Modification of the static potential in the presence of a hadron

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Motivation

- Experimental discovery of charmonium-like "X, Y, Z" resonances which cannot be explained by the quarkonium model ("exotic"), see [Olsen, 1411.7738]
- LHCb found pentaquark candidates P_c^+ of exotic quark content $uudc\bar{c}$ in the decay $\Lambda_b \to (J/\psi\ p)\ K$ [LHCb: Aaij et al, 1507.03414, 1604.05708]
- Conjecture of attractive forces between charmonium and pp systems [Brodsky, Schmidt and de Teramond, PRL64 (90) 1011]
- Many possible intepretations: tetra- (penta-) quarks, molecules, hybrid states, hadro-quarkonium, see [Esposito, Pilloni, Polosa, 1611.07920]

Motivation

- 5 (4 q, 1 $\bar{q}$) quark systems are very difficult to study directly on the lattice
- $20\,\mathrm{MeV}$ binding energy for charmonium-nucleon system for $m_\pi \approx 800\,\mathrm{MeV}$ [NPLQCD Collaboration: Beane et al, 1410.7069]

Hadro-quarkonia

- Hadro-quarkonium model: quarkonium core embedded in a light hadron cloud [Dubynskiy and Voloshin, 0803.2224]
- Could explain the LHCb pentaquark, examples of close-by charmonium-baryon systems:

  $J^P = \frac{3}{2}^- : m(\Delta) + m(J/\psi) \approx 4329\,\mathrm{MeV}$ vs. $P_c^+(4380)$ (width $200\,\mathrm{MeV}$)

  $J^P = \frac{5}{2}^+ : m(N) + m(\chi_{c2}) \approx 4496\,\mathrm{MeV}$ vs. $P_c^+(4450)$ (width $40\,\mathrm{MeV}$)

Static quarks

We consider quarkonium $\bar{Q}Q$ in the static limit $m_Q \to \infty$. To leading order in (p)NRQCD, quarkonia can be approximated by the non-relativistic Schrödinger equation with a static potential $V_0(r)$. Does this become more or less attractive, when a light hadron H is "added"? $\Rightarrow$ Lattice QCD study [Alberti, Bali, Collins, Knechtli, Moir and Söldner, 1608.06537]

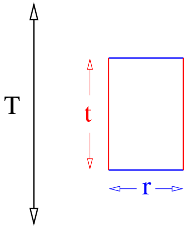
Static potential V_0 in the vacuum

$\mathcal{Q}^\dagger_{\mathbf{r}}(\mathbf{z})$ is a lattice operator which creates a static quark at $\mathbf{z} + \mathbf{r}/2$ and an anti-quark at $\mathbf{z} - \mathbf{r}/2$

$$V_0(r) = -\lim_{t\to\infty} \frac{\mathrm{d}}{\mathrm{d}t} \ln\langle 0|\mathcal{Q}_{\mathbf{r}} \mathcal{T}^{t/a} \mathcal{Q}^\dagger_{\mathbf{r}}|0\rangle$$

assuming the theory has a transfer matrix $\mathcal{T} = e^{-a\mathbb{H}}$

Wilson loops



T is the temporal size of the lattice

Planar Wilson loop:

$$\langle W(r,t)\rangle = \Big\langle \mathrm{tr}\, \Big\{ P(0,\vec{0};0,r\hat{k})P(0,r\hat{k};t,r\hat{k}) \\ P^\dagger(t,\vec{0};t,r\hat{k})P^\dagger(0,\vec{0};t,\vec{0}) \Big\} \Big\rangle$$

$P(x_0,\vec{0};x_0,r\hat{k})$ is the product of spatial links joining the static sources at time x_0; it represents a string-like state

$P^\dagger(0,\vec{x};t,\vec{x})$ is the product of temporal links at position $\vec{x}$; it represents the propagator of the static quark

$$\langle W(r,t)\rangle \overset{T\to\infty}{\sim} \langle 0|\mathcal{Q}_r \mathcal{T}^{t/a} \mathcal{Q}_r^\dagger|0\rangle = \sum_n c_n c_n^* \mathrm{e}^{-V_n(r)t}$$

Extrapolate $t \to \infty$ to extract the ground state potential V_0

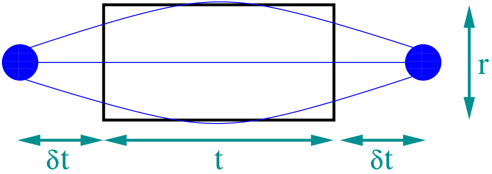
Static quarks in background of a hadron

Static potential V_H in the presence of a hadron

$\overline{\mathcal{H}}$ is a lattice operator which creates a hadron $|H\rangle = \overline{\mathcal{H}}|0\rangle$

$$V_H(r) = -\lim_{t\to\infty} \frac{\mathrm{d}}{\mathrm{d}t} \ln\langle H|\mathcal{Q}_{\mathbf{r}}\mathcal{T}^{t/a}\mathcal{Q}_{\mathbf{r}}^{\dagger}|H\rangle$$

We project to zero-momentum $\mathcal{H} \equiv \sum_{\mathbf{x}} \mathcal{H}(\mathbf{x})$



Create a hadronic state $|H\rangle$ at the time 0.
Let it propagate to δt, create a quark-antiquark “string”.
Destroy this at $t + \delta t$ and the light hadron at $t + 2\delta t$

Correlation function

Modification of the static potential ΔV_H

We compute

$$C_H(r,\delta t,t) = \frac{\langle W(r,t) C_{H,2\mathrm{pt}}(t+2\delta t)\rangle}{\langle W(r,t)\rangle \langle C_{H,2\mathrm{pt}}(t+2\delta t)\rangle}$$

where we average over the spatial positions of the Wilson loop $W(r,t)$ and over the light hadronic sink positions in the two-point function $\langle C_{H,2\mathrm{pt}}(t+2\delta t)\rangle = \langle 0|\mathcal{H}\mathcal{T}^{(t+2\delta t)/a}\overline{\mathcal{H}}|0\rangle$.

We obtain the potential shift $\Delta V_H(r) = V_H(r) - V_0(r)$ from

$$\Delta V_H(r,\delta t) \equiv V_H(r,\delta t) - V_0(r) = -\lim_{t\to\infty} \frac{\mathrm{d}}{\mathrm{d}t} \ln[C_H(r,\delta t,t)]$$

and extrapolating $\delta t \to \infty$.

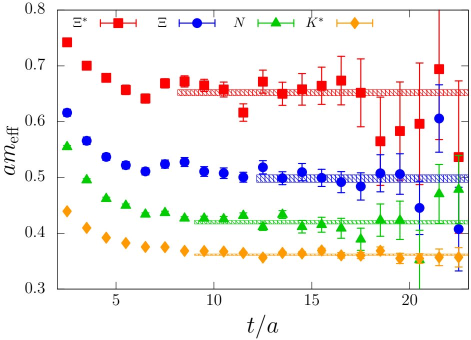
Lattice details

We analyse the $N_f = 2 + 1$ **CLS** ensemble C101 (96×48^3 sites) [Bruno et al, 1411.3982]:
$m_\pi = 220\,\mathrm{MeV}$, $m_\mathrm{K} = 470\,\mathrm{MeV}$, $Lm_\pi \approx 4.6$, $L \approx 4.1\,\mathrm{fm}$,
$t_0/a^2 = 2.9085(51)$, $a = 0.0854(15)\,\mathrm{fm}$ from $\sqrt{8t_0}$ extrapolated to physical point [G.S. Bali et al, 1606.09039] and $\sqrt{8t_0} = 0.4144(59)(37)\,\mathrm{fm}$ [Borsanyi et al, 1203.4469]

High statistics: 1552 configs, separated by 4 MDUs, times 12 hadron sources (1 forward, 1 backward, 11 forward and backward propagating $\Rightarrow$ 24 2-point functions).
Wilson loops at all positions and in all directions.

Hadronic two-point functions have improved overlap with the ground state. We measure ΔV_H for π, K, ρ, $K^\star$ and ϕ mesons; for N, Σ, Λ and Ξ octet baryons with $J^P = \frac{1}{2}^\pm$; and for Δ, $\Sigma^\star$, $\Xi^\star$ and Ω decuplet baryons with $J^P = \frac{3}{2}^\pm$.

Hadronic two-point functions

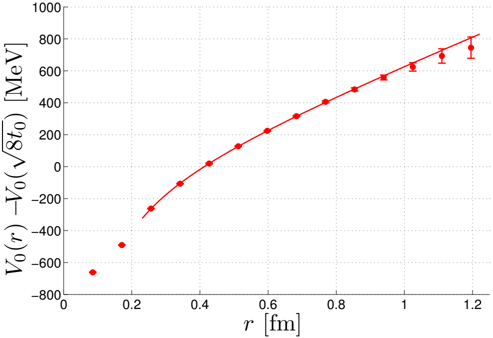


Plateaus of $am_{H,\rm eff}(t+a/2) \equiv \ln \frac{\langle C_{H,2\rm pt}(t)\rangle}{\langle C_{H,2\rm pt}(t+a)\rangle}$ start at $t < 10a$ in almost all cases.

When computing $C_H(r,\delta t,t)$ we will use $\delta t = 5a \approx 0.43\,\mathrm{fm}$ and $t \le t_{\max} = 10a \longrightarrow$ hadron propagates over time $t_{\max} + 2\delta t = 20a \longrightarrow$ plateau is reached early enough.

Static potential $V_0(r)$

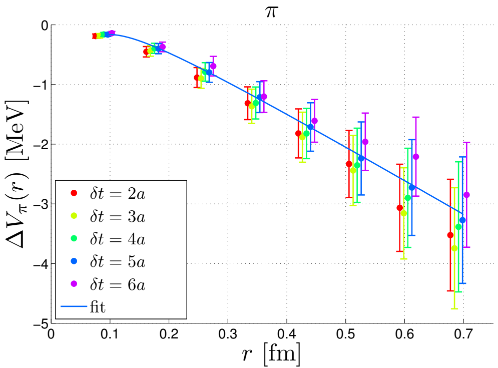
We determine V_0 using the methods of [Donnellan, Knechtli, Leder and Sommer 1012.3037]: variational basis with $0, 5, 7, 12$ spatial HYP levels; due to open boundary conditions, average Wilson loops between $t = 24a$ and $t = 72a$.



Cornell parametrization $V_0(r) = \mu - c/r + \sigma r$

Note: for ΔV_H we will only use the highest HYP level

Potential shift “within” a pion

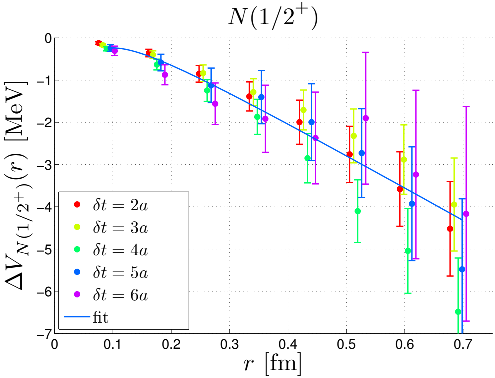


We can resolve small energy differences.

Cornell parametrization of $\Delta V_H(r, \delta t = 5a)$

$$\Delta V_H(r) = \Delta\mu_H - \frac{\Delta c_H}{r} + \Delta\sigma_H r$$

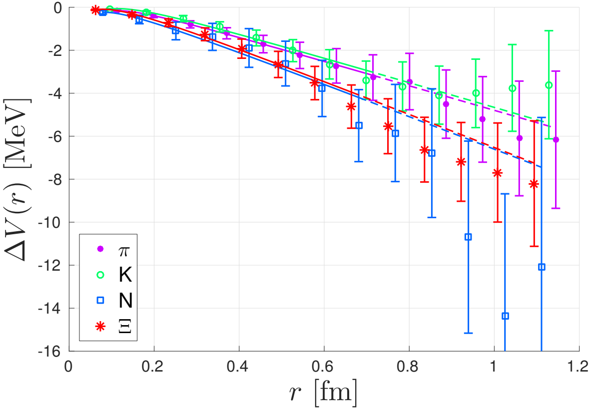
Potential shift “within” a nucleon $N(\frac{1}{2}^+)$



For a baryon of parity P, parity P is taken in the forward propagator and parity $-P$ in the backward propagator

Larger r, several particles

What happens at $r > 0.7\,\mathrm{fm}$? String breaking occurs around $r_\mathrm{b} \approx 1.25\,\mathrm{fm}$ [Bali et al., 0505012]



Reduction of the linear slope $\Delta\sigma_H \approx -(6\text{–}8)\,\mathrm{MeV}/\,\mathrm{fm}$ persists

The baryon decuplet

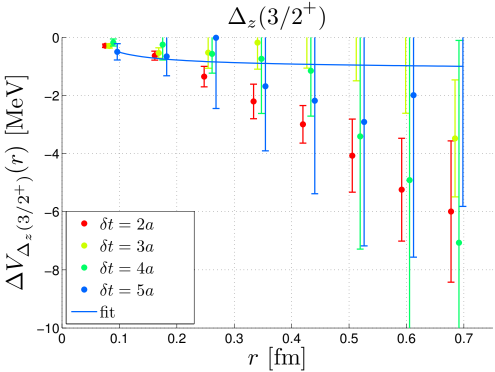
Hadro-quarkonium candidates for LHCb pentaquark states

We measure decuplet baryons with **helicity** $\pm\frac{3}{2}$.
Quarkonium in S-wave has $J^P = 0^-$ and 1^-.

Combining 0^- or 1^- quarkonium with a $\frac{3}{2}^+$ decuplet baryon gives a $J^P = \frac{3}{2}^-$ pentaquark state.

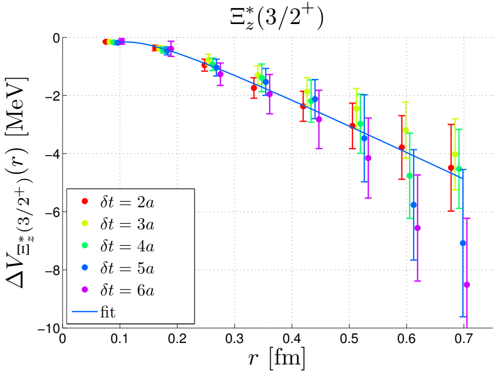
Combining 1^- quarkonium with a $\frac{3}{2}^-$ decuplet baryon gives a $J^P = \frac{5}{2}^+$ pentaquark state.

Potential shift "within" a $\Delta(\frac{3}{2}^+)$



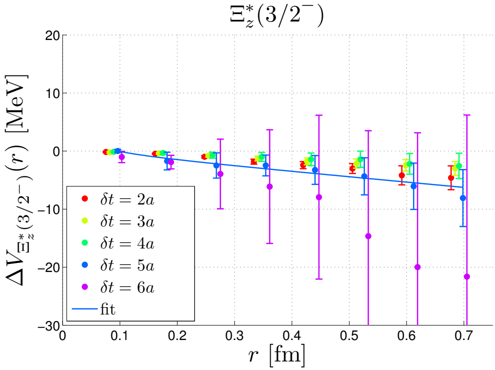
Polarization: correlation of baryon maximally polarized in z direction with Wilson loops in z direction to guarantee $\Lambda = |J_z| = 3/2$

Potential shift “within” a $\Xi^*(\frac{3}{2}^+)$



Better signal than the Δ, reduction of the static potential is similar in size to that of the octet baryons and mesons

Potential shift “within” a $\Xi^*(\frac{3}{2}^-)$



The signal for $\frac{3}{2}^-$ decuplet baryons is noisier. Also they do not match the mass of the $\frac{5}{2}^+$ LHCb pentaquark.

Cornell parametrization of energy shifts

Baryon H	$\Delta\mu_H\,[\mathrm{MeV}]$	$\Delta c_H\,[10^{-4}]$	$\Delta\sigma_H\,[\mathrm{MeV/\,fm}]$
$N\,(1/2^+)$	1.17(37)	3.21(1.30)	-7.83(97)
$\Sigma\,(1/2^+)$	1.62(21)	4.63(73)	-7.99(60)
$\Lambda\,(1/2^+)$	1.28(20)	3.46(69)	-8.49(57)
$\Xi\,(1/2^+)$	1.54(19)	4.32(75)	-7.81(55)
$\Delta\,(3/2^+)$	-0.99(1.75)	-2.22(6.16)	-0.10(4.77)
$\Sigma^\star\,(3/2^+)$	2.15(37)	6.14(1.30)	-11.38(1.01)
$\Xi^\star\,(3/2^+)$	1.74(36)	4.90(1.41)	-9.40(1.03)
$\Omega\,(3/2^+)$	2.34(49)	6.77(1.68)	-11.02(1.41)

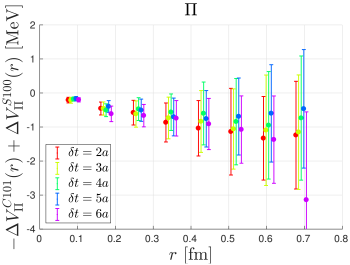
- In all cases we observe $\Delta V_H(r,\delta t) < 0$.
- The Cornell fits $\Delta V_H(r) = \Delta\mu_H - \Delta c_H/r + \Delta\sigma_H r$ describe well the data. They yield a reduction of the slope $\Delta\sigma_H < 0$ and increases of the Coulomb coefficient c and off-set μ.

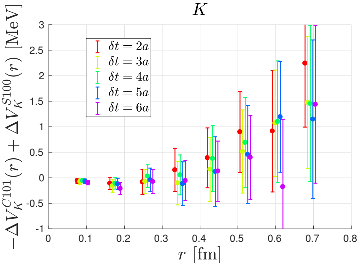
Volume check on mesons

To check for finite volume effects we computed

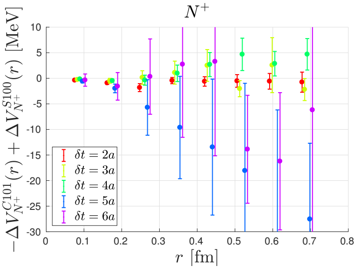
$$\Delta V_H^{\mathrm{S100}} - \Delta V_H^{\mathrm{C101}}$$

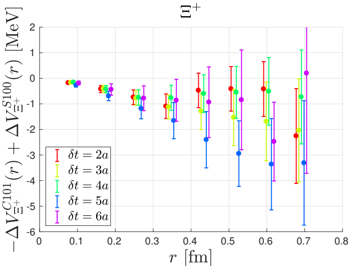
CLS ensemble S100: 128×32^3 sites, same lattice spacing and quark masses as ensemble C101 but smaller $L \approx 2.7\,\mathrm{fm}$. Statistics is 940 configurations times 10 hadron sources (forward and backward propagating $\Rightarrow$ 20 2-point functions).





Volume check on octet baryons





No significant finite volume effects are visible for distances $r > 0.3\,\mathrm{fm}$.

Note: only statistical errors are shown.

Phenomenological implications

Schrödinger equation for the shifted potential

$m_Q, m_Q v \gg \Lambda_{\mathrm{QCD}}$, $v \ll 1 \longrightarrow$ Non-relativistic approach (potential NRQCD) to describe quarkonia $\overline{Q}Q$

$$H\psi_{nL} = M_{nL}^{(0)}\psi_{nL}\,, \quad H = 2(m_Q - \delta m_Q) + \frac{p^2}{m_Q} + V_0(r) + \ldots$$

- Using Cornell fit to V_0 we computed $M_{nL}^{(0)}$ (n radial, L angular momentum quantum numbers) and adjusted m_c and δm_c to reproduce 1S and 2S charmonia.
- Replace $V_0(r) \longrightarrow V_H(r) = V_0(r) + \Delta V_H$ and compute $M_{nL}^{(H)}$ and mass differences $\Delta M_{nL}^{(H)} = M_{nL}^{(H)} - M_{nL}^{(0)}$.
- Caveats: relativistic corrections are not small for charmonium and $m_H \approx m_c$ for baryons.

Phenomenological implications

Mass/Mass difference	$1S\,[\mathrm{MeV}]$	$1P\,[\mathrm{MeV}]$	$2S\,[\mathrm{MeV}]$
M_{nL} (experiment)	3068.6	3525.3	3674.4
$M_{nL}^{(0)}$ (Schrödinger)	3068.6	3483.3	3674.4
$\Delta M^{(\pi)}$	-1.7	-3.1	-4.0
$\Delta M^{(K)}$	-1.5	-2.9	-3.8
$\Delta M^{(\rho)}$	-2.5	-4.9	-6.5
$\Delta M^{(K^*)}$	-1.6	-3.2	-4.2
$\Delta M^{(\phi)}$	-1.6	-3.2	-4.3
$\Delta M^{(N)}$	-2.4	-4.3	-5.5
$\Delta M^{(\Xi)}$	-2.0	-3.9	-5.1
$\Delta M^{(\Delta)}$	-0.9	-1.0	-1.0
$\Delta M^{(\Xi^*)}$	-2.6	-4.8	-6.3

We find $\Delta M^{(H)} < 0 \Rightarrow$ charmonium “within” a hadron H is energetically favourable.

Summary

Modification of the static potential “inside” light hadrons

- $\Delta V_H(r) < 0$, the size is 2–$3\,\mathrm{MeV}$ at $r = 0.5\,\mathrm{fm}$. The main effect is a 0.7–0.9% reduction of the “slope”.
- There is a similar attraction in all of the channels investigated. The effect is mostly independent of the volume. Also it does not change when momentum is injected to the hadron parallel or perpendicular to the direction of the Wilson loop.
- Interesting for hadron physics in medium.

Test of the hadro-quarkonium

Solving the Schrödinger equation with the modified potential gives a stronger binding of charmonium. However it is like deuterium binding which is somewhat inconsistent with the original hadro-charmonium.