First simultaneous extraction of spin PDFs and FFs from a global QCD analysis [arXiv:1705.05889 accepted to PRL]

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Jefferson Lab
Thomas Jefferson National Accelerator Facility

JAM

QCD Observables

- Collinear factorization → QCD observables split into hard scattering cross section and universal, non-perturbative functions
- Deep-inelastic scattering (DIS) $e^- N \to e^- X$

$$d\sigma = \sum_f \int d\xi f(\xi) d\hat{\sigma}$$

- Semi-inclusive DIS (SIDIS) $e^- N \to e^- hX$

$$d\sigma = \sum_f \int d\xi d\zeta f(\xi) D(\zeta) d\hat{\sigma}$$

- Single-inclusive annihilation (SIA) $e^+ e^- \to hX$

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Parton distribution functions (PDFs)

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Fragmentation functions (FFs)

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PDFs and FFs must be determined through analyses of experimental data

- Single-inclusive annihilation (SIA) $e^+ e^- \to h X$

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Proton spin structure from DIS

- Spin sum rule: $\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta g + \mathcal{L}$
- Spin PDFs: $\Delta q^+ = \Delta q + \Delta\bar{q}$

→ Quark contribution: $\Delta\Sigma(Q^2) = \int_0^1 dx\left(\Delta u^+(x,Q^2) + \Delta d^+(x,Q^2) + \Delta s^+(x,Q^2)\right) \approx 0.3_{[10^{-3},1]}$

→ Gluon contribution: $\Delta g(Q^2) = \int_0^1 dx \Delta g(x,Q^2) \approx 0.1_{[0.05,0.2]}$

→ Orbital angular momentum: requires information from GPDs

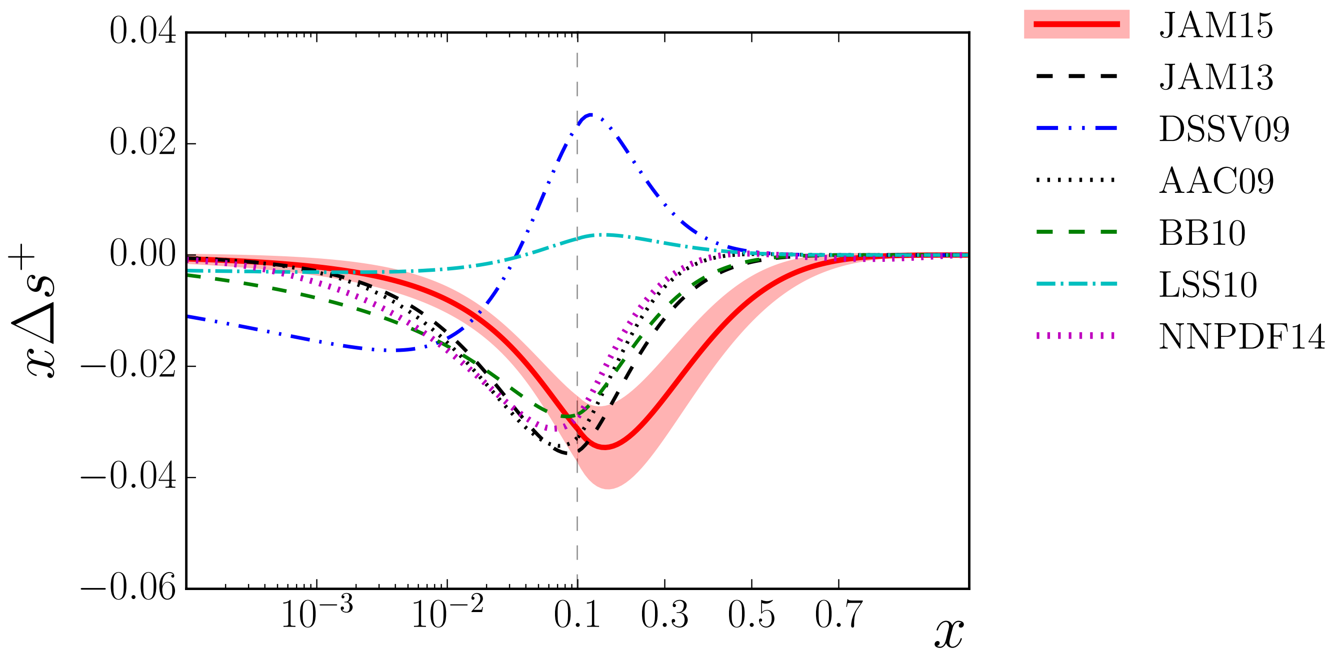
- Polarized DIS experiments extract information on the polarized structure function g_1

$$\int_0^1 dx g_1^p(x,Q^2) = \frac{1}{36}\left[8\Delta\Sigma + 3\underline{g_A} + \underline{a_8}\right]\left(1 - \frac{\alpha_s}{\pi} + \mathcal{O}(\alpha_s^2)\right) + \mathcal{O}\left(\frac{1}{Q^2}\right)$$

- First moment requires additional knowledge of g_A and a_8 quantities to extract $\Delta\Sigma$

→ Determined from weak baryon decays under exact SU(2) and SU(3) flavor symmetries

Strange polarization



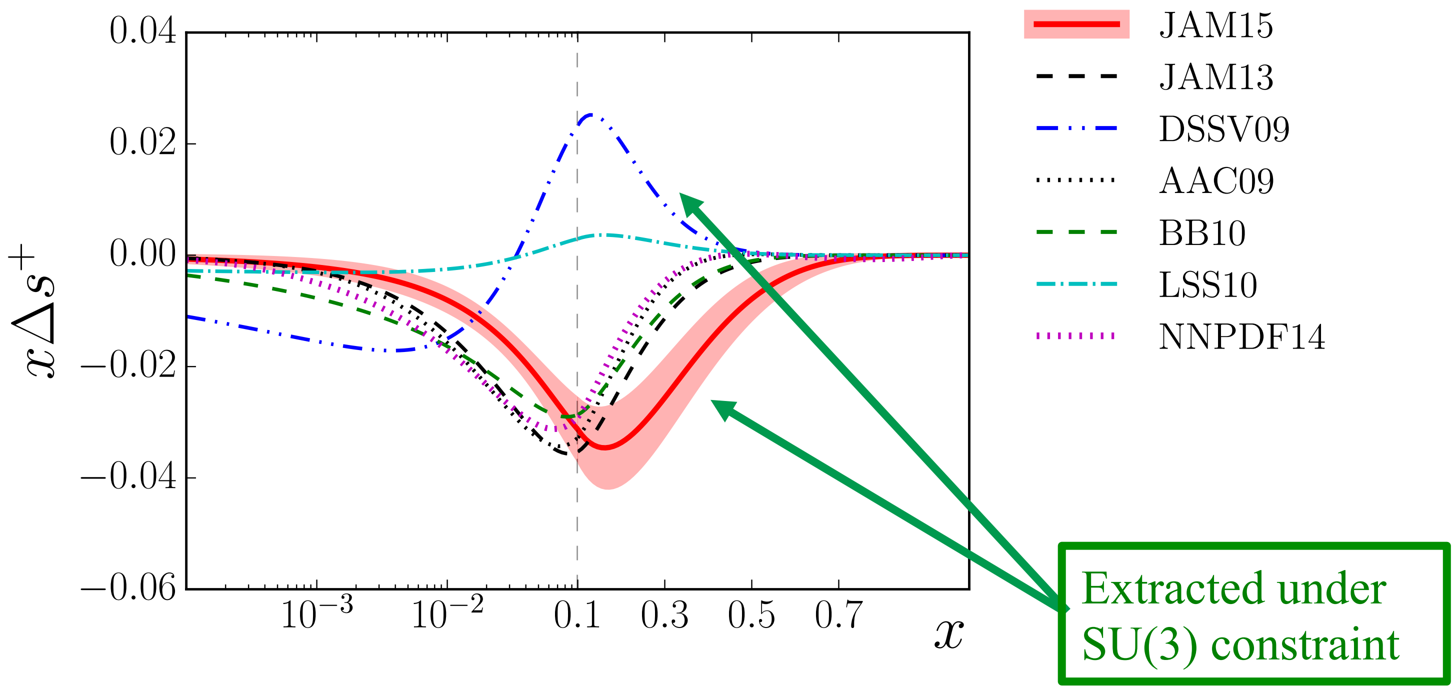
$$\Delta \mathbf{s}^+(Q^2) = \int_0^1 dx \Delta s^+(x, Q^2)$$

JAM15: $\Delta \mathbf{s}^+ = -0.1 \pm 0.01$ DSSV09: $\Delta \mathbf{s}^+ = -0.11$ $Q^2 = 1\,\mathrm{GeV}^2$

- Discrepancy in shape of strange polarization (DSSV, LSS)!

  → How does semi-inclusive DIS affect the shape of Δs+?

Strange polarization



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Semi-inclusive DIS

- SIDIS observables require information on FFs → contains information about quark to hadron fragmentation.

$$d\sigma^{SIDIS} = \sum_f \int d\xi d\zeta \, \Delta f(\xi) D_f(\zeta) d\hat{\sigma}$$

- Which SIDIS observable is most sensitive to strange PDF?

  → Kaon valence structure contains strange flavor $K^+(u\bar{s})$ $K^-(u\bar{s})$

- From proton target:

$$d\sigma^{K^+} \sim 4\Delta u D_u^{K^+} + \Delta\bar{s} D_{\bar{s}}^{K^+}$$

$$d\sigma^{K^-} \sim 4\Delta\bar{u} D_{\bar{u}}^{K^-} + \Delta s D_s^{K^-} + 4\Delta u D_u^{K^-}$$

- From deuteron target:

$$d\sigma^{K^+} \sim 4(\Delta u + \Delta d) D_u^{K^+} + 2\Delta\bar{s} D_{\bar{s}}^{K^+}$$

$$d\sigma^{K^-} \sim 4(\Delta\bar{u} + \Delta\bar{d}) D_{\bar{u}}^{K^-} + 2\Delta s D_s^{K^-} + 4(\Delta u + \Delta d) D_u^{K^-}$$

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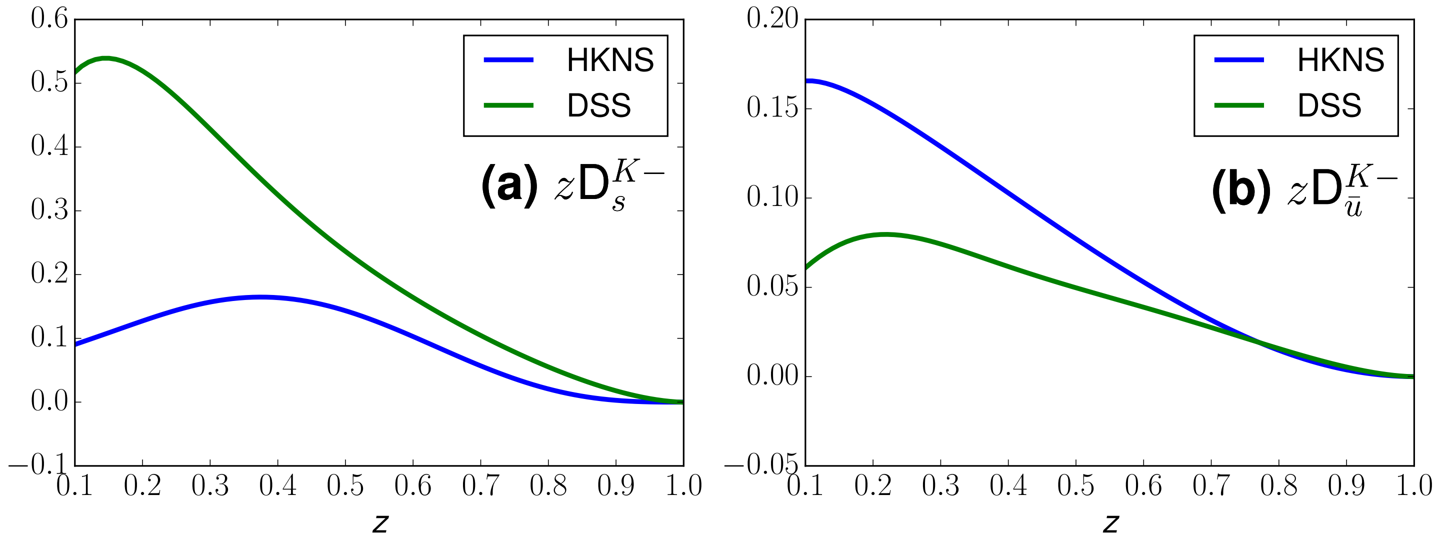
Fragmentation Functions

- SIDIS observables require information on FFs → contains information about quark to hadron fragmentation.

$$d\sigma^{SIDIS} = \sum_f \int d\xi d\zeta \, \Delta f(\xi) D_f(\zeta) d\hat{\sigma}$$

→ Choice of kaon FF parameterization influences shape of strange polarization density in SIDIS analysis (Leader, et al)



→ Recent JAM analysis extracted FFs from single-inclusive e^+e^- annihilation using the iterative Monte Carlo technique (arXiv:1609:00899)

JAM17 Combined Analysis

- We perform the first ever combined Monte Carlo analysis of polarized DIS, polarized SIDIS, and SIA data (at NLO)

$$d\sigma^{DIS} = \sum_f \int d\xi \, \Delta f(\xi) d\hat{\sigma} \qquad d\sigma^{SIA} = \sum_f \int d\zeta \, D_f(\zeta) d\hat{\sigma}$$

$$d\sigma^{SIDIS} = \sum_f \int d\xi d\zeta \, \Delta f(\xi) D_f(\zeta) d\hat{\sigma}$$

- Spin PDFs and FFs are fitted simultaneously
- SU(2) and SU(3) constraints used in DIS only analyses are released

$$\int_0^1 dx \left(\Delta u^+ - \Delta d^+\right) \stackrel{?}{=} g_A$$

$$\int_0^1 dx \left(\Delta u^+ + \Delta d^+ - 2\Delta s^+\right) \stackrel{?}{=} a_8$$

Traditional Fitting Method

- Functional form for PDF and FF, e.g.

$$xf(x) = Nx^a(1-x)^b(1+c\sqrt{x}+dx)$$

- Single χ^2 fit of parameters

  → Typically fix parameters that are difficult to constrain

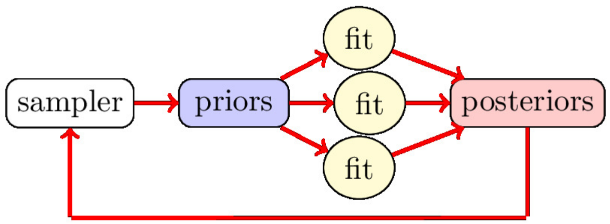
- Uncertainties determined by Hessian or Lagrange multiplier methods

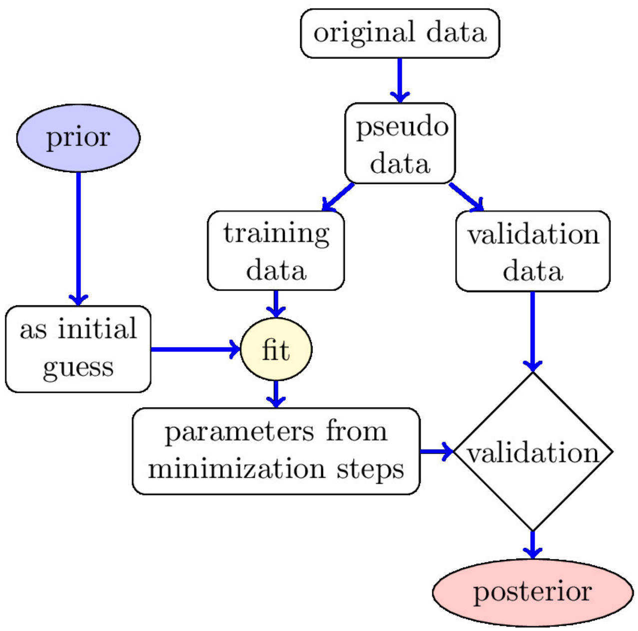
  → Introduces tolerance criteria

- Since χ^2 is a highly non-linear function of the fit parameters, there can be various local minima

- We can efficiently explore the parameter space using an iterative Monte Carlo procedure

Iterative Monte Carlo (IMC) Fitting Methodology





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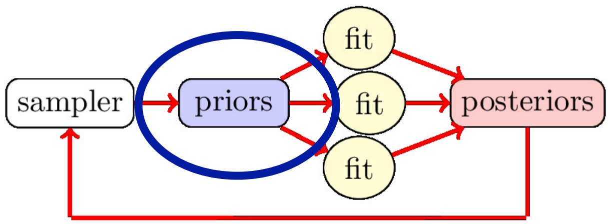

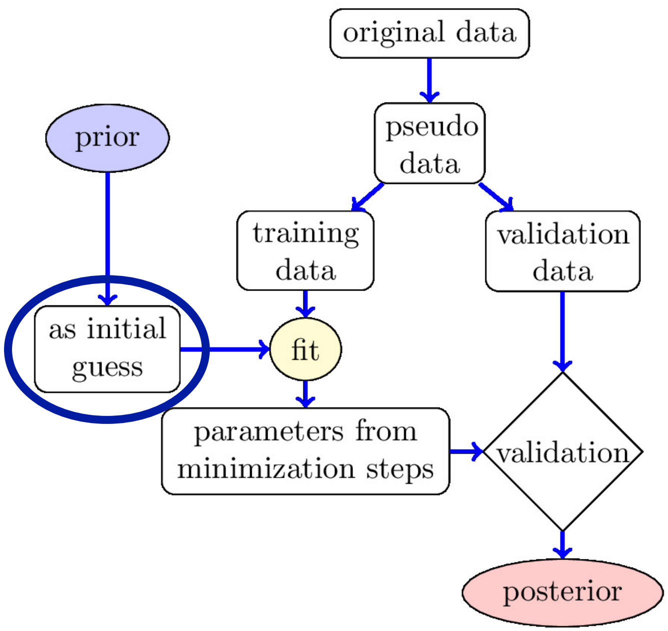

Initial iteration: flat sample priors

→ Set of parameters used as initial guess for least-squares fits

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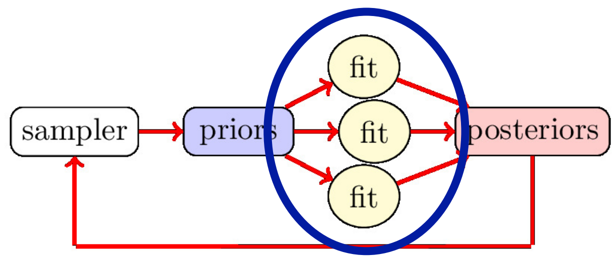
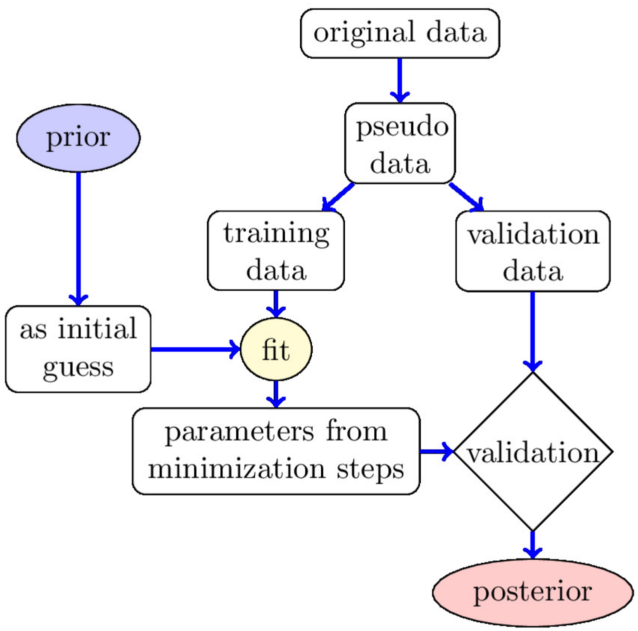
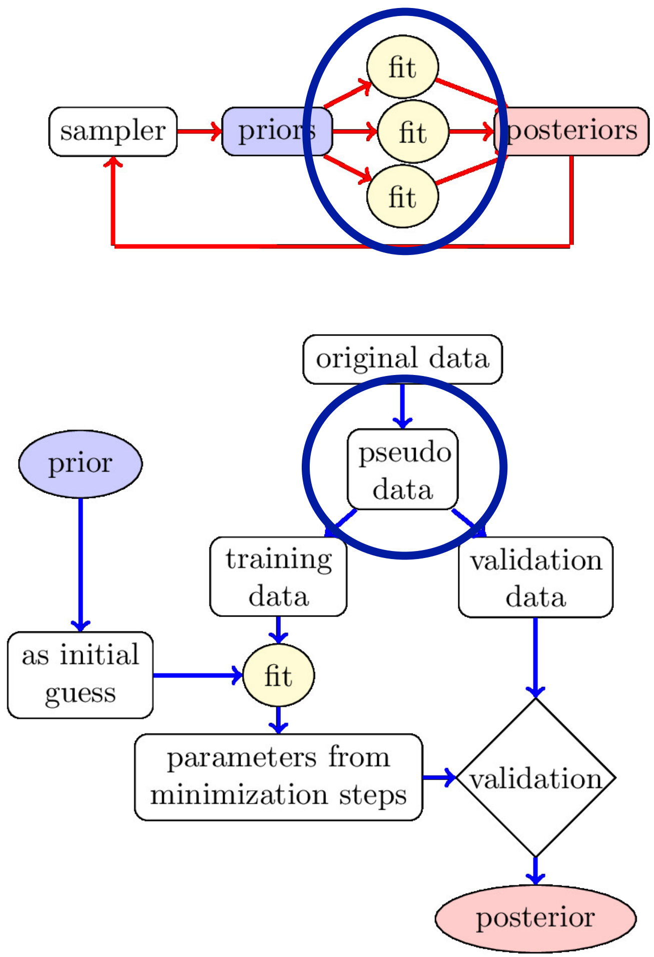




Perform thousands of fits

Iterative Monte Carlo (IMC) Fitting Methodology



Perform thousands of fits

→ Pseudo-data constructed by bootstrap method

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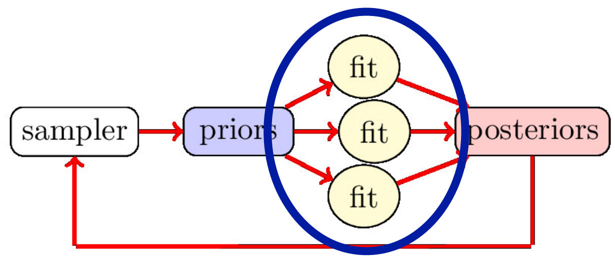

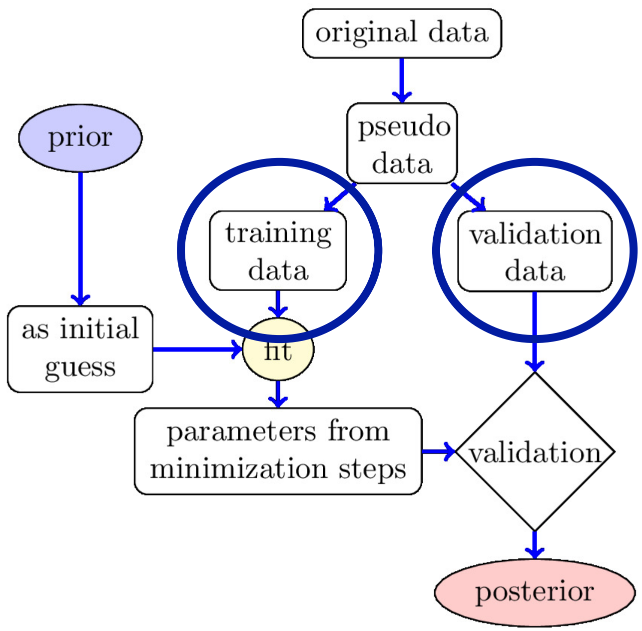

Perform thousands of fits

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→ Data is partitioned for cross-validation – training set is fitted via chi-square minimization

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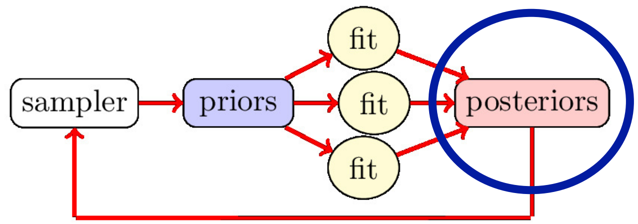

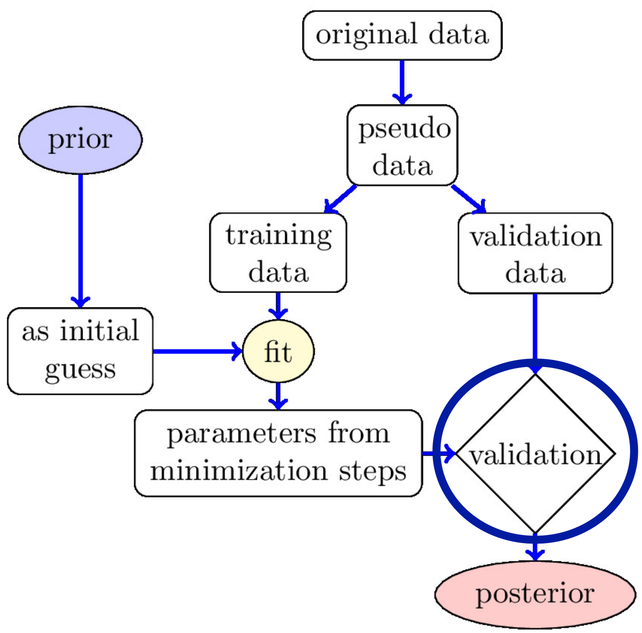

Obtain a set of posteriors

→ Set of parameters that minimize validation chi-square are chosen as posteriors

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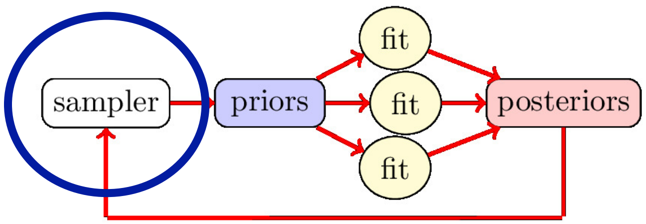
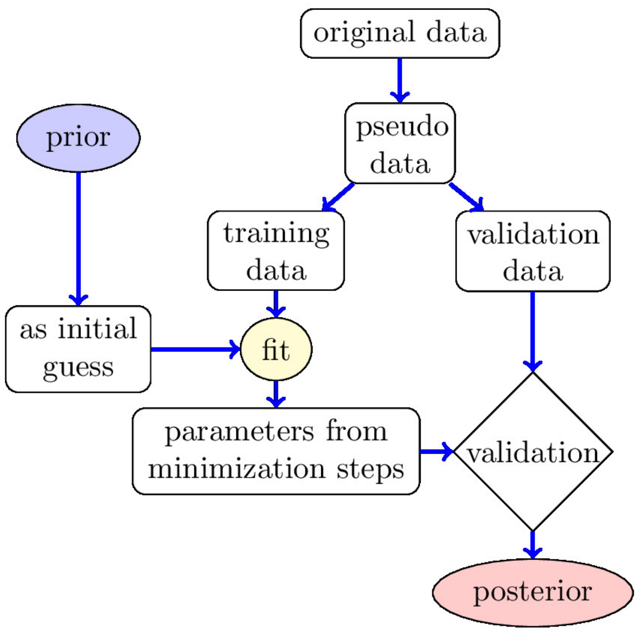




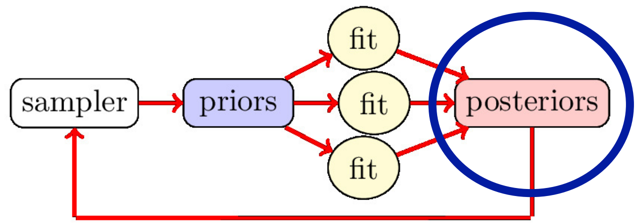
Posteriors are sent through a sampler

→ Kernel density estimation (KDE): estimates the multi-dimensional probability density function of the parameters

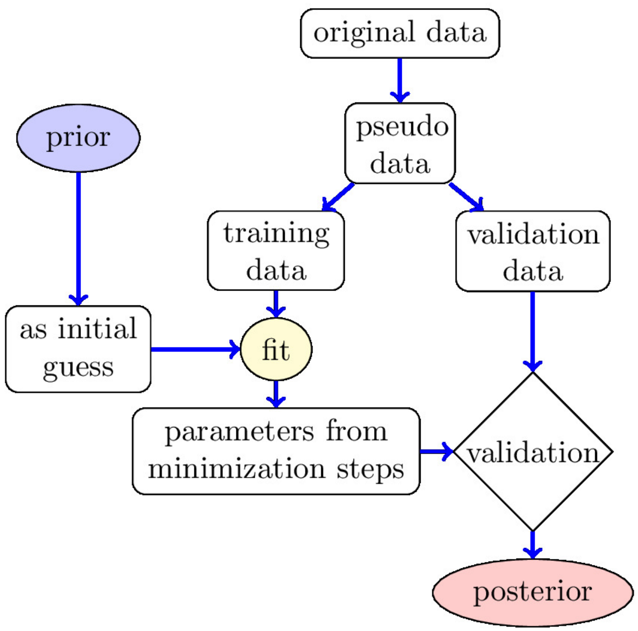
→ A sample of parameters is chosen from the KDE and used as starting priors for the next iteration

→ Iterated until distributions are converged

Iterative Monte Carlo (IMC) Fitting Methodology



Obtain final set of parameters



→ Compute mean and standard deviation of observables

$$\mathrm{E}[\mathcal{O}] = \frac{1}{n}\sum_{k=1}^{n}\mathcal{O}(\boldsymbol{a}_k)$$

$$\mathrm{V}[\mathcal{O}] = \frac{1}{n}\sum_{k=1}^{n}(\mathcal{O}(\boldsymbol{a}_k) - \mathrm{E}[\mathcal{O}])^2$$

Parameterizations and Chi-square

Template function: $$\mathrm{T}(x;\boldsymbol{a}) = \frac{M\, x^{a}(1-x)^{b}(1+c\sqrt{x})}{B(n+a,1+b)+cB(n+\frac{1}{2}+a,1+b)}$$

- PDFs: n = 1 $\Delta q^{+}, \Delta\bar{q}, \Delta g = \mathrm{T}(x;\boldsymbol{a})$
- FFs: n = 2, c = 0 Favored: $D^{h}_{q^{+}} = \mathrm{T}(z;\boldsymbol{a}) + \mathrm{T}(z;\boldsymbol{a}')$

  Unfavored: $D^{h}_{q^{+},g} = \mathrm{T}(z;\boldsymbol{a})$

Pions:

$$D^{\pi^{+}}_{\bar{u}} = D^{\pi^{+}}_{d} = \mathrm{T}(z;\boldsymbol{a})$$
$$D^{\pi^{+}}_{s} = D^{\pi^{+}}_{\bar{s}} = \frac{1}{2}D^{\pi^{+}}_{s^{+}}$$

Kaons:

$$D^{K^{+}}_{\bar{u}} = D^{K^{+}}_{d} = \frac{1}{2}D^{K^{+}}_{d^{+}}$$
$$D^{K^{+}}_{s} = \mathrm{T}(z;\boldsymbol{a})$$

- Chi-squared definition:

$$\chi^{2}(\boldsymbol{a}) = \sum_{e}\left[\sum_{i}\left(\frac{\mathcal{D}^{(e)}_{i}N^{(e)}_{i} - T^{(e)}_{i}(\boldsymbol{a})}{\alpha^{(e)}_{i}N^{(e)}_{i}}\right)^{2} + \sum_{k}\left(r^{(e)}_{k}\right)^{2}\right] + \sum_{\ell}\left(\frac{a^{(\ell)}-\mu^{(\ell)}}{\sigma^{(\ell)}}\right)^{2}$$

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Penalty for fitting normalizations

Parameterizations and Chi-square

Template function: $\mathrm{T}(x;\boldsymbol{a}) = \dfrac{M\, x^{a}(1-x)^{b}(1+c\sqrt{x})}{B(n+a,1+b)+cB(n+\frac{1}{2}+a,1+b)}$

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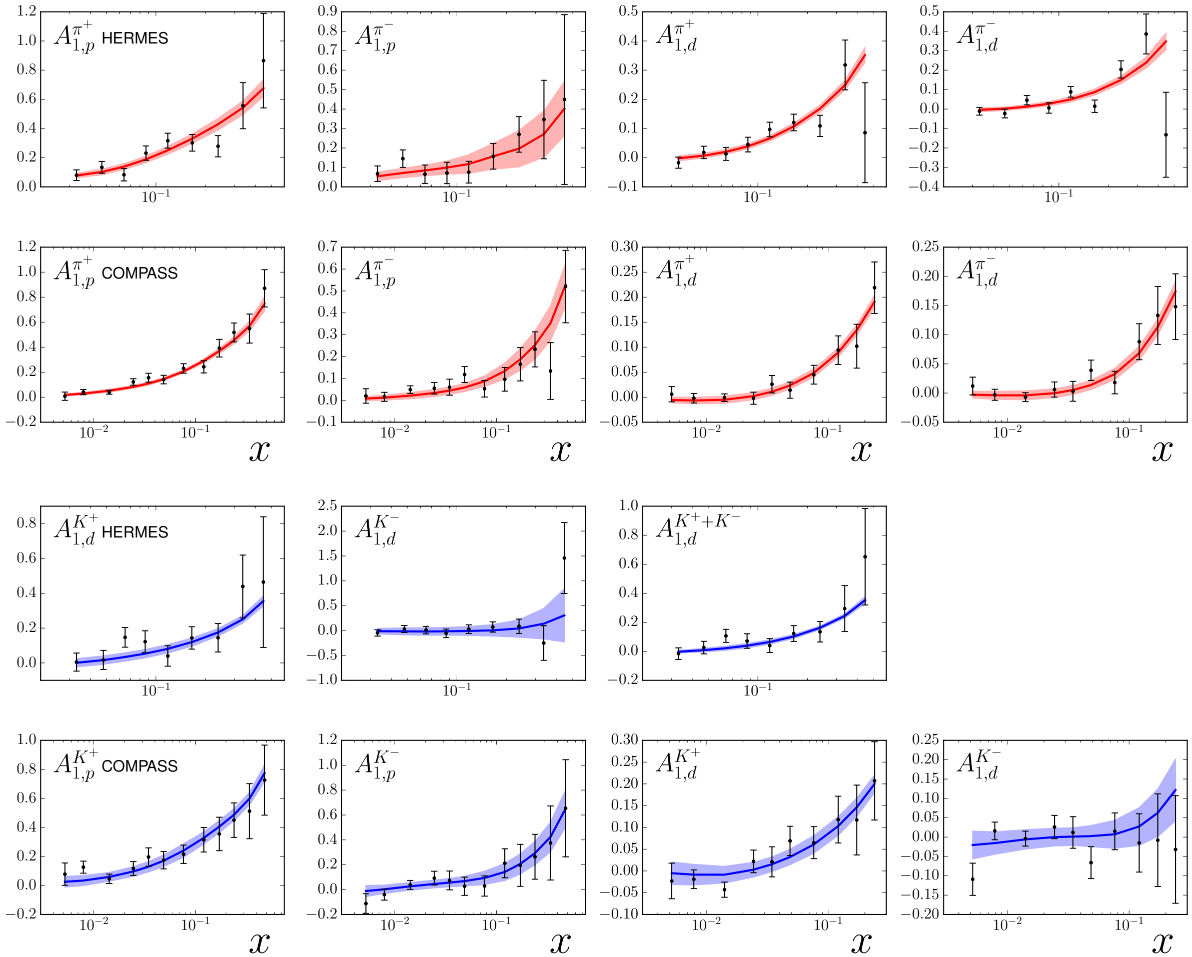
Modified likelihood to include prior information

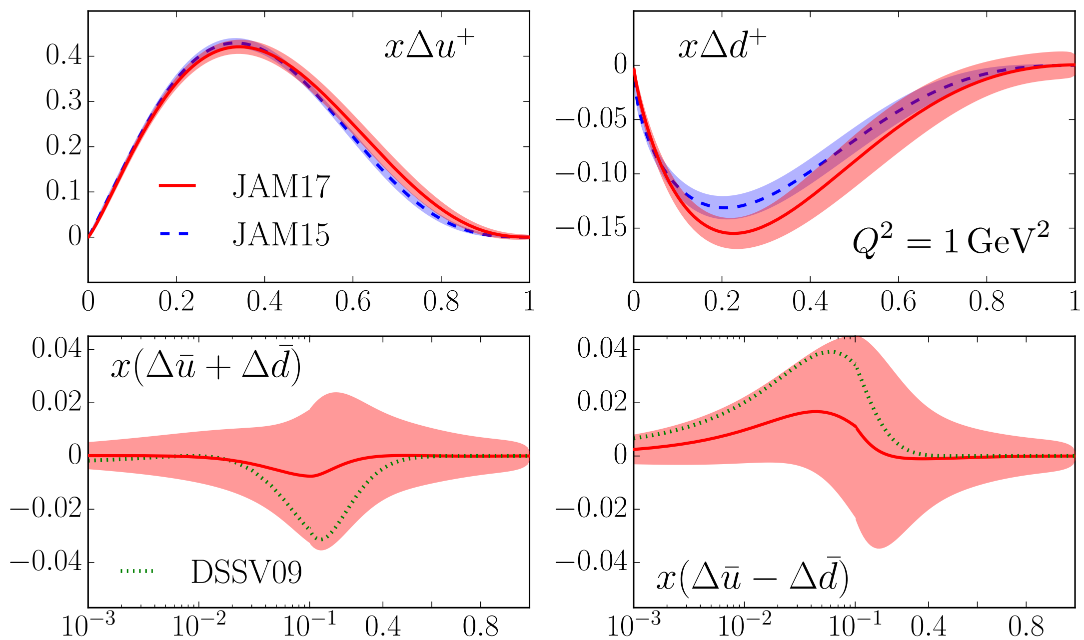
Data vs Theory – SIDIS



process	target	N_{dat}	χ^2
DIS	$p, d, {}^3\text{He}$	854	854.8
SIA $(\pi^\pm, K^\pm)$		850	997.1
SIDIS $(\pi^\pm)$			
HERMES	d	18	28.1
HERMES	p	18	14.2
COMPASS	d	20	8.0
COMPASS	p	24	18.2
SIDIS $(K^\pm)$			
HERMES	d	27	18.3
COMPASS	d	20	18.7
COMPASS	p	24	12.3
Total:		**1855**	**1969.7**

- Good agreement overall with all data!

Polarized PDF Distributions

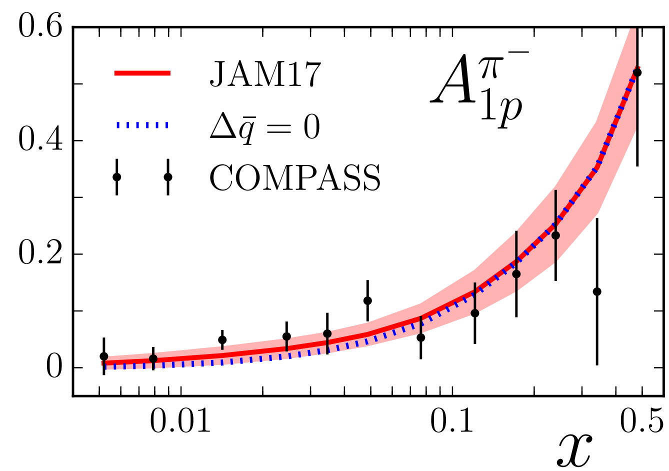


- Δu+ consistent with previous analysis
- Δd+ slightly larger in magnitude

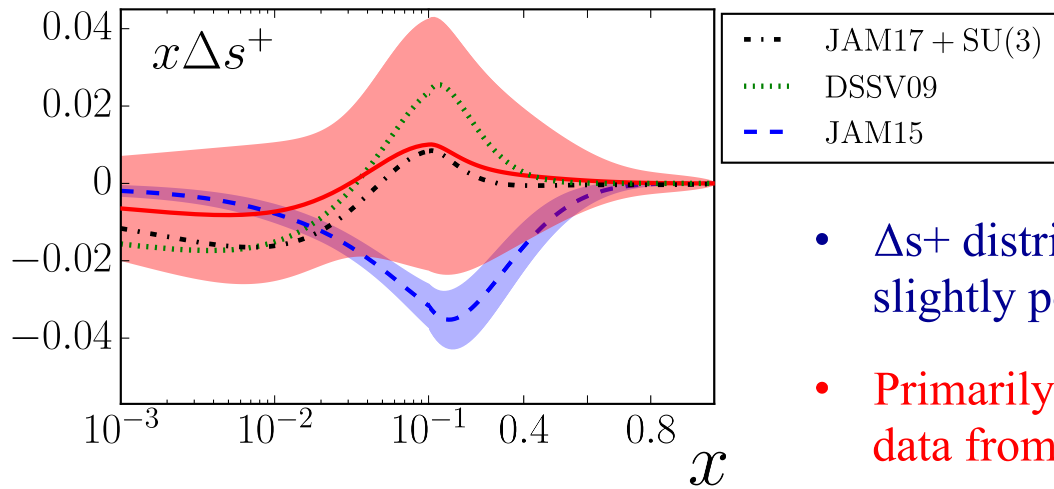
→ Anti-correlation with Δs+, which is less negative than JAM15 at $x \sim 0.2$

- Isoscalar sea distribution consistent with zero
- Isovector sea slightly prefers positive shape at low x

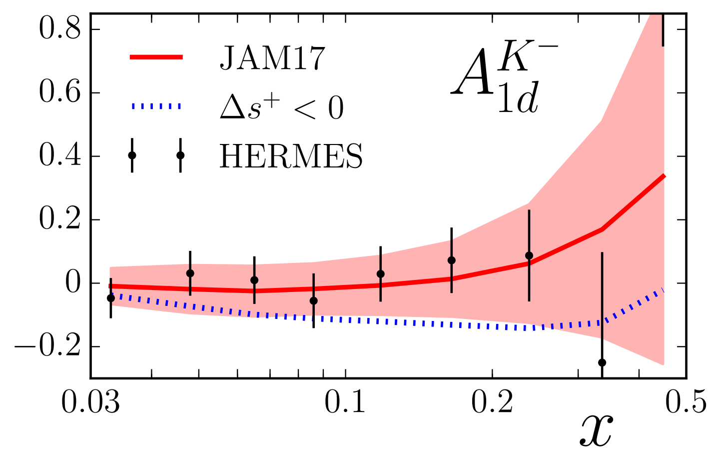
→ Non-zero asymmetry given by small contributions from SIDIS asymmetries



Strange polarization



- Δs+ distribution consistent with zero, slightly positive in intermediate *x* range
- Primarily influenced by HERMES K- data from deuterium target



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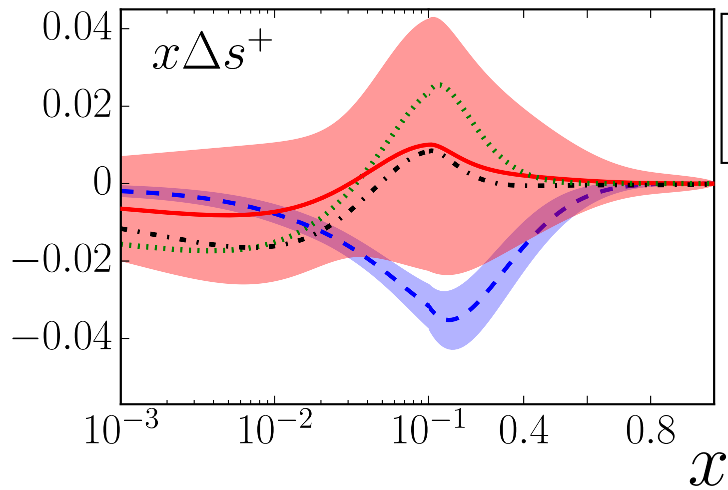

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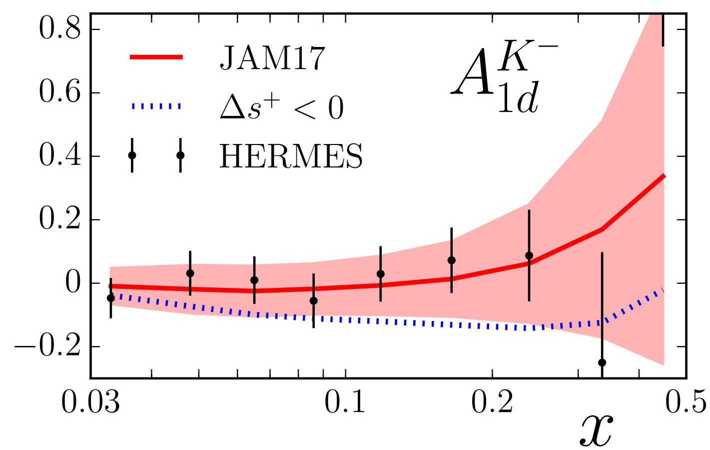
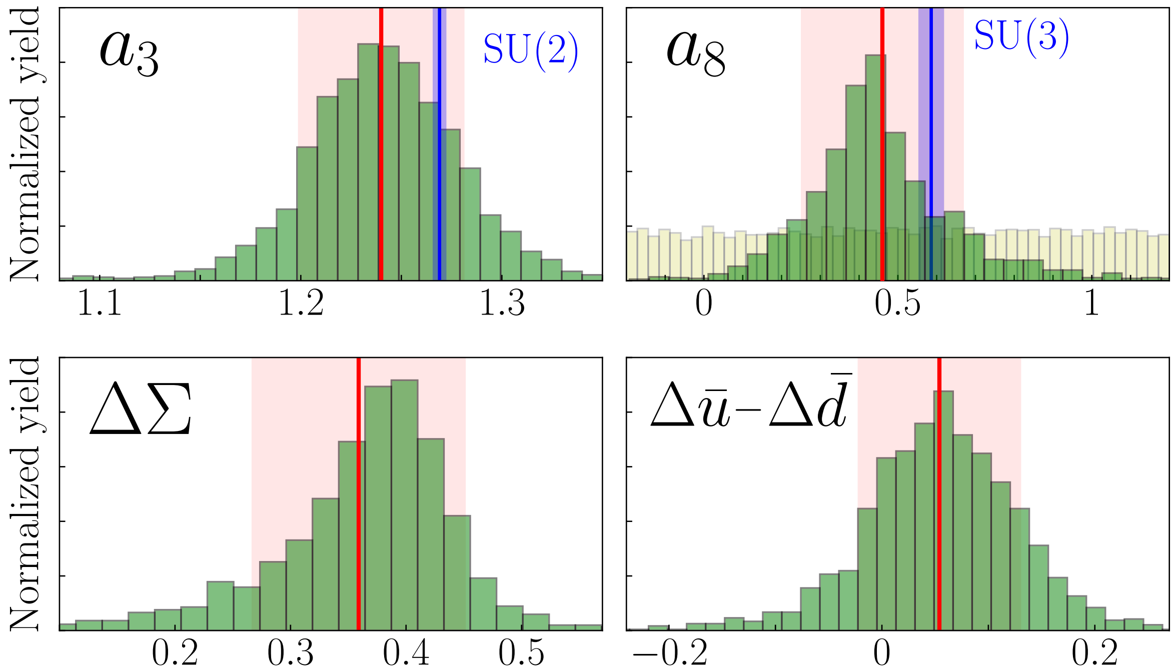
Why does DIS+SU(3) give large negative Δs+?

- Low *x* DIS deuterium data from COMPASS prefers small negative Δs+
- Needs to be more negative in intermediate region to satisfy SU(3) constraint
- b parameter for Δs+ typically fixed to values ~6-10, producing a peak at x~0.1



Moments



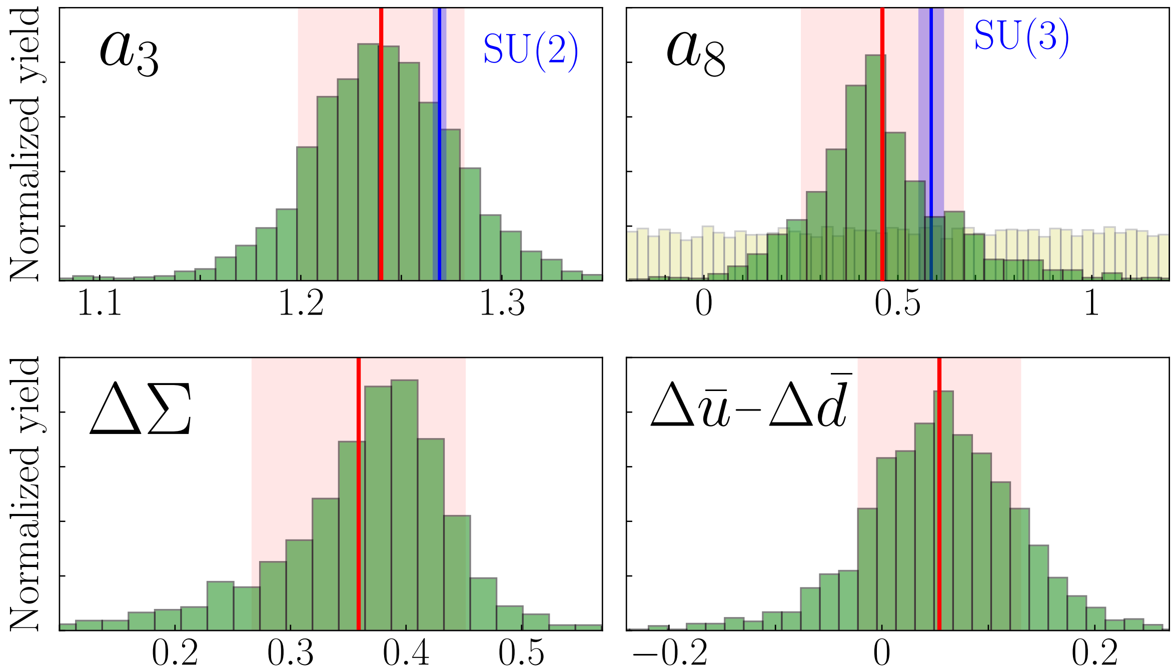
$$g_A = 1.24 \pm 0.04$$ Confirmation of SU(2) symmetry to ~2%

$$a_8 = 0.46 \pm 0.21$$ ~20% SU(3) breaking ± ~20%; large uncertainty

- Need better determination of Δs+ moment to reduce a_8 uncertainty!

$$\Delta \mathbf{s}^+ = -0.03 \pm 0.09$$

Moments



$$\Delta\Sigma = 0.36 \pm 0.09$$

Slightly larger central value than previous analyses, but consistent within uncertainty

Preference for slightly positive sea asymmetry; not very well constrained by SIDIS

$$\Delta\bar{u} - \Delta\bar{d} = 0.05 \pm 0.08$$

Summary and Outlook

- Analysis suggests the resolution of the "strange polarization puzzle"

  → Shape of $\Delta s+$ in DIS+SU(3) analyses is artificial (caused by SU(3) constraint + large-x shape parameter)

- Data sensitive to $\Delta s+$ distribution give result consistent with zero with large uncertainties

  → Need higher precision polarized SIDIS kaon data

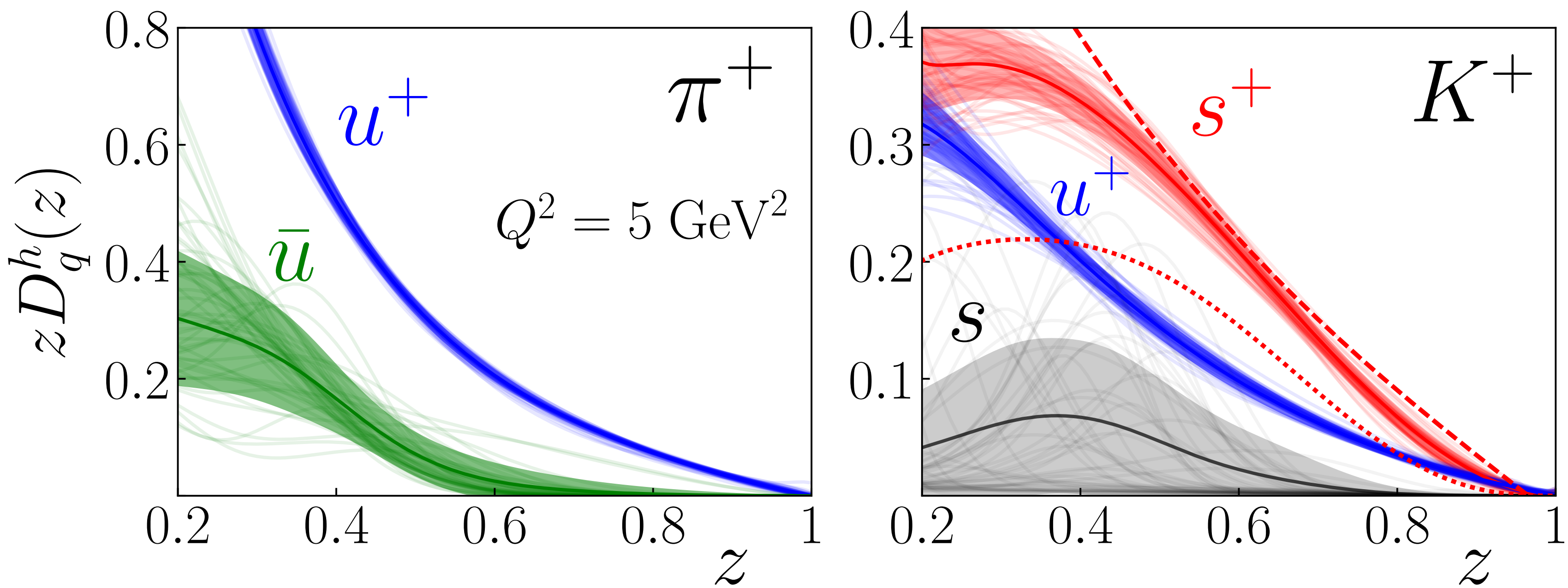
- Difficult to determine a_8 with DIS+SIDIS, but results confirm SU(2) symmetry to ~2%

- QCD observables yet to be implemented:
  - *W* asymmetries for constraints on up and down sea polarization
  - Unpolarized SIDIS and single-inclusive *pp* collision for FFs

- Working towards a universal fit of quark helicity distributions q↑, q↓
  - Global analyses of combined unpolarized and polarized data

BACKUP SLIDES

Fragmentation Functions



- Little change in ‘plus’ distributions from JAM16
  - →s+ to K+ FF marginally smaller at low-z compared to JAM16
- Agreement with DSS’s strange FF (dashed red line)
- Uncertainty for unfavored ū to π distribution smaller than s to K
  - →Due to lower precision kaon production data

Polarized DIS

- We fit measurements of parallel and perpendicular spin asymmetries

$$A_{\parallel} = \frac{\sigma^{\uparrow\Downarrow} - \sigma^{\uparrow\Uparrow}}{\sigma^{\uparrow\Downarrow} + \sigma^{\uparrow\Uparrow}} = D\left(A_1 + \eta A_2\right)$$

$$A_{\perp} = \frac{\sigma^{\uparrow\Rightarrow} - \sigma^{\uparrow\Leftarrow}}{\sigma^{\uparrow\Rightarrow} + \sigma^{\uparrow\Leftarrow}} = d\left(A_2 + \zeta A_1\right)$$

$$A_1 = \frac{(g_1 - \gamma^2 g_2)}{F_1} \qquad A_2 = \gamma \frac{(g_1 + g_2)}{F_1} \qquad \gamma^2 = \frac{4M^2x^2}{Q^2}$$

- Define our polarized structure functions:

$$g_{1,2}(x,Q^2) = g_{1,2}^{\mathrm{LT+TMC}}(\Delta u^+, \Delta d^+, \Delta g, \ldots)$$

Leading Twist Structure Functions

- Leading twist structure function defined in collinear factorization as

$$g_1^{(\tau 2)}(x,Q^2) = \frac{1}{2}\sum_q e_q^2 \left[(\Delta C_q \otimes \Delta q^+)(x,Q^2) + \ (\Delta C_g \otimes \Delta g)(x,Q^2) \right]$$

- Leading twist + target mass corrections $\xi = \frac{2x}{1+\rho}, \qquad \rho^2 = 1+\gamma^2$

J. Blümlein and A. Tkabladze
Nucl. Phys. B553, 427 (1999)

$$g_2^{(\tau 2+\mathrm{TMC})}(x,Q^2) = -\frac{x}{\xi\rho^3}\, g_1^{(\tau 2)}(\xi,Q^2) + \frac{1}{\rho^4}\int_\xi^1 \frac{dz}{z}\left[\frac{x}{\xi} - (\rho^2-1) + \frac{3(\rho^2-1)}{2\rho}\ln\frac{z}{\xi}\right] g_1^{(\tau 2)}(z,Q^2)$$

$$g_1^{(\tau 2+\mathrm{TMC})}(x,Q^2) = \frac{x}{\xi\rho^3}\, g_1^{(\tau 2)}(\xi,Q^2) + \frac{(\rho^2-1)}{\rho^4}\int_\xi^1 \frac{dz}{z}\left[\frac{(x+\xi)}{\xi} - \frac{(3-\rho^2)}{2\rho}\ln\frac{z}{\xi}\right] g_1^{(\tau 2)}(z,Q^2)$$

- In Bjorken limit $\left(Q^2 \to \infty\right)$:

$$g_1^{(\tau 2+\mathrm{TMC})} = g_1^{(\tau 2)} \qquad g_2^{(\tau 2)}(x,Q^2) = -g_1^{(\tau 2)}(x,Q^2) + \int_x^1 \frac{dz}{z} g_1^{(\tau 2)}(z,Q^2)$$

Semi-inclusive DIS

- In SIDIS, we fit the photon-nucleon spin asymmetries (assuming Bjorken limit)

$$A_1^h\left(x, z, Q^2\right) = \frac{g_1^h(x, z, Q^2)}{F_1(x, z, Q^2)}$$

- The (un)polarized structure functions are defined

$$g_1^h(x, z, Q^2) = \frac{1}{2}\sum_q e_q^2 \Bigg\{ \Delta q(x, \mu_F) D_q^h(z, \mu_{FF}) + \frac{\alpha_s(\mu_R)}{2\pi}$$
$$\times \left(\Delta q \otimes \Delta C_{qq} \otimes D_q^h + \Delta q \otimes \Delta C_{gq} \otimes D_g^h + \Delta g \otimes \Delta C_{qg} \otimes D_q^h \right) \Bigg\}$$

$$F_1^h(x, z, Q^2) = \frac{1}{2}\sum_q e_q^2 \Bigg\{ q(x, \mu_F) D_q^h(z, \mu_{FF}) + \frac{\alpha_s(\mu_R)}{2\pi}$$
$$\times \left(q \otimes C_{qq} \otimes D_q^h + q \otimes C_{gq} \otimes D_g^h + g \otimes C_{qg} \otimes D_q^h \right) \Bigg\}$$

Single-inclusive Annihilation (SIA)

- Cross sections for $e^+e^- \to hX$

$$\frac{d\sigma^h}{dz} = \frac{d\sigma^h_L}{dz} + \frac{d\sigma^h_T}{dz} \qquad\qquad z = \frac{2E_h}{\sqrt{s}}$$

- Transverse and longitudinal cross sections

$$\frac{d\sigma^h_T}{dz} = \sum_i \sigma_i \left[D_i(z,Q^2) + \frac{\alpha_s}{2\pi}\big(C_i^T \otimes D_i\big)(z,Q^2)\right] + \sigma_0 \frac{\alpha_s}{2\pi}\big(C_g^T \otimes D_g\big)(z,Q^2)$$

$$\frac{d\sigma^h_L}{dz} = \sum_i \sigma_i \frac{\alpha_s}{2\pi}\big(C_i^L \otimes D_i\big)(z,Q^2) + \sigma_0 \frac{\alpha_s}{2\pi}\big(C_g^L \otimes D_g\big)(z,Q^2)$$

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- Electroweak cross section $e^+e^- \to \gamma, Z \to q\bar{q}$:

$$\sigma_i(s) = \sigma_0\left[e_i^2 + 2e_i\, g_V^i g_V^e\, \rho_1(s) + \left(g_A^{e\,2} + g_V^{e\,2}\right)\left(g_A^{i\,2} + g_V^{i\,2}\right)\rho_2(s)\right]$$

Single-inclusive Annihilation (SIA)

- Cross sections for $e^+e^- \to hX$

$$\frac{d\sigma^h}{dz} = \frac{d\sigma_L^h}{dz} + \frac{d\sigma_T^h}{dz} \qquad\qquad z = \frac{2E_h}{\sqrt{s}}$$

- Transverse and longitudinal cross sections

$$\frac{d\sigma_T^h}{dz} = \sum_i \sigma_i \left[D_i(z,Q^2) + \frac{\alpha_s}{2\pi}\big(C_i^T \otimes D_i\big)(z,Q^2)\right] + \sigma_0 \frac{\alpha_s}{2\pi}\big(C_g^T \otimes D_g\big)(z,Q^2)$$

$$\frac{d\sigma_L^h}{dz} = \sum_i \sigma_i \frac{\alpha_s}{2\pi}\big(C_i^L \otimes D_i\big)(z,Q^2) + \sigma_0 \frac{\alpha_s}{2\pi}\big(C_g^L \otimes D_g\big)(z,Q^2)$$

- Electroweak cross section $e^+e^- \to \gamma, Z \to q\bar{q}$:

$$\sigma_i(s) = \sigma_0 \left[e_i^2 + 2e_i\, g_V^i g_V^e\, \rho_1(s) + \left(g_A^{e\,2} + g_V^{e\,2}\right)\left(g_A^{i\,2} + g_V^{i\,2}\right) \rho_2(s)\right]$$

- Typically, observables are normalized by total hadronic cross section

$$\sigma_{\rm tot}(s) = \sum_i \sigma_i \left(1 + \frac{\alpha_s}{\pi}\right)$$