Short-range correlations as spatio-temporal fluctuations – forging links between data and models

Jan Ryckebusch

Department of Physics and Astronomy, Ghent University

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- **Is there a comprehensive picture of nuclear SRC? (mass (A), isospin (pp/nn/pn), asymmetry ($N - Z$) dependence of SRC)?**
- **How to forge links between nuclear-structure theory (models) and $A(e, e'pX)$observables sensitive to nuclear SRC?**

STRATEGY (and OUTLINE of this presentation)

1. Low-order correlation operator approximation (LCA): transition matrix elements between short-range correlated wave functions
2. Apply LCA to the computation of nuclear momentum distributions and find the driving physical mechanisms
   (Compare results to those of "ab-initio" approaches)
3. Compute aggegrated effect of SRC for any $A(N, Z)(A \geq 4)$
   (a_2 data from $A(e, e')$; magnitude of EMC effect)
4. Compute isospin and asymmetry dependence of SRC
   ($A(e, e'pp)/A(e, e'pn)$ data)
5. Develop a proper reaction theory for SRC-driven two-nucleon knockout
   - proper factorization properties of cross sections
     (data for c.m. distributions of SRC pairs)
   - FSI corrections (elastic and charge-exchange)
   - $A(e, e'NN)$ for $N \gtrsim Z$ and $p(A, pNN(A-2))$ for $N > Z$

Universal physics from short-distance correlations

1. Vicszek model for understanding emergent collective motion from local interactions: neighboring particles tend to align their velocities
2. Competition between an aligning force and a stochastic force
3. Two different energy ("time") scales emerge:
   - Particles in high-density zones tend to align their velocities (liquid phase, SRC nucleons)
   - Particles in low-density zones move in a disorderly fashion (gas phase, IPM nucleons)

Nuclear transition matrix elements with SRC (I)

- Shift complexity from wave functions to operators

$$| \Psi \rangle = \frac{1}{\sqrt{\mathcal{N}}} \widehat{\mathcal{G}} | \Phi \rangle \qquad \text{with,} \qquad \mathcal{N} \equiv \langle \Phi | \widehat{\mathcal{G}}^{\dagger} \widehat{\mathcal{G}} | \Phi \rangle$$

$| \Phi \rangle$ is an IPM single Slater determinant

- Nuclear SRC correlation operator $\widehat{\mathcal{G}}$

$$\widehat{\mathcal{G}} \approx \widehat{\mathcal{S}} \left(\prod_{i<j=1}^{A} \left[1 + \hat{l}(i,j) \right] \right) ,$$

- Major source of correlations: central (Jastrow), tensor and spin-isospin (universal scaling functions in r_{ij})

$$\hat{l}(i,j) = -g_c(r_{ij}) + f_{t\tau}(r_{ij}) \widehat{S}_{ij} \vec{\tau}_i \cdot \vec{\tau}_j \,. + f_{\sigma\tau}(r_{ij}) \vec{\sigma}_i \cdot \vec{\sigma}_j \vec{\tau}_i \cdot \vec{\tau}_j$$

Nuclear transition matrix elements with SRC (II)

- Turn expectation values between correlated states Ψ into expectation values between uncorrelated states Φ

$$\langle \Psi \mid \widehat{\Omega} \mid \Psi \rangle = \frac{1}{\mathcal{N}} \langle \Phi \mid \widehat{\Omega}^{\text{eff}} \mid \Phi \rangle$$

- "Conservation Law of Misery": $\widehat{\Omega}^{\text{eff}}$ is an A-body operator

$$\widehat{\Omega}^{\text{eff}} = \widehat{\mathcal{G}}^{\dagger} \, \widehat{\Omega} \, \widehat{\mathcal{G}} = \left(\sum_{i<j=1}^{A} \left[1 - \hat{l}(i,j) \right] \right)^{\dagger} \widehat{\Omega} \left(\sum_{k<l=1}^{A} \left[1 - \hat{l}(k,l) \right] \right)$$

- Truncation procedure for short-distance phenomena

$$\text{SCALING:} \quad \Psi^{\dagger}(\vec{R} - \frac{\vec{r}}{2}) \Psi(\vec{R} + \frac{\vec{r}}{2}) \approx \sum_{n} c_n(\vec{r}) O_n(\vec{R}) \quad (\mid \vec{r} \mid \approx 0)$$

Low-order correlation operator approximation (LCA)

- LCA: N-body operators receive SRC-induced $(N+1)$-body corrections

Norm $\mathcal{N} \equiv \langle \Phi \mid \widehat{\mathcal{G}}^{\dagger}\widehat{\mathcal{G}} \mid \Phi \rangle$: aggregated SRC effect

- LCA expansion of the norm $\mathcal{N}$

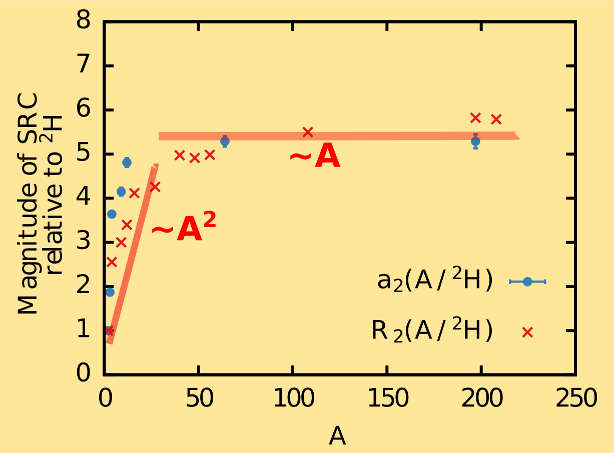
$$\mathcal{N} = 1 + \frac{2}{A} \sum_{\alpha<\beta} {}_{\text{nas}}\langle \alpha\beta \mid \hat{l}^{\dagger}(1,2) + \hat{l}^{\dagger}(1,2)\hat{l}(1,2) + \hat{l}(1,2) \mid \alpha\beta \rangle_{\text{nas}}.$$

  1. $\mid \alpha\beta\rangle_{\text{nas}}$: normalized and anti-symmetrized two-nucleon IPM-state
  2. $\sum_{\alpha<\beta}$ extends over all IPM states $\mid \alpha\rangle \equiv \mid n_\alpha l_\alpha j_\alpha m_{j_\alpha} t_\alpha\rangle$,
- $(\mathcal{N} - 1)$: measure for aggregated effect of SRC in g.s. of A
- Aggregated quantitative effect of SRC in A relative to ^{2}H

$$R_2(A/^2\text{H}) = \frac{\mathcal{N}(A) - 1}{\mathcal{N}(^2\text{H}) - 1} = \frac{\text{measure for SRC effect in } A}{\text{measure for SRC effect in } ^2\text{H}} .$$

- Input to the calculations for $R_2(A/^2\text{H})$
  1. HO IPM states with $\hbar\omega = 45A^{-1/3} - 25A^{-2/3}$
  2. correlation functions $[g_c(r), f_{t\tau}(r), f_{\sigma\tau}(r)]$ = UNIVERSAL SCALING FUNCTIONS

$a_2(A/^2\mathrm{H})$ from $A(e, e')$ at $x_B \gtrsim 1.5$ and $R_2(A/^2\mathrm{H})$



1. $A \lesssim 40$: strong mass dependence in SRC effect
2. $A > 40$: soft mass dependence
3. SRC effect saturates for A large *(for large A aggregated SRC effect per nucleon is about 5× larger than in ^2H)*

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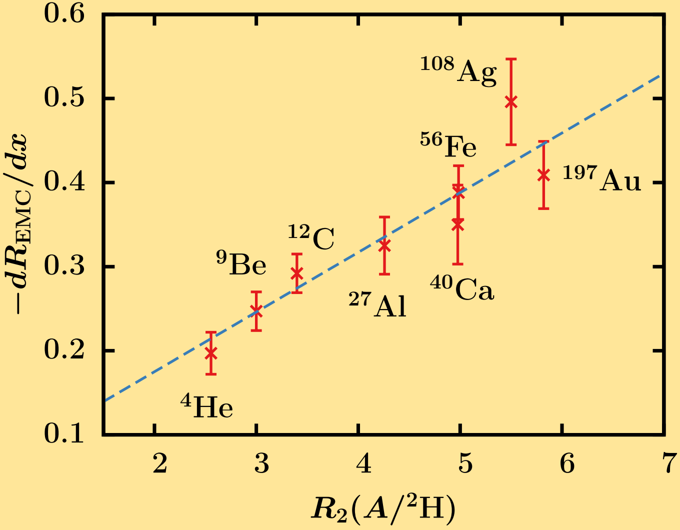
Magnitude of EMC effect versus $R_2(A/^2\text{H})$



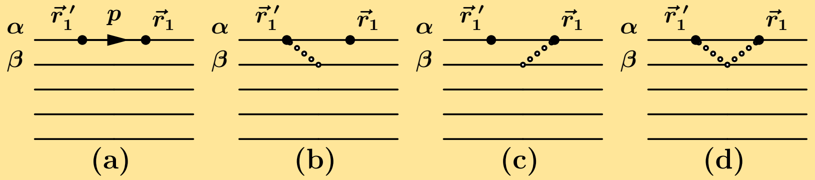
LCA can predict magnitude of EMC effect for any $A(N,Z) \geq 4$

Single-nucleon momentum distribution $n^{[1]}(p)$

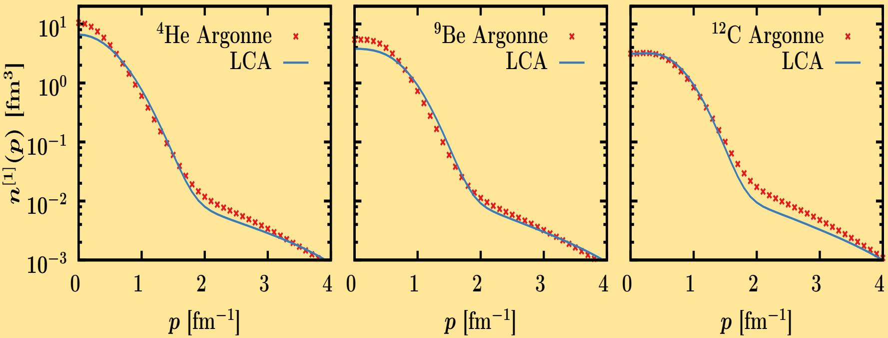
- Probability to find a nucleon with momentum p

$$n^{[1]}(p) = \int \frac{d^2\Omega_p}{(2\pi)^3} \int d^3\vec{r}_1 \; d^3\vec{r}_1{}' \; d^{3(A-1)}\{\vec{r}_{2-A}\} e^{-i\vec{p}\cdot(\vec{r}_1{}'-\vec{r}_1)}$$
$$\times \Psi^*(\vec{r}_1, \vec{r}_{2-A}) \Psi(\vec{r}_1{}', \vec{r}_{2-A}).$$

- SRC-induced corrections to IPM $n^{[1]}(p)$ are of two-body type
- Normalization property $\int dp \; p^2 n^{[1]}(p) = 1$ can be preserved



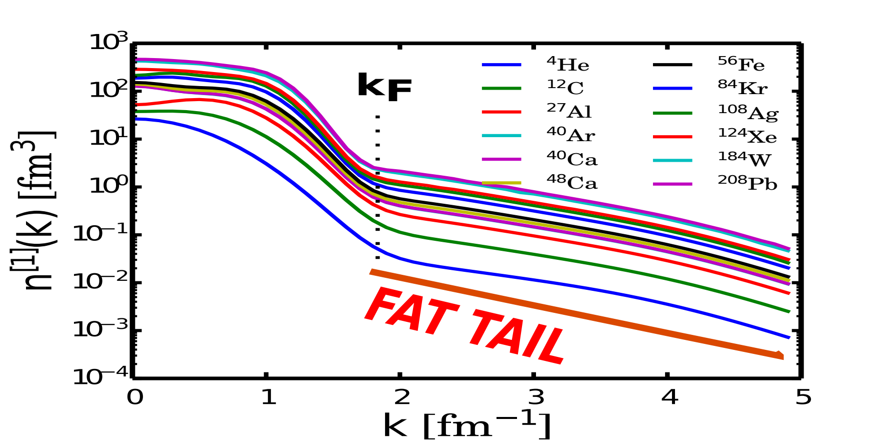
(a): IPM contribution **(b)-(d): SRC contributions in LCA**



QMC: PRC89(2014)024305 LCA: JPG42(2015)055104

1. $p \lesssim p_F = 1.25$ fm^{-1}: $n^{[1]}(p)$ is "Gaussian" (IPM PART)
2. $p \gtrsim p_F$: $n^{[1]}(p)$ has an "exponential" fat tail (CORRELATED PART)
3. fat tail of $n^{[1]}(p)$ in QMC and LCA are comparable

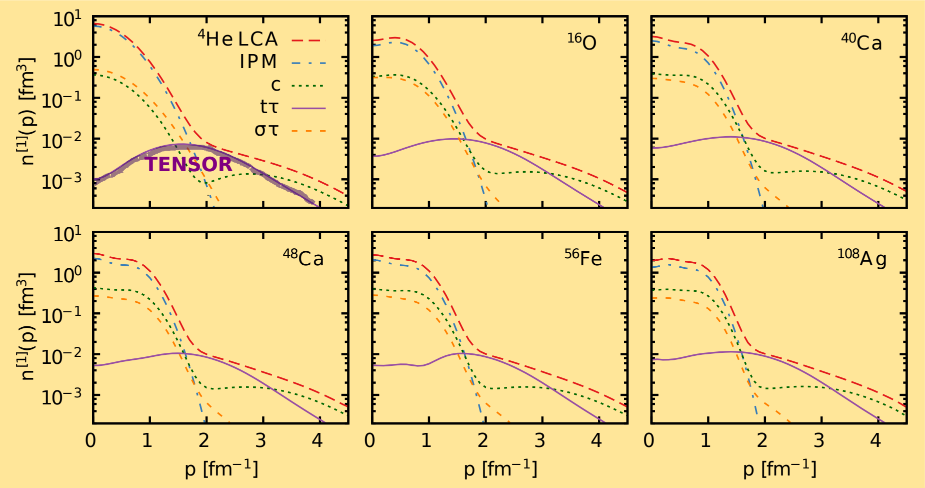
$n^{[1]}(k)$ in LCA: from light to heavy



LCA: JPG42(2015)055104

1. momentum dependence of fat tail of $n^{[1]}$ is "universal" (universal SRC 2N correlation functions)

Major source of correlated strength in $n^{[1]}(p)$?



1. $1.5 \lesssim p \lesssim 3$ fm^{-1} is dominated by tensor correlations
2. central correlations substantial at $p \gtrsim 3.5$ fm^{-1}

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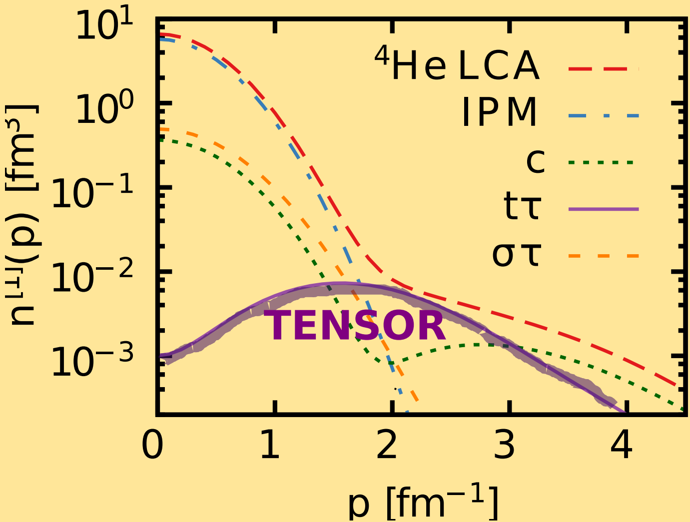

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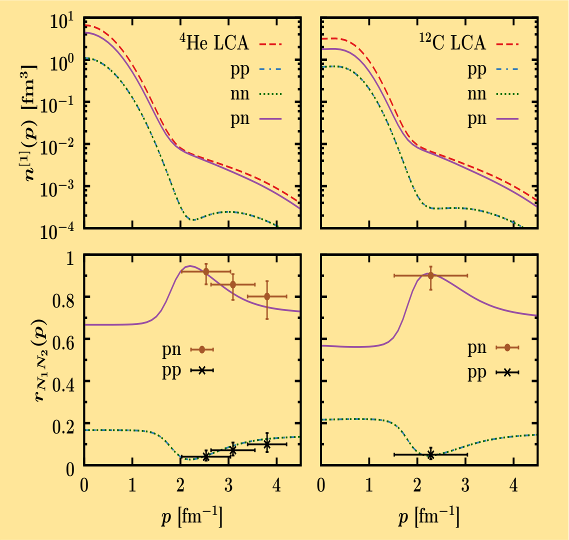
Isospin dependence of SRC: pp, nn and pn

$r_{N_1N_2}(p)$: relative contribution of (N_1N_2) pairs to $n^{[1]}(p)$

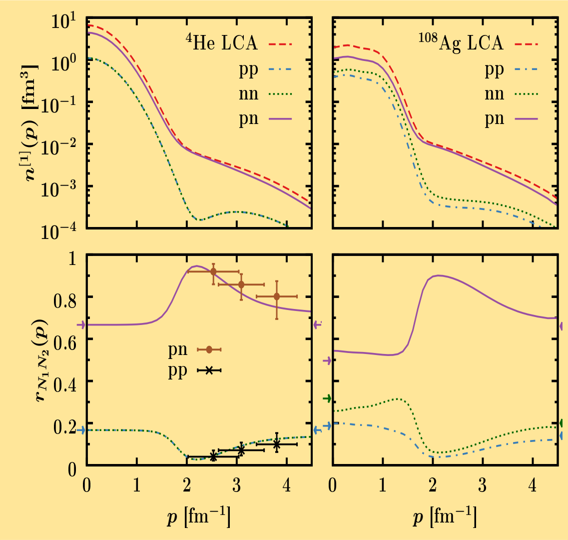


- Naive IPM:

$$r_{pp} = \frac{Z(Z-1)}{A(A-1)},$$
$$r_{nn} = \frac{N(N-1)}{A(A-1)},$$
$$r_{pn} = \frac{2NZ}{A(A-1)}.$$

- Data extracted from ^{4}He$(e,e'pp)/(e,e'pn)$ (PRL 113, 022501) and $\frac{^{12}C(p,ppn)}{^{12}C(p,pp)}$ (Science 320, 1476) assuming that $r_{pp} \approx r_{nn}$
- **Fat tail is dominated by "pn" (momentum dependent)**

$r_{N_1N_2}(p)$: relative contribution of (N_1N_2) pairs to $n^{[1]}(p)$



- Naive IPM:
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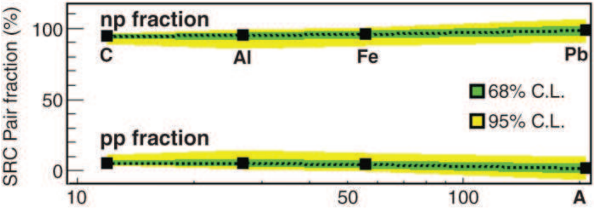
- **Fat tail is dominated by "pn" (momentum dependent)**

Imbalanced strongly interacting Fermi systems (I)

Sciencexpress

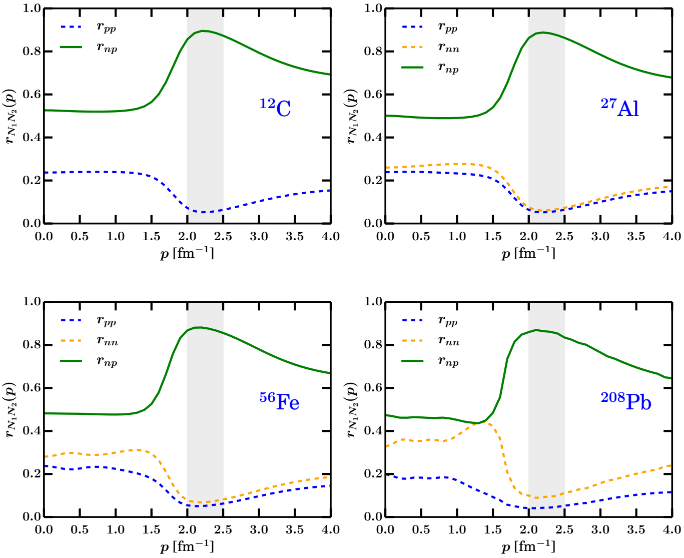
Momentum sharing in imbalanced Fermi systems

O. Hen,[1*] M. Sargsian,[2] L. B. Weinstein,[3] E. Piasetzky,[1] H. Hakobyan,[4,5] D. W. Higinbotham,[6] M



LCA predicts that ≈90% of correlated pairs is “pn”, and ≈5% is “pp” (UNIVERSAL: A independent)

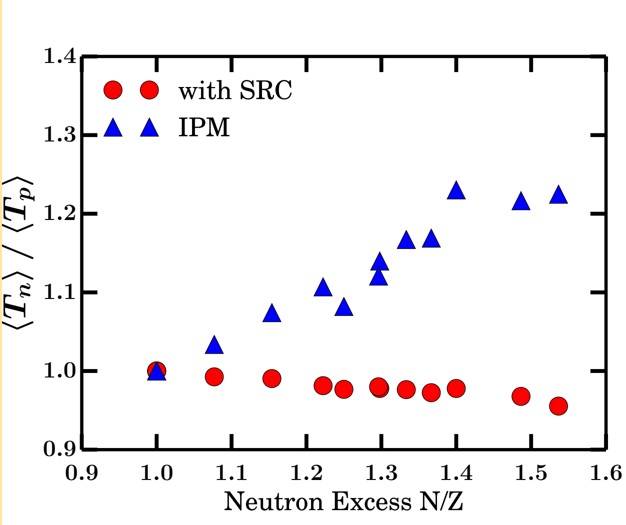
Imbalanced strongly interacting Fermi systems (II)



-The pp and pn SRC pair fractions are momentum dependent

-Semi-exclusive $A(e, e'pN)$ measurements mainly probe the low p part of the SRC region

The ratio $\langle T_n = p_n^2/(2M_n)\rangle \,/\, \langle T_p = p_p^2/(2M_p)\rangle$



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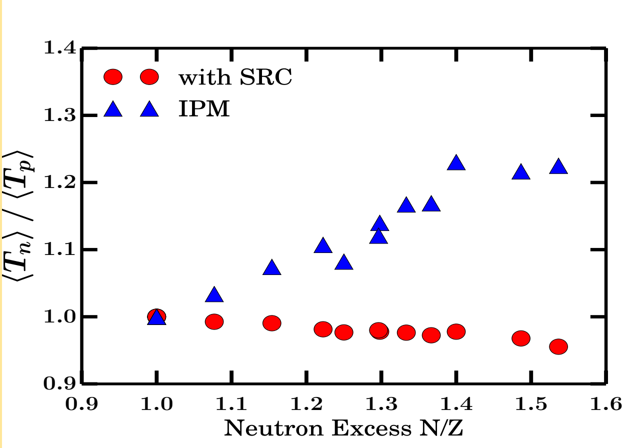
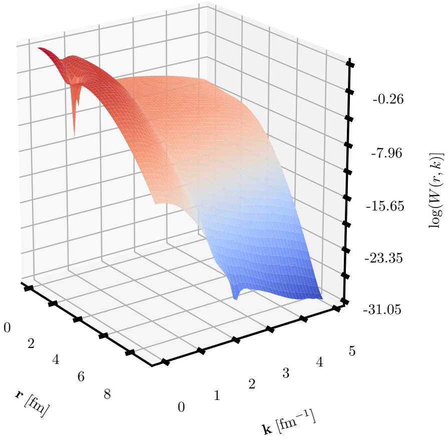


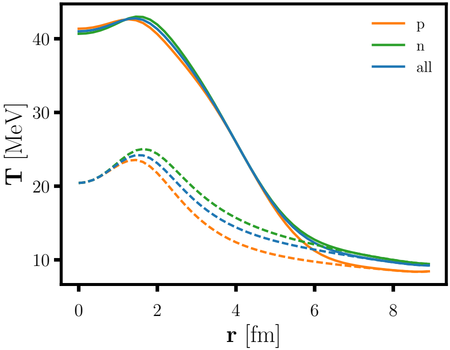
After correcting for SRC, minority component has largest kinetic energy (depends on the ratio of N/Z)

Wigner quasiprobability distribution $W(r,p)$ (^{48}Ca)

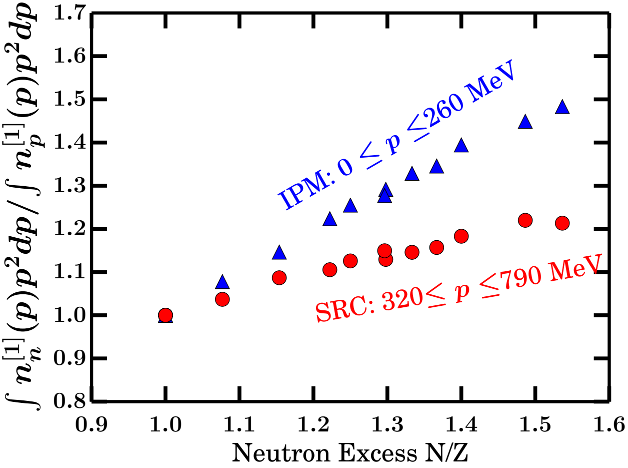


$$\langle T(r)\rangle = \frac{\int dp p^2 \widehat{T} W(r,p)}{\int dp p^2 W(r,p)}$$

- - -: IPM ——: including SRC



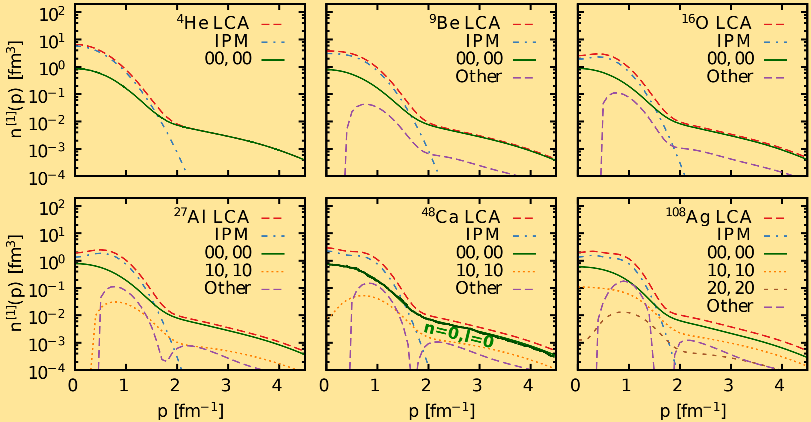
Isospin asymmetry dependence of the SRC?



Can be "measured" by evaluating the ratio $A(e,e'n)/A(e,e'p)$ at "low" and "high" missing momenta (Axel Schmidt's talk)

Quantum numbers of SRC-susceptible IPM pairs?

Fat tails: correlation operators acting on IPM pairs.
What are relative quantum numbers (nl) of those IPM pairs?



$$\sum_{nl} \sum_{n'l'} n^{[1],\mathrm{corr}}_{nl,n'l'}(p) = n^{[1],\mathrm{corr}}(p)$$

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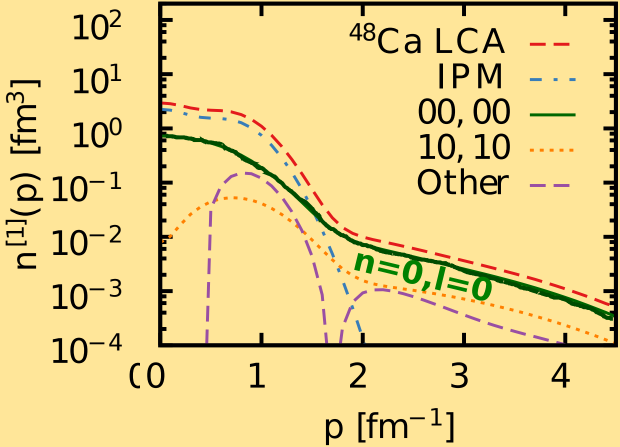


($n = 0$, $l = 0$) IPM pairs are very susceptible to SRC

Nucleon knockout data and nuclear models (I)

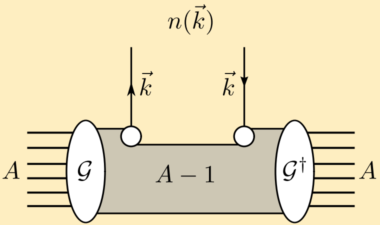
The quasi-free one-nucleon knockout case

- Link between $A(e,e'N)$ cross section and single-nucleon spectral function can be derived

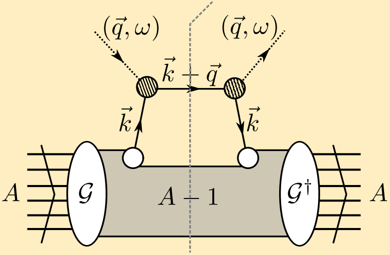
$$\frac{d^5\sigma}{d\epsilon' d\Omega_{\epsilon'} d\Omega_N dE_N}(e,e'N) = K\sigma_{eN} S\left(E_m, p_m\right)$$

- Factorization is approximate: relativity, final state interactions, spin effects, ...

NUCLEAR STRUCTURE



$A(e,e'p)$ OBSERVABLES

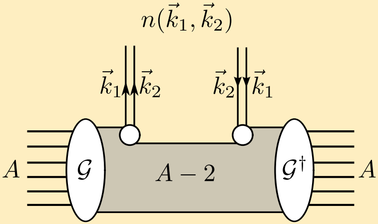


Nucleon knockout data and nuclear models (II)

The quasi-free two-nucleon knockout case

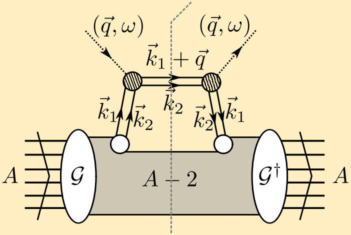
- Connection between SRC driven $A(e, e'NN)$ observables and high-momentum part of two-nucleon momentum distribution?

NUCLEAR STRUCTURE



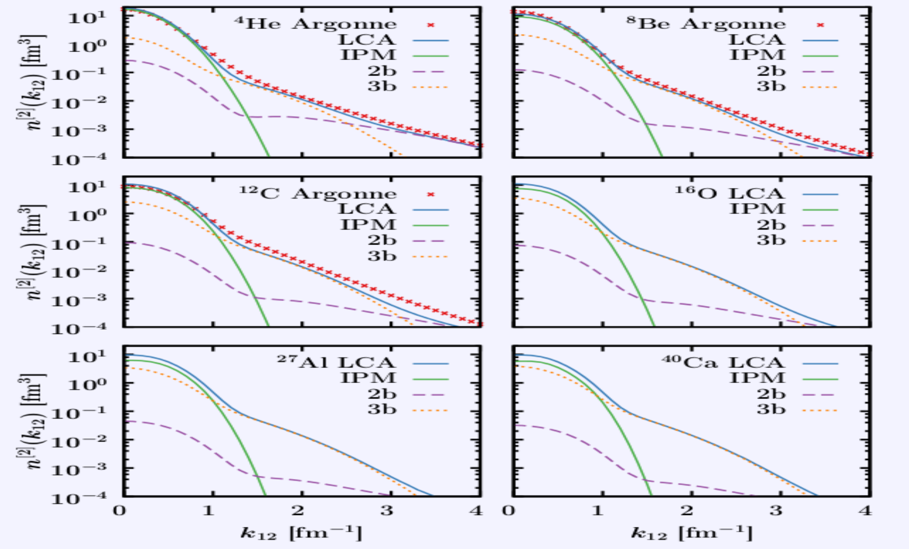
Two-nucleon momentum distribution

$A(e, e'NN)$ OBSERVABLES

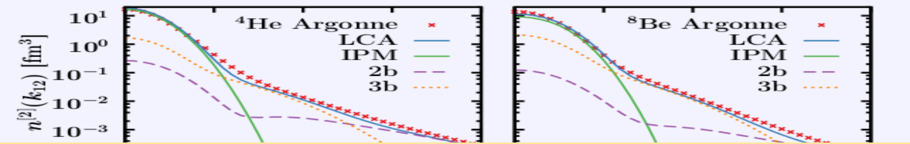


Measurements with specific kinematic cuts

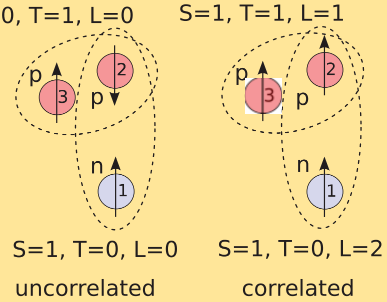
Relative two-nucleon momentum distribution in LCA: tail is dominated by "3-body" SRC effects



Relative two-nucleon momentum distribution in LCA: tail is dominated by "3-body" SRC effects



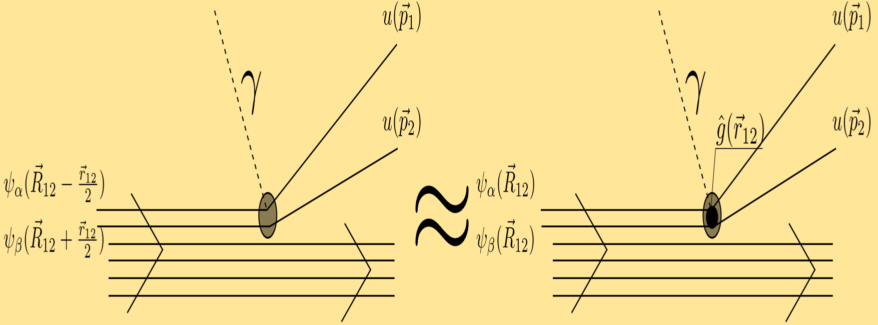
Correlations through the mediation of a third particle:



Feldmeier *et al.*, PRC 84 (2011), 054003

Exclusive SRC-driven $A(e, e'NN)$ (I)

- SRC-prone IPM pairs: close-proximity $(n_{12} = 0, l_{12} = 0)$ state
- The EXCLUSIVE $A(e, e'NN)$ cross sections can be factorized [PLB383,1 ; PRC89,024603 ; PRC96,034608]



ZRA: Zero-range approximation

Exclusive SRC-driven $A(e, e'NN)$ (II)

1. **$A(e, e'NN)$ cross section factorizes according to**

$$\frac{d^8\sigma}{d\epsilon' d\Omega_{\epsilon'} d\Omega_1 d\Omega_2 dT_{p_2}}(e, e'NN) = K\sigma_{eNN}(k_+, k_-, q)F^{(D)}(P)$$

$F^{(D)}(P)$: FSI corrected conditional probability to find a dinucleon with c.m. momentum P in a relative $(n_{12} = 0, l_{12} = 0)$ state

2. **A dependence of the $A(e, e'pp)$ cross sections is soft**

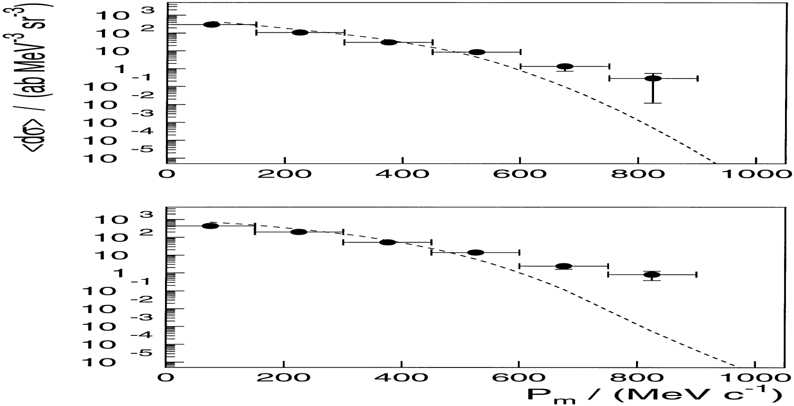
(much softer than predicted by naive $Z(Z-1)$ counting)

$$\frac{A(e, e'pp)}{^{12}\mathrm{C}(e, e'pp)} \approx \frac{N_{pp}(A)}{N_{pp}\left(^{12}\mathrm{C}\right)} \times \left(\frac{T_A(e, e'p)}{T_{^{12}\mathrm{C}}(e, e'p)}\right)^{1-2}$$

3. **C.m. width of SRC susceptible pairs is "large" (in p-space)**

Factorization of the $A(e, e'pp)$ cross sections

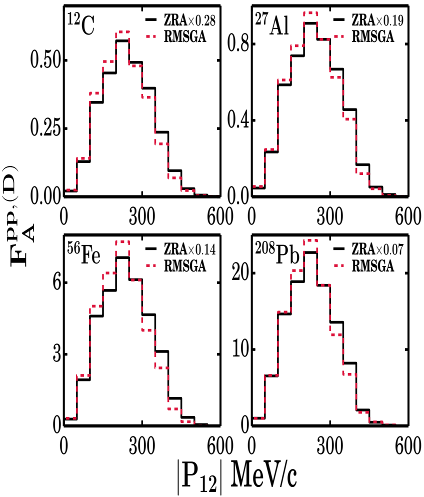
$^{12}C(e, e'pp)$ @ MAMI (Mainz) (Physics Letters B **421** (1998) 71.)



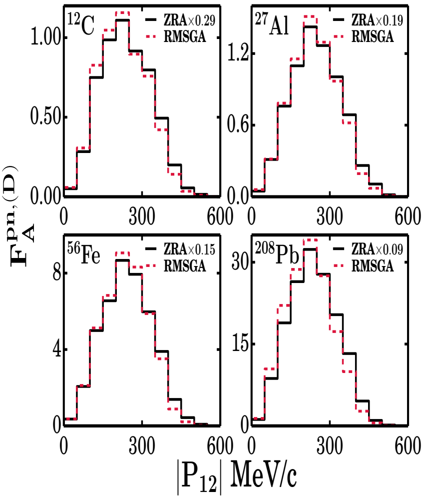
For P $\lesssim$ 0.5 GeV c.m. motion of correlated pairs in ^{12}C is mean-field like $\left(\exp \frac{-P^2}{2\sigma^2_{c.m}}\right)$! Data prove the proposed factorization in terms of $F^{(D)}(P)$.

Effect of FSI on factorization of $A(e, e'pN)$ c.s.?

A(e, e′pp)

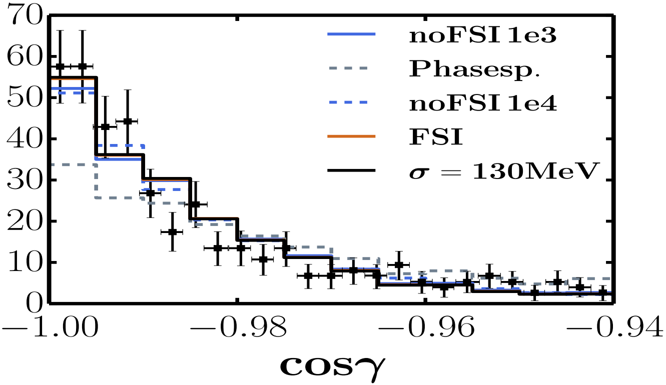


A(e, e′pn)



$A(e, e'NN)$: Effect of the final-state interactions?

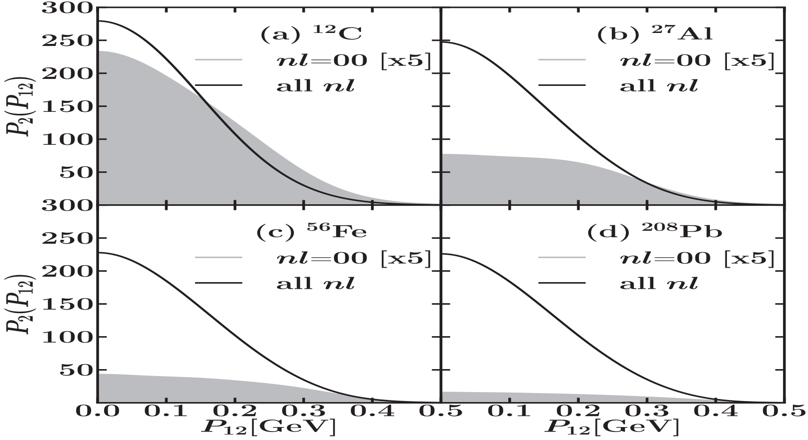
Opening-angle distribution of ^{4}He$(e, e'pp)$



1. **FSI (eikonal model) reduces the cross sections**
2. **FSI marginally affects the angular distributions**
   (FSI preserves factorization properties)

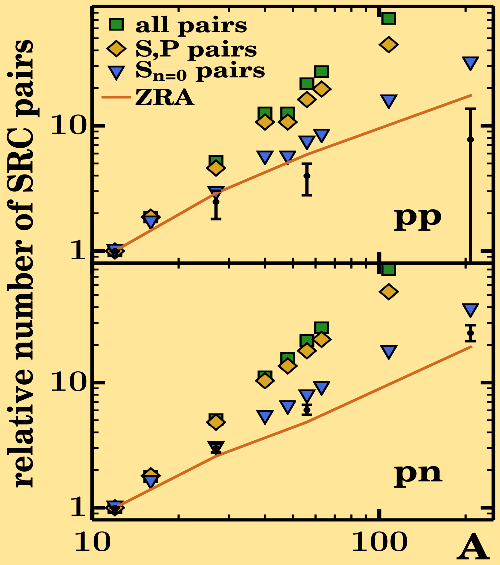
C.m. motion of correlated pp pairs

PHYSICAL REVIEW C **89**, 024603 (2014)



Width of c.m. distribution is a lever to discriminate between SRC-prone and other IPM pairs (Axel Schmidt's talk)

A dependence of number of pp and pn SRC pairs

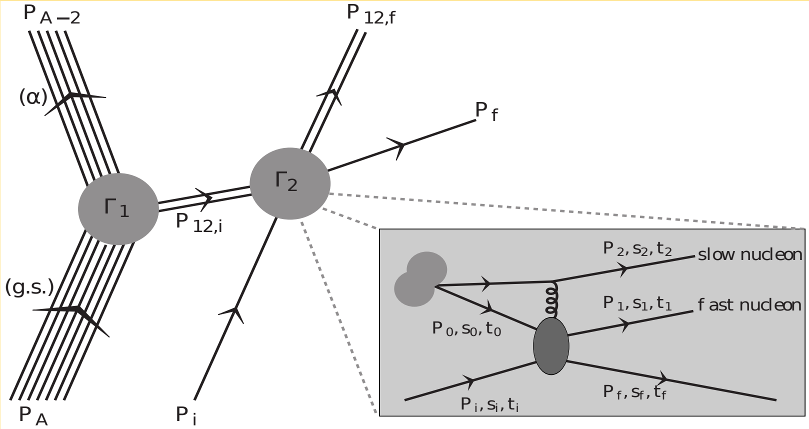


- Analysis of $A(e,e'pp)$ and $A(e,e'p)$ (A=^{12}C, ^{27}Al, ^{56}Fe, ^{208}Pb) in "SRC" kinematics (Data Mining Collaboration @JLAB)
- FSI corrections applied to the data
- $(n=0, l=0)$ pair counting
- ZRA: MC reaction-model calculations in the large phase space

PRC92, 024604 (2015)

$p(A, pNN\ A-2)$ with radioactive beams

SRC in neutron-rich matter? Success of program hinges on existence of a proper factorization expression for cross section.



S. Stevens *et al.*, PLB777 (2018) 374

$p(A, pNN\ A-2)$ with radioactive beams

SRC in neutron-rich matter? Success of program hinges on existence of a proper factorization expression for cross section.

$$\frac{d\sigma^{(pN_1N_2)}}{d\Omega_f\, dE_1\, d\Omega_1\, dE_2\, d\Omega_2} = 2^{2|M_T|-1}\, \mathcal{S}(J_A,\beta\gamma)\, \mathcal{K}\, \frac{d\sigma^{pN_1}}{dt} \left\{ \frac{E_2}{E_m + m_{N_1}} \sum_J \frac{1}{2J+1} \sum_M \sum_T \frac{1}{2T+1} F^{\beta\gamma}_{JM,T}(\vec{P},\vec{k}) \right\}_{\text{PF}}, \quad (11)$$

with $\mathcal{K}$ a kinematic factor

$$\mathcal{K} = \frac{1}{(2\pi)^8} \frac{(P_f\cdot P_1)^2 - m_p^2 m_{N_1}^2}{\sqrt{(P_i\cdot P_A)^2 - m_p^2 m_A^2}}\, m_A m_R^* \frac{p_f p_1 p_2}{E_R} \left| 1 - \frac{E_f}{E_R}\frac{\vec{p}_R\cdot\vec{p}_f}{p_f^2} \right|^{-1} \quad (12)$$

$$F^{\beta\gamma}_{JM,T}(\vec{P},\vec{k}) = \sum_{\mu=T-1}^{1-T} \Bigg| \mathcal{F}^{(0)}_\nu[f_c - 3f_{\sigma\tau}](k)\, \mathcal{P}^{\varepsilon\beta\gamma}_{JMT\mu}(\vec{P}) - \delta_{T,0}\, 12\sqrt{2\pi}\, \mathcal{F}^{(2)}_\nu[f_{t\tau}](k) \sum_{m_l=-2}^{2} \langle 2m_l 1\mu | 1(m_l+\mu)\rangle\, \mathcal{P}^{\varepsilon\beta\gamma}_{JMT(m_l+\mu)}(\vec{P})\, Y_{2,m_l}(\Omega_k) \Bigg|^2 .$$

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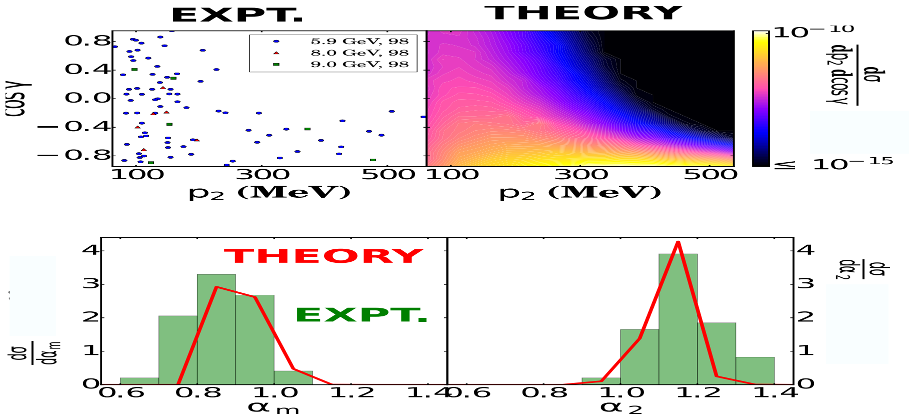
Postdictions for $^{12}C(p, ppn)$ from BNL



- DATA: A. Tang *et al.*, Phys. Rev. Lett. **90**, 042301 (2003)
- Calculations based on a factorized form of the cross section

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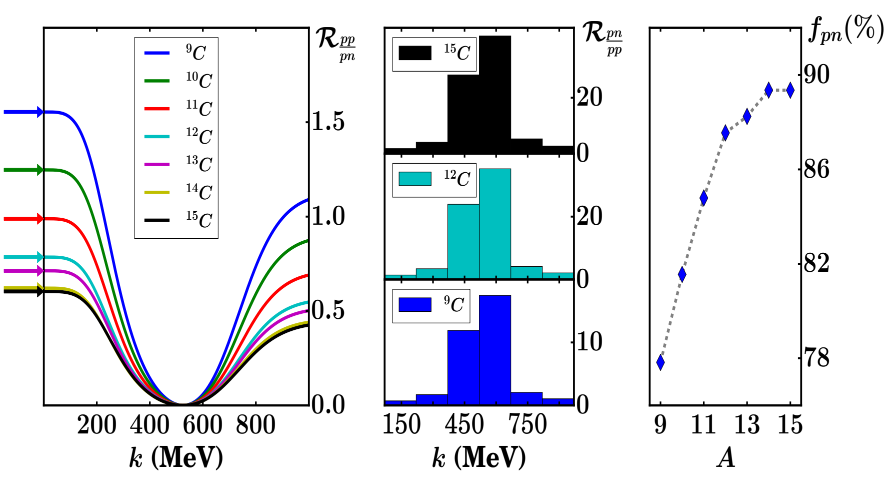
$p(A, pNN\ A-2)$ with radioactive beams: asymmetry dependence of nuclear SRC

Ratios of SRC pp to pn pairs for various carbon isotopes



CONCLUSIONS (I) - Nuclear Structure Theory

- **Nuclear SRC can be captured by general and robust principles**
- **LCA: efficient way of computing the SRC contributions to NMDs**
  1. **Magnitude of EMC effect and $A(e,e')/D(e,e')$ scaling factor ($x_B \gtrsim 1.5$) can be predicted in LCA**
  2. **$A \leq 12$: LCA predictions for fat tails are in line with those of QMC**
  3. **LCA predictions for $\langle T_N \rangle$ and radii are "realistic" (consistency checks)**
  4. **Natural explanation for the universal behavior of the NMD tails**
- **MAJOR contribution to SRC strength: correlation operators acting on IPM pairs in a nodeless relative S state**

CONCLUSIONS (II)- - Nuclear Reactions Theory

- **Insights from study of SRC contribution to NMD has implications for SRC-driven $A(e,e'NN)A-2$ and $p(A,pNN\ A-2)$**
  1. **Scaling behavior of cross section ($\sim F(P)$) (CONFIRMED)**
  2. **Very soft mass dependence of cross section (CONFIRMED)**
  3. **Peculiar c.m. width of the SRC-susceptible pairs (CONFIRMED)**
- **Generally applicable techniques for quantifying SRC: two-body effects in neutrino reactions, role of SRC in exotic forms of hadronic matter, . . .**
- **SRC induced spatio-temporal fluctuations are measurable, are significant and are quantifiable**


THANK YOU!

Selected publications

- C. Colle, W. Cosyn, J. Ryckebusch
  "Final-state interactions in two-nucleon knockout reactions"
  arXiv:1512.07841 and PRC **93** (2016) 034608.
- J. Ryckebusch, M. Vanhalst, W. Cosyn
  "Stylized features of single-nucleon momentum distributions"
  arXiv:1405.3814 and Journal of Physics G **42** (2015) 055104.
- C. Colle, O. Hen, W. Cosyn, I. Korover, E. Piasetzky, J. Ryckebusch, L.B. Weinstein
  "Extracting the Mass Dependence and Quantum Numbers of Short-Range Correlated Pairs from $A(e,e'p)$ and $A(e,e'pp)$ Scattering" arXiv:1503.06050 and PRC **92** (2015), 024604.
- C. Colle, W. Cosyn, J. Ryckebusch, M. Vanhalst
  "Factorization of electroinduced two-nucleon knockout reactions"
  arXiv:1311.1980 and PRC **89** (2014), 024603.
- Maarten Vanhalst, Jan Ryckebusch, Wim Cosyn
  "Quantifying short-range correlations in nuclei"
  arXiv:1206.5151 and PRC **86** (2012), 044619.