Unpolarized TMDs in hard scattering experiments

Andrea Signori

CLAS
collaboration meeting

Oct. 4th 2017

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DEPARTMENT OF ENERGY
UNITED STATES OF AMERICA

Jefferson Lab

JSA

Outline of the talk

1) Transverse-Momentum-Dependent distributions (**TMDs**)

2) **formalism**

2) **extractions** of unpolarized quark TMDs

3) polarized case

4) how to access **gluon** TMDs

5) TMDs in **spin-1** hadrons

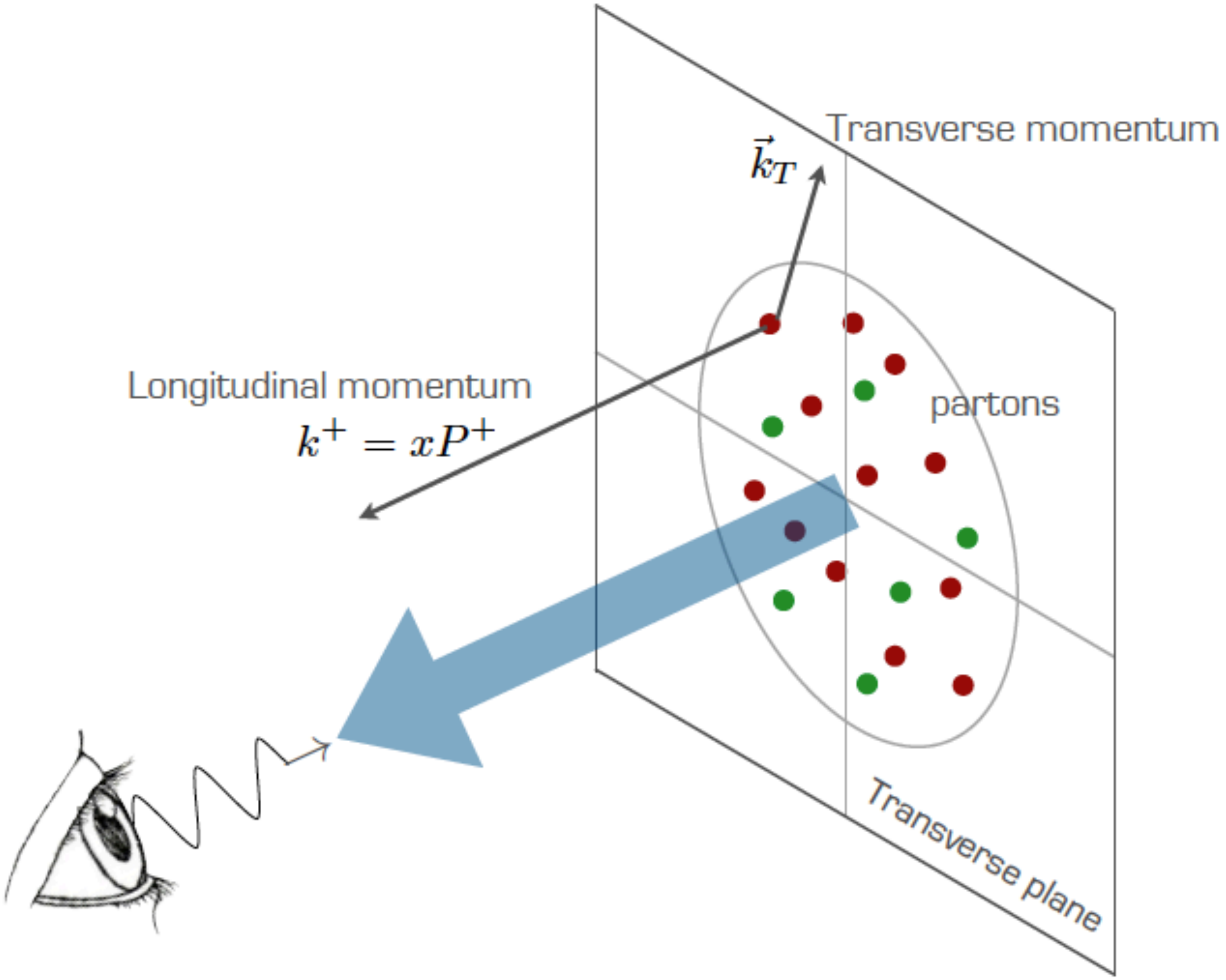
TMDs

References (intro and reviews) :

- "The 3D structure of the nucleon" **EPJ A (2016) 52**
- J.C. Collins "**Foundations of perturbative QCD**"
- material from the TMD collaboration **summer school**, e.g. :
  * P.J. Mulders' **lecture notes**
  * T. Rogers' **lecture notes**
  * A. Bacchetta's **lecture notes**
  * and all the other lecture notes/references on the webpage

quark TMD PDFs

$$\Phi_{ij}(k, P; S \quad) \sim \mathrm{F.T.}\ \langle PS \quad |\ \bar{\psi}_j(0)\ U_{[0,\xi]}\ \psi_i(\xi)\ |PS \quad \rangle_{|LF}$$



extraction of a **quark**
not collinear with the proton

quark TMD PDFs

$$\Phi_{ij}(k,P;S\quad)\sim \text{F.T.}\,\langle PS\quad|\,\bar{\psi}_j(0)\;U_{[0,\xi]}\;\psi_i(\xi)\;|PS\quad\rangle_{|LF}$$

$\mathbf{Quarks}$	γ^+	$\gamma^+\gamma^5$	$i\sigma^{i+}\gamma^5$
U	$\boldsymbol{f_1}$		$h_1^\perp$
L		$\boldsymbol{g_1}$	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$\boldsymbol{h_1}$, $h_{1T}^\perp$

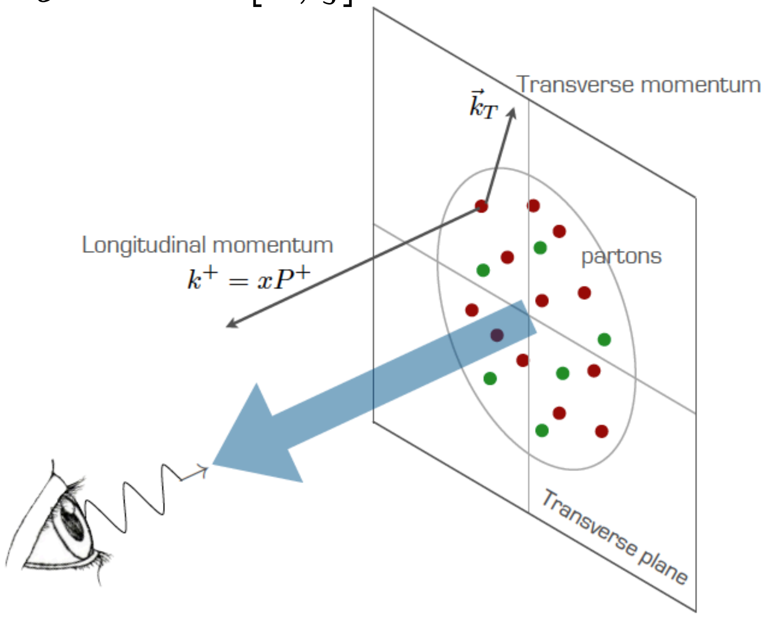
Sivers TMD PDF

unpolarized TMD PDF

similar table for **gluons** and for **fragmentation**

bold : also collinear

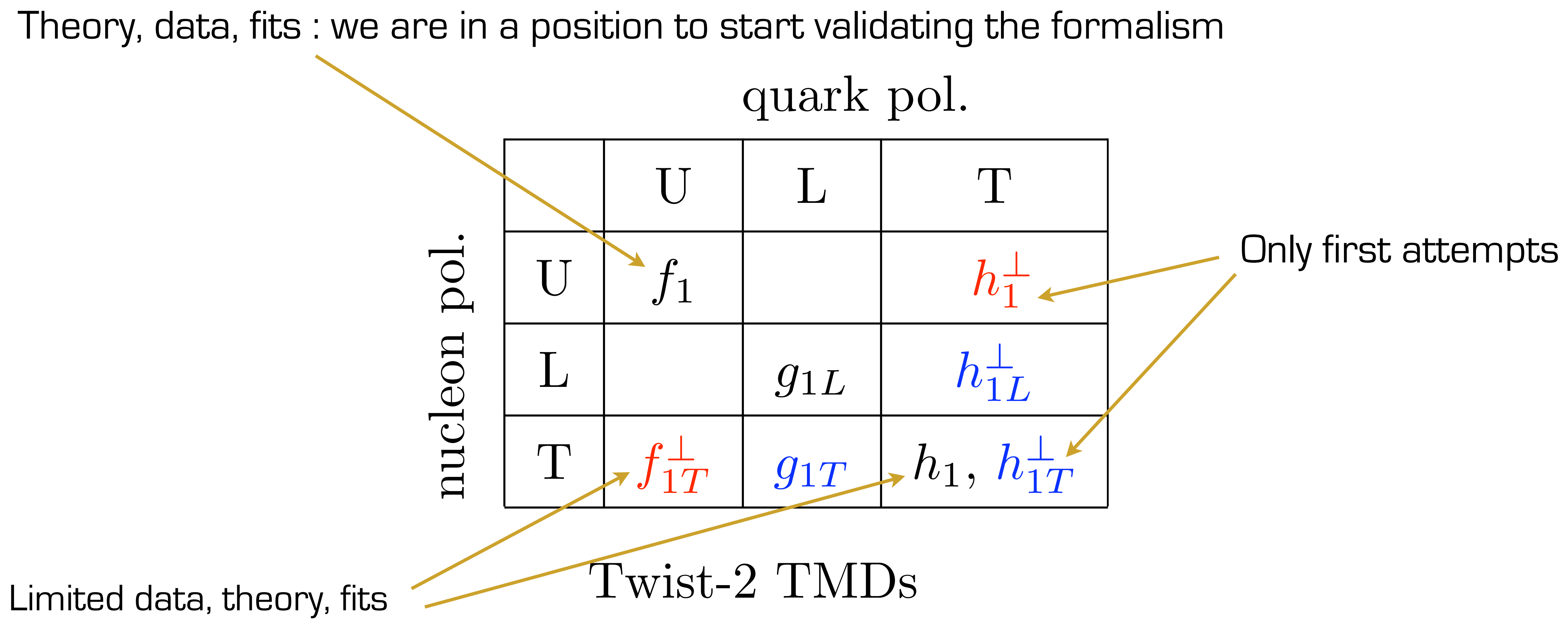
red : time-reversal odd (universality properties)



extraction of a **quark**
not collinear with the proton

encode all the possible
spin-spin and **spin-momentum**
correlations
between the proton
and its constituents

Status of TMD phenomenology



Theory, data, fits : we are in a position to start validating the formalism

quark pol.

nucleon pol.

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_1,\ h_{1T}^\perp$

Twist-2 TMDs

Only first attempts

Limited data, theory, fits

see, e.g, Bacchetta, Radici, arXiv:1107.5755
Anselmino, Boglione, Melis, PRD86 (12)
Echevarria, Idilbi, Kang, Vitev, PRD 89 (14)
Anselmino, Boglione, D'Alesio, Murgia, Prokudin, arXiv:1612.06413
Anselmino et al., PRD87 (13)
Kang et al. arXiv:1505.05589

Lu, Ma, Schmidt, arXiv:0912.2031
Lefky, Prokudin arXiv:1411.0580
Barone, Boglione, Gonzalez, Melis, arXiv:1502.04214

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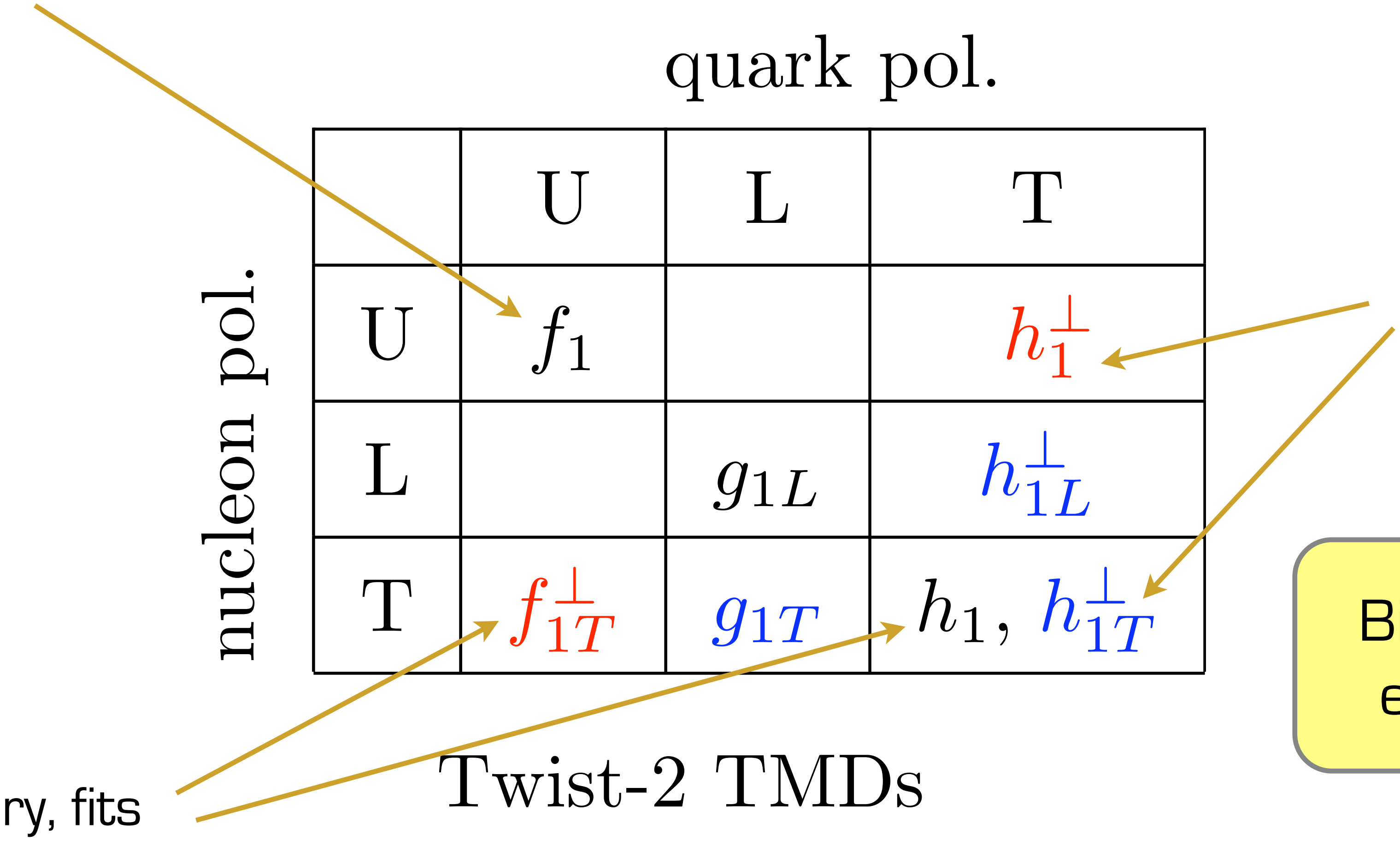

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T	$f_{1T}^\perp$	g_{1T}	$h_1,\ h_{1T}^\perp$

Twist-2 TMDs

Only first attempts

Big effort at JLab to explore all of them

Limited data, theory, fits

see, e.g, Bacchetta, Radici, arXiv:1107.5755
Anselmino, Boglione, Melis, PRD86 (12)
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TMD & collinear factorization

References:

- J.C. Collins "**Foundations of perturbative QCD**"
- SCET literature

Collinear and TMD factorization

Let’s consider a process with
three separate scales:

(SIDIS, Drell-Yan, e+e- to hadrons,
pp to quarkonium, ...)

hadronic mass scale

$$\boxed{\Lambda_{\mathrm{QCD}} \ll q_T \ll Q}$$

hard scale

(related to the)
transverse momentum of the observed particle

The ratios $\boxed{\Lambda_{\mathrm{QCD}}/Q}$ $\boxed{\Lambda_{\mathrm{QCD}}/q_T}$ $\boxed{q_T/Q}$

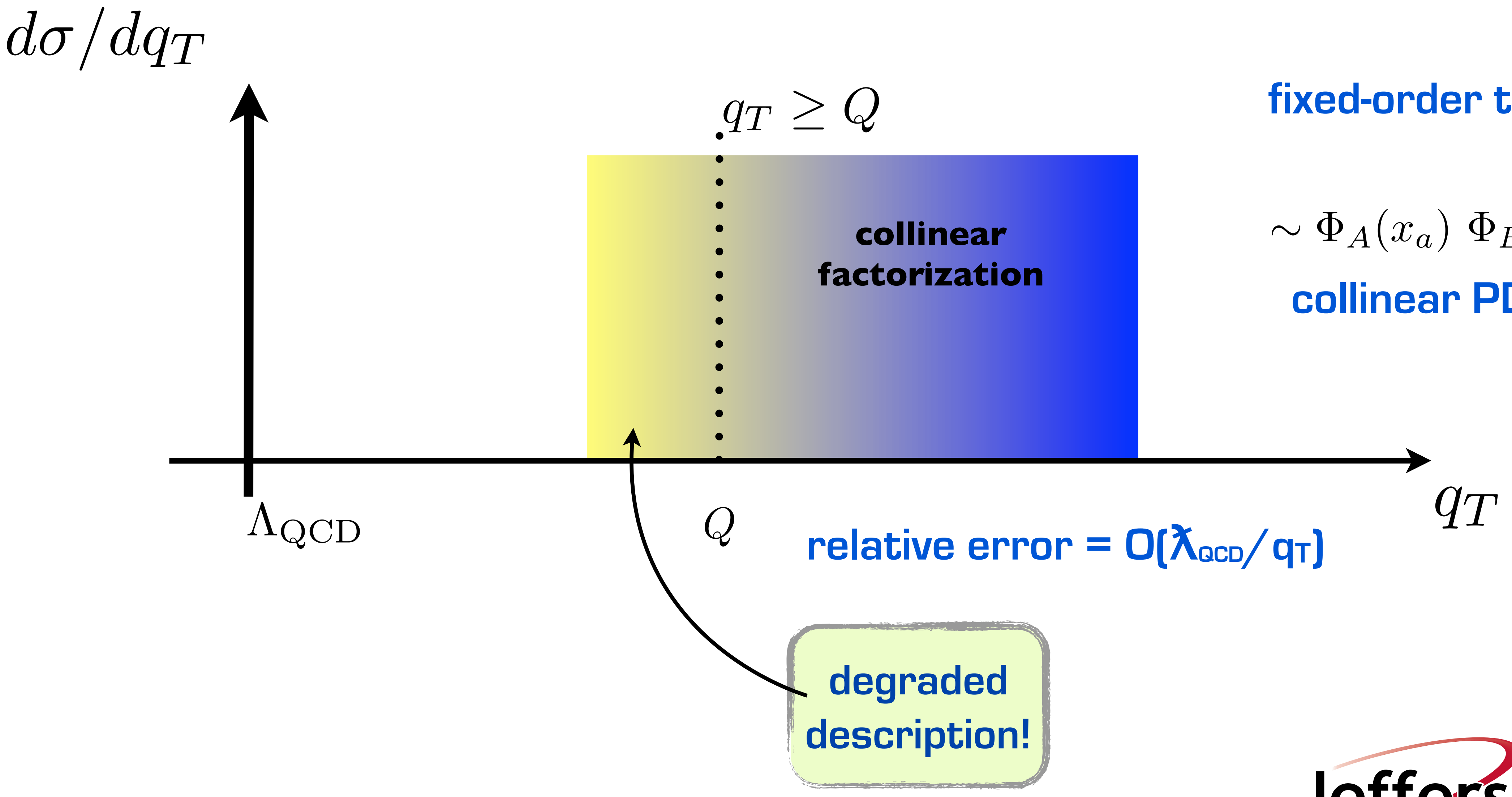
select the factorization theorem that we rely on.

According to their **values** we can access **different “projections”** of hadron structure

Collinear and TMD factorization

The key of phenomenology : emergence of TMD and collinear distributions from **factorization theorems**

fixed Q, variable q_T



fixed-order term

$\sim \Phi_A(x_a)\ \Phi_B(x_b)$

collinear PDFs

Collinear and TMD factorization

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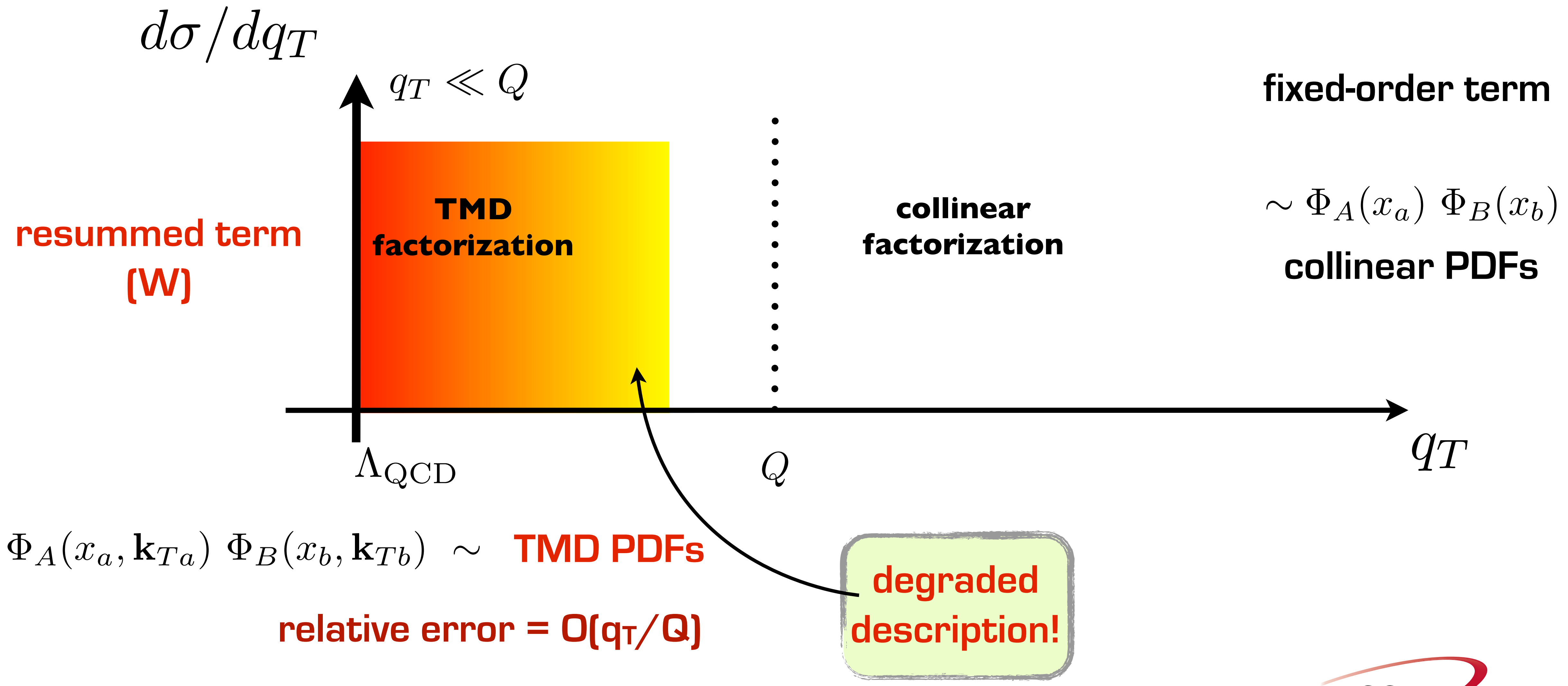
fixed Q, variable q_T



$d\sigma / dq_T$

$q_T \ll Q$

resummed term (W)

TMD factorization

collinear factorization

fixed-order term

$\sim \Phi_A(x_a)\ \Phi_B(x_b)$

collinear PDFs

$\Lambda_{\rm QCD}$ Q q_T

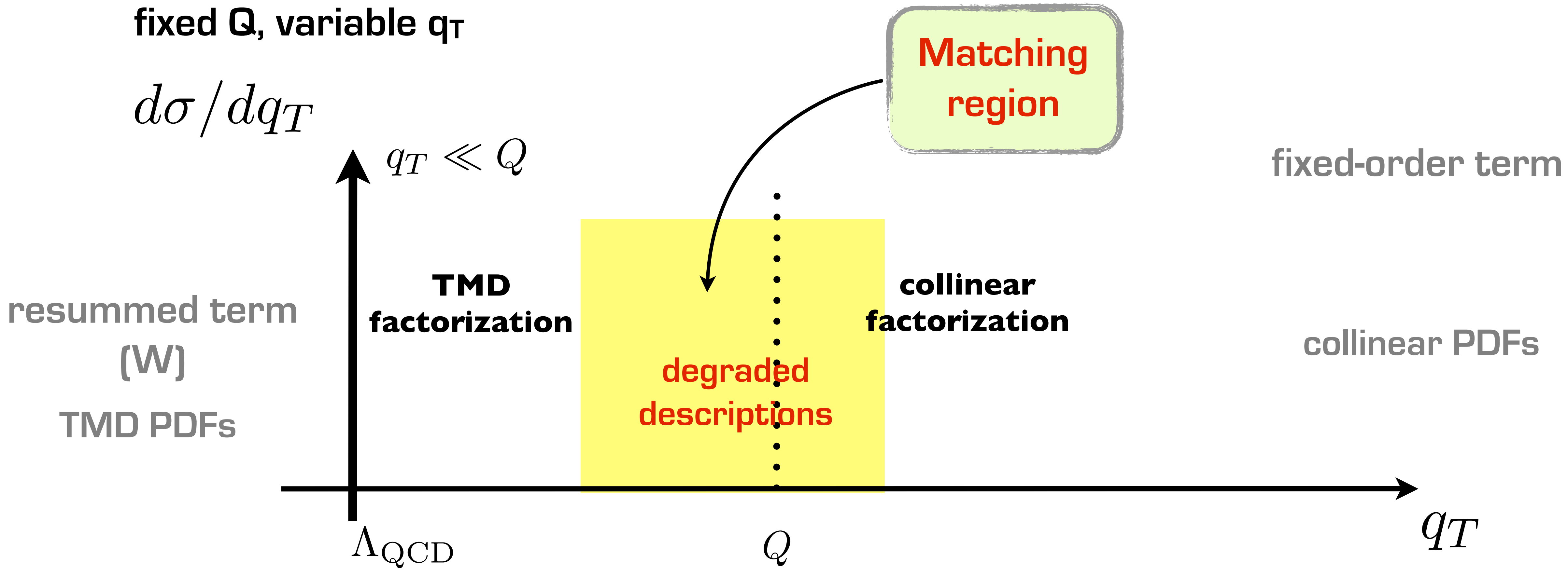
$\Phi_A(x_a, \mathbf{k}_{Ta})\ \Phi_B(x_b, \mathbf{k}_{Tb})\ \sim$ **TMD PDFs**

relative error = O(q_T/Q)

degraded description!

Collinear and TMD factorization

The key of phenomenology : emergence of TMD and collinear distributions from **factorization theorems**



fixed Q, variable q_T

$d\sigma/dq_T$

$q_T \ll Q$

Matching region

fixed-order term

TMD factorization

collinear factorization

resumed term (W)

TMD PDFs

collinear PDFs

degraded descriptions

$\Lambda_{\rm QCD}$

Q

q_T

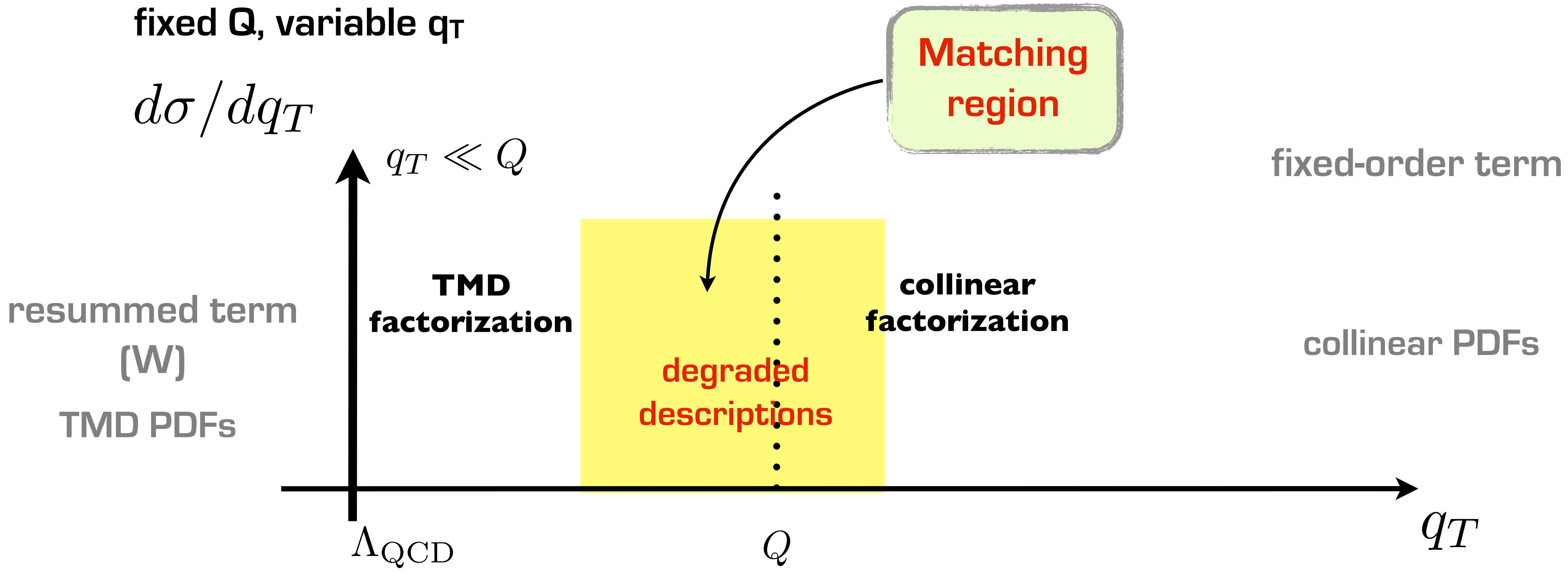
W, relative error = $O(q_T/Q)$

F.O., relative error = $O(\lambda_{\rm QCD}/q_T)$

We need a prescription to deal with the region where both descriptions are not good

Collinear and TMD factorization

The key of phenomenology : emergence of TMD and collinear distributions from **factorization theorems**



fixed Q, variable q_T

$d\sigma/dq_T$

$q_T \ll Q$

Matching region

fixed-order term

resummed term (W)

TMD PDFs

TMD factorization

degraded descriptions

collinear factorization

collinear PDFs

Λ_{QCD}

Q

q_T

The extraction of the **nonperturbative part of TMDs** is affected by the **description of the whole q_T range**

→ Crucial, especially at **low Q** (e.g. JLab kinematics), where the **regions shrink**

→ polarization ?

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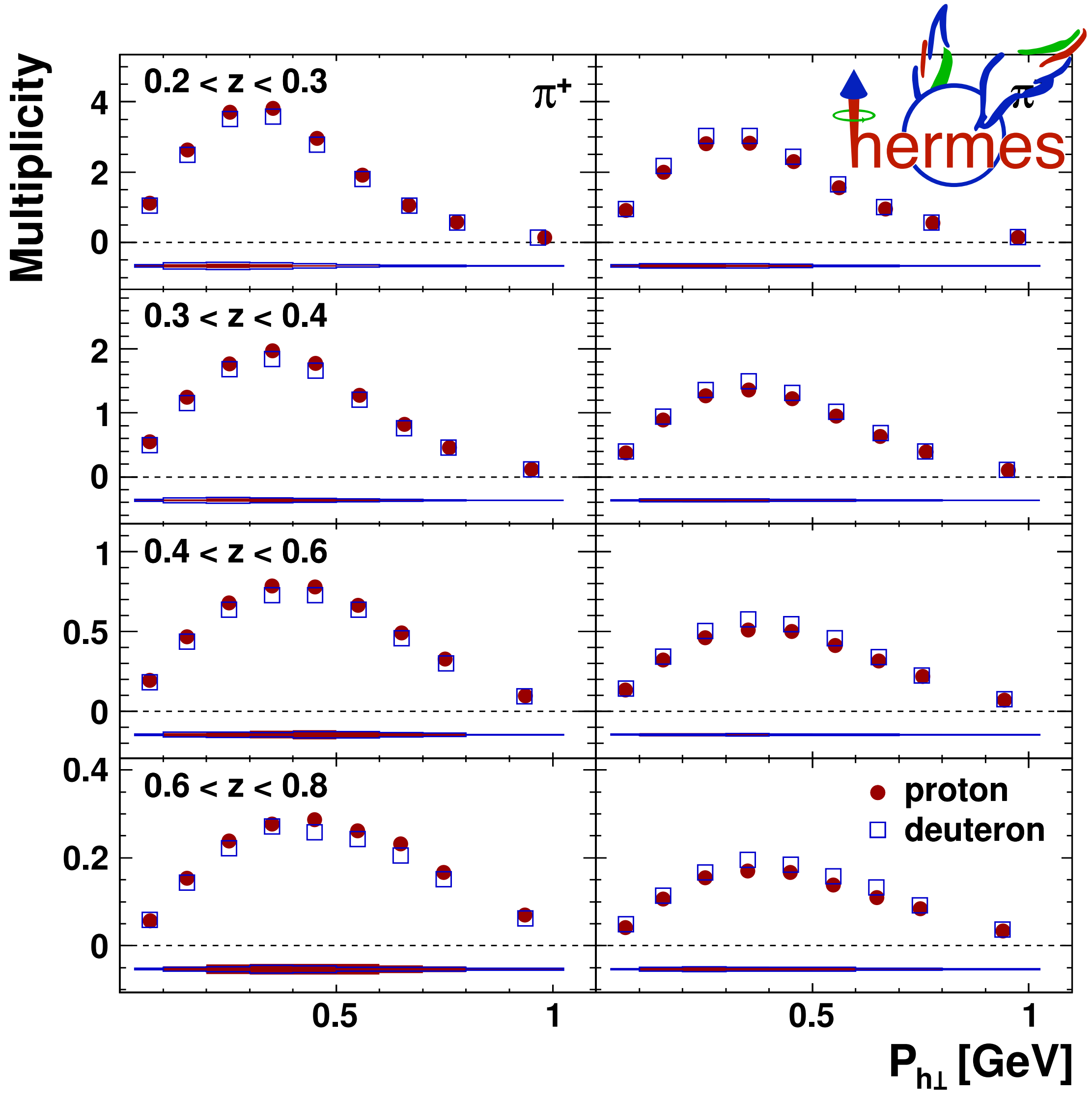
Need of TMD evolution

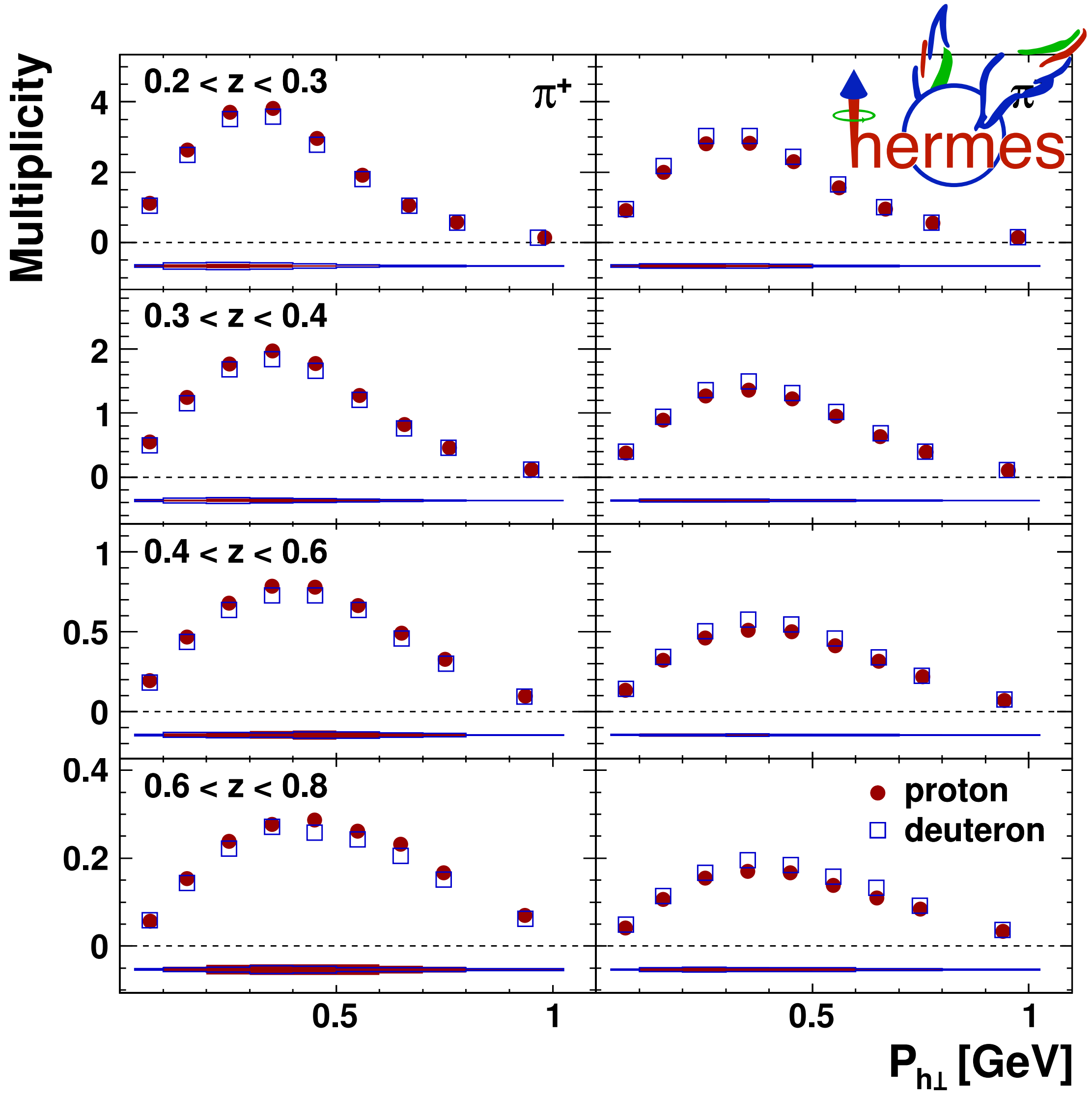
HERMES, $Q \approx 1.5$ GeV



Airapetian et al., PRD87 (2013)

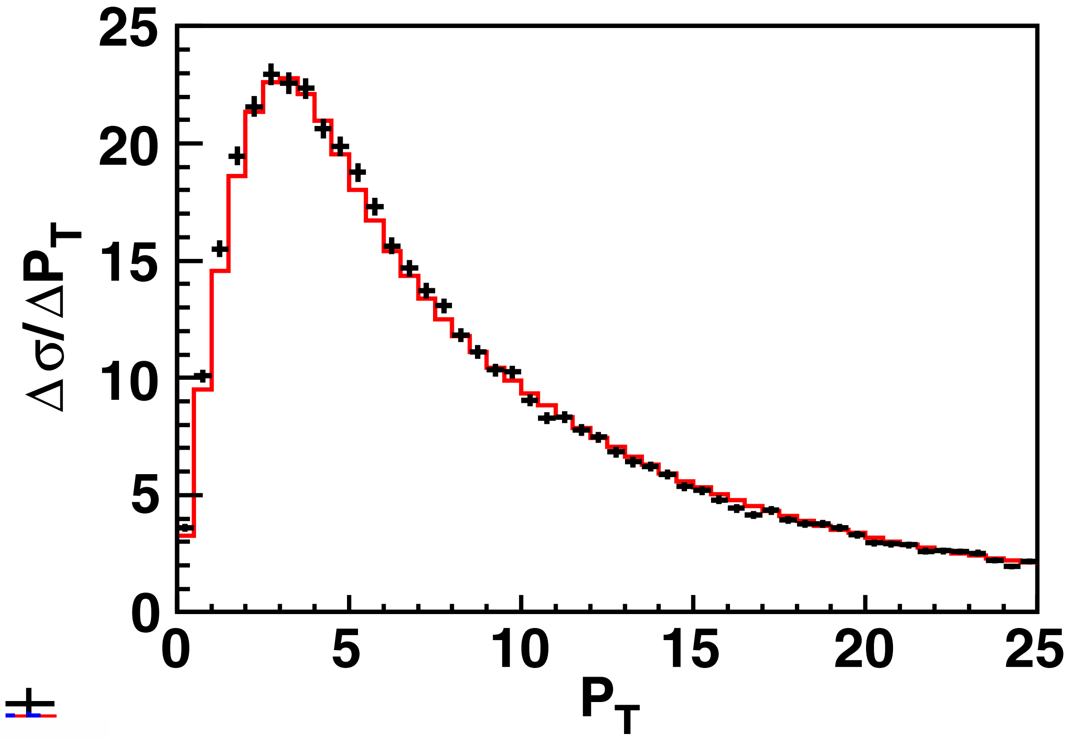
Need of TMD evolution

HERMES, Q ≈ 1.5 GeV



Airapetian et al., PRD87 (2013)

CDF, Q ≈ 91 GeV

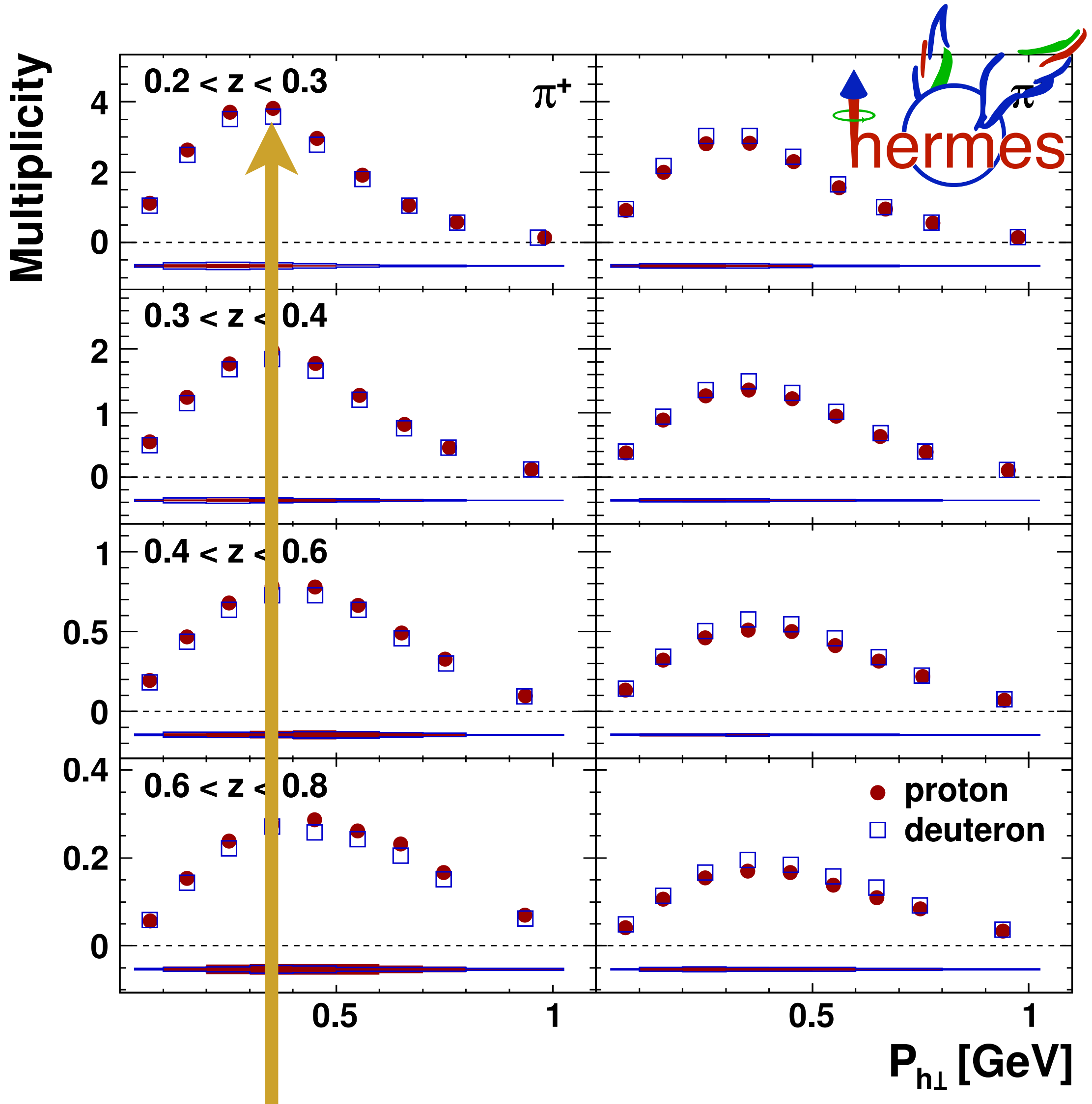


Aaltonen et al., PRD86 (2012)

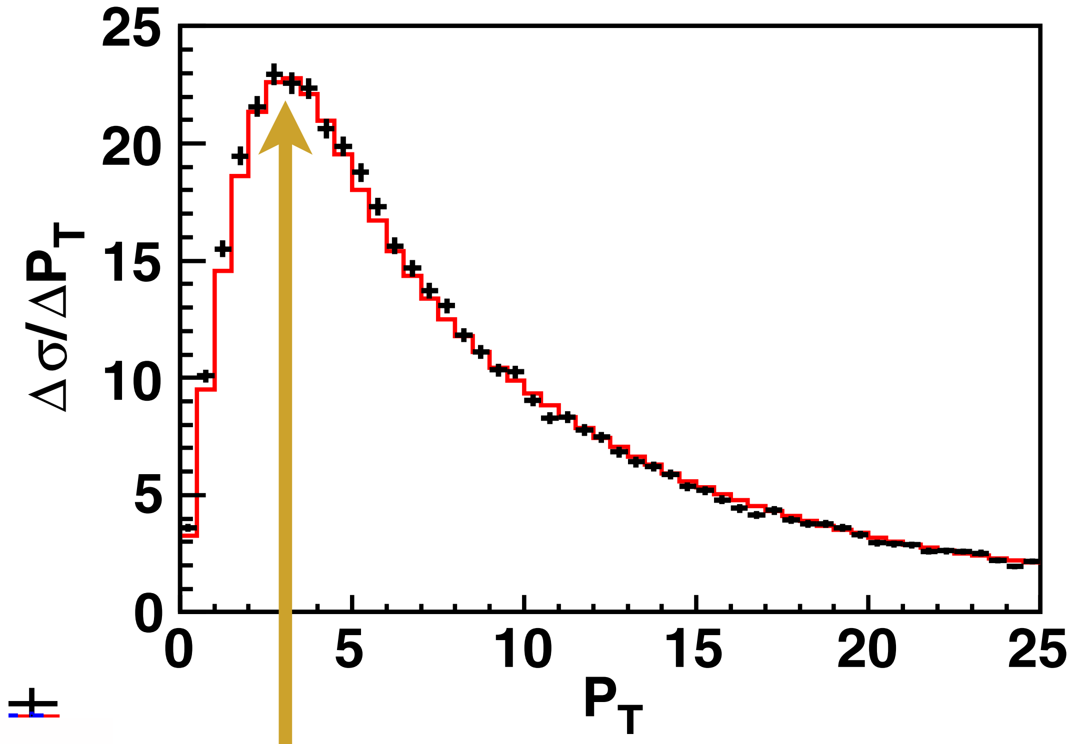
Need of TMD evolution

HERMES, Q ≈ 1.5 GeV

CDF, Q ≈ 91 GeV



Airapetian et al., PRD87 (2013)



Aaltonen et al., PRD86 (2012)

Width of TMDs changes of one order of magnitude:
we can we explain this with TMD evolution

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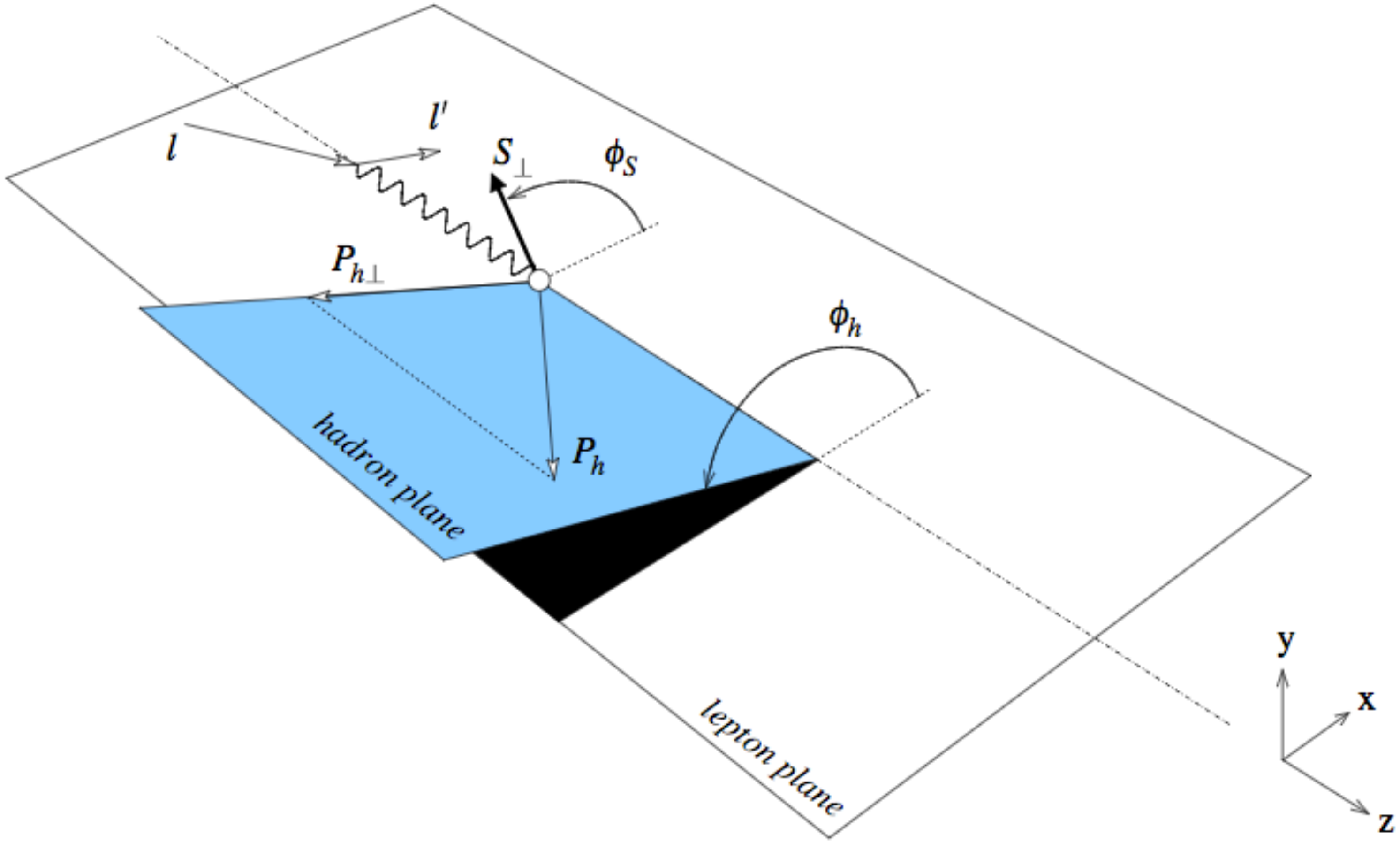

TMDs in SIDIS

Some references:

- Bacchetta, Diehl, Goeke, Metz, Mulders, Schlegel **JHEP 0702 (2007) 093**
- Bacchetta, Boer, Diehl, Mulders **JHEP 0808 (2008) 023**
- Boglione, Collins, Gamberg, Gonzalez, Rogers, Sato **Phys.Lett. B766 (2017) 245-253**
- ...

Structure functions

$\ell P \to \ell' h X$

$$
\begin{aligned}
&\frac{d\sigma}{dx\,dy\,d\psi\,dz\,d\phi_h\,dP_{h\perp}^2} = \\
&\quad \frac{\alpha^2}{xyQ^2}\,\frac{y^2}{2\,(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\Bigg\{F_{UU,T}+\varepsilon F_{UU,L}+\sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\,F_{UU}^{\cos\phi_h} \\
&\qquad +\varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h}+\lambda_e\,\sqrt{2\,\varepsilon(1-\varepsilon)}\,\sin\phi_h\,F_{LU}^{\sin\phi_h} \\
&\qquad +S_{\|}\left[\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_h\,F_{UL}^{\sin\phi_h}+\varepsilon\sin(2\phi_h)\,F_{UL}^{\sin 2\phi_h}\right] \\
&\quad +S_{\|}\lambda_e\left[\sqrt{1-\varepsilon^2}\,F_{LL}+\sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_h\,F_{LL}^{\cos\phi_h}\right] \\
&\quad +|\boldsymbol{S}_\perp|\Bigg[\sin(\phi_h-\phi_S)\left(F_{UT,T}^{\sin(\phi_h-\phi_S)}+\varepsilon\,F_{UT,L}^{\sin(\phi_h-\phi_S)}\right) \\
&\qquad +\varepsilon\,\sin(\phi_h+\phi_S)\,F_{UT}^{\sin(\phi_h+\phi_S)}+\varepsilon\,\sin(3\phi_h-\phi_S)\,F_{UT}^{\sin(3\phi_h-\phi_S)} \\
&\qquad +\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_S\,F_{UT}^{\sin\phi_S}+\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin(2\phi_h-\phi_S)\,F_{UT}^{\sin(2\phi_h-\phi_S)}\Bigg] \\
&\quad +|\boldsymbol{S}_\perp|\lambda_e\Bigg[\sqrt{1-\varepsilon^2}\,\cos(\phi_h-\phi_S)\,F_{LT}^{\cos(\phi_h-\phi_S)}+\sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_S\,F_{LT}^{\cos\phi_S} \\
&\qquad +\sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos(2\phi_h-\phi_S)\,F_{LT}^{\cos(2\phi_h-\phi_S)}\Bigg]\Bigg\}, \qquad (2.7)
\end{aligned}
$$

Structure functions

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$$
\begin{aligned}
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&\quad+\varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h}+\lambda_e\,\sqrt{2\,\varepsilon(1-\varepsilon)}\,\sin\phi_h\,F_{LU}^{\sin\phi_h} \\
&\quad+S_\parallel\left[\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_h\,F_{UL}^{\sin\phi_h}+\varepsilon\sin(2\phi_h)\,F_{UL}^{\sin 2\phi_h}\right] \\
&\quad+S_\parallel\lambda_e\left[\sqrt{1-\varepsilon^2}\,F_{LL}+\sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_h\,F_{LL}^{\cos\phi_h}\right] \\
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&\qquad+\varepsilon\,\sin(\phi_h+\phi_S)\,F_{UT}^{\sin(\phi_h+\phi_S)}+\varepsilon\,\sin(3\phi_h-\phi_S)\,F_{UT}^{\sin(3\phi_h-\phi_S)} \\
&\qquad+\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_S\,F_{UT}^{\sin\phi_S}+\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin(2\phi_h-\phi_S)\,F_{UT}^{\sin(2\phi_h-\phi_S)}\Bigg] \\
&\quad+|\boldsymbol{S}_\perp|\lambda_e\Bigg[\sqrt{1-\varepsilon^2}\,\cos(\phi_h-\phi_S)\,F_{LT}^{\cos(\phi_h-\phi_S)}+\sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_S\,F_{LT}^{\cos\phi_S} \\
&\qquad+\sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos(2\phi_h-\phi_S)\,F_{LT}^{\cos(2\phi_h-\phi_S)}\Bigg]\Bigg\}, \qquad (2.7)
\end{aligned}
$$

SFs :

convolutions of

TMD PDFs and FFs!

quark pol.

nucleon pol.

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1, $h_{1T}^\perp$

Twist-2 TMDs

+ higher-twist contributions

Structure functions

$\ell P \to \ell' h X$

unpolarized

$$\frac{d\sigma}{dx\,dy\,d\psi\,dz\,d\phi_h\,dP_{h\perp}^2} =$$
$$\frac{\alpha^2}{xyQ^2}\,\frac{y^2}{2\,(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\Bigg\{\boxed{F_{UU,T}} + \varepsilon F_{UU,L} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\,F_{UU}^{\cos\phi_h}$$
$$+\,\varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h} + \lambda_e\,\sqrt{2\,\varepsilon(1-\varepsilon)}\,\sin\phi_h\,F_{LU}^{\sin\phi_h}$$
$$+\,S_{\|}\left[\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_h\,F_{UL}^{\sin\phi_h} + \varepsilon\sin(2\phi_h)\,F_{UL}^{\sin 2\phi_h}\right]$$
$$+\,S_{\|}\lambda_e\left[\sqrt{1-\varepsilon^2}\,F_{LL} + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_h\,F_{LL}^{\cos\phi_h}\right]$$
$$+\,|\boldsymbol{S}_\perp|\Bigg[\sin(\phi_h-\phi_S)\left(F_{UT,T}^{\sin(\phi_h-\phi_S)} + \varepsilon\,F_{UT,L}^{\sin(\phi_h-\phi_S)}\right)$$
$$+\,\varepsilon\,\sin(\phi_h+\phi_S)\,F_{UT}^{\sin(\phi_h+\phi_S)} + \varepsilon\,\sin(3\phi_h-\phi_S)\,F_{UT}^{\sin(3\phi_h-\phi_S)}$$
$$+\,\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_S\,F_{UT}^{\sin\phi_S} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin(2\phi_h-\phi_S)\,F_{UT}^{\sin(2\phi_h-\phi_S)}\Bigg]$$
$$+\,|\boldsymbol{S}_\perp|\lambda_e\Bigg[\sqrt{1-\varepsilon^2}\,\cos(\phi_h-\phi_S)\,F_{LT}^{\cos(\phi_h-\phi_S)} + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_S\,F_{LT}^{\cos\phi_S}$$
$$+\,\sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos(2\phi_h-\phi_S)\,F_{LT}^{\cos(2\phi_h-\phi_S)}\Bigg]\Bigg\}, \qquad (2.7)$$

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Twist-2 TMDs

+ higher-twist

contributions

Structure functions

$\ell P \to \ell' h X$

$$\frac{d\sigma}{dx\,dy\,d\psi\,dz\,d\phi_h\,dP_{h\perp}^2} =$$

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$$+\,\varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h}+\lambda_e\,\sqrt{2\,\varepsilon(1-\varepsilon)}\,\sin\phi_h\,\overset{\text{e(x) - twist 3}}{F_{LU}^{\sin\phi_h}}$$

$$+\,S_\parallel\left[\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_h\,F_{UL}^{\sin\phi_h}+\varepsilon\sin(2\phi_h)\,F_{UL}^{\sin 2\phi_h}\right]$$

$$+\,S_\parallel\lambda_e\left[\sqrt{1-\varepsilon^2}\,F_{LL}+\sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_h\,F_{LL}^{\cos\phi_h}\right]$$

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$$+\,\varepsilon\,\sin(\phi_h+\phi_S)\,F_{UT}^{\sin(\phi_h+\phi_S)}+\varepsilon\,\sin(3\phi_h-\phi_S)\,F_{UT}^{\sin(3\phi_h-\phi_S)}$$

$$+\,\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_S\,F_{UT}^{\sin\phi_S}+\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin(2\phi_h-\phi_S)\,F_{UT}^{\sin(2\phi_h-\phi_S)}\Bigg]$$

$$+\,|\boldsymbol{S}_\perp|\lambda_e\Bigg[\sqrt{1-\varepsilon^2}\,\cos(\phi_h-\phi_S)\,F_{LT}^{\cos(\phi_h-\phi_S)}+\sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_S\,F_{LT}^{\cos\phi_S}$$

$$+\,\sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos(2\phi_h-\phi_S)\,F_{LT}^{\cos(2\phi_h-\phi_S)}\Bigg]\Bigg\}, \qquad (2.7)$$

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Twist-2 TMDs

+ higher-twist contributions

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\tag{2.7}
$$

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&\quad+S_{\|}\left[\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_h\,F_{UL}^{\sin\phi_h}+\varepsilon\sin(2\phi_h)\,F_{UL}^{\sin 2\phi_h}\right] \\
&\quad+S_{\|}\lambda_e\left[\sqrt{1-\varepsilon^2}\,F_{LL}+\sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_h\,F_{LL}^{\cos\phi_h}\right] \\
&\quad+|\boldsymbol{S}_\perp|\Bigg[\sin(\phi_h-\phi_S)\Big(\overset{\text{Sivers}}{F_{UT,T}^{\sin(\phi_h-\phi_S)}}+\varepsilon\,F_{UT,L}^{\sin(\phi_h-\phi_S)}\Big) \\
&\qquad+\varepsilon\,\sin(\phi_h+\phi_S)\,F_{UT}^{\sin(\phi_h+\phi_S)}+\varepsilon\,\sin(3\phi_h-\phi_S)\,F_{UT}^{\sin(3\phi_h-\phi_S)} \\
&\qquad+\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_S\,F_{UT}^{\sin\phi_S}+\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin(2\phi_h-\phi_S)\,F_{UT}^{\sin(2\phi_h-\phi_S)}\Bigg] \\
&\quad+|\boldsymbol{S}_\perp|\lambda_e\Bigg[\sqrt{1-\varepsilon^2}\,\cos(\phi_h-\phi_S)\,F_{LT}^{\cos(\phi_h-\phi_S)}+\sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_S\,F_{LT}^{\cos\phi_S} \\
&\qquad+\sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos(2\phi_h-\phi_S)\,F_{LT}^{\cos(2\phi_h-\phi_S)}\Bigg]\Bigg\}, \qquad (2.7)
\end{aligned}
$$

SFs :

convolutions of

TMD PDFs and FFs!

quark pol.

nucleon pol.	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1, $h_{1T}^\perp$

Twist-2 TMDs

+ higher-twist
contributions

Structure functions

$\ell P \to \ell' h X$

$$
\begin{aligned}
&\frac{d\sigma}{dx\,dy\,d\psi\,dz\,d\phi_h\,dP_{h\perp}^2} = \\
&\frac{\alpha^2}{xyQ^2}\,\frac{y^2}{2\,(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\Bigg\{F_{UU,T}+\varepsilon F_{UU,L}+\sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\,F_{UU}^{\cos\phi_h} \\
&\quad +\varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h}+\lambda_e\,\sqrt{2\,\varepsilon(1-\varepsilon)}\,\sin\phi_h\,F_{LU}^{\sin\phi_h} \\
&\quad +S_\parallel\left[\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_h\,F_{UL}^{\sin\phi_h}+\varepsilon\sin(2\phi_h)\,F_{UL}^{\sin 2\phi_h}\right] \\
&\quad +S_\parallel\lambda_e\left[\sqrt{1-\varepsilon^2}\,F_{LL}+\sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_h\,F_{LL}^{\cos\phi_h}\right] \\
&\quad +|\boldsymbol{S}_\perp|\Bigg[\sin(\phi_h-\phi_S)\left(F_{UT,T}^{\sin(\phi_h-\phi_S)}+\varepsilon\,F_{UT,L}^{\sin(\phi_h-\phi_S)}\right) \\
&\qquad +\varepsilon\,\sin(\phi_h+\phi_S)\,\overset{\text{transversity}}{F_{UT}^{\sin(\phi_h+\phi_S)}}+\varepsilon\,\sin(3\phi_h-\phi_S)\,F_{UT}^{\sin(3\phi_h-\phi_S)} \\
&\qquad +\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_S\,F_{UT}^{\sin\phi_S}+\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin(2\phi_h-\phi_S)\,F_{UT}^{\sin(2\phi_h-\phi_S)}\Bigg] \\
&\quad +|\boldsymbol{S}_\perp|\lambda_e\Bigg[\sqrt{1-\varepsilon^2}\,\cos(\phi_h-\phi_S)\,F_{LT}^{\cos(\phi_h-\phi_S)}+\sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_S\,F_{LT}^{\cos\phi_S} \\
&\qquad +\sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos(2\phi_h-\phi_S)\,F_{LT}^{\cos(2\phi_h-\phi_S)}\Bigg]\Bigg\}, \qquad (2.7)
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&\quad+\varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h}+\lambda_e\,\sqrt{2\,\varepsilon(1-\varepsilon)}\,\sin\phi_h\,F_{LU}^{\sin\phi_h} \\
&\quad+S_{\|}\left[\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_h\,F_{UL}^{\sin\phi_h}+\varepsilon\sin(2\phi_h)\,F_{UL}^{\sin 2\phi_h}\right] \\
&\quad+S_{\|}\lambda_e\left[\sqrt{1-\varepsilon^2}\,F_{LL}+\sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_h\,F_{LL}^{\cos\phi_h}\right] \\
&\quad+|\boldsymbol{S}_\perp|\left[\sin(\phi_h-\phi_S)\left(F_{UT,T}^{\sin(\phi_h-\phi_S)}+\varepsilon\,F_{UT,L}^{\sin(\phi_h-\phi_S)}\right)\right. \\
&\qquad+\varepsilon\,\sin(\phi_h+\phi_S)\,F_{UT}^{\sin(\phi_h+\phi_S)}+\varepsilon\,\sin(3\phi_h-\phi_S)\,\overset{\text{pretzelosity}}{F_{UT}^{\sin(3\phi_h-\phi_S)}} \\
&\qquad\left.+\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_S\,F_{UT}^{\sin\phi_S}+\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin(2\phi_h-\phi_S)\,F_{UT}^{\sin(2\phi_h-\phi_S)}\right] \\
&\quad+|\boldsymbol{S}_\perp|\lambda_e\left[\sqrt{1-\varepsilon^2}\,\cos(\phi_h-\phi_S)\,F_{LT}^{\cos(\phi_h-\phi_S)}+\sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_S\,F_{LT}^{\cos\phi_S}\right. \\
&\qquad\left.+\sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos(2\phi_h-\phi_S)\,F_{LT}^{\cos(2\phi_h-\phi_S)}\right]\Bigg\},
\end{aligned}
\qquad (2.7)
$$

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	T	$f_{1T}^\perp$	g_{1T}	h_1, $h_{1T}^\perp$

Twist-2 TMDs

+ higher-twist contributions

Some motivations

f_1

unpolarized TMD PDF:
- test of factorization formalism
- improve our description of qT spectra (e.g. at **W at LHC**)
- baseline to extract polarized TMDs from asymmetries

e

collinear twist 3 PDF e(x):
- insights in quark-gluon-quark correlations
- scalar charge of the nucleon
- nucleon sigma term ?

$h_1^\perp \;,\quad f_{1T}^\perp$

T-odd Boer-Mulders and Sivers TMD PDFs:
- rigorous tests of the symmetry properties of QCD
(sign change between SIDIS and Drell-Yan)

h_1

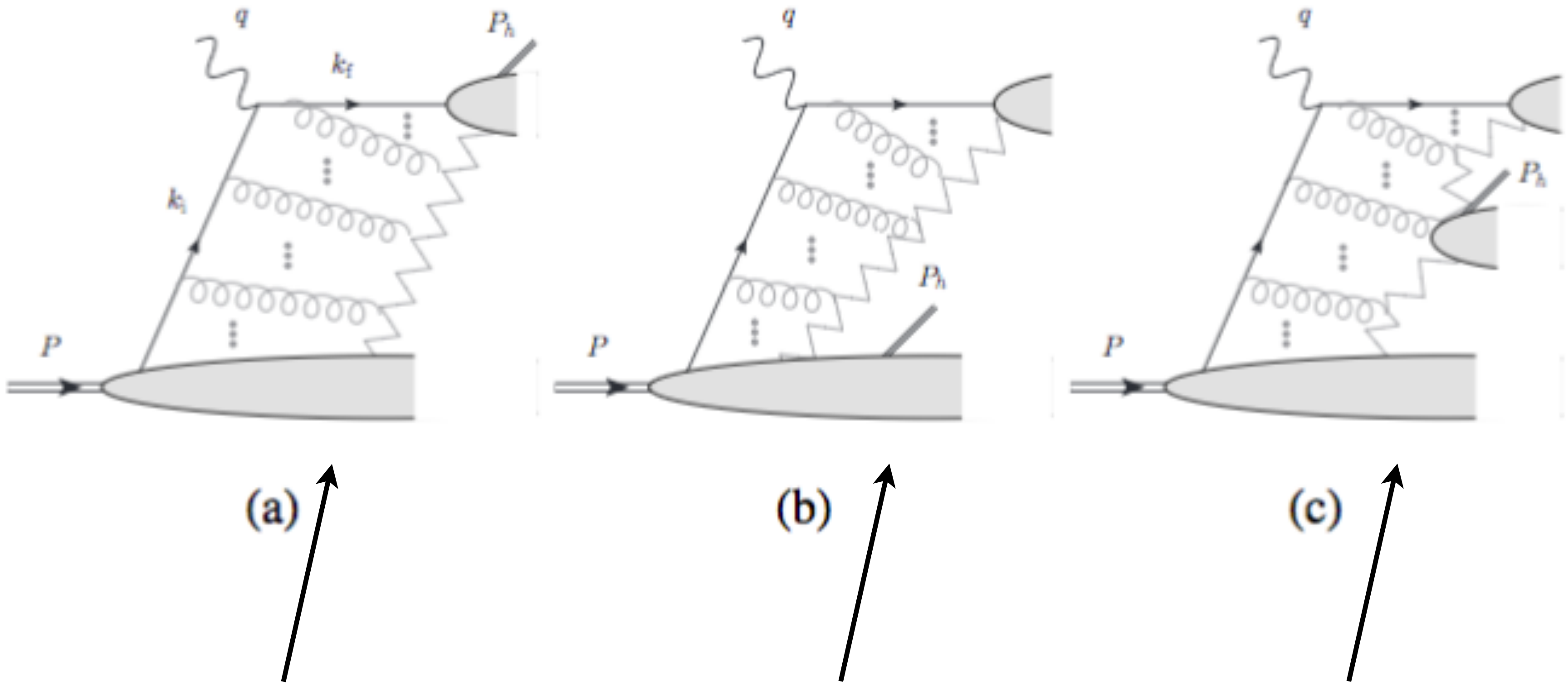
transversity (TMD) PDF:
- access to the tensor charge of the nucleon
- window on BSM physics
- also accessible in inclusive DIS ?

h_{1LT}

collinear (?) Bacchetta function:
- another rigorous test of QCD symmetries
- T-odd effects in **spin-1** hadrons

Target vs current vs central regions

Boglione, Collins, Gamberg, Gonzalez, Rogers, Sato **Phys.Lett. B766 (2017) 245-253**



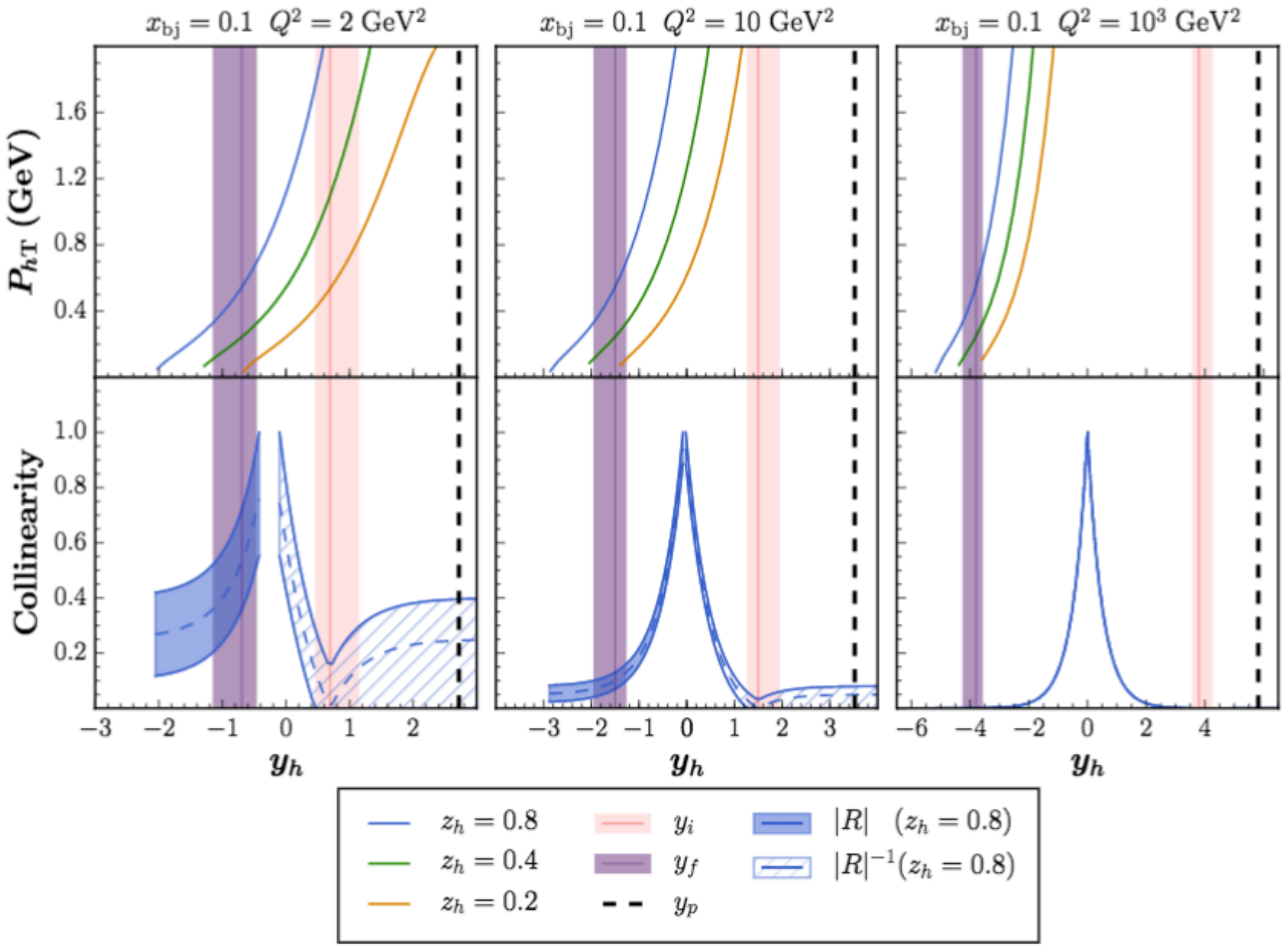
PDFs, TMDs

fracture
functions

?

Target vs current vs central regions

Boglione, Collins, Gamberg, Gonzalez, Rogers, Sato **Phys.Lett. B766 (2017) 245-253**



Extraction of quark unpolarized TMDs

References :

- “The 3D structure of the nucleon” **EPJ A (2016) 52**
- Bacchetta et al. **JHEP 1706 (2017) 081**
- A. Signori , **PhD thesis**
- Angelez-Martinez et al. **arXiv:1507.05267**
- EIC white paper, JLab 12 GeV white paper, …
- …

The frontier

Nucleon tomography in momentum space:
to understand how hadrons are built in terms of the elementary degrees of freedom of QCD

High-energy phenomenology:
to improve our understanding of high-energy scattering experiments and their potential to explore BSM physics

More open questions (phenomenology) :

1) what is the **functional form** of TMDs at low transverse momentum ?

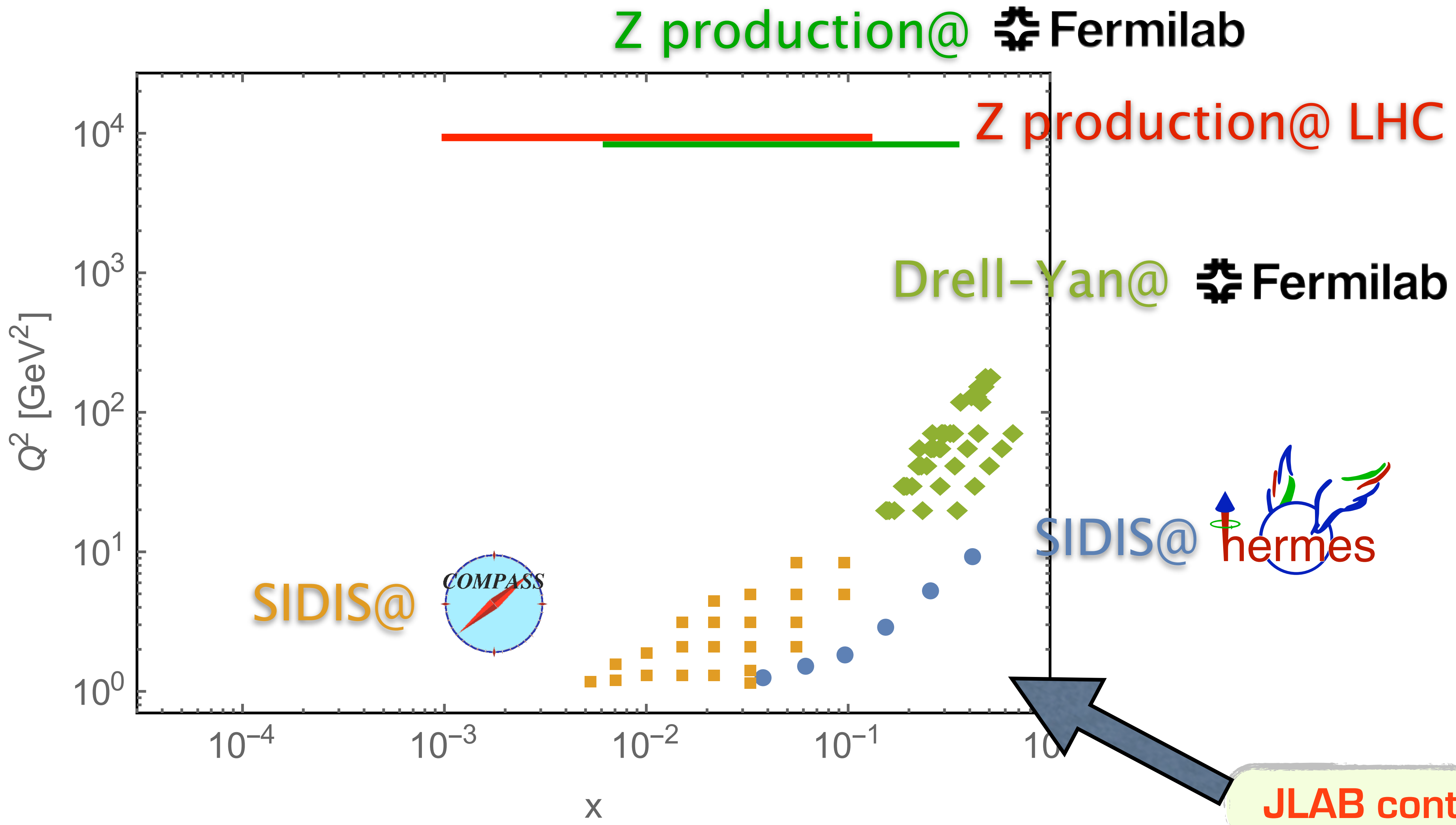
2) what is its **kinematic** and **flavor** dependence ?

3) can we attempt a global fit of TMDs ?

4) can we test the generalized **universality** of TMDs ?

5) what's the impact of hadron structure on the determination of Standard Model parameters ?

Experimental measurements

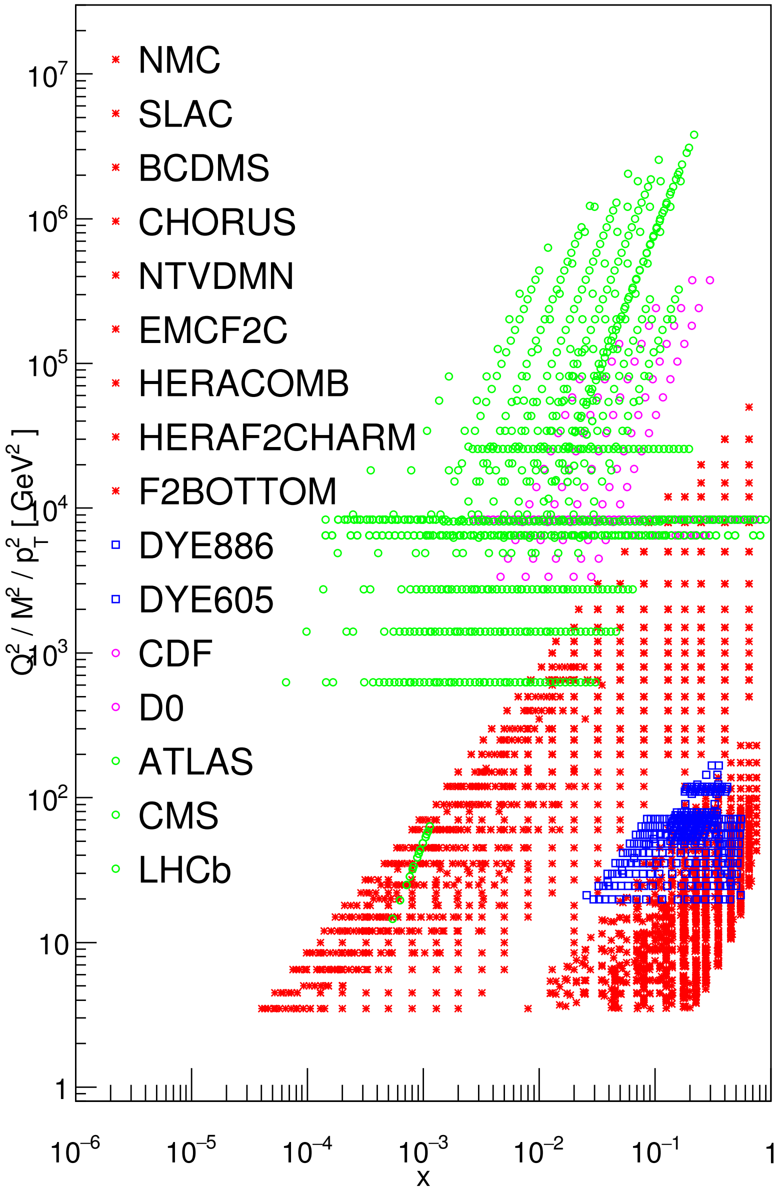


JLAB contribution!

Electron-positron annihilation data are still **missing**
(only some azimuthal asymmetries are available)

Comparison with collinear PDF fits

see talk by E. Nocera at POETIC2016

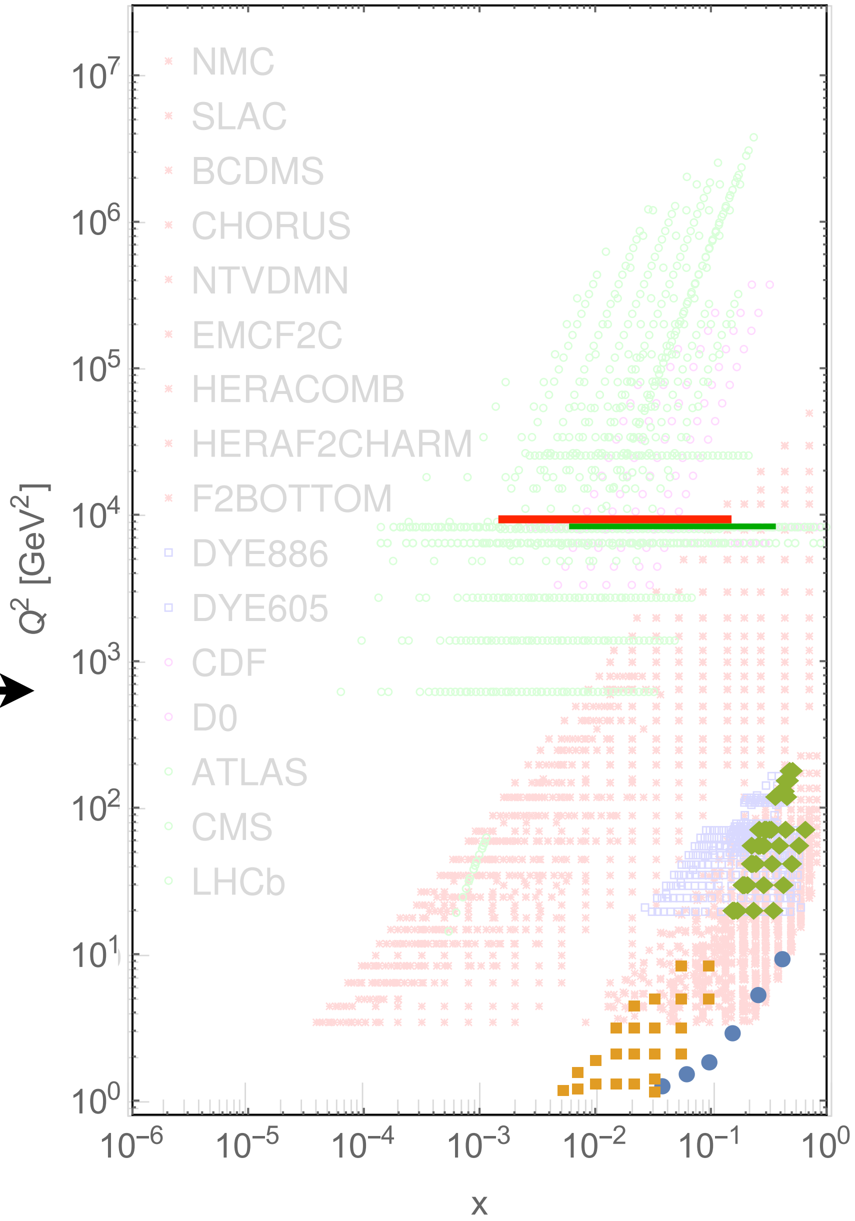


data sets available:

collinear PDFs

vs

TMD PDFs



What do we know ?

(only a selection of results!)

	Framework	HERMES	COMPASS	DY	Z production	N of points
KN 2006 hep-ph/0506225	LO-NLL	✘	✘	✔	✔	98
Pavia 2013 (+Amsterdam, Bilbao) arXiv:1309.3507	No evo (QPM)	✔	✘	✘	✘	1538
Torino 2014 (+JLab) arXiv:1312.6261	No evo (QPM)	✔ (separately)	✔ (separately)	✘	✘	576 (H) 6284 (C)
DEMS 2014 arXiv:1407.3311	NLO-NNLL	✘	✘	✔	✔	223
EIKV 2014 arXiv:1401.5078	LO-NLL	1 (x,Q^2) bin	1 (x,Q^2) bin	✔	✔	500 (?)
Pavia/JLab 2017 arXiv:1703.10157	LO-NLL	✔	✔	✔	✔	8059
SV 2017 arXiv:1706.01473	NNLO-NNLL	✘	✘	✔	✔	309

(courtesy A. Bacchetta)

What do we know ?

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	Framework	HERMES	COMPASS	DY	Z production	N of points
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Torino 2014 (+JLab) arXiv:1312.6261	No evo (QPM)	✓ (separately)	✓ (separately)	✗	✗	576 (H) 6284 (C)
DEMS 2014 arXiv:1407.3311	NLO-NNLL	✗	✗	✓	✓	223
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Pavia/JLab 2017 arXiv:1703.10157	LO-NLL	✓	✓	✓	✓	8059
SV 2017 arXiv:1706.01473	NNLO-NNLL	✗	✗	✓	✓	309

(courtesy A. Bacchetta)

Features

	Framework	HERMES	COMPASS	DY	Z production	N of points
Pavia/JLab 2017 arXiv:1703.10157	LO-NLL	✓	✓	✓	✓	8059

PROs

almost a **global fit** of
quark unpolarized TMDs

includes **TMD evolution**

replica (bootstrap)
fitting methodology

kinematic dependence
in intrinsic part of TMDs

intrinsic momentum: **beyond the Gaussian** assumption

CONs

no "pure" info on TMD FFs

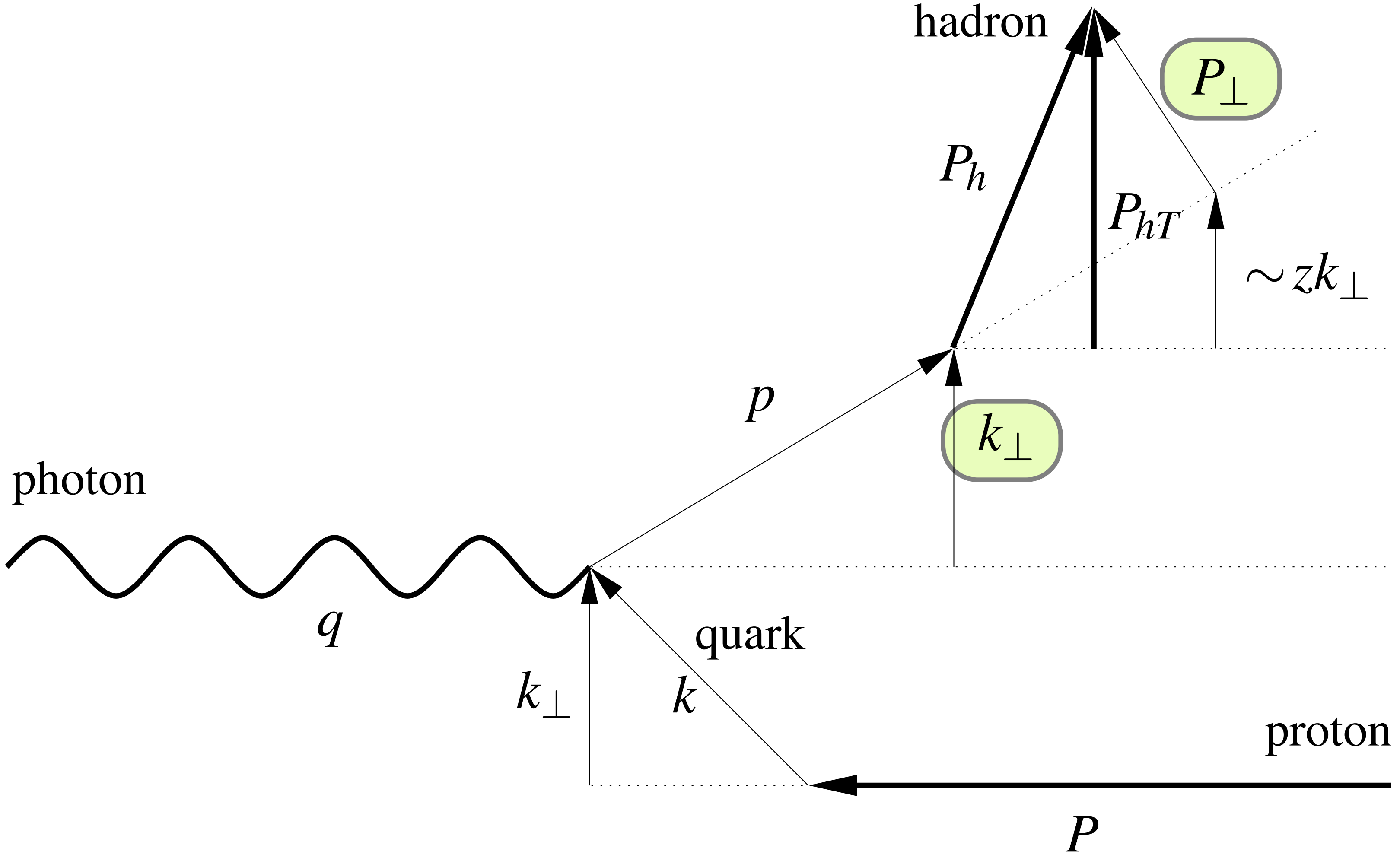
accuracy of TMD evolution :
not the state of the art

only "low" transverse momentum
(no fixed order and Y-term)

flavor separation in
the transverse
plane : problematic

Transverse momenta

SIDIS



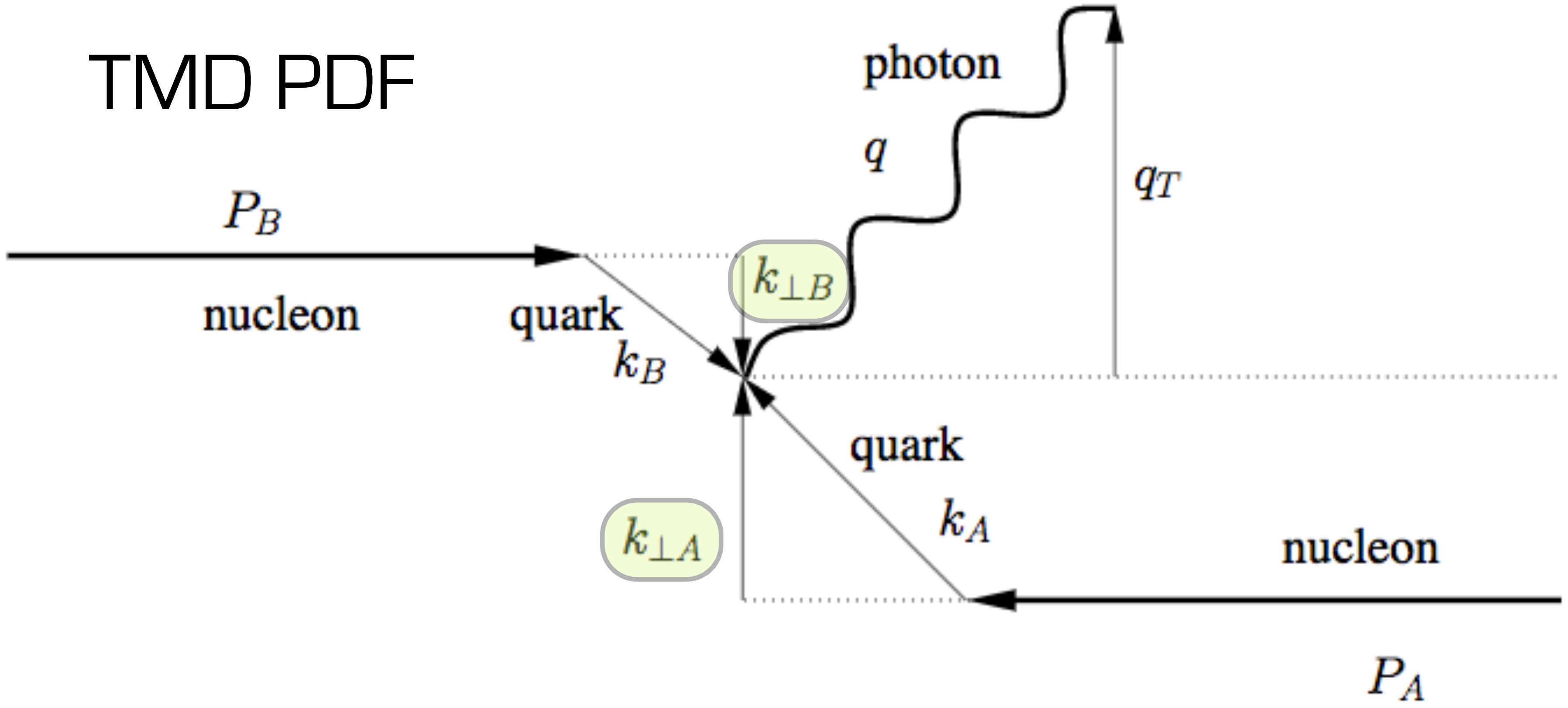
TMD FF

TMD PDF

Transverse momenta

DY

TMD PDF



TMD PDF

TMD PDFs at 1 GeV

$$\widetilde{f}^{a}_{1\mathrm{NP}}(x, b_T^2) = \frac{1}{2\pi} e^{-g_{1a} \frac{b_T^2}{4}} \left(1 - \frac{\lambda g_{1a}^2}{1 + \lambda g_{1a}} \frac{b_T^2}{4} \right)$$

$$f^{a}_{1\mathrm{NP}}(x, \boldsymbol{k}_\perp^2) = \frac{1}{\pi} \frac{\left(e^{-\frac{\boldsymbol{k}_\perp^2}{g_{1a}}} + \lambda \boldsymbol{k}_\perp^2 e^{-\frac{\boldsymbol{k}_\perp^2}{g_{1a}}} \right)}{g_{1a} + \lambda \; g_{1a}^2}$$

TMD PDFs at 1 GeV

$$\widetilde{f}^a_{1\mathrm{NP}}(x,b_T^2) = \frac{1}{2\pi} e^{-g_{1a}\frac{b_T^2}{4}}\left(1-\frac{\lambda g_{1a}^2}{1+\lambda g_{1a}}\frac{b_T^2}{4}\right)$$

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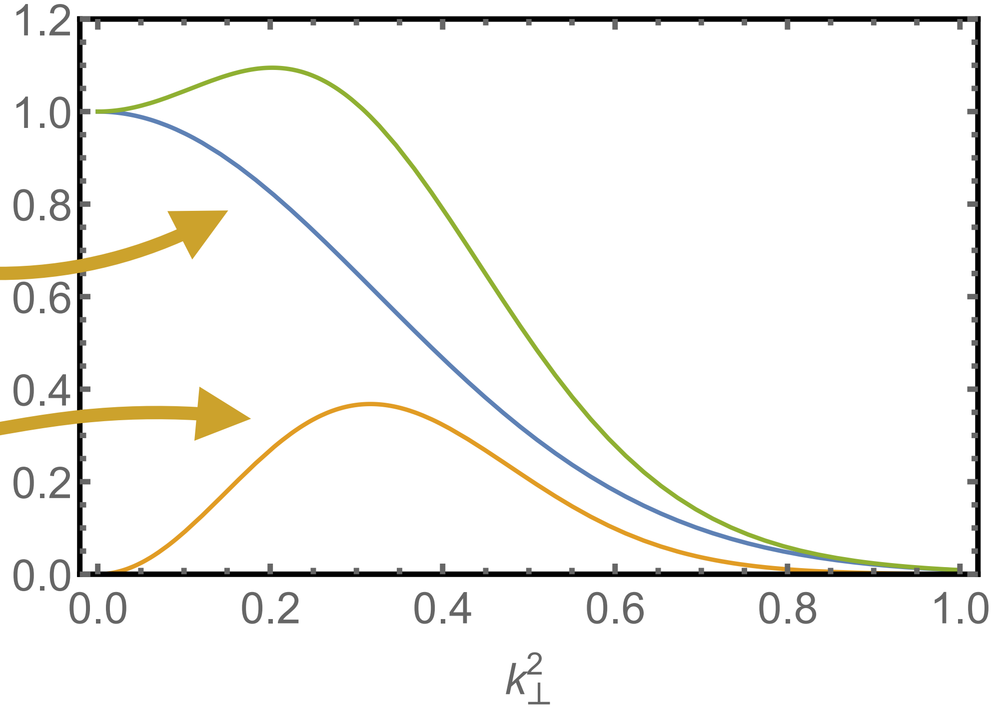

TMD PDFs at 1 GeV

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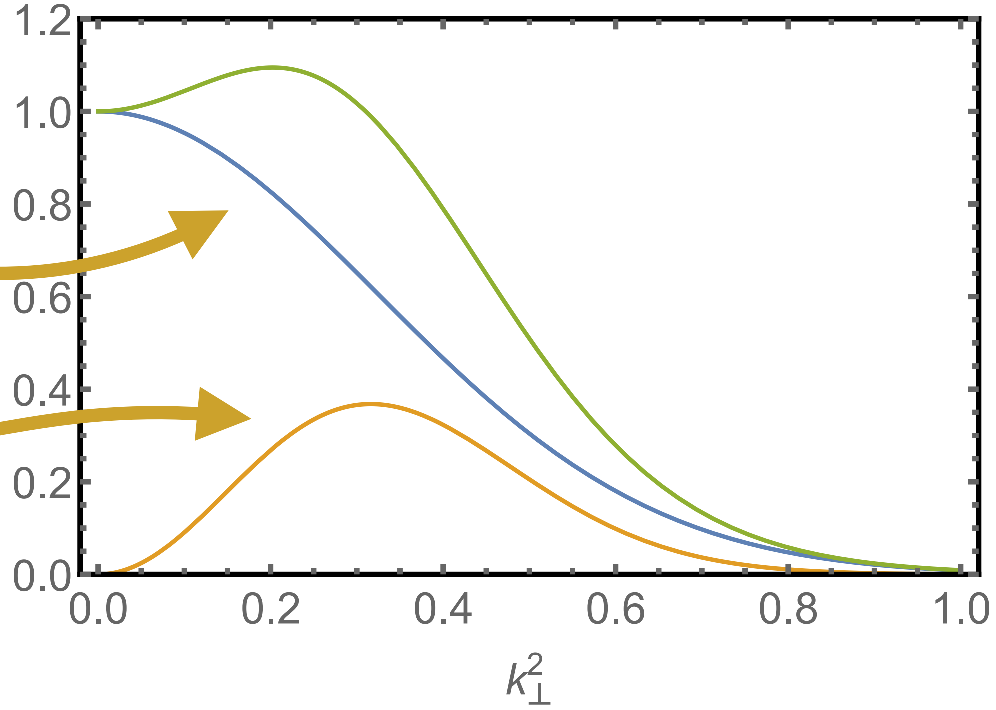

x-dependent width

$$g_1(x) = N_1 \frac{(1-x)^\alpha\ x^\sigma}{(1-\hat{x})^\alpha\ \hat{x}^\sigma}$$

TMD PDFs at 1 GeV

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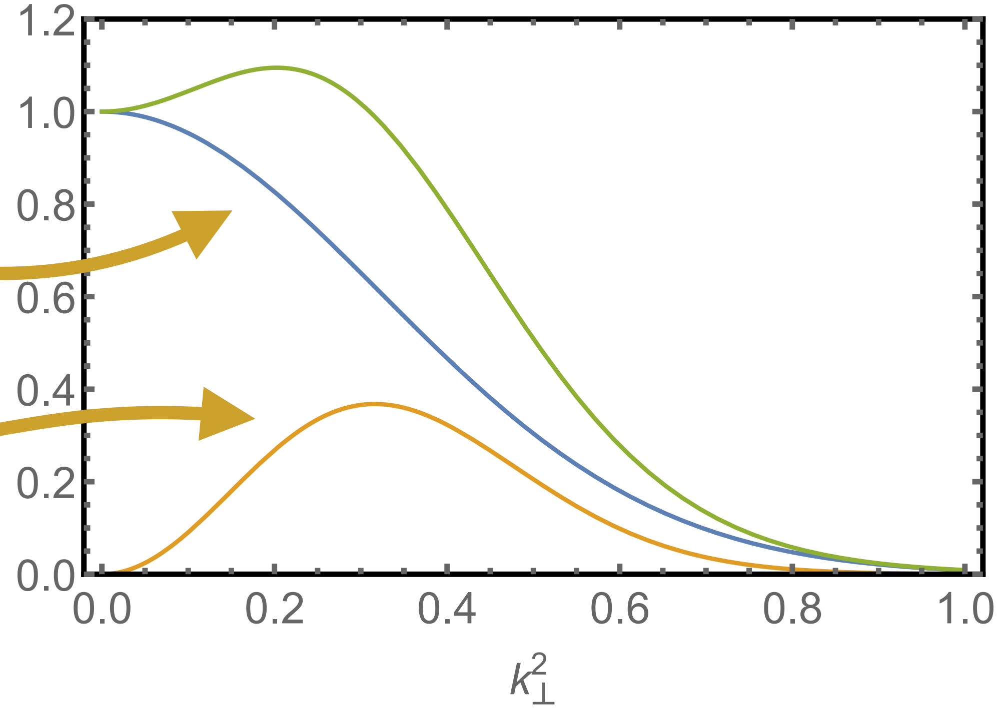


x-dependent width $$g_1(x) = N_1 \frac{(1-x)^\alpha\ x^\sigma}{(1-\hat{x})^\alpha\ \hat{x}^\sigma}$$

Fragmentation function is similar

Including TMD PDFs and FFs, in total: 11 free parameters

(4 for TMD PDFs, 6 for TMD FFs, 1 for TMD evolution)

Agreement data-theory

Flavor independent scenario

Flavor independent configuration | 11 parameters

Points	Parameters	χ^2	χ^2/d.o.f.
8059	11	12629 ± 363	1.55 ± 0.05

	HERMES $p \to \pi^+$	HERMES $p \to \pi^-$	HERMES $p \to K^+$	HERMES $p \to K^-$
Points	190	190	189	187
χ^2/points	4.83	2.47	0.91	0.82

Hermes kaons better than pions: larger uncertainties from FFs

	HERMES $D \to \pi^+$	HERMES $D \to \pi^-$	HERMES $D \to K^+$	HERMES $D \to K^-$	COMPASS $D \to h^+$	COMPASS $D \to h^-$
Points	190	190	189	189	3125	3127
χ^2/points	3.46	2.00	1.31	2.54	1.11	1.61

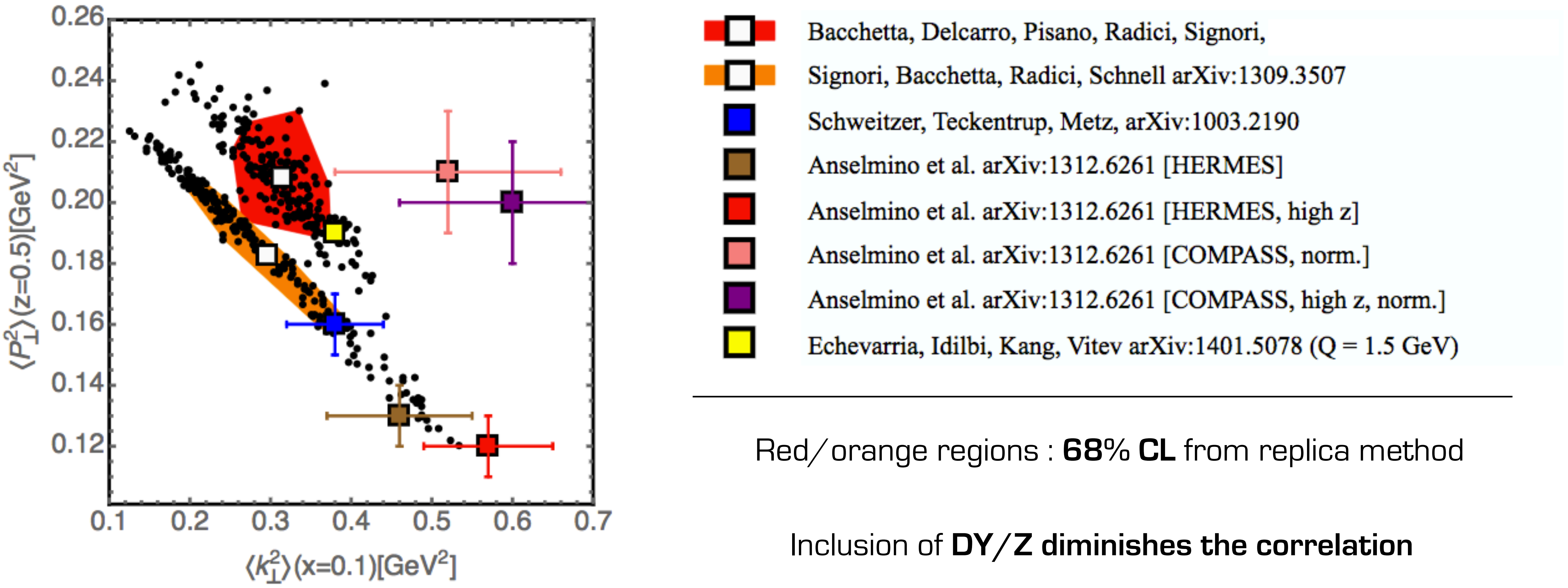
	E288 [200]	E288 [300]	E288 [400]	E605
Points	45	45	78	35
χ^2/points	0.99	0.84	0.32	1.12

Compass : better agreement due to #points and normalization

	CDF Run I	D0 Run I	CDF Run II	D0 Run II
Points	31	14	37	8
χ^2/points	1.36	1.11	2.00	1.73

Best-fit values

Flavor independent scenario



Red/orange regions : **68% CL** from replica method

Inclusion of **DY/Z diminishes the correlation**

Inclusion of **Compass** increases the $\langle P_\perp^2\rangle$ and reduces its spread

e+e- would further reduce the correlation

Caveat for comparisons :
NP effects (as the intrinsic momentum) always depend on the accuracy of the perturbative part ;

determined as observed - calculable

Polarized case

References :

- “The 3D structure of the nucleon” **EPJ A (2016) 52**
- STAR **arXiv:1511.06003**
- Compass: **arxiv:1704.00488**
- Accardi, Bacchetta arXiv:1706.02000
- …

Sivers: process dependence

Gauge invariance and T-reversal invariance generate a **sign change** between the **Sivers** TMD PDF in **Drell-Yan** and **Semi-Inclusive DIS**

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Collins, PLB 536 (02)

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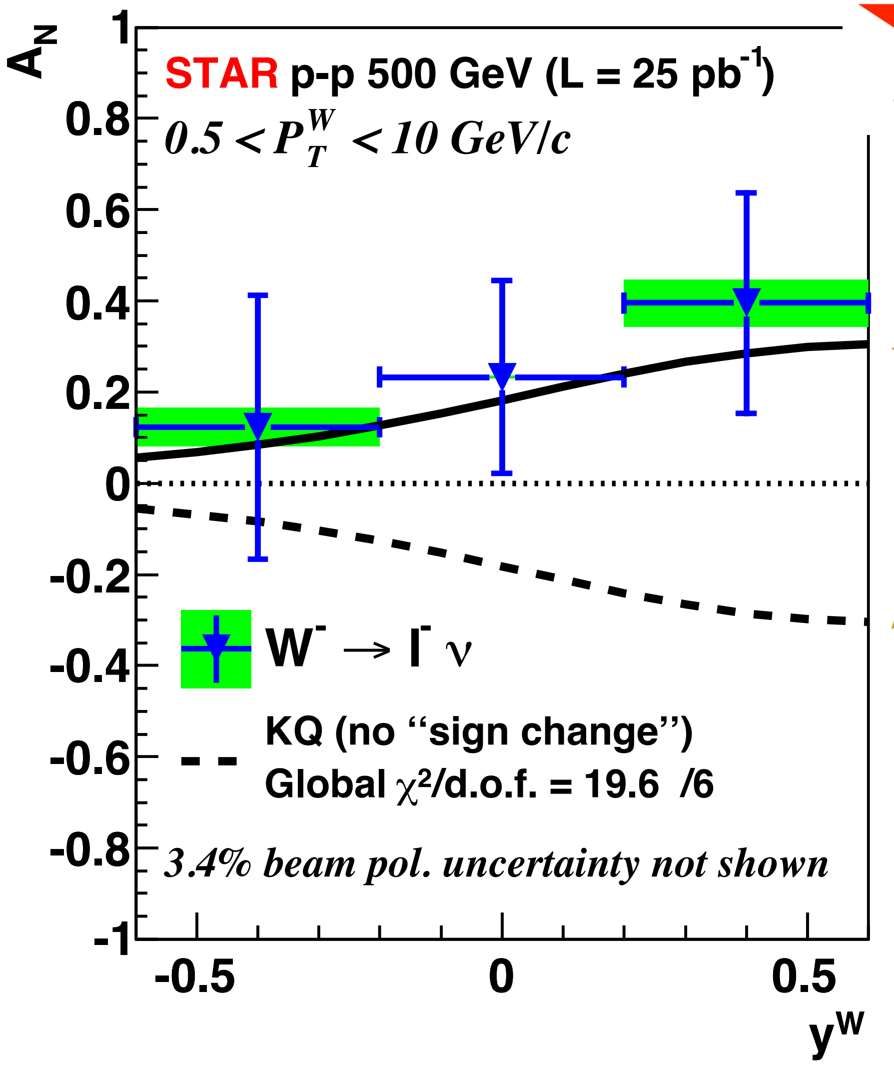
Collins, PLB 536 (02)



first evidence of sign change?

STAR Collab. arXiv:1511.06003

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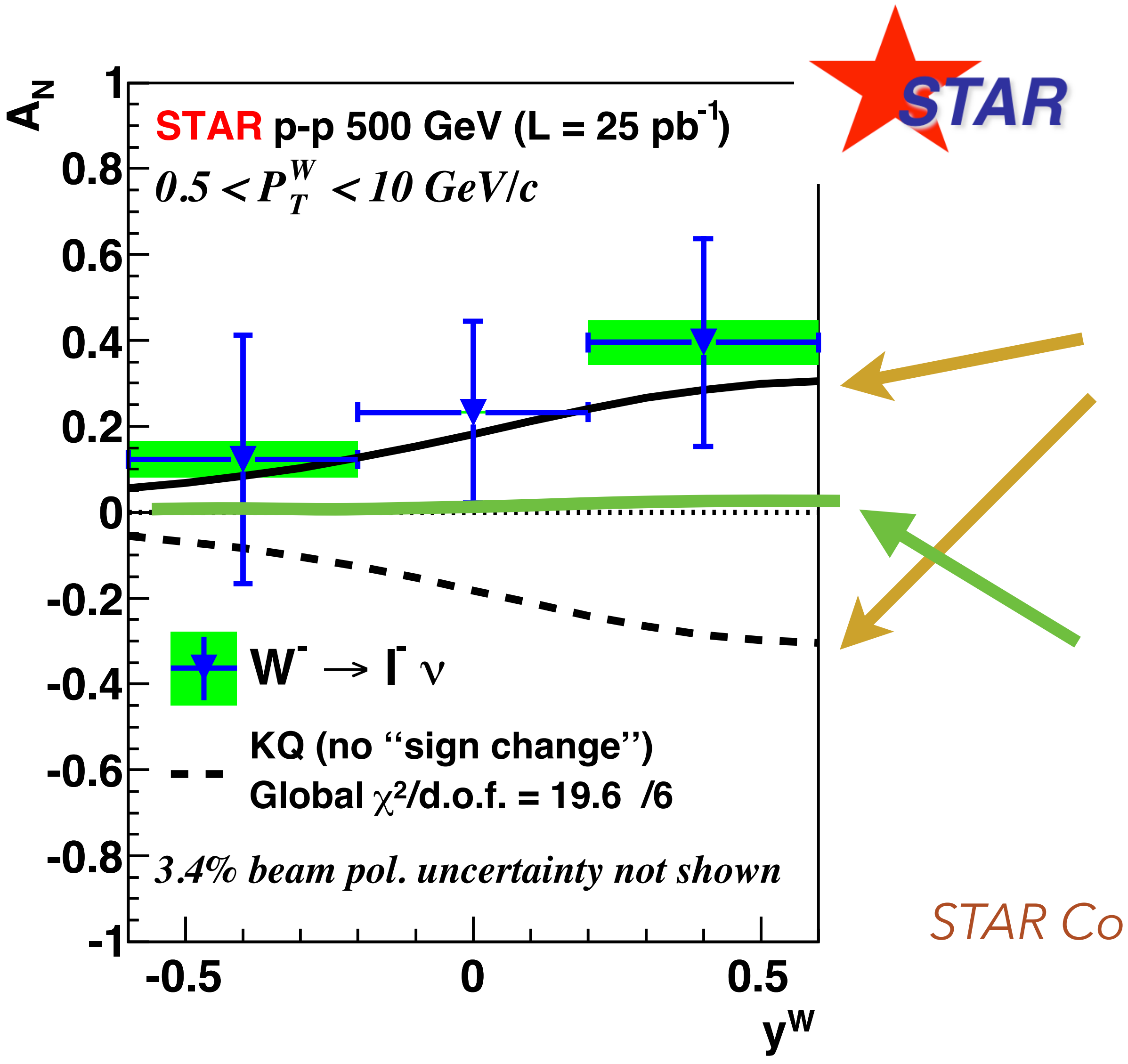


first evidence
of sign change?

prediction with TMD
evolution equations

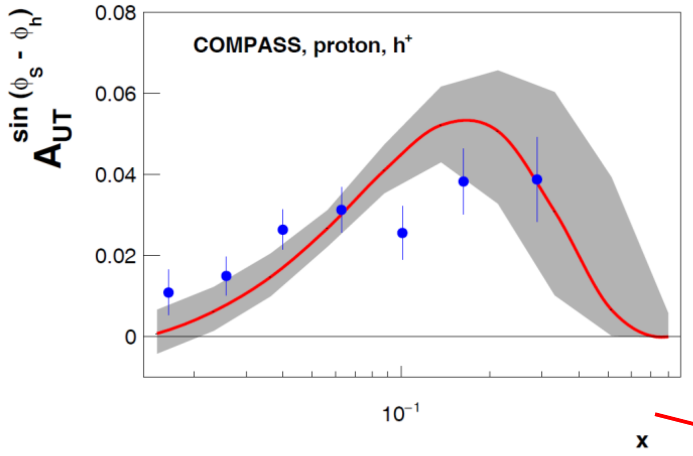
STAR Collab. arXiv:1511.06003

Jefferson Lab

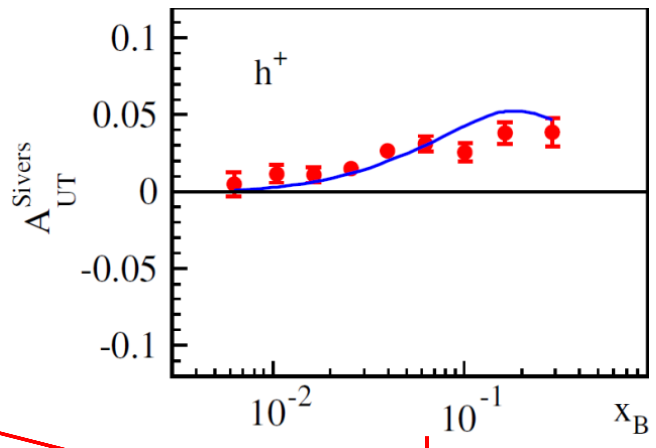
Sivers: process dependence

Sivers asymmetry in Semi-Inclusive DIS

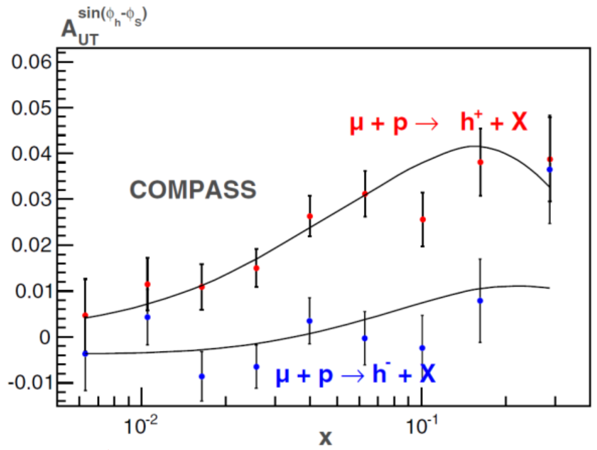
DGLAP (2016)
M. Anselmino et al., **arXiv:1612.06413**



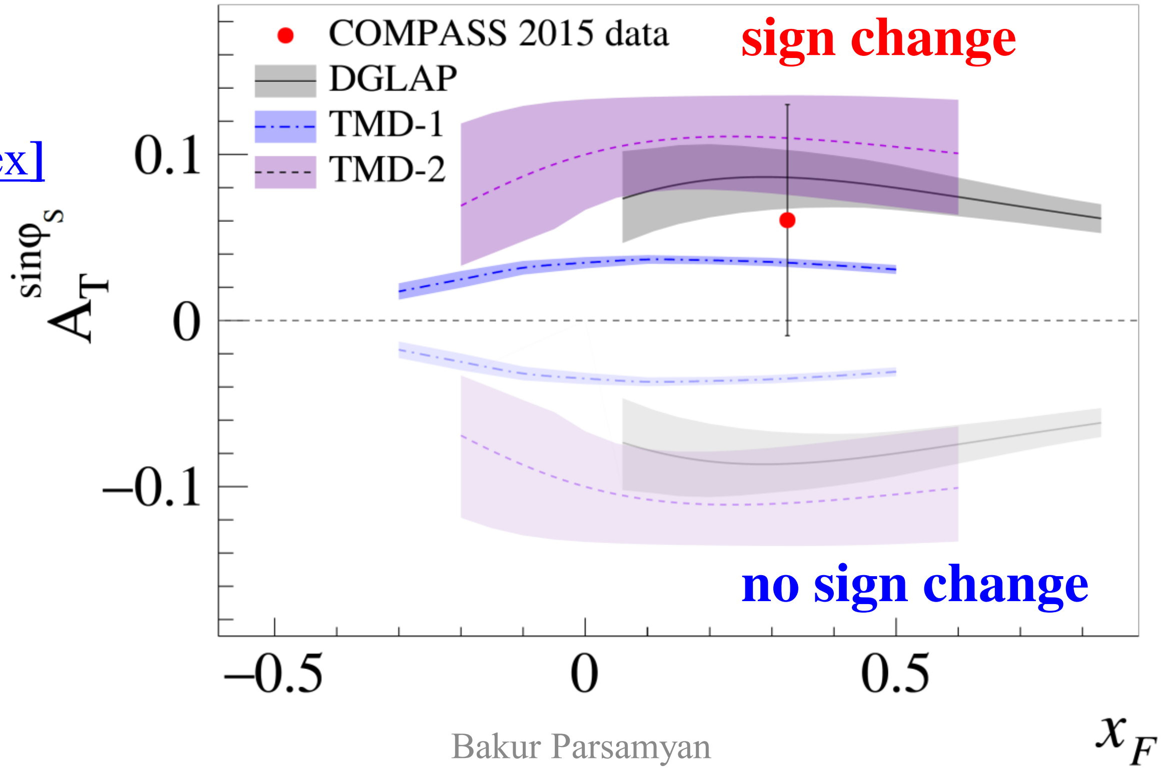
TMD-1 (2014)
M. G. Echevarria et al. **PRD89,074013**



TMD-2 (2013)
P. Sun, F. Yuan, **PRD88, 114012**



New! 03 April 2017
COMPASS
CERN-EP-2017-059
arXiv:1704.00488[hep-ex]



sign change

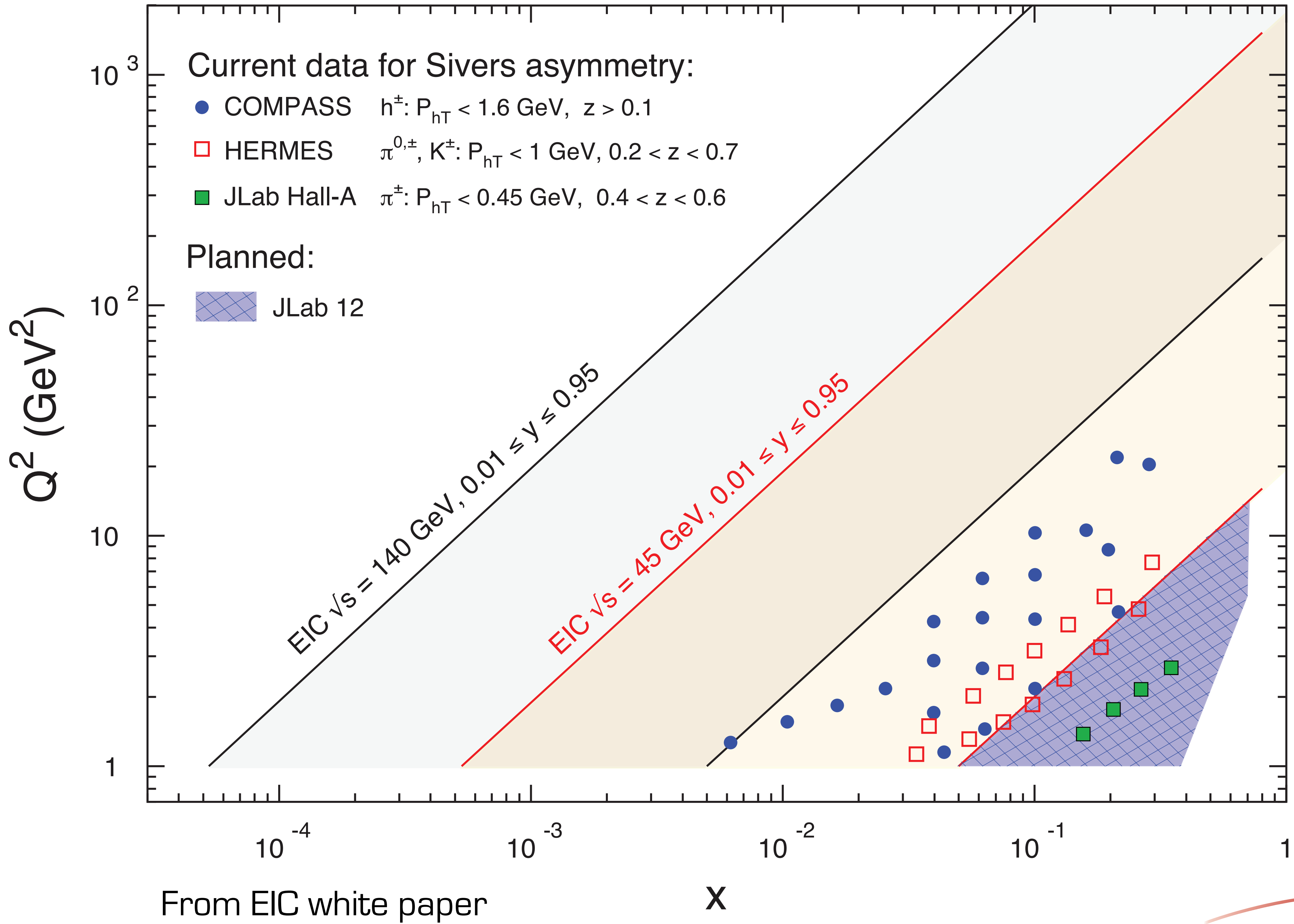
no sign change

Sivers asymmetry in Drell-Yan

courtesy B. Parsamyan

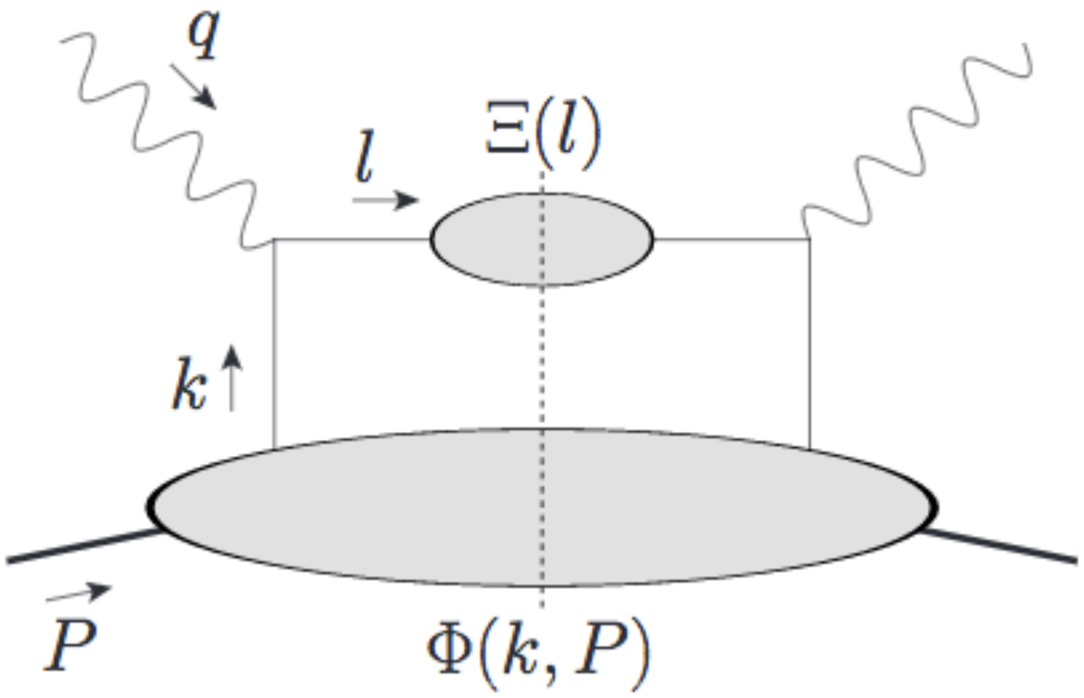
n Lab

Sivers: kinematic coverage



From EIC white paper

Transversity in DIS



transversity PDF couples to
a **chiral odd jet fragmentation function**
in inclusive DIS

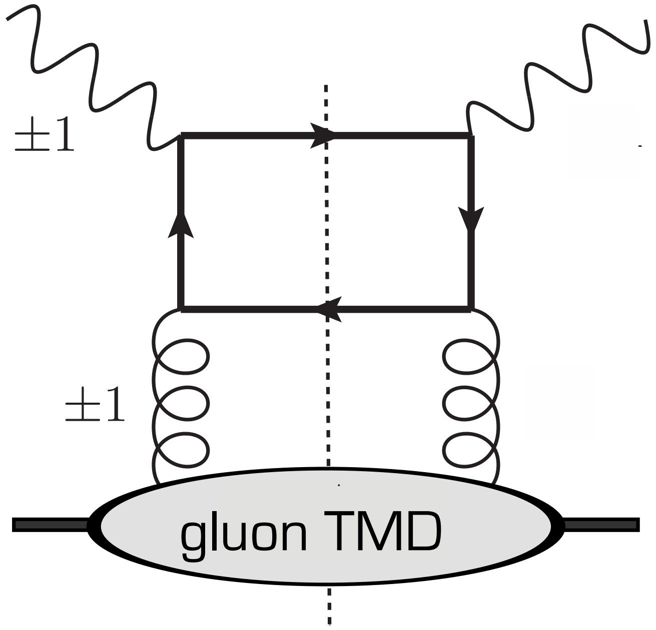
Gluon TMDs

see, e.g.,
- Boer, Mulders, Pisano, Zhou JHEP 1608 (2016) 001
- Boer, den Dunnen, Pisano, Schlegel, Vogelsang, PRL108 (12)
- den Dunnen, Lansberg, Pisano, Schlegel, PRL 112 (14)
- AS: PhD thesis , arXiv:1602.03405
- AFTER@LHC working group: arXiv:1702.01546 , arXiv:1610.05228 , ...
- Echevarria et al. arXiv:1502.05354
- ...

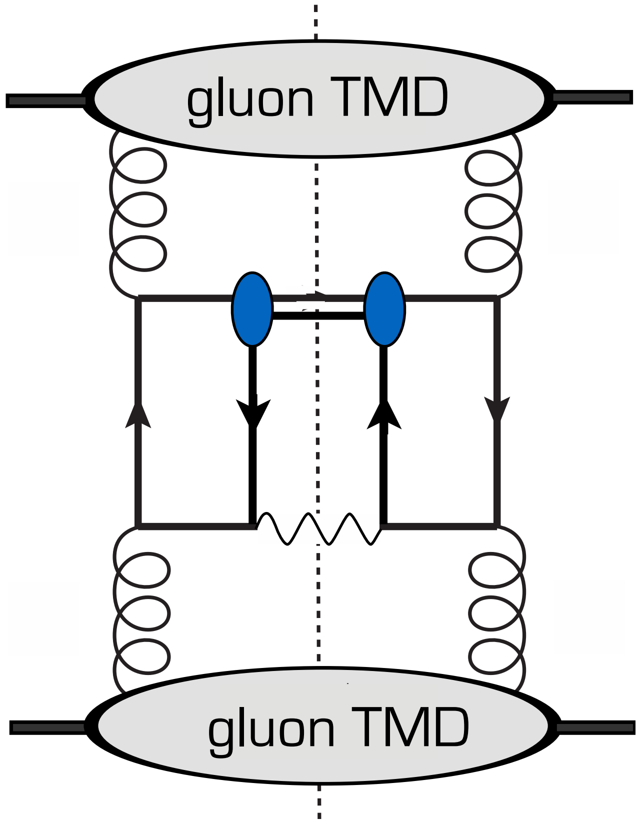
Gluon TMDs

$e\,p \to e\ \text{jet jet}\ X$

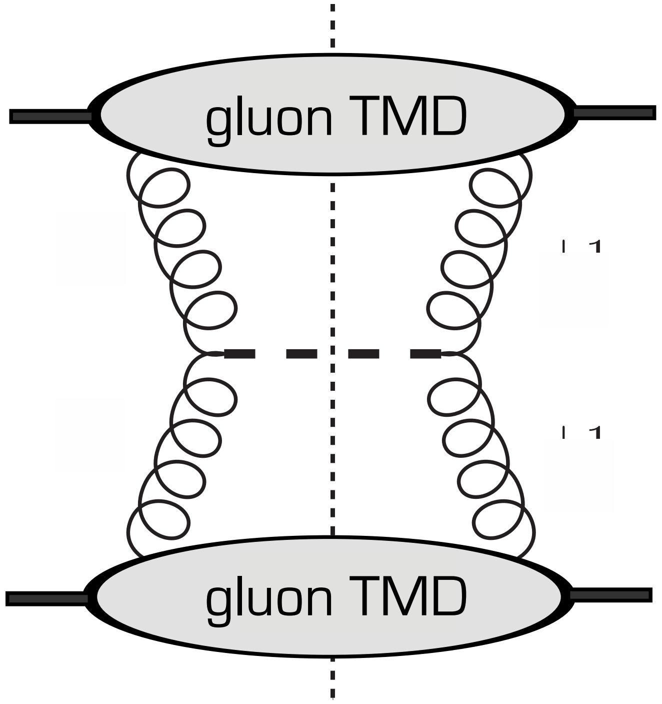
EIC !



$p\,p \to J/\psi\,\gamma\,X$



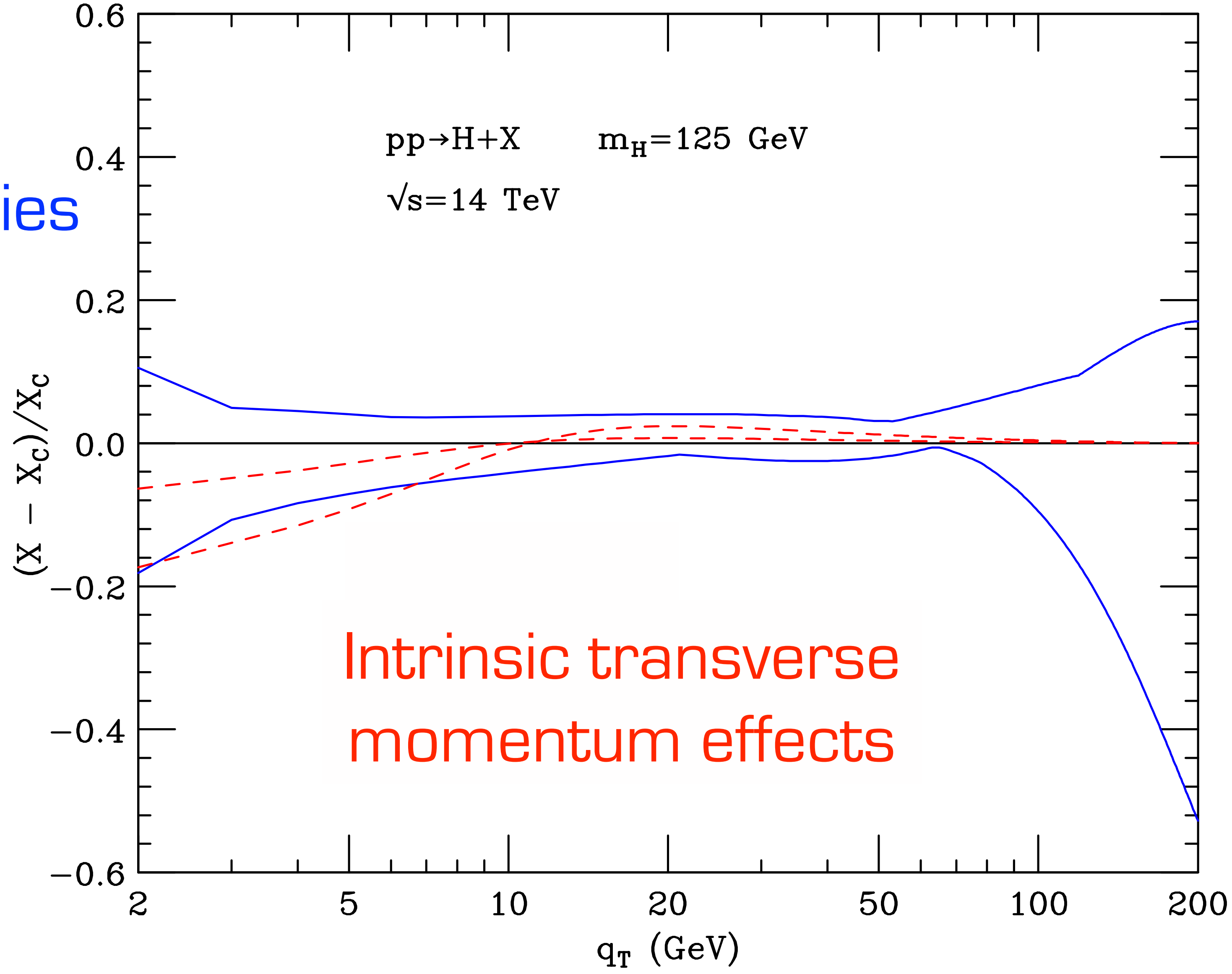
$p\,p \to \eta_c\,X$



Higgs transverse momentum

G. Ferrera, talk at REF 2014, Antwerp, https://indico.cern.ch/event/330428/

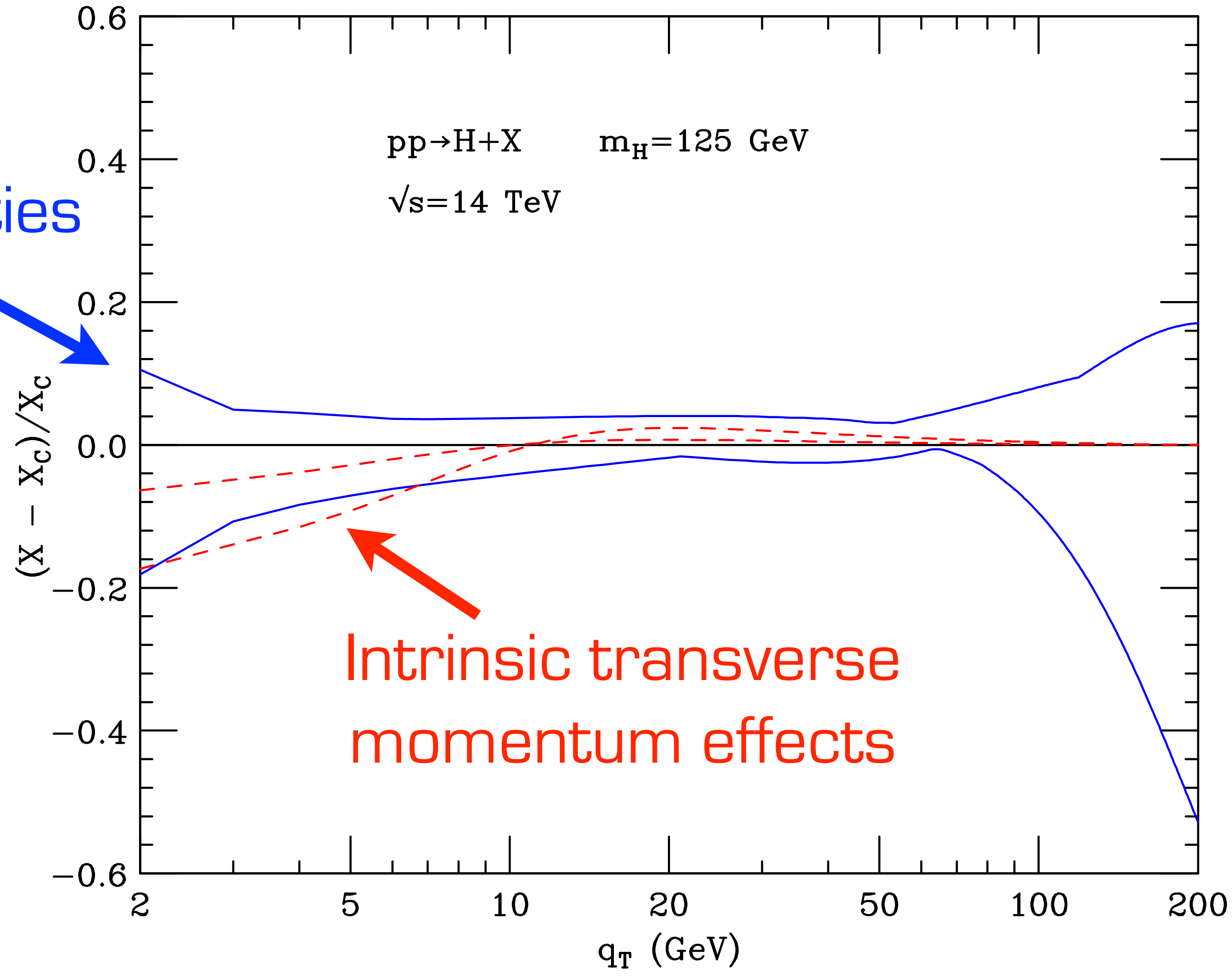
PDF uncertainties



Intrinsic transverse momentum effects

Higgs transverse momentum

G. Ferrera, talk at REF 2014, Antwerp, https://indico.cern.ch/event/330428/



PDF uncertainties

Intrinsic transverse momentum effects

Spin 1 TMDs

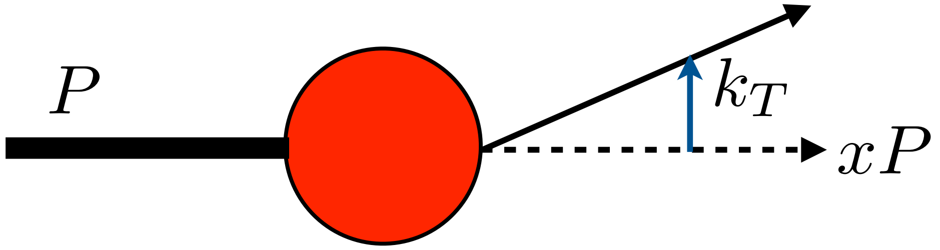
References :

- quark TMDs : Phys.Rev. D62 (2000) 114004
- gluon TMDs : JHEP 1610 (2016) 013
- ...

quark TMD PDFs

$$\Phi_{ij}(k,P;S,T) \sim \text{F.T.}\ \langle PST|\ \bar{\psi}_j(0)\ U_{[0,\xi]}\ \psi_i(\xi)\ |PST\rangle_{|LF}$$

Quarks	γ^+	$\gamma^+\gamma^5$	$i\sigma^{i+}\gamma^5$
U	$\boldsymbol{f_1}$		$h_1^\perp$
L		$\boldsymbol{g_1}$	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$\boldsymbol{h_1}$, $h_{1T}^\perp$
LL	$\boldsymbol{f_{1LL}}$		$h_{1LL}^\perp$
LT	f_{1LT}	g_{1LT}	h_{1LT}, $h_{1LT}^\perp$
TT	f_{1TT}	g_{1TT}	h_{1TT}, $h_{1TT}^\perp$



extraction of a **quark**
not collinear with the proton

a similar scheme holds for
TMD FFs and gluons

bold : also collinear
red : time-reversal odd (universality properties)

quark TMD PDFs

recent investigations of the T-even
TMDs in the context of DSE
arXiv:1707.03787

Quarks	γ^+	$\gamma^+\gamma^5$	$i\sigma^{i+}\gamma^5$
U	$\boldsymbol{f_1}$		$h_1^\perp$
L		$\boldsymbol{g_1}$	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$\boldsymbol{h_1}$, $h_{1T}^\perp$
LL	$\boldsymbol{f_{1LL}}$		$h_{1LL}^\perp$
LT	f_{1LT}	g_{1LT}	h_{1LT}, $h_{1LT}^\perp$
TT	f_{1TT}	g_{1TT}	h_{1TT}, $h_{1TT}^\perp$

Collinear, related to b1(x)
(under scrutiny at JLab)

Collinear & T-odd : should be zero!
(to be investigated)

bold : also collinear
red : time-reversal odd (universality properties)

Conclusions : a path to move forward

1) Phenomenology of TMDs is well underway ...

2) ... but there are a lot of theoretical challenges to be addressed: definition of kinematic regions in SIDIS, matching, perturbative accuracy, a better understanding of hadronization, context for gluon TMDs , ...

3) we definitely need more data (CLAS, EIC, ...), at the moment especially for e+e-

4) Working with some approximations, we are getting closer to a global fit analysis of TMDs

5) polarized structure functions unexplored from the point of view of QCD, but we have guidance from parton model studies (see JLab activities)

Backup

TMDs and their evolution

FT of TMDs :

$$\tilde{F}_i(x, b_T; Q, Q^2) = \tilde{F}_i(x, b_T, \mu_{\hat{b}}, \mu_{\hat{b}}^2) \times \exp\left\{ \int_{\mu_{\hat{b}}}^{Q} \frac{d\mu}{\mu} \gamma_F[\alpha_s(\mu), Q^2/\mu^2] \right\} \left(\frac{Q^2}{\mu_{\hat{b}}^2} \right)^{-K(\hat{b}_T;\mu_{\hat{b}}) - g_K(\overline{b_T};\{\lambda\})}$$

Sudakov form factor : perturbative and **nonperturbative** contributions

Collinear distribution!

(input) TMD distribution : Wilson coefficients and **intrinsic part**

$$\tilde{F}_i(x, b_T; \mu_{\hat{b}}, \mu_{\hat{b}}^2) = \sum_{j=q,\bar{q},g} C_{i/j}(x, \hat{b}_T; \mu_{\hat{b}}, \mu_{\hat{b}}^2) \otimes f_j(x; \mu_{\hat{b}}) \tilde{F}_{i,NP}(x, \overline{b}_T; \{\lambda\})$$

Nonperturbative parts defined in a “negative” way : **observed-calculable**

TMDs and their evolution

Distribution for intrinsic transverse momentum (and its FT):

$$\tilde{F}_{i,NP}(x, \bar{b}_T; \{\lambda\})$$

a Gaussian ?

Soft gluon emission

$$g_K(\bar{b}_T; \{\lambda\})$$

TMDs and their evolution

Distribution for intrinsic transverse momentum (and its FT):

$$\tilde{F}_{i,NP}(x, \bar{b}_T; \{\lambda\})$$

a Gaussian ?

Soft gluon emission

$$g_K(\bar{b}_T; \{\lambda\})$$

Separation of **b$_T$ regions**

$$\hat{b}_T(b_T; b_{\min}, b_{\max}) \begin{array}{ll} \nearrow & b_{\max} \ , \quad b_T \to +\infty \\ \sim & b_T \ , \quad b_{\min} \ll b_T \ll b_{\max} \\ \searrow & b_{\min} \ , \quad b_T \to 0 \end{array}$$

High b$_T$ limit : avoid Landau pole

Low b$_T$ limit : recover fixed order expression

Models - evolution and b_T regions

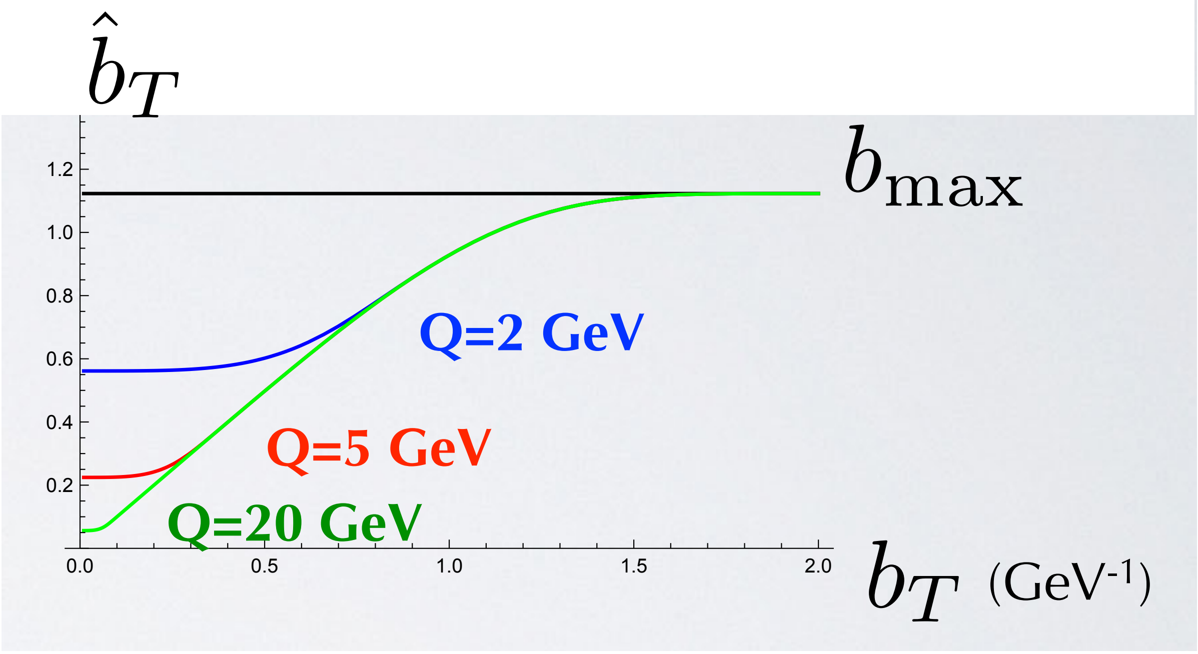
$$g_K(b_T; g_2) = -g_2 \frac{b_T^2}{2}$$

$$\hat{b}(b_T; b_{\min}, b_{\max}) = b_{\max} \left(\frac{1 - e^{-b_T^4/b_{\max}^4}}{1 - e^{-b_T^4/b_{\min}^4}} \right) \begin{array}{l} \nearrow \; b_{\max} \; , \quad b_T \to +\infty \\ \\ \searrow \; b_{\min} \; , \quad b_T \to 0 \end{array}$$

$$b_{\max} = 2e^{-\gamma_E}$$

$$b_{\min} = 2e^{-\gamma_E}/Q$$

These choices guarantee that for Q=1 GeV the TMD coincides with the NP model



Models - evolution and b_T regions

$$g_K(b_T; g_2) = -g_2 \frac{b_T^2}{2}$$

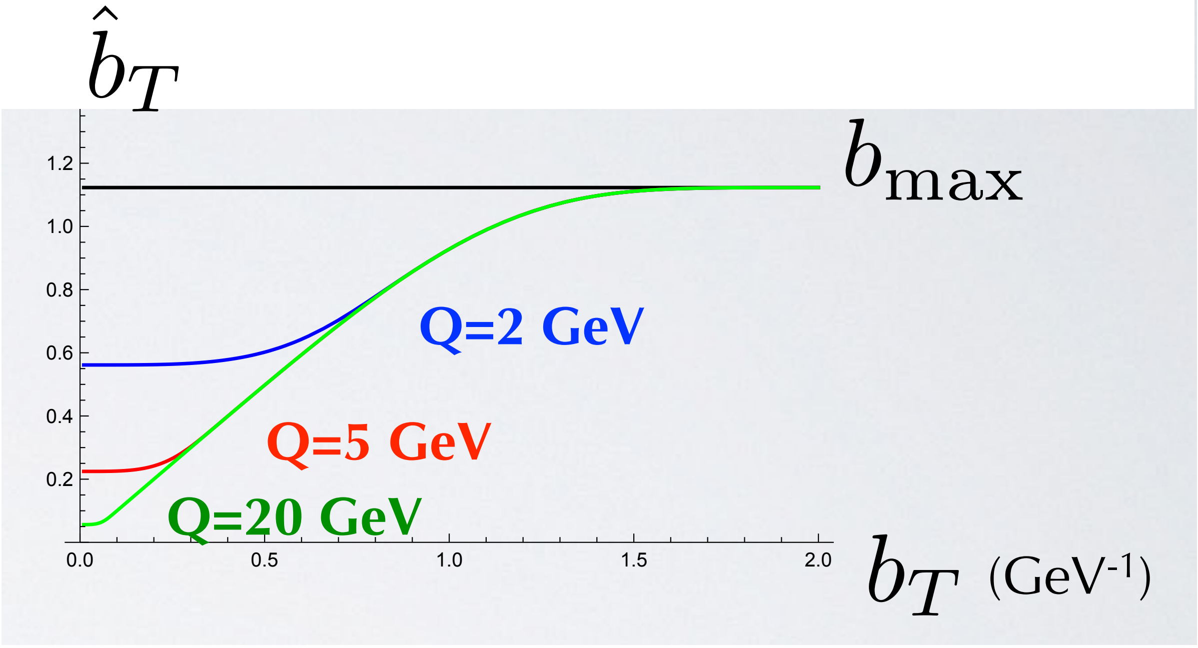
$$\hat{b}(b_T; b_{\rm min}, b_{\rm max}) = b_{\rm max} \left(\frac{1 - e^{-b_T^4/b_{\rm max}^4}}{1 - e^{-b_T^4/b_{\rm min}^4}} \right)$$

$\nearrow$ $b_{\rm max}\ , \quad b_T \to +\infty$

$\searrow$ $b_{\rm min}\ , \quad b_T \to 0$

$b_{\rm min} \sim 1/Q\ , \quad \mu_{\hat{b}} < Q$

The phenomenological importance of $b_{\rm min}$ is a signal that -especially in SIDIS data at **low Q**- we are exiting the proper TMD region and approaching the region of collinear factorization



Intrinsic transverse momentum

$$f^a_{1\mathrm{NP}}(x,\boldsymbol{k}_\perp^2) = \frac{1}{\pi}\,\frac{(1+\lambda \boldsymbol{k}_\perp^2)}{\langle \boldsymbol{k}_{\perp a}^2\rangle + \lambda\,\langle \boldsymbol{k}_{\perp a}^2\rangle^2}\; e^{-\frac{\boldsymbol{k}_\perp^2}{\langle \boldsymbol{k}_{\perp a}^2\rangle}}$$

$$\langle \boldsymbol{k}_{\perp a}^2\rangle(x) = \langle \hat{\boldsymbol{k}}_{\perp a}^2\rangle\,\frac{(1-x)^\alpha\; x^\sigma}{(1-\hat{x})^\alpha\;\hat{x}^\sigma}$$

$$\hat{x} = 0.1$$

weighted sum of two Gaussians

same widths for distributions, **different widths** fragmentations

$$D^{a\to h}_{1\mathrm{NP}}(z,\boldsymbol{P}_\perp^2) = \frac{1}{\pi}\,\frac{1}{\langle \boldsymbol{P}^2_{\perp a\to h}\rangle + \big(\lambda_F/z^2\big)\langle \boldsymbol{P}'^2_{\perp a\to h}\rangle^2}\left(e^{-\frac{\boldsymbol{P}_\perp^2}{\langle \boldsymbol{P}^2_{\perp a\to h}\rangle}} + \big(\lambda_F/z^2\big)\boldsymbol{P}_\perp^2\; e^{-\frac{\boldsymbol{P}_\perp^2}{\langle \boldsymbol{P}'^2_{\perp a\to h}\rangle}}\right)$$

$$\langle \boldsymbol{P}^2_{\perp a\to h}\rangle(z) = \langle \hat{\boldsymbol{P}}^2_{\perp a\to h}\rangle\,\frac{(z^\beta+\delta)\;(1-z)^\gamma}{(\hat{z}^\beta+\delta)\;(1-\hat{z})^\gamma}$$

$$\hat{z} = 0.5$$

Inspired from diquark models
(Eur.Phys.J. A45 (2010) 373-388)

For $f_{1\mathrm{NP}}$ and $D_{1\mathrm{NP}}$ we have 10 free parameters
(flavor independent case)

Best-fit values

TMD PDFs	$\langle \hat{\boldsymbol{k}}_\perp^2 \rangle$ [GeV^2]	α	σ		λ [GeV^{-2}]	
All replicas	0.28 ± 0.06	2.95 ± 0.05	0.17 ± 0.02		0.86 ± 0.78	
Replica 105	0.285	2.98	0.173		0.39	
TMD FFs	$\langle \hat{\boldsymbol{P}}_\perp^2 \rangle$ [GeV^2]	β	δ	γ	λ_F [GeV^{-2}]	$\langle \hat{\boldsymbol{P}}_\perp'^2 \rangle$ [GeV^2]
All replicas	0.21 ± 0.02	1.65 ± 0.49	2.28 ± 0.46	0.14 ± 0.07	5.50 ± 1.23	0.13 ± 0.01
Replica 105	0.212	2.10	2.52	0.094	5.29	0.135

TABLE XI: 68% confidence intervals of best-fit values for parametrizations of TMDs at $Q = 1$ GeV.

Flavor independent scenario:

$$\langle \hat{k}_\perp^2 \rangle = 0.28 \pm 0.06 \text{ GeV}^2$$

$$\langle \hat{P}_\perp^2 \rangle = 0.21 \pm 0.02 \text{ GeV}^2$$

$$\langle \hat{P}_\perp'^2 \rangle = 0.13 \pm 0.01 \text{ GeV}^2$$

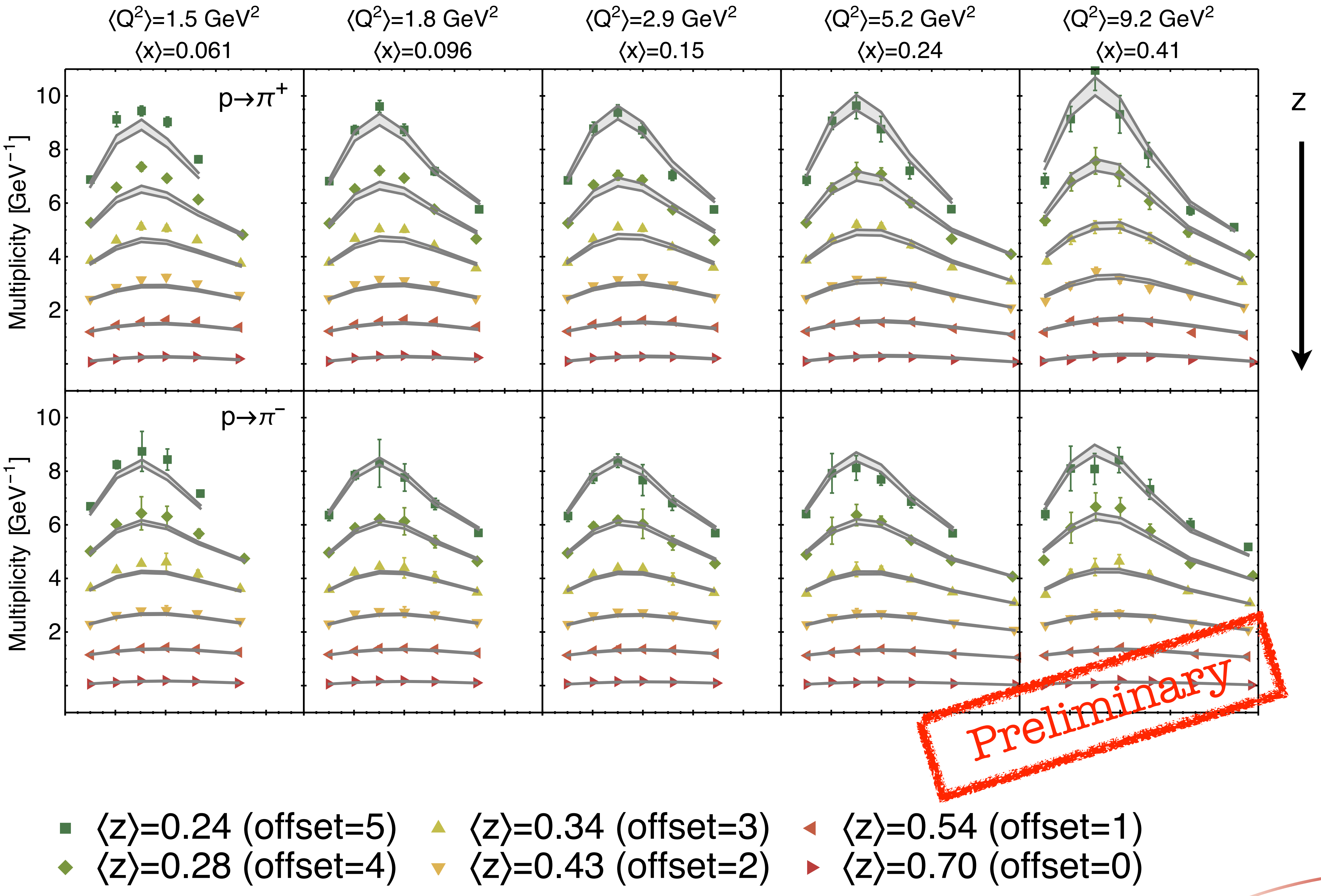
$$g_2 = 0.13 \pm 0.01 \text{ GeV}^2$$

best value from 200 replicas

compatible with other extractions

SIDIS @ Hermes

$\{P, \pi^{\pm}\}$

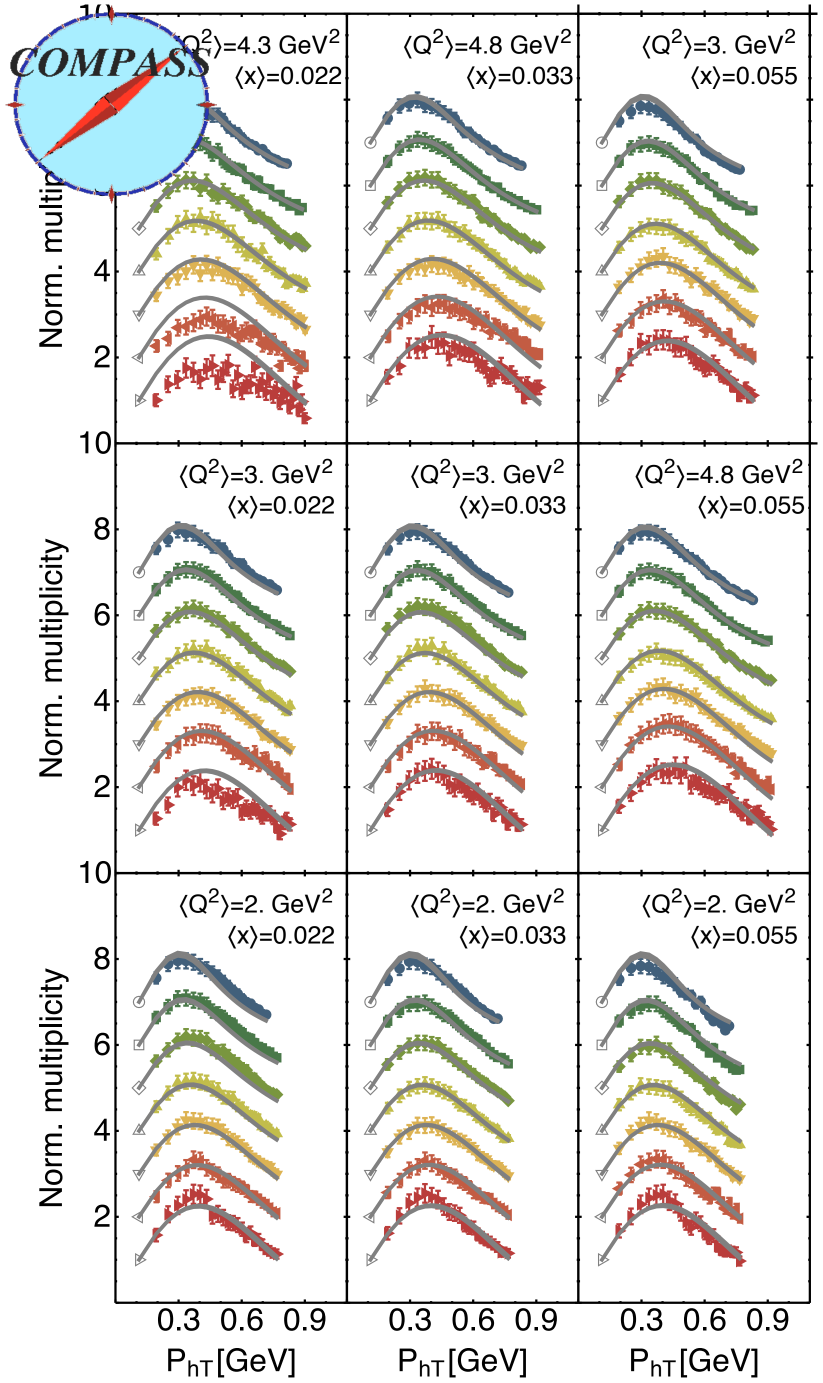


agreement improves at high z and high Q2

strong x-Q2 correlation

Global fit

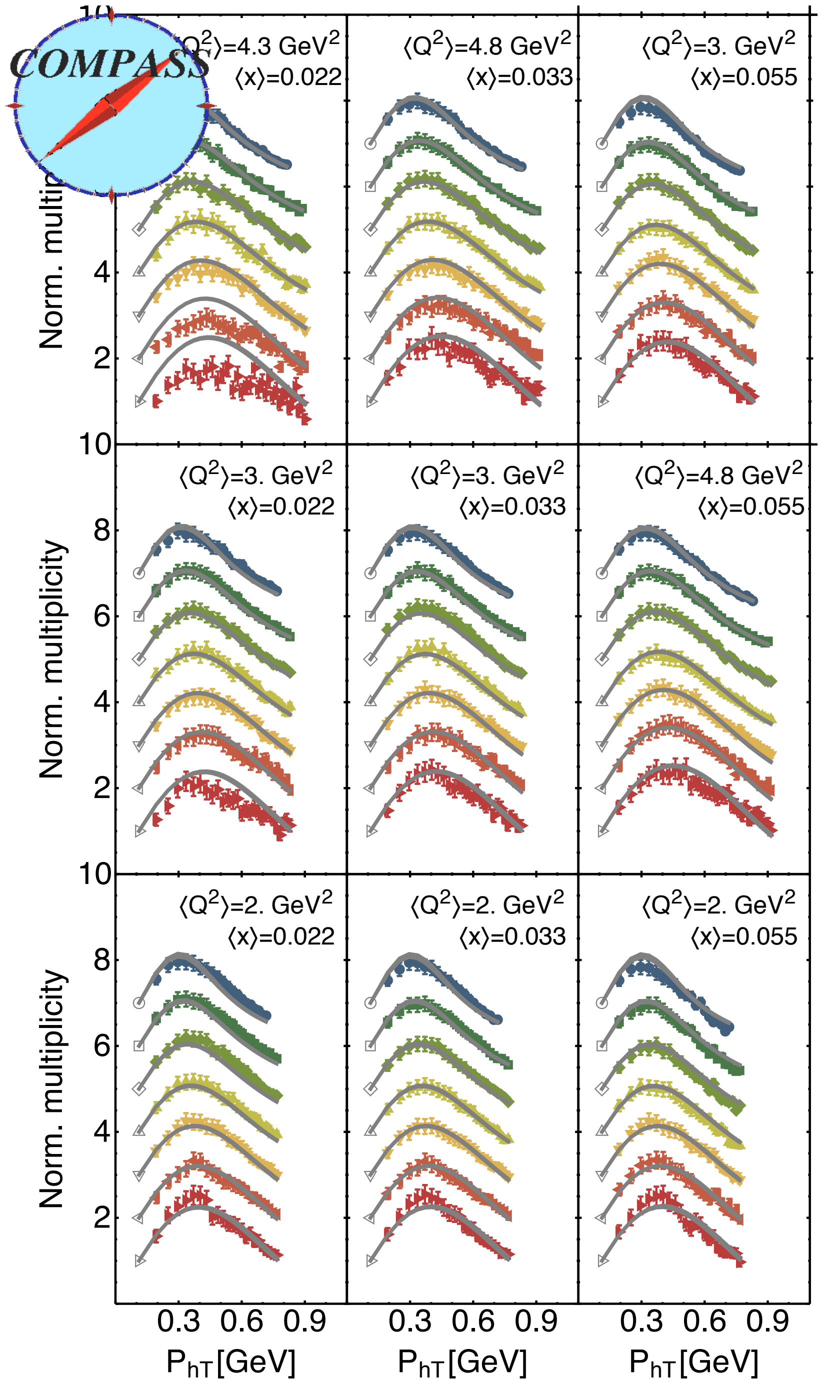
SIDIS



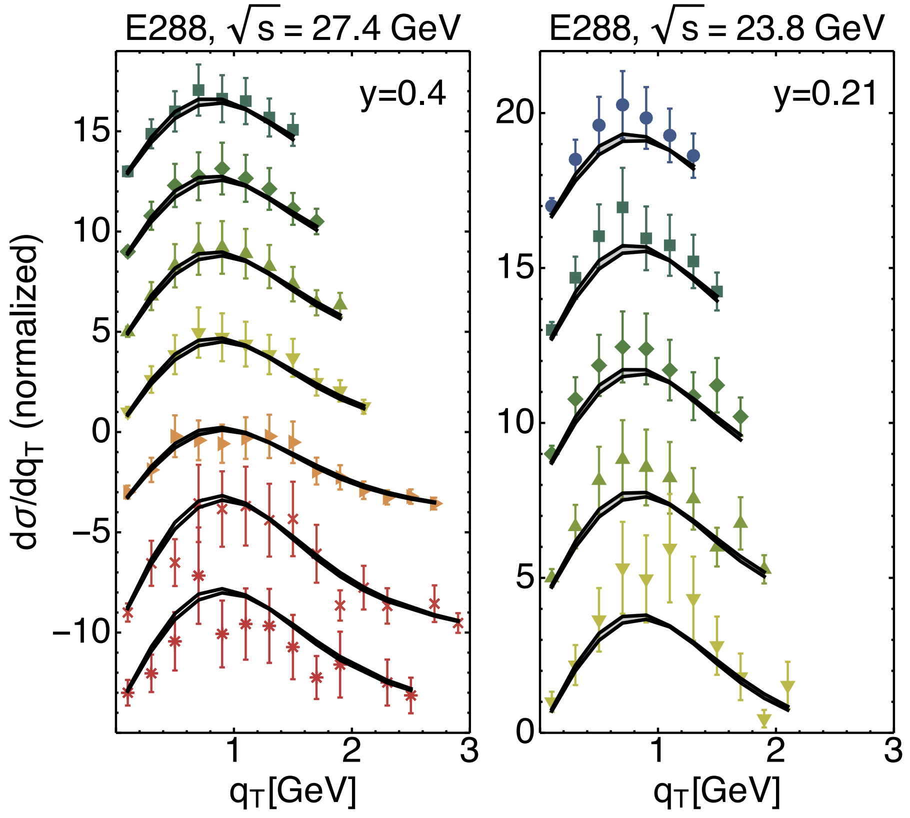
Bacchetta et al. **JHEP 1706 (2017) 081**

Global fit

SIDIS



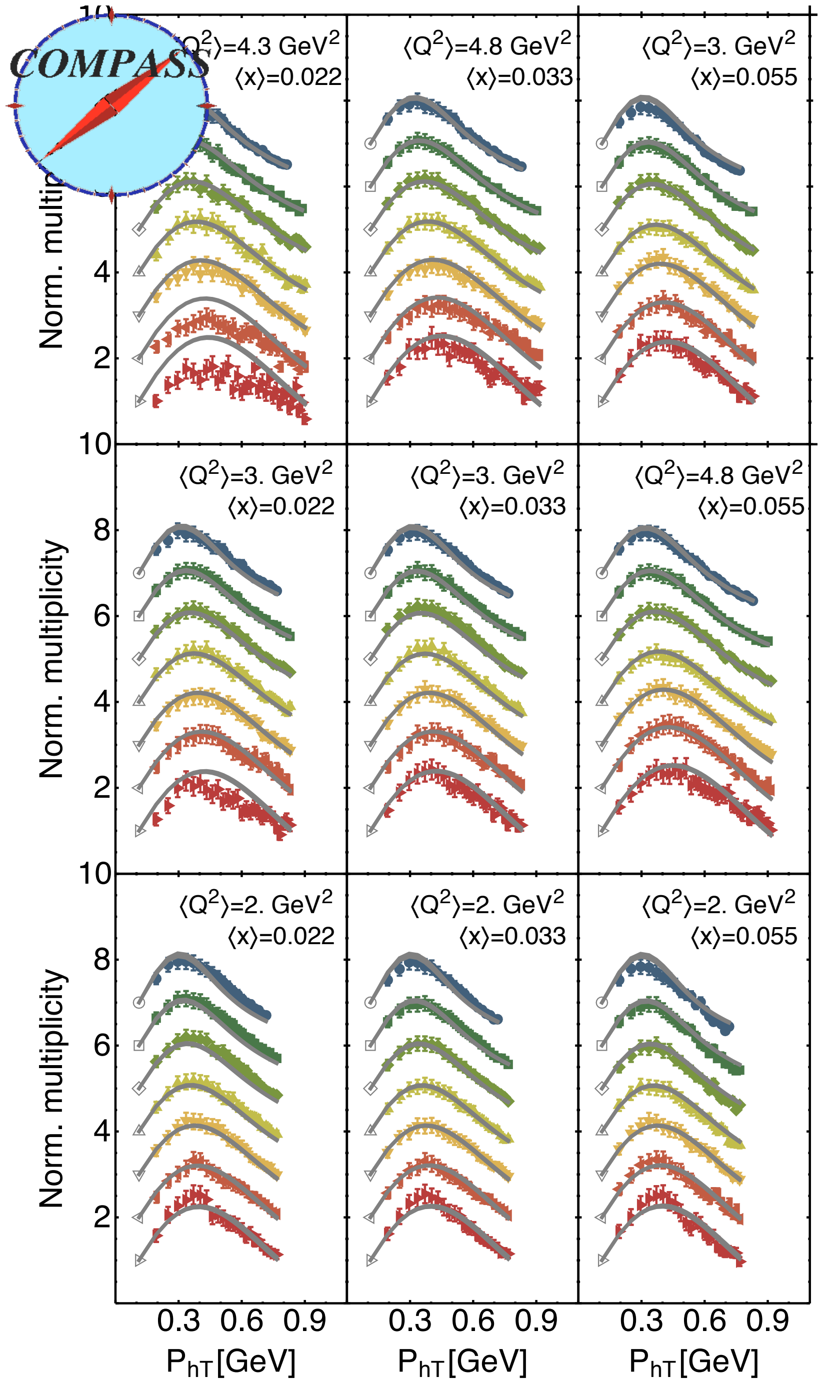
Drell-Yan



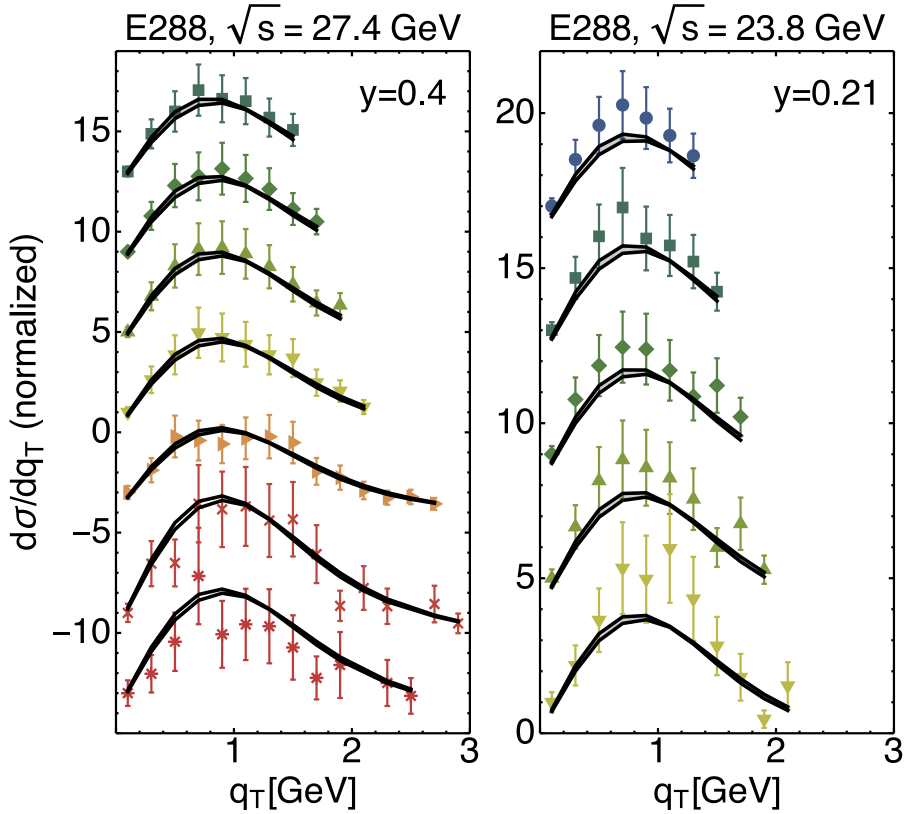
Bacchetta et al. **JHEP 1706 (2017) 081**

Global fit

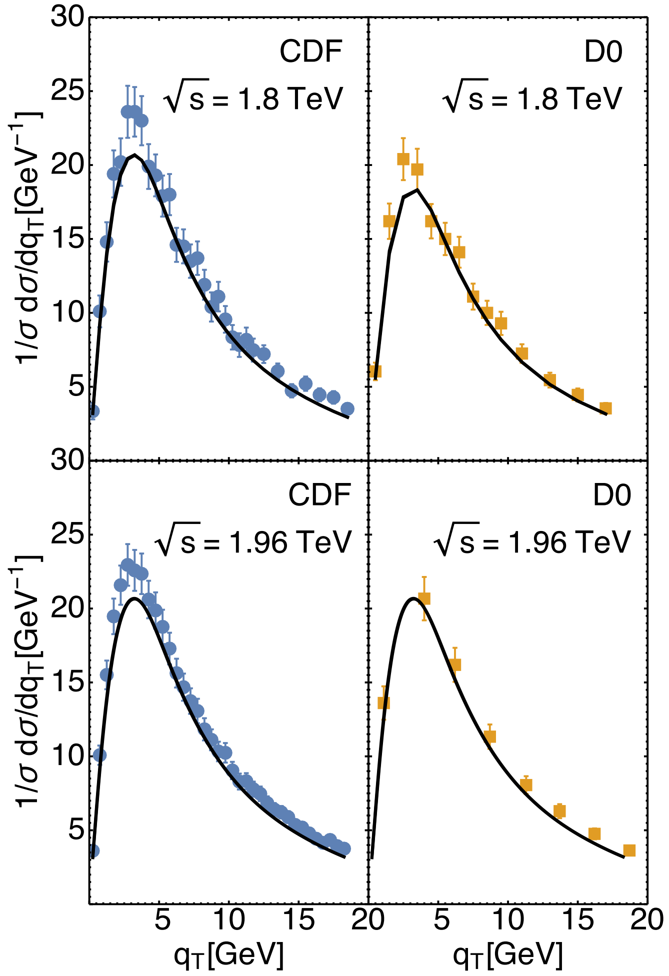
SIDIS



Drell-Yan

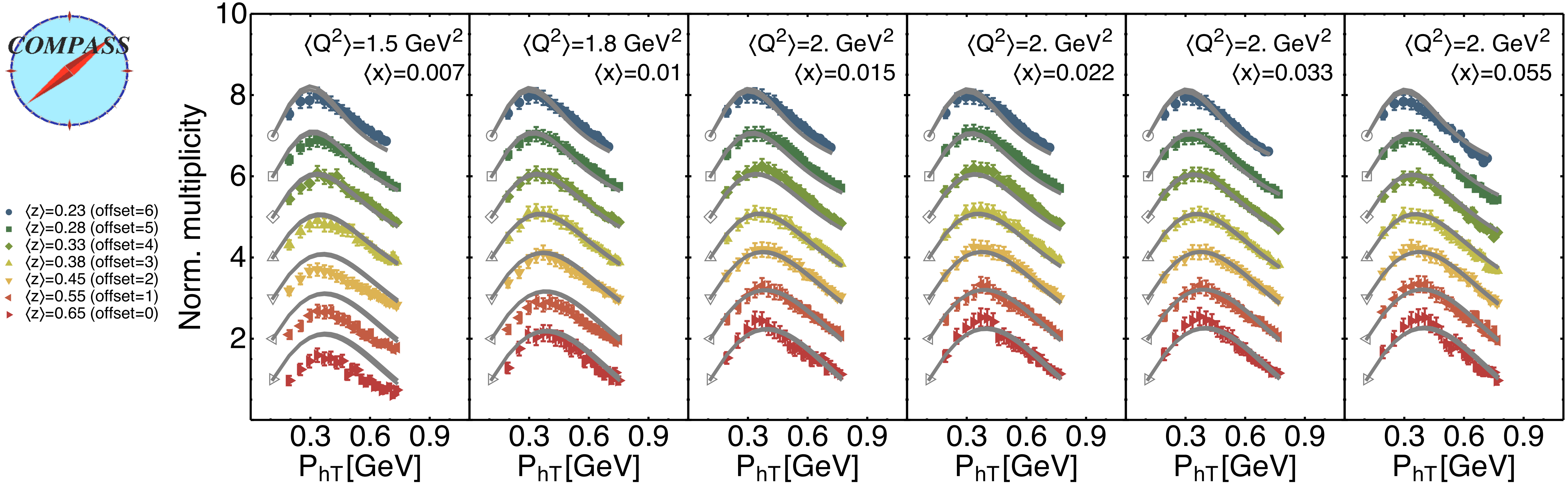


Z production



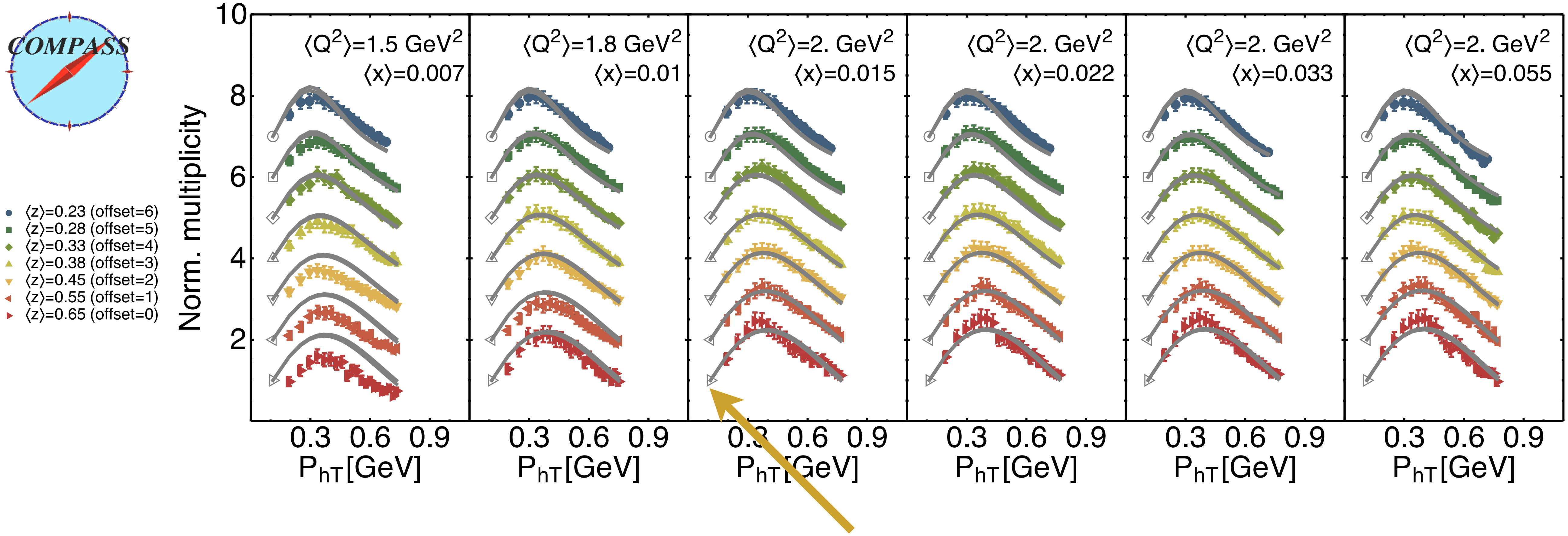
Bacchetta et al. **JHEP 1706 (2017) 081**

COMPASS, selected bins



Deuteron h^- $\chi^2/\text{dof} = 1.58$

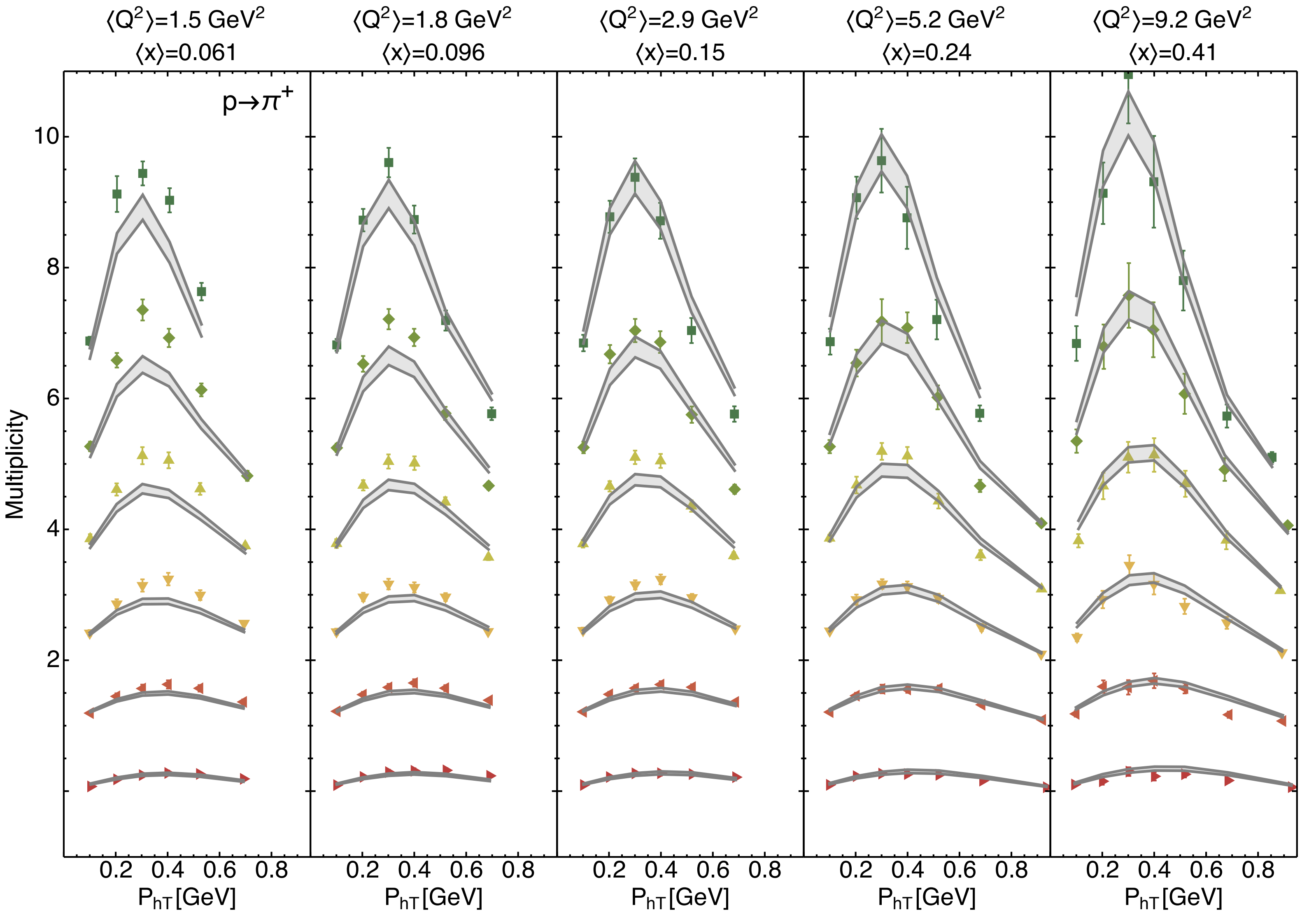
COMPASS, selected bins



First points are not fitted, but used as normalization

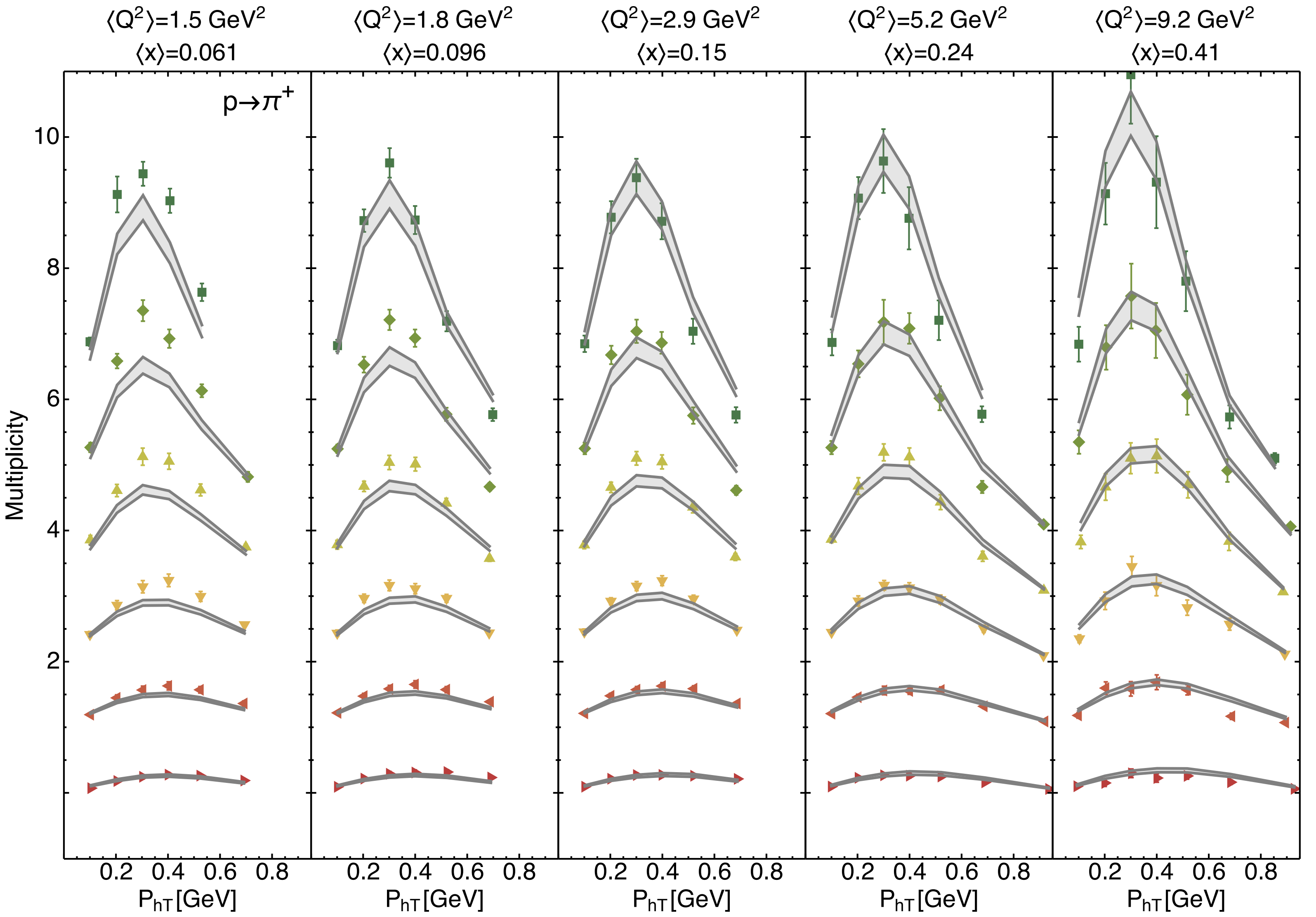
Deuteron h^- $\chi^2/\text{dof} = 1.58$

HERMES, selected bins



Contributions to chi2 mainly from **normalization**, not shape
(also in Z-boson production)

HERMES, selected bins

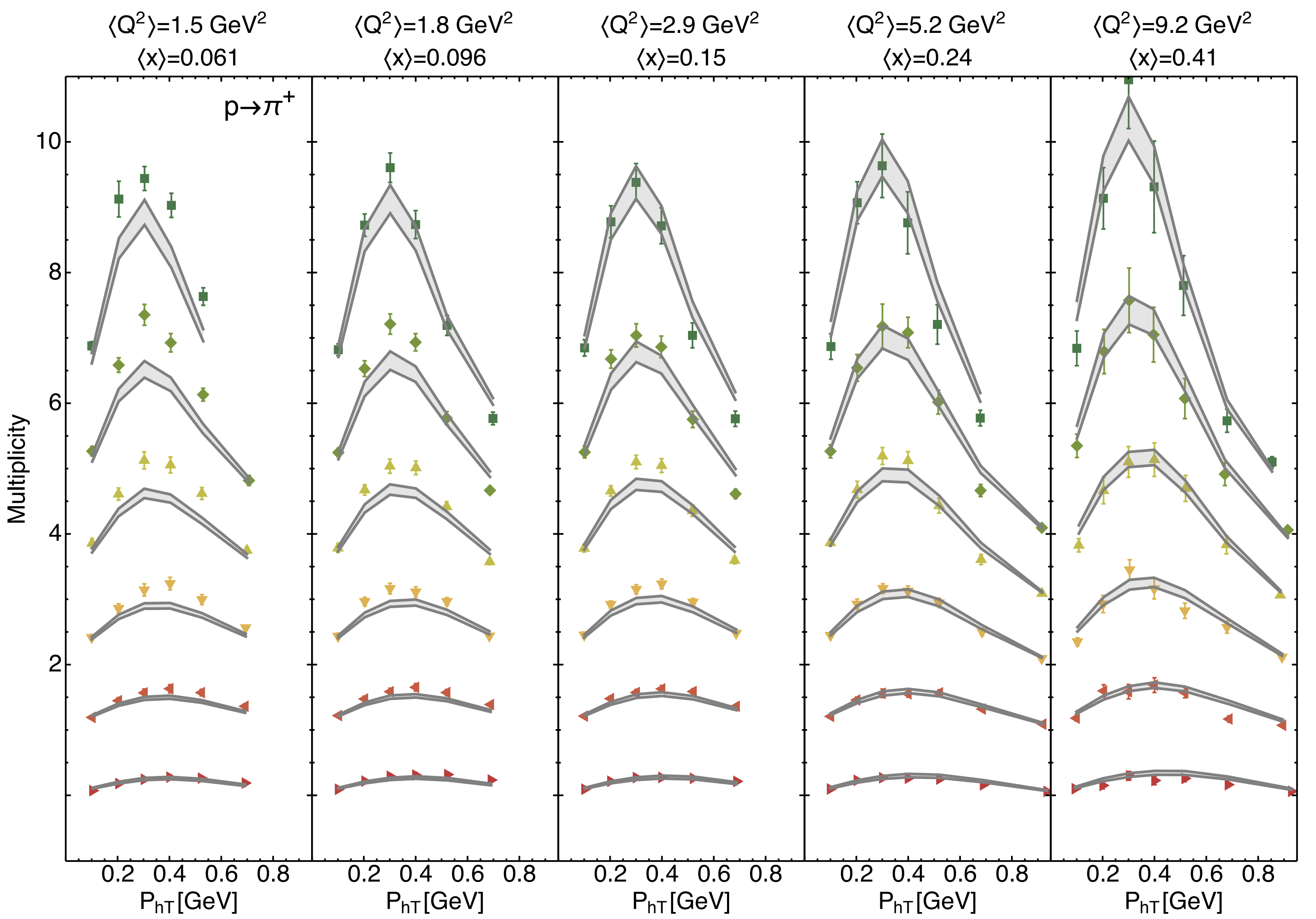


$\chi^2/\text{dof} = 4.80$

The worst of all channels...

Contributions to chi2 mainly from **normalization**, not shape (also in Z-boson production)

HERMES, selected bins



$\chi^2/\text{dof} = 4.80$

The worst of all channels…

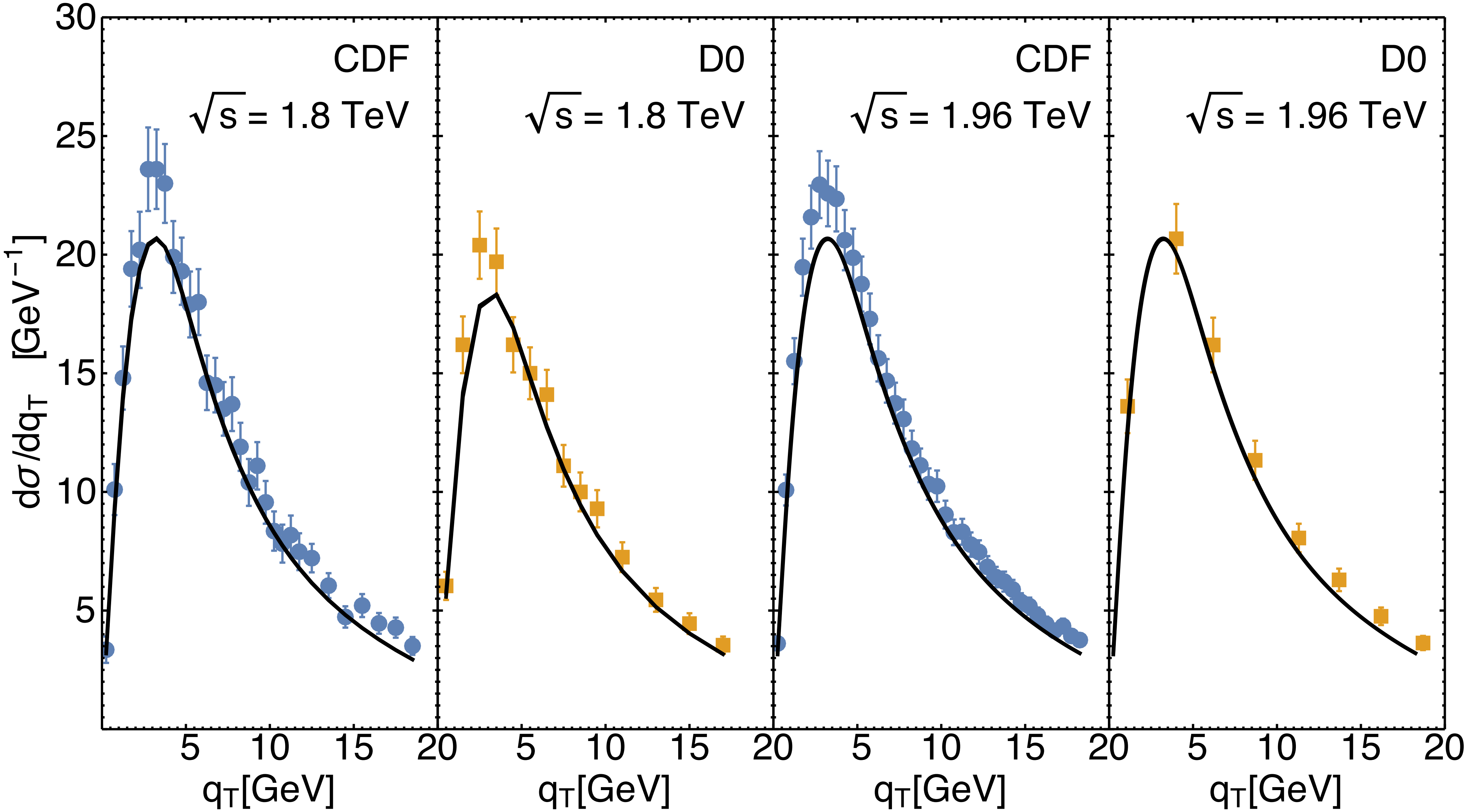
However **normalizing** the theory curves to the first bin, without changing the parameters of the fit, χ^2/dof becomes good

Contributions to chi2 mainly from **normalization**, not shape (also in Z-boson production)

Z-boson @ Fermilab

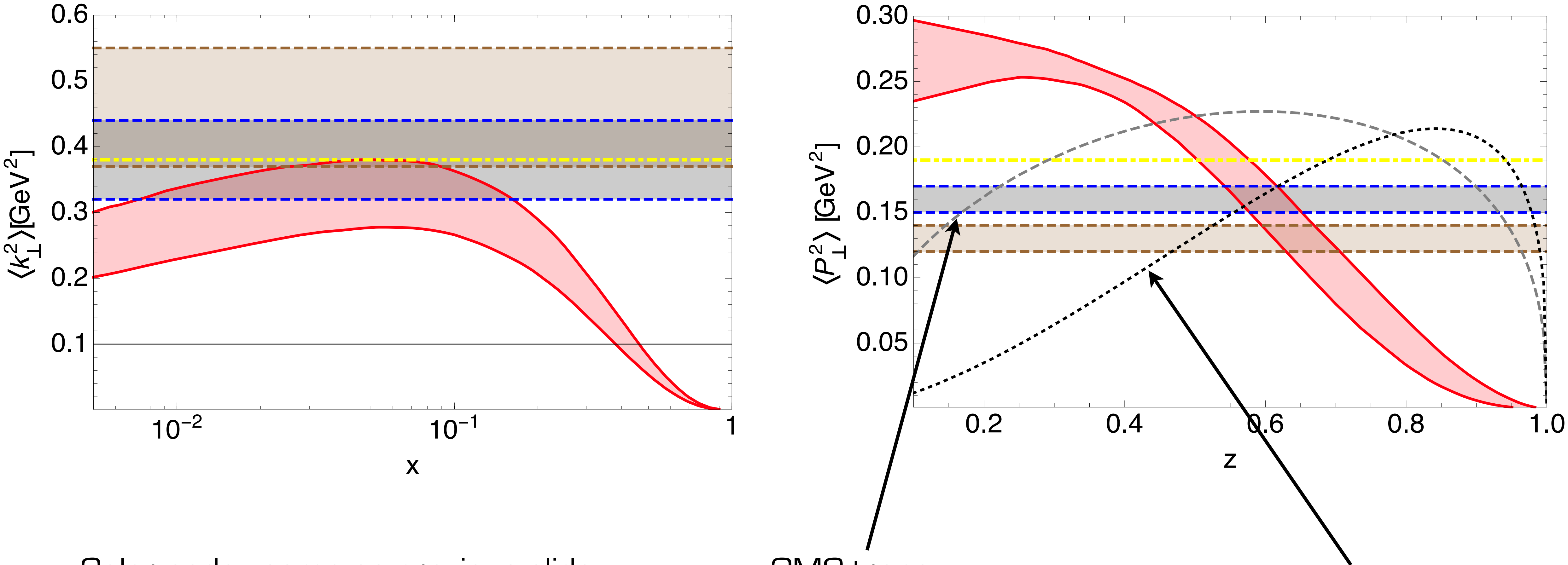
Narrow bands, driven mainly by **g_2 values** (reduced sensitivity to intrinsic k_T)

Contributions to chi2 mainly from **normalization**, not shape



Kinematic dependence

Comparison with other extractions :



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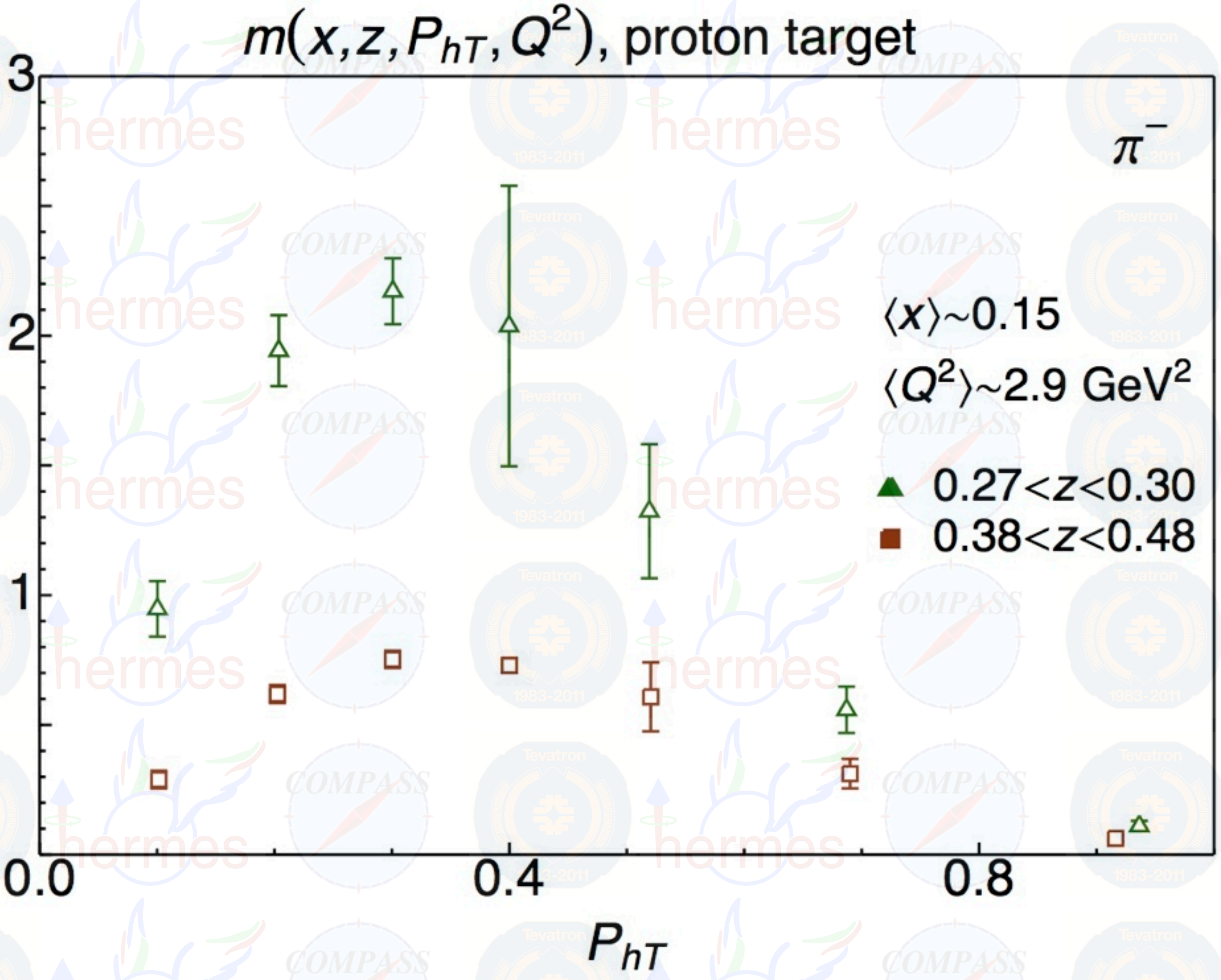
GMC trans

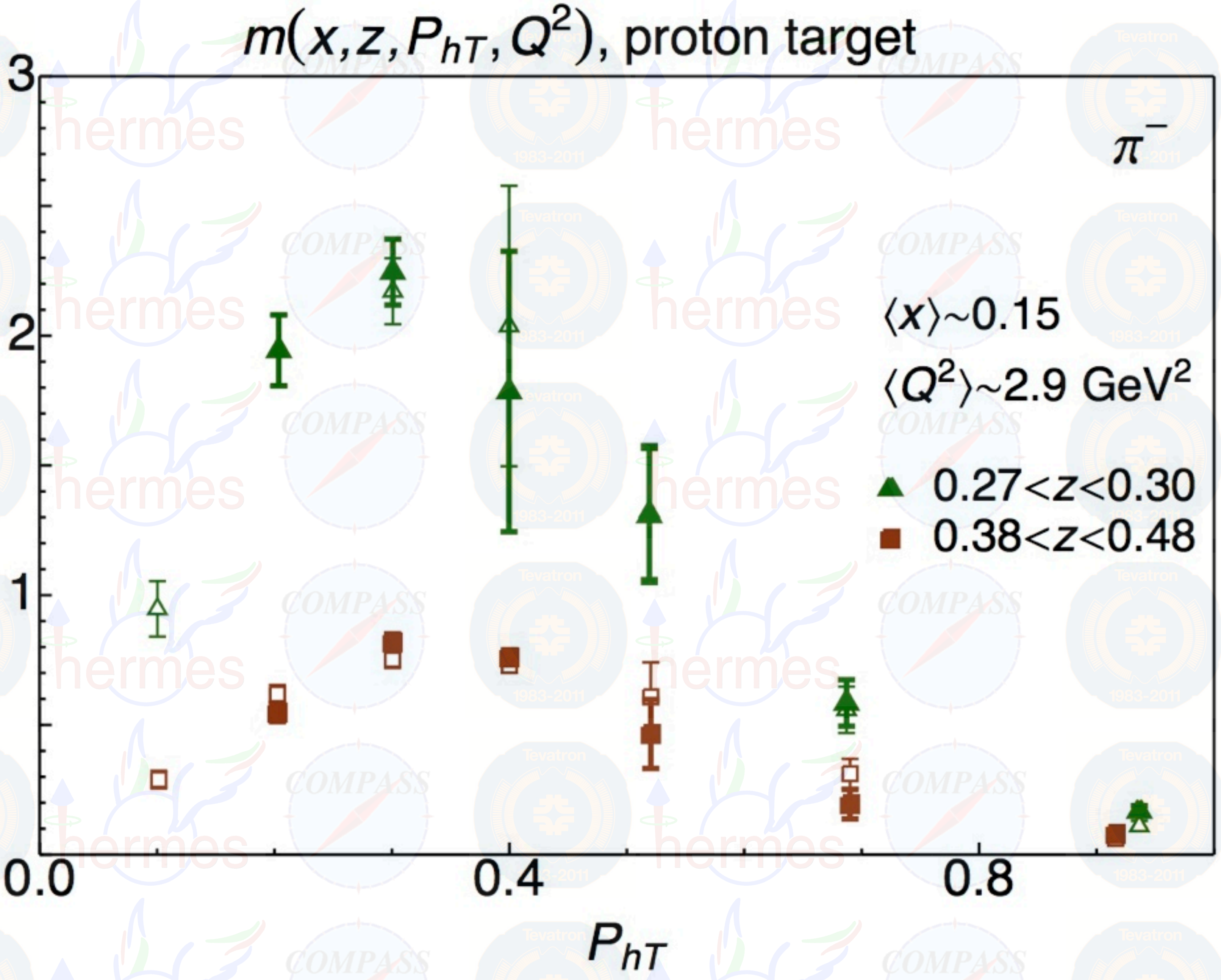
Anselmino et al.
hep-ph/9901442

The replica method



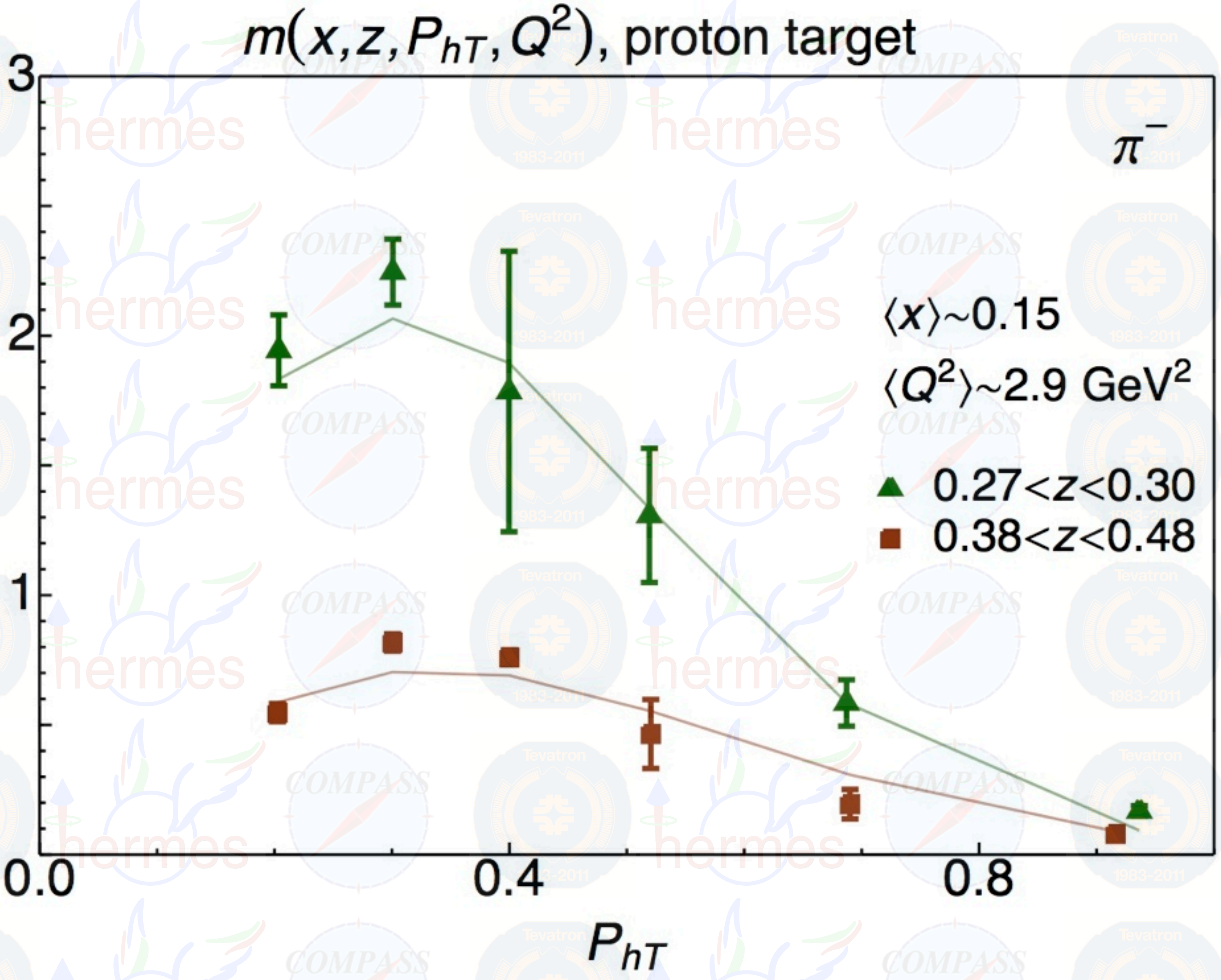
Sample of original data

The replica method



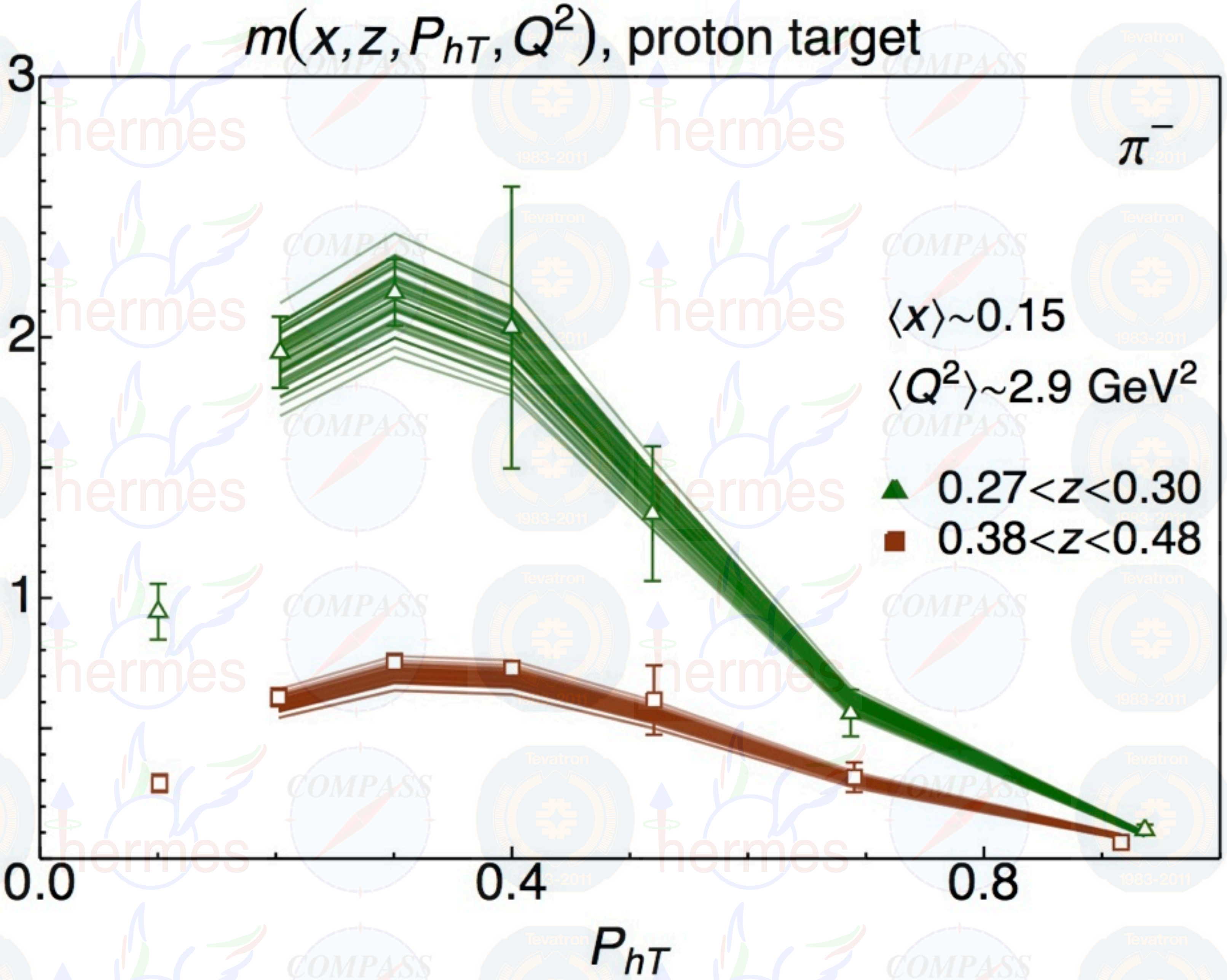
Replica of the original data with Gaussian noise

The replica method



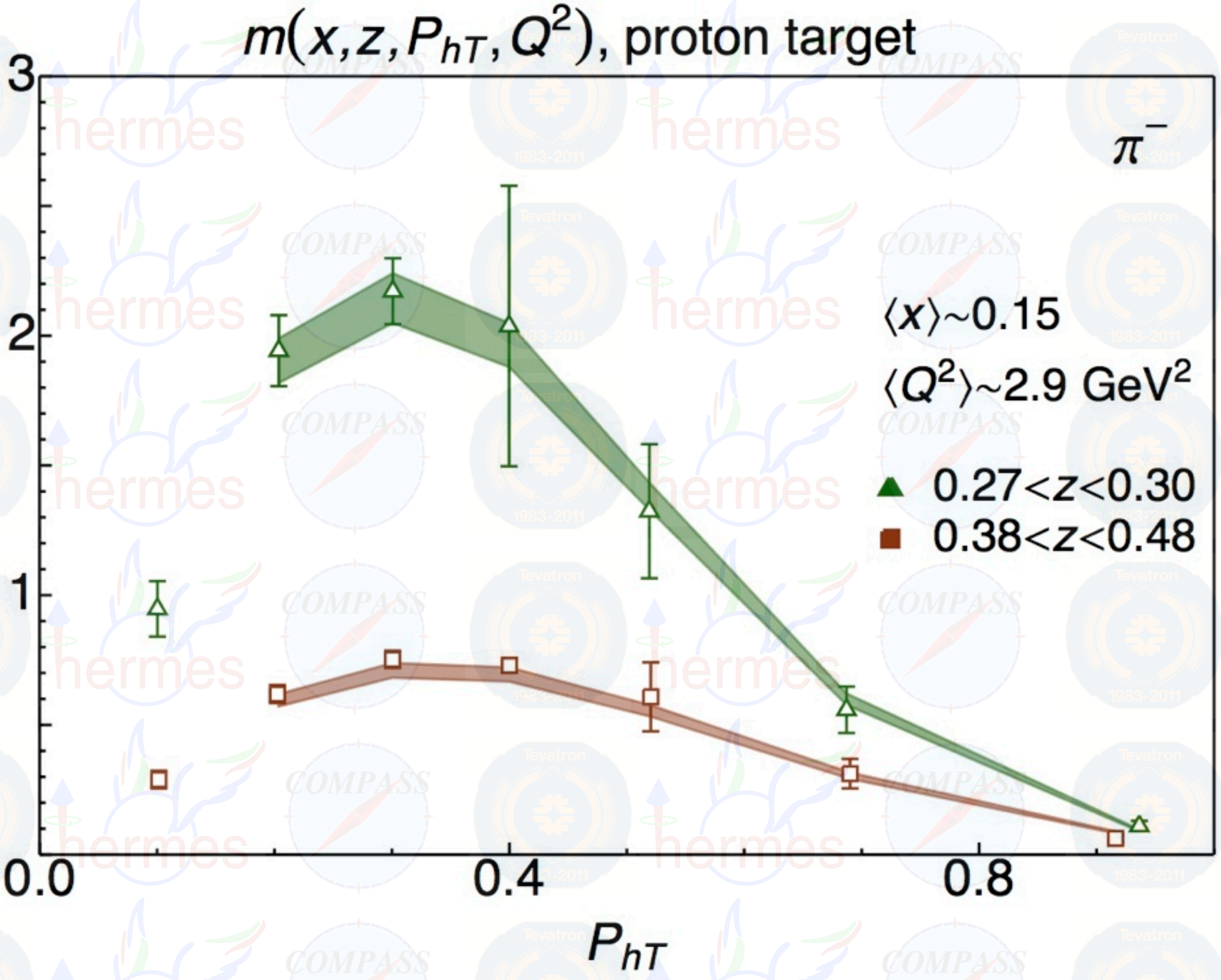
Fit of the replicated data

The replica method



Repeat the generation and the fit N times

The replica method



Obtain **distributions of best values** - calculate **68% CL bands**

Data sets and selections

SIDIS

	HERMES $p \to \pi^+$	HERMES $p \to \pi^-$	HERMES $p \to K^+$	HERMES $p \to K^-$
Reference	[61]			
Cuts	$Q^2 > 1.4$ GeV^2 $0.2 < z < 0.7$ $P_{hT} < \text{Min}[0.2\ Q, 0.7\ Qz] + 0.5$ GeV			
Points	190	190	189	187
Max. Q^2	9.2 GeV^2			
x range	$0.06 < x < 0.4$			

TMD factorization ($P_{hT}/z \ll Q^2$)

avoid target fragmentation (low z) and exclusive contributions (high z)

In order to avoid the problems with the normalization in COMPASS data (see Compass coll., Erratum)

	HERMES $D \to \pi^+$	HERMES $D \to \pi^-$	HERMES $D \to K^+$	HERMES $D \to K^-$	COMPASS $D \to h^+$	COMPASS $D \to h^-$
Reference	[61]				[62]	
Cuts	$Q^2 > 1.4$ GeV^2 $0.2 < z < 0.7$ $P_{hT} < \text{Min}[0.2\ Q, 0.7\ Qz] + 0.5$ GeV					
Points	190	190	189	189	3125	3127
Max. Q^2	9.2 GeV^2				10 GeV^2	
x range	$0.06 < x < 0.4$				$0.006 < x < 0.12$	
Notes					Observable: $m_{\text{norm}}(x, z, \boldsymbol{P}_{hT}^2, Q^2)$, eq. (38)	

Data sets and selections

	E288 200	E288 300	E288 400	E605
Reference	[65]	[65]	[65]	[66]
Cuts	$q_T < 0.2\ Q + 0.5$ GeV			
Points	45	45	78	35
$\sqrt{s}$	19.4 GeV	23.8 GeV	27.4 GeV	38.8 GeV
Q range	4-9 GeV	4-9 GeV	5-9, 11-14 GeV	7-9, 10.5-18 GeV
Kin. var.	y=0.4	y=0.21	y=0.03	$-0.1 < x_F < 0.2$

TMD factorization ($q_T \ll Q^2$)

Drell-Yan

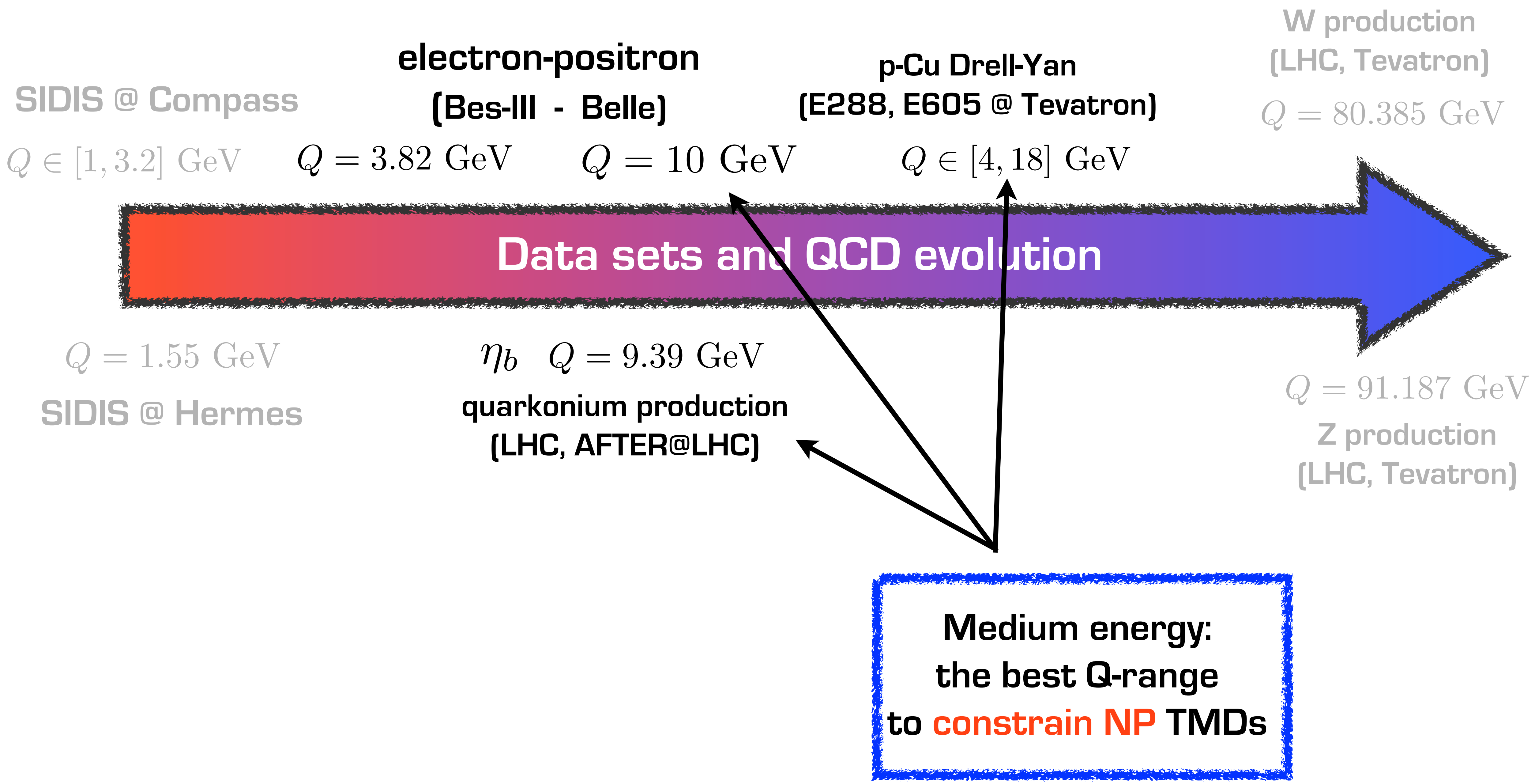
	CDF Run I	D0 Run I	CDF Run II	D0 Run II
Reference	[67]	[68]	[69]	[70]
Cuts	$q_T < 0.2\ Q + 0.5$ GeV $= 18.7$ GeV			
Points	31	14	37	8
$\sqrt{s}$	1.8 TeV	1.8 TeV	1.96 TeV	1.96 TeV
Normalization	1.114	0.992	1.049	1.048

Z

normalization :
fixed from DEMS fit,
different from exp.
(not really relevant for TMD
parametrizations)

Jefferson Lab

Evolution at work



SIDIS @ Compass

$Q \in [1, 3.2]$ GeV

electron-positron

(Bes-III - Belle)

$Q = 3.82$ GeV $Q = 10$ GeV

p-Cu Drell-Yan

(E288, E605 @ Tevatron)

$Q \in [4, 18]$ GeV

W production

(LHC, Tevatron)

$Q = 80.385$ GeV

Data sets and QCD evolution

$Q = 1.55$ GeV

SIDIS @ Hermes

η_b $Q = 9.39$ GeV

quarkonium production

(LHC, AFTER@LHC)

$Q = 91.187$ GeV

Z production

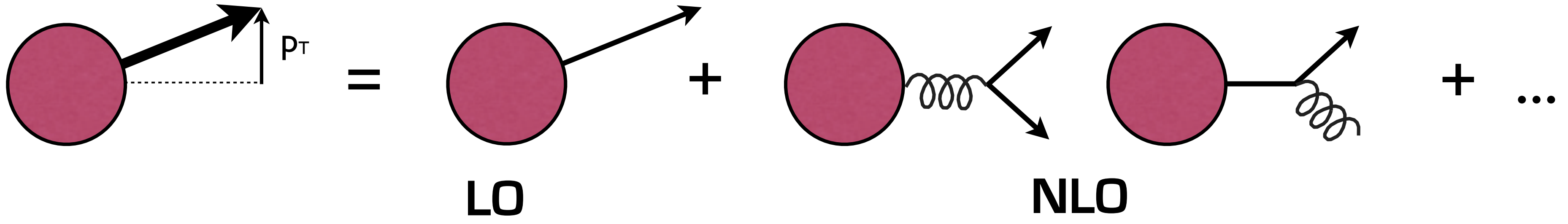
(LHC, Tevatron)

Medium energy:
the best Q-range
to constrain NP TMDs

Perturbative accuracy

Overview of the terminology

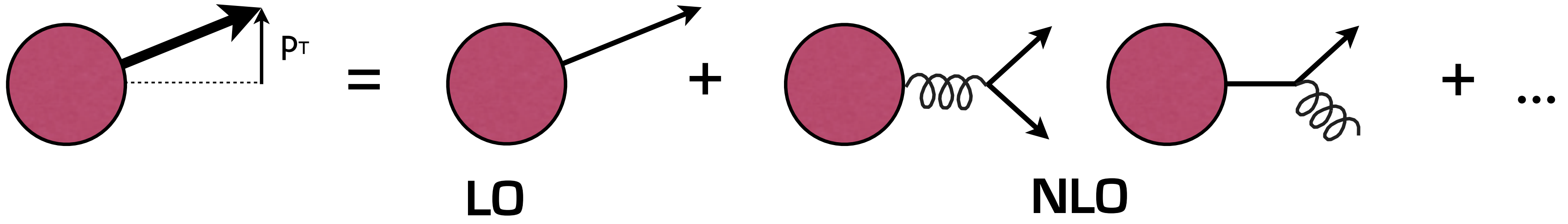
$C_{i/j}$ **Wilson coefficients** : expansion of the TMD distribution on a basis of collinear PDFs



Perturbative accuracy

Overview of the terminology

$C_{i/j}$ **Wilson coefficients** : expansion of the TMD distribution on a basis of collinear PDFs



Anomalous dimension of the TMD and **logarithmic expansion**

$$\gamma_F[\alpha_s(\mu),\zeta/\mu^2] \sim \underbrace{\alpha_s L}_{\mathrm{LL}} + \underbrace{(\alpha_s + \alpha_s^2 L)}_{\mathrm{NLL}} + \underbrace{(\alpha_s^2 + \alpha_s^3 L)}_{\mathrm{NNLL}} + \cdots$$

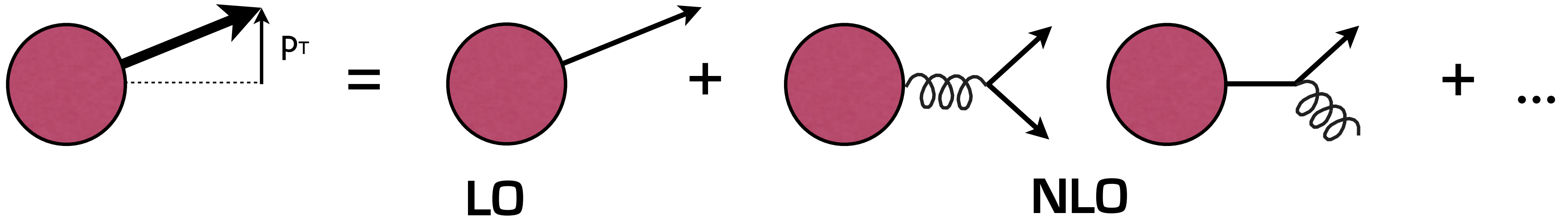
$$\sim \; 1 + \alpha_s + \alpha_s^2 + \cdots$$

$$L = \ln \frac{Q^2}{\mu} \; , \quad \alpha_s L \sim 1$$

Perturbative accuracy

Overview of the terminology

$C_{i/j}$ **Wilson coefficients** : expansion of the TMD distribution on a basis of collinear PDFs



Anomalous dimension of the TMD and **logarithmic expansion**

$$\gamma_F[\alpha_s(\mu),\zeta/\mu^2] \sim \underbrace{\alpha_s L}_{\text{LL}} + \underbrace{(\alpha_s + \alpha_s^2 L)}_{\text{NLL}} + \underbrace{(\alpha_s^2 + \alpha_s^3 L)}_{\text{NNLL}} + \cdots$$

$$\sim \ 1 + \alpha_s + \alpha_s^2 + \cdots$$

$$L = \ln \frac{Q^2}{\mu} \ , \quad \alpha_s L \sim 1$$

Collins-Soper kernel : a power series in the coupling

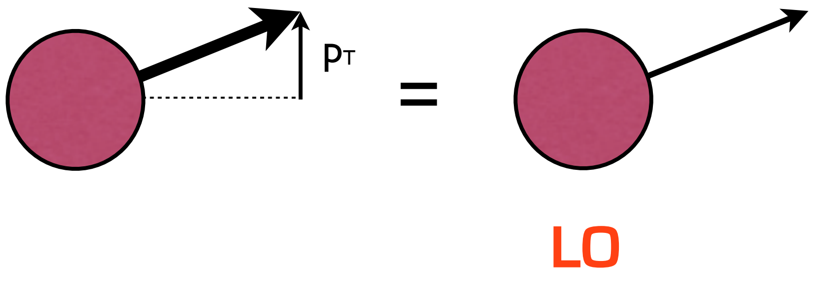
$$K(b_T;\mu_b) \sim 1 + \alpha_s + \alpha_s^2 \cdots$$

accuracy chosen consistently
with Wilson coefficients
and anomalous dimension

Jefferson Lab

Perturbative accuracy

$C_{i/j}$ **Wilson coefficients** : expansion of the TMD distribution on a basis of collinear PDFs



Anomalous dimension of the TMD and **logarithmic expansion**

$$\mu_{\hat{b}} = 2e^{-\gamma_E}/\bar{b}_\star$$

$$\gamma_F[\alpha_s(\mu), \zeta/\mu^2] \sim \underbrace{\alpha_s L}_{\text{LL}} + \underbrace{(\alpha_s + \alpha_s^2 L)}_{\text{NLL}} + \cdots$$

$$\sim 1 + \alpha_s + \cdots$$

Collins-Soper kernel : a power series in the coupling

$$K(b_T; \mu_b) \sim 1 + \alpha_s + \cdots$$

$C_{i/j}$	γ_{nc}	Γ_{cusp}	K	*accuracy*
0	0	0	0	QPM
0	0	1	0	LO-LL
0	1	2	1	LO-NLL
0	2	3	2	LO-NNLL
1	1	2	1	NLO-NLL
1	2	3	2	NLO-NNLL
2	2	3	2	NNLO-NNLL

Pavia / Amsterdam / Bilbao 2013

proton target global χ^2 / d.o.f. = 1.63 ± 0.12

no flavor dep. 1.72 ± 0.11

π^-

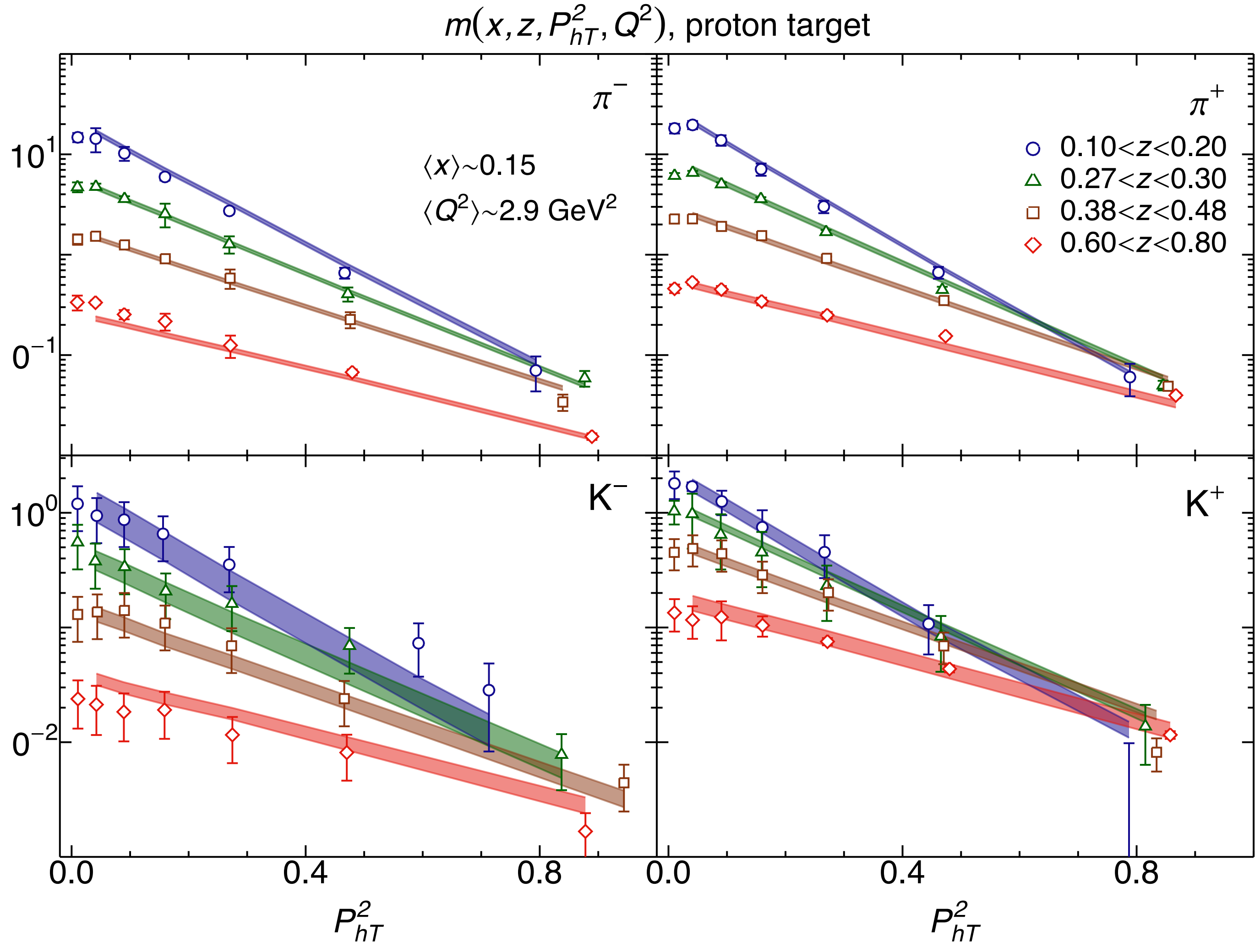
1.80 ± 0.27

1.83 ± 0.25

K^-

0.78 ± 0.15

0.87 ± 0.16



π^+

2.64 ± 0.21

2.89 ± 0.23

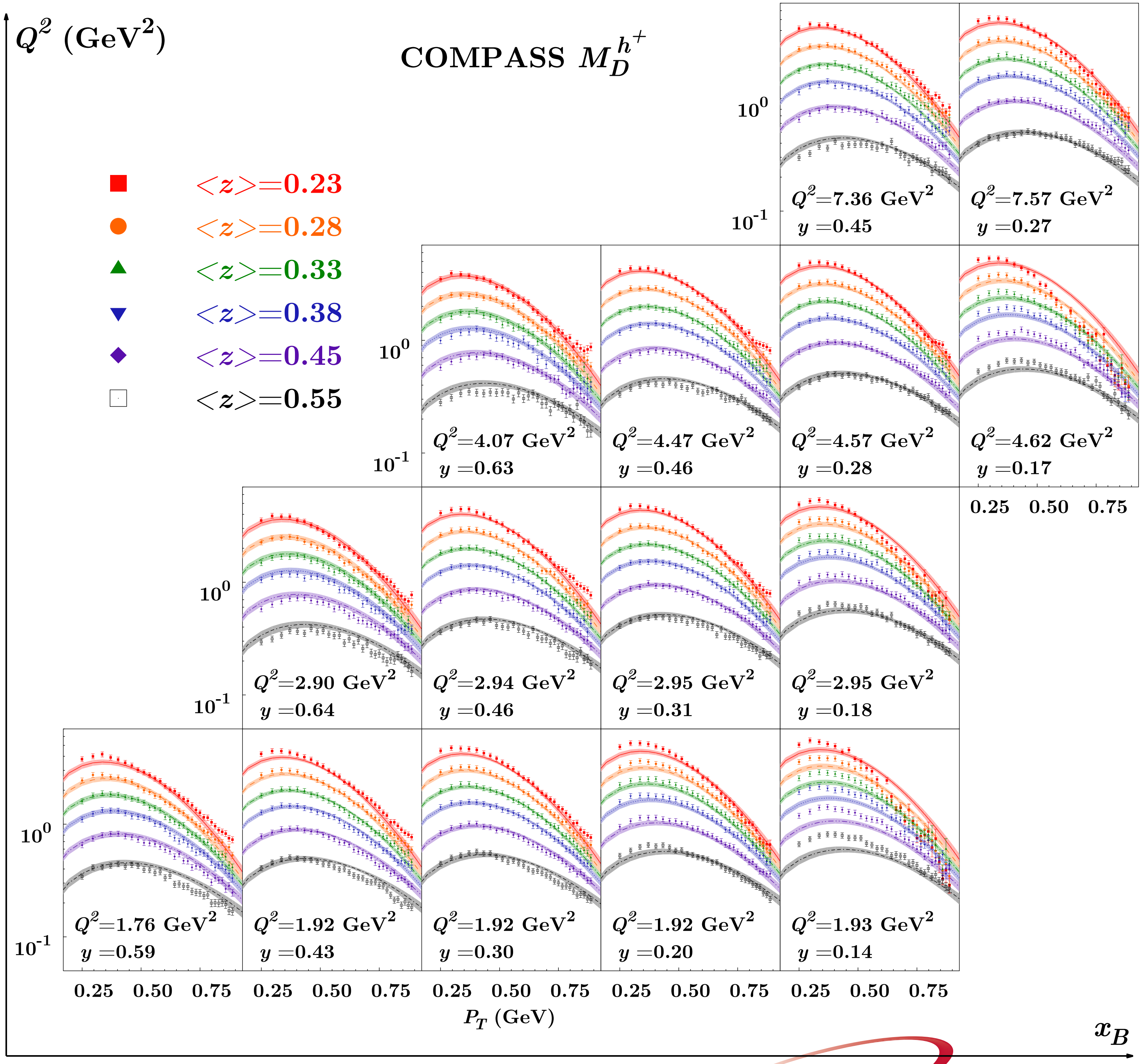
K^+

0.46 ± 0.07

0.43 ± 0.07

Torino / JLab 2014

COMPASS $M_D^{h^+}$



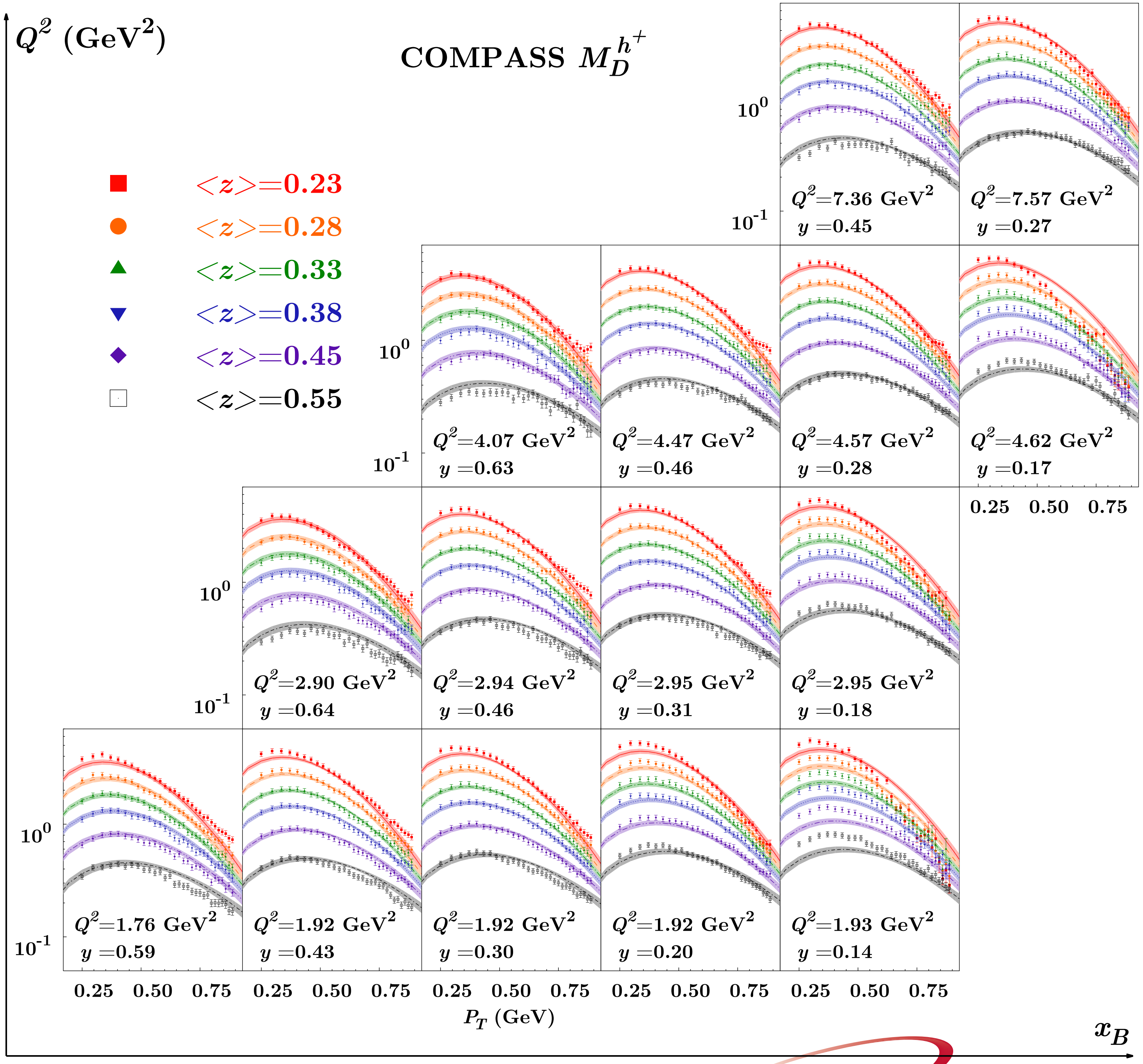
Anselmino, Boglione, Gonzalez, Melis, Prokudin, JHEP 1404 (14)

Torino / JLab 2014

COMPASS $M_D^{h^+}$

$\chi^2/\mathrm{dof} = 3.79$

with ad-hoc normalization



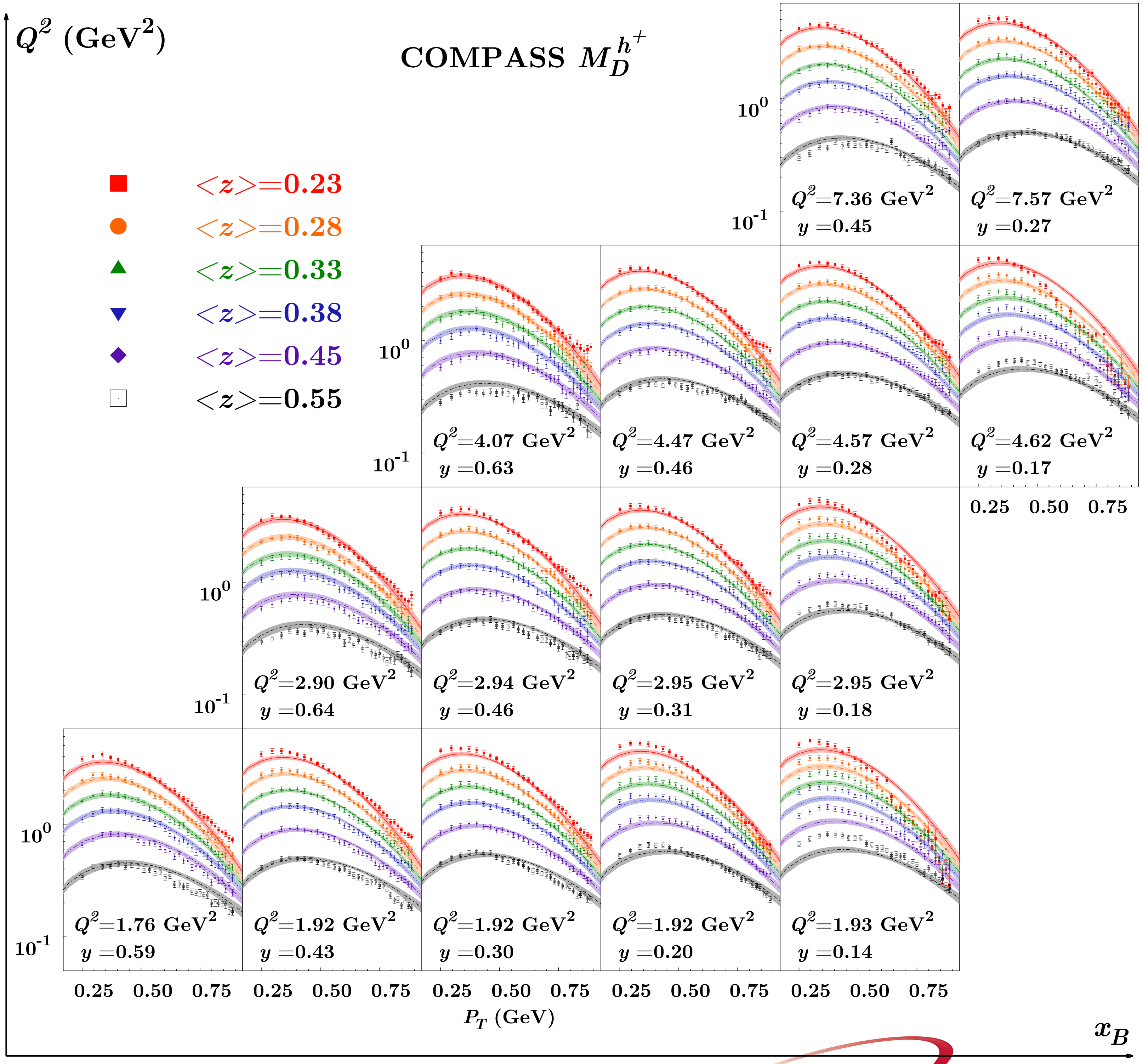
Anselmino, Boglione, Gonzalez, Melis, Prokudin, JHEP 1404 (14)

Torino / JLab 2014

COMPASS $M_D^{h^+}$

χ^2/dof = 3.79
with ad-hoc
normalization

see Compass coll.
Erratum



Anselmino, Boglione, Gonzalez, Melis, Prokudin, JHEP 1404 (14)

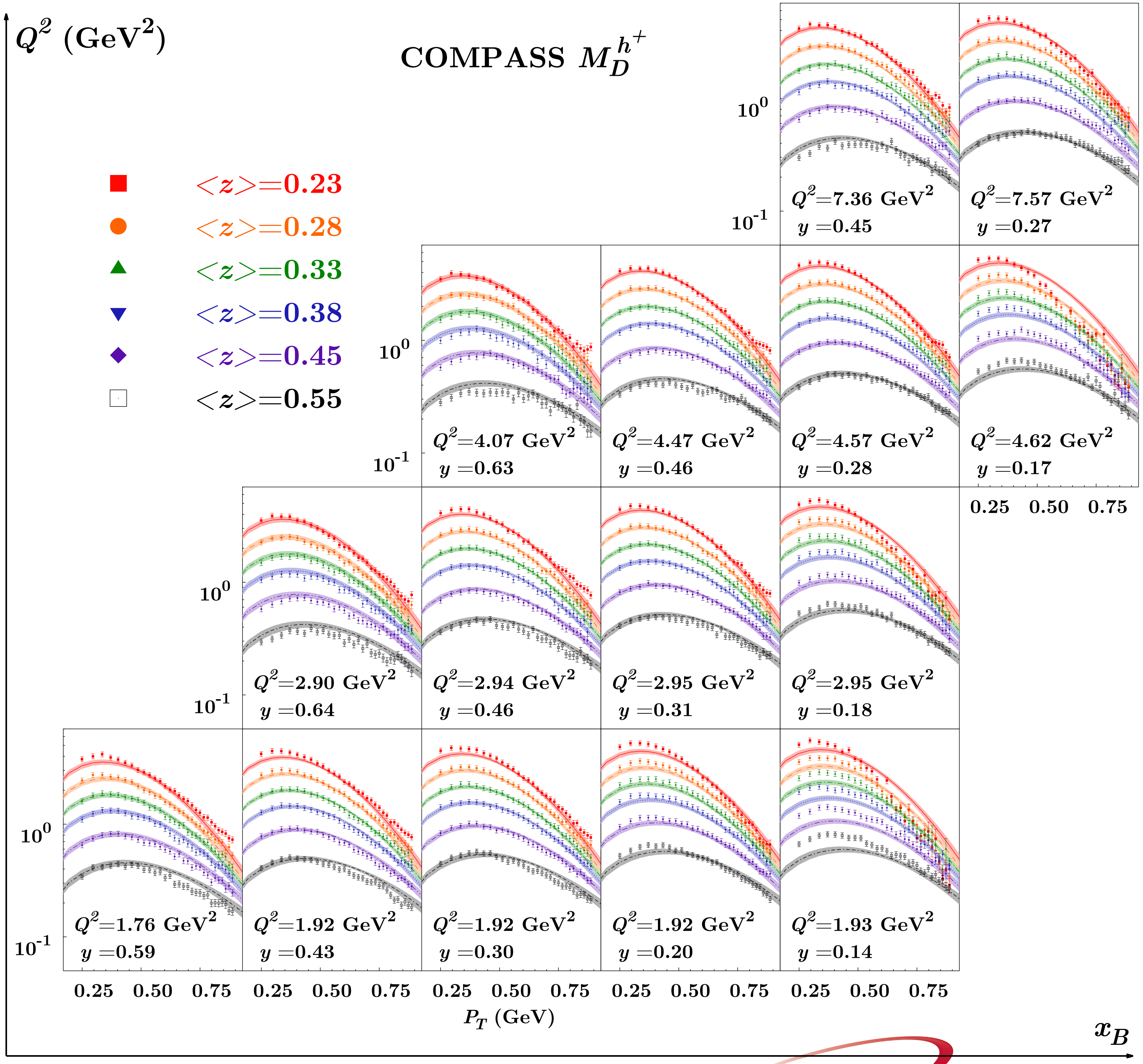
Torino / JLab 2014

COMPASS $M_D^{h^+}$

simple Gaussian ansatz

$\chi^2/\text{dof} = 3.79$
with ad-hoc
normalization

see Compass coll.
Erratum

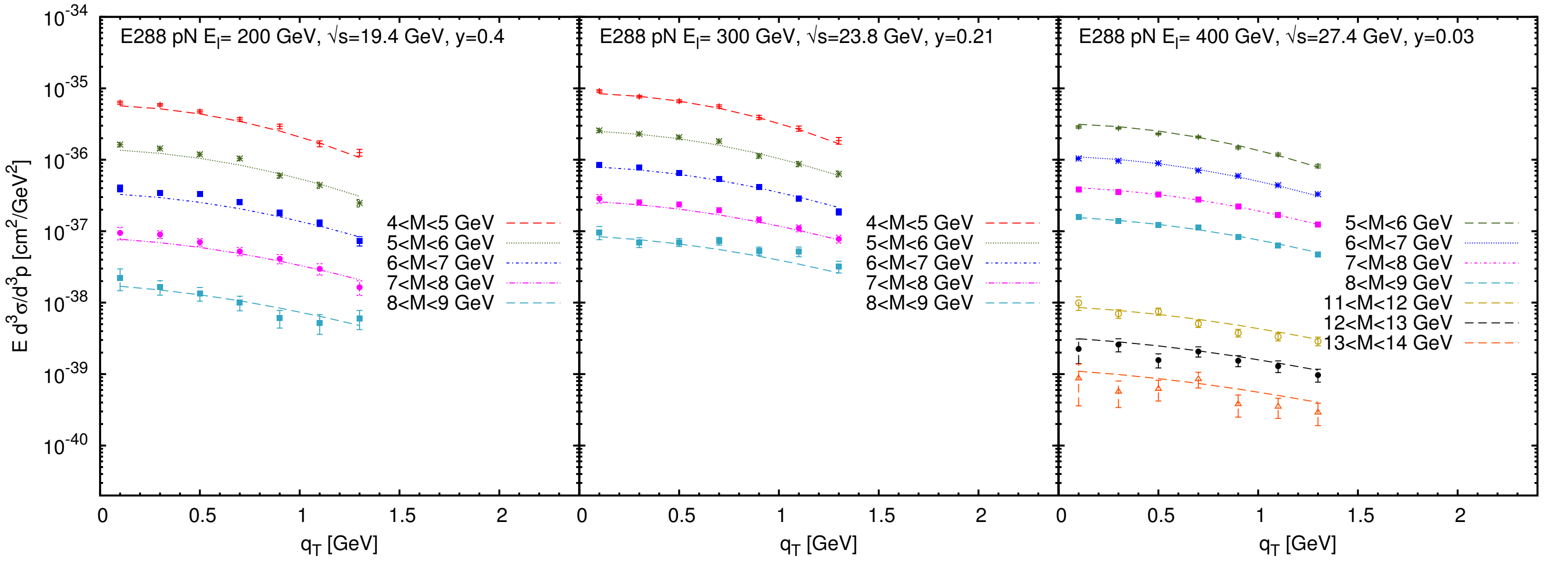


Anselmino, Boglione, Gonzalez, Melis, Prokudin, JHEP 1404 (14)

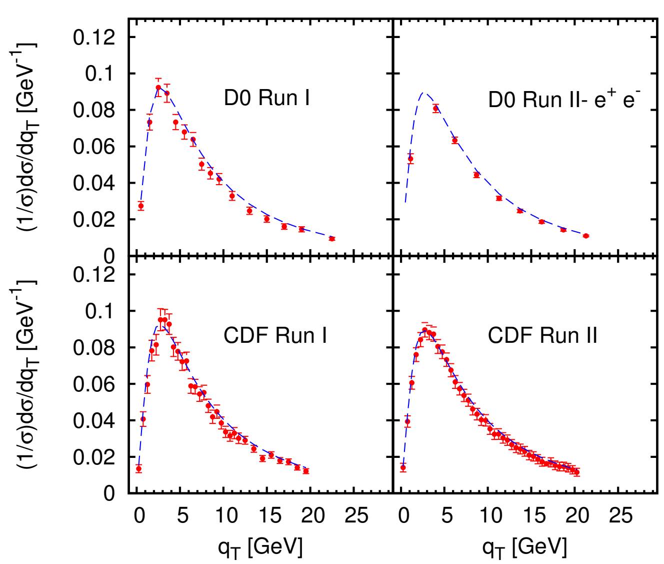
DEMS 2014

D'Alesio, Echevarria, Melis, Scimemi, JHEP 1411 (14)

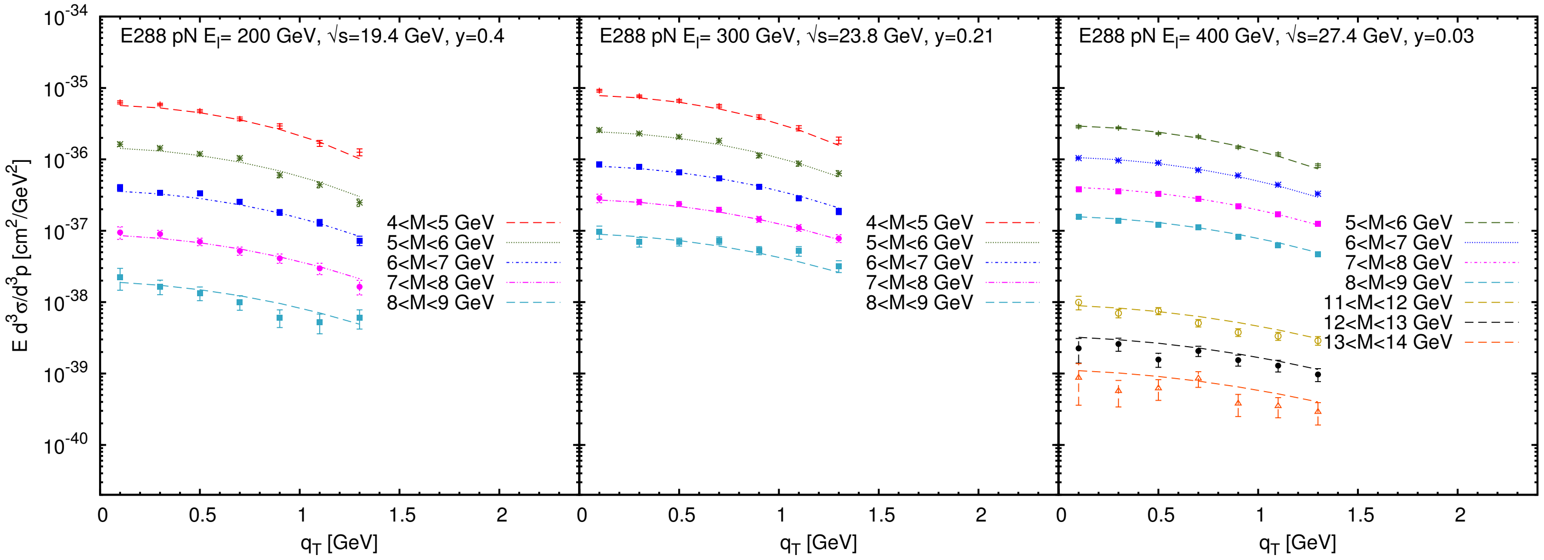
NLL-NLO



NLL-NLO



NNLL-NNLO



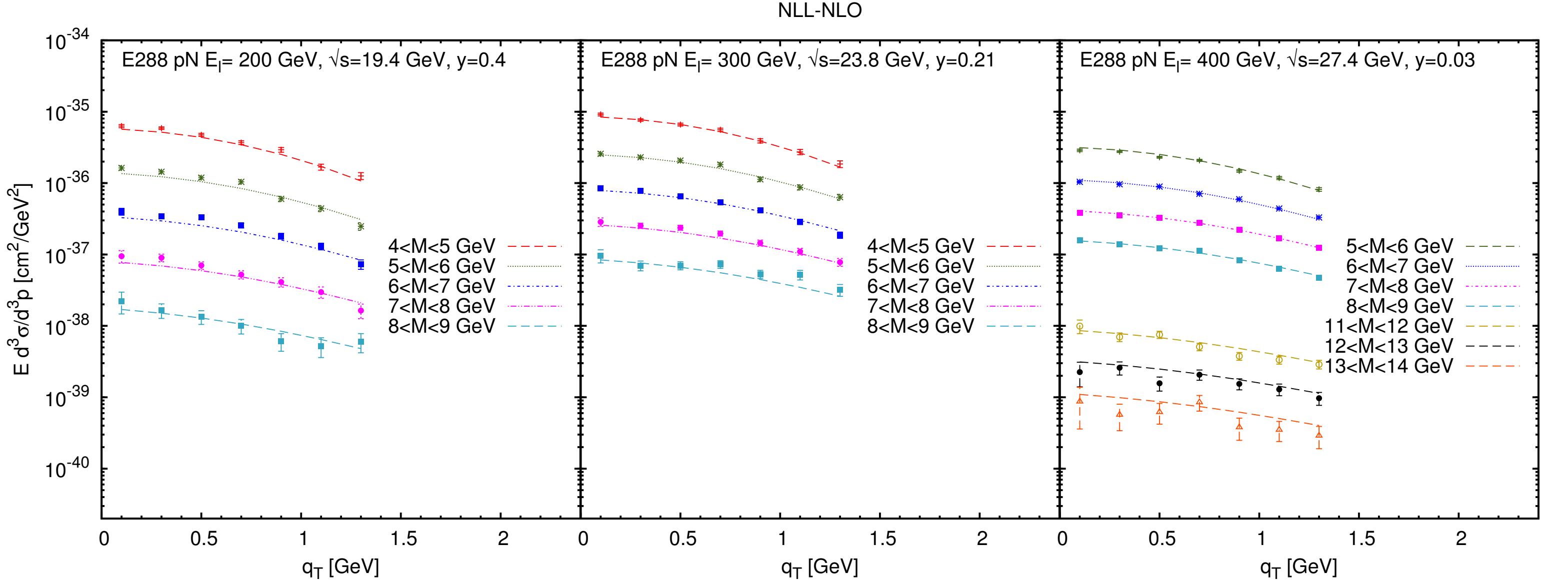
NLO-NNLL analysis
with evaluation of
theoretical uncertainties

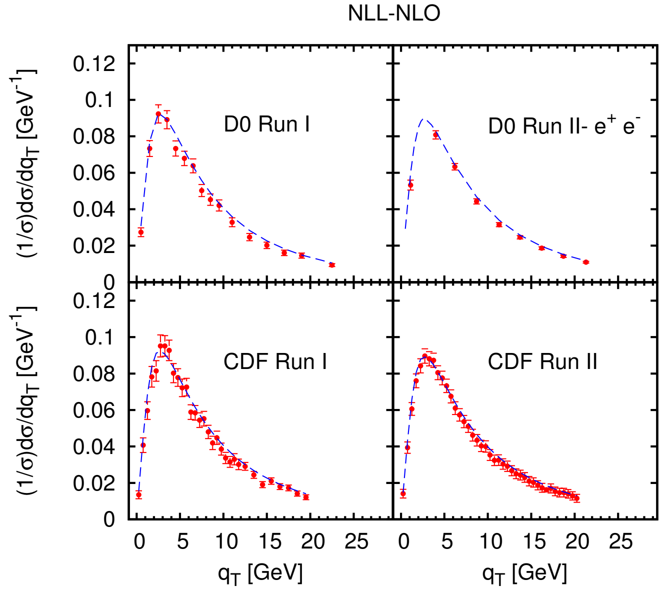
very good

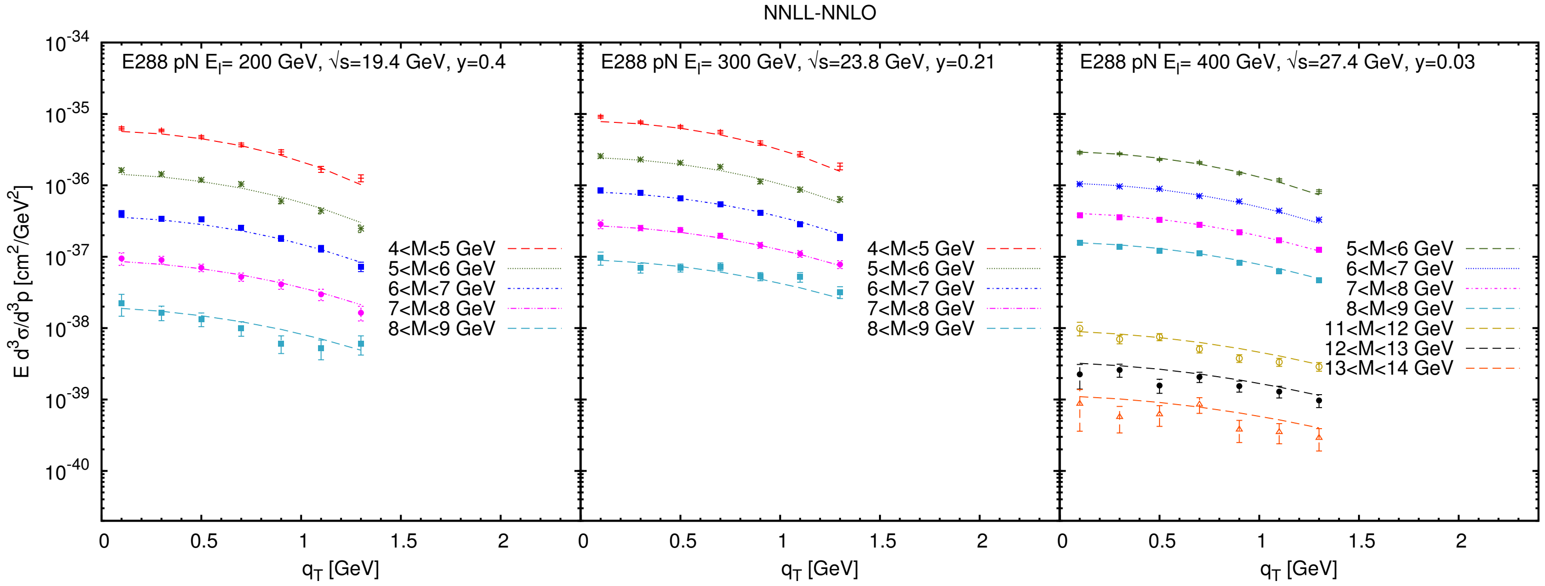
DEMS 2014

$\chi^2/\mathrm{dof} = 0.81$

D’Alesio, Echevarria, Melis, Scimemi, JHEP 1411 (14)







NLO-NNLL analysis
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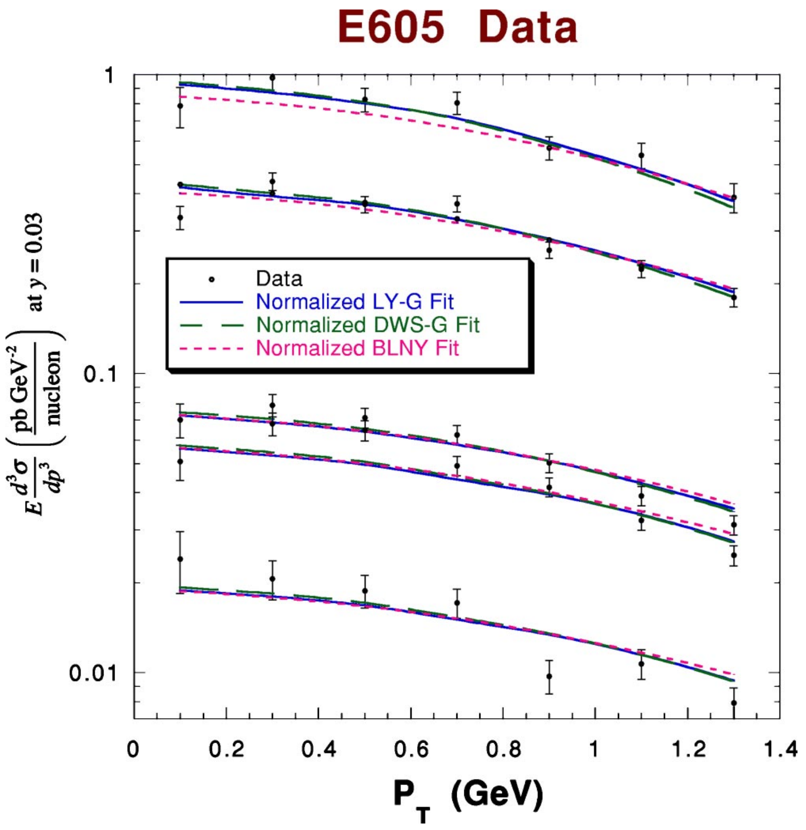
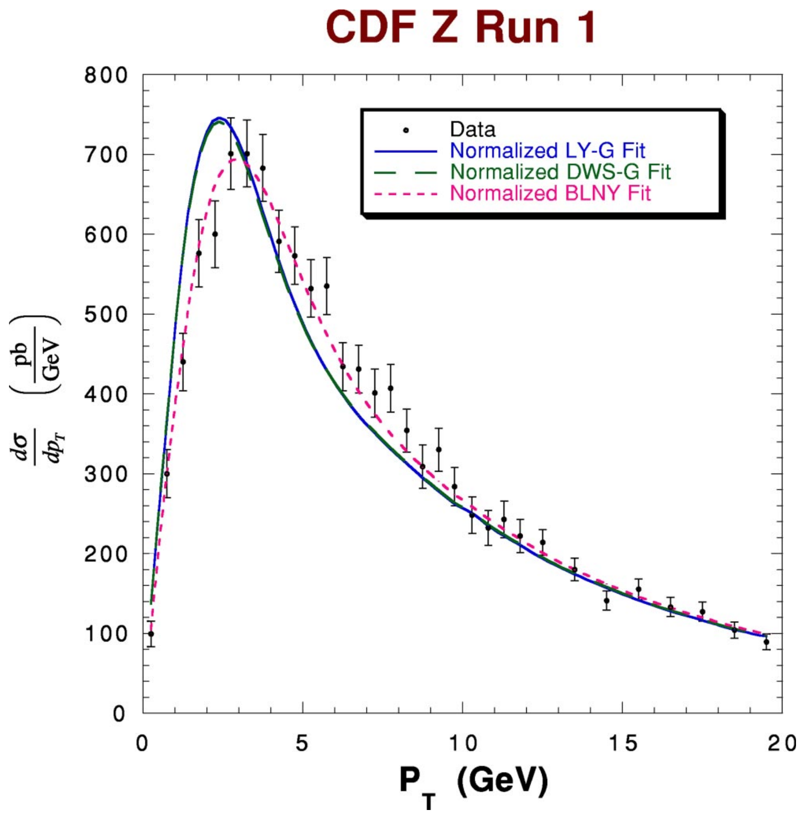
KN 2006

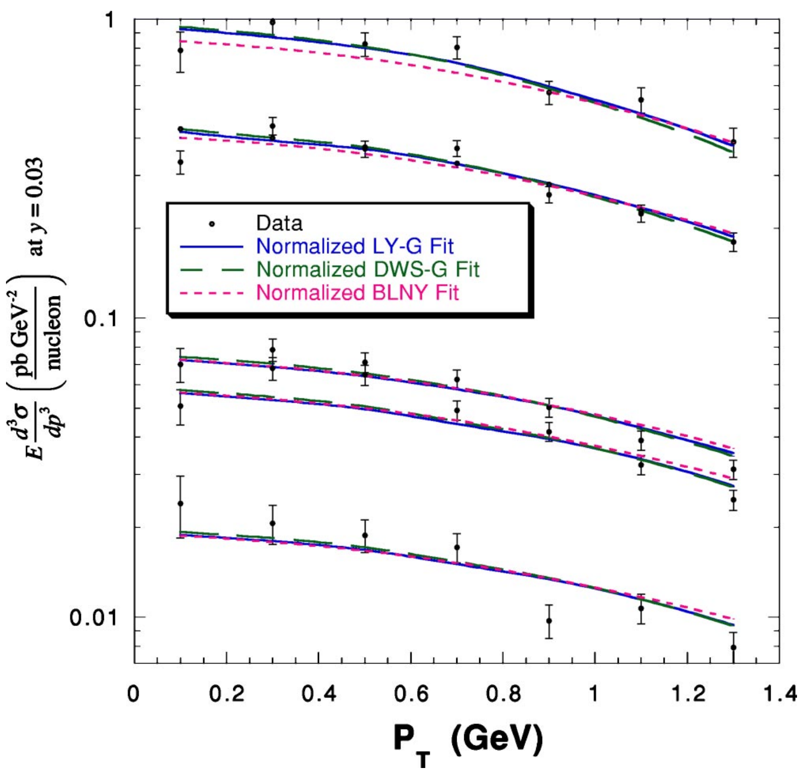

E605 Data


CDF Z Run 1

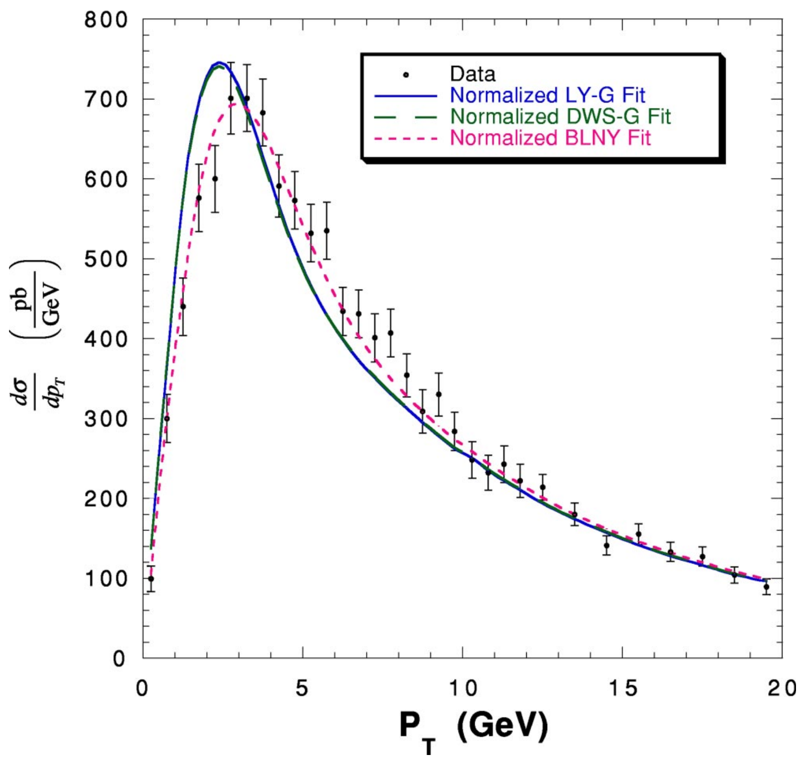
≈100 data points
Q^2>4 GeV

KN 2006

E605 Data



CDF Z Run 1



≈100 data points

$Q^2 > 4$ GeV

$$Q_0 = 3.2 \text{ GeV}$$

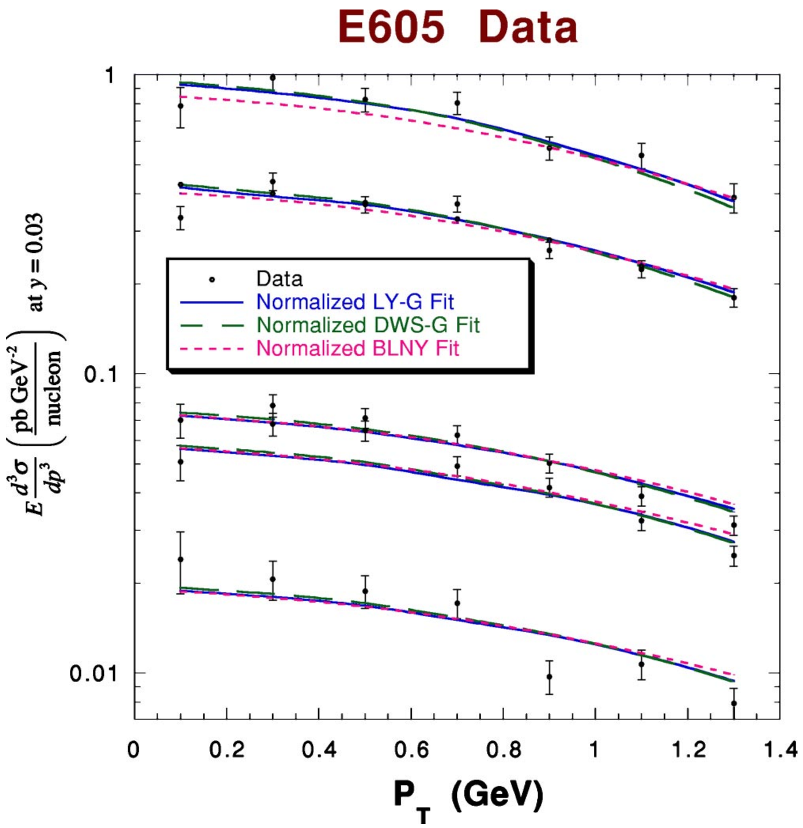
$$b_{\max} = 0.5 \text{ GeV}^{-1}$$

$$\frac{1}{\langle b_T^2 \rangle} = \frac{1}{2}\left(0.21 + 0.68 \log\left(\frac{Q}{2Q_0}\right) - 0.25 \log(10x)\right)$$

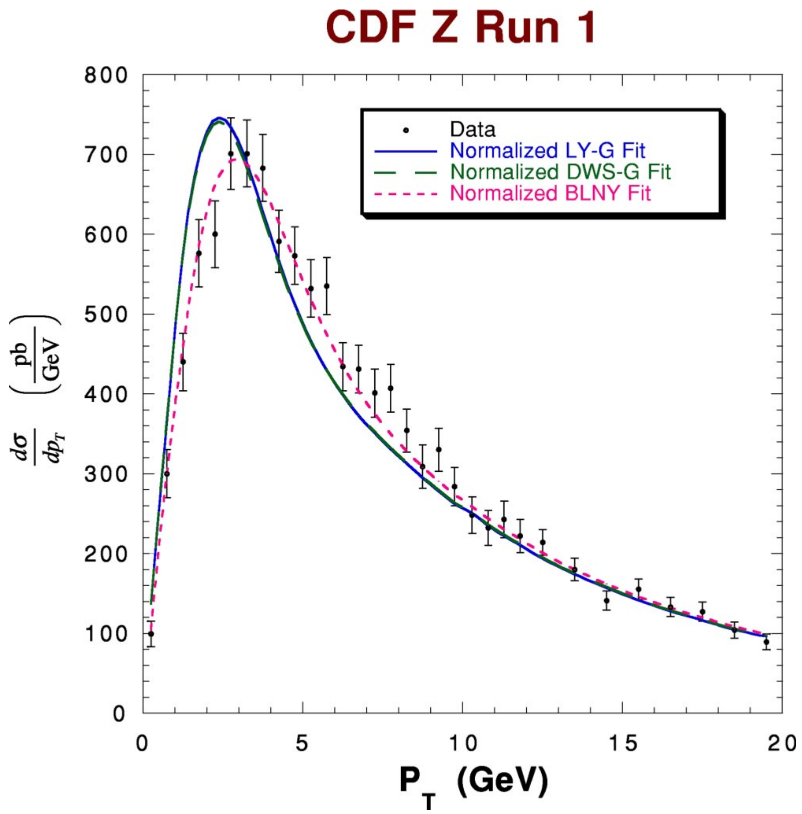
Brock, Landry, Nadolsky, Yuan, PRD67 (03)

KN 2006

E605 Data



CDF Z Run 1



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$$b_{\max} = 0.5 \text{ GeV}^{-1}$$

Brock, Landry, Nadolsky, Yuan, PRD67 (03)

$$\frac{1}{\langle b_T^2\rangle} = \frac{1}{2}\left(0.20 + 0.184 \log\left(\frac{Q}{2Q_0}\right) - 0.026 \log(10x)\right)$$

$$b_{\max} = 1.5 \text{ GeV}^{-1}$$

EIKV 2014

Parametrizations for intrinsic momenta
and soft gluon emission :

$$F_{NP}(b_T,Q)^{\text{pdf}} = \exp\left[-b_T^2\left(g_1^{\text{pdf}} + \frac{g_2}{2}\ln(Q/Q_0)\right)\right]$$

$$F_{NP}(b_T,Q)^{\text{ff}} = \exp\left[-b_T^2\left(g_1^{\text{ff}} + \frac{g_2}{2}\ln(Q/Q_0)\right)\right]$$

Pros and Cons :

1) a global analysis of SIDIS and DY/Z/W data

2) TMD evolution at LO-NLL

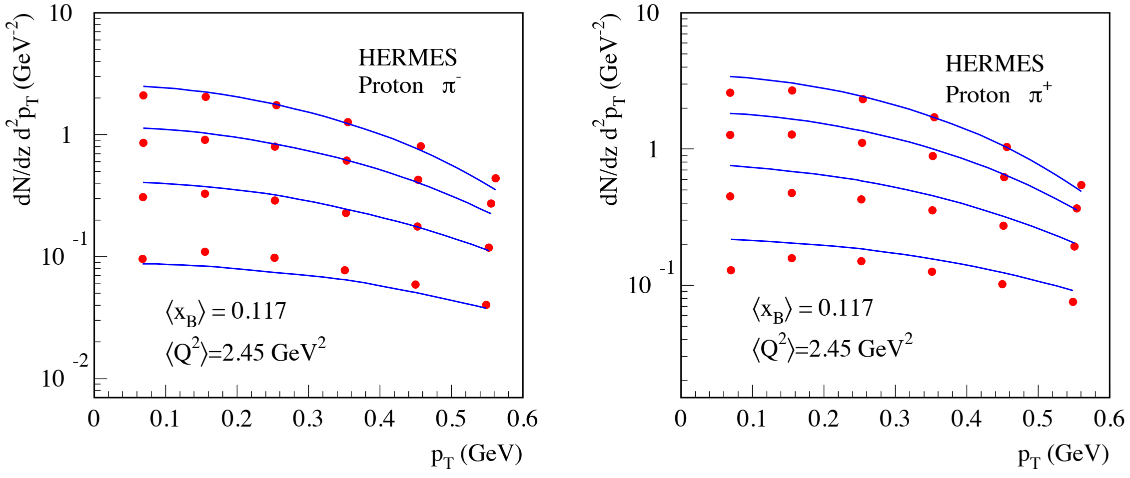
3) multidimensionality not exploited

4) chi-square not provided

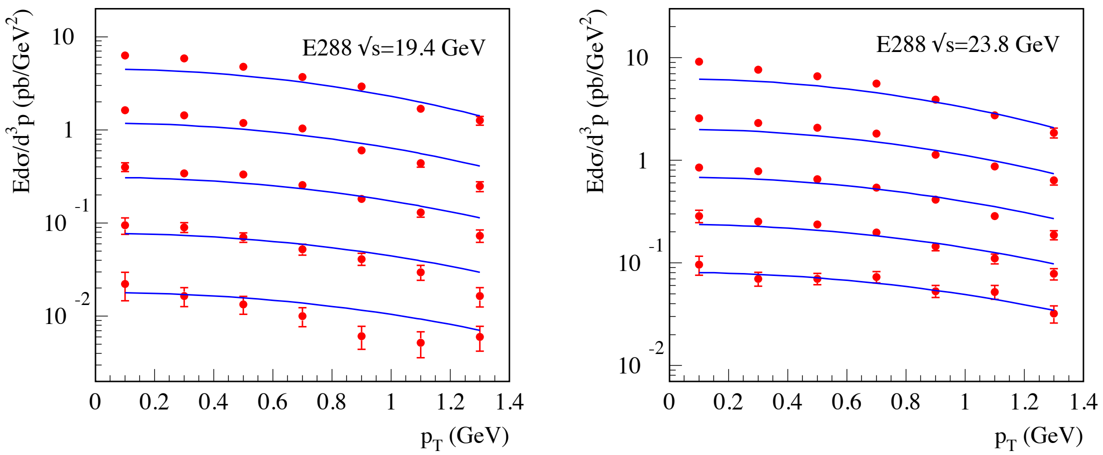
5) can't be considered as a "complete" fit

EIKV 2014

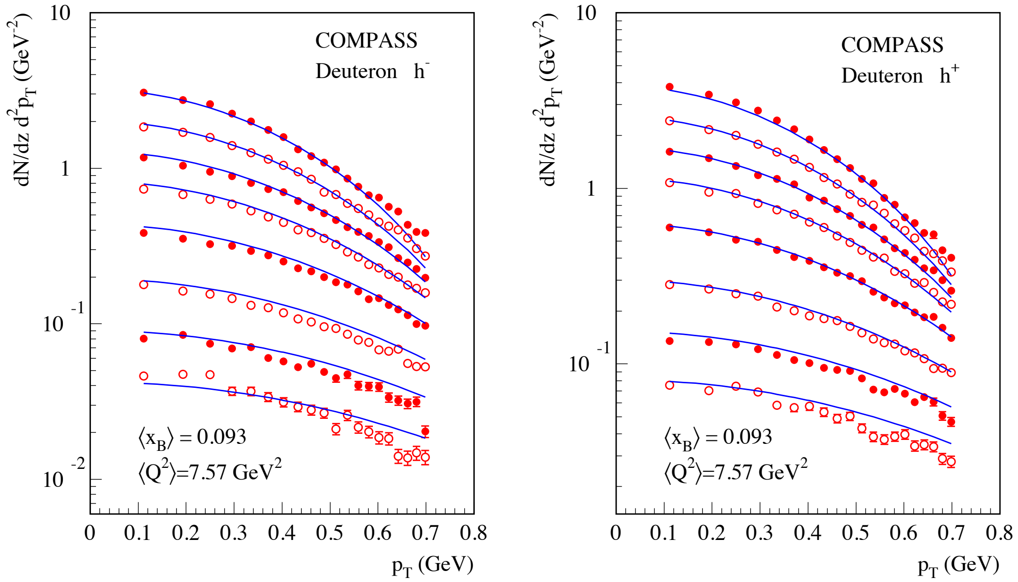
SIDIS



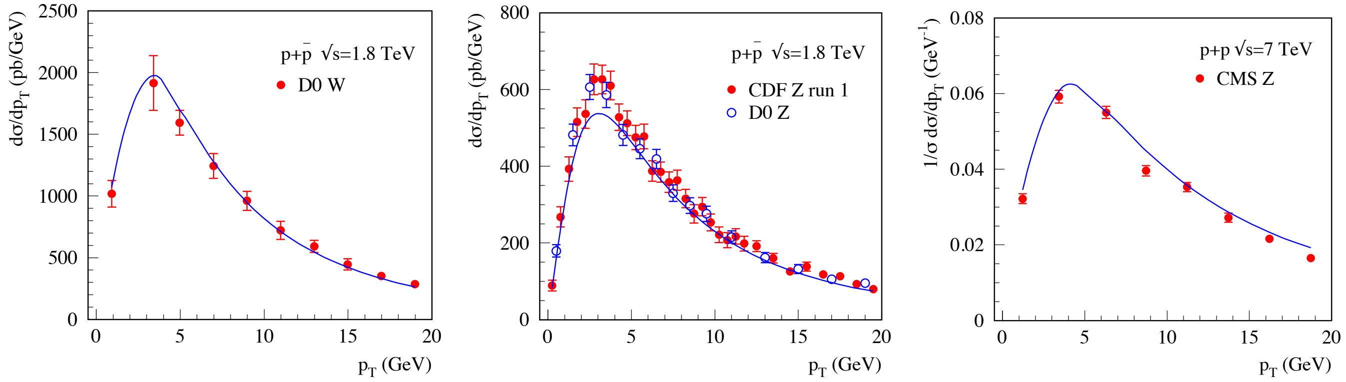
DRELL-YAN



SIDIS



W AND Z PRODUCTION



$$b_{\max} = 1.5\ \text{GeV}^{-1}$$

$$g_2 = 0.16$$

Echevarria et al. arXiv:1401.5078

Jefferson Lab

Other studies

CSS formalism on DY/Z/W data:

1) Davies-Webber-Stirling (DOI: 10.1016/0550-3213(85)90402-X)

2) Ladinsky-Yuan (DOI: 10.1103/PhysRevD.50.R4239)

3) BLNY (DOI: 10.1103/PhysRevD.63.013004)

4) Hirai, Kawamura, Tanaka (DOI: 10.3204/DESY-PROC-2012-02/136) - complex-b prescription

…

combined SIDIS/DY/W/Z :

5) Sun, Yuan (arXiv:1308.5003)

6) Isaacson, Sun, Yuan, Yuan (arXiv:1406.3073)

…

… and the next challenges

The goal is not only to fit data,
but to answer fundamental questions in QCD in the best possible way

11) identification of the current fragmentation region in SIDIS ?

12) rise the accuracy of transverse momentum resummation

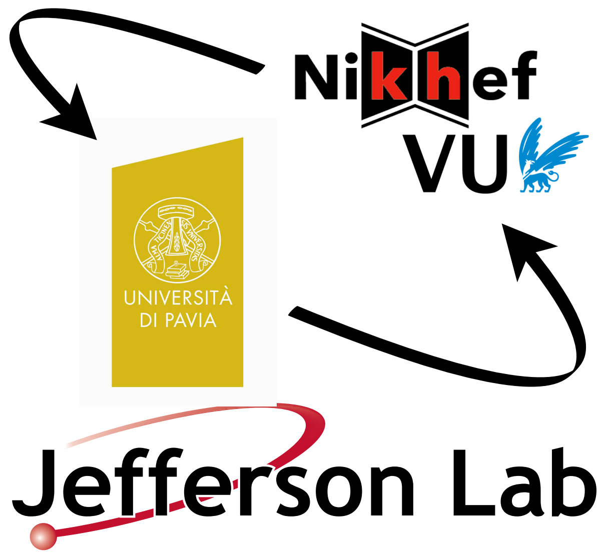
13) match TMD and collinear factorization : fixed-order description of the high transverse momentum region and its matching to the low transverse momentum one

14) order the hadronic tensor in terms of definite rank

---

15) include electron-positron annihilation, LHC and JLab data
16) address the flavor decomposition in transverse momentum
17) address the polarized structure functions
18) Monte Carlo generators and TMDs
19) what about spin 1 targets ?

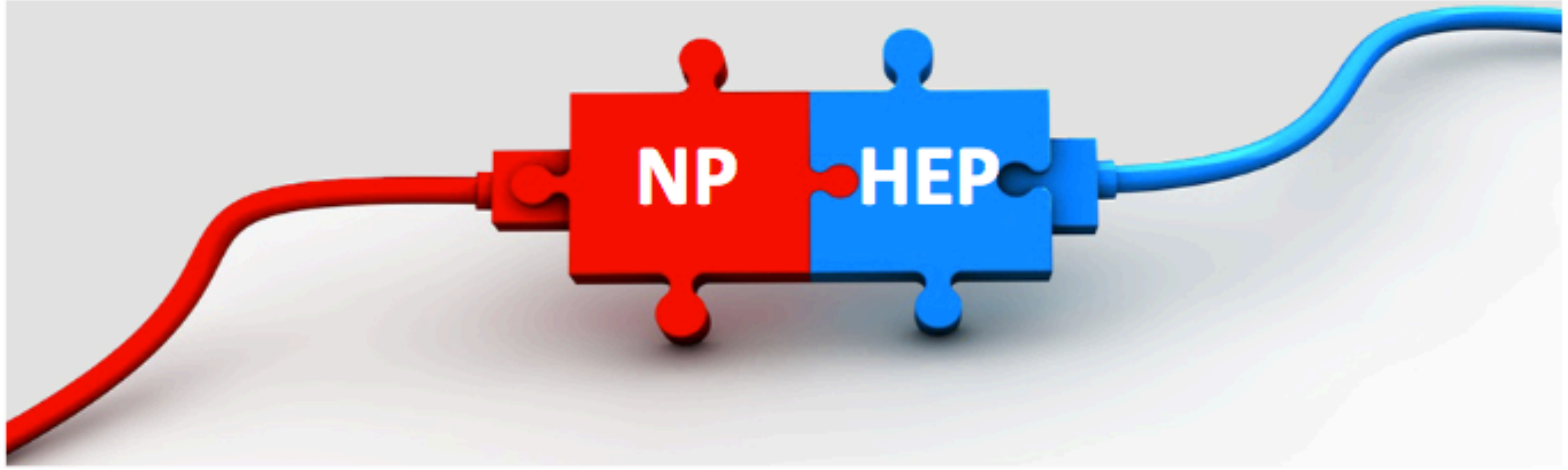
20) …



Monte Carlo generators

LDRD

Mapping the hadronization description in the Pythia MCEG to the correlation functions of TMD factorization



see the talk by M. Diefenthaler