Coarse graining short range correlations

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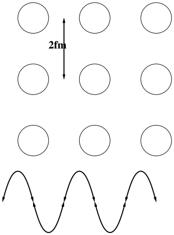
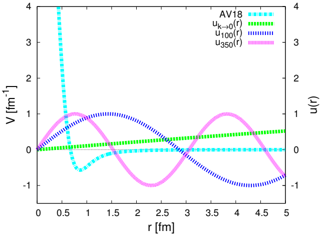
Scales in nuclei

- Nuclear matter density

$$\rho = \frac{A}{V} = \frac{1}{d^3} = 0.17\text{fm}^3 \qquad \rightarrow \qquad d = 1.8\text{fm}$$

- Fermi momentum $k_F \sim 270\text{MeV}$
- Range of the interaction $\sim 1/m_\pi \sim 1.4\text{fm}$
- Size of the nucleon $r_N \sim 0.9\text{fm}$
- Most energetic process for two nucleons with $p_1 + p_2 = 0$ (back-to-back) is $p = |p_1 - p_2| = 2k_F$





- Pion production $p \sim \sqrt{M_N m_\pi} \sim 400\text{MeV}$
- We need the nuclear force to describe scattering below $2k_F$
- FINITE RESOLUTION $\Delta r \sim 1/2k_F \sim 0.4\text{fm} \sim r_{\text{core}}$.

The Nuclear Many Body Proplem

THE DIRECT PROBLEM IN NUCLEAR PHYSICS

- The Nuclear Hamiltonian

$$H = T + V_{2N} + V_{3N} + V_{4N} + \dots$$

- Solve Schrödinger Equation (technical problem)

$$H(A)\Psi_n(x_1, \dots, x_A) = E_n \Psi_n(x_1, \dots, x_A)$$

THE INVERSE PROBLEM IN NUCLEAR PHYSICS

- V_{2N} from NN scattering data and deuteron d$\equiv{}^2$H
- V_{3N} from nd data and triton t$\equiv{}^3$H , ^{3}He
- V_{4N} from dd , t n , t p , ^{4}He

Correlations depend on the interaction. Do we know the interaction ?
Elusive evidence for short range correlations

The energy funtional

- The total energy (central potential)

$$
\begin{aligned}
E(A) &= A\int \frac{d^3p}{(2\pi)^3}\frac{p^2}{2M}\,n_1(p) + \frac{A(A-1)}{2}\int d^3r\, n_2(r)\,V_2(r) \\
&+ \frac{A(A-1)(A-3)}{2}\int d^3r\, n_3(r,s)\,V_3(r,s) + \dots
\end{aligned} \tag{1}
$$

- Distribution functions

$$
\begin{aligned}
n_1(p) &= \int \prod_{i=1}^{A}\frac{d^3p_i}{(2\pi)^3}|\Psi(p_1,\dots,p_A)|^2\delta(\sum_i p_i)\delta(p_1-p) \\
n_2(r) &= \int \prod_{i=1}^{A} d^3x_i|\Psi(x_1,\dots,x_A)|^2\delta(\sum_i x_i)\delta(x_1-x_2-r) \\
n_3(r,s) &= \int \prod_{i=1}^{A} d^3x_i|\Psi(x_1,\dots,x_A)|^2\delta(\sum_i x_i)\delta(x_1-x_2-r)\delta\left(\frac{x_1+x_2)}{2}-x_3-s\right)
\end{aligned} \tag{2}
$$

The Independent particle model

- Long distances $r_1 \to \infty$ (Separation energy, $S_A = E_A - E_{A-1}$)

$$\Psi(x_1,\dots,x_A) \to \chi(x_1)\Psi(x_2,\dots,x_A)\,, \qquad -\frac{\nabla^2}{2\mu}\chi(x_1) = S_A\chi(x_1)\,, \quad \frac{1}{\mu} = \frac{1}{M}+\frac{1}{(A-1)M}$$

- Average field

$$H = \sum_{i=1}^{A} \frac{p_i^2}{2M} + \sum_{i=1}^{A} U(x_i) \equiv \sum_{i=1}^{A} H_i$$

- Product Solution

$$\begin{aligned} H_i\phi_{\alpha_i}(x_i) &= \epsilon_{\alpha_i}\phi_{\alpha_i}(x_i) \to E = \sum_{i=1}^{A}\epsilon_{\alpha_i} \\ \Psi(x_1,\dots,x_A) &= \phi_{\alpha_1}(x_1)\dots\phi_{\alpha_1}(x_A) \to |\Psi(x_1,\dots,x_A)|^2 = |\phi_{\alpha_1}(x_1)|^2\dots|\phi_{\alpha_1}(x_A)|^2 \end{aligned}$$

- Fermi Statistics $\to$ Antisymetrize (correlations)

$$\begin{aligned} [H,P]=0 \quad P\in S_A \qquad \Psi \to \mathcal{A}\left[\phi_{\alpha_1}(x_1)\dots\phi_{\alpha_1}(x_A)\right] \\ |\Psi(x_1,\dots,x_A)|^2 = |\phi_{\alpha_1}(x_1)|^2\dots|\phi_{\alpha_1}(x_A)|^2 + \dots \end{aligned} \tag{3}$$

The Independent pair model

- Short distances $r_{12} \to 0$ for $V_{12} \gg E_A$ (repulsive core)

$$\Psi_A(x_1, \dots, x_A) \to \chi_2(x_1, x_2)\chi_{A-2}(x_3, \dots, x_A)$$

- Universal behaviour

$$n_{2,A}(r) \underbrace{\to}_{r\to 0} N_A|\chi_2(r)|^2 \qquad N_A = \int |\chi_{A-2}(R_{12}, x_3, \dots, x_A)|^2 d^3R \dots$$

$$n_{2,A}(p) \underbrace{\to}_{p\to\infty} N_A|\chi_2(p)|^2$$

- Independent pair approximation

$$\Psi_A(x_1, \dots, x_A) = \mathcal{A}\left[\chi(r_{12}, R_{12})\chi_{A-2}(R_{12}, x_3, \dots, x_A)\right]$$

- Bethe-Goldstone equation Independent pair approximation

$$\left[H_0 + QV_{12}\right]\psi = e\psi \qquad Q = \sum_{k_1,k_2>k_F} |k_1 k_2\rangle\langle k_1 k_2|$$

- Effective interaction G-matrix

$$G = V + \frac{Q}{e - H_0} G \to |\psi\rangle = |\varphi\rangle + \frac{Q}{E - H_0}|\psi\rangle$$

Bethe-Goldstone S-wave equation

- Two particles at rest (back-to-back scattering)
- For S-wave and $E = 0$ we have an integral equation

$$\begin{aligned} u_k(r) &= \sin(kr) + \frac{2}{\pi}\int_0^\infty ds U(s)\, u_k(s) \int_{k_F}^\infty dq\, \frac{\sin(qs)\sin(qr)}{k^2-q^2} \\ &\to \sin(kr) + Z\frac{\cos(k_F r)}{r} \qquad \text{(no scattering)} \end{aligned} \tag{4}$$

- In momentum space ($k < k_F$, $p > k_F$)

$$\widetilde{\Phi}_k(p) = \frac{1}{p}\frac{\sqrt{4\pi}}{(2\pi)^3}\left\{\frac{\pi}{2}\,\delta(p-k) + \frac{\theta(p-k_F)}{k^2-p^2}\int_0^\infty dr U(r)\, u_k(r)\, \sin(pr)\right\}$$

- The defect function

$$\zeta_k(r) = u_k(r) - \sin(kr)$$

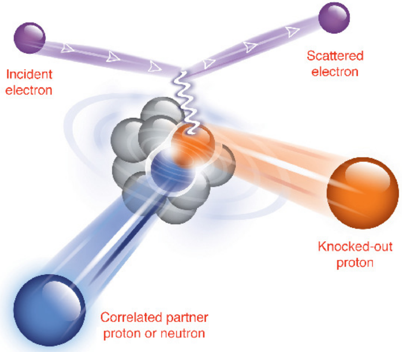
- Number of back-to-back particles in the s-channel

$$N_k(p) \sim |\widetilde{\Phi}_k(p)|^2$$

Hall A JLAB Experiment

- $^{12}C(e,e'pN)$ (Back-to-back kinematics)
- $E_e = 4.627\text{GeV}$, $I = 5 - 40\mu A$, $(q,\omega) = (1.65, 0.865)$GeV, $Q^2 = 2\text{GeV}^2$

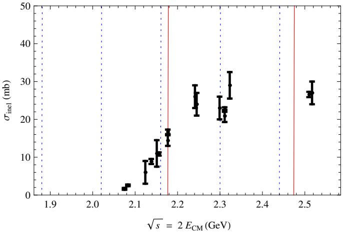
$$\frac{N_{pn}}{N_{pp}} = 18 \pm 5\,, \qquad 300 \le p \le 600\,\text{MeV}$$

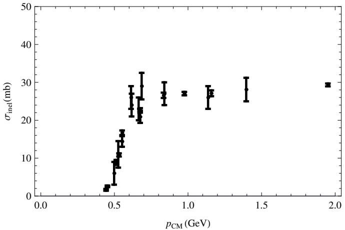


The nuclear force

- What are the relevant scales for back-to-back $300 \leq p \leq 600\,\mathrm{MeV}$?
- Fermi momentum $k_F \sim 270\mathrm{MeV}$
- Δ-production , $NN \to N\Delta$

$$p_\Delta = \sqrt{M_N(M_\Delta - M_N)} \sim 2k_F \sim 540\mathrm{MeV}$$





- We need the nuclear force to describe *elastic* scattering below p_Δ
- FINITE RESOLUTION $\Delta r \sim 1/p_\Delta \sim 0.4\mathrm{fm} \sim r_{\rm core}$.

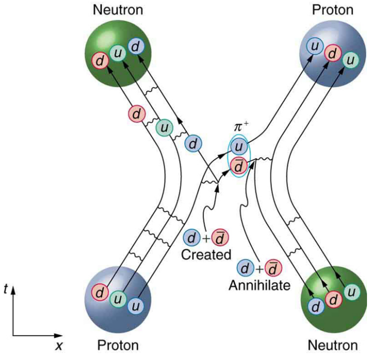
FUNDAMENTAL VS EFFECTIVE FORCES

Fundamental Forces-Elementary particles

- The particle exchange picture "explains" all interactions for elementary particles
- Quarks and gluons are elementary and interact strongly
- Most of the masses of hadrons comes from the interaction

- Hadrons are composite. When are they effectively elementary ?
- Hadrons have a size. When are they effectively point-like ?
- Hadrons can be internally excited. When are they effectively
- Hadrons interact via van der Waals forces

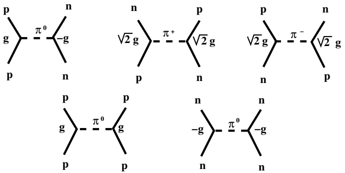
Fundamental approach: QCD



- Lattice form factor $g_{\pi NN} \sim 10 - 12$
- Lattice NN potential $g^2_{\pi NN}/(4\pi) = 12.1 \pm 2.7$
- QCD sum rules $g_{\pi NN} \sim 13(1)$

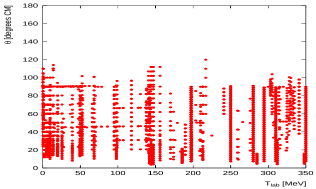
Long distances

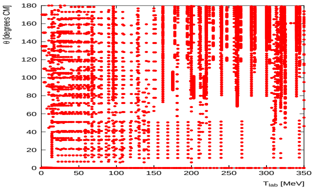
- Nucleons exchange JUST one pion



- Low energies (about pion production) 8000 pp + np scattering data (polarizations etc.)
- Granada coarse grained analysis (2016) (isospin breaking !!)

$$g_p^2/(4\pi) = 13.72(7) \neq g_n^2/(4\pi) = 14.91(39) \neq g_c^2/(4\pi) = 13.81(11)$$





Effective Field Theory Paradigm

- Low energy physics (long wavelength $\lambda \gg r_e, a$
- Effective elementarity $\to \varphi(x)$ (effective field)

$$\sigma(x) = Z\bar{q}(x)q(x) + Z_2\lambda^2\bar{q}D^2q + Z_3\lambda^3\left[\bar{q}(x)q(x)\right]^2 + Z_4\lambda^6\left[\bar{q}(x)D_\mu q(x)\right]^2 + \dots$$

- Effective interactions

$$\mathcal{L} = \frac{1}{2}\left[(\partial\sigma)^2 - m^2\sigma^2\right] + C_0\lambda^{-2}\sigma^4 + C_2\lambda^{-4}\sigma^2(\partial\sigma)^2 + \dots$$

- Loops are suppressed $p\lambda \ll 1 \to$ Power counting

EFT (polynomial power counting of potential)

- Scattering problem: Lippmann-Schwinger equation

$$
\begin{aligned}
T_l(p',p) &= V_l(p',p) + \frac{2}{\pi}\int_0^\infty dq V_l(p',q)\frac{q^2}{k^2-q^2}T_l(q,p) \\
&= V_l^\Lambda(p',p) + \frac{2}{\pi}\int_0^\Lambda dq V_l^\Lambda(p',q)\frac{q^2}{k^2-q^2}T_l(q,p)
\end{aligned}
$$

- In the partial wave amplitude we have a left-hand branch cut due to particle π exchange

$$
|p|,|p'| < m_\pi/2 \quad (1\pi) \qquad |p|,|p'| < m_\pi \quad (2\pi) \qquad |p|,|p'| < 3m_\pi/2 \quad (3\pi)
$$

- Low momentum expansion

$$
V_l(p',p) = p'^l p^l \left[C_0(m_\pi) + C_2(m_\pi)(p^2+p'^2) + \dots\right] \qquad C_0(m_\pi^2) = c_0 + c_0' m_\pi^2 + \dots
$$

- Count powers of $Q = p, p', m_\pi$ and use $\Lambda_\chi = 4\pi f_\pi \sim M_N$ but keep $\Lambda \sim n m_\pi/2$ for $n-\pi$ exchange
- Full chiral potential to $\mathcal{O}(Q^n/\Lambda_\chi^n)$

$$
V_\chi(p',p) - V_{1\pi}(p'p) - V_{2\pi}(p',p) - \dots = p'^l p^l \left[c_0 + c_2(p^2+p'^2) + c_0' m_\pi^2 + \dots\right]
$$

Fitting EFT

- We do not know the EFT constants $\to$ fitting $N_{\rm Dat}$ data with $N_{\rm Par}$ parameters

$$\mathcal{O}_i^{\rm exp} = \mathcal{O}_i(c_1, c_2, \dots) \pm \Delta\mathcal{O}_i^{\rm exp} ? \qquad i = 1, \dots, N_{\rm Dat}$$

- Standard χ^2 minimization

$$\chi^2_{\rm min} = \min_{c_1, c_2, \dots} \chi^2(c_1, c_2, \dots) = \sum_{i=1}^{N_{\rm Dat}} \left[\frac{\mathcal{O}_i(c_1, c_2, \dots) - \mathcal{O}_i^{\rm exp}}{\Delta\mathcal{O}_i^{\rm exp}} \right]^2$$

- Probabilistic answer: p-value

$$p = 0.68 \qquad \chi^2_{\rm min}/\nu = 1 \pm \sqrt{2/\nu} \qquad \nu = N_{\rm Par} - N_{\rm Dat}$$

- Confidence interval for parameters

$$c_i = c_i^{\rm Fit} \pm \Delta c_i \qquad \Delta\chi^2 = 1$$

- What happens when there are incompatible data ?
- TRUNCATED EFT (with $\Delta\mathcal{O}^{\rm th}$) $\to$ Validate/falsify data vs validate/falsify EFT
- Do we need $N_{\rm Par} \to \infty$ when $\Delta\mathcal{O}^{\rm exp} \to 0$ or $N_{\rm Dat} \to \infty$?

Coarse graining approach

Particle exchange leads to small forces at long distances $rm_\pi \gg 1$
$\to$ Perturbative definition of QM potential stemming from QFT

$$f^{\rm Born}_{\rm QFT}(\theta, E) = -\frac{2\mu}{4\pi} \int d^3 x e^{-i\vec{k}'\cdot\vec{x}} V_{\rm QFT}(\vec{x},\vec{p}) e^{i\vec{k}\cdot\vec{x}} \, ,$$

- What is the elementarity radius r_e?

$$V(r) = V_{\rm QFT}(r) \qquad r > r_e$$

- What is the maximum CM momentum $p_{\rm CM,max}$?

$$\Delta r = \hbar / p_{\rm CM,max} \to r_n = n \Delta r \quad l_{\rm max} = p r_e$$

- Centrifugal barrier

$$p^2 > \frac{l(l+1)}{r^2} \to pr < l + 1/2$$

- Use $V(r_n)$ as fitting parameters

Coarse graining approach

- The complete fitting potential (delta-shells)

$$V(r) = \underbrace{\left\{ \sum_i \Delta r V(r_i) \delta(r - r_i) \right\}}_{\text{Short(Coarse grained)}} \theta(r_e - r) + \underbrace{V_{\text{QFT}}(r)}_{\text{Long(Particle exchange)}} \theta(r - r_e)$$

- The number of fitting parameters

$$N_{\text{Par}} = \sum_{n,l} \sim \frac{1}{2} (p_{\max} r_c)^2 \times \text{spin, isospin}$$

- Compare to data with χ^2. Check p-value
- Confidence interval

$$V(r_n)|_{\text{Fit}} \pm \Delta V(r_n)$$

- If $N_{\text{Dat}} \to \infty$ we expect N_{Par}-FIXED but $\Delta V(r_n) \to 0$
- THE NUMBER OF PARAMETERS IS ALWAYS THE SAME: WE CAN USE COARSE GRAINING TO FIT AND SELECT DATA

COARSE GRAINING NN (LOW ENERGY)

Nucleon-Nucleon Scattering

- Scattering amplitude

$$
\begin{aligned}
M &= a + m(\sigma_1 \cdot \mathbf{n})(\sigma_2 \cdot \mathbf{n}) + (g-h)(\sigma_1 \cdot \mathbf{m})(\sigma_2 \cdot \mathbf{m}) \\
&+ (g+h)(\sigma_1 \cdot \mathbf{l})(\sigma_2 \cdot \mathbf{l}) + c(\sigma_1 + \sigma_2) \cdot \mathbf{n} \\
\mathbf{l} &= \frac{\mathbf{k}_f + \mathbf{k}_i}{|\mathbf{k}_f + \mathbf{k}_i|} \qquad \mathbf{m} = \frac{\mathbf{k}_f - \mathbf{k}_i}{|\mathbf{k}_f - \mathbf{k}_i|} \qquad \mathbf{n} = \frac{\mathbf{k}_f \wedge \mathbf{k}_i}{|\mathbf{k}_f \wedge \mathbf{k}_i|}
\end{aligned}
$$

- 5 complex amplitudes $\to$ 24 measurable cross-sections and polarization asymmetries
- Partial Wave Expansion

$$
\begin{aligned}
M^s_{m'_s, m_s}(\theta) &= \frac{1}{2ik} \sum_{J,l',l} \sqrt{4\pi(2l+1)} Y^{l'}_{m'_s - m_s}(\theta, 0) \\
&\times \quad C^{l',s,J}_{m_s - m'_s, m'_s, m_s} i^{l-l'} (S^{J,s}_{l,l'} - \delta_{l',l}) C^{l,s,J}_{0,m_s,m_s}, \qquad (5)
\end{aligned}
$$

- S-matrix

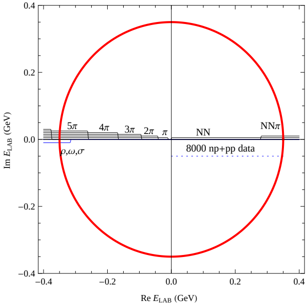
$$
S^J = \begin{pmatrix} e^{2i\delta^{J,1}_{J-1}} \cos 2\epsilon_J & ie^{i(\delta^{J,1}_{J-1} + \delta^{J,1}_{J+1})} \sin 2\epsilon_J \\ ie^{i(\delta^{J,1}_{J-1} + \delta^{J,1}_{J+1})} \sin 2\epsilon_J & e^{2i\delta^{J,1}_{J+1}} \cos 2\epsilon_J \end{pmatrix}, \qquad (6)
$$

Analytical Structure

- $s = 4(M_N^2 + p^2) \to E_{\rm LAB} = 2p^2/M_N$
- Partial Wave Scattering Amplitude analytical for $|p| \le m_\pi/2$

$$T^J_{ll'}(p) \equiv S^J_{ll'}(p) - \delta_{l,l'} = p^{l+l'} \sum_n C_{n,l,l'} p^{2n}$$

- Nucleons behave as elementary (AT WHAT SCALE ?)



- Nucleons are heavy → Local Potentials

$$V_{n\pi}(r) \sim \frac{g^{2n}}{r} e^{-n m_\pi r}$$

Charge dependent One Pion Exchange

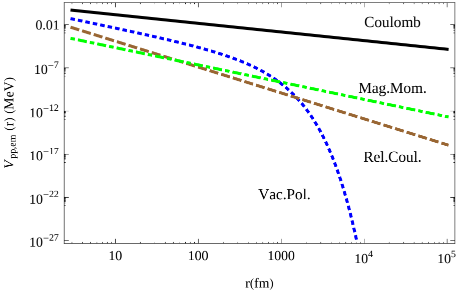
$$
\begin{aligned}
V_{\mathrm{OPE},pp}(r) &= f_{pp}^2 V_{m_{\pi^0},\mathrm{OPE}}(r), \\
V_{\mathrm{OPE},np}(r) &= -f_{nn} f_{pp} V_{m_{\pi^0},\mathrm{OPE}}(r) + (-)^{(T+1)} 2 f_c^2 V_{m_{\pi^\pm},\mathrm{OPE}}(r),
\end{aligned}
$$

where $V_{m,\mathrm{OPE}}$ is given by

$$
\begin{aligned}
V_{m,\mathrm{OPE}}(r) &= \left(\frac{m}{m_{\pi^\pm}}\right)^2 \frac{1}{3} m \left[Y_m(r) \sigma_1 \cdot \sigma_2 + T_m(r) S_{1,2} \right], \\
S_{1,2} &= 3\sigma_1 \cdot \hat{r} \sigma_2 \cdot \hat{r} - \sigma_1 \cdot \sigma_2 \\
Y_m(r) &= \frac{e^{-mr}}{mr} \\
T_m(r) &= \frac{e^{-mr}}{mr} \left[1 + \frac{3}{mr} + \frac{3}{(mr)^2} \right]
\end{aligned}
$$

Small but crucial long range

- Coulomb interaction (pp) e/r
- Magnetic moments $\sim \mu_p \mu_n / r^3, \mu_p \mu_p / r^3, \mu_n \mu_n / r^3$
  Lowered $\chi^2/\nu \sim 2 \to \chi^2/\nu \sim 1$
  Summing 1000-2000 partial waves
- Vacuum polarization (Uehling potential,Lamb-shift)
- Relativistic corrections $1/r^2$



Effective Elementary

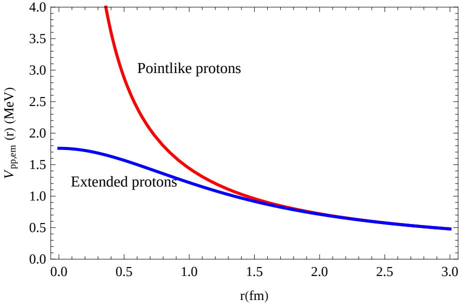
When are two protons interacting as point-like particles ?

- Electromagnetic Form factor

$$F_i(q) = \int d^3 r e^{iq\cdot r} \rho_i(r)$$

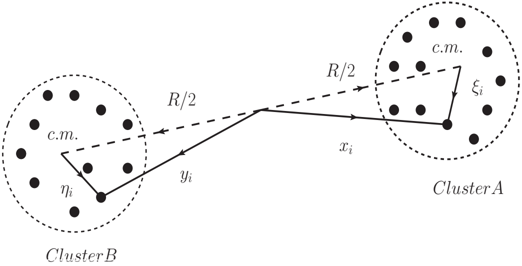
- Electrostatic interaction

$$V_{pp}^{\rm el}(r) = e^2 \int d^3 r_1 d^3 r_2 \frac{\rho_p(r_1)\rho_p(r_2)}{|\vec{r}_1 - \vec{r}_2 - \vec{r}|} \to \frac{e^2}{r} \qquad r > r_e \sim 2{\rm fm}$$



Quark Cluster Dynamics (qcd)

- Atomic analogue. Neutral atoms
- Non-overlapping atoms exchange TWO photons (Van der Waals force)
- Overlapping atoms are not locally neutral; ONE photon exchange is possible (Chemical bonding)

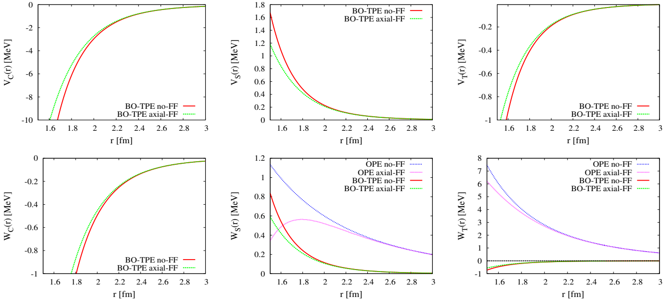


Finite size effects

- NN potential in the Born-Oppenheimer approximation Calle Cordon, RA, '12

$$\bar{V}^{1\pi+2\pi+\cdots}_{NN,NN}(\boldsymbol{r}) = V^{1\pi}_{NN,NN}(\boldsymbol{r}) + 2\,\frac{|V^{1\pi}_{NN,N\Delta}(\boldsymbol{r})|^2}{M_N - M_\Delta} + \frac{1}{2}\,\frac{|V^{1\pi}_{NN,\Delta\Delta}(\boldsymbol{r})|^2}{M_N - M_\Delta} + \mathcal{O}(V^3)\,,$$

- Bulk of TWO-Pion Exchange Chiral forces reproduced
- Finite size effects set in at $2\mathrm{fm}$ $\rightarrow$ exchange quark effects become explicit
- High quality potentials confirm these trends.



The number of parameters (for $E_{\mathrm{LAB}} \leq 350$ MeV)

- At what distance look nucleons point-like ?

$$r > 2\mathrm{fm}$$

- When is OPE the ONLY contribution ?

$$r_c > 3\mathrm{fm}$$

- What is the minimal resolution where interaction is elastic ?

$$p_{\max} \sim \sqrt{M_N m_\pi} \to \Delta r = 1/p_{\max} = 0.6\mathrm{fm}$$

- How many partial waves must be fitted ?

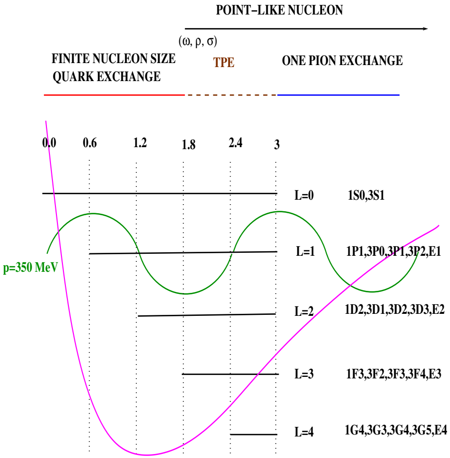
$$l_{\max} = p_{\max} r_c = r_c/\Delta r = 5$$

- Minimal distance where centrifugal barrier dominates

$$\frac{l(l+1)}{r_{\min}^2} \leq p^2$$

- How many parameters ?
$({}^1S_0, {}^3S_1)$, $({}^1P_1, {}^3P_0, {}^3P_1, {}^3P_2)$, $({}^1D_2, {}^3D_1, {}^3D_2, {}^3D_3)$, $({}^1F_3, {}^3F_2, {}^3F_3, {}^3F_4)$

$$2 \times 5 + 4 \times 4 + 4 \times 3 + 4 \times 2 + 4 \times 1 = 50$$


POINT–LIKE NUCLEON
(ω, ρ, σ)
FINITE NUCLEON SIZE
QUARK EXCHANGE
TPE
ONE PION EXCHANGE
0.0
0.6
1.2
1.8
2.4
3
L=0
1S0,3S1
L=1
1P1,3P0,3P1,3P2,E1
p=350 MeV
L=2
1D2,3D1,3D2,3D3,E2
L=3
1F3,3F2,3F3,3F4,E3
L=4
1G4,3G3,3G4,3G5,E4

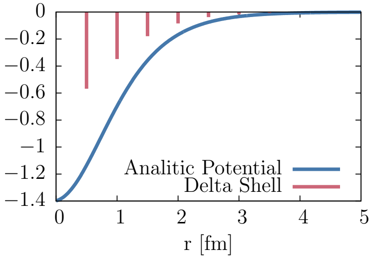
Delta Shell Potential

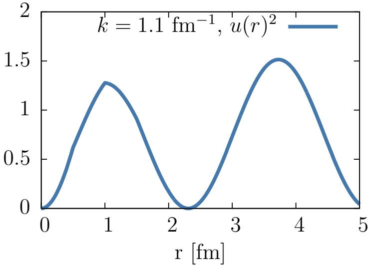
- A sum of delta functions

$$V(r) = \sum_i \frac{\lambda_i}{2\mu}\delta(r - r_i)$$

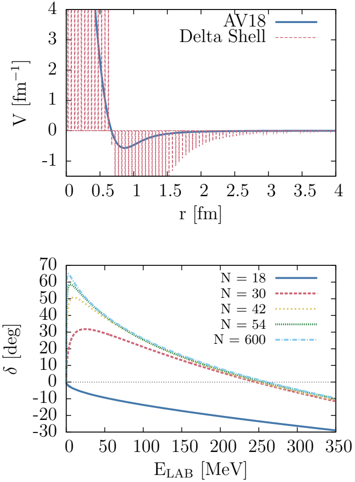
[Aviles, Phys.Rev. C6 (1972) 1467]

- Optimal and minimal sampling of the nuclear interaction
- Pion production threshold $\Delta k \sim 2$ fm^{-1}
- Optimal sampling, $\Delta r \sim 0.5$fm





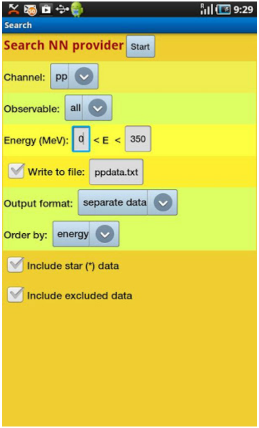
Coarse Graining vs Fine Graining the AV18 potential



Delta Shell Potential

- 3 well defined regions
- Innermost region $r \leq 0.5$ fm
  - Short range interaction
  - No delta shell (No repulsive core)
- Intermediate region $0.5 \leq r \leq 3.0$ fm
  - Unknown interaction
  - λ_i parameters fitted to scattering data
- Outermost region $r \geq 3.0$ fm
  - Long range interaction
  - Described by OPE and EM effects
    - Coulomb interaction V_{C1} and relativistic correction V_{C2} (pp)
    - Vacuum polarization V_{VP} (pp)
    - Magnetic moment V_{MM} (pp and np)

Fitting NN observables



- Database of NN scattering data obtained till 2013
  - http://nn-online.org/
  - http://gwdac.phys.gwu.edu/
  - NN provider for Android
    - Google Play Store

[J.E. Amaro, R. Navarro-Perez, and E. Ruiz-Arriola]

- 2868 pp data and 4991 np data
- 3σ criterion by Nijmegen to remove possible outliers

Fitting NN observables

- Delta shell potential in every partial wave

$$V_{l,l'}^{JS}(r) = \frac{1}{2\mu_{\alpha\beta}} \sum_{n=1}^{N} (\lambda_n)_{l,l'}^{JS} \delta(r - r_n) \qquad r \leq r_c = 3.0\text{fm}$$

- Strength coefficients λ_n as fit parameters
- Fixed and equidistant concentration radii $\Delta r = 0.6$ fm
- EM interaction is crucial for pp scattering amplitude

$$\begin{aligned}
V_{C1}(r) &= \frac{\alpha'}{r} , \\
V_{C2}(r) &\approx -\frac{\alpha\alpha'}{M_p r^2} , \\
V_{VP}(r) &= \frac{2\alpha\alpha'}{3\pi r} \int_1^\infty dx\, e^{-2m_e r x} \left[1 + \frac{1}{2x^2}\right] \frac{(x^2-1)^{1/2}}{x^2} , \\
V_{MM}(r) &= -\frac{\alpha}{4M_p^2 r^3} \left[\mu_p^2 S_{ij} + 2(4\mu_p - 1)\mathbf{L}\cdot\mathbf{S}\right]
\end{aligned}$$

STATISTICS

Self-consistent fits

- We test the assumption

$$O_i^{\rm exp} = O_i^{\rm th} + \xi_i \Delta O_i \qquad i = 1, \dots, N_{\rm Data} \qquad \xi_i \in N[0,1]$$

- Least squares minimization $\mathbf{p} = (p_1, \dots,)$

$$\chi^2(\mathbf{p}) = \sum_{i=1}^N \left(\frac{O_i^{\rm exp} - F_i(\mathbf{p})}{\Delta O_i^{\rm exp}} \right)^2 \to \min_{\lambda_i} \chi^2(\mathbf{p} \chi^2(\mathbf{p}_0) \tag{7}$$

- Are residuals Gaussian ?

$$R_i = \frac{O_i^{\rm exp} - O_i^{\rm th}}{\Delta O_i} \qquad O_i^{\rm th} = F_i(\mathbf{p}_0) \qquad i = 1, \dots, N \tag{8}$$

  If $R_i \in N[0,1]$ self-consistent fit.

- Normality test for a finite sample with N elements $\to$ Probability (Confidence level) p-value

$$\chi^2_{\rm min} = 1 \pm \sigma \sqrt{\frac{2}{\nu}} \qquad \nu = N_{\rm Dat} - N_{\rm Par} \qquad p = 1 - \int_\sigma^\sigma dt \frac{e^{-t^2}}{\sqrt{2\pi}}$$

  Histograms, Moments, Kolmogorov-Smirnov, **Tail Sentitive** QQ-plots

Normality tests

- Does the sequence

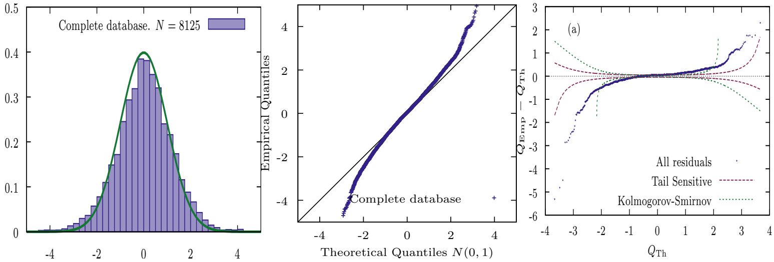
$$x_1^{\rm exp} \le x_2^{\rm exp} \le \cdots \le x_N^{\rm exp} \in N[0,1]$$

- We compute the theoretical points

$$\frac{n}{N+1} = \int_{-\infty}^{x_n^{\rm th}} dt \frac{e^{-t^2/2}}{\sqrt{2\pi}}$$

- The Q-Q plot is $x_n^{\rm th}$ vs $x_n^{\rm exp}$
- For large N

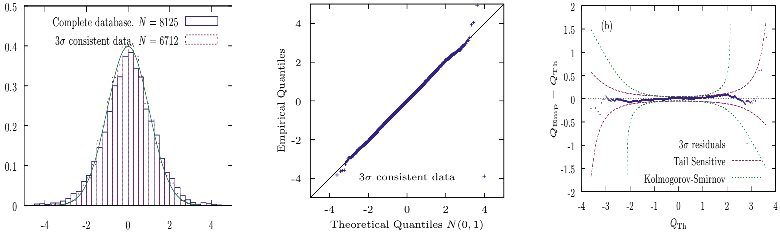
$$x_n^{\rm th} - x_n^{\rm exp} = \mathcal{O}(1/\sqrt{N})$$



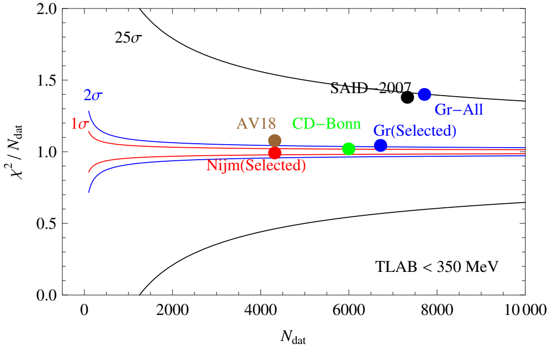
Granada-2013 np+pp database

Selection criterium

- Mutually incompatible data. Which experiment is correct? Is any of the two correct?
- Maximization of experimental consensus
- Exclude data sets inconsistent with the rest of the database
  1. Fit to all data $(\chi^2/\nu > 1)$
  2. Remove data sets with improbably high or low χ^2 (3σ criterion)
  3. Refit parameters
  4. Re-apply 3σ criterion to all data
  5. Repeat until no more data is excluded or recovered



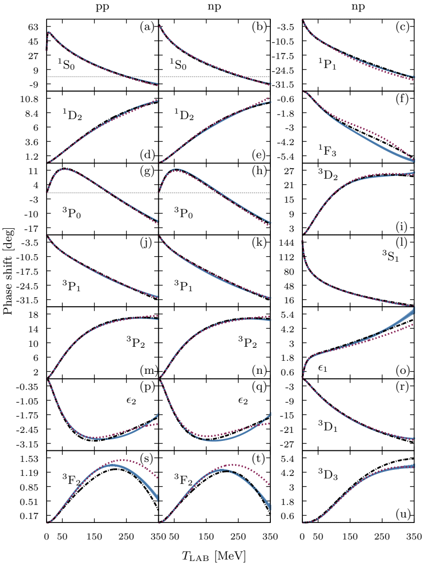
To believe or not to believe



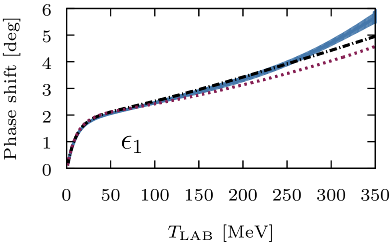
$$\chi^2_{\rm min}/\nu = 1 \pm \sqrt{2/\nu}$$

- Charge dependence in OPE
- Magnetic-Moments, Vacuum polarization, ...

Phase shifts



- Phase shifts for every partial
- Statistical uncertainty propagated directly from covariance matrix



Wolfenstein Parameters

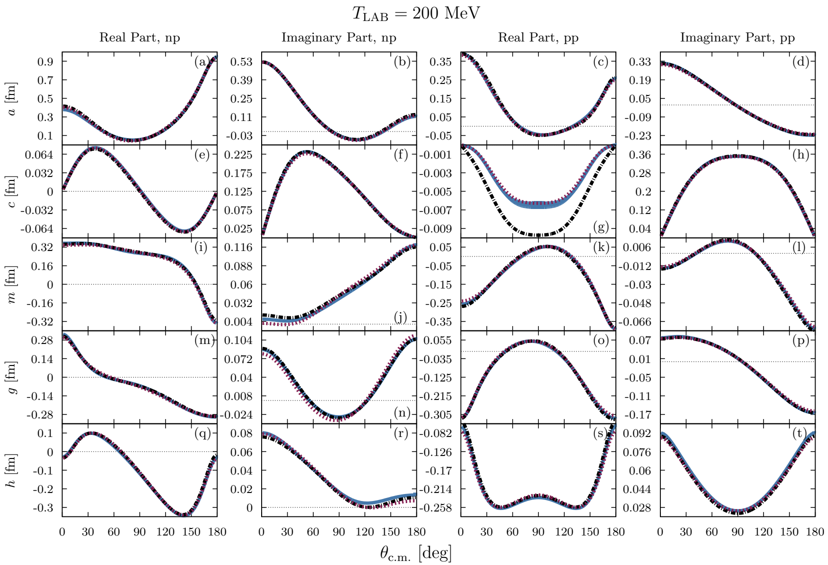
- A complete parametrization of the on-shell scattering amplitudes
- Five independent complex quantities
- Function of Energy and Angle

$$\begin{aligned} M(\mathbf{k}_f, \mathbf{k}_i) &= a + m(\sigma_1, \mathbf{n})(\sigma_2, \mathbf{n}) + (g - h)(\sigma_1, \mathbf{m})(\sigma_2, \mathbf{m}) \\ &\quad + (g + h)(\sigma_1, \mathbf{l})(\sigma_2, \mathbf{l}) + c(\sigma_1 + \sigma_2, \mathbf{n}) \end{aligned}$$

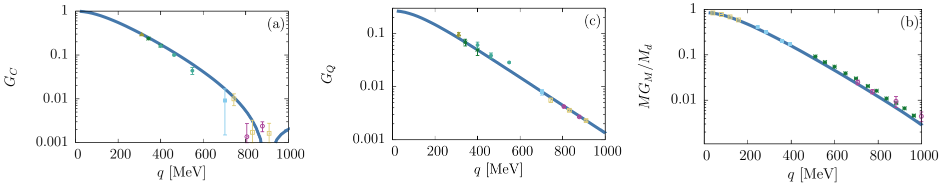
- Scattering observables can be calculated from M

[Bystricky, J. et al, Jour. de Phys. 39.1 (1978) 1]

Wolfenstein Parameters

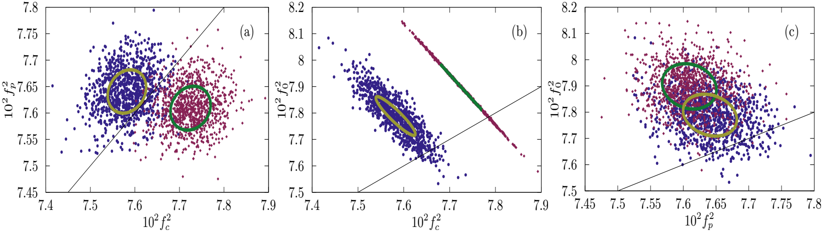


	Delta Shell	Empirical	Nijm I	Nijm II	Reid93	AV18	CD-Bonn
E_d(MeV)	Input	2.224575(9)	Input	Input	Input	Input	Input
η	0.02493(8)	0.0256(5)	0.0253	0.0252	0.0251	0.0250	0.0256
$A_S(\mathrm{fm}^{1/2})$	0.8829(4)	0.8781(44)	0.8841	0.8845	0.8853	0.8850	0.8846
$r_m(\mathrm{fm})$	1.9645(9)	1.953(3)	1.9666	1.9675	1.9686	1.967	1.966
$Q_D(\mathrm{fm}^2)$	0.2679(9)	0.2859(3)	0.2719	0.2707	0.2703	0.270	0.270
P_D	5.62(5)	5.67(4)	5.664	5.635	5.699	5.76	4.85
$\langle r^{-1}\rangle(\mathrm{fm}^{-1})$	0.4540(5)			0.4502	0.4515		



STATISTICAL CONSEQUENCES

Bootstrapping 6713 np+pp data (benchmark $\sim 0.5\%$)

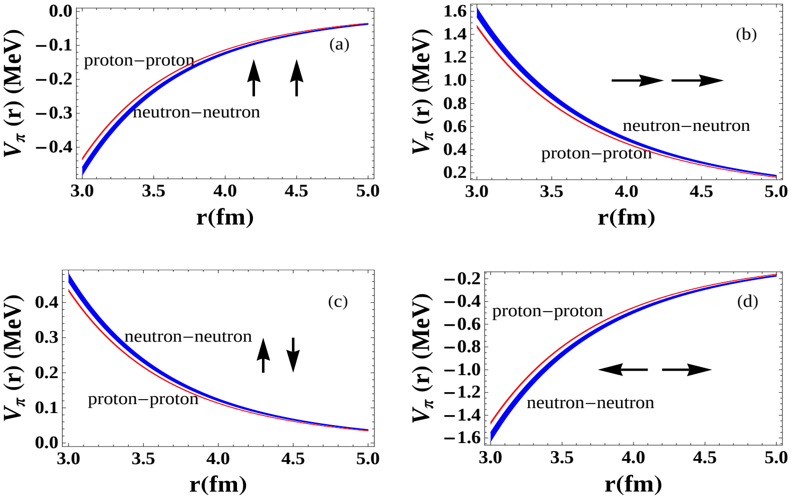


Fits to the Granada-2013 database.

f^2	f_0^2	f_c^2	CD-waves	χ^2_{pp}	χ^2_{np}	$N_{\rm Dat}$	$N_{\rm Par}$	χ^2/ν
0.075	idem	idem	$^1S_0$	3051	3951	6713	46	1.051
$0.0761(3)$	idem	idem	$^1S_0$	3051	3951	6713	46+1	1.051
-	-	-	$^1S_0, P$	2999	3951.40	6713	46+3	1.043
$0.0759(4)$	$0.079(1)$	$0.0763(6)$	$^1S_0, P$	3045	3870	6713	46+3+9	1.039

Neutron-Neutron vs Proton-Proton (Polarized)

nn interaction is more intense than pp interaction



Arqueological Flashback

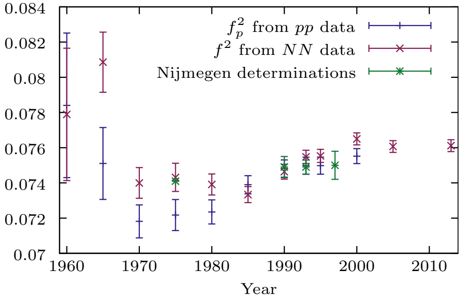
- Can we witness isospin breaking in the couplings ?

$$\frac{dg}{g}\Big|_{\rm QCD} = \mathcal{O}(\alpha, \frac{m_u - m_d}{\Lambda_{\rm QCD}}) = \mathcal{O}(\alpha, \frac{M_n - M_p}{\Lambda_{\rm QCD}}) \sim 0.01 - 0.02$$

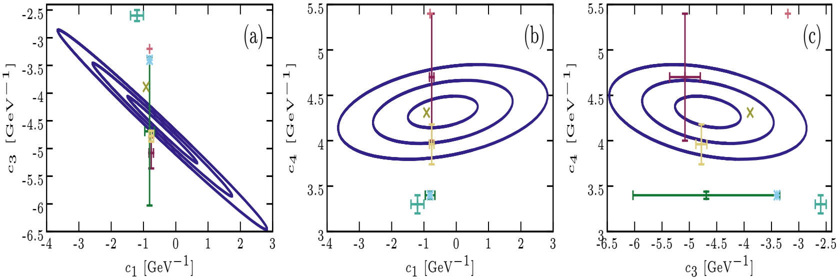
- Statistically yes ! Granada NN $N = 6713$

$$\frac{dg}{g}\Big|_{\rm stat} = \mathcal{O}(\frac{\Delta N_{\rm Dat}}{N_{\rm Dat}}) = \mathcal{O}(\frac{1}{\sqrt{N_{\rm Dat}}}) \to N \sim 7000 - 10000$$

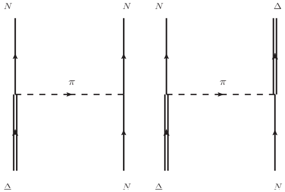
- Chronological recreation of pion-nucleon coupling constants



Chiral Two Pion Exchange from Granada-2013 np+pp database



$f_{\pi N\Delta}$ from Granada-2013 np+pp database



- NN potential in the Born-Oppenheimer approximation

$$\bar{V}_{NN,NN}^{1\pi+2\pi+\cdots}(\boldsymbol{r}) = V_{NN,NN}^{1\pi}(\boldsymbol{r}) + 2\,\frac{|V_{NN,N\Delta}^{1\pi}(\boldsymbol{r})|^2}{M_N - M_\Delta} + \frac{1}{2}\,\frac{|V_{NN,\Delta\Delta}^{1\pi}(\boldsymbol{r})|^2}{M_N - M_\Delta} + \mathcal{O}(V^3)\,,$$

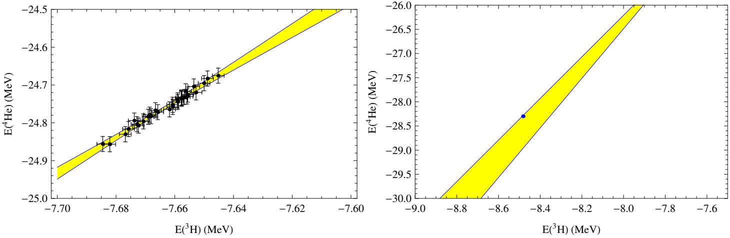
- Bulk of TWO-Pion Exchange Chiral forces reproduced
- Fit with $r_e = 1.8\mathrm{fm}$ to $N = 6713 pp + np$ scattering data

$$f_{\pi N\Delta}/f_{\pi NN} = 2.178(14) \qquad \chi^2/\nu = 1.12 \rightarrow h_A = 1.397(9)$$

Tjon-Lines: numerical accuracy of $A = 2, 3, 4$ Nuclei

(with A. Nogga)

$$\Delta E^{\rm stat}_{\rm triton} = 15{\rm KeV} \qquad \Delta E^{\rm stat}_{\alpha} = 50{\rm KeV}$$



- 4-Body forces are masked by numerical noise in the 3 and 4 body calculation if

$$\Delta^{\rm num}_t > 1{\rm KeV} \qquad \Delta^{\rm num}_t > 20{\rm KeV}$$

To count or not to count: The Falsification of Chiral Forces

- We can fit CHIRAL forces to ANY energy and look if counterterms are compatible with zero within errors
- We find that if $E_{\rm LAB} \le 125 {\rm MeV}$ Weinberg counting is INCOMPATIBLE with data.
- You have to promote D-wave counterterms.
  N2LO-Chiral TPE + N3LO-Counterterms → Residuals are normal
  Piarulli,Girlanda,Schiavilla,Navarro Pérez,Amaro,RA, PRC
- We find that if $E_{\rm LAB} \le 40 {\rm MeV}$ TPE is INVISIBLE
- We find that peripheral waves predicted by 6th-order chiral perturbation theory ARE NOT consistent with data within uncertainties

$$|\delta^{\rm Ch,N5LO} - \delta^{\rm PWA}| > \Delta\delta^{\rm PWA,stat}$$

5 σ incompatible

COARSE GRAINING SHORT RANGE CORRELATIONS

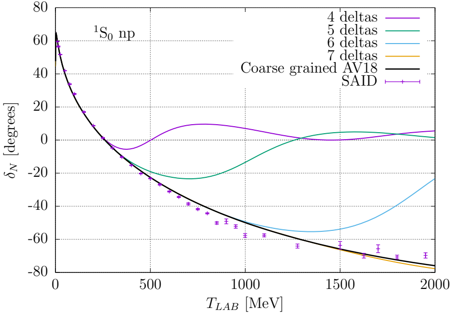
I. Ruiz Simó, R. Navarro Pérez, J. E. Amaro and E.R.A.

Coarse graining vs fine graining

- For S-wave and $E = 0$ we have the scattering problem

$$u_k(r) = \sin(kr) + \frac{2}{\pi}\sum_{i=1}^{N} \lambda_i \, u_k(r_i) \int_0^\infty dq \, \frac{\sin(qr)\sin(qr_i)}{k^2 - q^2}$$

$$\to A \sin(kr + \delta)$$



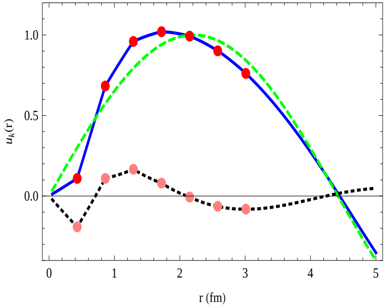
Bethe-Goldstone equation

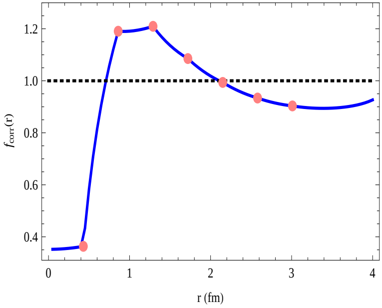
- For S-wave and $E = 0$ we have a $N \times N$ matrix equation

$$u_k(r_n) = \sin(kr_n) + \frac{2}{\pi} \sum_{i=1}^{N} \lambda_i \, u_k(r_i) \int_{k_F}^{\infty} dq \, \frac{\sin(qr_n) \sin(qr_i)}{k^2 - q^2}$$

- In momentum space

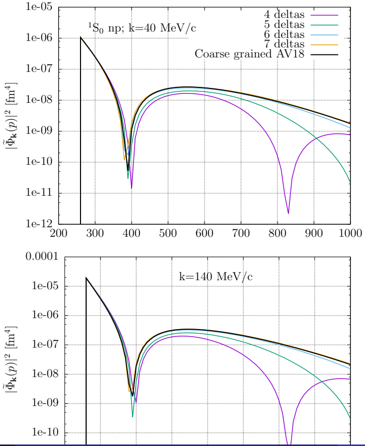
$$\widetilde{\Phi}_k(p) = \frac{1}{p} \frac{\sqrt{4\pi}}{(2\pi)^3} \left\{ \frac{\pi}{2} \delta(p-k) + \frac{\theta(p-k_F)}{k^2 - p^2} \sum_{i=1}^{N} \lambda_i \, u_k(r_i) \, \sin(pr_i) \right\}$$



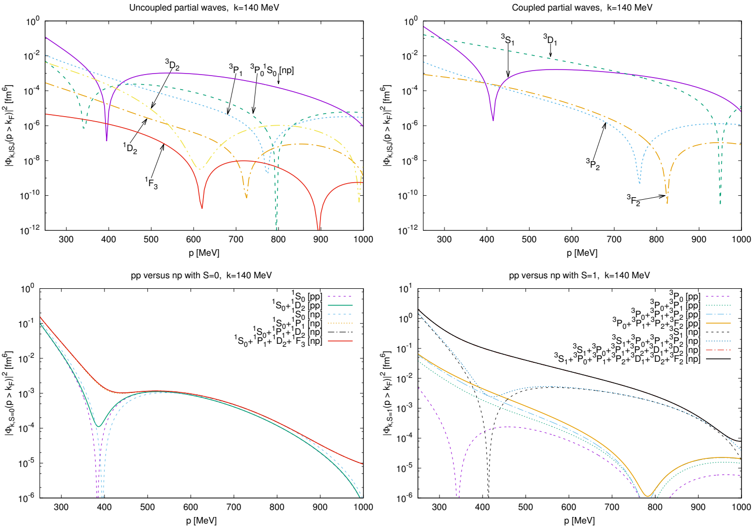


High momentum states, S-wave

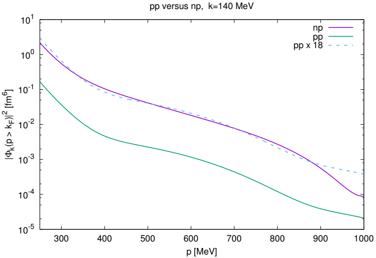
- 5 deltas are enough for $p \leq 0.5\mathrm{GeV}$!
- We can use the Granada-Analysis !

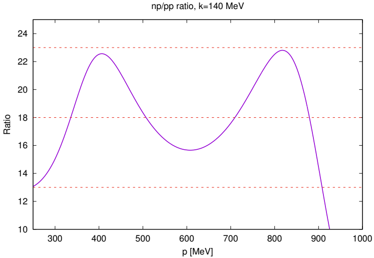


High momentum states: All partial waves



High momentum states: All partial waves





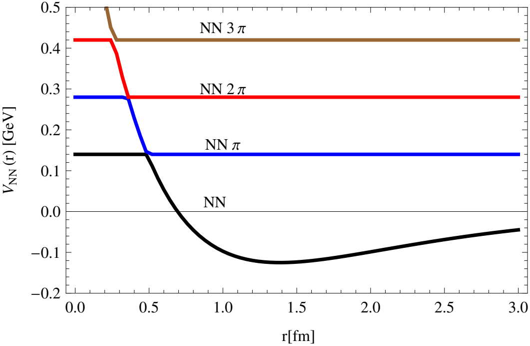
$$\left.\frac{N_{pn}}{N_{pp}}\right|_{\rm JLAB} = 18 \pm 5\,, \qquad 300 \le p \le 600\,\mathrm{MeV}$$

COARSE GRAINING INELASTIC NN

P. Fernández Soler and E.R.A.

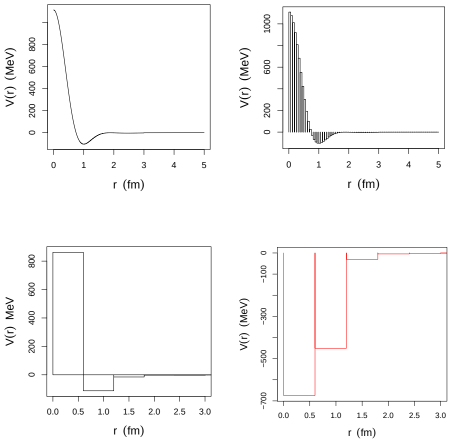
The repulsive core: pros and cons

- Flat differential pp cross section at $\sim 200\mathrm{MeV}$ Jastrow (1950)
- Stability of high density states
- Lattice QCD provides a core
- When $V(r) > m_\pi$ pions should be produced



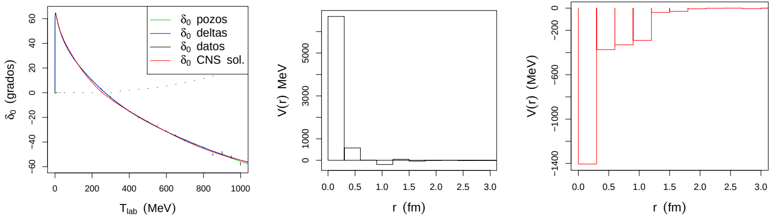
Coarse graining with square well potentials

- Inverse scattering problem ambiguities for $\leq 350\mathrm{MeV}$
- Attractive and Repulsive solutions. Which is the right one ?



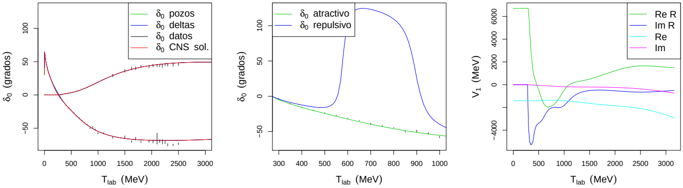
Coarse graining with square well potentials

- Inverse scattering problem ambiguities for $\leq 1\text{GeV}$
- Attractive and Repulsive solutions. Which is the right one ?



Coarse graining optical potential

- Inverse scattering problem ambiguities for $\leq 3\mathrm{GeV}$
- Attractive and Repulsive solutions. Which is the right one ?
- Make V_1 complex and energy dependent from data



- *The hard core is assumed to disappear with increasing energy and to be replaced by absorption.* (G. E. Brown, 1958)
- There was never a hard core $\rightarrow$ smooth behaviour and adiabatic onset of inelasticity

CONCLUSIONS

Conclusions

- Coarse graining is a simple method to analyze and select data using a statistical framework
- Self-consistent databases allow to determine coupling constants
- Validate power countings (Weinberg is not)
- It could be a possible way to do Nuclear Physics: First example High Momentum components.