Wandzura-Wilczek type Approximation in TMDs

Kemal Tezgin

University of Connecticut

kemal.tezgin@uconn.edu

October 5, 2017

SIDIS Cross Section

- SIDIS Cross Section consists of a linear combination of 18 Structure Functions.
- Each Structure Function is another linear combination of convolutions of a TMD PDF with a TMD FF. i.e.

$$\mathcal{C}[w\, f\, D] = x \sum_a e_a^2 \int d^2\boldsymbol{p}_T\, d^2\boldsymbol{k}_T\, \delta^{(2)}(\boldsymbol{p}_T - \boldsymbol{k}_T - \boldsymbol{P}_{h\perp}/z)\, w(\boldsymbol{p}_T, \boldsymbol{k}_T)\, f^a(x, p_T^2)\, D^a(z, z^2 k_T^2)$$

- Since each TMD PDF and FF contains information on different aspects of the nucleon, TMDs are independent objects in QCD.
- However, in quark models or in some approximations (as in WW-type) TMDs could be dependent on one another.

SIDIS Cross Section

$$\frac{d\sigma}{dx\,dy\,d\Psi\,dz\,d\phi_h\,dP_{hT}^2} = \frac{\alpha^2}{xyQ^2}\frac{y^2}{2(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\sum_{i=1}^{18} F_i(x,z,Q^2,P_{hT}^2)\beta_i$$

F_i	Standard label	β_i
F_1	$F_{UU,T}$	1
F_2	$F_{UU,L}$	ε
F_3	$F_{UU}^{\cos\phi_h}$	$\sqrt{2\varepsilon(1+\varepsilon)}\ \cos\phi_h$
F_4	$F_{UU}^{\cos 2\phi_h}$	$\varepsilon\ \cos(2\phi_h)$
F_5	$F_{LU}^{\sin\phi_h}$	$\lambda_e\sqrt{2\varepsilon(1-\varepsilon)}\sin\phi_h$
F_6	$F_{UL}^{\sin\phi_h}$	$S_{\|\|}\sqrt{2\varepsilon(1+\varepsilon)}\sin\phi_h$
F_7	$F_{UL}^{\sin\ 2\phi_h}$	$S_{\|\|}\varepsilon\sin(2\phi_h)$
F_8	F_{LL}	$S_{\|\|}\lambda_e\sqrt{1-\varepsilon^2}$
F_9	$F_{LL}^{\cos\phi_h}$	$S_{\|\|}\lambda_e\sqrt{2\varepsilon(1-\varepsilon)}\cos\phi_h$
F_{10}	$F_{UT,T}^{\sin(\phi_h-\phi_S)}$	$\|\vec{S}_\perp\|\sin(\phi_h-\phi_S)$
F_{11}	$F_{UT,L}^{\sin(\phi_h-\phi_S)}$	$\|\vec{S}_\perp\|\varepsilon\sin(\phi_h-\phi_S)$
F_{12}	$F_{UT}^{\sin(\phi_h+\phi_S)}$	$\|\vec{S}_\perp\|\varepsilon\sin(\phi_h+\phi_S)$
F_{13}	$F_{UT}^{\sin(3\phi_h-\psi_S)}$	$\|\vec{S}_\perp\|\varepsilon\sin(3\phi_h-\phi_S)$
F_{14}	$F_{UT}^{\sin\phi_S}$	$\|\vec{S}_\perp\|\sqrt{2\varepsilon(1+\varepsilon)}\sin\phi_S$
F_{15}	$F_{UT}^{\sin(2\phi_h-\phi_S)}$	$\|\vec{S}_\perp\|\sqrt{2\varepsilon(1+\varepsilon)}\sin(2\phi_h-\phi_S)$
F_{16}	$F_{LT}^{\cos(\phi_h-\phi_S)}$	$\|\vec{S}_\perp\|\lambda_e\sqrt{1-\varepsilon^2}\cos(\phi_h-\phi_S)$
F_{17}	$F_{LT}^{\cos\phi_S}$	$\|\vec{S}_\perp\|\lambda_e\sqrt{2\varepsilon(1-\varepsilon)}\cos\phi_S$
F_{18}	$F_{LT}^{\cos(2\phi_h-\phi_S)}$	$\|\vec{S}_\perp\|\lambda_e\sqrt{2\varepsilon(1-\varepsilon)}\cos(2\phi_h-\phi_S)$

SIDIS Cross Section

- Due to limited data, we only have a limited access to most of the TMDs.
- Then the question is: Can we express less known (higher-twist) TMDs by relatively better known (twist-2) ones?
- This question has an answer in WW-type approximations since
  1. It provides an order among TMDs (by neglecting "pure" twist-3 terms). So that a direct relation among 2 TMDs is possible.
  2. It relates a twist-3 TMD with a twist-2 one.

What is WW Approximation

- It was realized in PDFs that certain twist-3 PDFs, $g_T^q(x)$ and $h_L^q(x)$, obtained by $\bar{q}q$ correlators can be expressed in terms of a twist-2 PDF, $g_1^q(x)$ and $h_1^q(x)$ respectively, plus a negligibly small "pure" twist-3 term.
  [Refs: 1) S. Wandzura and F. Wilczek, Phys. Lett. B72 (1977) 195. 2) R. Jaffe and X.-D. Ji, Nucl.Phys. B375 (1992).]
- Here "pure" means PDFs obtained by $< \bar{q}gq >$ correlators.
- Those "pure" twist-3 PDFs are suppressed by the twist-3 PDFs obtained by the $< \bar{q}q >$ correlators

$$\left| \frac{< \bar{q}gq >}{< \bar{q}q >} \right| \ll 1.$$

- So when such a decomposition is possible, after neglecting the "pure" twist-3 term we relate a twist-2 PDF to a twist-3 PDF.

Procedure

- The Procedure can be summarized as follows;
  1. By using the Equations of Motion (EoM) of a quark field in QCD;

     $$(i\not{D} - m)\psi = 0$$

     decompose a twist-3 PDF obtained by $< \bar{q}q >$ correlator to a twist-2 PDF and a "pure" twist-3 PDF.
  2. Neglect the "pure" twist-3 part due to its small effect.

WW Approximation

- This "neglecting the pure twist-3 term" is known as Wandzura-Wilczek (WW) Approximation. So that a twist-3 function can be expressed by a twist-2 function. i.e.

$$g_T^q(x) = \int_x^1 \frac{dy}{y} g_1^q(y) + \tilde{g}_T^q(x) \approx \int_x^1 \frac{dy}{y} g_1^q(y)$$

$$h_L^q(x) = 2x \int_x^1 \frac{dy}{y^2} h_1^q(y) + \tilde{h}_L^q(x) \approx 2x \int_x^1 \frac{dy}{y^2} h_1^q(y).$$

- WW Approximation is supported by Semi-Classical approximations in QCD; like Chiral Quark Soliton Model. [Ref:J. Balla, M. V. Polyakov and C. Weiss, Nucl.Phys. B510 (1998)]
- Also supported by data [SLAC, JLAB].

WW-type Approximation in TMDs

- Similar to PDFs, an analogous approximation can be used in TMDs. By using the EoM in QCD, twist-3 TMDs can be decomposed in terms of a twist-2 TMD plus a tilde twist-3 term.
- Recall that quark-quark and quark-gluon-quark correlation functions are defined as

$$\Phi_{ij}(x,p_T) = \int \frac{d\xi^- d^2\boldsymbol{\xi}_T}{(2\pi)^3}\, e^{ip\cdot\xi}\, \langle P|\bar{\psi}_j(0)\,\mathcal{U}^{n_-}_{(0,+\infty)}\,\mathcal{U}^{n_-}_{(+\infty,\xi)}\,\psi_i(\xi)|P\rangle\Big|_{\xi^+=0}$$

$$\left(\Phi^{\mu}_{D}\right)_{ij}(x,p_T) = \int \frac{d\xi^- d^2\boldsymbol{\xi}_T}{(2\pi)^3}\, e^{ip\cdot\xi}\langle P|\bar{\psi}_j(0)\,\mathcal{U}^{n_-}_{(0,+\infty)}\,\mathcal{U}^{n_-}_{(+\infty,\xi)}\, iD^{\mu}(\xi)\,\psi_i(\xi)|P\rangle\Big|_{\xi^+=0}$$

  where D^{μ}, $\mathcal{U}^{n-}$ denote the covariant derivative and Wilson line respectively.
- Again by neglecting the "pure" twist-3 part we relate a twist-3 TMD with a twist-2 TMD. Due to this analogy with PDFs, it is called WW-type approximations.

WW-type Approximation in TMDs

- Under WW-type approximation, the whole set of SIDIS structure functions can be expressed by a basis of 6 twist-2 TMDs (f_1^q,$f_{1T}^{\perp q}$,g_1^q,h_1^q,$h_1^{\perp q}$,$h_{1T}^{\perp q}$) and 2 twist-2 FFs (D_1^q, $H_1^{\perp q}$).
  Recall the TMDs
  1. f_1^q: Unpolarized
  2. g_1^q: Helicity
  3. h_1^q: Transversity
  4. $f_{1T}^{\perp q}$: Sivers
  5. $h_1^{\perp q}$: Boer-Mulders
  6. $h_{1T}^{\perp q}$: Pretzelosity

  and the FFs
  1. D_1^q: Unpolarized
  2. $H_1^{\perp q}$: Collins

Example: $F_{UU}^{cos\phi_h}$

- Let us apply WW-type approximation to a specific structure function, $F_{UU}^{cos\phi_h}$. The original definition is given by

$$F_{UU}^{\cos\phi_h} = \frac{2M}{Q}\mathcal{C}\Big[\ \frac{\hat{\mathbf{h}}\cdot\mathbf{P}_\perp}{zm_h}\Big(xh\,H_1^\perp + \frac{m_h}{M}\,f_1\frac{\tilde{D}^\perp}{z}\Big) - \frac{\hat{\mathbf{h}}\cdot\mathbf{k}_\perp}{M}\Big(xf^\perp D_1 + \frac{m_h}{M}\,h_1^\perp\frac{\tilde{H}}{z}\Big)\Big]$$

- After neglecting the tilde ("pure" twist-3) terms and applying the following WW-type approximations

$$xf^{\perp q}(x,k_\perp^2) \approx f_1^q(x,k_\perp^2)$$

$$xh^q(x,k_\perp^2) \approx -\frac{k_\perp^2}{M^2}h_1^\perp(x,k_\perp^2)$$

We obtain

$$F_{UU}^{cos\phi_h} = \frac{2M}{Q}\mathcal{C}\Big[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{P}_\perp}{zm_h}\Big(\frac{k_\perp^2}{M^2}h_1^\perp H_1^\perp\Big) - \frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_\perp}{M}\Big(f_1 D_1\Big)\Big]$$

Conclusions

- Our current experimental knowledge on TMDs that appear in SIDIS Cross Section is insufficient to determine most of the TMDs.
- In order to get an access to those functions, physically motivated models/approximations are needed.
- WW-type Approximation will be the first one to describe the complete SIDIS Cross Section for pion production. [Ref: S. Bastami et al., in preparation]
- Such a complete description of SIDIS Cross Section is important for EVA framework. [see the talk by N. Sato]