Light spectroscopy on the lattice

Raúl Briceño

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OLD DOMINION UNIVERSITY

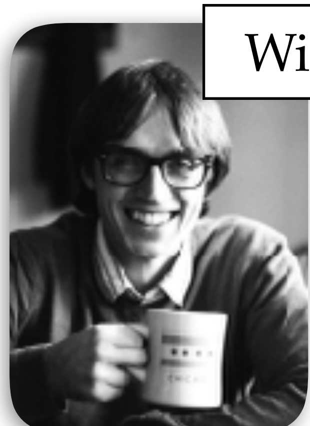
had spec

Lepton/Hadron workshop, 2017

JLab, VA

Lattice QCD calculations with multi-hadron states

in the mesonic isoscalar sector



Wilson (Marie Curie / Royal fellow / Trinity)

Dudek (W&M / JLab)

Edwards (JLab)

hadspec

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Isoscalar $\pi\pi$ Scattering and the σ Meson Resonance from QCD

Raul A. Briceño,[1,*] Jozef J. Dudek,[1,2,†] Robert G. Edwards,[1,‡] and David J. Wilson[3,§]

(for the Hadron Spectrum Collaboration)

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JLAB-THY-17-2534

Isoscalar $\pi\pi, K\overline{K}, \eta\eta$ scattering and the σ, f_0, f_2 mesons from QCD

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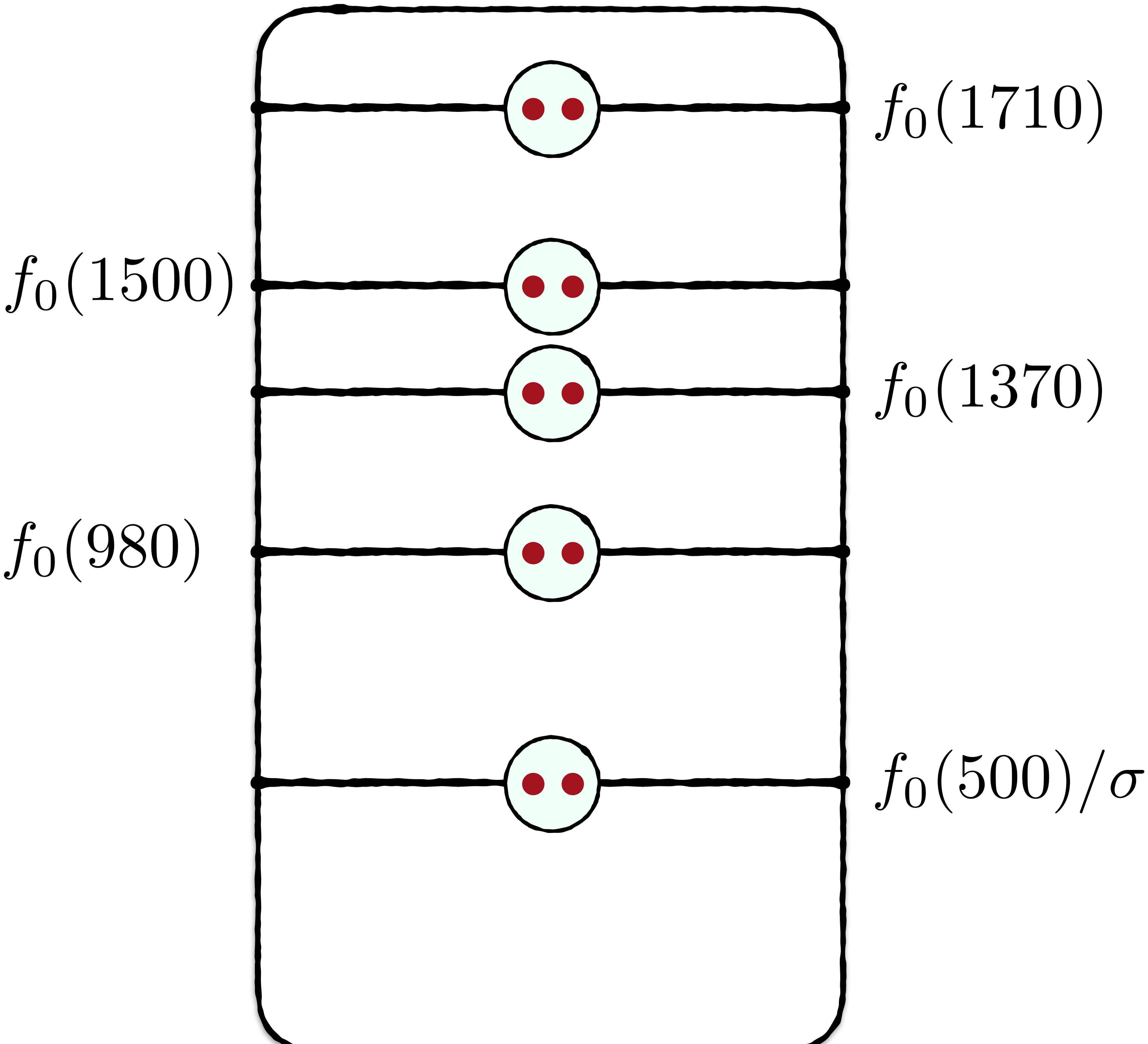
[3]*Department of Physics, College of William and Mary, Williamsburg, VA 23187, USA*
[4]*School of Mathematics, Trinity College, Dublin 2, Ireland*
(Dated: August 23, 2017)

We present the first lattice QCD study of coupled isoscalar $\pi\pi, K\overline{K}, \eta\eta$ S- and D-wave scattering extracted from discrete finite-volume spectra computed on lattices which have a value of the quark mass corresponding to $m_\pi \sim 391$ MeV. In the $J^P = 0^+$ sector we find analogues of the experimental σ and $f_0(980)$ states, where the σ appears as a stable bound-state below $\pi\pi$ threshold, and, similar to what is seen in experiment, the $f_0(980)$ manifests itself as a dip in the $\pi\pi$ cross section in the vicinity of the $K\overline{K}$ threshold. For $J^P = 2^+$ we find two states resembling the $f_2(1270)$ and $f_2'(1525)$, observed as narrow peaks, with the lighter state dominantly decaying to $\pi\pi$ and the heavier state to $K\overline{K}$. The presence of all these states is determined rigorously by finding the pole singularity content of scattering amplitudes, and their couplings to decay channels are established using the residues of the poles.

The isoscalar, scalar sector



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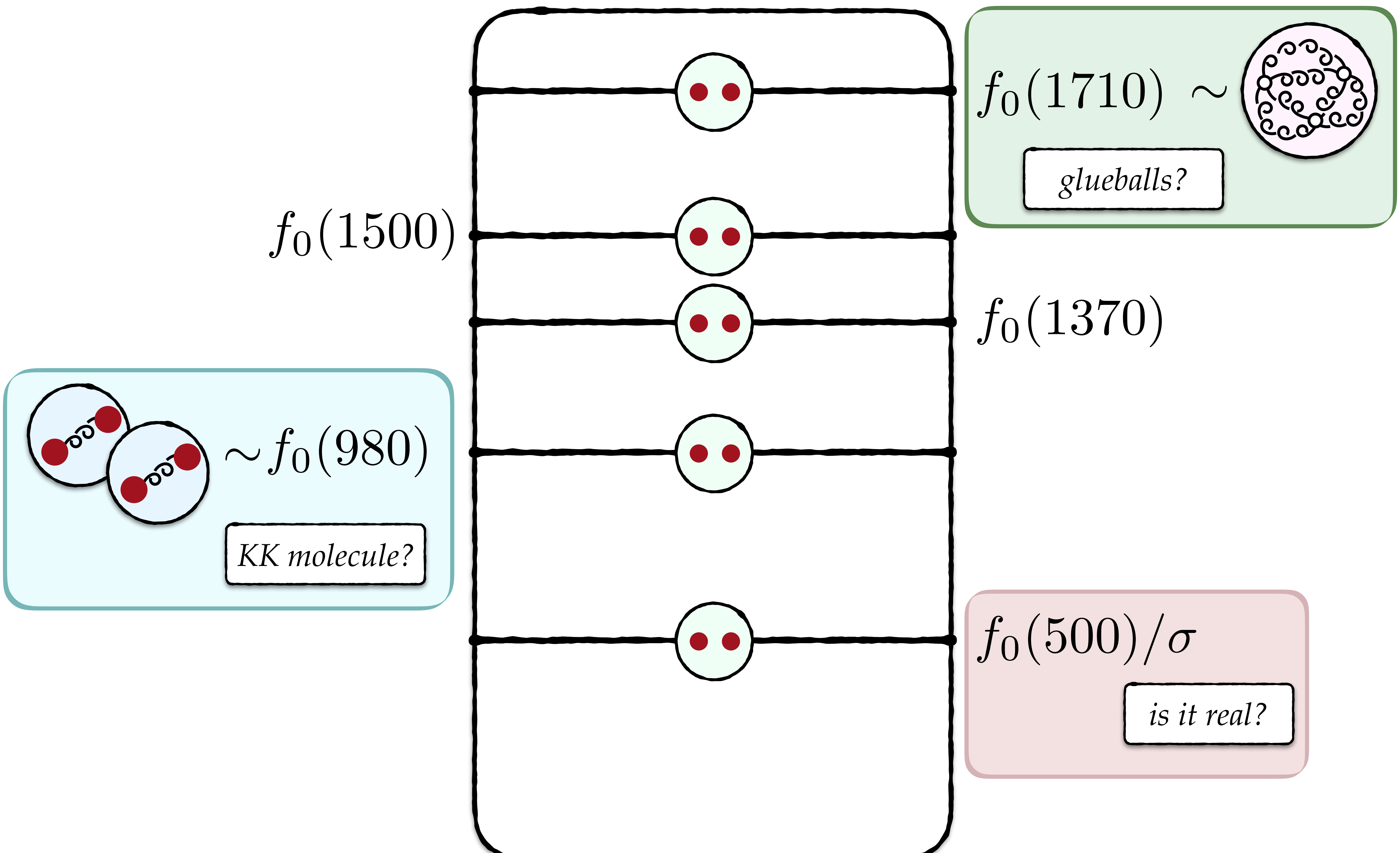

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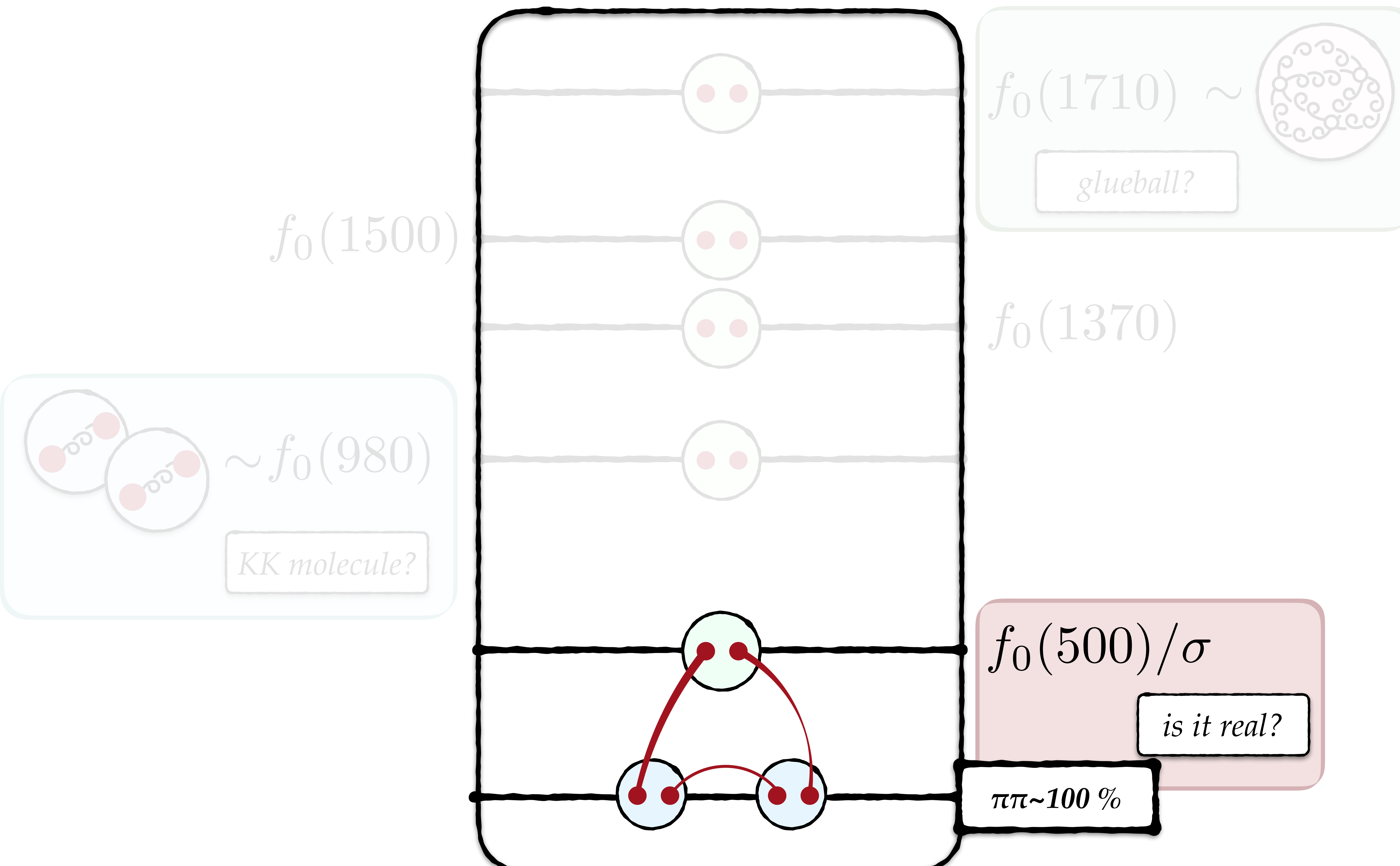

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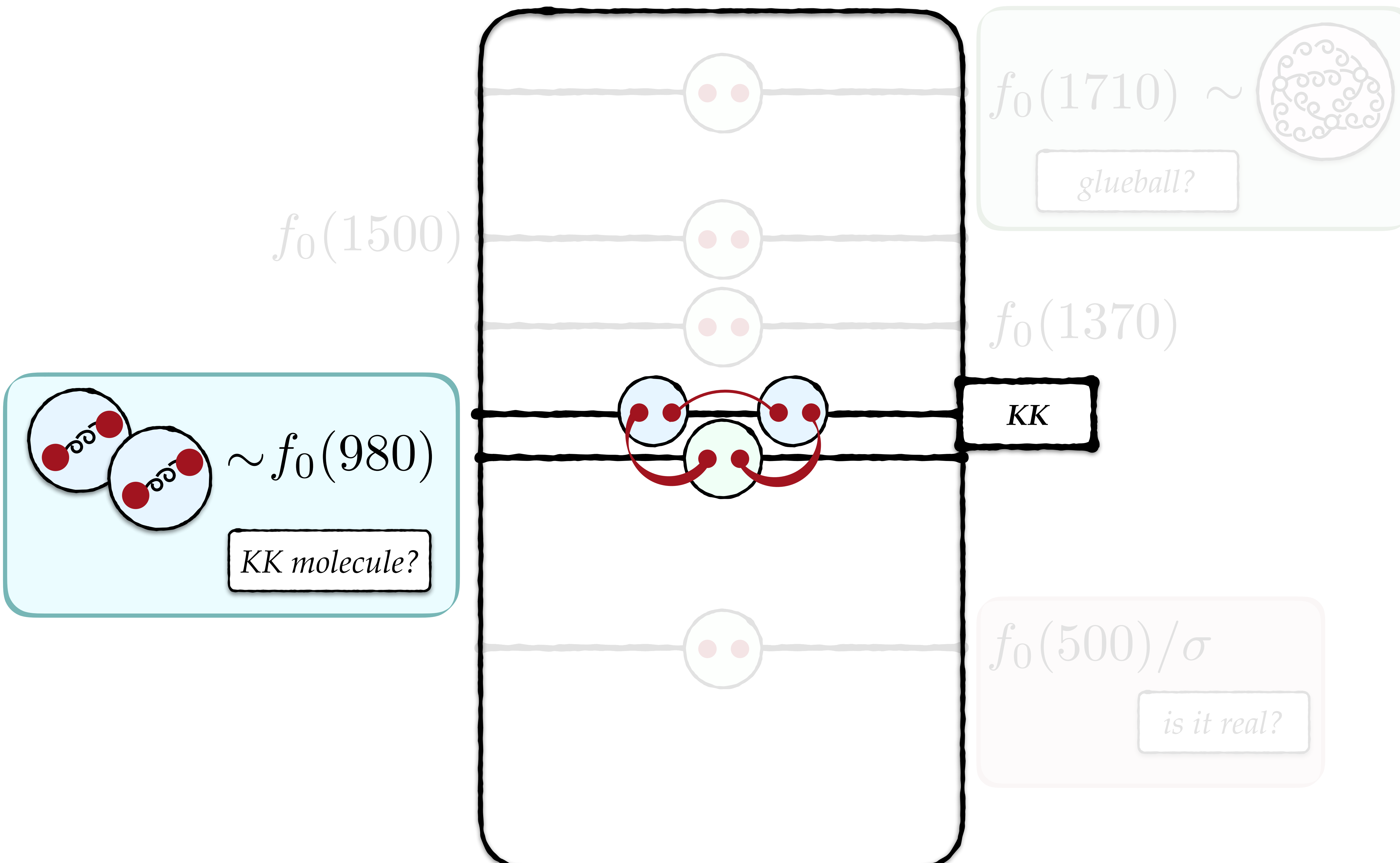
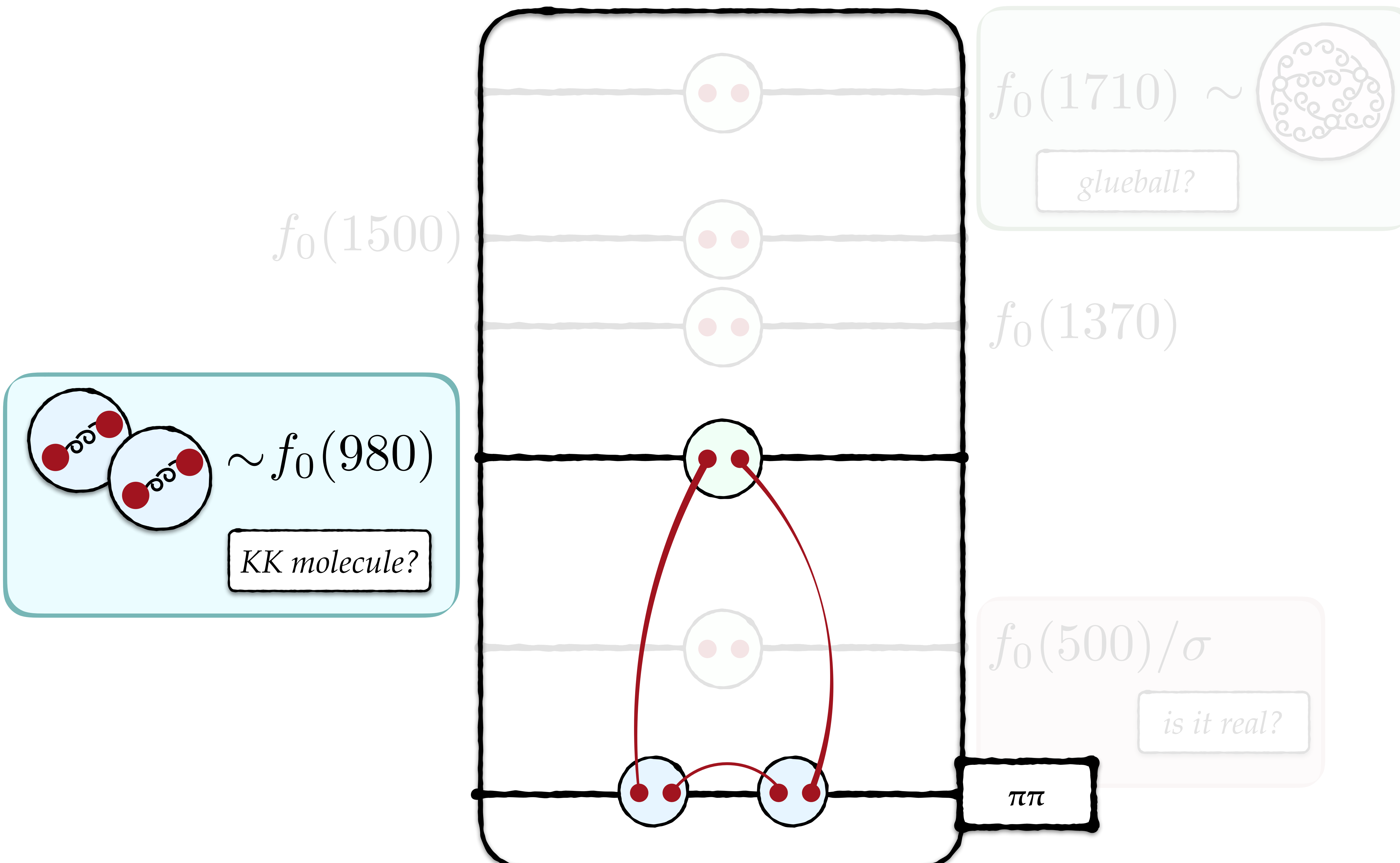
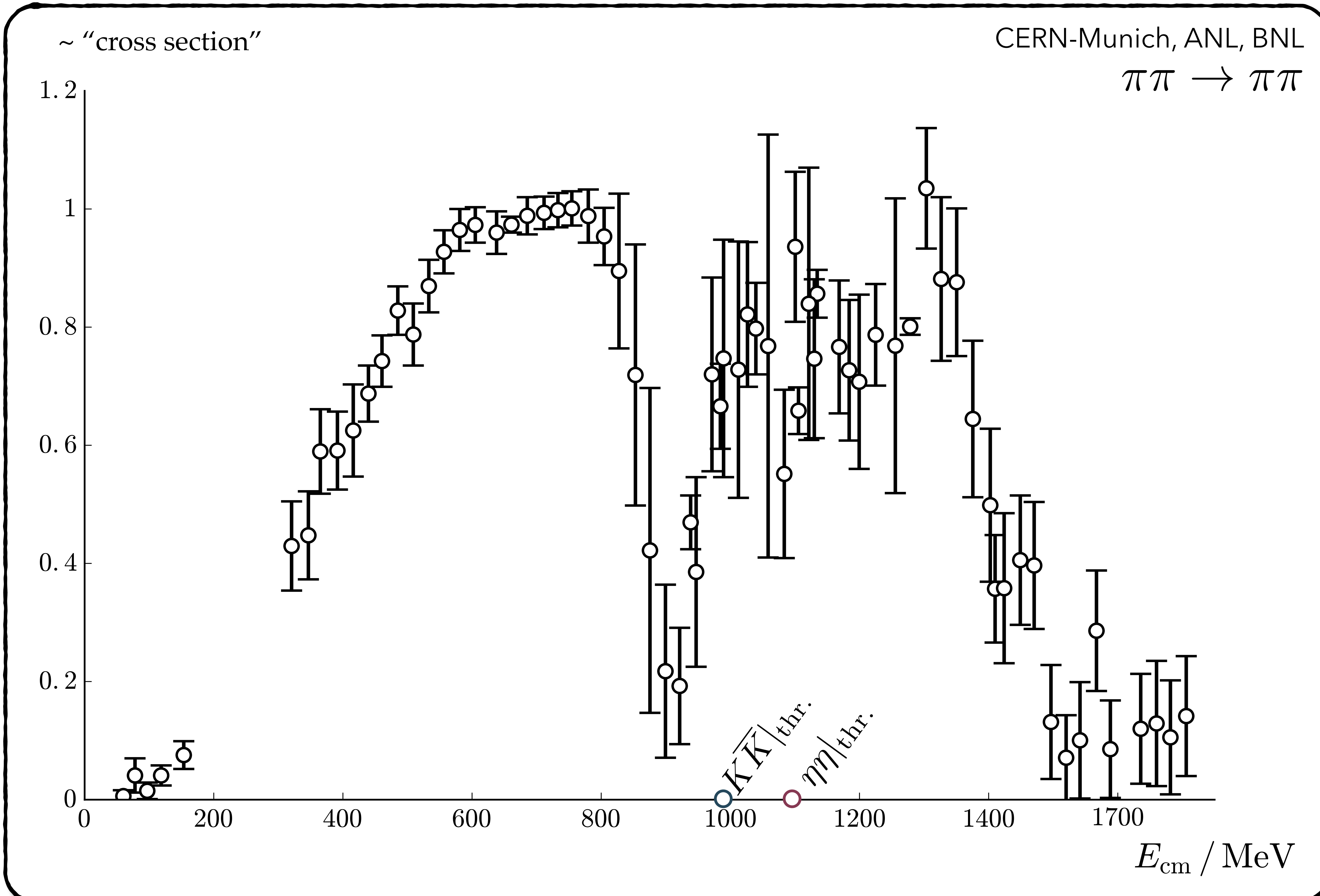


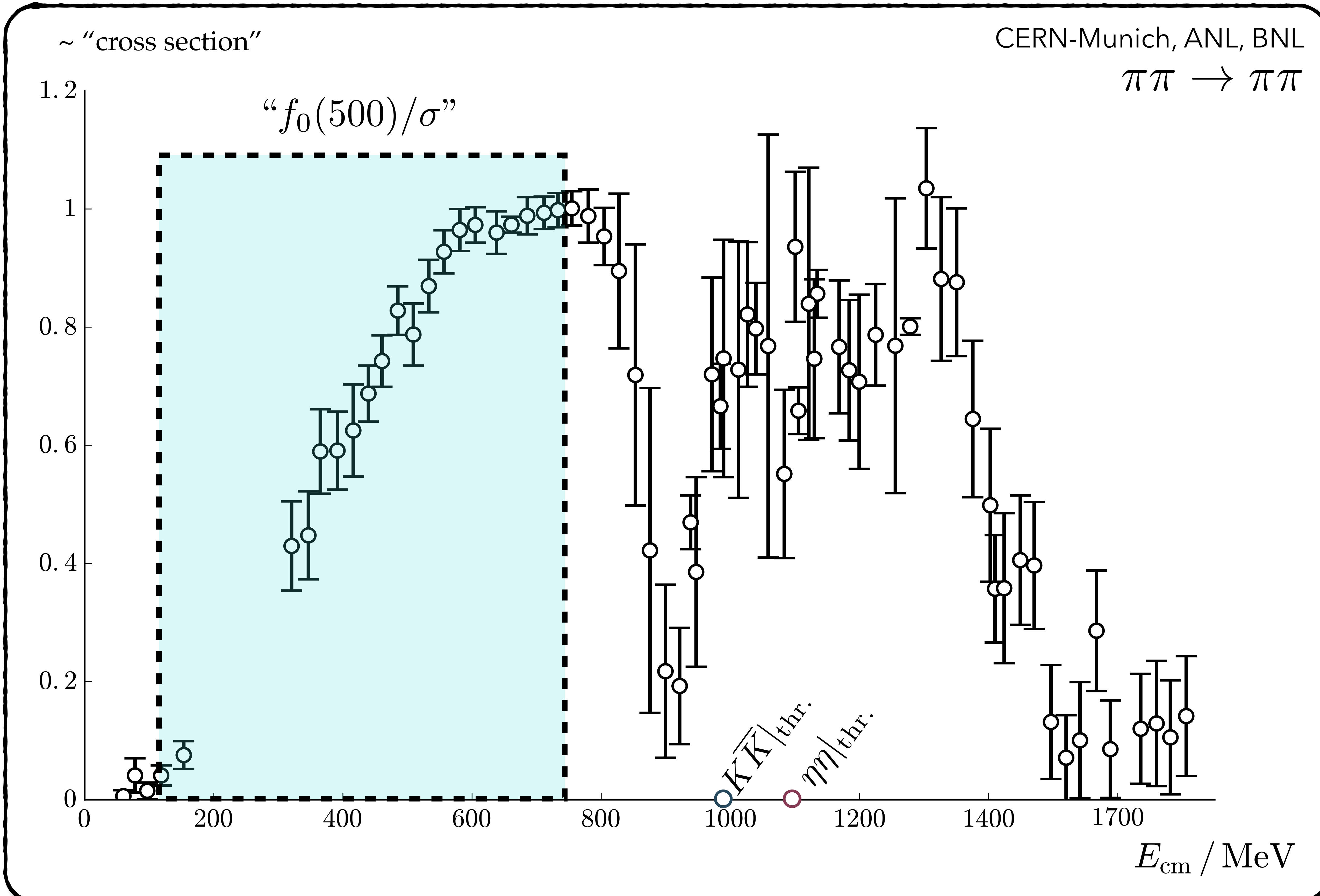
The isoscalar, scalar sector



Experimental manifestation



Experimental manifestation



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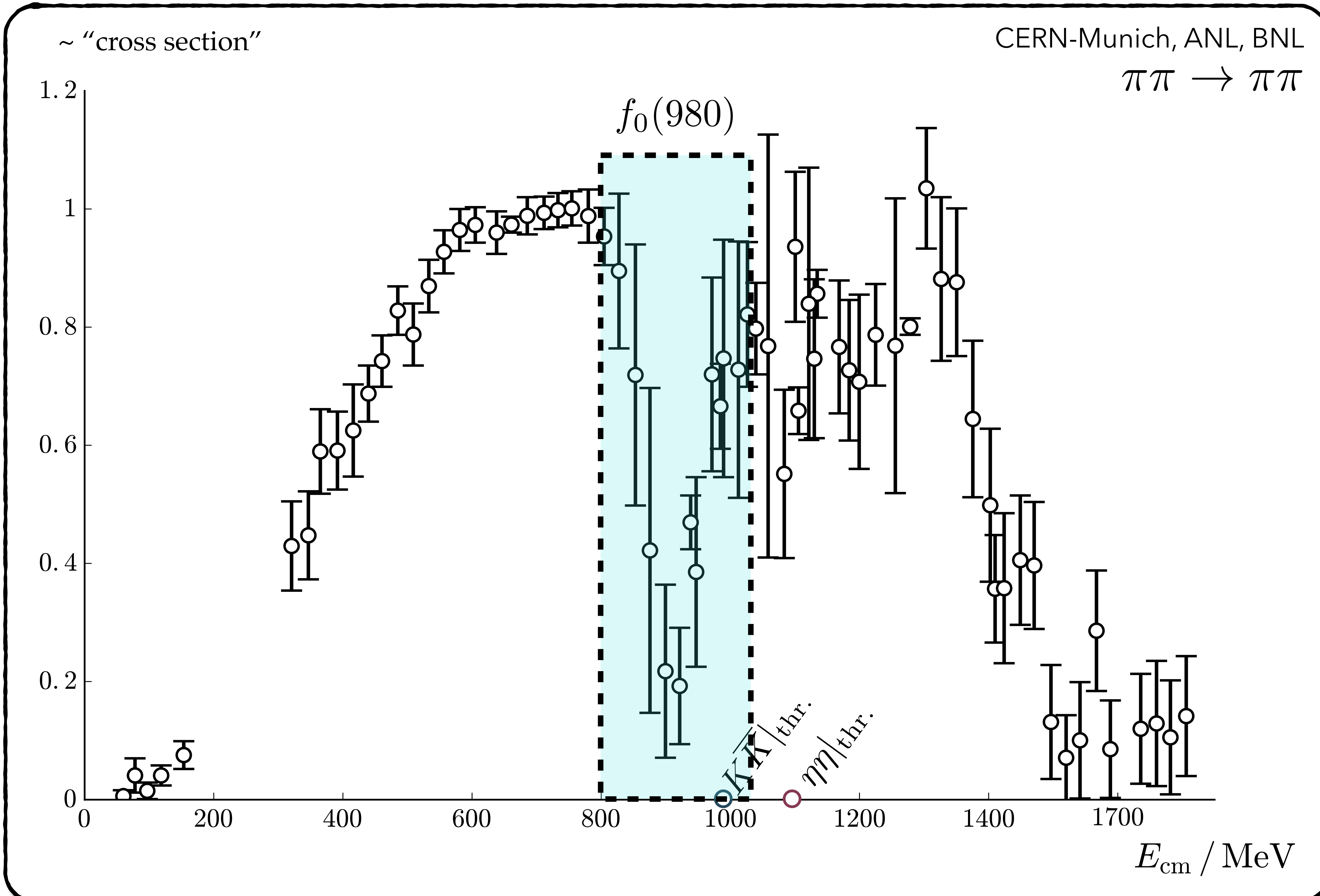
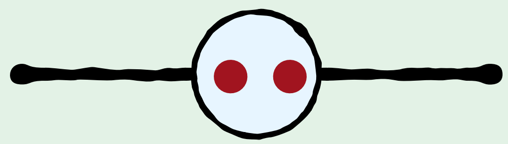


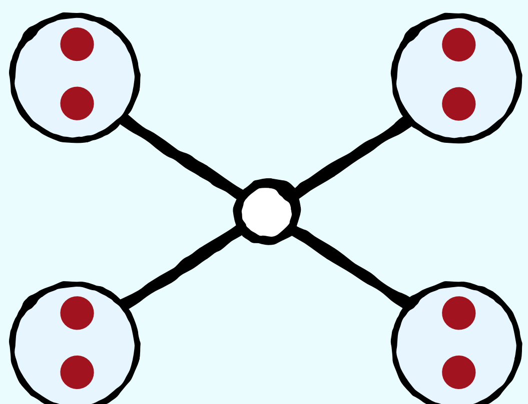
Quantitative definition

propagator:



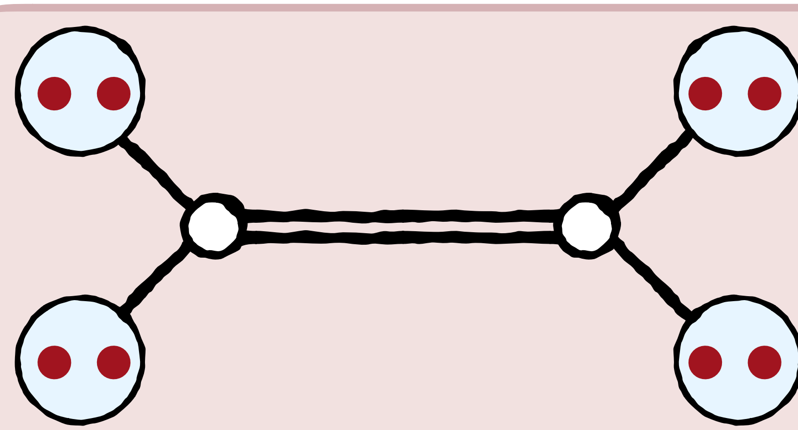
$$\sim \frac{iZ}{p^2 - m^2} = \frac{iZ}{s - m^2}$$

scattering amplitude:



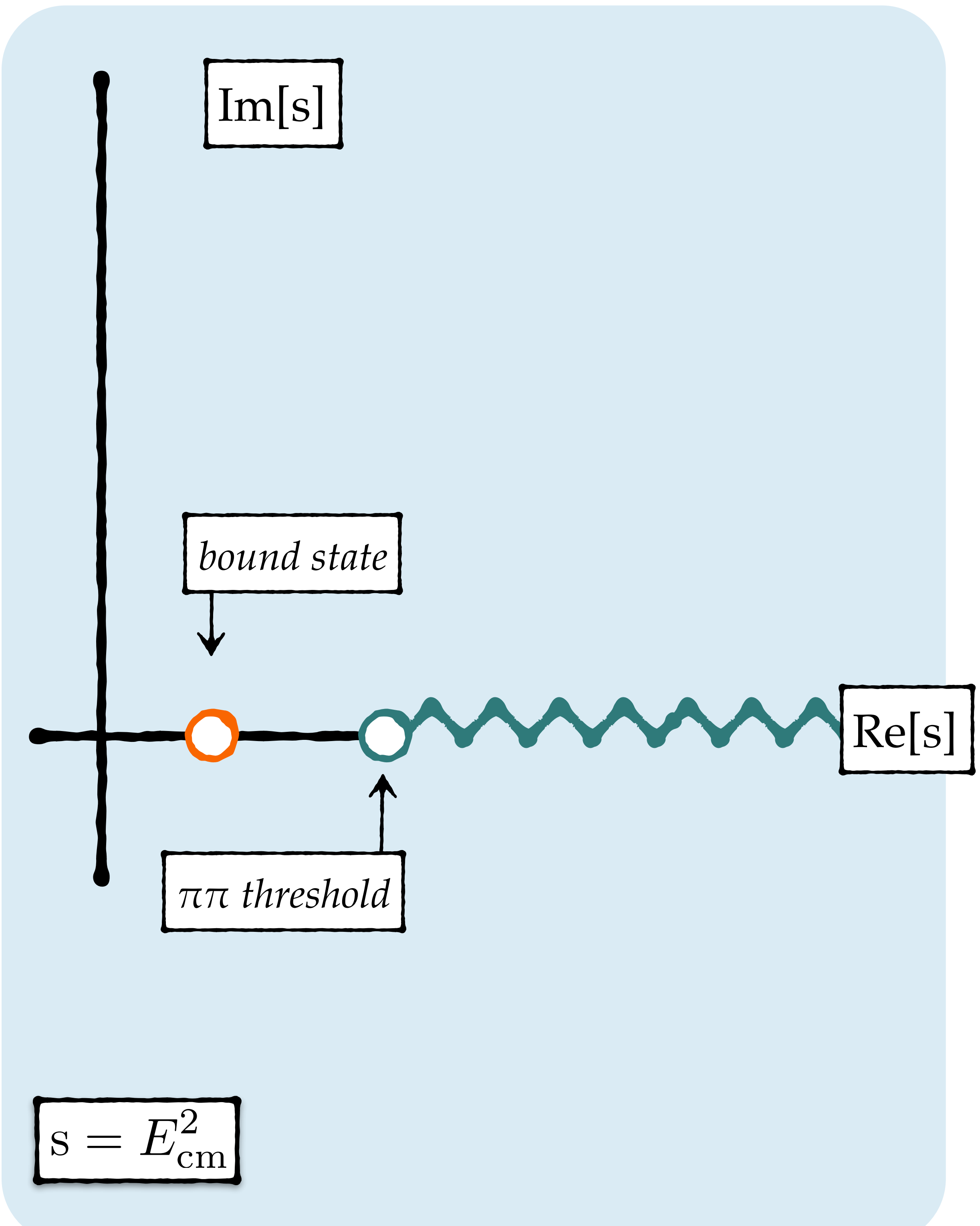
$$= i\mathcal{M}$$

…near bound state or resonance:



$$\sim \frac{ig^2}{s_0 - s}, \quad s_0 = (E_0 - \tfrac{i}{2}\Gamma)^2$$

Sheets and composite states



unitarity: $\mathcal{M}_{\pi\pi}^{-1} \propto \mathcal{K}_{\pi\pi}^{-1} - ip_{\pi\pi}$

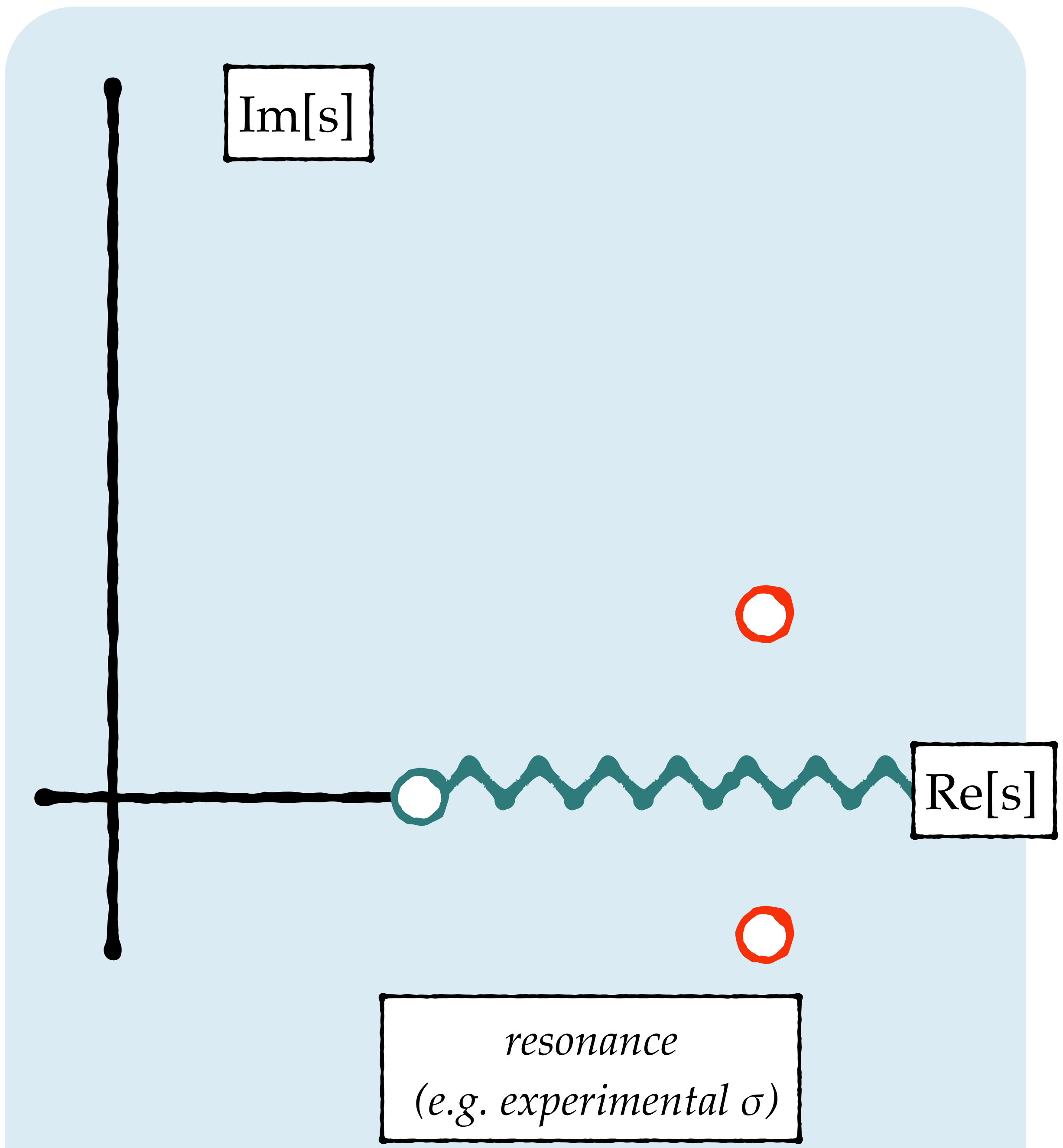
square-root singularity at each threshold:

$$p_{\pi\pi} = \frac{1}{2}\sqrt{s - s_{\pi\pi,th}}$$

unphysical sheet:

sheet	Im $p_{\pi\pi}$
I	$+$

Sheets and composite states



$$s_R = (E_R - \tfrac{i}{2}\Gamma_R)^2$$

unitarity: $\mathcal{M}_{\pi\pi}^{-1} \propto \mathcal{K}_{\pi\pi}^{-1} - i p_{\pi\pi}$

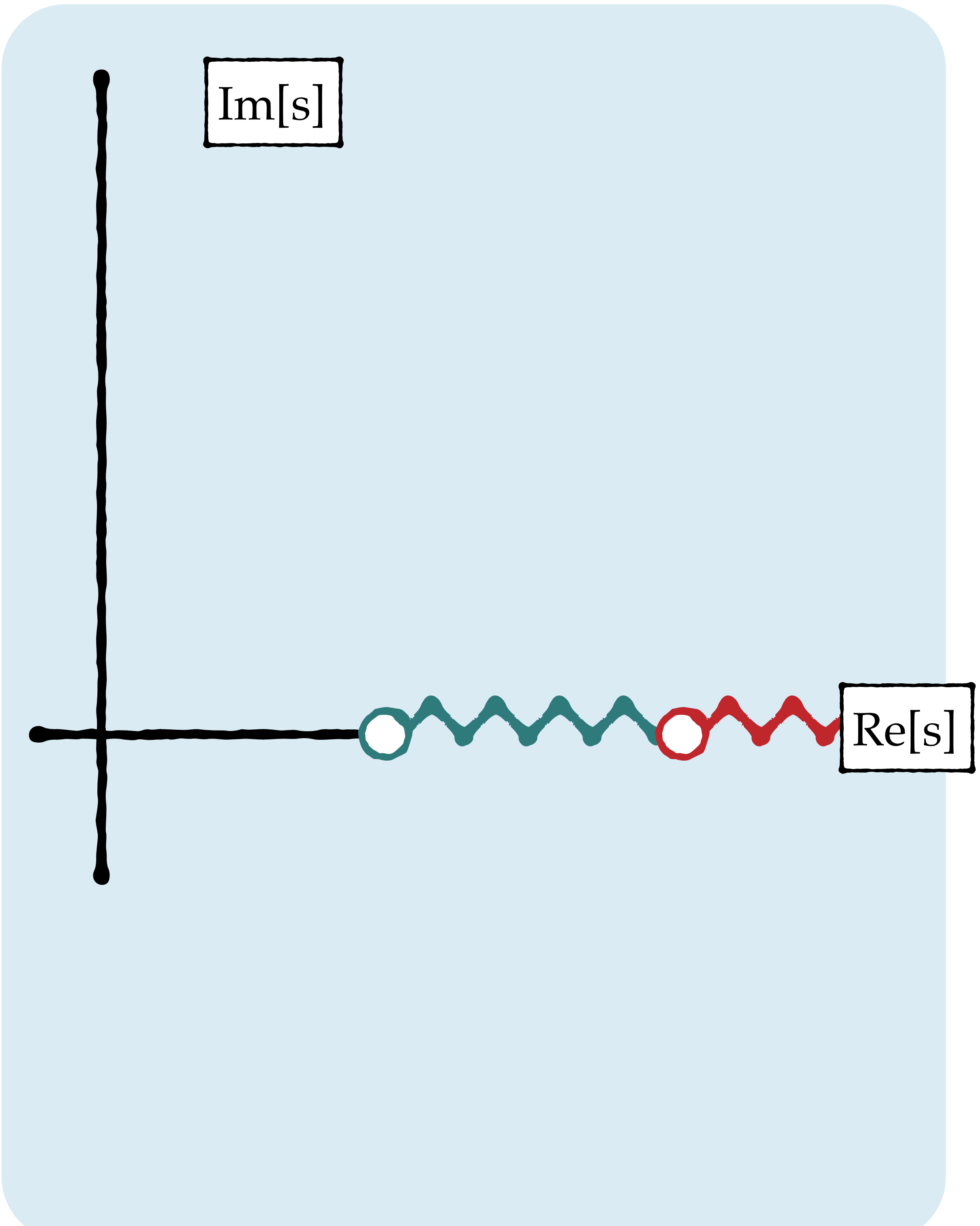
square-root singularity at each threshold:

$$p_{\pi\pi} = \frac{1}{2}\sqrt{s - s_{\pi\pi,th}}$$

unphysical sheet:

sheet	Im $p_{\pi\pi}$
I	$+$
II	$-$

Sheets and composite states



unitarity: $\mathcal{M}_{ab}^{-1} \propto \mathcal{K}_{ab}^{-1} - ip_a\,\delta_{ab}$

$a\,b = \pi\pi$ or $K\overline{K}$

square-root singularity at each threshold:

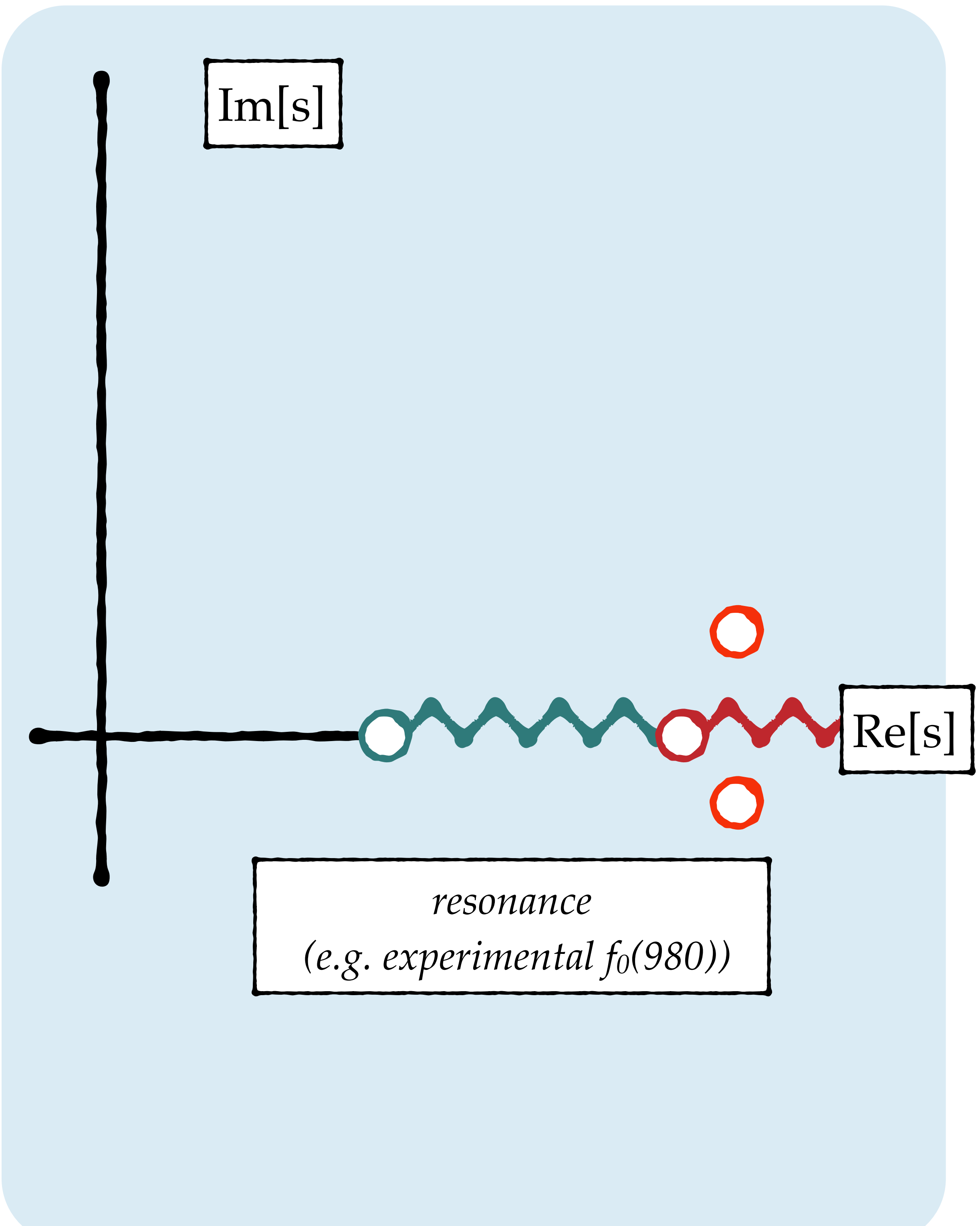
$$p_{\pi\pi} = \frac{1}{2}\sqrt{s - s_{\pi\pi,th}}$$

$$p_{K\overline{K}} = \frac{1}{2}\sqrt{s - s_{K\overline{K},th}}$$

physical sheet:

sheet	Im $p_{\pi\pi}$	Im $p_{K\overline{K}}$
I	$+$	$+$

Sheets and composite states



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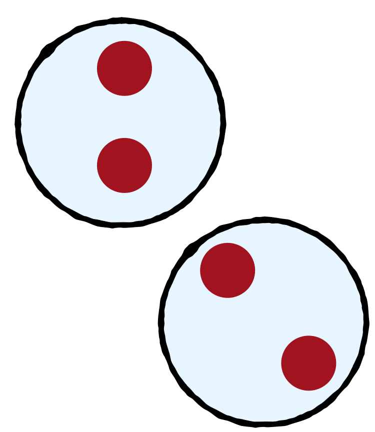
unphysical sheets:

sheet	Im $p_{\pi\pi}$	Im $p_{K\overline{K}}$
I	+	+
II	−	+
III	−	−
IV	+	−

Lattice QCD in a nutshell

- Wick rotation [Euclidean spacetime]: $t_M \to -it_E$
- Monte Carlo sampling
- quark masses: $m_q \to m_q^{\text{phys.}}$

Advantage over experiment!



Lattice QCD in a nutshell

- Wick rotation [Euclidean spacetime]: $t_M \to -it_E$
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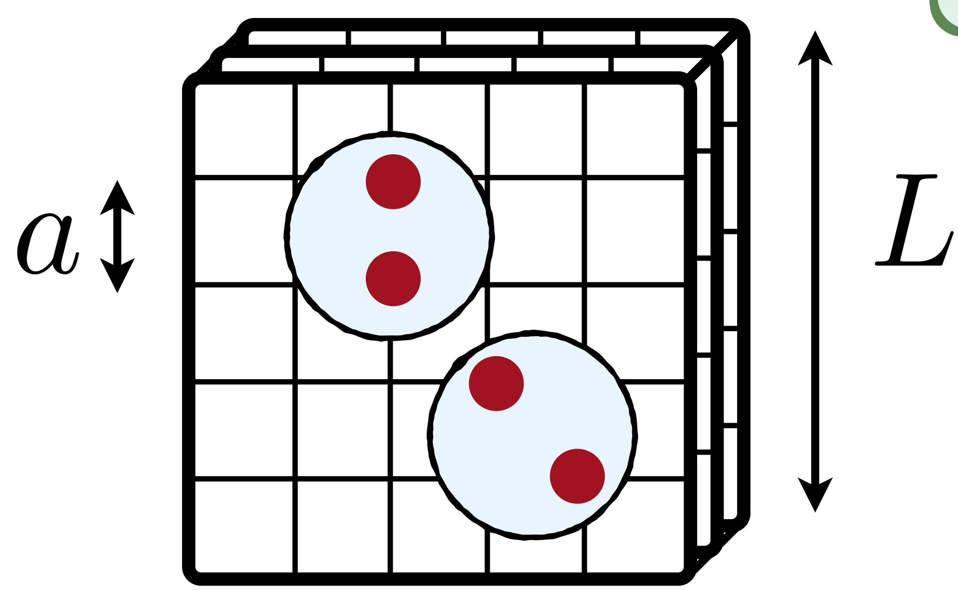
- quark masses: $m_q \to m_q^{\text{phys.}}$
- lattice spacing: $a \sim 0.03 - 0.15\ \text{fm}$
- finite volume

$$D_\mu = \left(\qquad\qquad \right) \updownarrow (L/a)^3 \times (T/a)$$



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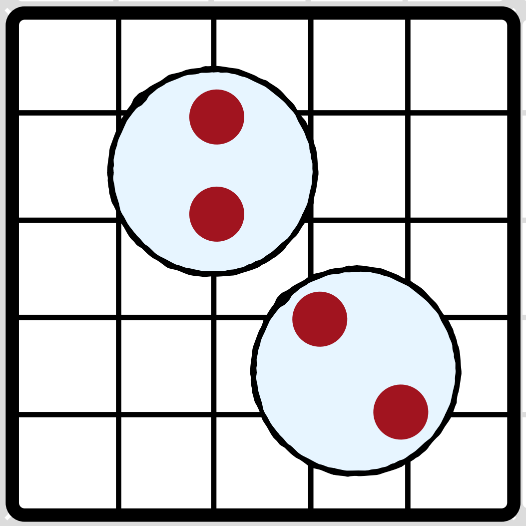

Never free!
No asymptotic states!
No scattering!

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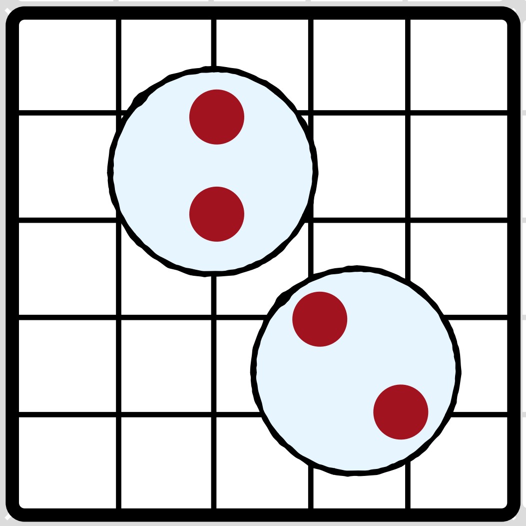

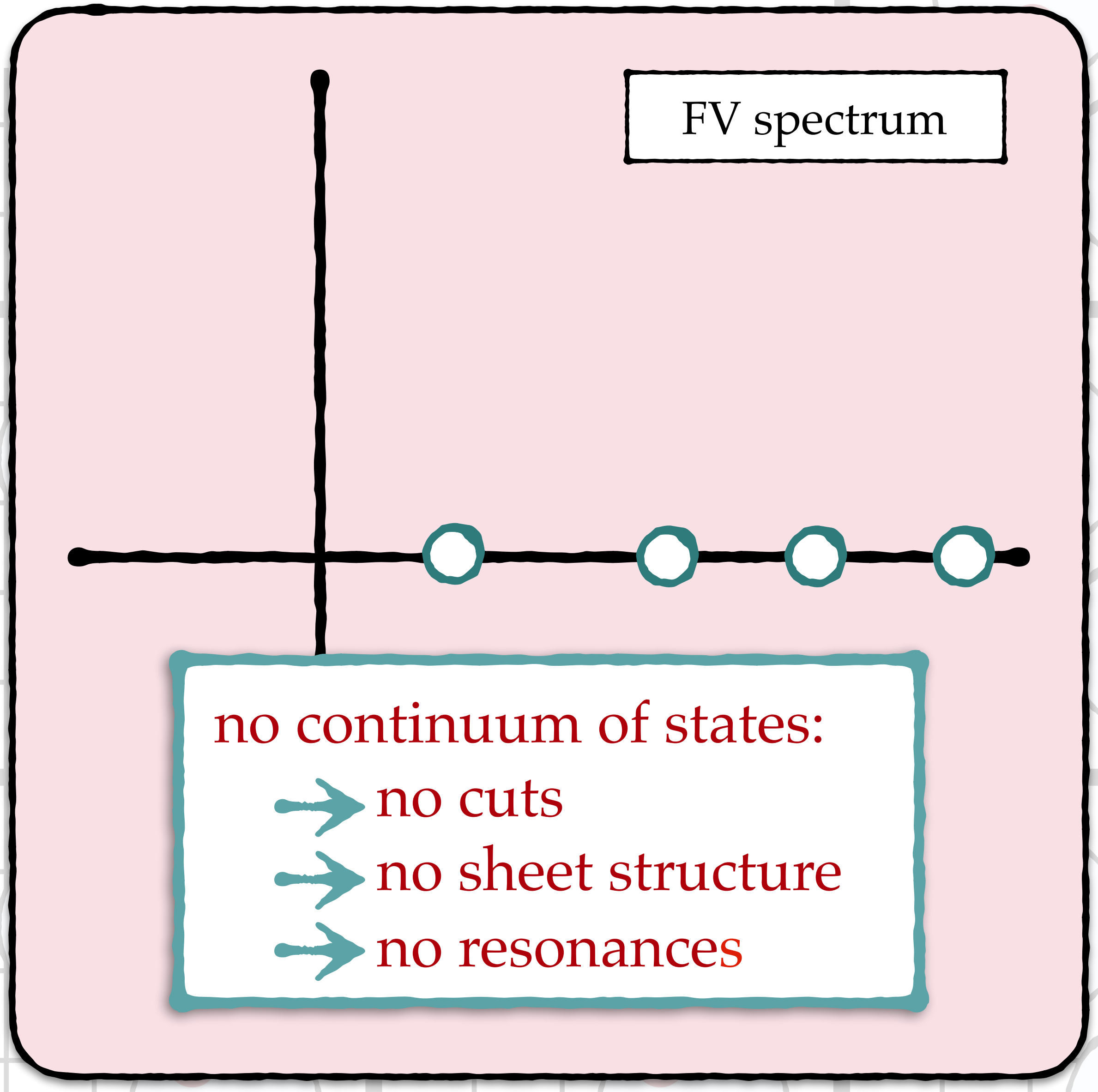

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- quark masses: $m_q \to m_q^{\text{phys.}}$
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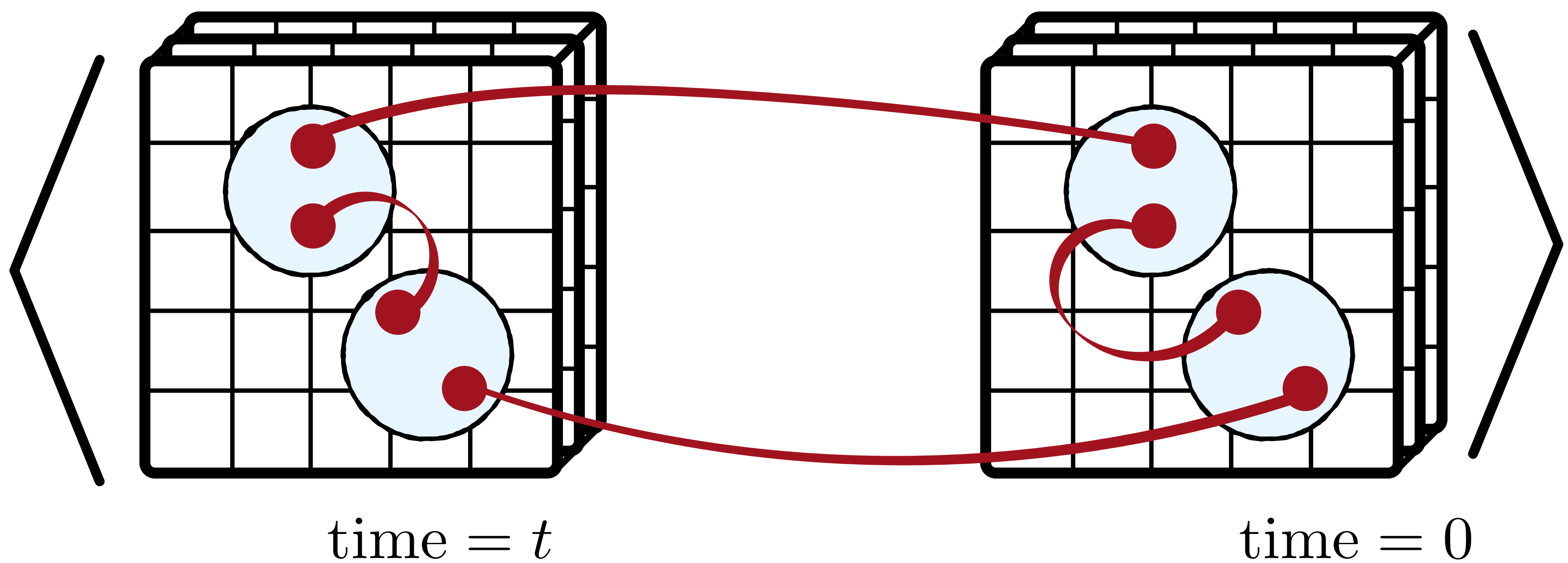
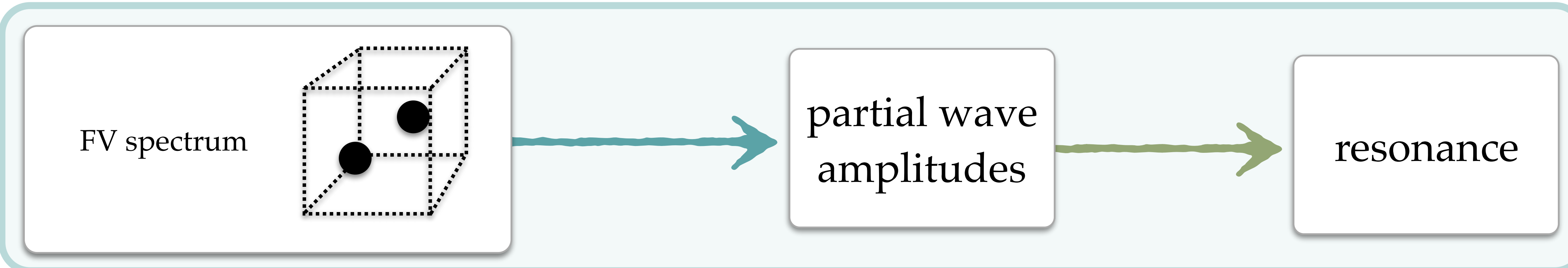
- finite volume
- Correlation functions: spectrum, matrix elements

$$C_L(t, \mathbf{P}) =$$



Scattering amplitudes



$$\det[F^{-1}(E_L, L) + \mathcal{M}(E_L)] = 0$$

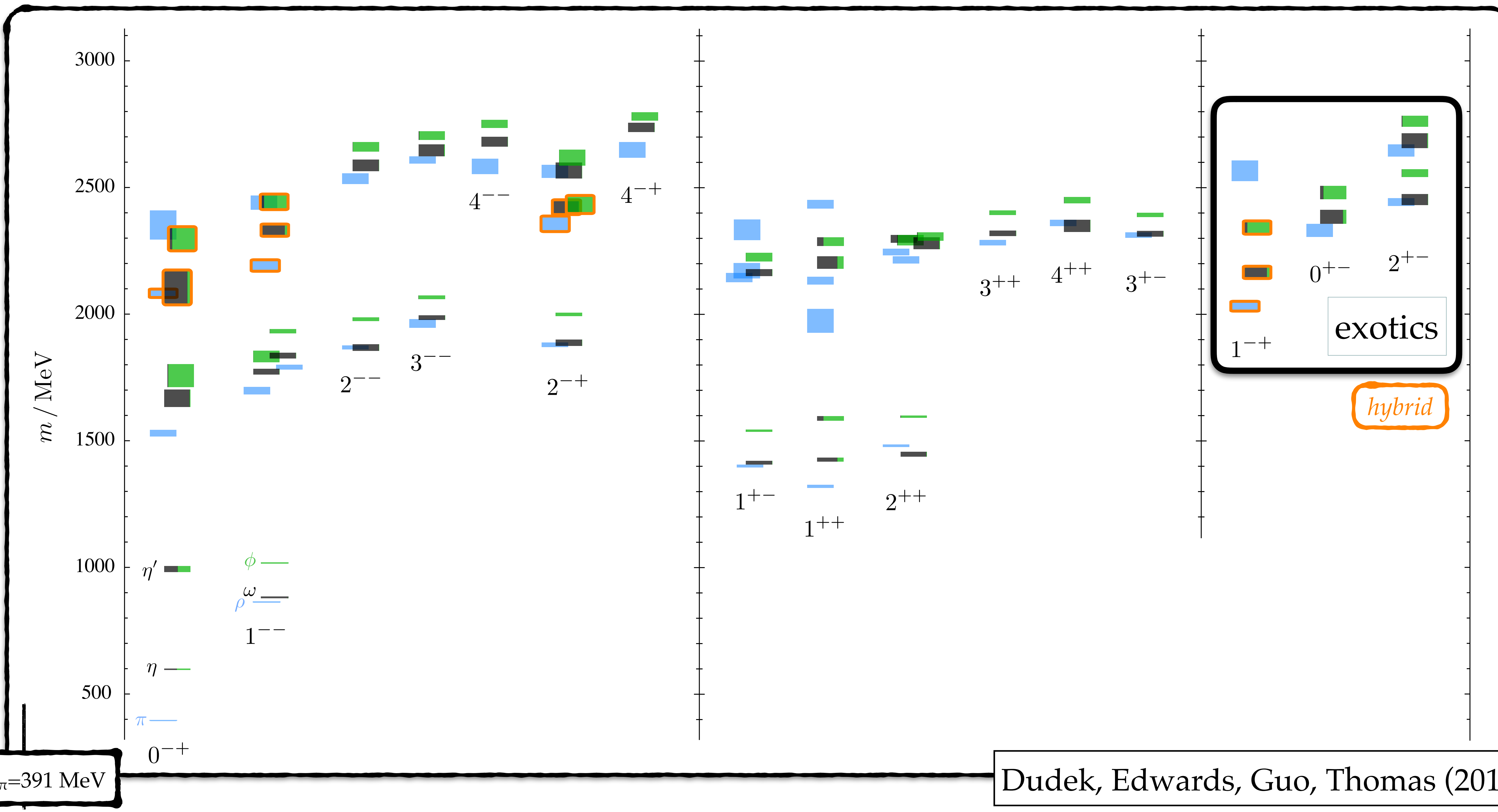
E_L = finite volume spec.
L = finite volume
F = known function
$\mathcal{M}$ = scattering amp.

- Lüscher (1986, 1991) [elastic scalar bosons]
- Rummukainen & Gottlieb (1995) [moving elastic scalar bosons]
- Kim, **Sachrajda**, & **Sharpe** / **Christ**, Kim & Yamazaki (2005) [QFT derivation]
- Feng, Li, & Liu (2004) [inelastic scalar bosons]
- **Hansen** & **Sharpe** / RB & Davoudi (2012) [moving inelastic scalar bosons]
- RB (2014) [general 2-body result]

New "old-school spectroscopy"

Evaluate: $C_{ab}^{2pt.}(t,\mathbf{P}) \equiv \langle 0|\mathcal{O}_b(t,\mathbf{P})\mathcal{O}_a^\dagger(0,\mathbf{P})|0\rangle = \sum_n Z_{b,n} Z_{a,n}^* e^{-E_n t}$

…a large number [10-30] of local ops, $\mathcal{O}_b \sim \bar{q}\,\Gamma_b\, q$



Dudek, Edwards, Guo, Thomas (2013)

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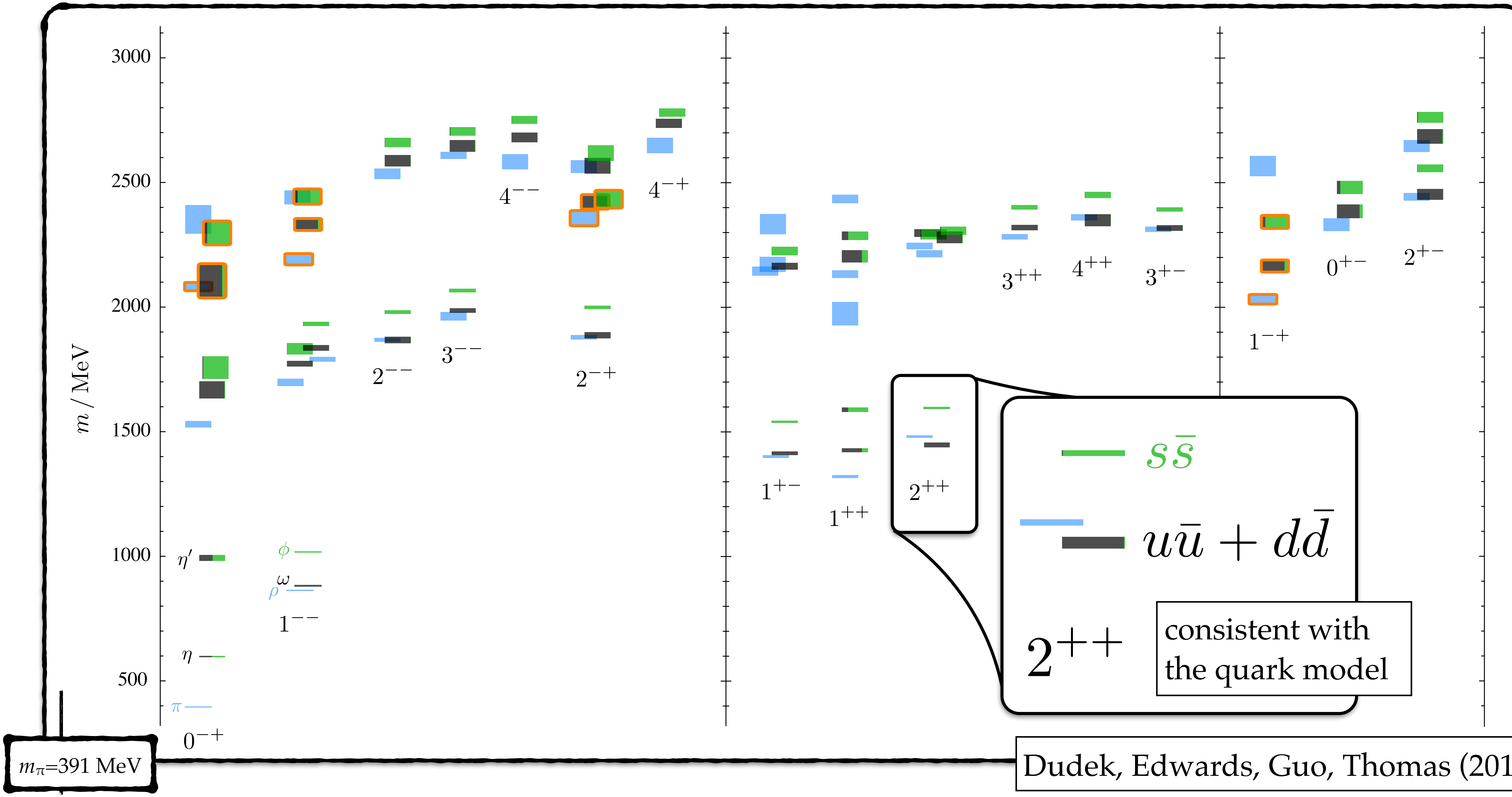

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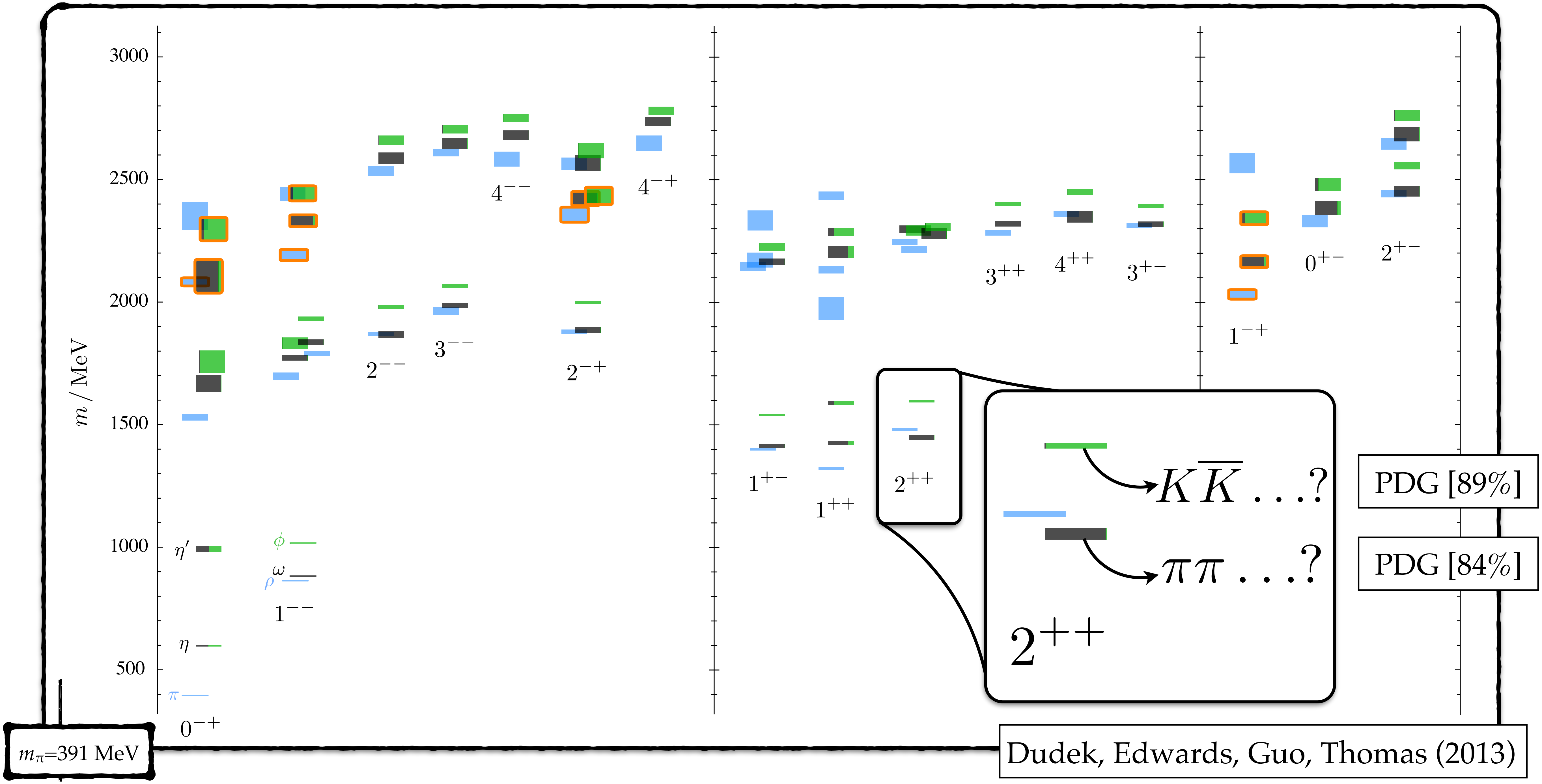
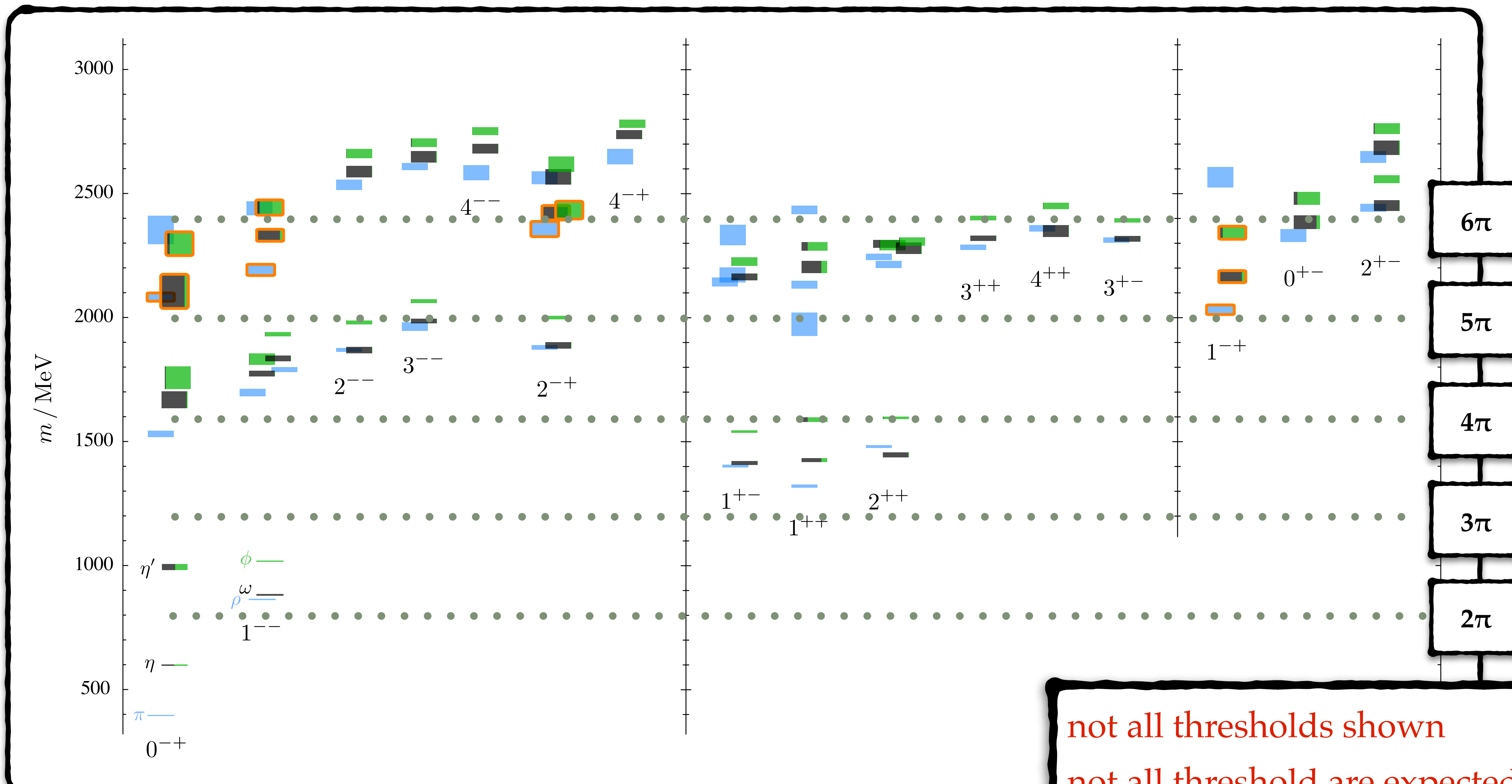


Dudek, Edwards, Guo, Thomas (2013)

Narrow width approximation

- Op. basis did not include multi-hadron ops: $\pi\pi,\ K\overline{K}, \eta\eta, \pi\pi\pi, \ldots$
- Unstable nature of the states ignored
  - Finite-volume states are *not* resonances
  - Must use Lüscher and its extensions
- **Spectrum does suggest where *some* resonance might lie**

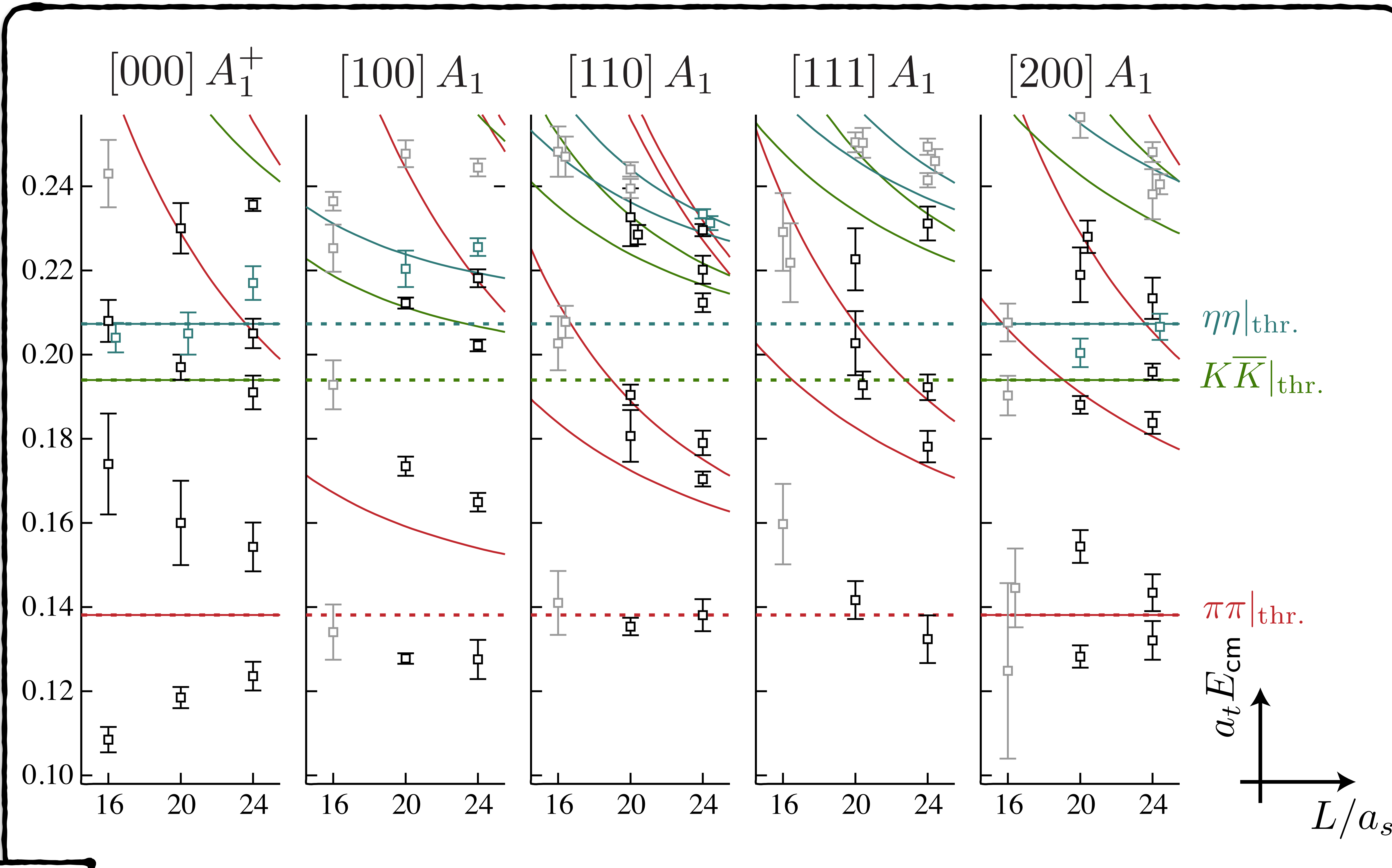


not all thresholds shown

not all threshold are expected to matter

Isoscalar spectra: S-wave dominant

- Multi-meson ops. are crucial
- Spectrum including a larger basis: $\{\pi\pi, K\overline{K}, \eta\eta, \ell\bar{\ell}, s\bar{s}\}$



m_π=391 MeV

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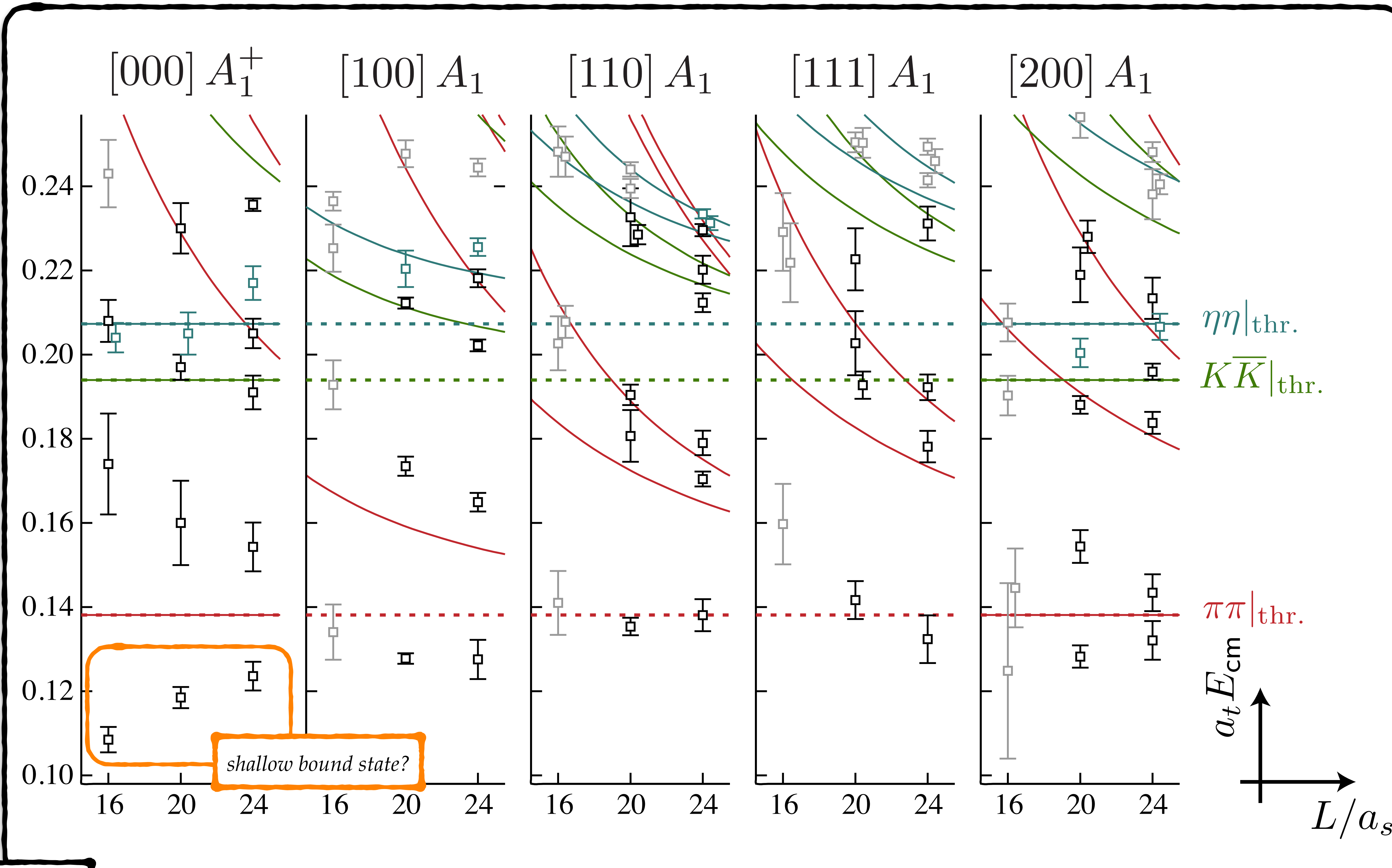

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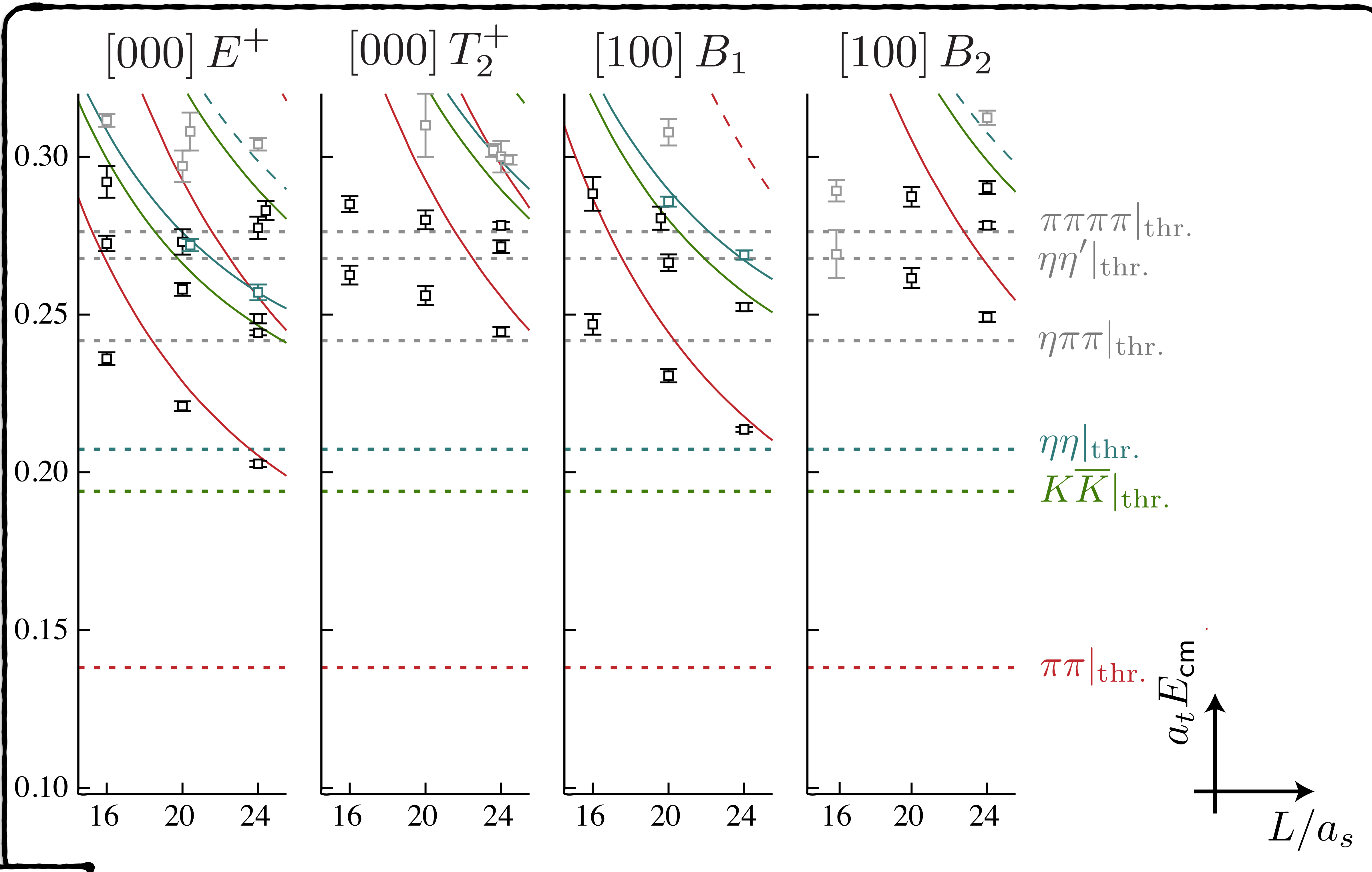

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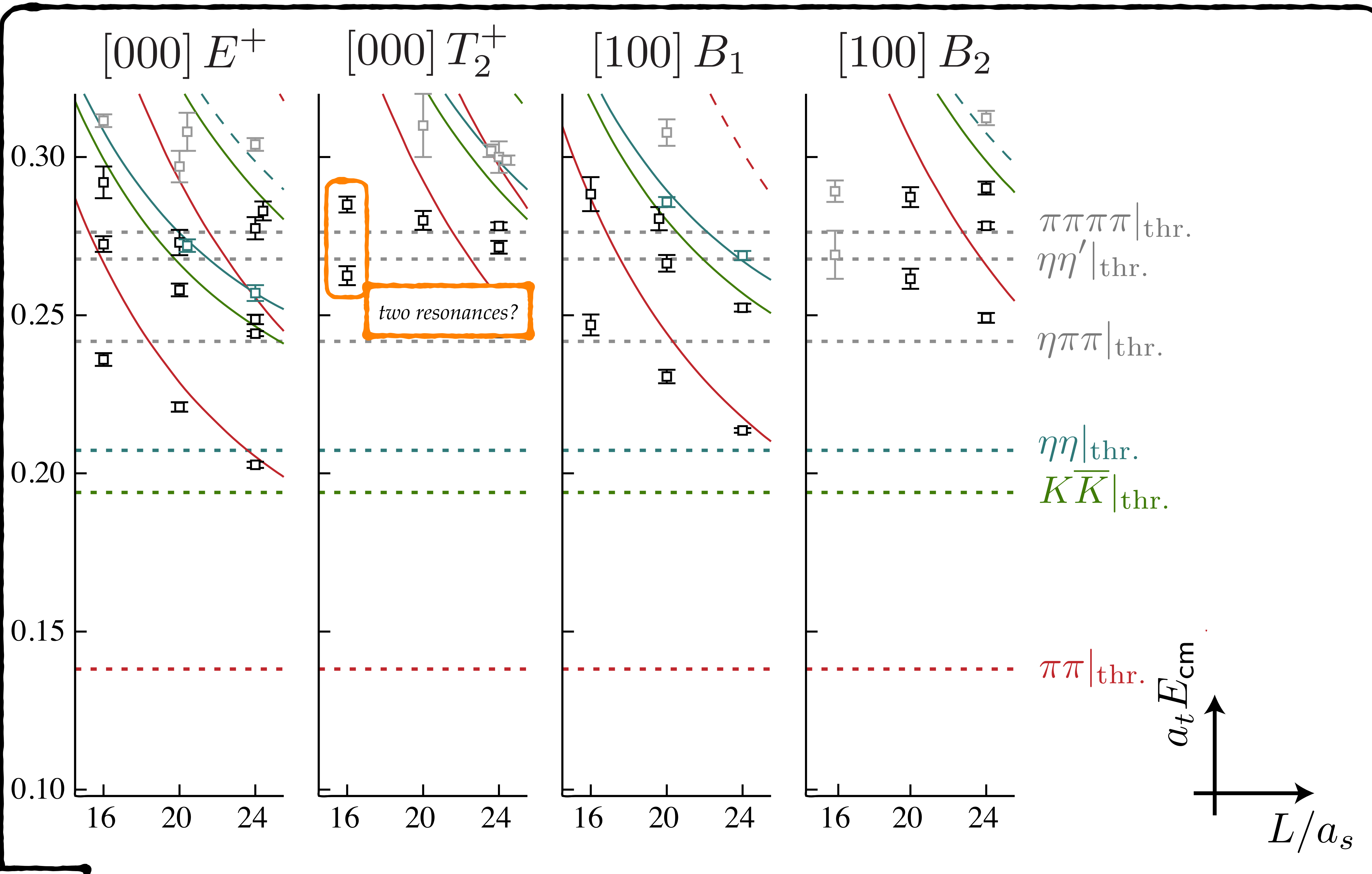
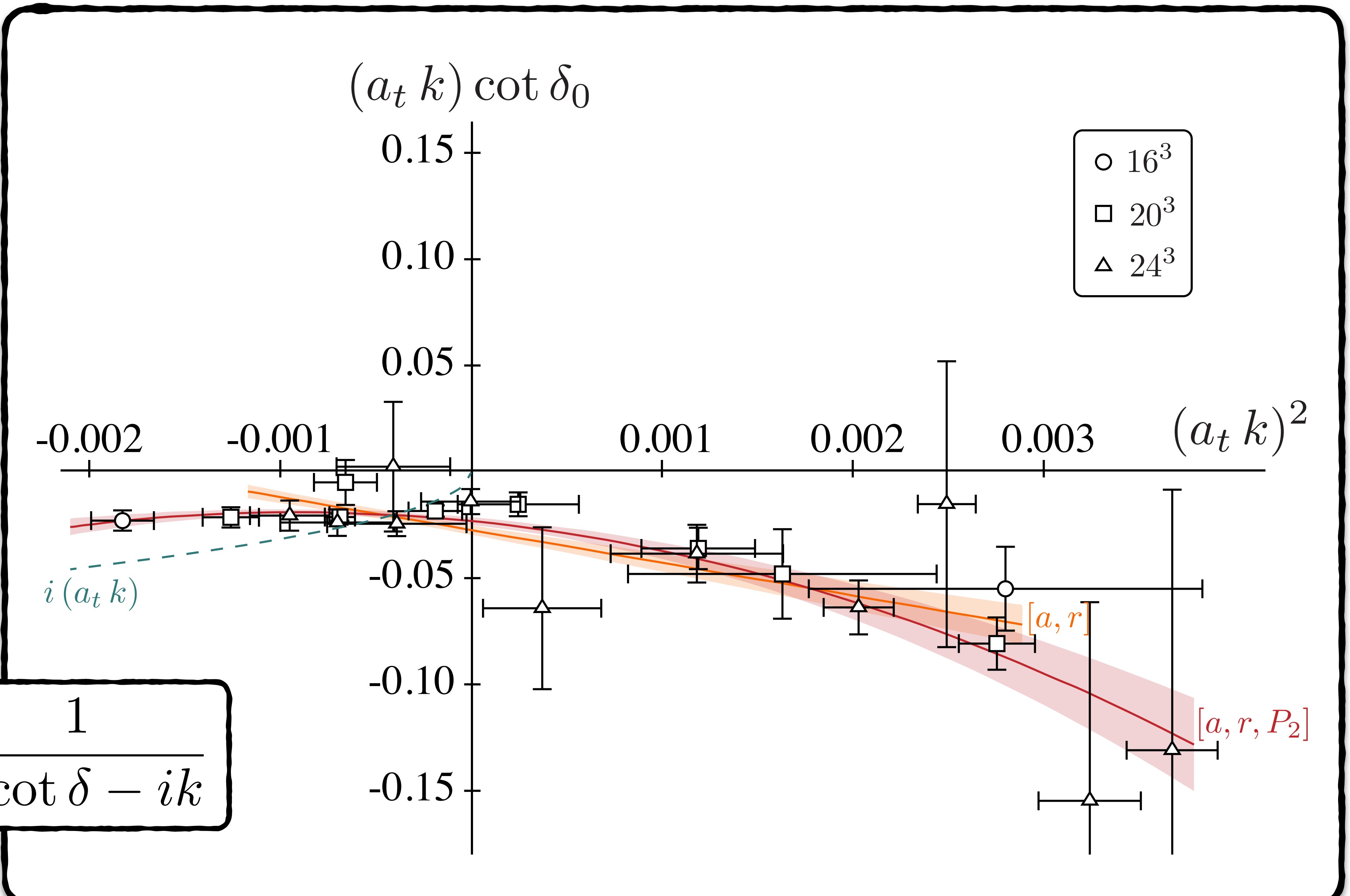


m_π=391 MeV

Elastic region

- Below $2m_K$, one directly determine the $\pi\pi$ amplitude



$$\mathcal{M} \sim \frac{1}{k\cot\delta - ik}$$

- Clear evidence of bound state below $\pi\pi$ threshold
- This correspondence only holds if partial-wave mixing is negligible [checked]

Coupled-channels analysis

- Above *2m_K*, there is not a one-to-one correspondence

$$\det \begin{bmatrix} F^{-1}_{\pi\pi} + \mathcal{M}_{\pi\pi,\pi\pi} & \mathcal{M}_{\pi\pi,K\overline{K}} \\ \mathcal{M}_{\pi\pi,K\overline{K}} & F^{-1}_{K\overline{K}} + \mathcal{M}_{K\overline{K},K\overline{K}} \end{bmatrix} = 0$$

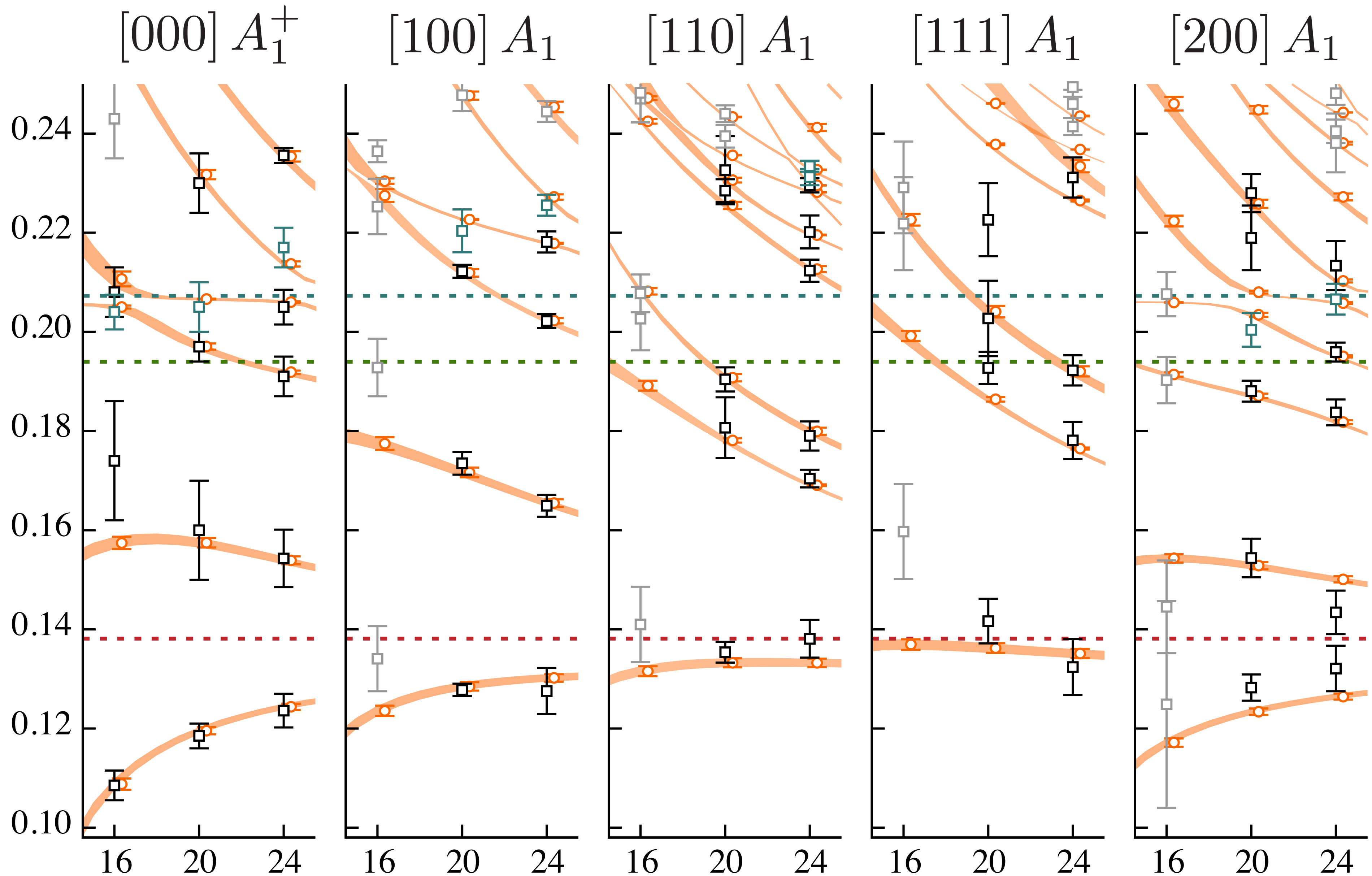
Feng, Li, & Liu (2004),
Hansen & Sharpe / RB & Davoudi (2012)

- In general, must constrain *(1/2) [N^2 + N]* functions of energy
- Need that many energy levels at the same energy
- Alternatively, parametrize scattering amplitude and do a global fit

Coupled-channels analysis

- S-wave above $2m_\pi$, $2m_K$, and $2m_\eta$
- Ansatz $\mathbf{K}^{-1}(s) = \begin{pmatrix} a+bs & c+ds & e \\ c+ds & f & g \\ e & g & h \end{pmatrix}$

Spectrum $\quad \chi^2/N_{\mathrm{dof}} = \frac{44.0}{57-8} = 0.90 \quad$ 57 energy levels



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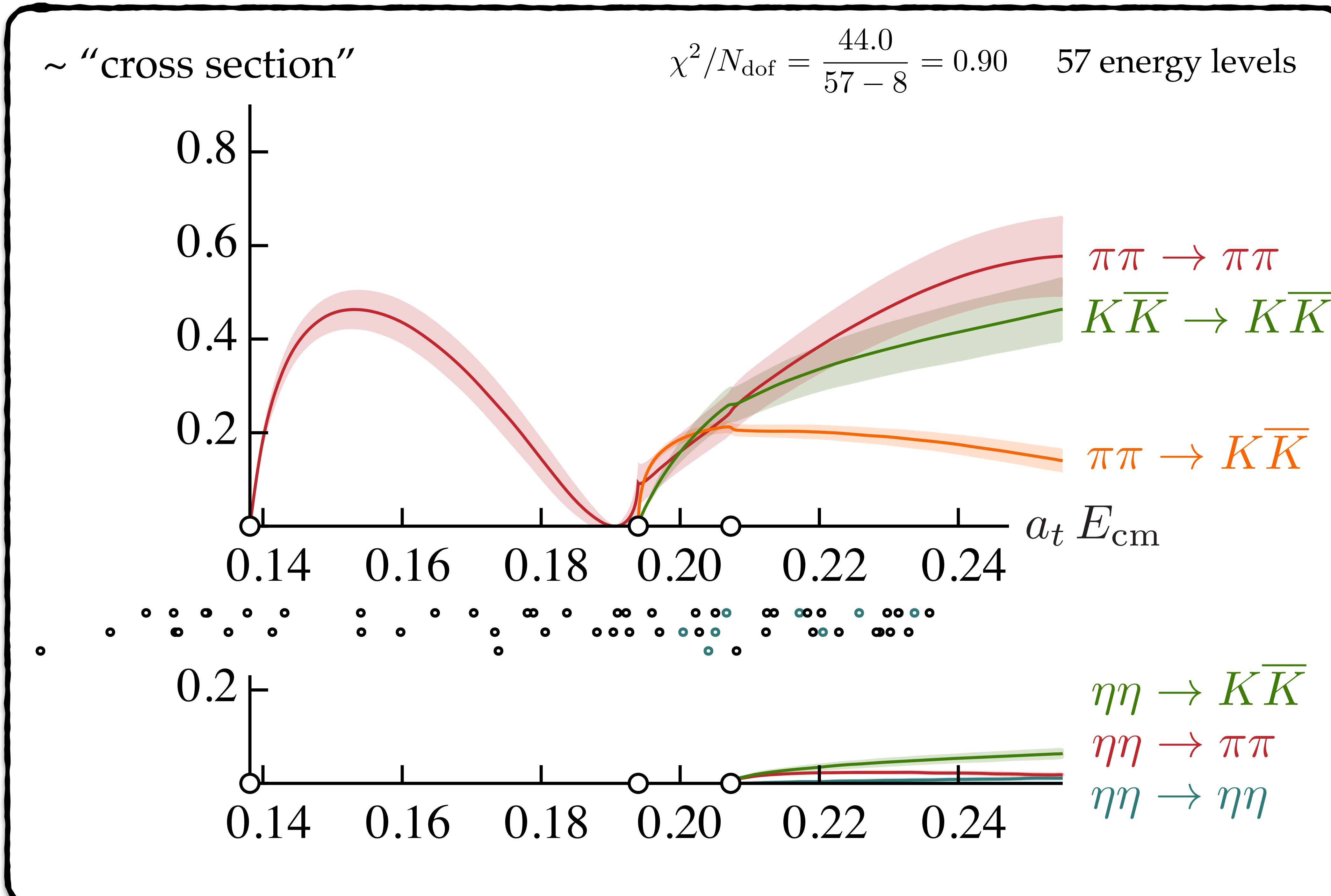
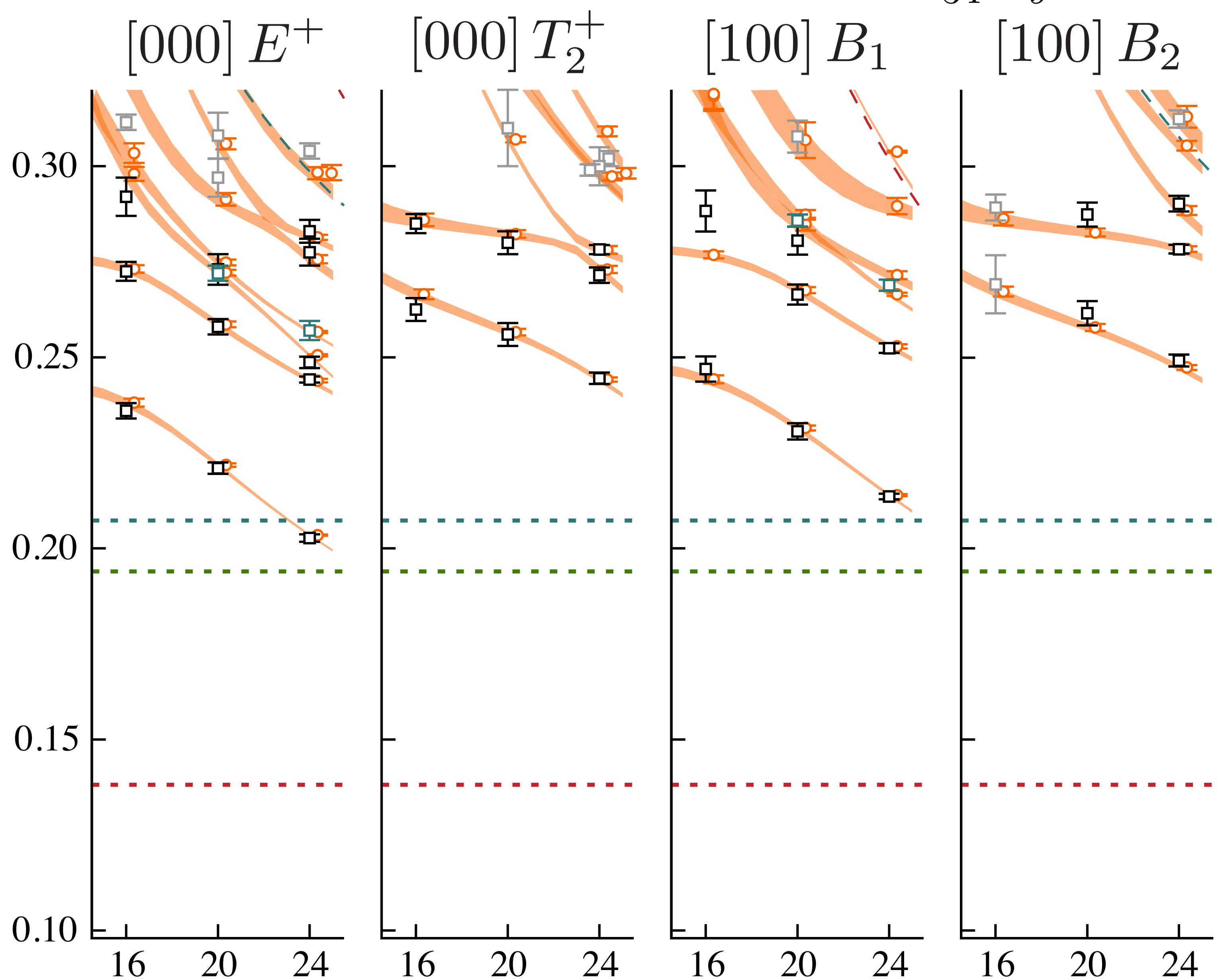


Coupled-channels analysis

- D-wave above $2m_\pi$, $2m_K$, and $2m_\eta$
- Ansatz $K_{ij}(s) = \frac{g_i^{(1)} g_j^{(1)}}{m_1^2 - s} + \frac{g_i^{(2)} g_j^{(2)}}{m_2^2 - s} + \gamma_{ij}$ $\quad \gamma_{\eta\eta} \neq 0$, $\gamma_{ij} = 0$ otherwise

Spectrum $\quad \chi^2/N_{\mathrm{dof}} = \frac{28.9}{34-9} = 1.15 \quad$ 34 energy levels



Coupled-channels analysis

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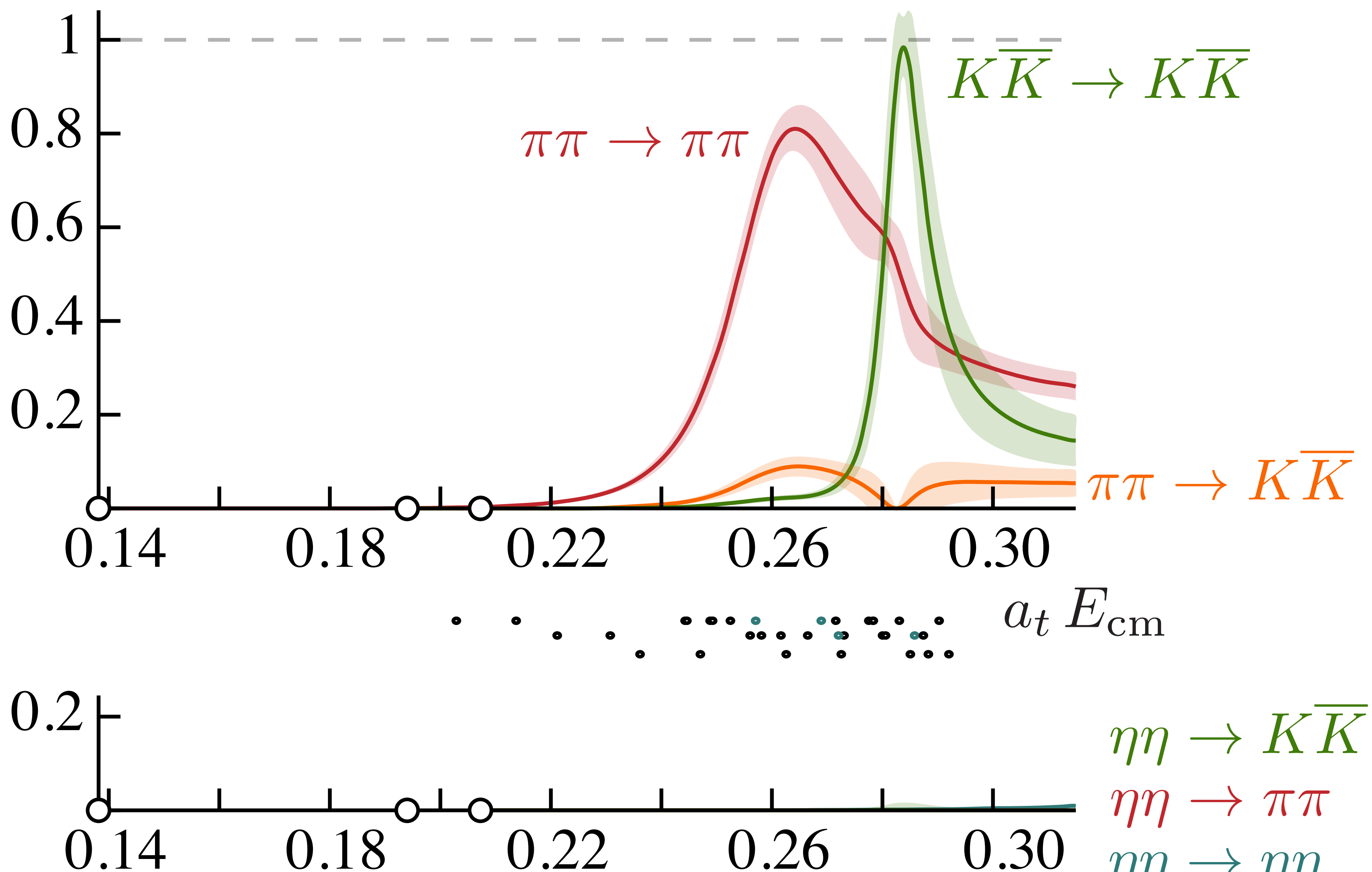
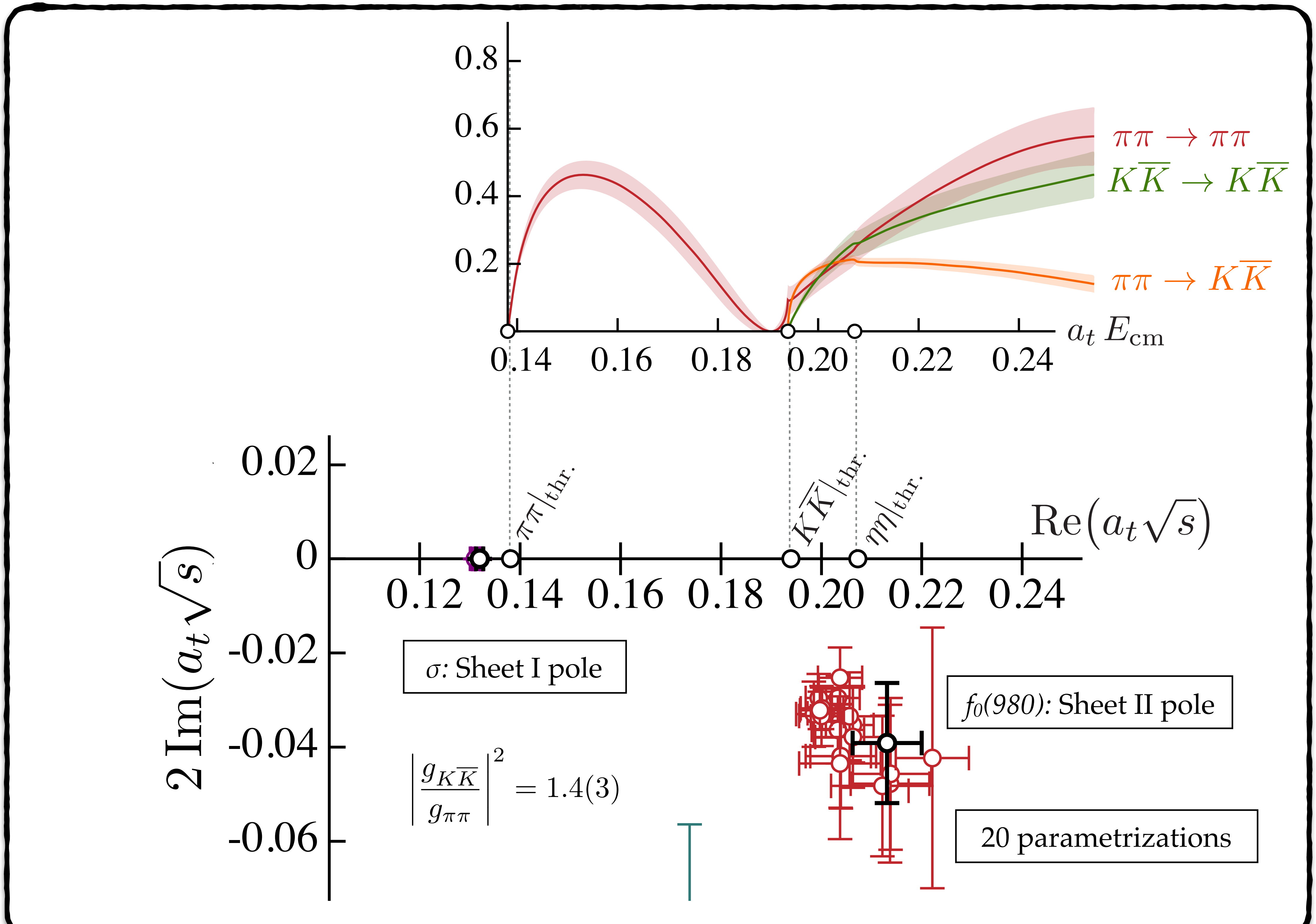
~ "cross section" $\quad \chi^2/N_{\mathrm{dof}} = \frac{28.9}{34-9} = 1.15 \quad$ 34 energy levels



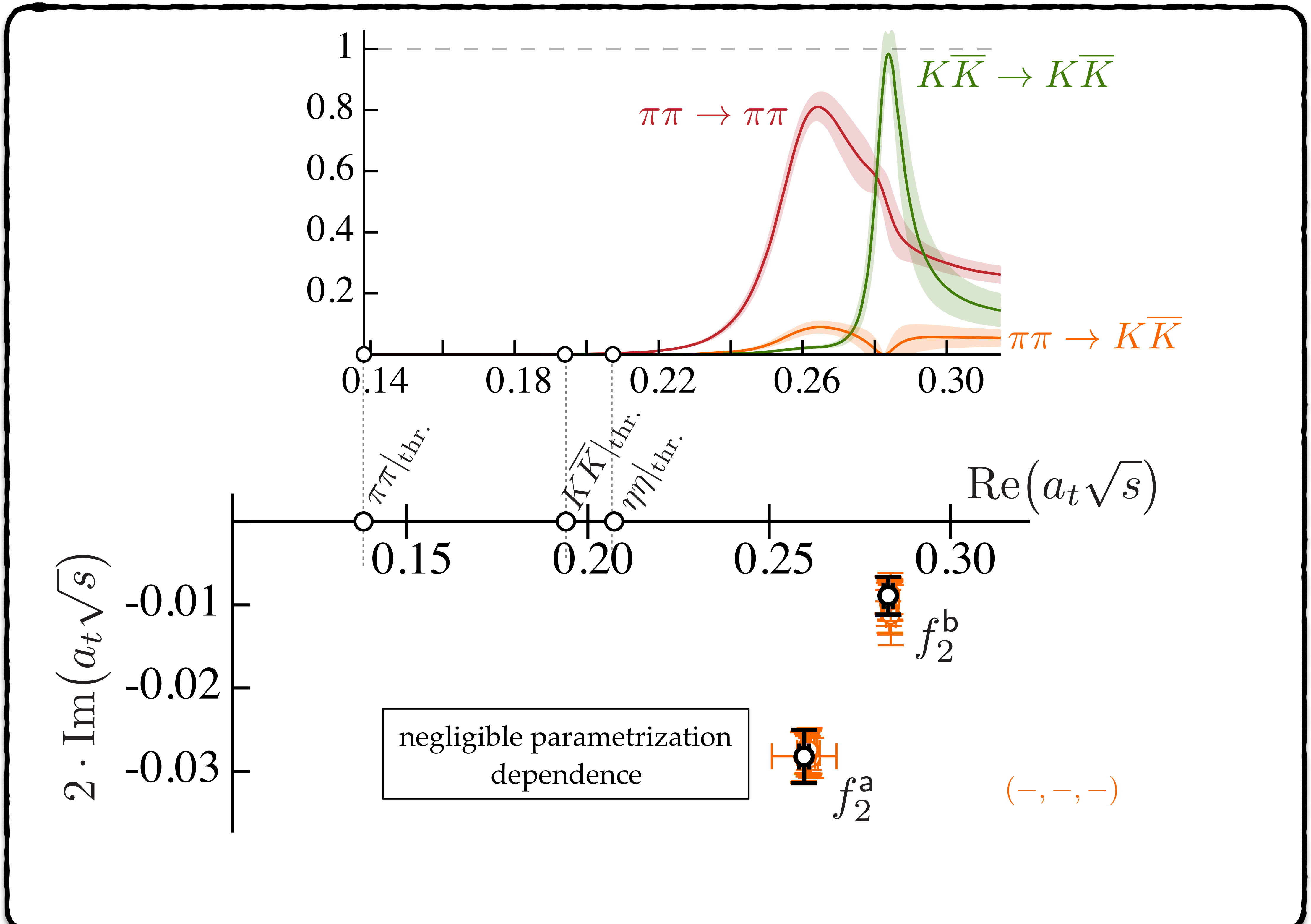
Scalar poles: σ and *f₀(980)*

- Near poles: $\mathcal{M} \sim \dfrac{g^2}{s_0 - s}$



Tensor poles: the f_2's

- Near poles: $\mathcal{M} \sim \dfrac{g^2}{s_0 - s}$



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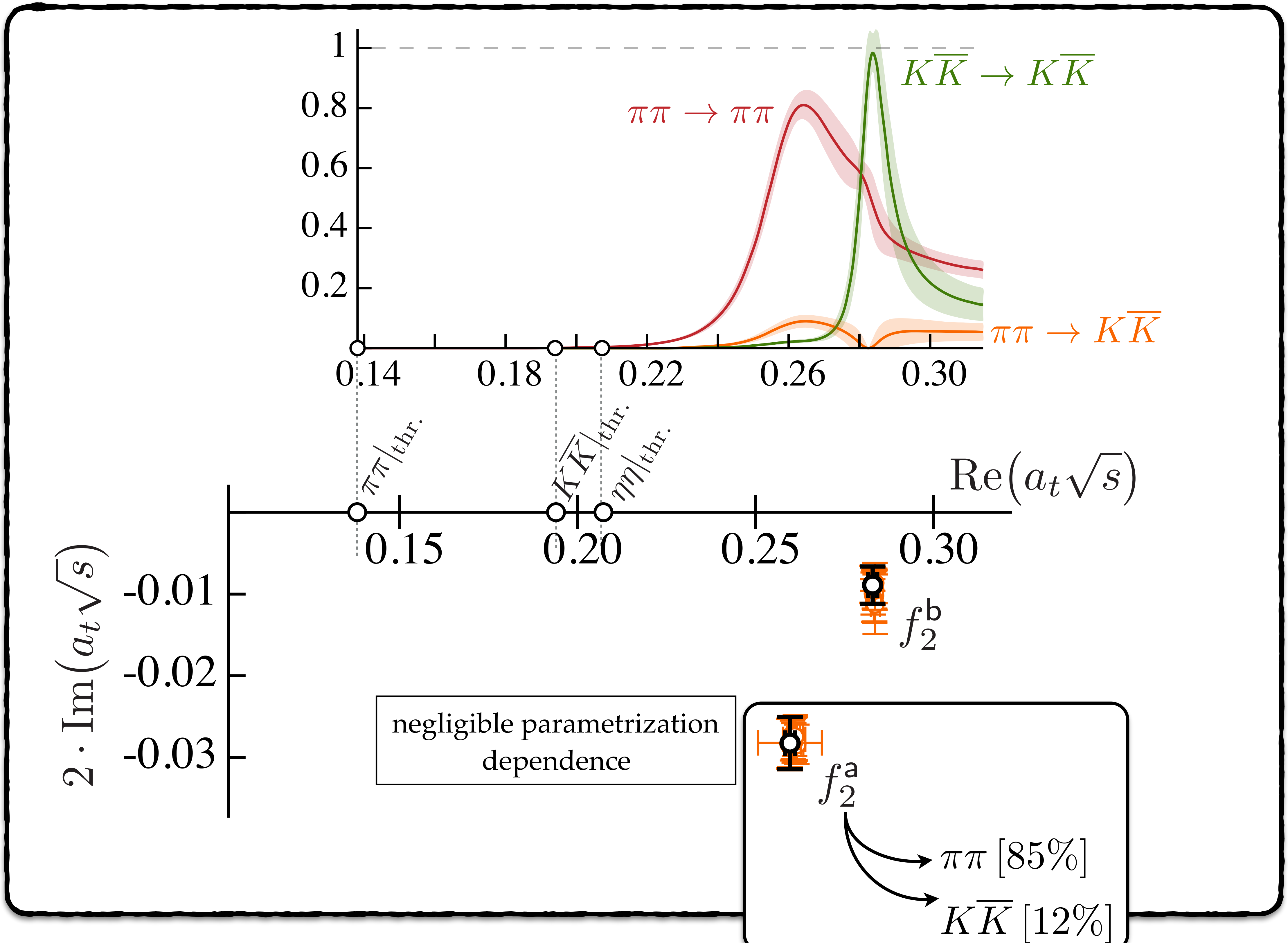

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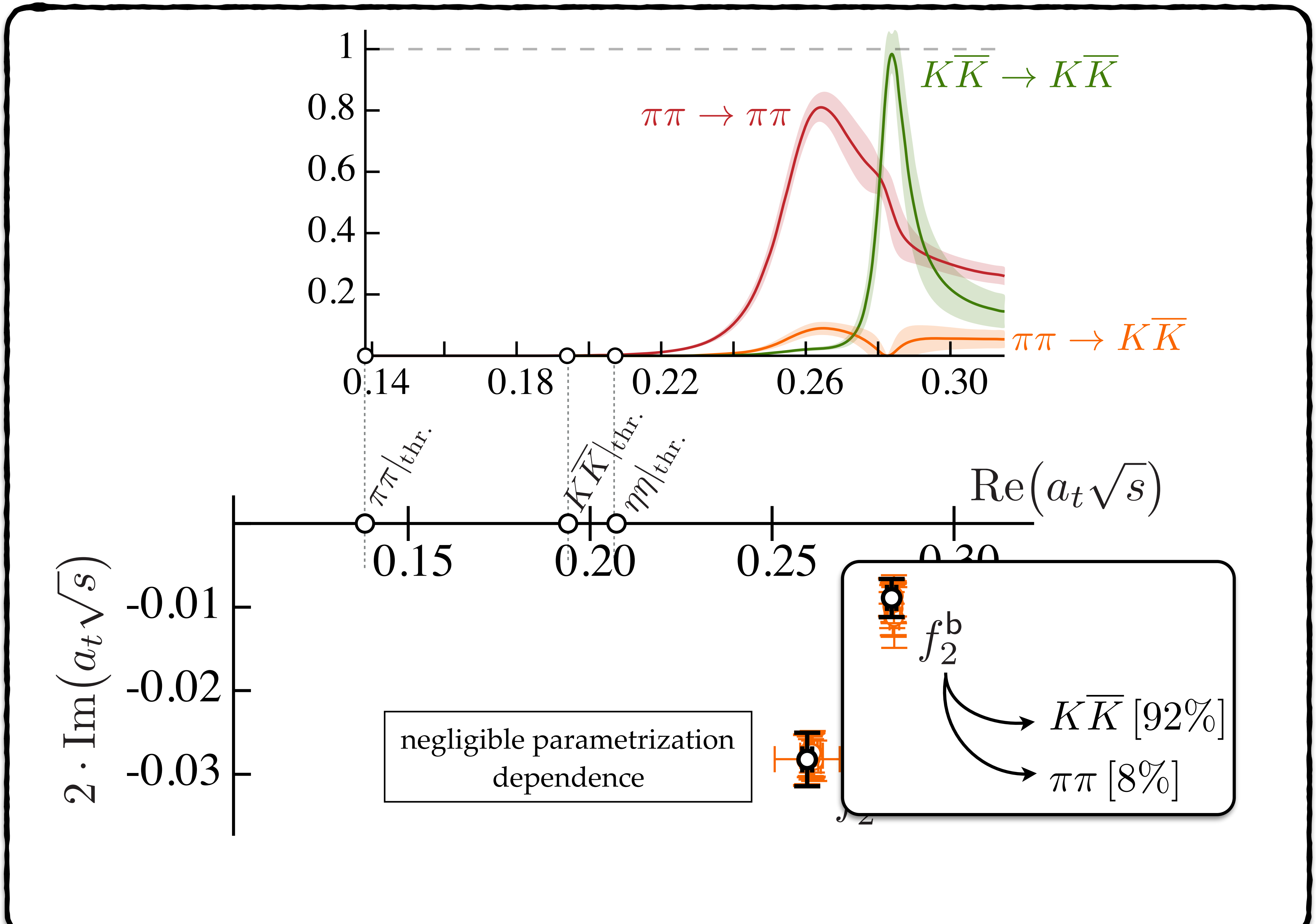
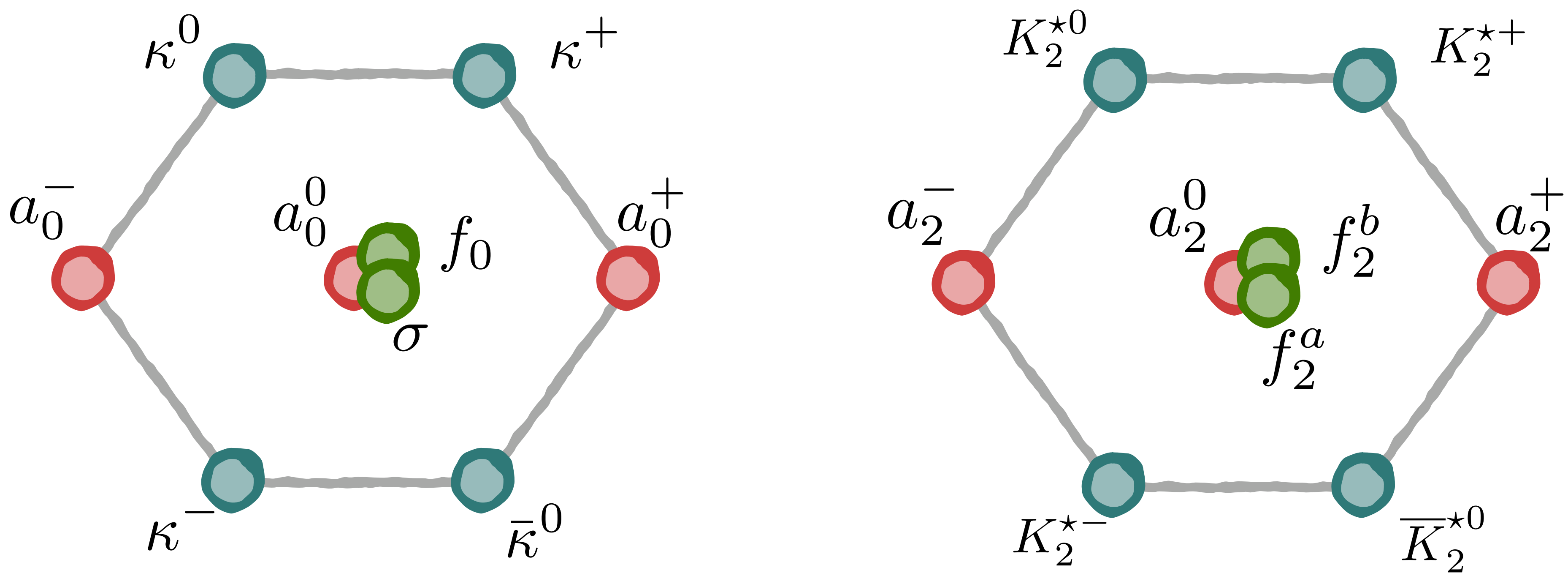


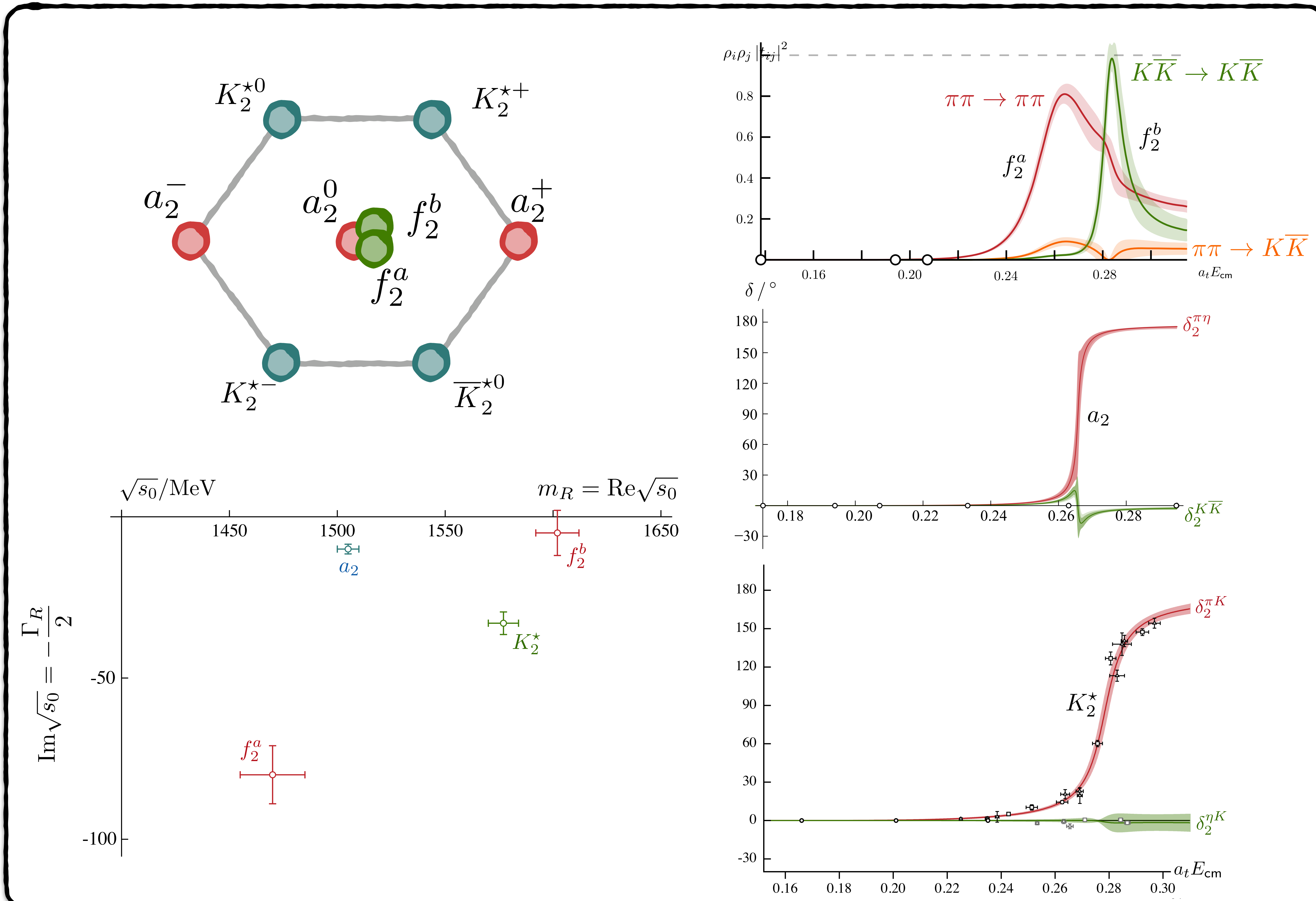
Tensor and scalar nonets

- First complete determination of the scalar and tensor nonets from LQCD :

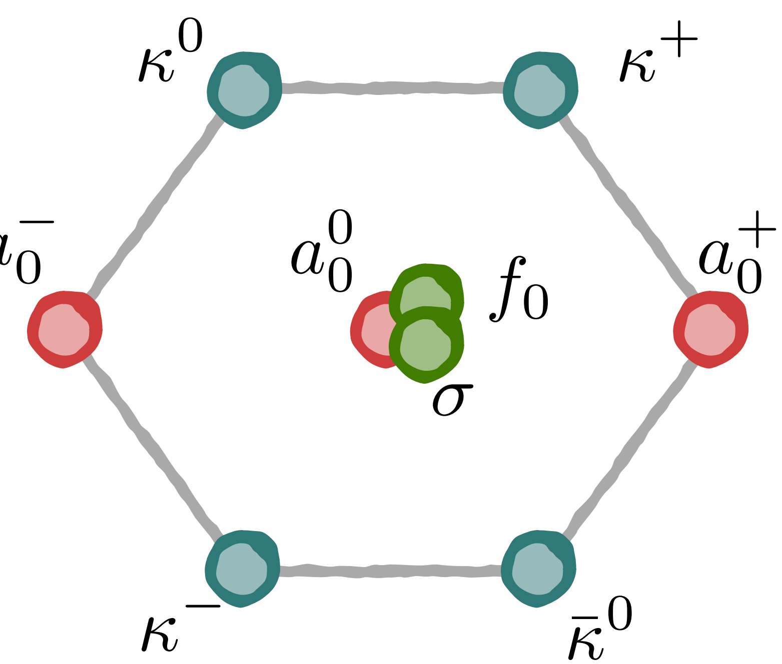


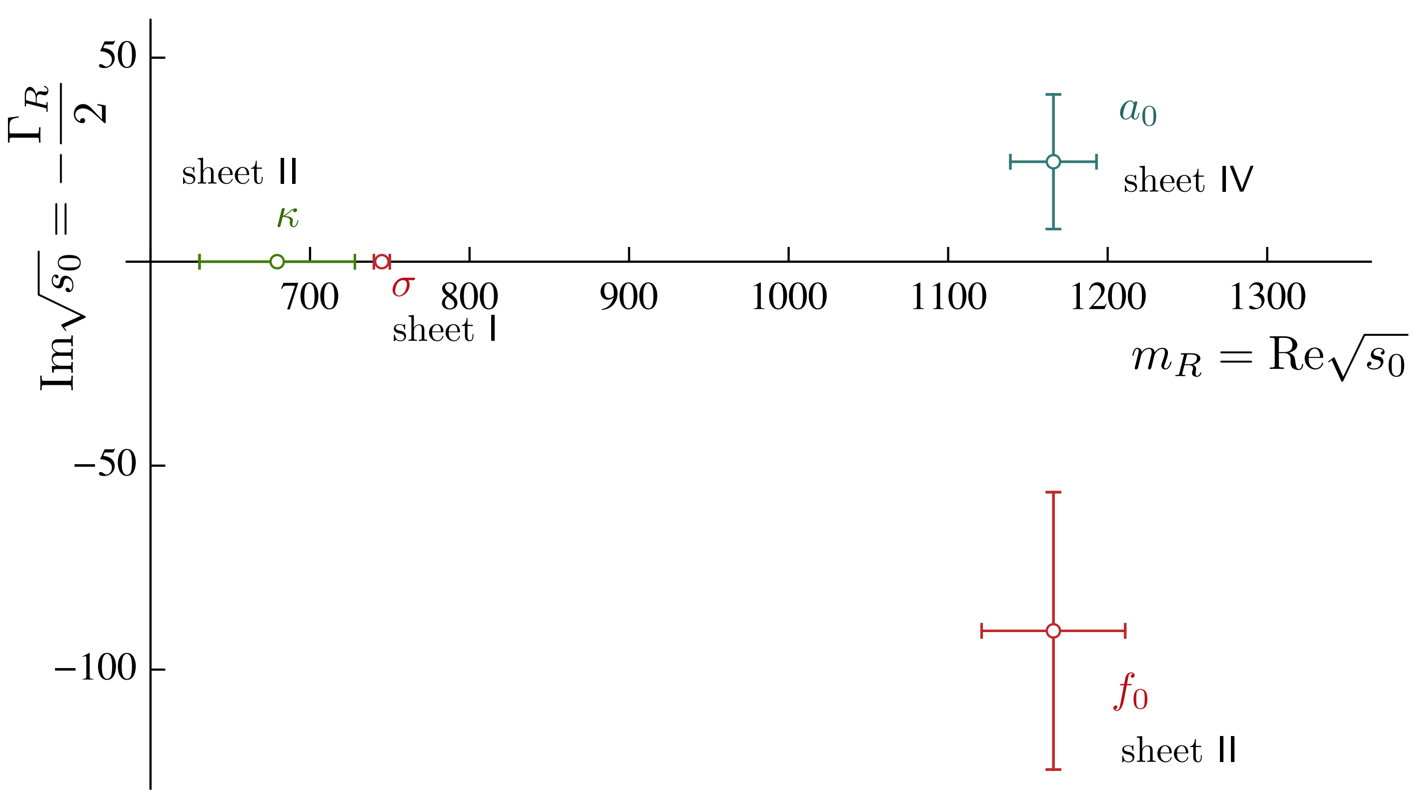
ππ, KK, ηη:	RB, Dudek, Edwards - PRL (2017)
	RB, Dudek, Edwards - arXiv (2017)
Kπ, Kη:	Dudek, Edwards, Thomas, Wilson - PRL (2015)
	Wilson, Dudek, Edwards, Thomas - PRD (2015)
πη, KK:	Dudek, Edwards, Wilson - PRD (2016)

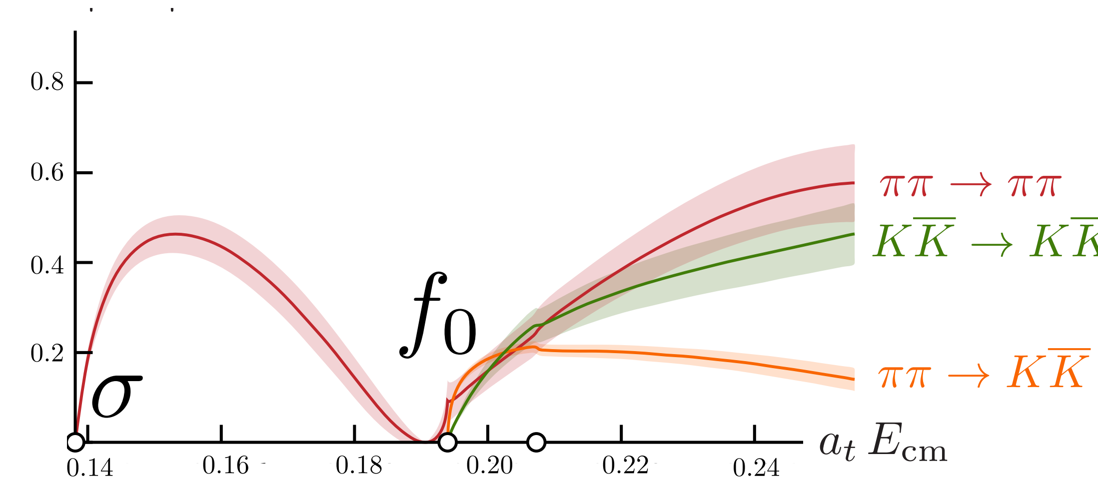
Tensor nonet

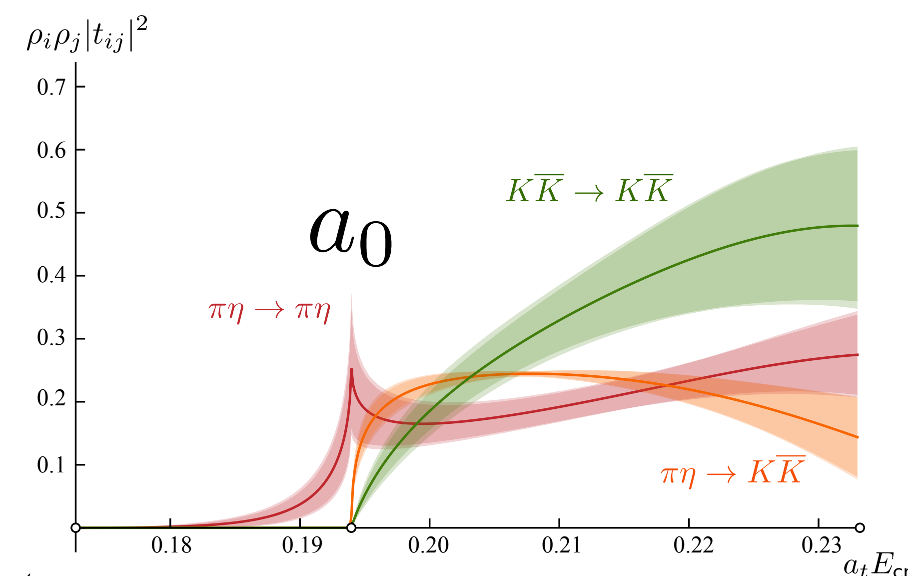


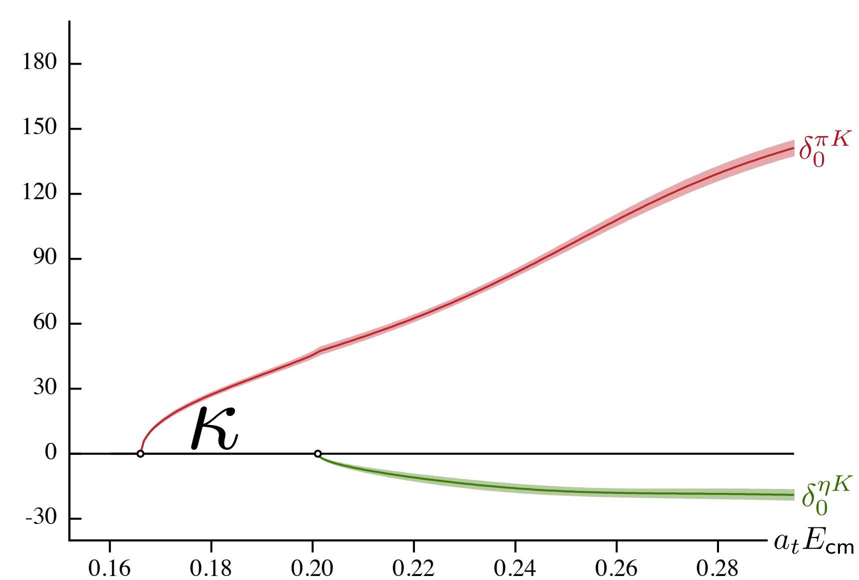
Scalar nonet











Remaining questions:

Operator basis:

- tetraquarks? on it!
- 4π?
- glueballs? harder
- …

p-lat] 5 Sep 2017

Tetraquark operators in lattice QCD and exotic flavour states in the charm sector

Gavin K. C. Cheung,[a] **Christopher E. Thomas,**[a] **Jozef J. Dudek,**[b,c] **Robert G. Edwards**[b] **(For the Hadron Spectrum Collaboration)**

[a] *DAMTP, University of Cambridge, Centre for Mathematical Sciences, Wilberforce Road, Cambridge CB3 0WA, UK*

[b] *Thomas Jefferson National Accelerator Facility, 12000 Jefferson Avenue, Newport News, VA 23606, USA*

[c] *Department of Physics, College of William and Mary, Williamsburg, VA 23187, USA*

E-mail: gkcc2@damtp.cam.ac.uk, c.e.thomas@damtp.cam.ac.uk, dudek@jlab.org, edwards@jlab.org

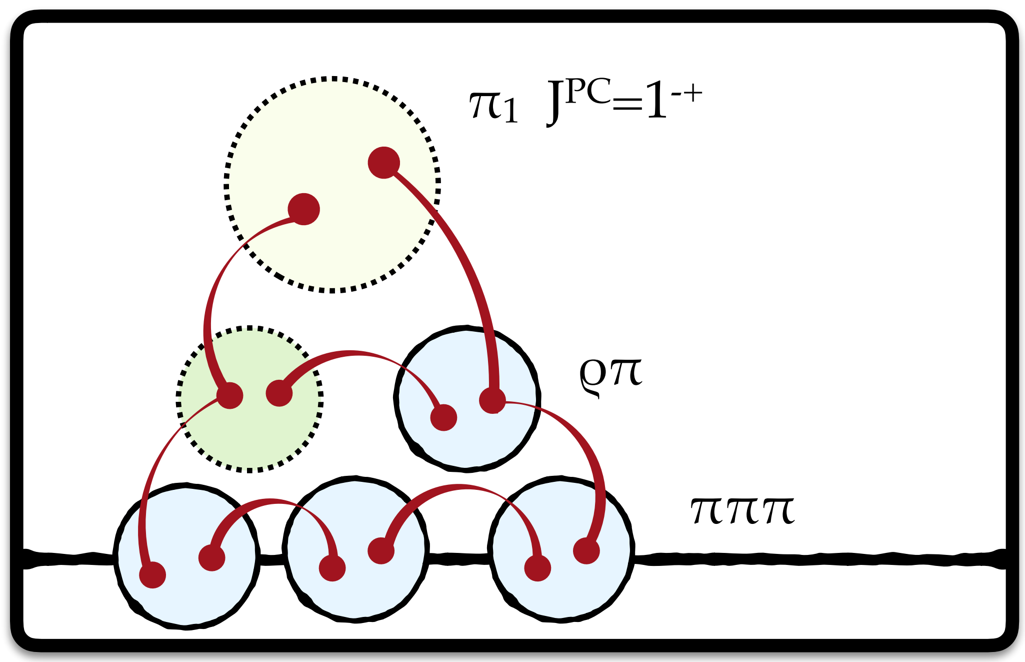
Remaining questions:

Operator basis:

- tetraquarks? on it!
- 4π?
- glueballs? harder
- …

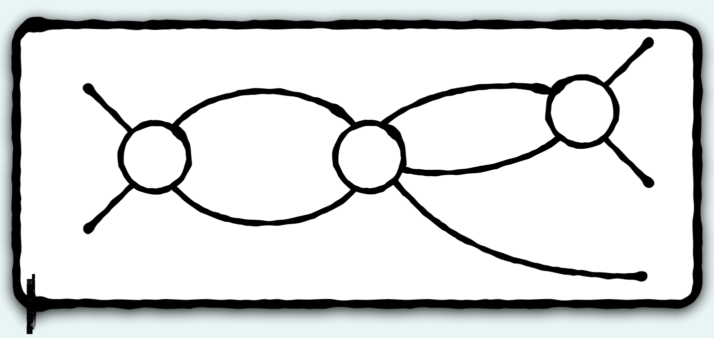
Amplitude analysis:

- 3 particles or more? on it!



$$\det\left[1+\begin{pmatrix} F_2 & 0 \\ 0 & F_3 \end{pmatrix}\begin{pmatrix} \mathcal{K}_2 & \mathcal{K}_{23} \\ \mathcal{K}_{32} & \mathcal{K}_{\mathrm{df},3} \end{pmatrix}\right]=0$$

RB, Hansen & Sharpe (2017)



Remaining questions:

Operator basis:

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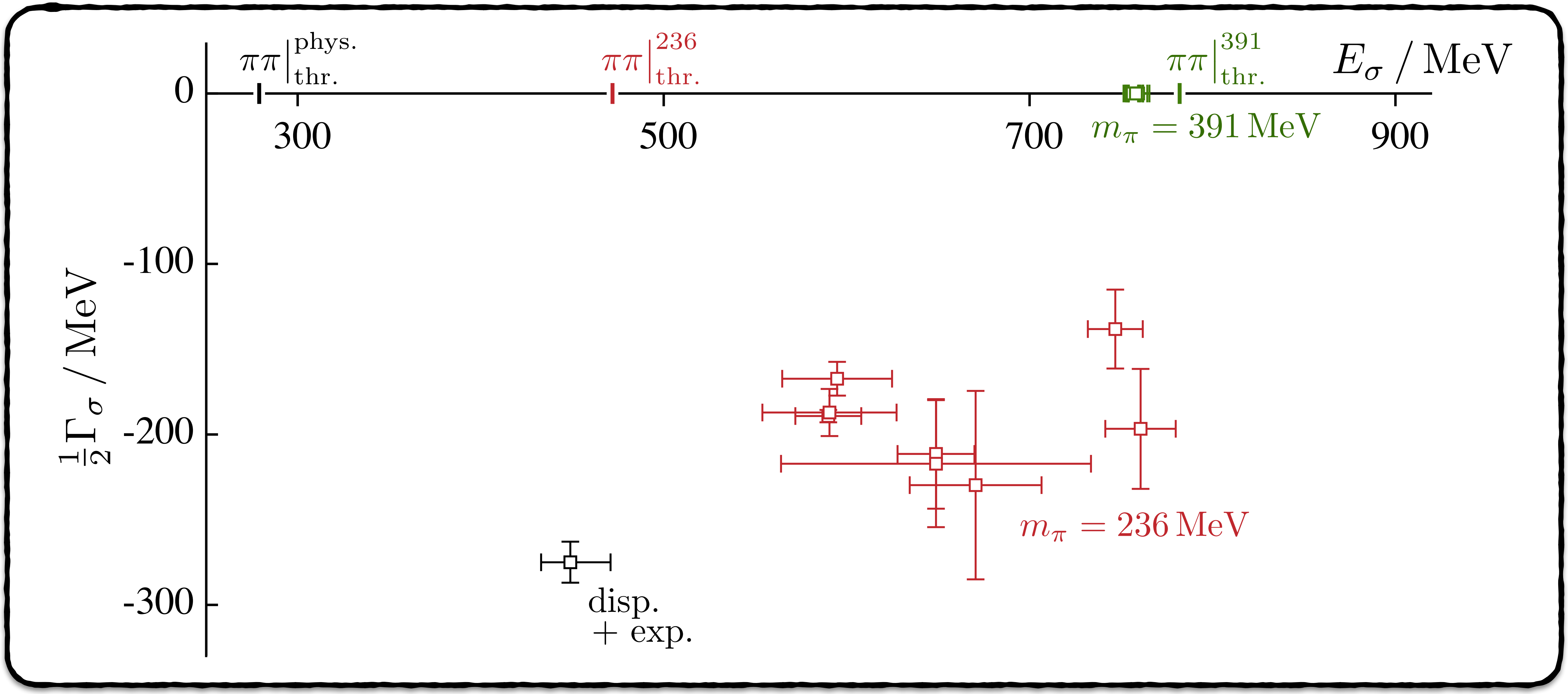
- 4π?
- glueballs? harder
- …

Amplitude analysis:

- 3 particles or more? on it!
- dispersive techniques?



Remaining questions:

Operator basis:

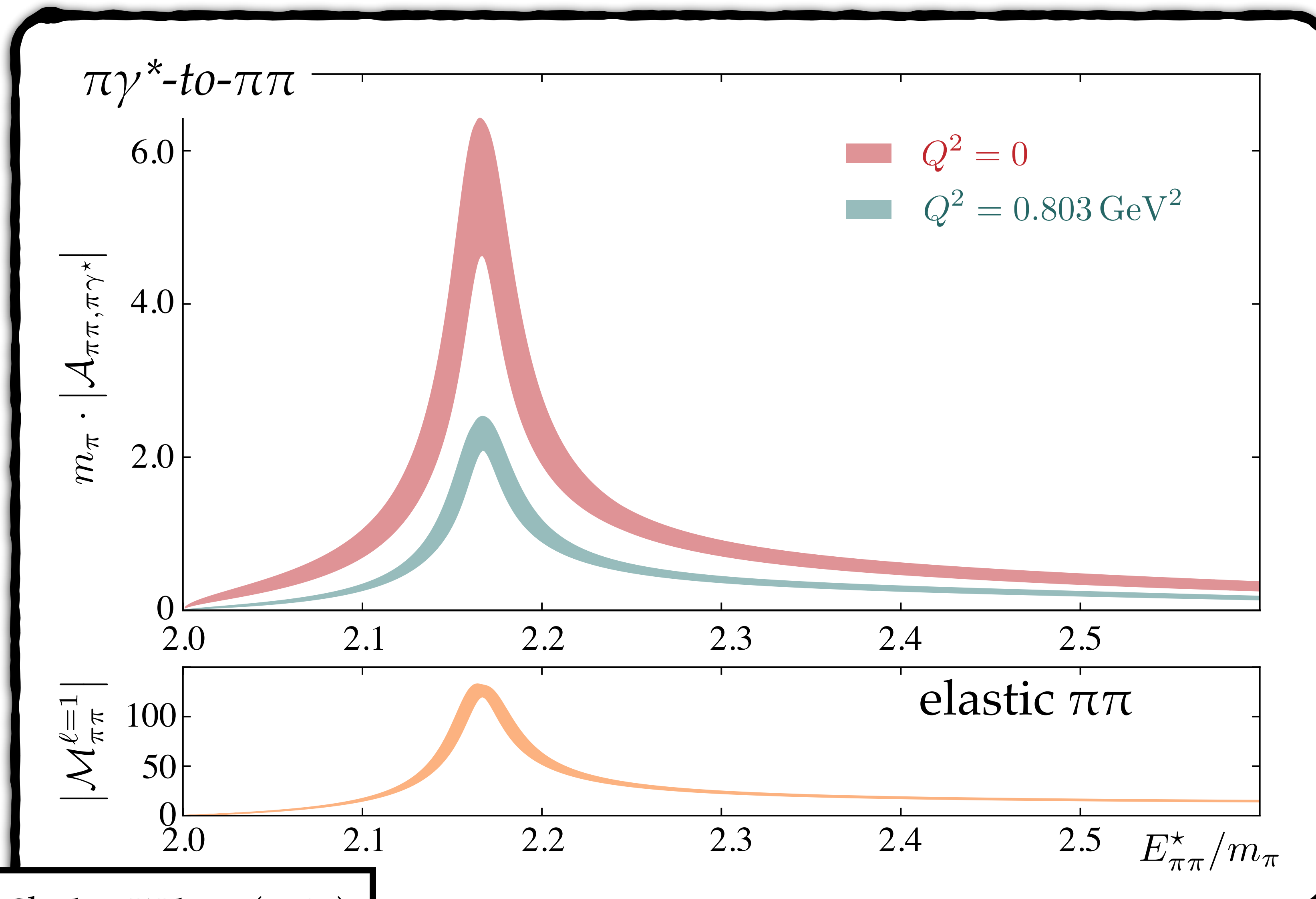
- tetraquarks? on it!
- 4π?
- glueballs? harder
- …

Amplitude analysis:

- 3 particles or more? on it!
- dispersive techniques?

Leptons:

- transition processes



RB, Dudek, Edwards, Thomas, Shultz, Wilson (2015)

Remaining questions:

Operator basis:

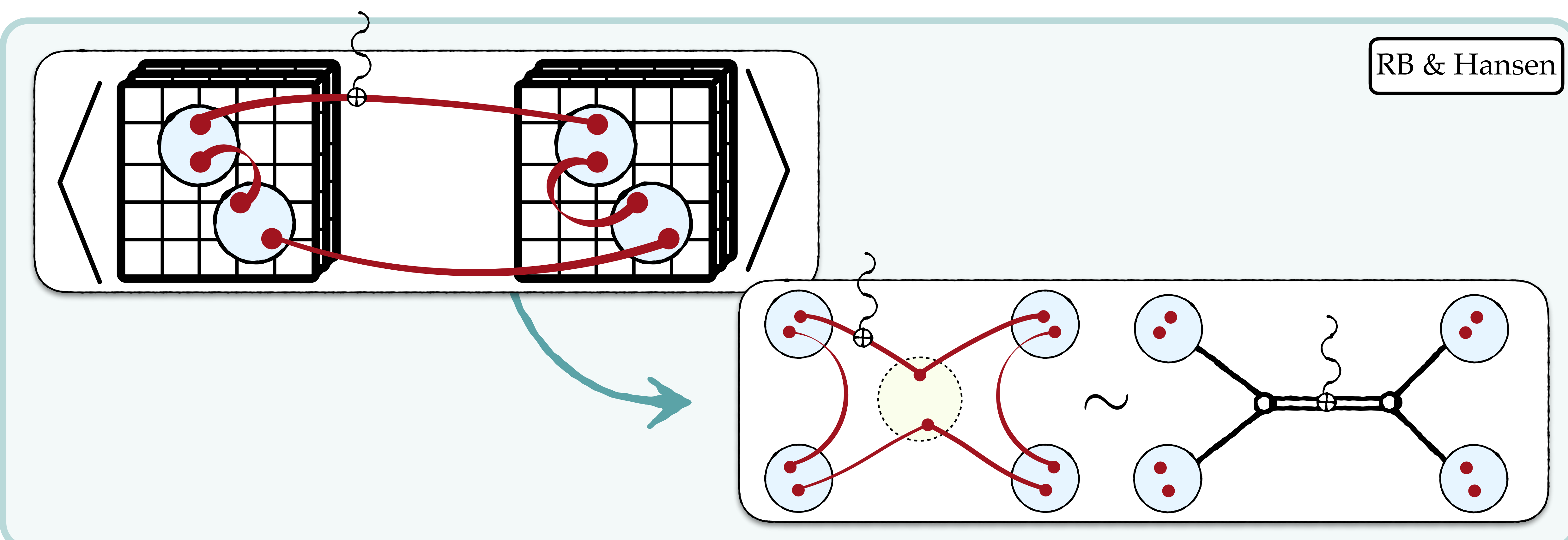
- tetraquarks? on it!
- 4π?
- glueballs? harder
- …

Amplitude analysis:

- 3 particles or more? on it!
- dispersive techniques?

Leptons:

- transition processes? on it!
- elastic processes (the future)? on it!

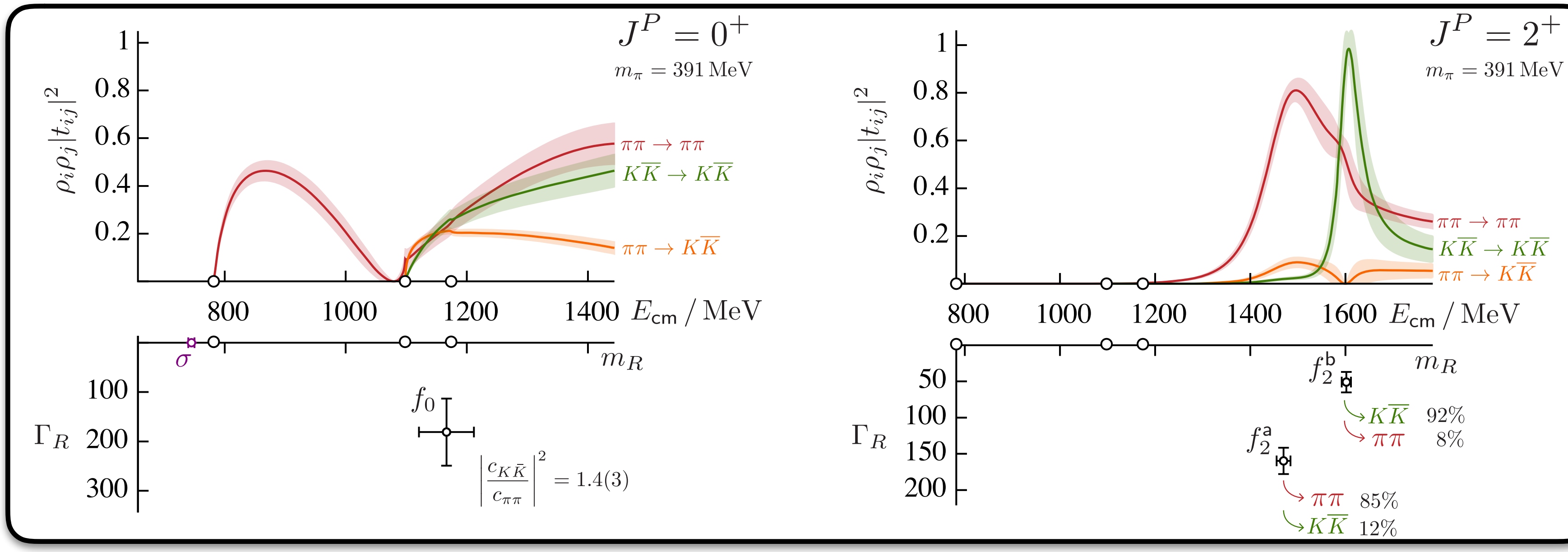


Collaborators and references

Wilson (Marie Curie / Royal fellow / Trinity)

Dudek (W&M / JLab)

Edwards (JLab)



PRL 118, 022002 (2017) PHYSICAL REVIEW LETTERS week ending 13 JANUARY 2017

Isoscalar $\pi\pi$ Scattering and the σ Meson Resonance from QCD

Raul A. Briceño,[1,*] Jozef J. Dudek,[1,2,†] Robert G. Edwards,[1,‡] and David J. Wilson[3,§]

(for the Hadron Spectrum Collaboration)

[1]*Thomas Jefferson National Accelerator Facility, 12000 Jefferson Avenue, Newport News, Virginia 23606, USA*
[2]*Department of Physics, College of William and Mary, Williamsburg, Virginia 23187-8795, USA*
[3]*Department of Applied Mathematics and Theoretical Physics, Centre for Mathematical Sciences,*

JLAB-THY-17-2534

Isoscalar $\pi\pi$, $K\overline{K}$, $\eta\eta$ scattering and the σ, f_0, f_2 mesons from QCD

Raul A. Briceño,[1,2,*] Jozef J. Dudek,[1,3,†] Robert G. Edwards,[1,‡] and David J. Wilson[4,§]
(for the Hadron Spectrum Collaboration)
[1]*Thomas Jefferson National Accelerator Facility, 12000 Jefferson Avenue, Newport News, VA 23606, USA*
[2]*Department of Physics, Old Dominion University, Norfolk, VA 23529, USA*
[3]*Department of Physics, College of William and Mary, Williamsburg, VA 23187, USA*
[4]*School of Mathematics, Trinity College, Dublin 2, Ireland*

(Dated: August 23, 2017)

A review / introduction

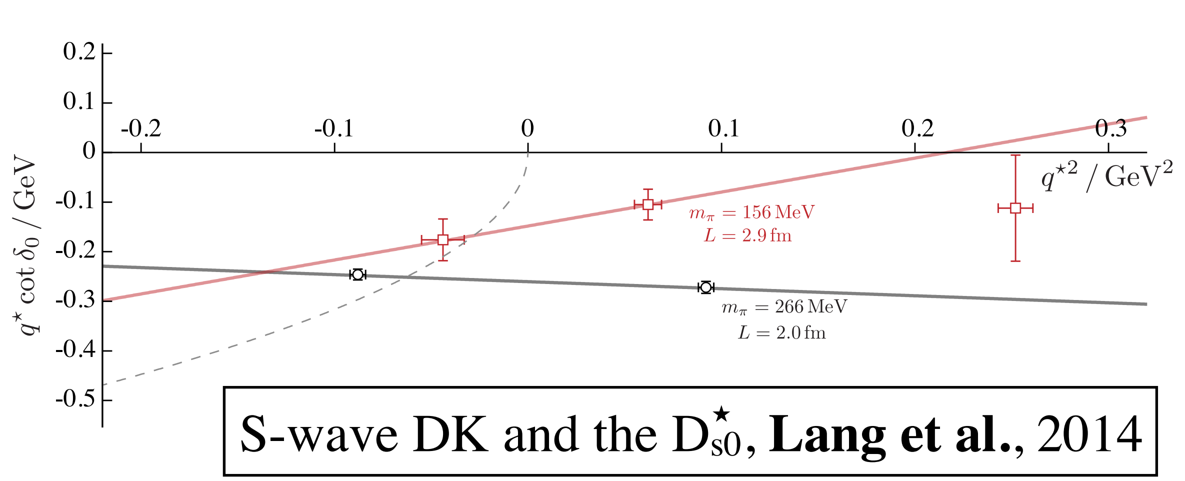
Scattering processes and resonances from lattice QCD

Raúl A. Briceño,[1,*] Jozef J. Dudek,[1,2,†] and Ross D. Young[3,‡]

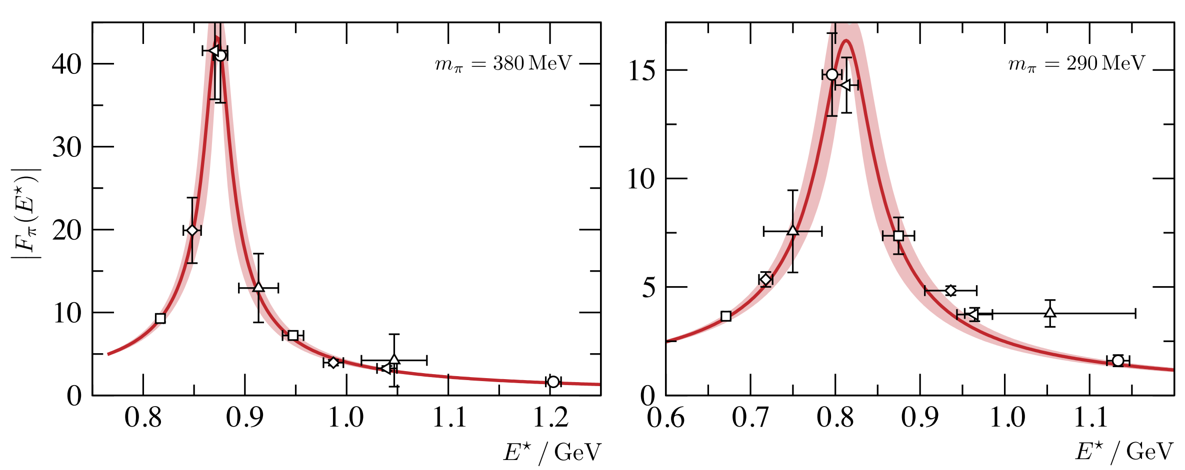
[1] *Thomas Jefferson National Accelerator Facility, 12000 Jefferson Avenue, Newport News, Virginia 23606, USA*

[2] *Department of Physics, College of William and Mary, Williamsburg, Virginia 23187, USA*

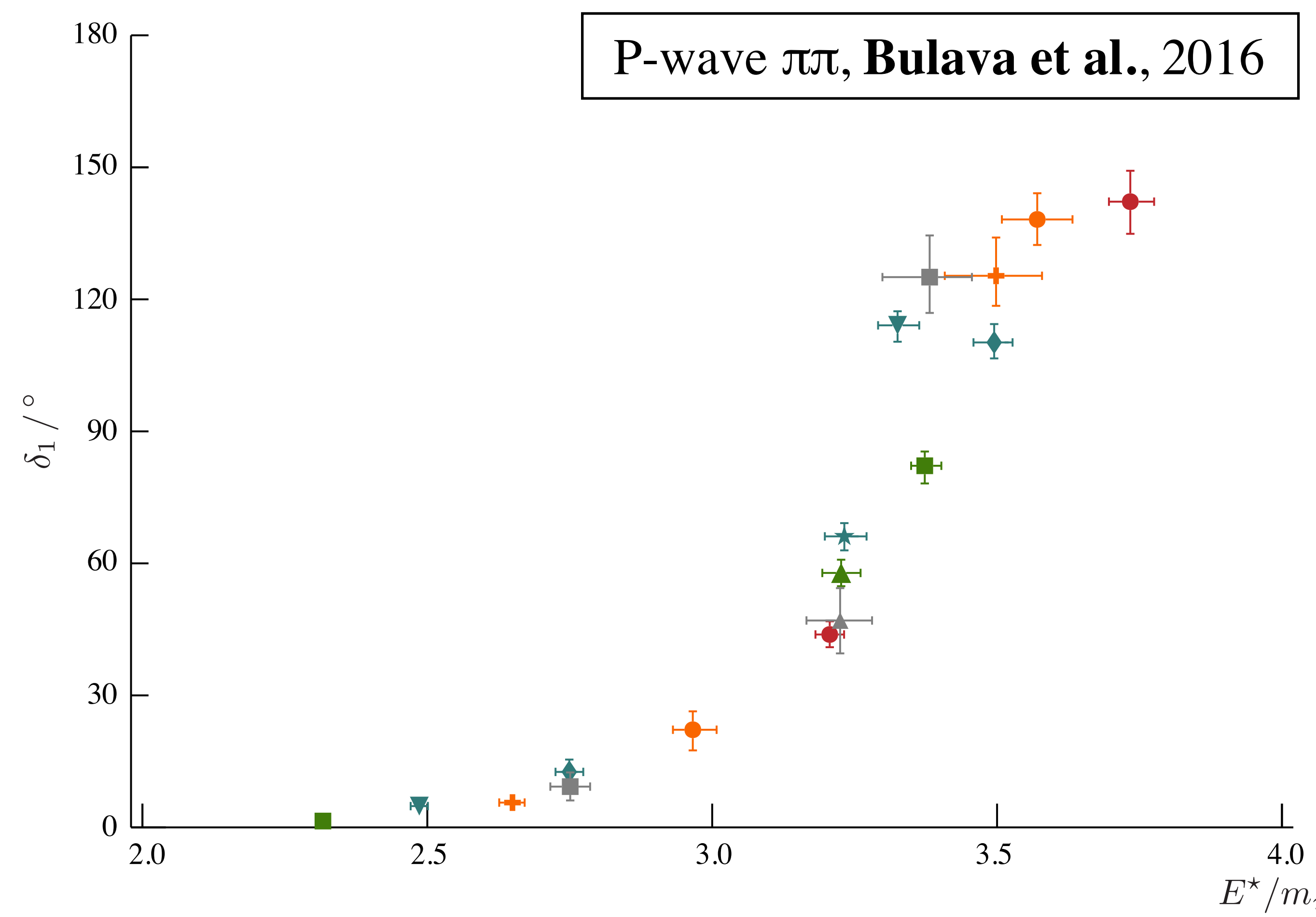
[3] *Special Research Center for the Subatomic Structure of Matter (CSSM), Department of Physics, University of Adelaide, Adelaide 5005, Australia*



S-wave DK and the $D^\star_{s0}$, **Lang et al.**, 2014



γ^*-to-$\pi\pi$, **Feng et al.**, 2015



P-wave $\pi\pi$, **Bulava et al.**, 2016

Multi-Hadron Systems from Lattice QCD

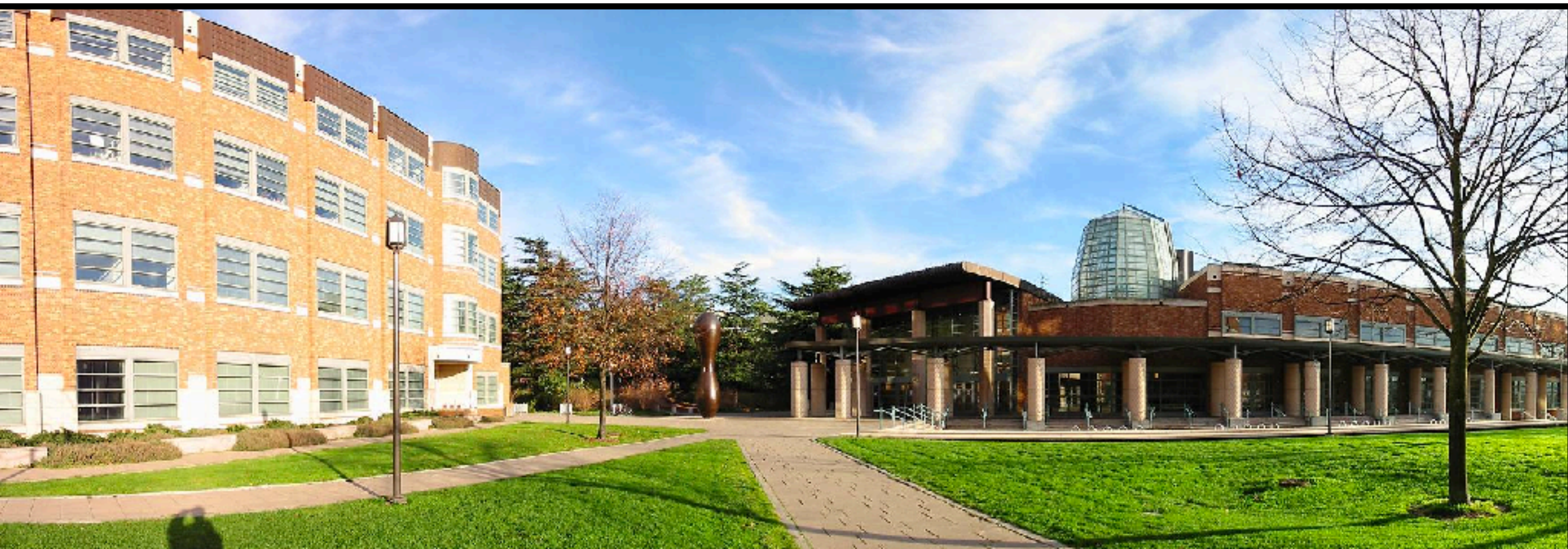
INT workshop: Seattle, WA

Feb 5-9th 2018

Hansen

Sharpe

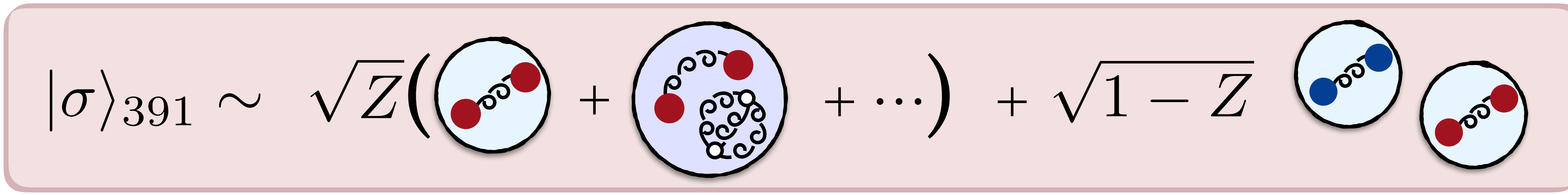
Wilson



back-up slides

Weinberg compositeness criterion for the σ

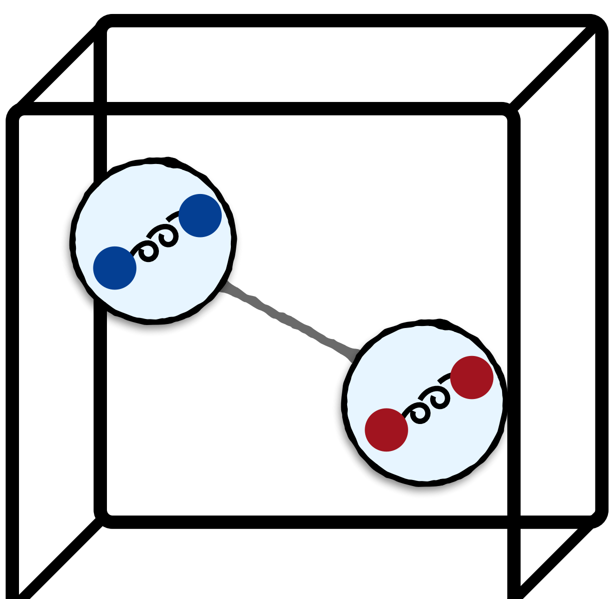
- The σ is a bound state, so we can apply Weinberg's criterion

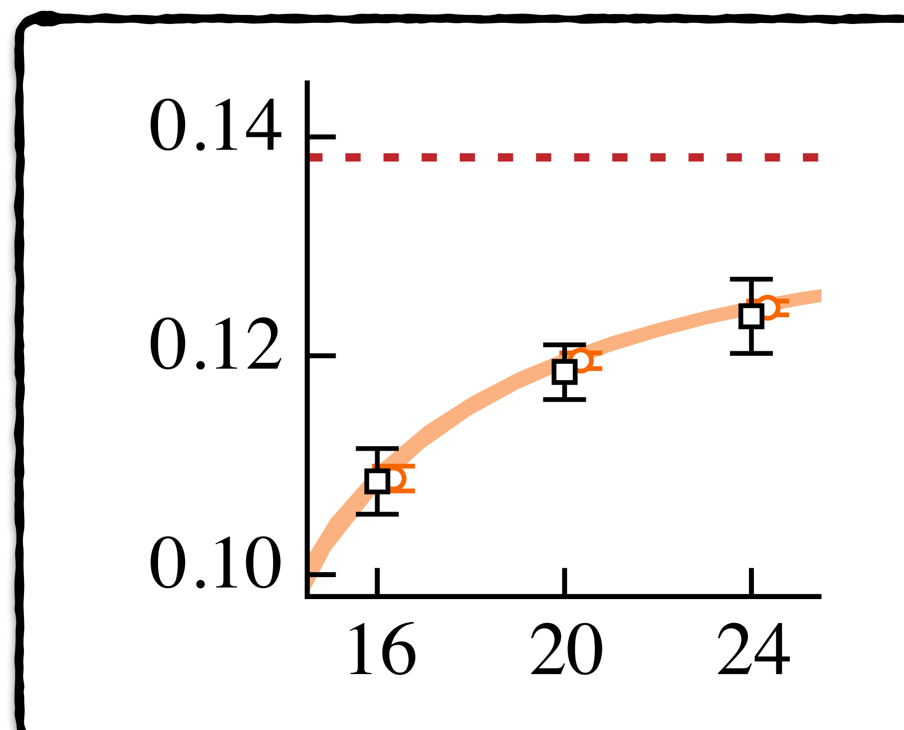


- Can relate Z to scattering information

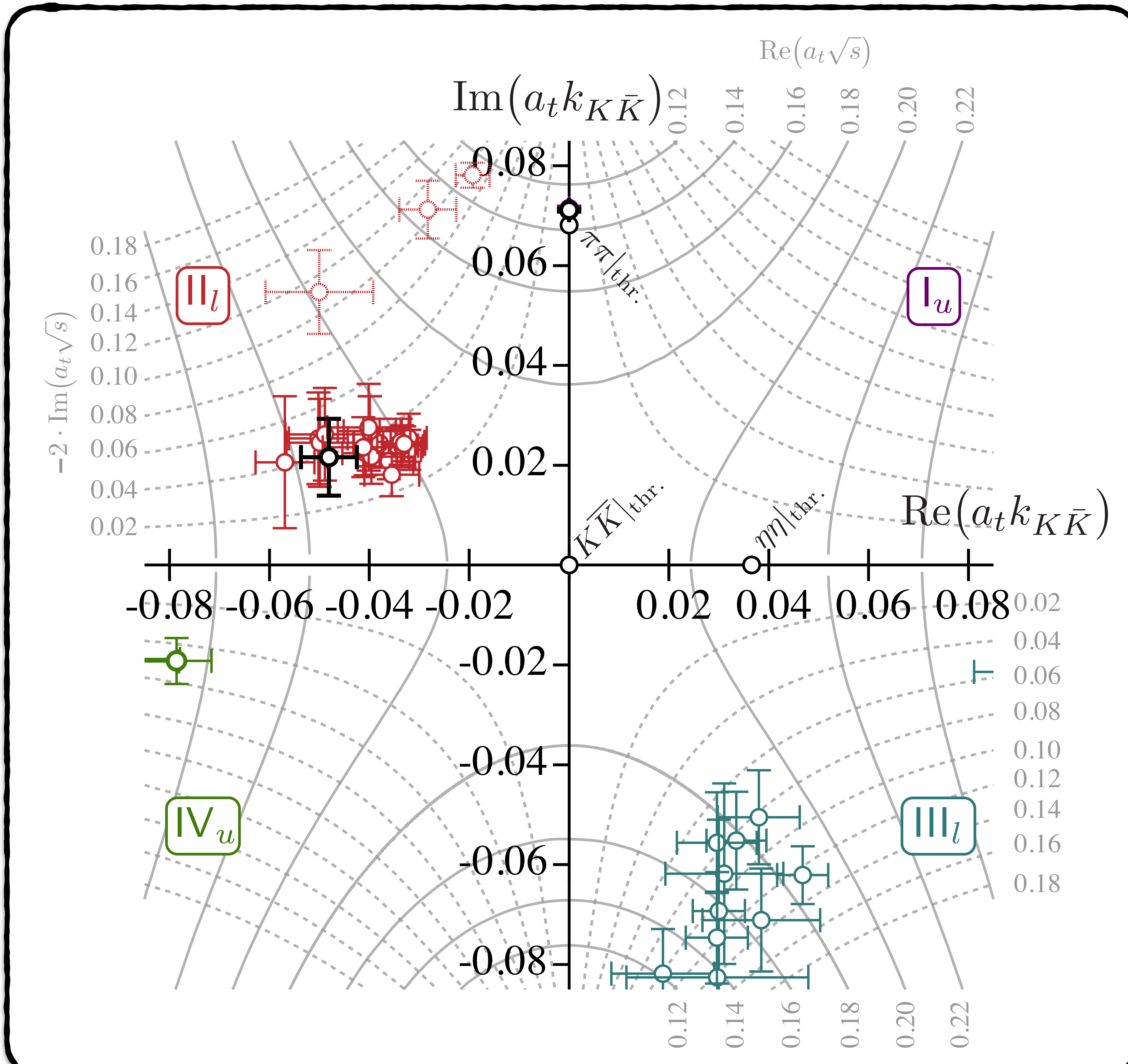
$$a = -2\frac{1-Z}{2-Z}\frac{1}{\sqrt{m_\pi B_\sigma}}, \qquad r = -\frac{Z}{1-Z}\frac{1}{\sqrt{m_\pi B_\sigma}}$$

- To obtain: $Z \sim 0.3(1)$
- Consistent with the large FV effects

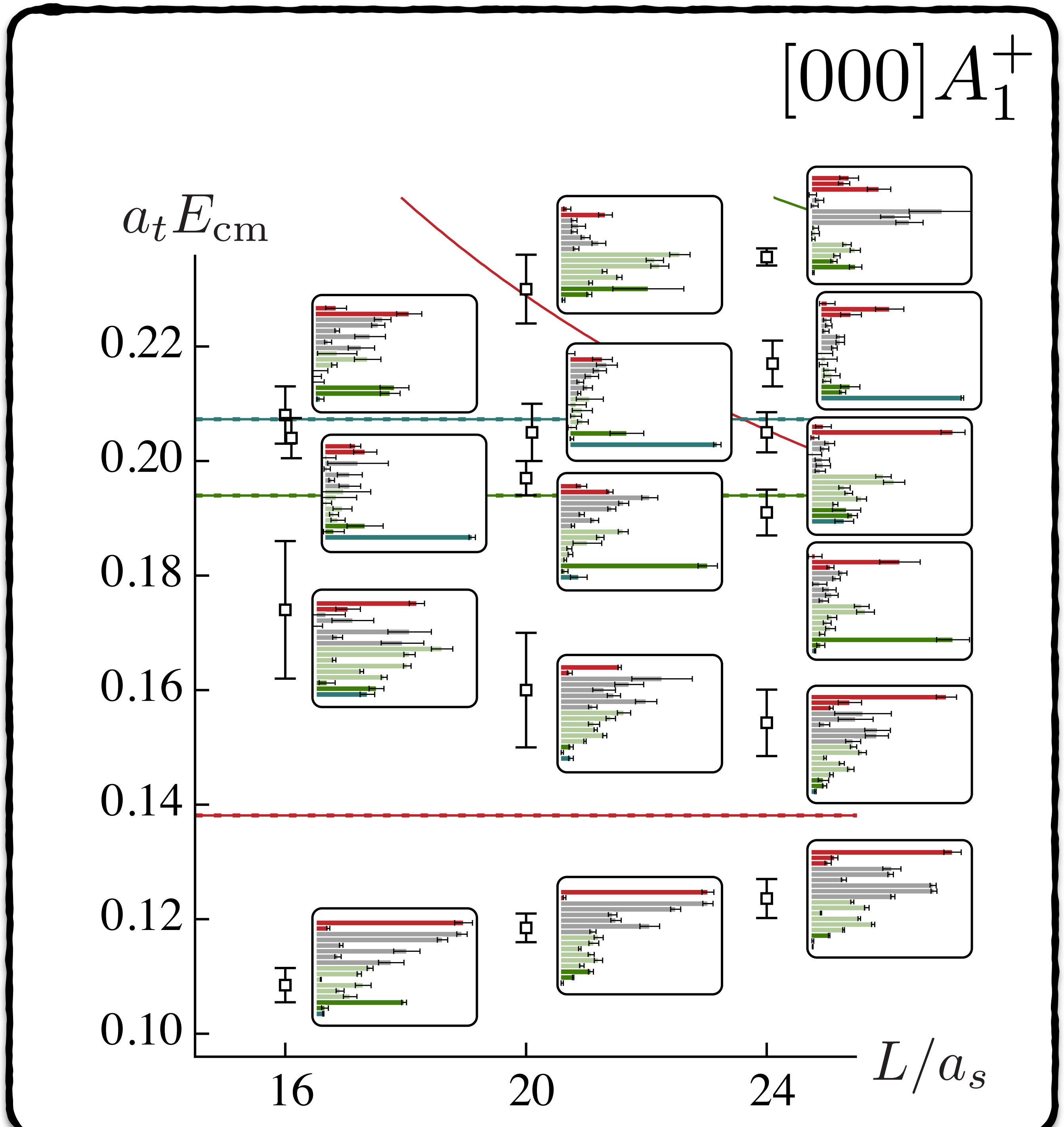




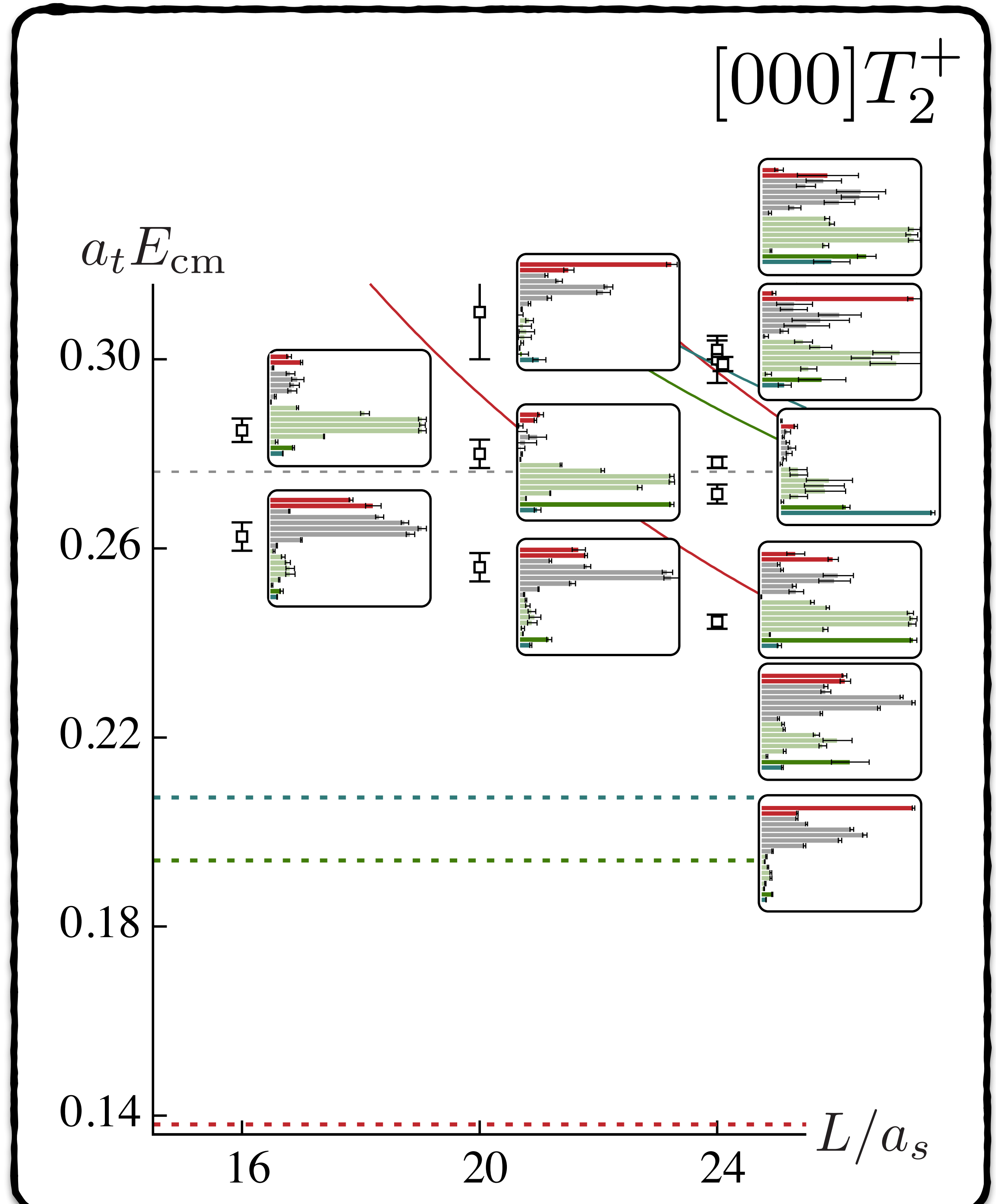
Complex momentum plane



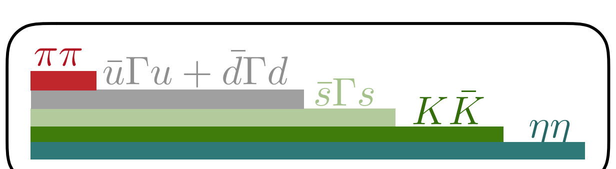
Overlaps



[000]A_1^+
$a_t E_{\mathrm{cm}}$
0.22
0.20
0.18
0.16
0.14
0.12
0.10
16
20
24
L/a_s



[000]T_2^+
$a_t E_{\mathrm{cm}}$
0.30
0.26
0.22
0.18
0.14
16
20
24
L/a_s

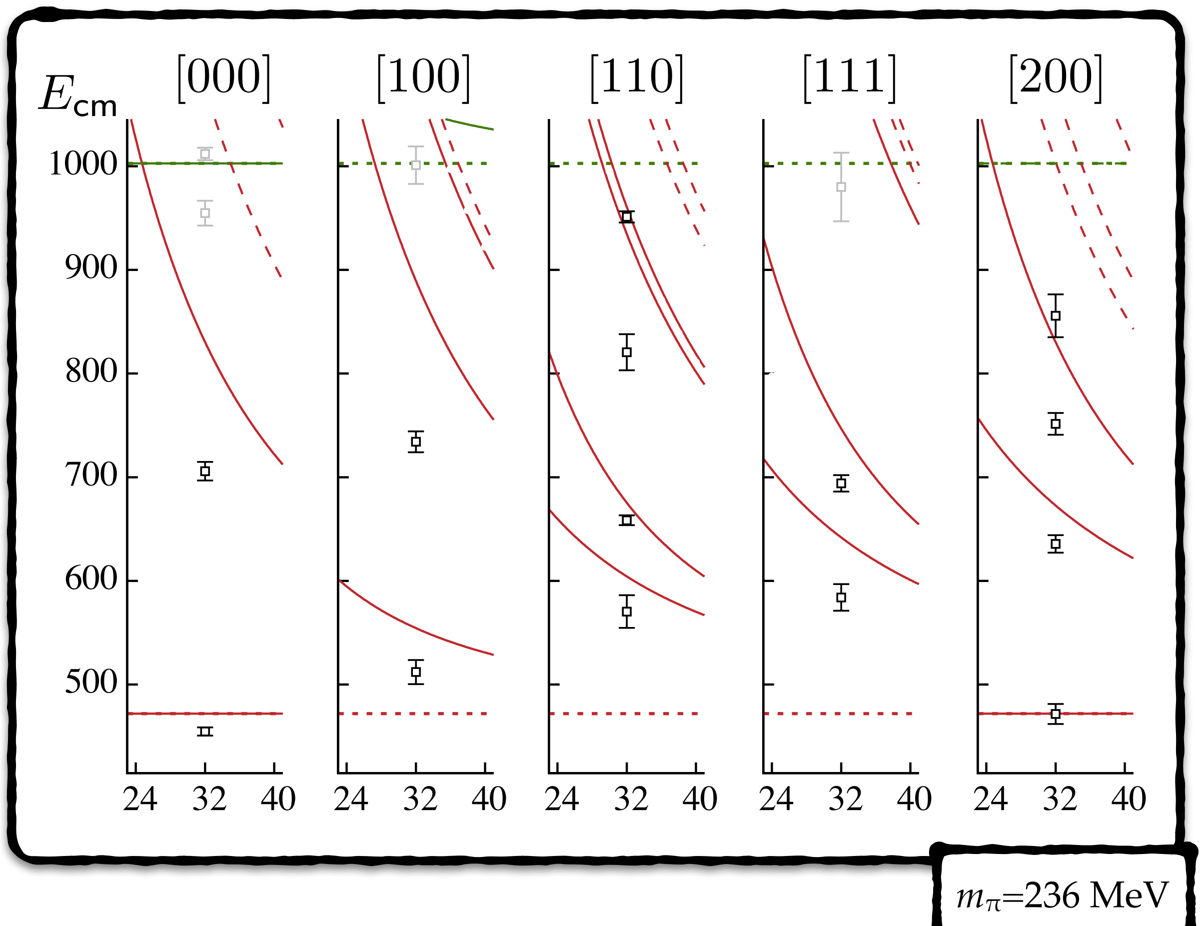


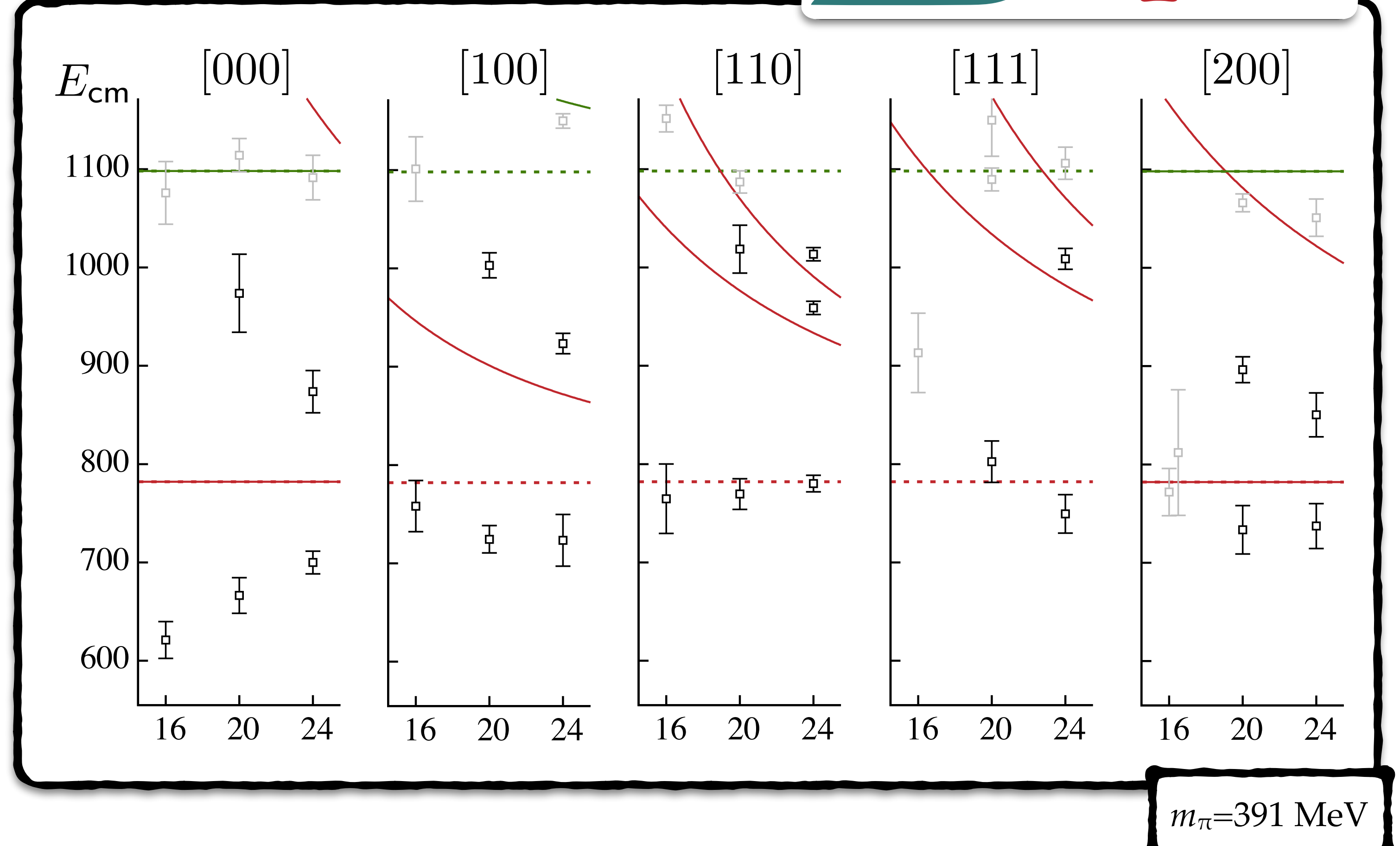
$\pi\pi$
$\bar{u}\Gamma u + \bar{d}\Gamma d$
$\bar{s}\Gamma s$
$K\bar{K}$
$\eta\eta$

Extracting the spectrum

$$C^{2pt.}_{ab}(t, \mathbf{P}) \equiv \langle 0|\mathcal{O}_b(t, \mathbf{P})\mathcal{O}^\dagger_a(0, \mathbf{P})|0\rangle = \sum_n Z_{b,n} Z^\dagger_{a,n} e^{-E_n t}$$

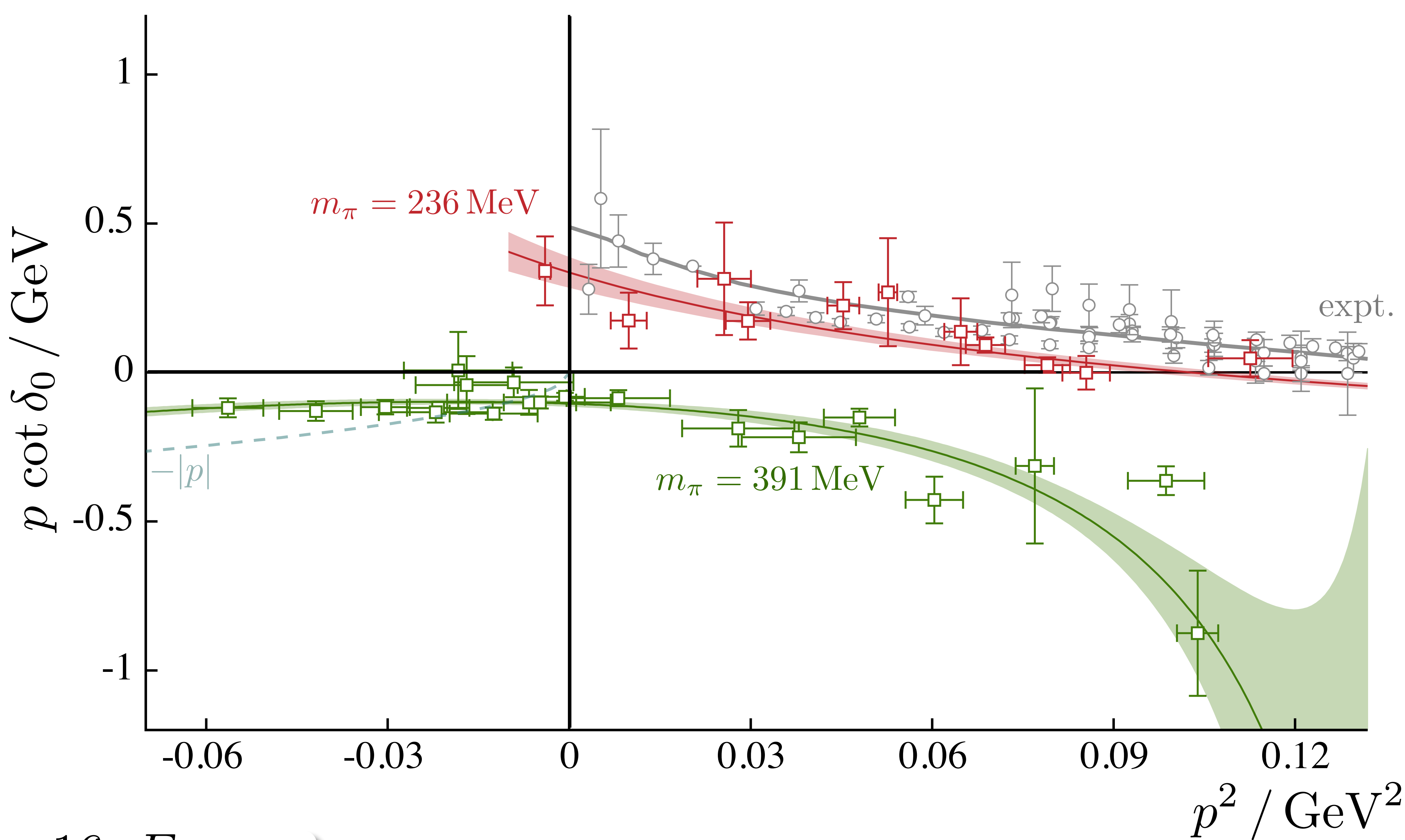
- *Use local and multi-hadron ops ~ 20-30 ops*
- Evaluate all Wick contraction: **distillation** [Peardon, *et al.* (2009)]
- *Variationally* optimize operators [Michael (1985), Lüscher & Wolff (1990)]
- extract ~ 30 - 100 energy levels
- e.g., isoscalar $\pi\pi$ below the $2m_K$ threshold





RB, Dudek, Edwards, Wilson - PRL (2017)

Isoscalar $\pi\pi$ scattering



$$\mathcal{M}_0 = \frac{16\pi E_{\mathrm{cm}}}{p \cot \delta_0 - ip}$$

RB, Dudek, Edwards, Wilson - PRL (2017)

The σ / f₀(500) vs m_π