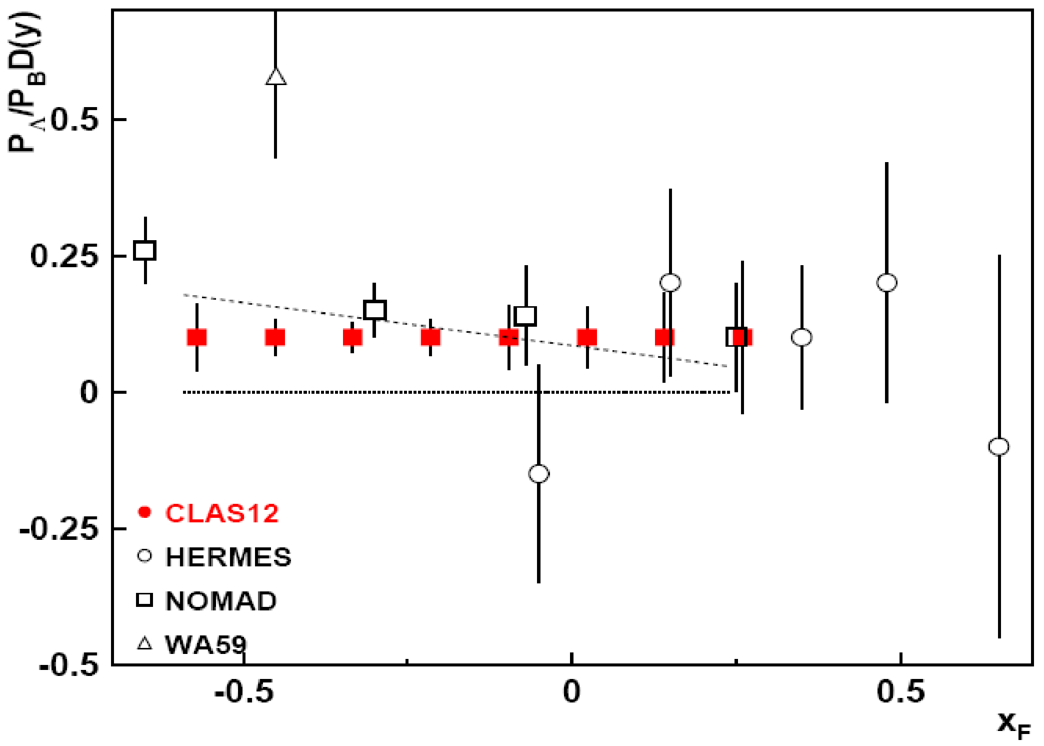
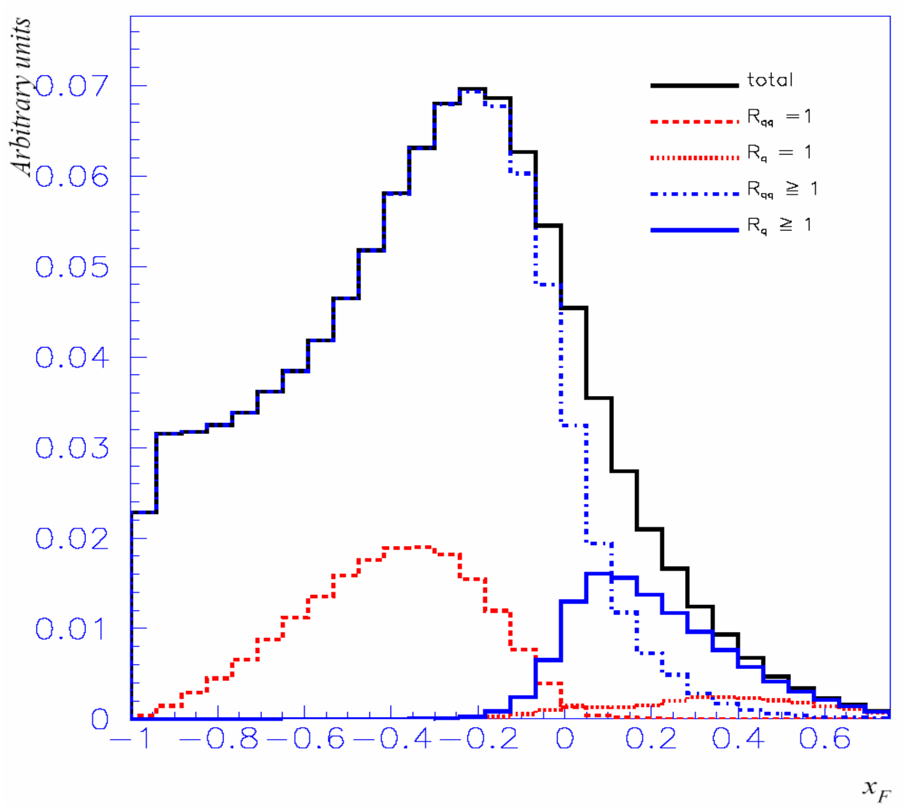


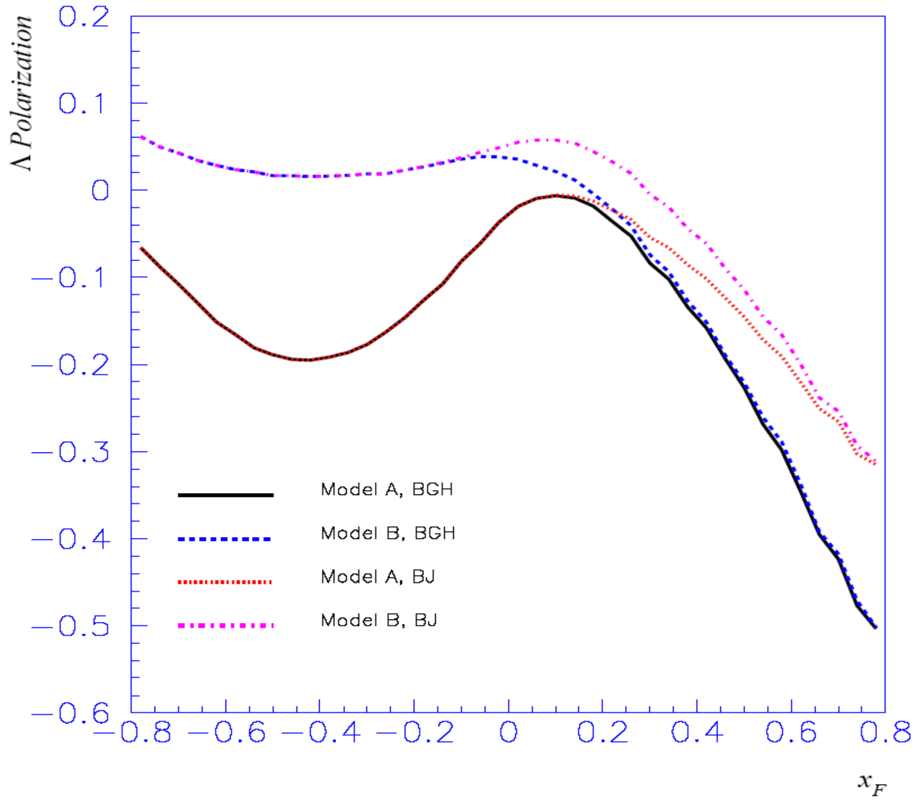
Figure 5: Our model predictions (model A - solid line, model B - dashed line) for polarization of Λ hyperons produced in ν_μ charged current DIS interactions off nuclei as functions of W^2, Q^2, x_{Bj}, y_{Bj}, x_F and z (at $x_F > 0$). The points with error bars are from NOMAD.



Predictions for EIC

5 GeV/c electron + 50 GeV/c proton, $P_{\text{Beam}} = 1,\ Q^2 > 1.\text{GeV/c},\ 0.4 < y < 0.9$





Good separation of the quark and diquark fragmentation allows to distinguish between different spin transfer mechanisms in the quark fragmentation