

Heavy quark spectroscopy in Lattice QCD.

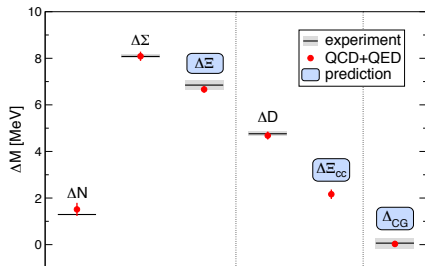
Daniel Mohler

Newport News, Sep 7th, 2017



Recent progress in Lattice QCD

- Dynamical simulations with 2+1(+1) flavors of sea quarks
- Simulations at physical pion (light-quark) masses
- Isospin splitting and QCD+QED simulations
- Improved heavy quark actions for charm



BMW Collaboration, Borsanyi et al. Science 347 1452 (2015)

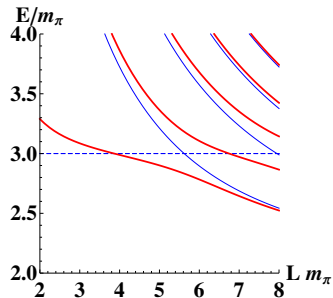
Progress from an old idea: Lüscher's finite-volume method

M. Lüscher Commun. Math. Phys. 105 (1986) 153;
Nucl. Phys. B 354 (1991) 531; Nucl. Phys. B 364 (1991) 237.

Basic observation: Finite volume, multi-particle energies are shifted with regard to the free energy levels due to the interaction

$$E = E(p_1) + E(p_2) + \Delta E$$

- Energy shifts encode scattering amplitude(s)
- Original method: Elastic scattering in the rest-frame in multiple spatial volumes L^3
- Coupled 2-hadron channels well understood
- $2 \leftrightarrow 1$ and $2 \leftrightarrow 2$ transitions well understood (example $\pi\pi \rightarrow \pi\gamma^*$)
- significant progress for 3-particle scattering



Reviews by R. A. Briceño arXiv:1411.6944 and M. Hansen arXiv:1511.04737

Fully systematic calculation vs. exploratory study

Important lattice systematics from

- Taking the *continuum limit*: $a(g, m) \rightarrow 0$
- Taking the *infinite volume limit*: $L \rightarrow \infty$
- Calculation at (or extrapolation to) the physical pion mass

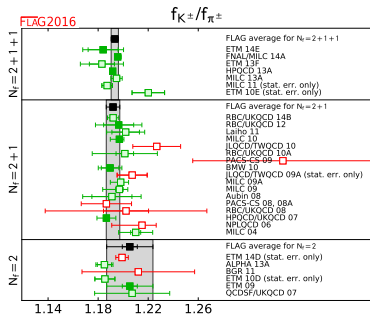
I cover many *exploratory* results

- Should be compared only qualitatively to experiment
- Provide an outlook on future Lattice QCD results

Example for fully systematic results:

- Flavor physics results listed in the FLAG review

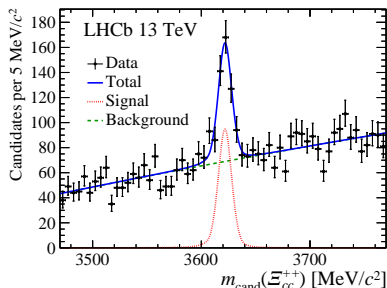
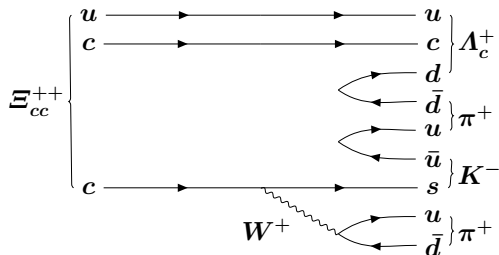
<http://itpwiki.unibe.ch/flag/>



- 1 Charmed baryons
- 2 D , D_s , and B_s
- 3 Charmonium(-like) states
 - Charmonium-like states: Z_c
 - Charmonium-like states: $X(3915)$
 - Charmonium-like states: $X(3872)$
- 4 Searches for exotic b-quark hadrons from Lattice QCD
 - $B_s\pi^+$ scattering and search for the $X(5568)$
 - Search for a deeply bound $bb\bar{b}\bar{b}$ state
- 5 Exotic doubly-heavy states

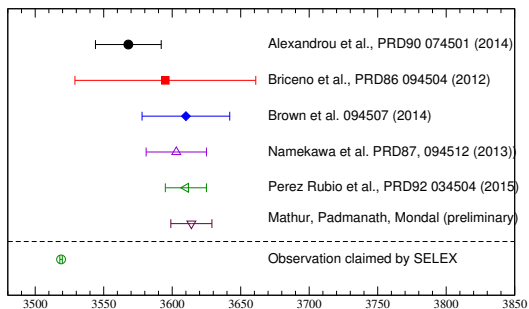
Observation of a doubly-charmed Ξ_{cc}^{++} by LHCb

Roel Aaij et al., LHCb collaboration arXiv:1707.01621



- Ξ_{cc}^{++} with mass $3621.40 \pm 0.72 \pm 0.27 \pm 0.14$ seen in both 13 TeV and 8 TeV data
- Previous claim of Ξ_{cc} by SELEX with mass ≈ 3519 MeV not seen by BaBar, Belle, LHCb
- What about Lattice QCD Predictions?

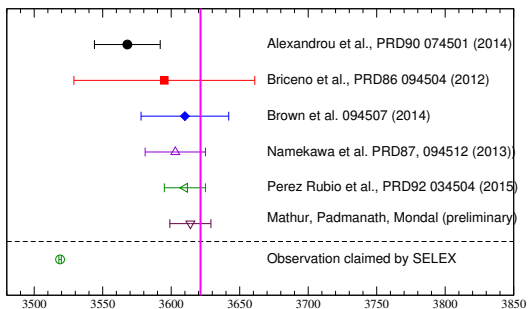
Ξ_{cc} – Recent Lattice QCD predictions



- Full symbols: Good control of systematic uncertainty
Empty symbols: Continuum extrapolation missing
- All simulations neglect isospin splittings $\Xi_{cc}^{++} - \Xi_{cc}^{+}$
- Publications also contain a number of further predictions and successful postdictions
- History of earlier calculations, most notably

Mathur, Lewis, Woloshyn, PRD 64 094509 (2001); PRD 66 014502

Ξ_{cc} – Recent Lattice QCD predictions

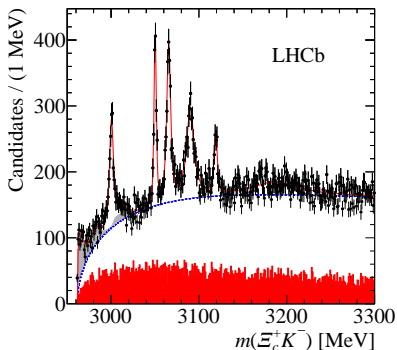


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Observation of 5 narrow Ω_c states

Roel Aaij et al., LHCb collaboration PRL 118 182001 (2017)

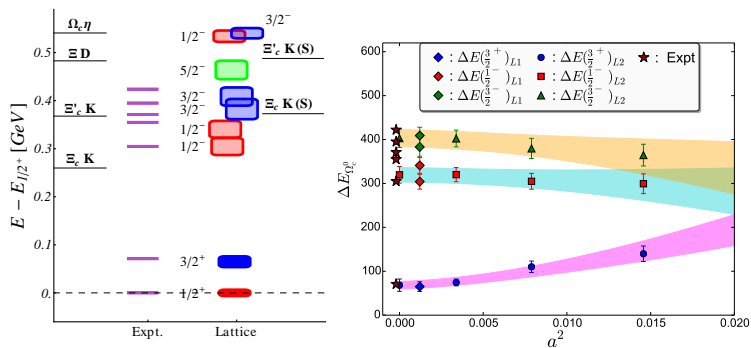


Resonance	Mass [MeV]	Γ [MeV]
$\Omega_c(3000)^0$	$3000.4 \pm 0.2 \pm 0.1^{+0.3}_{-0.5}$	$4.5 \pm 0.6 \pm 0.3$
$\Omega_c(3050)^0$	$3050.2 \pm 0.1 \pm 0.1^{+0.3}_{-0.5}$	$0.8 \pm 0.2 \pm 0.1$
$\Omega_c(3066)^0$	$3065.6 \pm 0.1 \pm 0.3^{+0.3}_{-0.5}$	$3.5 \pm 0.4 \pm 0.2$
$\Omega_c(3090)^0$	$3090.2 \pm 0.3 \pm 0.5^{+0.3}_{-0.5}$	$8.7 \pm 1.0 \pm 0.8$
$\Omega_c(3119)^0$	$3119.1 \pm 0.3 \pm 0.9^{+0.3}_{-0.5}$	$1.1 \pm 0.8 \pm 0.4$

- Observation of 5 new Ω_c resonances
- J^P not identified
- All states are narrow \rightarrow can compare to Lattice QCD simulations treating them as stable

What can Lattice QCD say about their spin-parity?

Padmanath, Mathur, PRL 119 042001 (2017)



- Pattern of lattice states agrees well with experiment
- second study with smaller basis can not resolve two states with same J^P ; checks systematics
- Scattering thresholds somewhat unphysical

Exotic D_s and B_s candidates

Established s and p-wave D_s and B_s hadrons:

$D_s (J^P = 0^-)$ and $D_s^* (1^-)$
 $D_{s0}^* (2317) (0^+)$, $D_{s1} (2460) (1^+)$,
 $D_{s1} (2536) (1^+)$, $D_{s2}^* (2573) (2^+)$

$B_s (J^P = 0^-)$ and $B_s^* (1^-)$
 $B_{s1} (5830) (1^+)$, $B_{s2}^* (5840) (2^+)$

- Corresponding $D_0^* (2400)$ and $D_1 (2430)$ are broad resonances
- Peculiarity: $M_{c\bar{s}} \approx M_{c\bar{d}} \rightarrow$ exotic structure? (tetraquark, molecule)
- B_s cousins of the $D_{s0}^* (2317)$ and $D_{s1} (2460)$ not (yet) seen in experiment
- The LHCb experiment at CERN should be able to see these

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$$\begin{array}{ll} D_s (J^P = 0^-) \text{ and } D_s^* (1^-) & B_s (J^P = 0^-) \text{ and } B_s^* (1^-) \\ D_{s0}^* (2317) (0^+), D_{s1} (2460) (1^+), & ? \\ D_{s1} (2536) (1^+), D_{s2}^* (2573) (2^+) & B_{s1} (5830) (1^+), B_{s2}^* (5840) (2^+) \end{array}$$

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$D_{s0}^*(2317)$: D-meson – Kaon s-wave scattering

M. Lüscher Commun. Math. Phys. 105 (1986) 153;
Nucl. Phys. B 354 (1991) 531; Nucl. Phys. B 364 (1991) 237.

Charm-light hadrons

$D_{s0}^*(2317)^\pm$

$I(J^P) = 0(0^+)$
 J, P need confirmation.

J^P is natural, low mass consistent with 0^+ .

Mass $m = 2317.7 \pm 0.6$ MeV

$m_{D_{s0}^*(2317)^\pm} - m_{D^\pm} = 349.4 \pm 0.6$ MeV

Full width $\Gamma < 3.8$ MeV, CL = 95%

$$p \cot \delta_0(p) = \frac{2}{\sqrt{\pi}L} Z_{00} \left(1; \left(\frac{L}{2\pi} p \right)^2 \right) \\ \approx \frac{1}{a_0} + \frac{1}{2} r_0 p^2$$

Mohler et al. PRL 111 222001 (2013)

Lang, DM et al. PRD 90 034510 (2014)

Results for ensembles (1) and (2)

$$a_0 = -0.756 \pm 0.025 \text{ fm} \quad (1)$$

$$r_0 = -0.056 \pm 0.031 \text{ fm}$$

$$a_0 = -1.33 \pm 0.20 \text{ fm} \quad (2)$$

$$r_0 = 0.27 \pm 0.17 \text{ fm}$$

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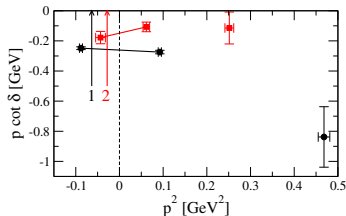
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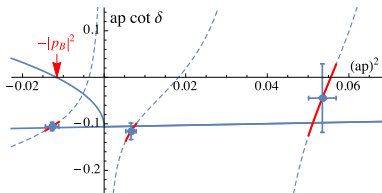
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B_{s0}^* and B_{s1} : Results

Lang, Mohler, Prelovsek, Woloshyn PLB 750 17 (2015)

B_{s0}^*

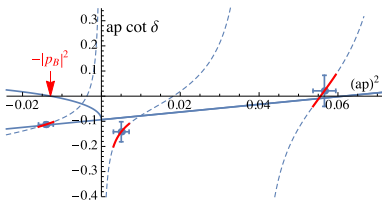


$$a_0^{BK} = -0.85(10) \text{ fm}$$

$$r_0^{BK} = 0.03(15) \text{ fm}$$

$$M_{B_{s0}^*} = 5.711(13) \text{ GeV}$$

B_{s1}



$$a_0^{B^*K} = -0.97(16) \text{ fm}$$

$$r_0^{B^*K} = 0.28(15) \text{ fm}$$

$$M_{B_{s1}} = 5.750(17) \text{ GeV}$$

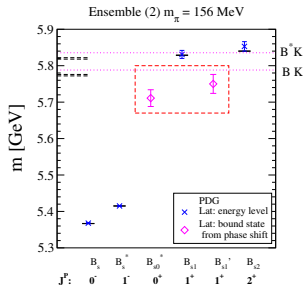
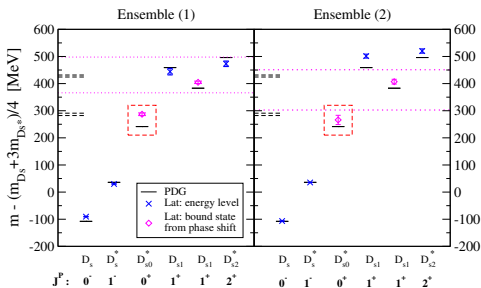
- Energy from the difference to the $B^{(*)}K$ threshold

D_s and B_s : Spectrum results

Mohler et al. PRL 111 222001 (2013)

Lang, Mohler et al. PRD 90 034510 (2014)

Lang, Mohler, Prelovsek, Woloshyn PLB 750 17 (2015)

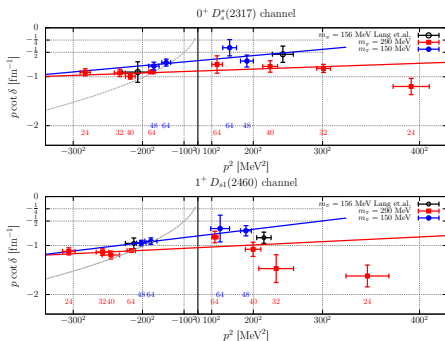


- Discretization uncertainties sizeable for charm
- Many improvements possible for the D_s states

- Full uncertainty estimate only for magenta B_s states
- Prediction of exotic states from Lattice QCD!

Positive parity D_s : More comprehensive results from RQCD

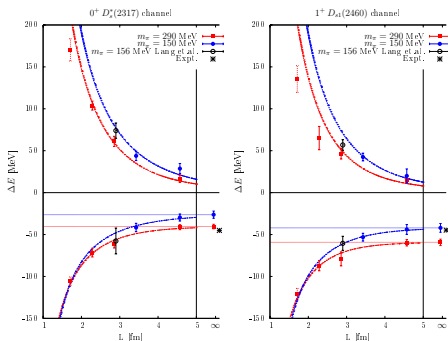
Bali, Collins, Cox, Schäfer, arXiv:1706.01247



- Study with different volumes at pion masses of 150, 290 MeV
- Remaining discretization effects non-negligible
- Results confirm basic behavior seen in previous simulation

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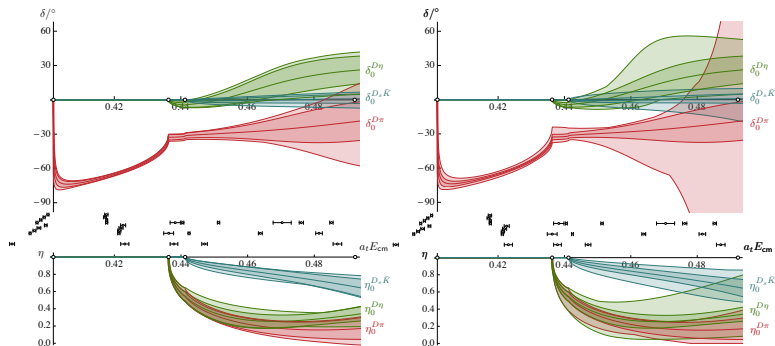
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Coupled-channel study of $D\pi$, $D\eta$, $D_s K$ scattering

Moir et al., JHEP 1610 011 (2016)



- Lattice data from multiple volumes at $m_\pi = 391$ MeV
- Shallow bound state seen in coupled channel s-wave
- Narrow spin-2 D-wave resonance seen as well
- For older single-channel results see

DM, Prelovsek, Woloshyn PRD 87 034501 (2013)

Search for a Z_c^+ state from Lattice QCD

Prelovsek, Lang, Leskovec, DM, Phys.Rev. D91 014504 (2015)

- Search for a Z_c^+ in the $I^G J^{PC} = 1^+ 1^{+-}$ channel
- Aim at simulating all meson-meson states below $\approx 4.3\text{GeV}$
- Caveat: Neglects 3-particle states
- Include tetraquark interpolators of type $3_c \times \bar{3}_c$
- Count energy levels and identify them according to their overlaps
- Hope: See an extra level, as would be expected for a (narrow) resonance

More rigorous approach (a la Lüscher) quite challenging

- Coupled channel system with many channels
- Small shifts in finite volume and (largish) discretization effects
- Thresholds should be close to physical
- Suitable ensembles are (probably) not available at the moment.

A look at the spectrum of scattering states

- Expect level close to non-interacting scattering states

$J/\Psi\pi$

$\eta_c\rho$

$J\Psi(1)\pi(-1)$

DD^*

$\Psi_{2S}\pi$

D^*D^*

$\Psi_{3770}\pi$

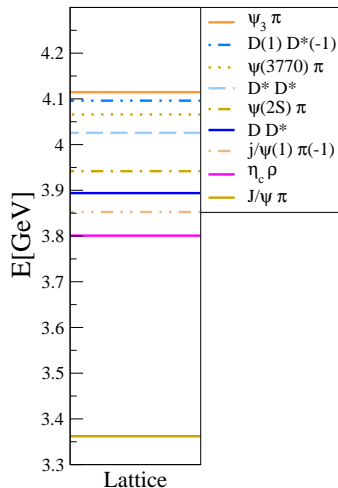
$D(1)D^*(-1)$

$\Psi_3\pi$

$J\Psi(2)\pi(-2)$

$D^*(1)D^*(-1)$

$D(2)D^*(-2)$



Search for Z_c^+ with $I^G J^{PC} = 1^+ 1^{+-}$

X(3900)

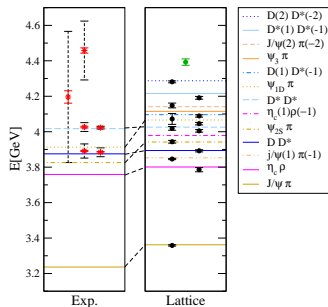
$$I^G(J^{PC}) = 1^+(1^{+-})$$

Mass $m = 3886.6 \pm 2.4$ MeV ($S = 1.6$)

Full width $\Gamma = 28.1 \pm 2.6$ MeV

Prelovsek, Lang, Leskovec, DM,

Phys.Rev. D91 014504 (2015)

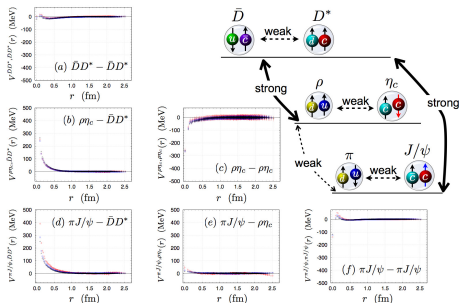


- Simple level counting approach
- We find 13 two meson states as expected
- We find no extra energy level that could point to a Z_c candidate

$Z_c(3900)$ with the HALQCD method I

Ikeda et al. PRL 117 242001 (2016); Ikeda arXiv:1706.07300

- Coupled-channel scattering $J/\psi\pi$, $\eta_c\rho$, $\bar{D}D^*$, $I^G(J^{PC}) = 1^+1^{+-}$
- Uses 2+1 flavor gauge configurations with $a = 0.907(13)$ and $m_\pi = 410, 570, 700$
- HALQCD method Ishii et al. PLB 712, 437 (2012)
 - Calculate a potential as a function of distance r
 - Solve Schrödinger equation with given $V(r)$ and determine scattering phase shifts



$Z_c(3900)$ with the HALQCD method II

$X(3900)$

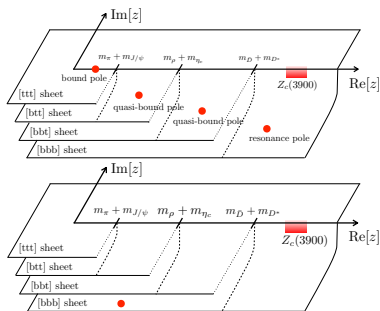
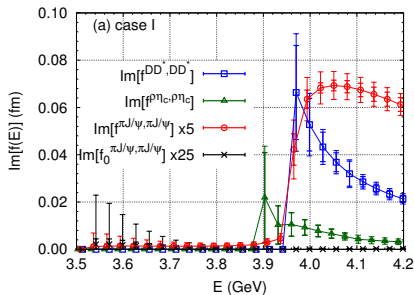
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Ikeda arXiv:1706.07300



- Pole found far below DD^* threshold for all quark masses
- Authors conclude $Z_c(3900)$ not a usual resonance but a threshold cusp
- Structure comes from strong $\pi J/\Psi - \bar{D}D^*$ coupling
- Analysis at close-to-physical pion mass planned

χ'_{c0} and $X/Y(3915)$

$X(3915)$
was $\chi_{c0}(3915)$

$$J^{PC} = 0^{+(0 \text{ or } 2^{++})}$$

Mass $m = 3918.4 \pm 1.9$ MeV

Full width $\Gamma = 20 \pm 5$ MeV ($S = 1.1$)

PDG interpreted $X(3915)$ as a **regular charmonium** (χ'_{c0})

- Some of the reasons to doubt this assignment:

Guo, Meissner Phys. Rev. **D86**, 091501 (2012)

Olsen, PRD 91 057501 (2015)

- No evidence for fall-apart mode $X(3915) \rightarrow \bar{D}D$
- Spin splitting $m_{\chi_{c2}(2P)} - m_{\chi_{c0}(2P)}$ too small
- Large OZI suppressed $X(3915) \rightarrow \omega J/\psi$
- Width should be significantly larger than $\Gamma_{\chi_{c2}(2P)}$
- Zhou *et al.* (PRL 115 2, 022001 (2015)) argue that what is dubbed $X(3915)$ is the spin 2 state already known and suggests that a broader state is hiding in the experiment data.
- Observation of an alternative $\chi_{c0}(2P)$ by Belle:

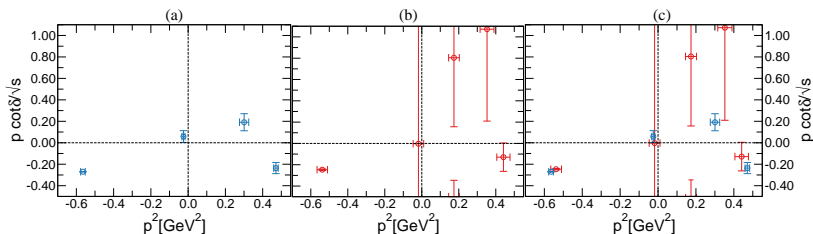
Chilikin *et al.* PRD 95 112003 (2017)

$$M = 3862^{+26+40}_{-32-13}$$

$$\Gamma = 201^{+154+88}_{-067-82}$$

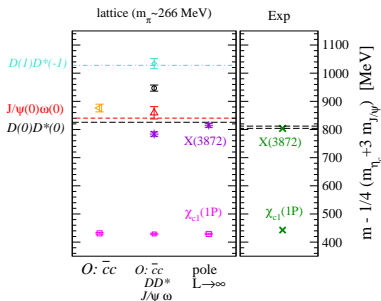
χ'_{c0} : Exploratory lattice calculation

Lang, Leskovec, DM, Prelovsek, JHEP 1509 089 (2015)

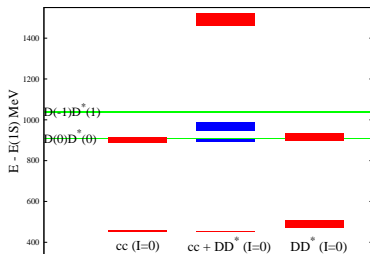


- Assumes only $\bar{D}D$ is relevant
- Lattice data suggests a fairly narrow resonance with $3.9\text{GeV} < M < 4.0\text{GeV}$ and $\Gamma < 100\text{MeV}$
- Future experiment and lattice QCD results needed to clarify the situation

An $X(3872)$ candidate from Lattice QCD



Prelovsek, Leskovec, PRL 111
192001 (2013)

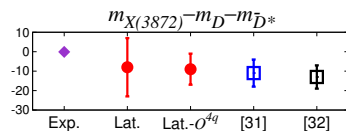
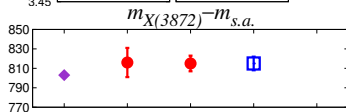
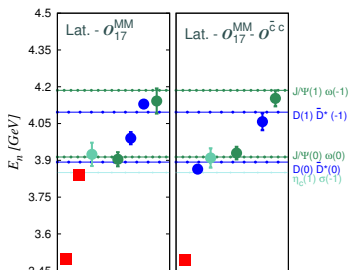


Lee, DeTar, DM, Na,
arXiv:1411.1389

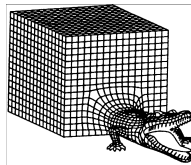
- Neglects charm annihilation and $J/\psi\omega$
- Seen only when $\bar{q}q$ and \bar{D}^*D are used
- The two simulations have vastly different systematics (yet results are similar)

An $X(3872)$ candidate from Lattice QCD II

Padmanath, Lang, Prelovsek, PRD 92 034501 (2015)



- Without $\bar{q}q$ interpolators signal vanishes
- Simulations still unphysical in many ways
- Discretization and finite volume effects sizable!



- Makes interpretation as pure molecule or pure tetraquark unlikely

The X(5568)

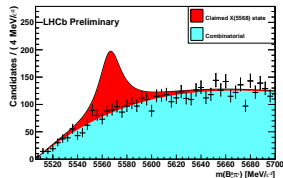
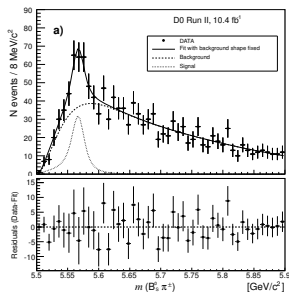
Abazov *et al.* PRL 117, 022003 (2016).

- The D0 collaboration is reporting evidence for a peak in the $B_s\pi^+$ invariant mass not far above threshold
- D0 attributes this to resonance X(5568)

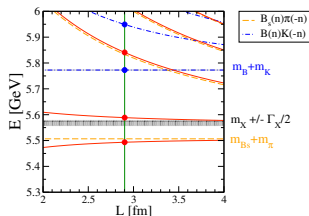
$$m_X = 5567.8 \pm 2.9^{+0.9}_{-1.9} \text{ MeV}$$

$$\Gamma_X = 21.9 \pm 6.4^{+5.0}_{-2.5} \text{ MeV}$$

- Decay to $B_s\pi^+$ implies exotic flavor structure $\bar{b}s\bar{d}u$
- Most model studies accommodating a X(5568) propose $J^P = 0^+$
- LHCb did not find any peak in the $B_s\pi^+$ invariant mass (with increased statistics)



Expected signatures of the X(5568)

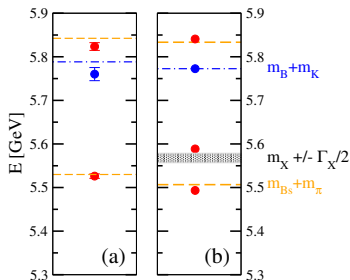


Analytic predictions for energies of eigenstates for an elastic resonance in $B_s\pi$ (with $J^P = 0^+$) as a function of the lattice size L .

- Orange (blue) dashed lines show the $B_s\pi$ (BK) eigenstates when B_s and π (B and K) do not interact
- Red lines show the expectation for lattice energy levels in elastic $B_s\pi$ scattering (decoupled from BK) for a resonance with a mass and width of the X(5568).
- In the scenario of a deeply bound BK state, the simulation would result in an eigenstate with $E \approx m_X$ up to exponentially small corrections in L .

Lattice results and conclusions

Lang, DM, Prelovsek, PRD 94 074509 (2016)

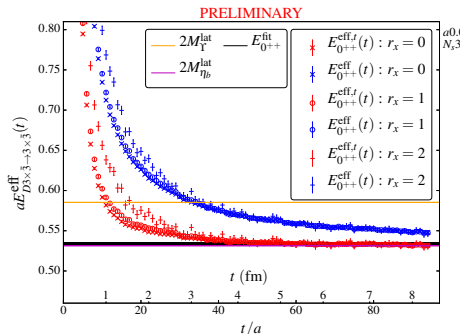
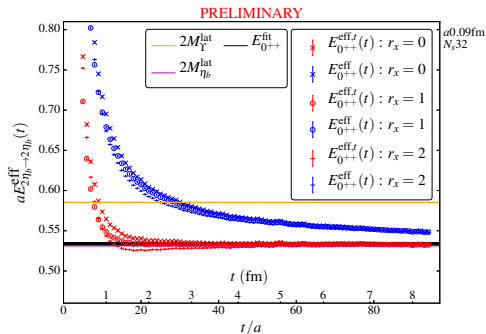


- Exploratory calculation with a single ensemble at $M_\pi = 156$ MeV
- Left pane: eigenenergies of the $\bar{b}s\bar{d}u$ system with $J^P = 0^+$
- Right pane: Analytic prediction based on the $X(5568)$
- Results stable under variations of the fit methodology
- Lattice simulation at close-to-physical quark masses does not yield a second low-lying energy level (expected for the case of the $X(5568)$)
- Results do not support the existence of $X(5568)$ with $J^P = 0^+$.

Search for a deeply bound $bb\bar{b}\bar{b}$ state

C. Hughes, C. Davies, E. Eichten, presented at Lattice 2017

- A number of recent potential model predictions of a very deeply bound $bb\bar{b}\bar{b}$ state
- Lattice study using NRQCD b quarks, 3 different lattice spacings
- Upshot: No deeply bound states but deficiencies in models understood



Recent simulations of charm or beauty tetraquarks

- Searches for charmed tetraquarks
 - Doubly charmed and charmed-strange tetraquarks by HALQCD
Ikeda et al. PLB 729 85–90 (2014)
 - Search for doubly charmed tetraquarks on CLS lattices (preliminary)
Guerrieri et al. arXiv:1411.2247
- HHLL systems with bottom quarks
 - Tetraquark bound states in heavy-light heavy-light systems
Brown and Orginos PRD 86 114506 (2012)
 - Lattice QCD results for a bottom-bottom tetraquark
Bicudo and Wagner PRD 87 114511 (2013)
 - Search for $ud\bar{b}\bar{b}$ $ss\bar{b}\bar{b}$ and $cc\bar{b}\bar{b}$ tetraquarks
Bicudo et al., PRD 92 014507 (2015)
 - ***BB* interactions with static bottom quarks**
Bicudo, Cichy, Peters, Wagner, PRD 93 034501 (2016);
Bicudo, Scheunert, Wagner, PRD 95 034502 (2017)
 - **Deeply bound doubly-heavy tetraquarks with NRQCD b-quarks**
Francis, Hudspith, Lewis, Maltman, PRL 118 142001 (2017)
Junnarkar, Mathur, Padmanath, presented at Lattice 2017

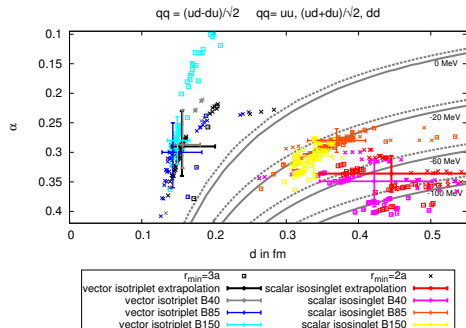
BB interactions with static bottom quarks

Bicudo, Cichy, Peters, Wagner, PRD 93 034501 (2016)

Bicudo, Scheunert, Wagner, PRD 95 034502 (2017)

- Potentials of two static antiquarks in the presence of two light quarks
- Search for bound states (rather than resonances)
- Lattices with $a = 0.079$ fm and $m_\pi \approx 650, 480, 340$
- Fit function used for the lattice QCD potentials

$$V(r) = -\frac{\alpha}{r} \exp\left(-\left(\frac{r}{d}\right)^p\right) + V_0$$



Resulting binding energy:

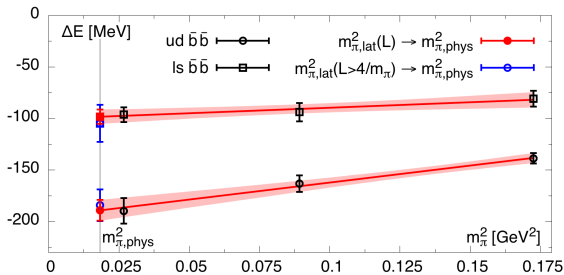
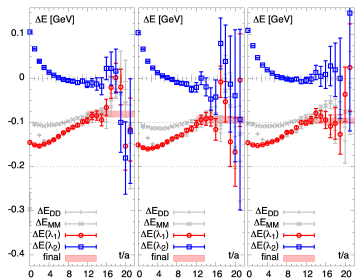
$$E_B = 90_{-36}^{+43} \text{ MeV}$$

Including heavy \bar{b} spins:

$$E_B = 59_{-38}^{+30} \text{ MeV}$$

Doubly bottom $J^P = 1^+$ tetraquarks with NRQCD b-quarks

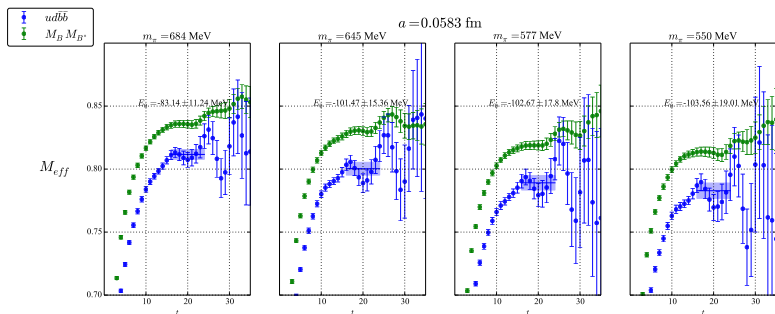
Francis, Hudspith, Lewis, Maltman, PRL 118 142001 (2017)



- Study at a single lattice spacing and three pion masses
 $164\text{MeV} \leq M_\pi \leq 415\text{MeV}$
- Authors obtain bound 4-quark states for both $ud\bar{b}\bar{b}$ and $ls\bar{b}\bar{b}$
- Potential issues
 - Binding energies extracted from ratios can be misleading
 - For a bound state, excited state naively expected above threshold
 - Finite volume effects alter the binding energy

$ud\bar{b}b$ $J^P = 1^+$ tetraquarks on HISQ lattices

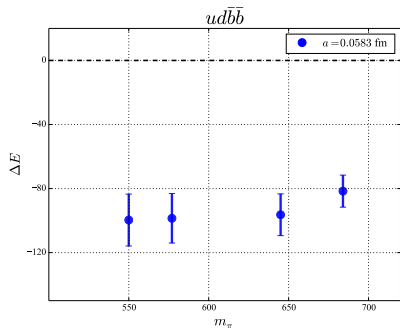
Junnarkar, Padmanath, Mathur, reported at Lattice 2017



- Results from $48^3 \times 144$ HISQ ensemble with $M_{\pi, sea} \approx 310$ MeV
- Four valence pion masses in range $550 \text{ MeV} \leq M_\pi \leq 684 \text{ MeV}$
- Same quantum numbers than Francis *et al.*
(but notable difference in binding energy)
- Preliminary results confirm existence of a bound state

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Thank you!

Thanks to Parikshit Junnarkar, Ciaran Hughes and Nilmani Mathur for sending me unpublished updates

Search for charmed tetraquarks by HALQCD

Ikeda et al. PLB 729 85–90 (2014)

- Search for bound states or resonances in DD , $\bar{K}D$, DD^* and $\bar{K}D^*$ interactions with flavor structure $cc\bar{u}\bar{d}$ and $cs\bar{u}\bar{d}$
- These contain no quark line diagrams with quark annihilation
- Uses 2+1 flavor gauge configurations with $a = 0.907(13)$ and $m_\pi = 410, 570, 700$
- HALQCD method

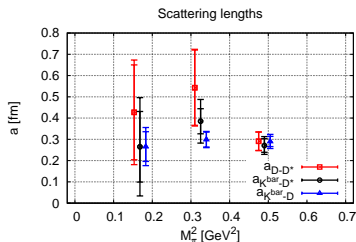
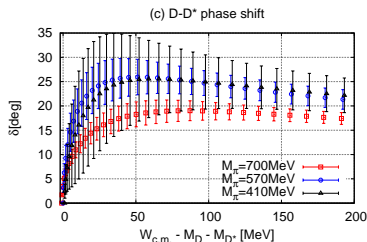
Ishii et al. PLB 712, 437 (2012)

- Calculate a potential as a function of distance r
- Solve Schrödinger equation with given $V(r)$ and determine scattering phase shifts
- Uses variant of the Fermilab method (relativistic heavy quark action)

Tetraquarks with the HALQCD method: Results

Ikeda et al. PLB 729 85–90 (2014)

- Repulsive interaction in all $I = 1$ channels considered
- Attractive interaction in all $I = 0$ channels considered



- No bound states or resonances at simulated m_π
- Attraction becomes more prominent at light pion masses
- Authors have some indication that BB^* with $IJ^P = 01^+$ is bound

Lattice simulation

- We use lattices with 2+1 flavors of Wilson-Clover quarks by PACS-CS

$N_L^3 \times N_T$	N_f	$a[\text{fm}]$	$L[\text{fm}]$	#configs	$m_\pi[\text{MeV}]$	$m_K[\text{MeV}]$
$32^3 \times 64$	2+1	0.0907(13)	2.90	196	156(7)(2)	504(1)(7)

- For a description of the heavy-quark methodology see

Lang, DM, Prelovsek, Woloshyn PLB 750 17 (2015)

- Interpolator basis

$$O_{1,2}^{B_s(0)\pi(0)} = [\bar{b}\Gamma_{1,2}s](\mathbf{p}=0) [\bar{d}\Gamma_{1,2}u](\mathbf{p}=0)$$
$$O_{1,2}^{B_s(1)\pi(-1)} = \sum_{\mathbf{p}=\pm\mathbf{e}_{x,y,z} 2\pi/L} [\bar{b}\Gamma_{1,2}s](\mathbf{p}) [\bar{d}\Gamma_{1,2}u](\mathbf{-p})$$
$$O_{1,2}^{B(0)K(0)} = [\bar{b}\Gamma_{1,2}u](\mathbf{p}=0) [\bar{d}\Gamma_{1,2}s](\mathbf{p}=0)$$