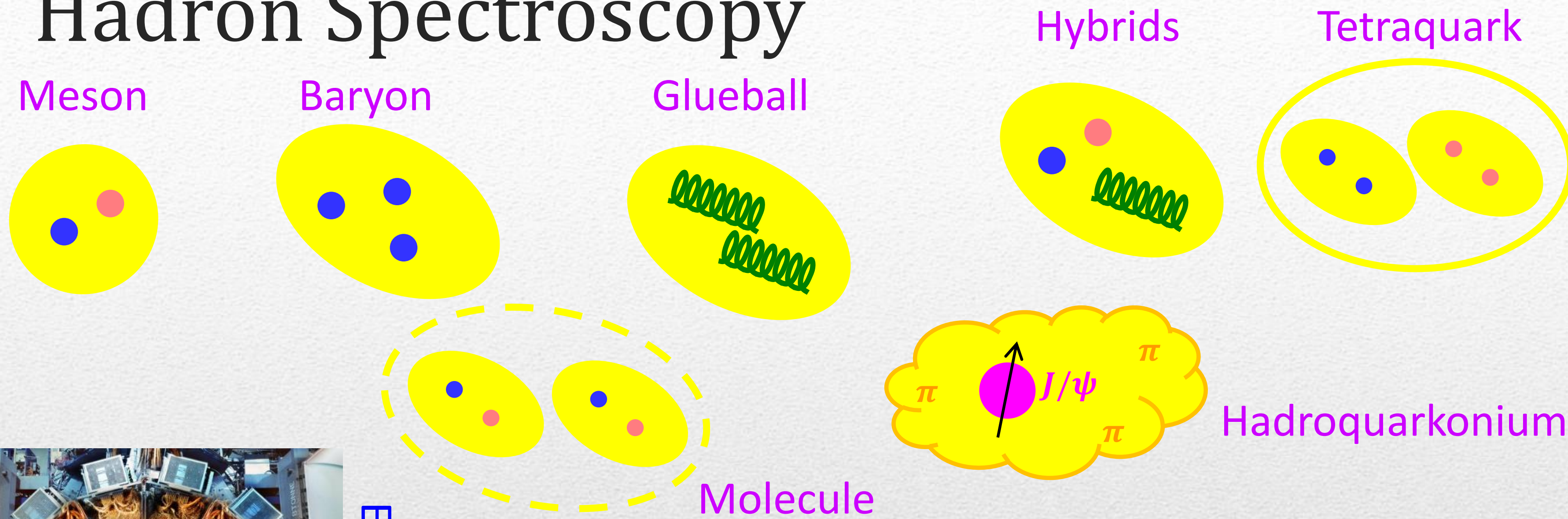
JPAC program for Hadron Spectroscopy

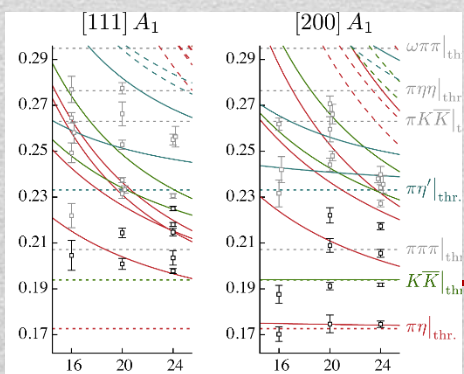
Alessandro Pilloni

Hadronic Physics with Lepton and Hadron Beams, JLab, September 5th, 2017

Jefferson Lab

Hadron Spectroscopy





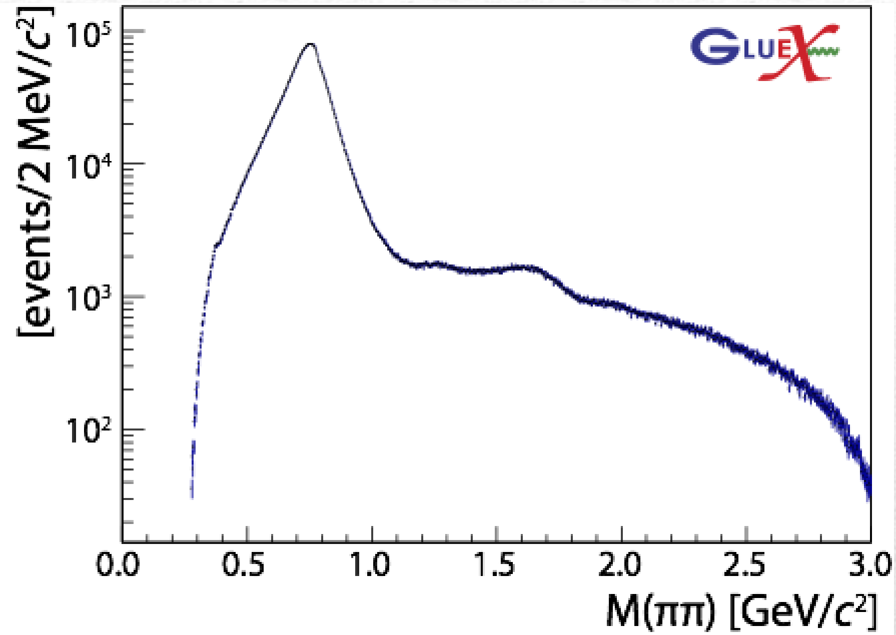
Experiment

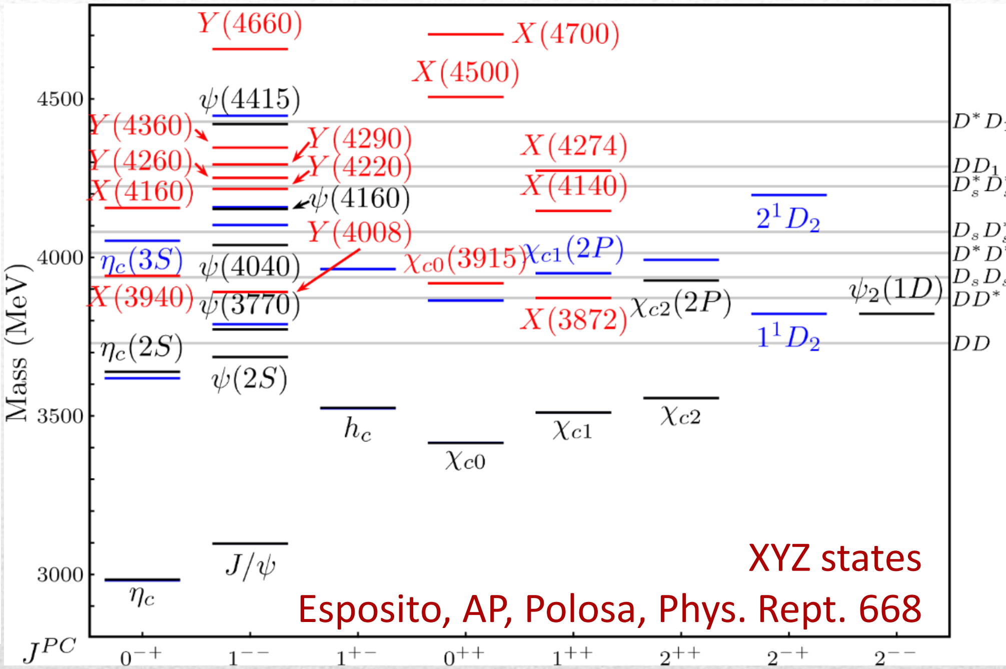
Lattice QCD

Data → ? → Fundamental properties, Model building

Interpretations on the spectrum leads to understanding fundamental laws of nature

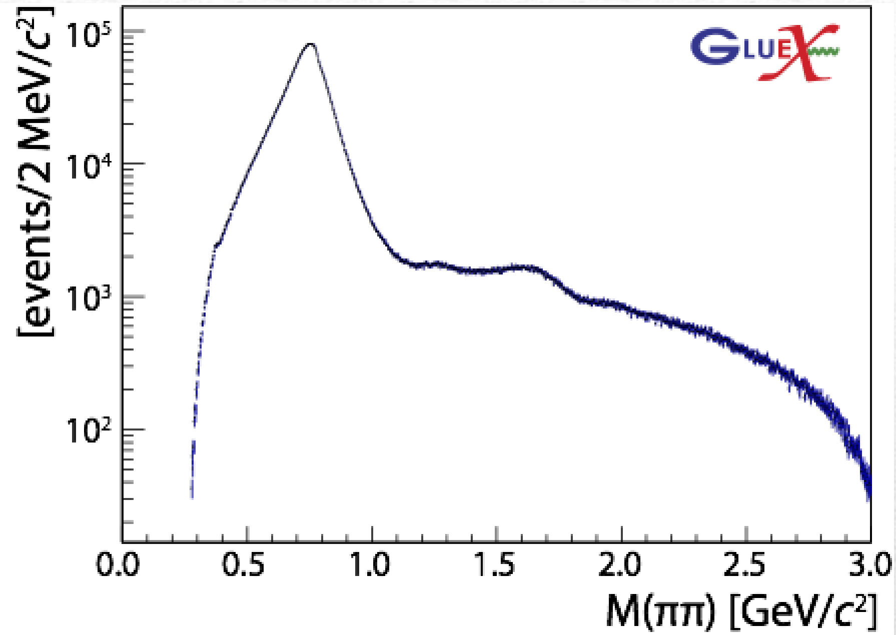
Hadron Spectroscopy

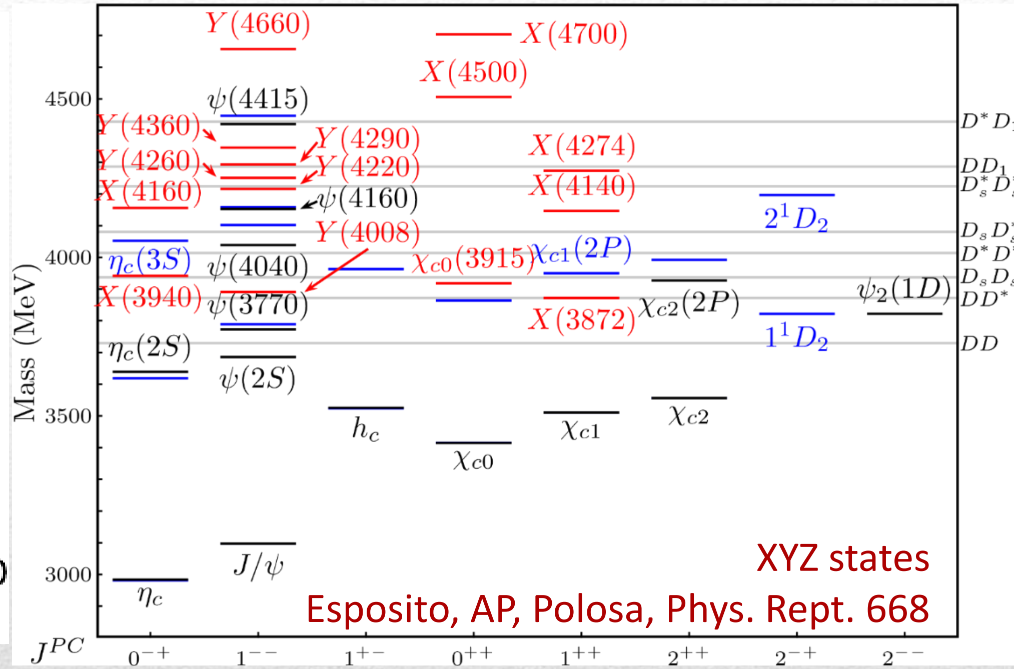


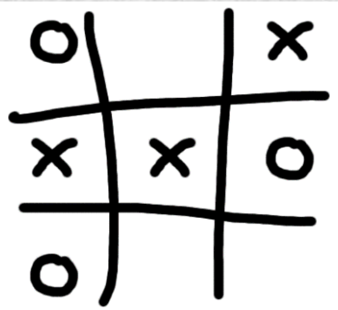


Data → ? → Fundamental properties, Model building

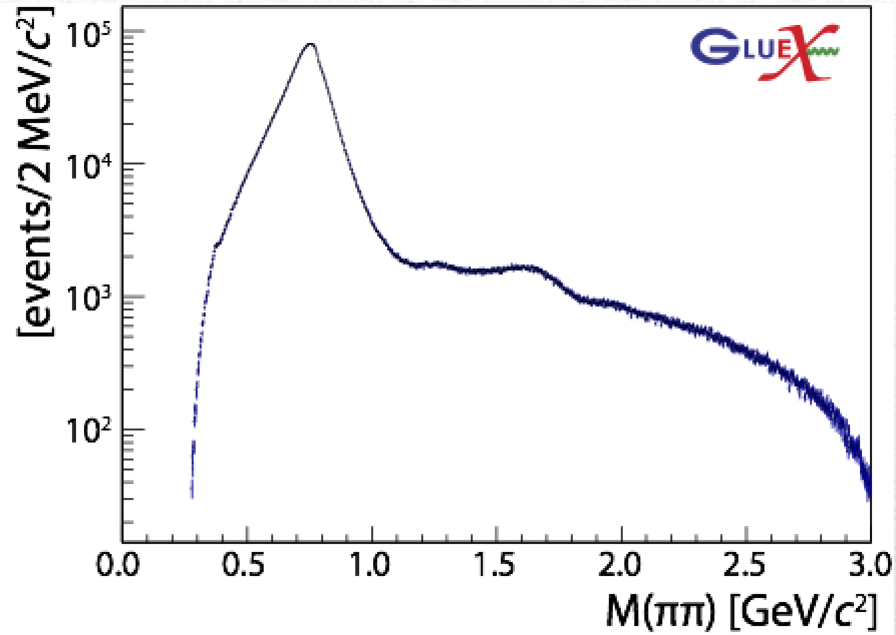
Hadron Spectroscopy

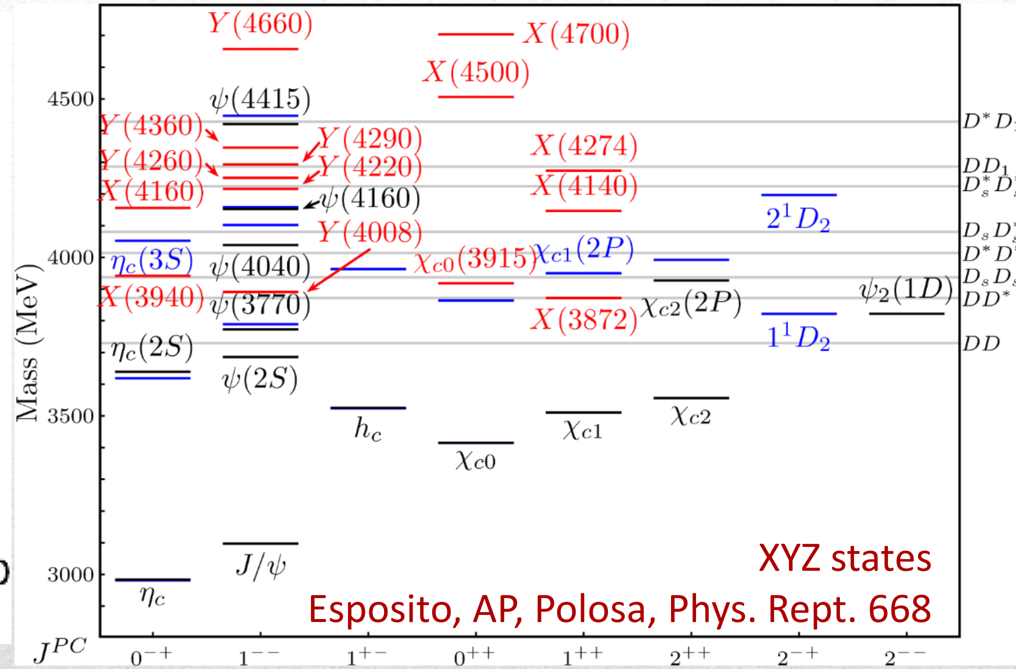


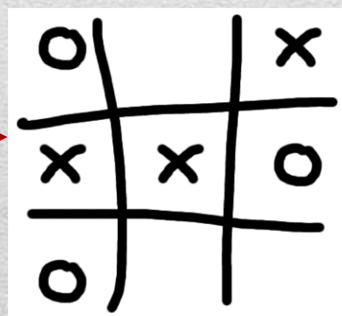


Data →  → Fundamental properties, Model building

Hadron Spectroscopy



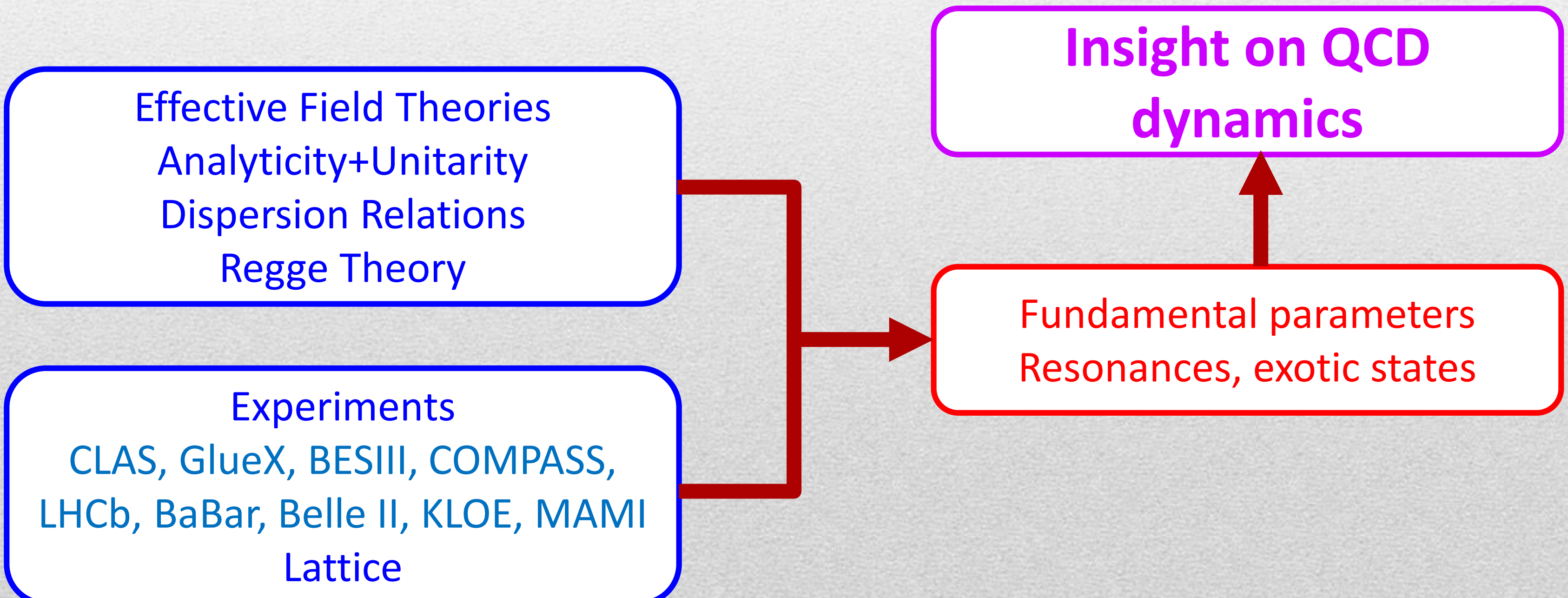


Data →  → Fundamental properties, Model building

Improvement needed! With great statistics comes great responsibility!

Joint Physics Analysis Center

- Joint effort between theorists and experimentalists to work together to make the best use of the next generation of very precise data taken at JLab and in the world
- Created in 2013 by JLab & IU agreement
- It is engaged in education of further generations of hadron physics practitioners



Joint Physics Analysis Center

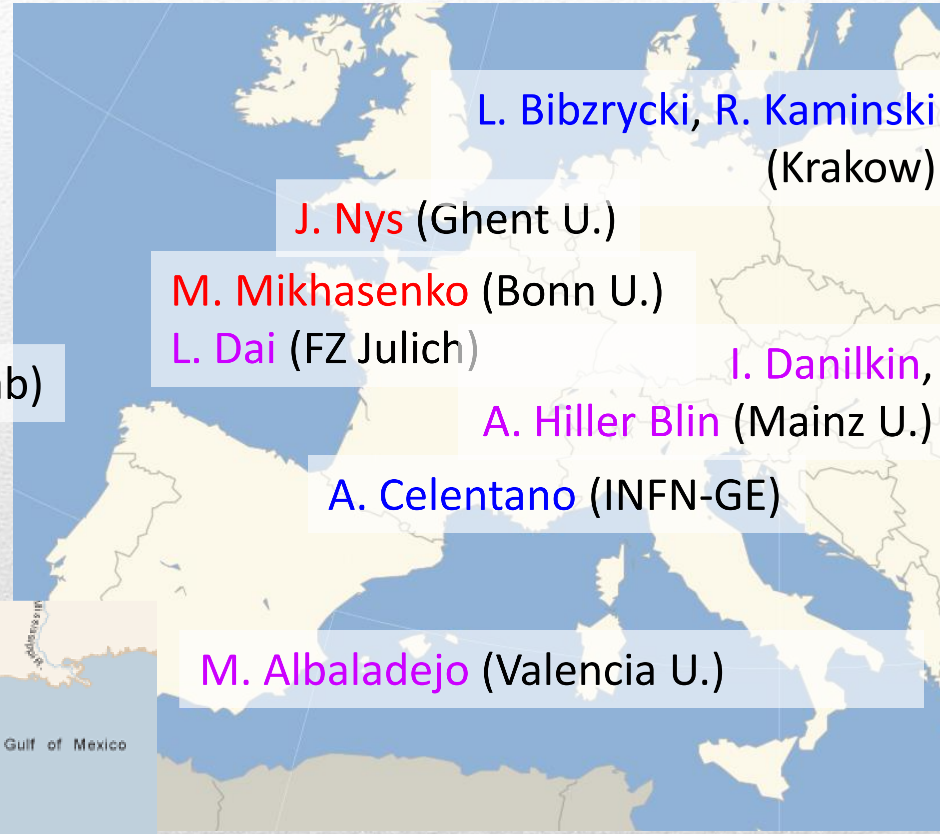
A. Jackura, N. Sherrill, G. Fox, T. Londergan (IU), E. Passemar, A. Szczepaniak (IU/JLab)

R. Workman (GWU), M. Döring (GWU/JLab)

V. Mathieu, V. Pauk, A. Pilloni, V. Mokeev (JLab)

P. Guo (Cal. State U.)

J. Castro, C. Fernandez-Ramirez (UNAM)



L. Bibzrycki, R. Kaminski (Krakow)

J. Nys (Ghent U.)

M. Mikhasenko (Bonn U.)

L. Dai (FZ Julich)

I. Danilkin, A. Hiller Blin (Mainz U.)

A. Celentano (INFN-GE)

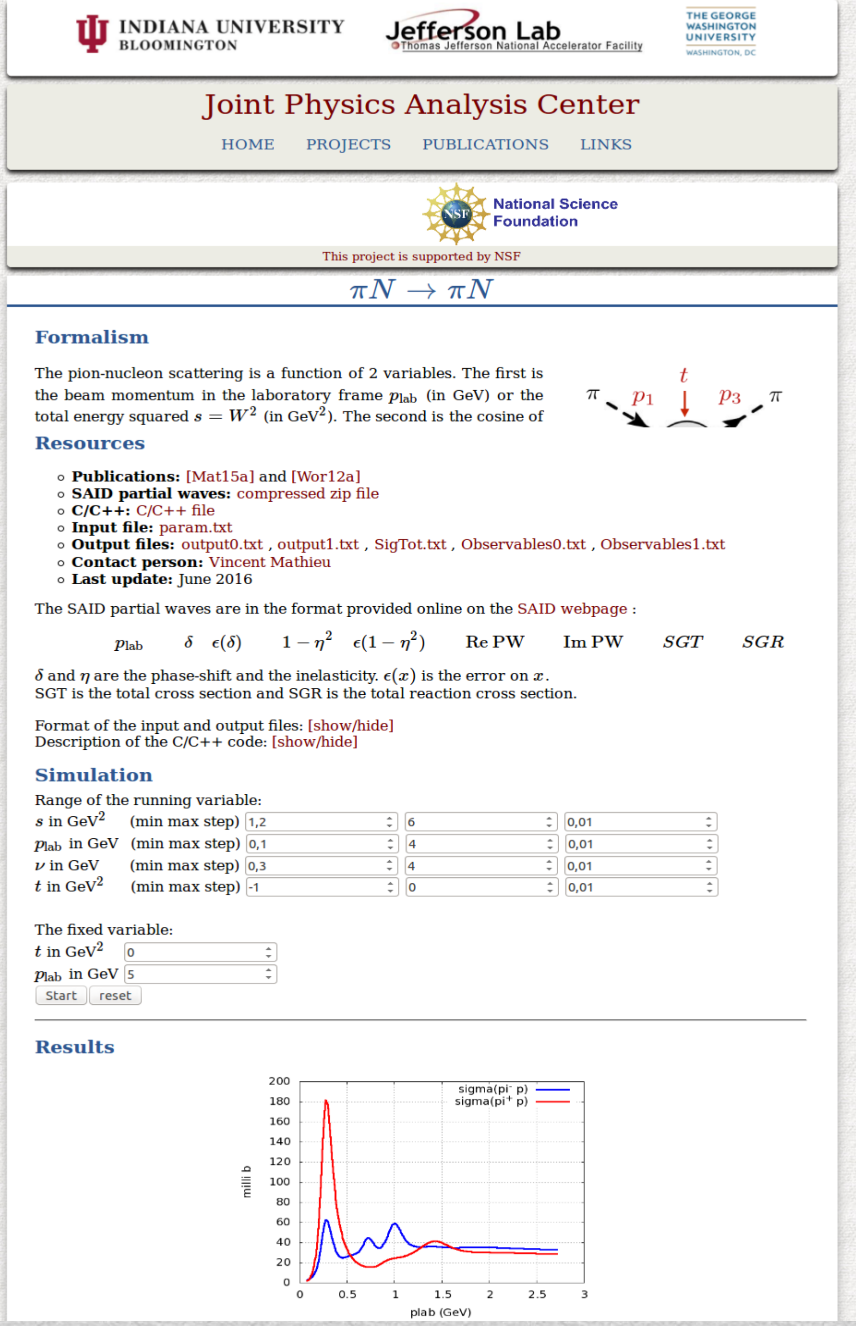
M. Albaladejo (Valencia U.)

Students, Postdocs, Faculties

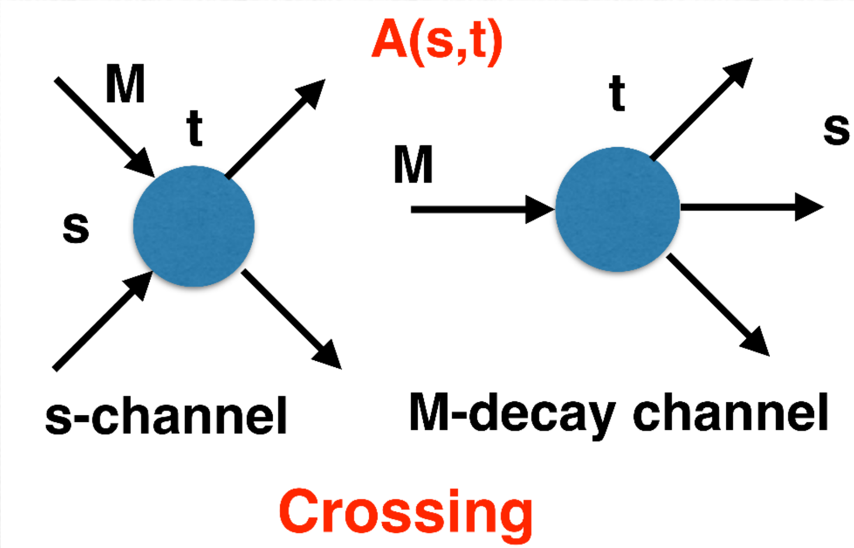
Interactive tools

- Completed projects are fully documented on interactive portals
- These include description on physics, conventions, formalism, etc.
- The web pages contain source codes with detailed explanation how to use them. Users can run codes online, change parameters, display results.

http://www.indiana.edu/~jpac/



S-Matrix principles

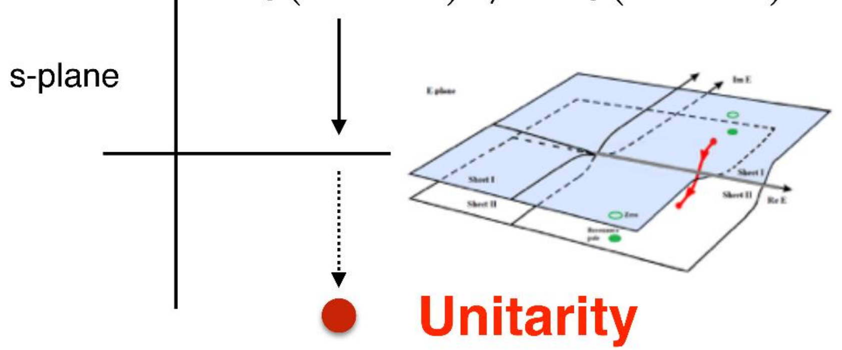


$$A(s,t) = \sum_l A_l(s) P_l(z_s)$$

Analyticity

$$A_l(s) = \lim_{\epsilon \to 0} A_l(s + i\epsilon)$$

$$A_l(s + i\epsilon) \neq A_l(s - i\epsilon)$$

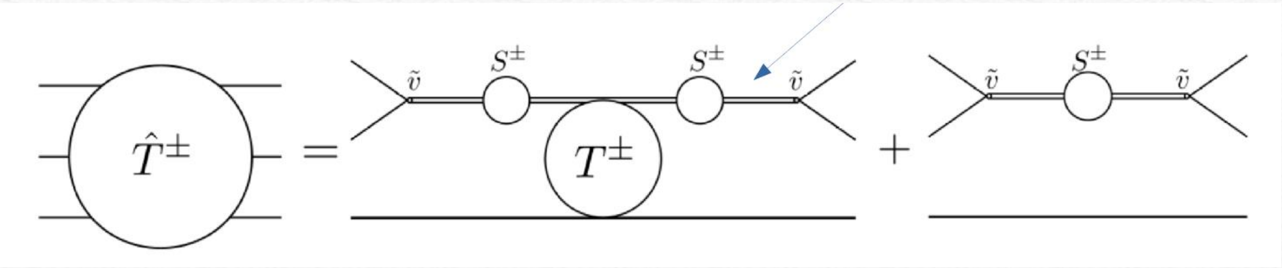


These are constraints the amplitudes have to satisfy, but do not fix the dynamics

Resonances (QCD states) are poles in the unphysical Riemann sheets

Three-Body Unitarity

Hu, Mai, Doring, AP, Szczepaniak, EPJA, arXiv:1707.06118



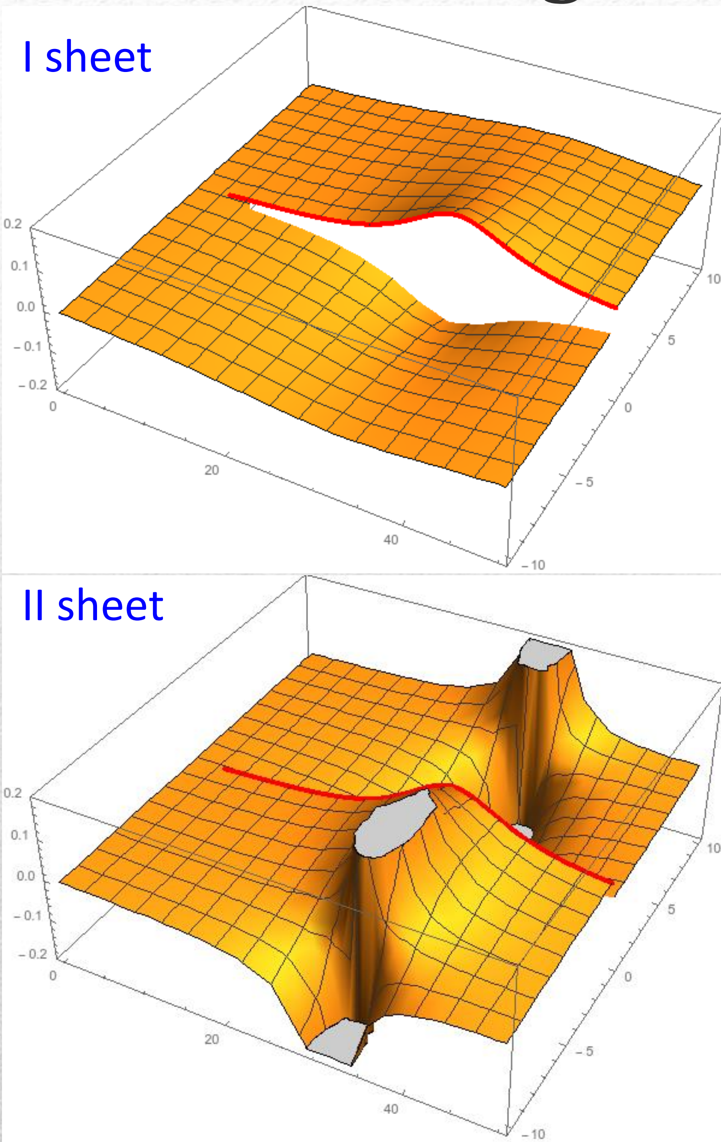
The full implementation of three-body unitarity is a major step for understanding the states appearing in such final states

e.g. $a_1(1260)^+ \to \pi^+\pi^-\pi^+, \pi_1(1400)^+ \to \pi^+\pi^-\pi^+, X(3872) \to D^0\overline{D^0}\pi^0$

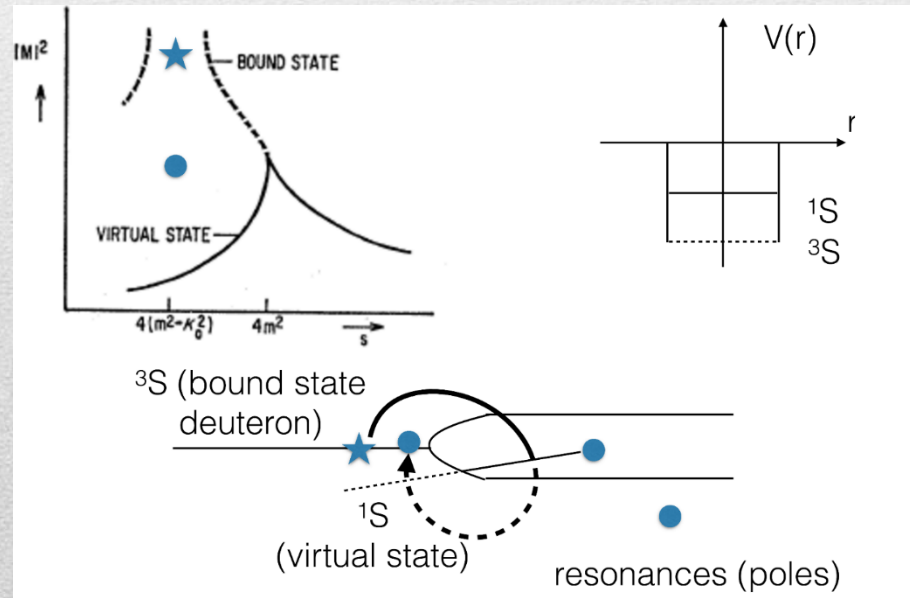
We completed the proof of the Amado model,
based on the isobar approximation
and a Bethe-Salpeter ansatz for the amplitude

See M. Doring's talk at 11:30am

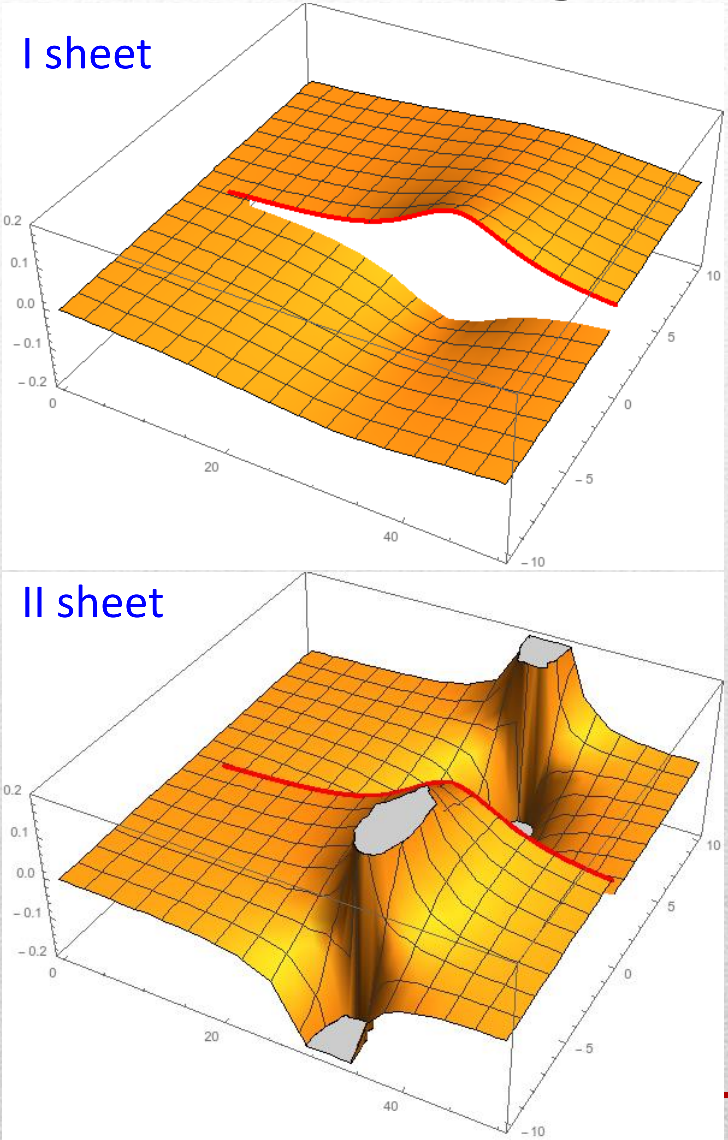
Pole hunting



Bound states on the real axis 1st sheet
Not-so-bound (virtual) states on the real axis 2nd sheet

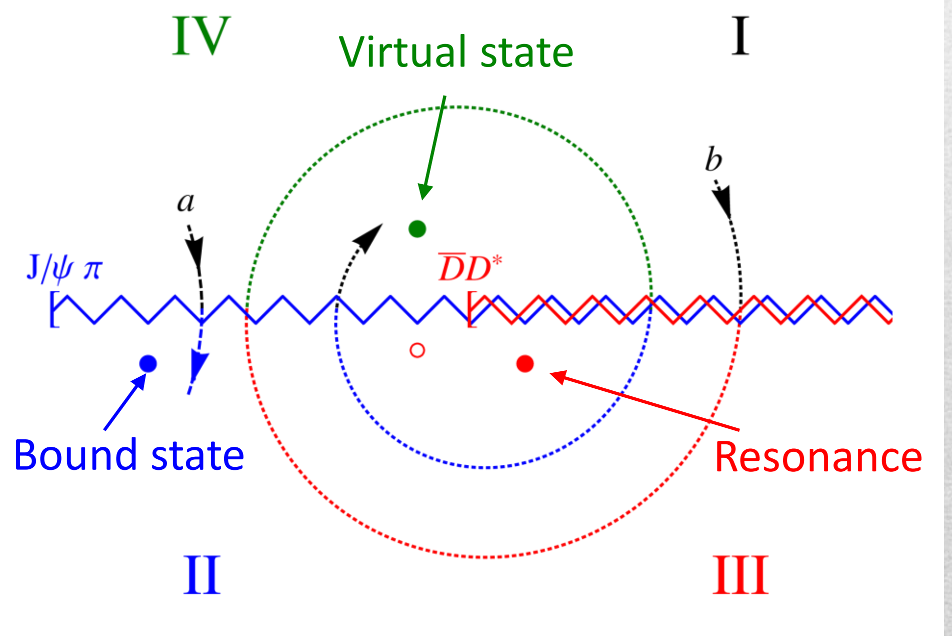


Pole hunting



More complicated structure when more thresholds arise:
two sheets for each new threshold

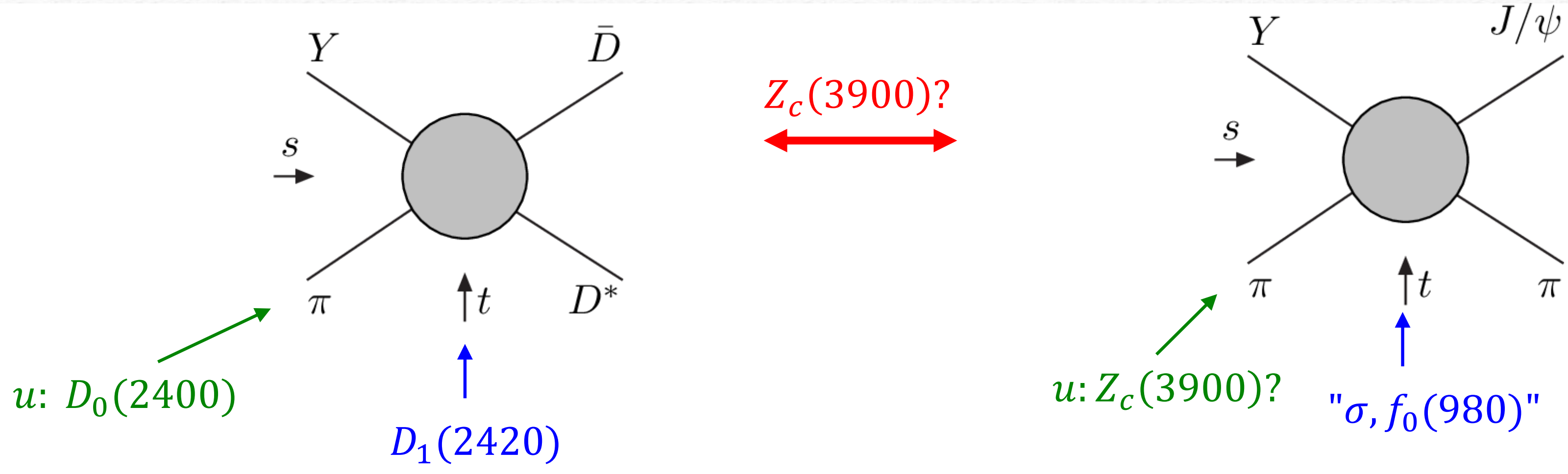
III sheet: usual resonances
IV sheet: cusps (virtual states)



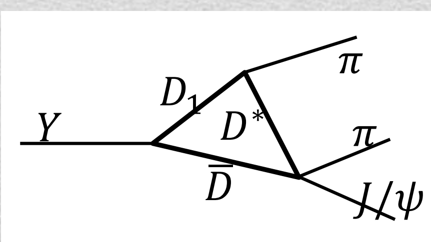
Amplitude analysis for $Z_c(3900)$

One can test different parametrizations of the amplitude, which correspond to
different singularities → different natures

AP *et al.* (JPAC), PLB772, 200



Triangle rescattering,
logarithmic branching point

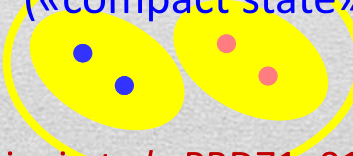


Szczepaniak, PLB747, 410

(anti)bound state,
II/IV sheet pole
(«molecule»)

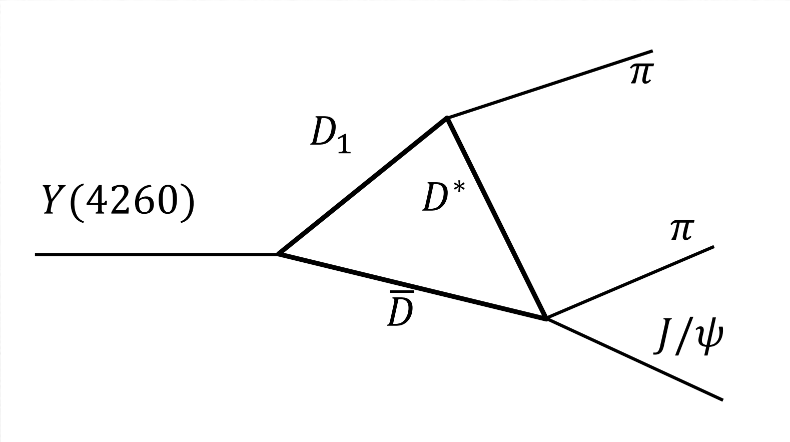
Tornqvist, Z.Phys. C61, 525
Swanson, Phys.Rept. 429
Hanhart *et al.* PRL111, 132003

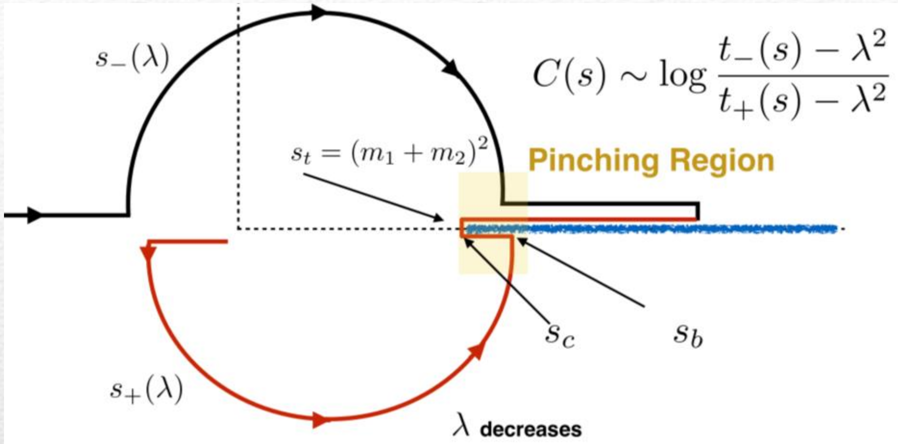
Resonance,
III sheet pole
(«compact state»)



Maiani *et al.*, PRD71, 014028
Faccini *et al.*, PRD87, 111102
Esposito *et al.*, Phys.Rept. 668

Triangle singularity





- Logarithmic branch points due to exchanges in the cross channels can simulate a resonant behavior, only in very special kinematical conditions (Coleman and Norton, Nuovo Cim. 38, 438)
- However, this effects cancels in Dalitz projections, no peaks (Schmid, Phys.Rev. 154, 1363)
- But the cancellation can be spread in different channels, you might still see peaks in other channels!

Testing scenarios

- We approximate all the particles to be scalar – this affects the value of couplings, which are not normalized anyway – but not the position of singularities.
  This also limits the number of free parameters

$$f_i(s,t,u) = 16\pi \left[a_{0,i}^{(t)}(t) + a_{0,i}^{(u)}(u) + \sum_j t_{ij}(s) \left(c_j + \frac{s}{\pi} \int_{s_j}^{\infty} ds' \frac{\rho_j(s') b_{0,j}(s')}{s'(s'-s)} \right) \right],$$

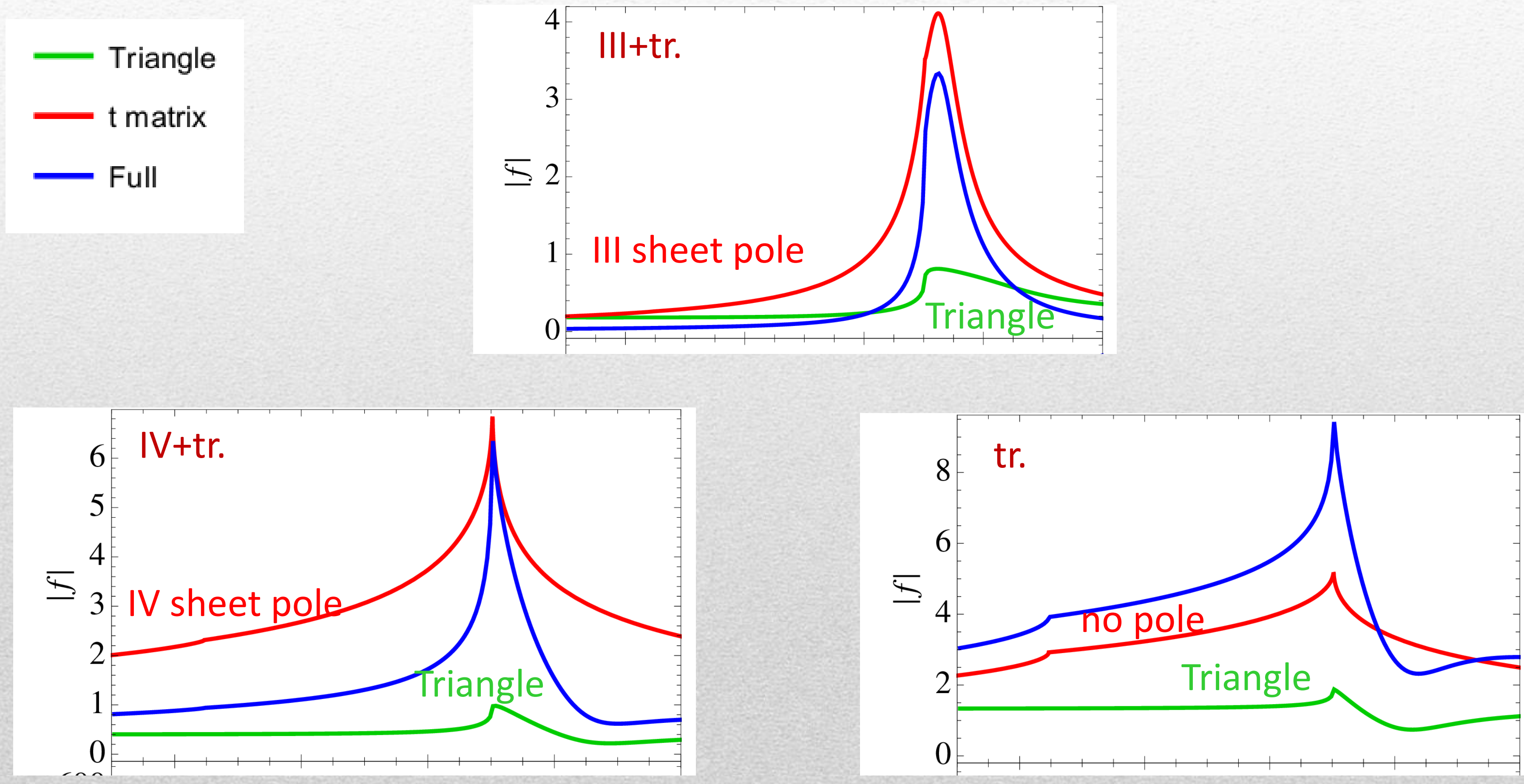
The scattering matrix is parametrized as $(t^{-1})_{ij} = K_{ij} - i\,\rho_i\,\delta_{ij}$

Four different scenarios considered:

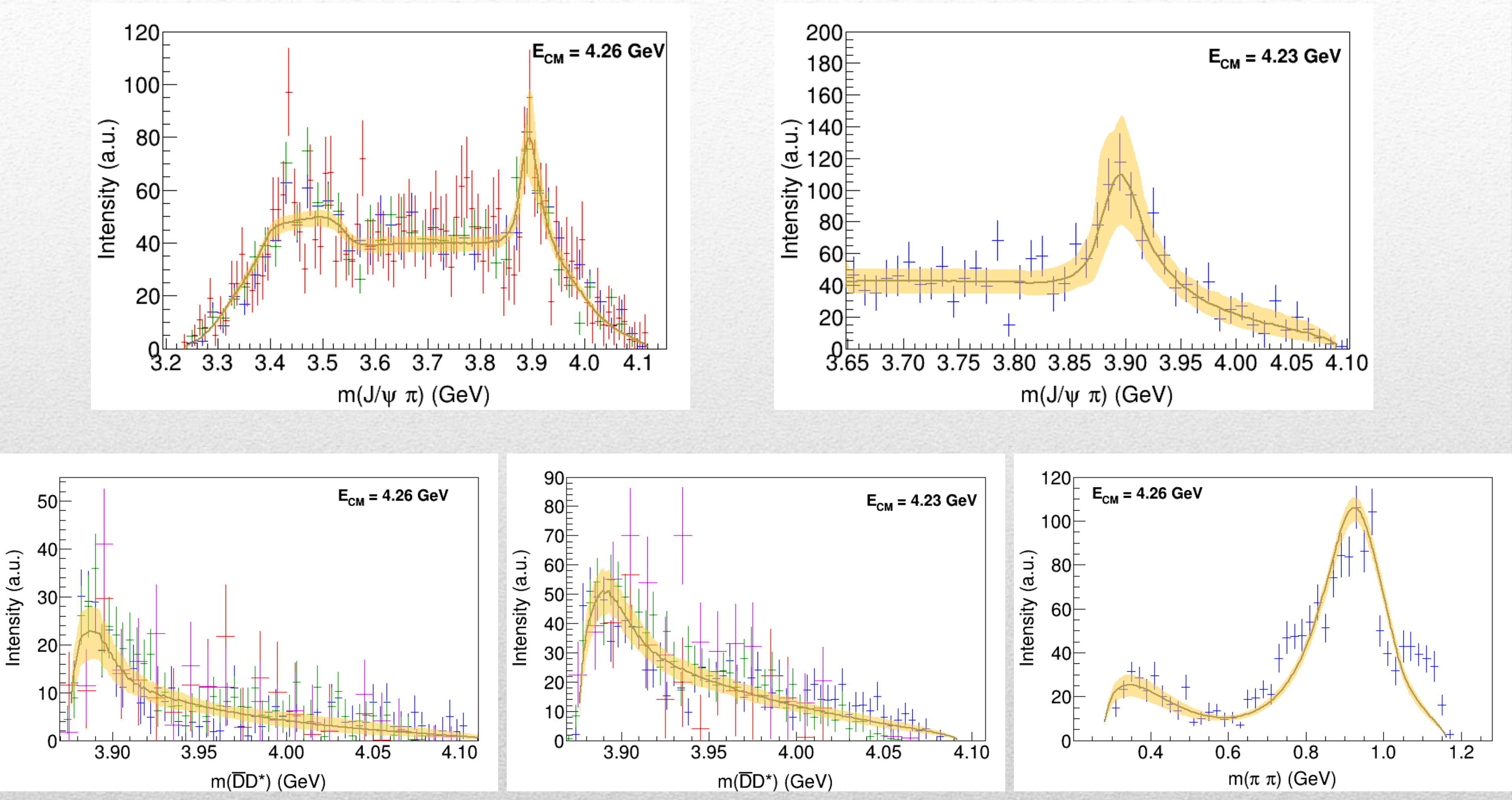
- «III»: the K matrix is $\frac{g_i\,g_j}{M^2 - s}$, this generates a pole in the closest unphysical sheet the rescattering integral is set to zero
- «III+tr.»: same, but with the correct value of the rescattering integral
- «IV+tr.»: the K matrix is constant, this generates a pole in the IV sheet
- «tr.»: same, but the pole is pushed far away by adding a penalty in the χ^2

Singularities and lineshapes

Different lineshapes according to different singularities



Fit: III


E_CM = 4.26 GeV
Intensity (a.u.)
m(J/ψ π) (GeV)
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Intensity (a.u.)
m(D̄D*) (GeV)
E_CM = 4.23 GeV
Intensity (a.u.)
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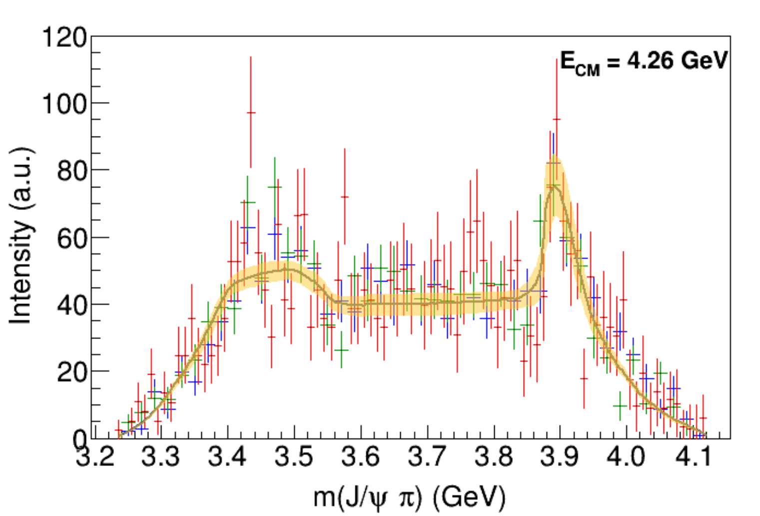
E_CM = 4.26 GeV
Intensity (a.u.)
m(π π) (GeV)

Fit: III+tr.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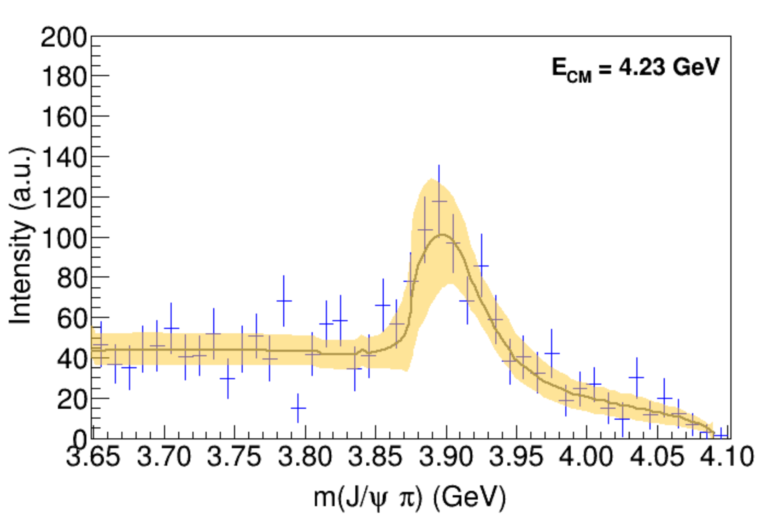
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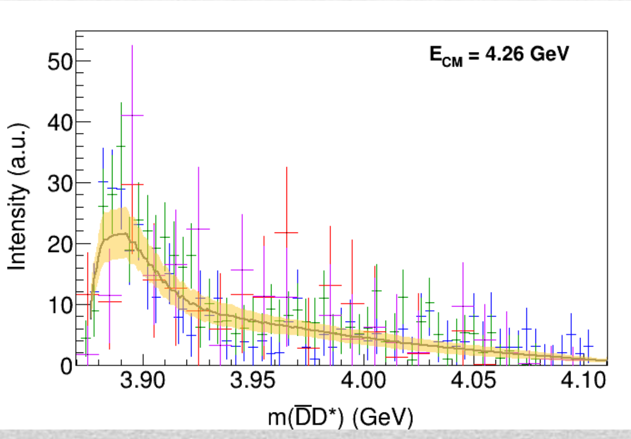
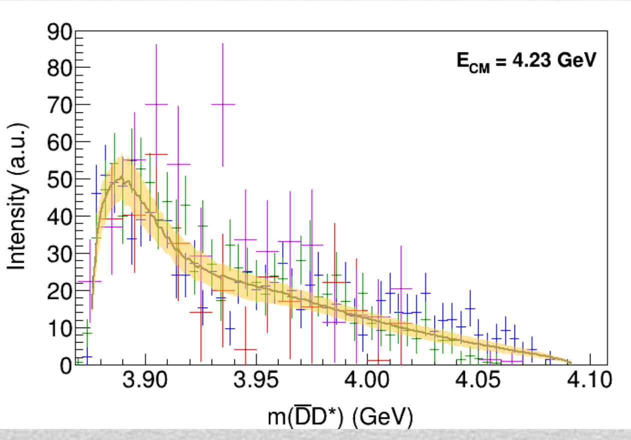

E_CM = 4.26 GeV
Intensity (a.u.)
m(D̄D*) (GeV)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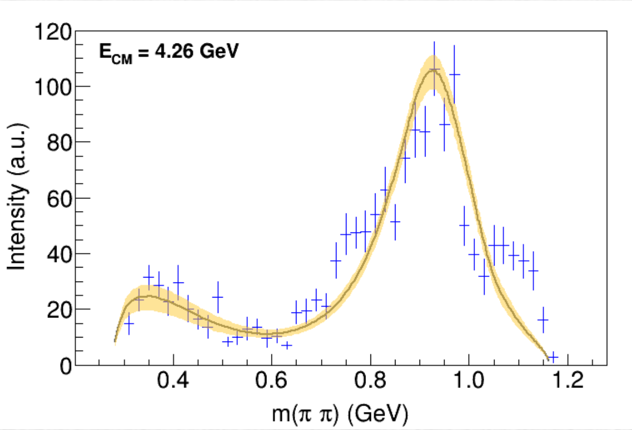
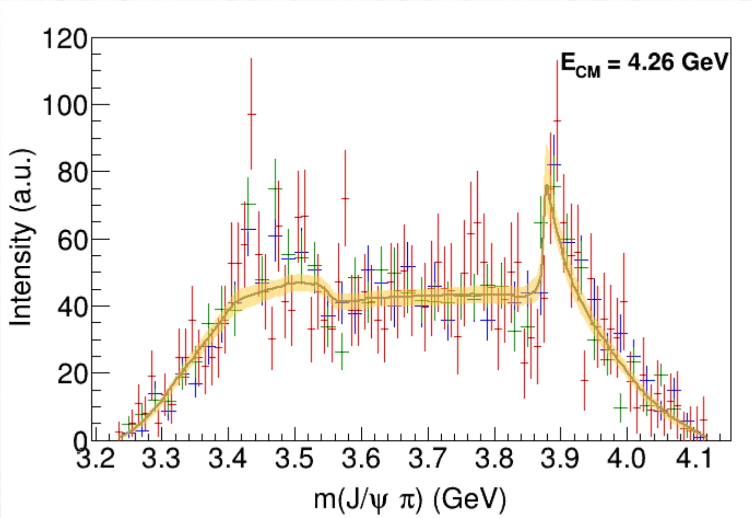
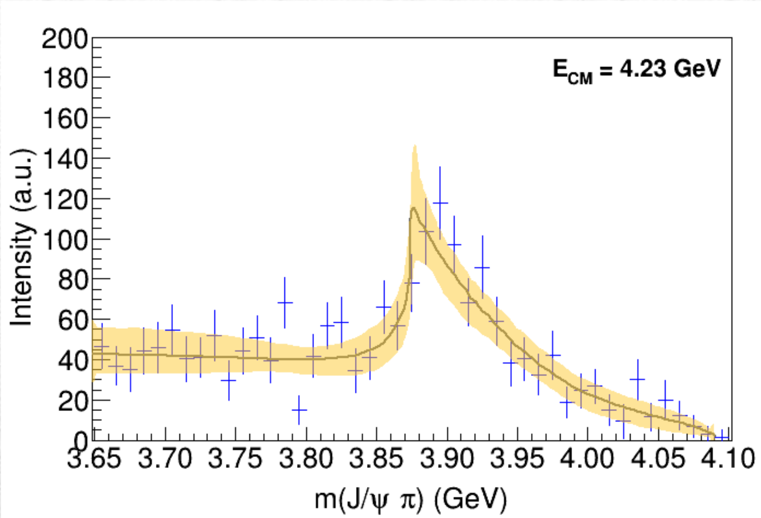

E_CM = 4.26 GeV
Intensity (a.u.)
m(π π) (GeV)

Fit: IV+tr.



E_CM = 4.26 GeV
Intensity (a.u.)
m(J/ψ π) (GeV)



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Intensity (a.u.)
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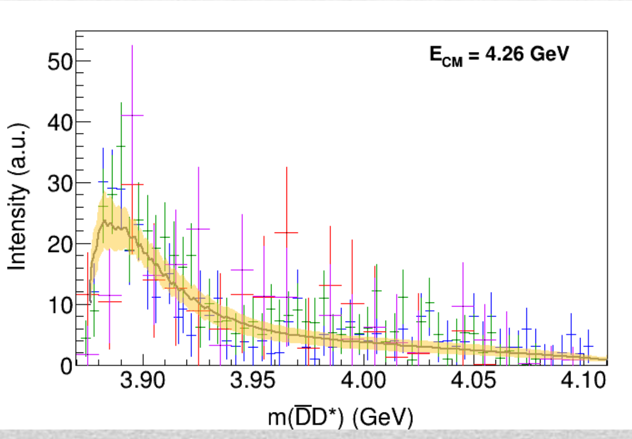
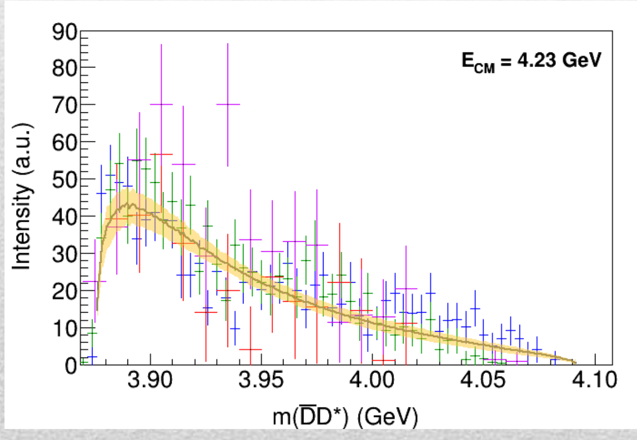


E_CM = 4.26 GeV
Intensity (a.u.)
m(D̄D*) (GeV)



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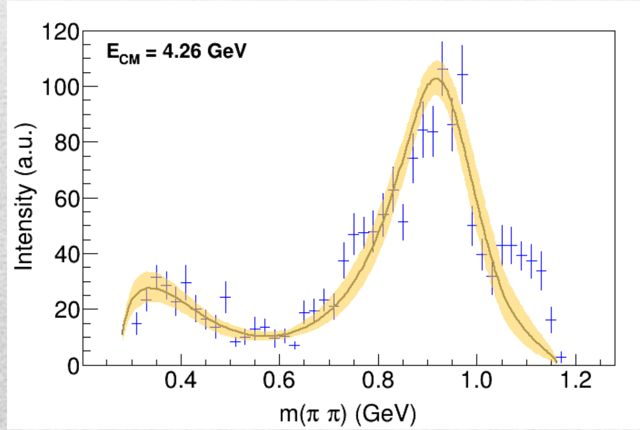
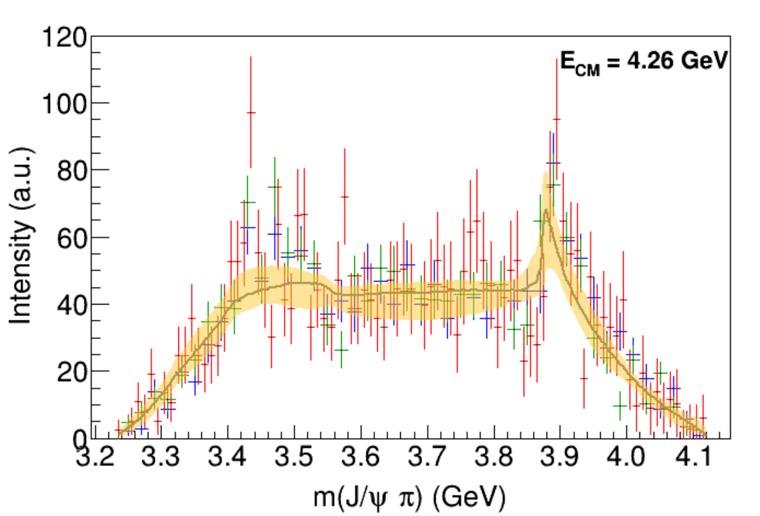
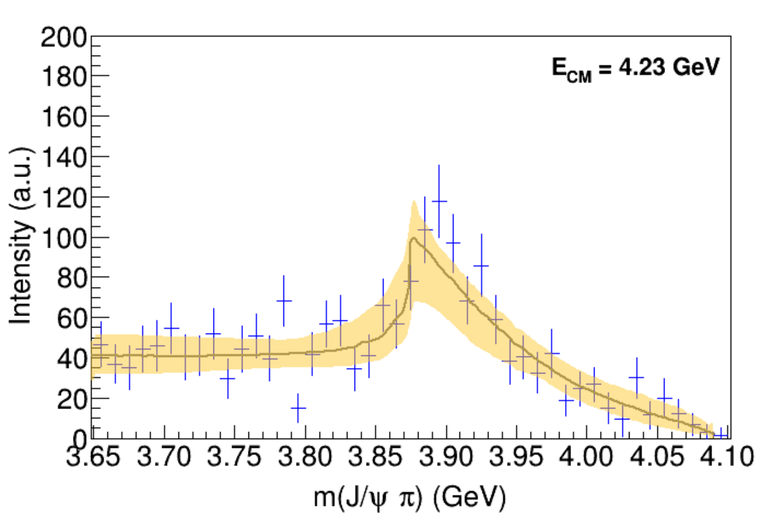


E_CM = 4.26 GeV
Intensity (a.u.)
m(π π) (GeV)

Fit: tr.



E_{CM} = 4.26 GeV
Intensity (a.u.)
m(J/ψ π) (GeV)



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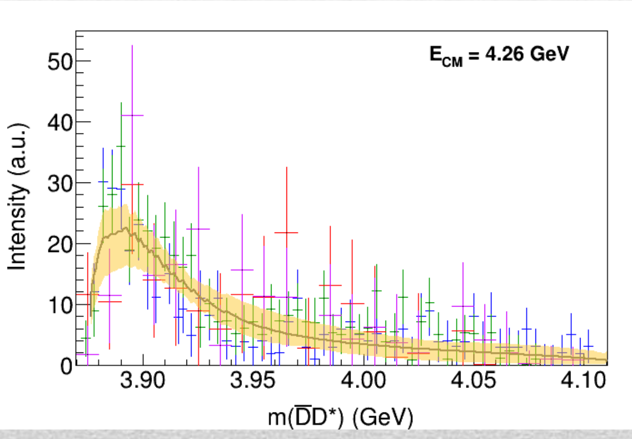
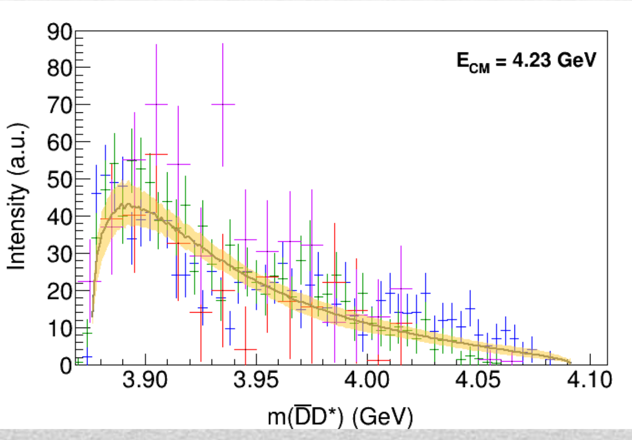


E_{CM} = 4.26 GeV
Intensity (a.u.)
m($\bar{D}D^*$) (GeV)



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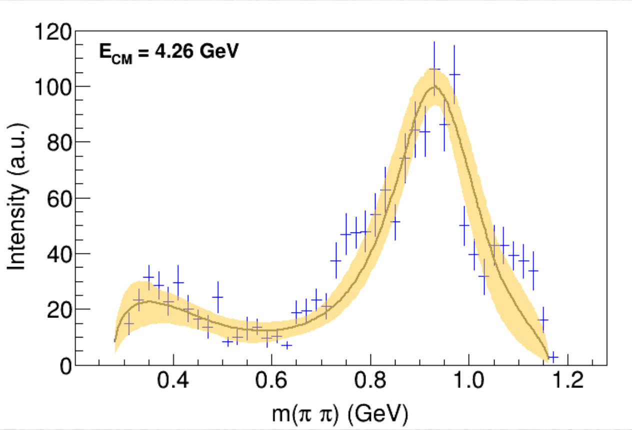
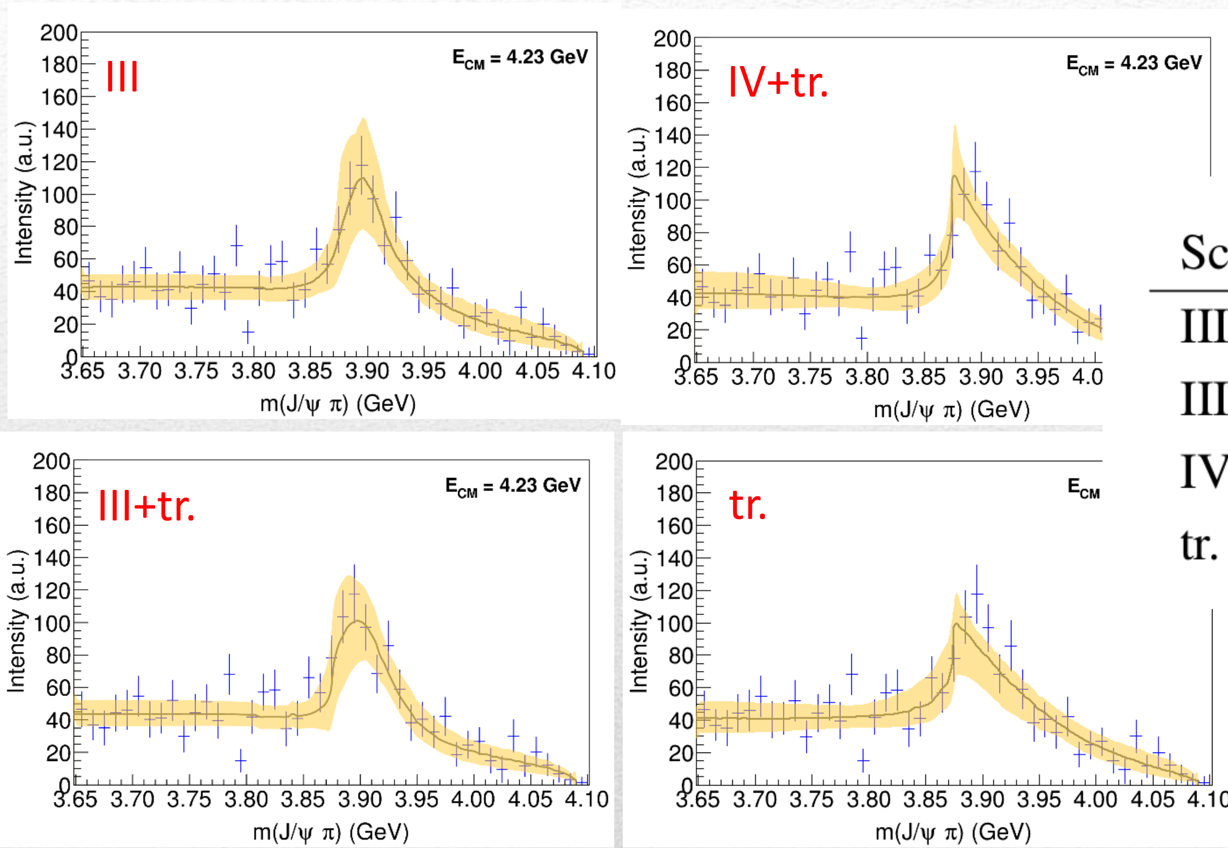


E_{CM} = 4.26 GeV
Intensity (a.u.)
m(π π) (GeV)

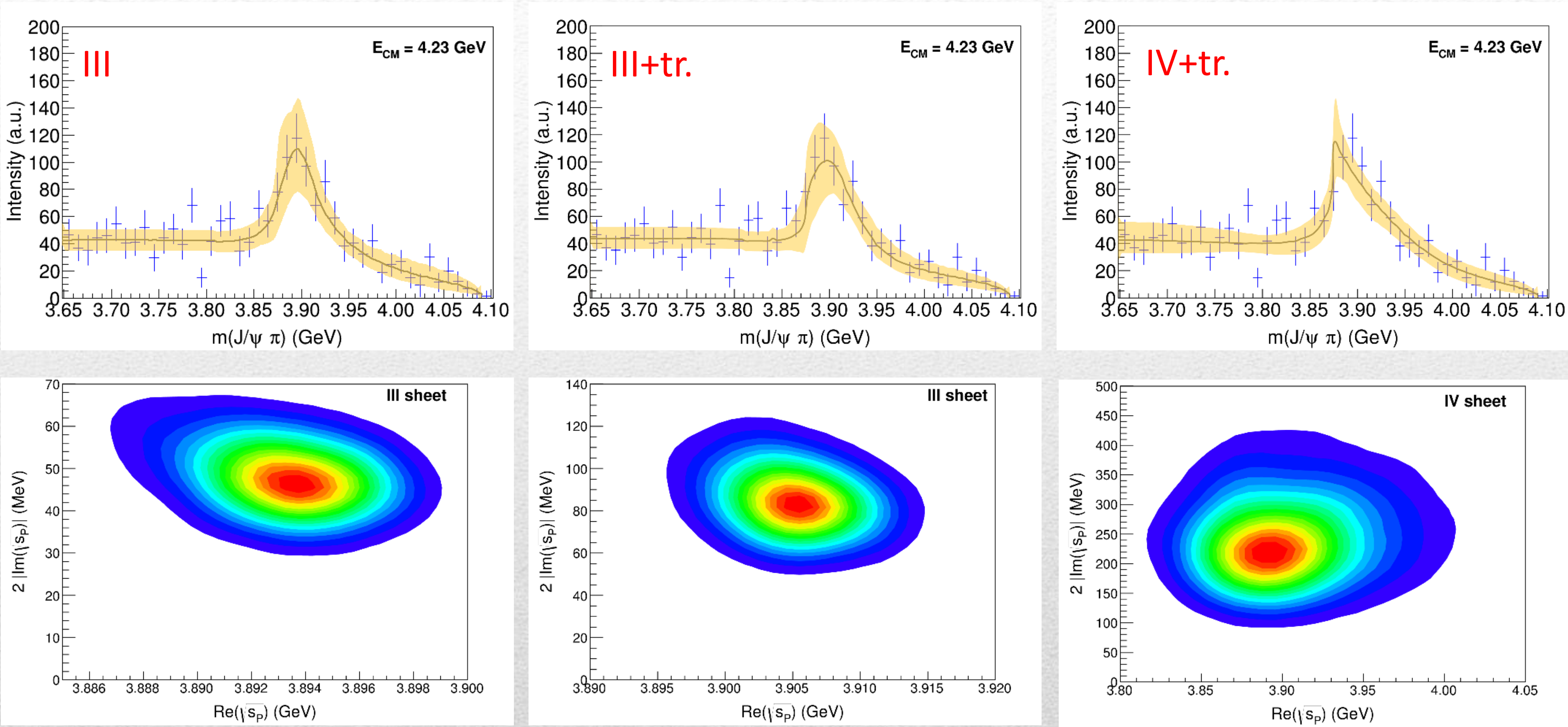
Fit summary



Scenario	χ^2	DOF	χ^2/DOF
III	644	532	1.21
III+tr.	642	532	1.21
IV+tr.	666	532	1.25
tr.	695	532	1.31

Naive loglikelihood ratio test give a $\sim 4\sigma$ significance of the scenario III+tr. over IV+tr., looking at plots it looks too much – better using some more solid test

Pole extraction



Scenario	III+tr.	IV+tr.	tr.
III	1.5σ (1.5σ)	1.5σ (2.7σ)	"2.4σ" ("1.4σ")
III+tr.	–	1.5σ (3.1σ)	"2.6σ" ("1.3σ")
IV+tr.	–	–	"2.1σ" ("0.9σ")

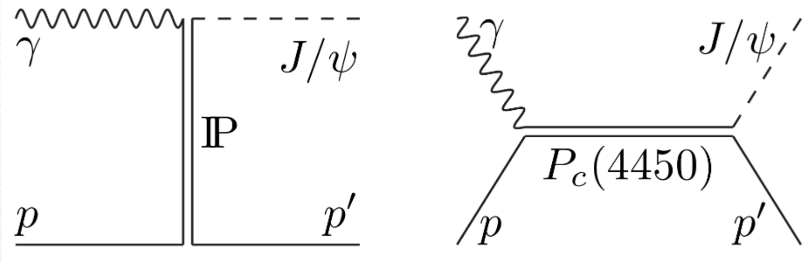
	III	III+tr.	IV+tr.
M (MeV)	$3893.2^{+5.5}_{-7.7}$	3905^{+11}_{-9}	3900^{+140}_{-90}
Γ (MeV)	48^{+19}_{-14}	85^{+45}_{-26}	240^{+230}_{-130}

Not conclusive at this stage

Pentaquark photoproduction

To exclude any rescattering mechanism, we propose to search the $P_c(4450)$ state in photoproduction

We use the (few) existing data and VMD + pomeron inspired bkg to estimate the cross section

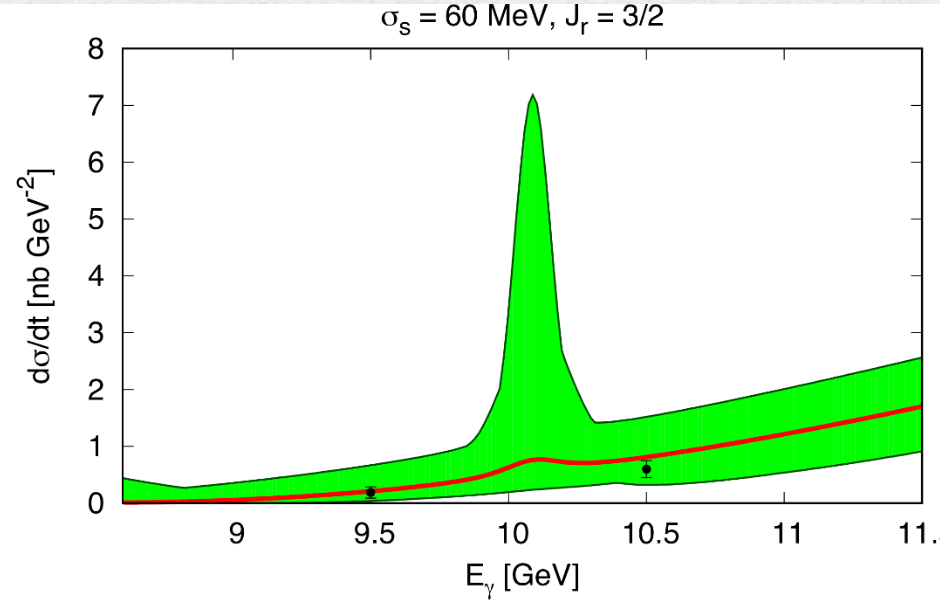


(a) Pomeron exchange (b) Resonant contribution

GlueX data coming soon!

$J^P = (3/2)^-$

σ_s (MeV)	0	60	120
A	$0.156^{+0.029}_{-0.020}$	$0.157^{+0.039}_{-0.021}$	$0.157^{+0.037}_{-0.022}$
α_0	$1.151^{+0.018}_{-0.020}$	$1.150^{+0.018}_{-0.026}$	$1.150^{+0.015}_{-0.023}$
α' (GeV^{-2})	$0.112^{+0.033}_{-0.054}$	$0.111^{+0.037}_{-0.064}$	$0.111^{+0.038}_{-0.054}$
s_t (GeV^2)	$16.8^{+1.7}_{-0.9}$	$16.9^{+2.0}_{-1.6}$	$16.9^{+2.0}_{-1.1}$
b_0 (GeV^{-2})	$1.01^{+0.47}_{-0.29}$	$1.02^{+0.61}_{-0.32}$	$1.03^{+0.49}_{-0.31}$
$\mathcal{B}_{\psi p}$ (95% CL)	≤ 29 %	≤ 30 %	≤ 23 %



Hiller Blin, AP *et al.* (JPAC), PRD94, 034002

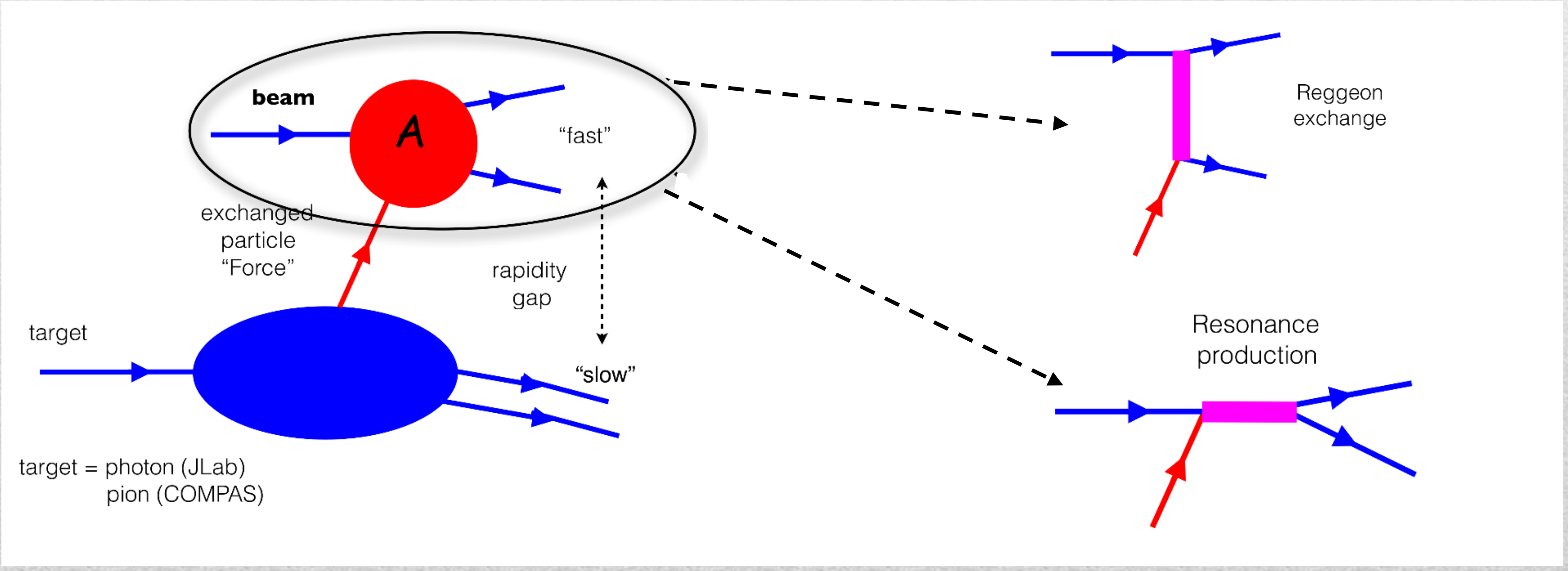
Higher energies: Regge exchange

Resonances are poles in s for fixed l
dominate low energy region

$$A_l \sim \frac{g_1 g_2}{s_p - s}$$

Reggeons are poles in l for fixed s
dominate high energy region

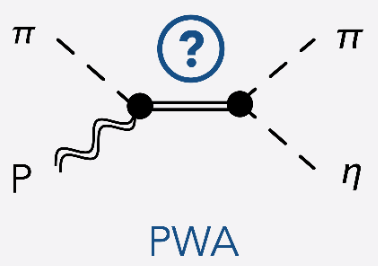
$$A \sim \sum s^l \sim \beta(t)\, s^{\alpha(t)}$$



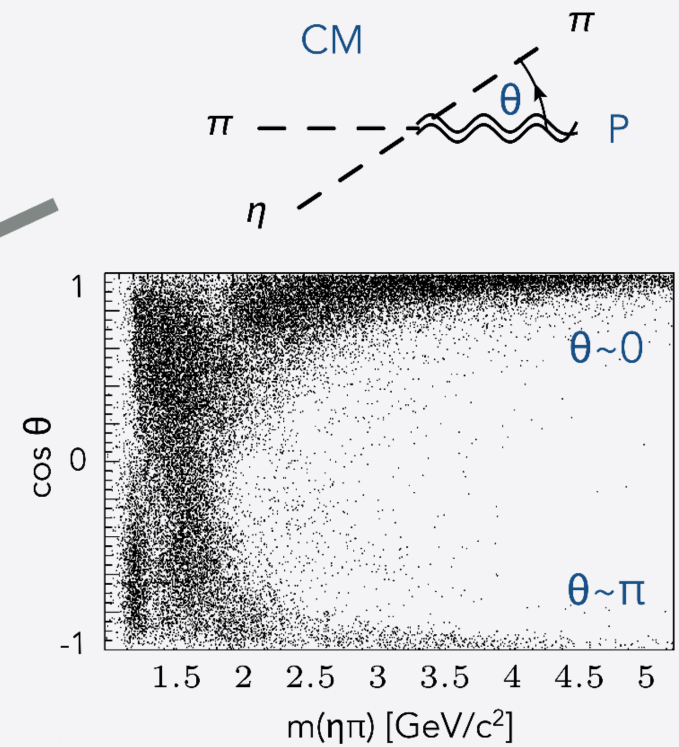
Finite energy sum rules

See J. Nys talk at 12pm

$m(\eta\pi) < 3\ (GeV/c^2)^2$

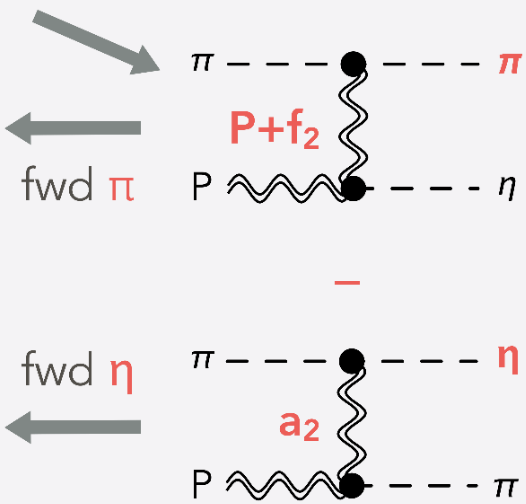


PWA in the low energy region
Resonance extraction



Analytically connected

$m(\eta\pi) \in [5\text{-}6]\ (GeV/c^2)^2$



Regge exchanges at high energy

Searching for resonances in $\eta\pi$

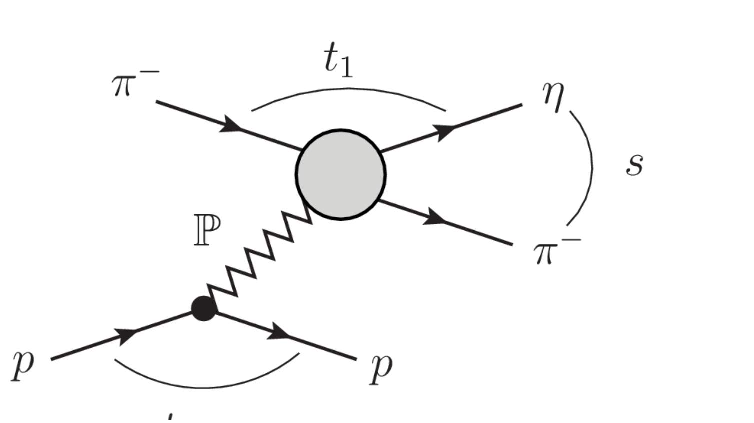
- The $\eta\pi$ system is one of the golden modes for hunting hybrid mesons
- We build the partial waves amplitude according to the N/D method
- We test against the D-wave data, where the a_2 and the a_2' show up

Resonant content

A. Jackura, AP *et al.* (JPAC & COMPASS), 1707.02848

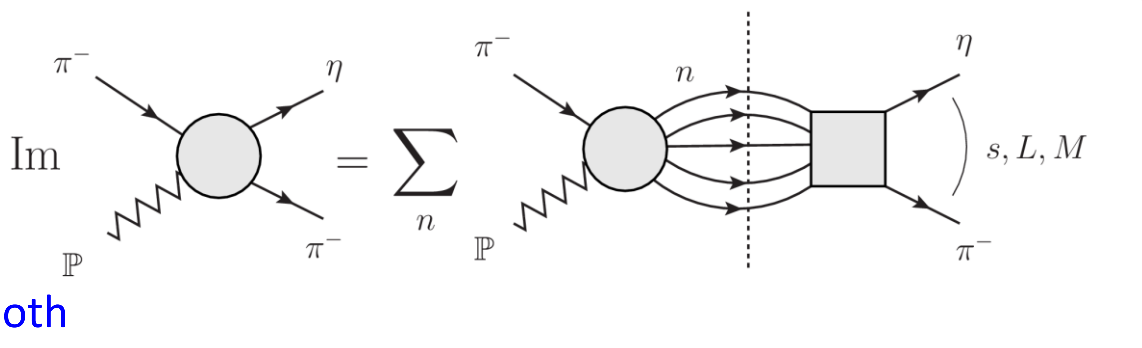
$$D(s) = c_0 - c_1 s - \frac{c_2}{c_3 - s} - \frac{s}{\pi}\int_{s_{th}}^{\infty} ds' \frac{\rho(s')N(s')}{s'(s'-s)}$$

$$a(s) = p^2 q \frac{n(s)}{D(s)} \qquad \rho(s)N(s) = g\,\frac{\lambda^{5/2}(s, m_\eta^2, m_\pi^2)}{(s+s_R)^n}$$

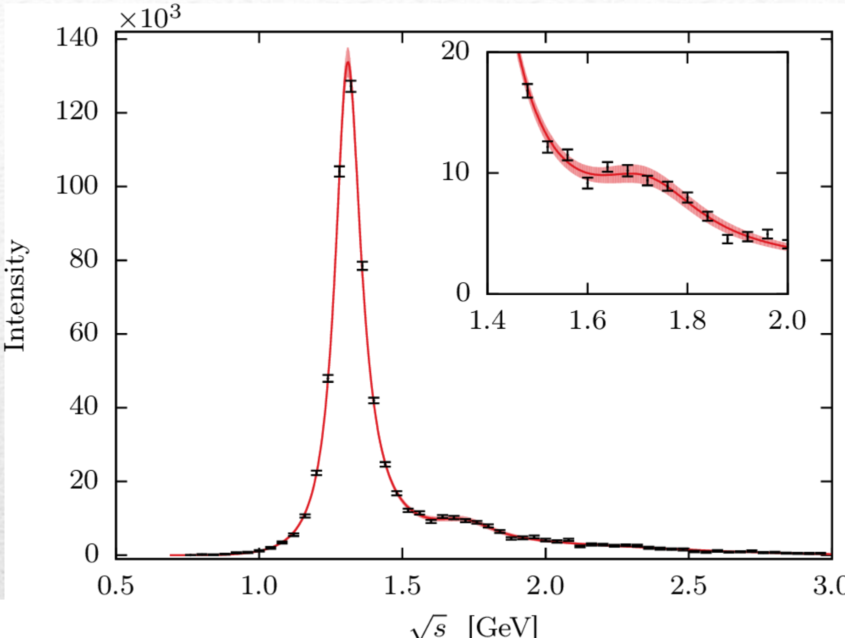


The denominator $D(s)$ contains all the Final State Interactions constrained by unitarity → universal

The numerator $n(s)$ depends on the exchanges → process-dependent, smooth

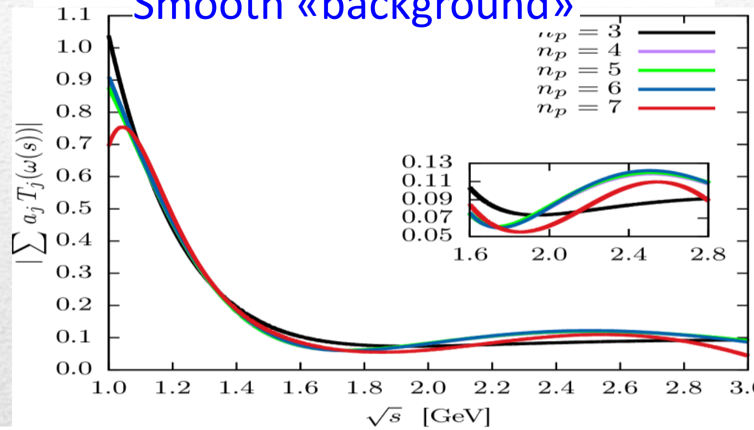


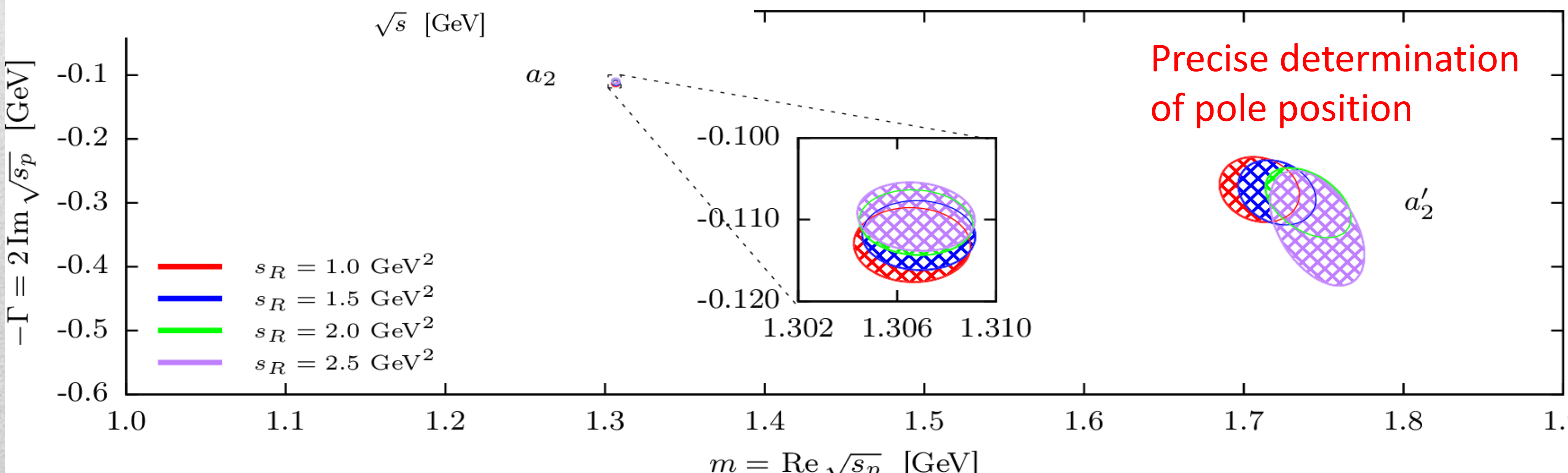
Searching for resonances in $\eta\pi$



$$n(s) = \frac{1}{c_3 - s} \sum^{n_p} a_j \, T_j\left(\frac{s}{s+\Lambda}\right)$$

Smooth «background»

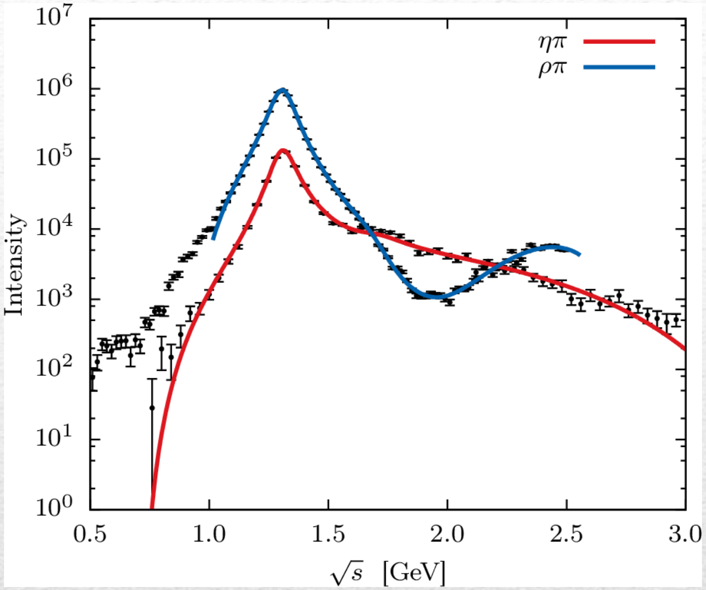




Precise determination of pole position

Searching for resonances in $\eta\pi$

- We implemented the two-channel fit to estimate the systematic dependence on coupled-channel effects
- Other systematic uncertainties include the variation of the number of terms in $n(s)$, and in the barrier factor radius s_R



$$m(a_2) = (1307 \pm 1 \pm 6)\ \text{MeV} \qquad m(a_2') = (1720 \pm 10 \pm 60)\ \text{MeV}$$

$$\Gamma(a_2) = (112 \pm 1 \pm 8)\ \text{MeV} \qquad \Gamma(a_2') = (280 \pm 10 \pm 70)\ \text{MeV}$$

- The coupled channel analysis involving the exotic P-wave is ongoing, as well as the extention to the GlueX production mechanism and kinematics

Conclusions & prospects

- JPAC is a joint effort between theorists and experimentalists to work together to make the best use of the next generation of very precise data taken at JLab and in the world
- We aim at developing new theoretical tools, to get insight on QCD using first principles of QFT (unitarity, analyticity, crossing symmetry, low and high energy constraints,…) to extract the physics out of the data
- Codes are public and available
- Many other ongoing projects (both for meson and baryon spectroscopy, and for high energy observables), with a particular attention to producing complete reaction models for the golden channels in exotic meson searches

Thank you

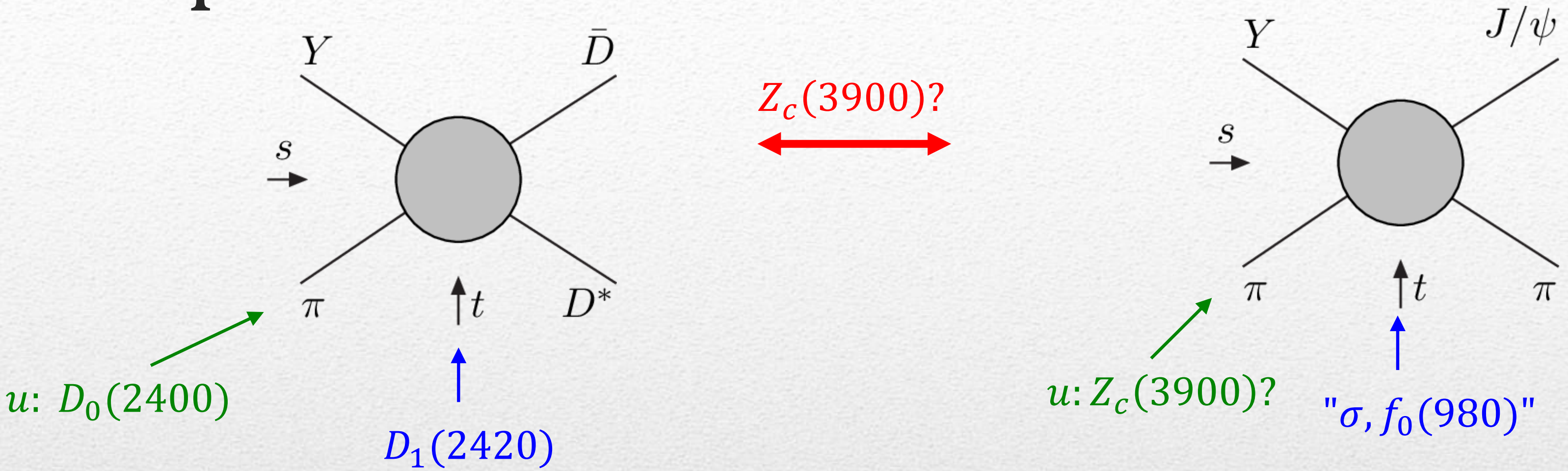
BACKUP

Production

- $>$ 40 Research Papers (Phys.Rev., Phys.Lett, Eur.J. Phys.)
- $\sim$120 Invited Talks and Seminars
- $O(10)$ ongoing analyses
- Summer Schools on Reaction Theory (IU, 2015 and 2017)
- Workshop “Future Directions in Hadron Spectroscopy” (JLab, 2014 and UNAM 2017)

FESR	V. Mathieu *et al.*,	arXiv:1708.07779
$\pi N \to \eta \pi N$	A. Jackura *et al.*,	arXiv:1707.02848
$\gamma N \to \eta N$ vs. $\to \eta' N$	V. Mathieu *et al.*,	arXiv:1704.07684
$Z_c(3900)$	A. Pilloni *et al.*,	PLB772, 200
$\gamma N \to \eta N$	J. Nys *et al.*,	PRD95, 034014
$\gamma p \to J/\psi p$	A. Blin *et al.*,	PRD94, 034002
$K N \to K N$	C. Fernandez-Ramirez *et al.*,	PRD93, 034029; PRD93, 074015
$\gamma p \to \pi^0 p$	V. Mathieu *et al.*,	PRD92, 074013
$\pi N \to \pi N$	V. Mathieu *et al.*,	PRD92, 074004
$\eta \to \pi^+ \pi^- \pi^0$	P. Guo *et al.*,	PRD92, 054016; PLB771, 497
$\omega, \phi \to \pi^+ \pi^- \pi^0$	I. Danilkin *et al.*,	PRD91, 094029
$\gamma p \to K^+ K^- p$	M. Shi *et al.*,	PRD91, 034007

Amplitude model



$$f_i(s,t,u) = 16\pi \sum_{l=0}^{L_{\max}} (2l+1)\left(a_{l,i}^{(s)}(s) P_l(z_s) + a_{l,i}^{(t)}(t) P_l(z_t) + a_{l,i}^{(u)}(u) P_l(z_u)\right)$$

Khuri-Treiman

$$f_{0,i}(s) = \frac{1}{32\pi}\int_{-1}^{1} dz_s\, f_i\left(s, t(s,z_s), u(s,z_s)\right) = a_{0,i}^{(s)} + \frac{1}{32\pi}\int_{-1}^{1} dz_s \left(a_{0,i}^{(t)}(t) + a_{0,i}^{(u)}(u)\right) \equiv a_{0,i}^{(s)} + b_{0,i}(s)$$

$$f_{l,i}(s) = \frac{1}{32\pi}\int_{-1}^{1} dz_s\, P_l(z_s)\left(a_{0,i}^{(t)}(t) + a_{0,i}^{(u)}(u)\right) \equiv b_{l,i}(s) \quad \text{for } l > 0. \quad f_{0,i}(s) = b_{0,i}(s) + \sum_j t_{ij}(s) \frac{1}{\pi}\int_{s_j}^{\infty} ds' \frac{\rho_j(s') b_{0,j}(s')}{s' - s},$$

$$f_i(s,t,u) = 16\pi \left[a_{0,i}^{(t)}(t) + a_{0,i}^{(u)}(u) + \sum_j t_{ij}(s)\left(c_j + \frac{s}{\pi}\int_{s_j}^{\infty} ds' \frac{\rho_j(s') b_{0,j}(s')}{s'(s'-s)}\right)\right],$$

Strategy

AP *et al.* (JPAC), arXiv:1612.06490

- We fit the following invariant mass distributions:
  - BESIII PRL110, 252001 $J/\psi\,\pi^+, J/\psi\,\pi^-, \pi^+\pi^-$ at $E_{CM} = 4.26$ GeV
  - BESIII PRL110, 252001 $J/\psi\,\pi^0$ at $E_{CM} = 4.23, 4.26,$ ~~4.36~~ GeV
  - BESIII PRD92, 092006 $\overline{D^0}D^{*+}, \overline{D^{*0}}D^+$ (double tag) at $E_{CM} = 4.23, 4.26$ GeV
  - BESIII PRL115, 222002 $\overline{D^0}D^{*0}, \overline{D^{*0}}D^0$ at $E_{CM} = 4.23, 4.26$ GeV
  - ~~BESIII PRL112, 022001 $\overline{D^0}D^{*+}, \overline{D^{*0}}D^+$ (single tag) at $E_{CM} = 4.26$ GeV~~
  - ~~Belle PRL110, 252002 $J/\psi\,\pi^\pm$ at $E_{CM} = 4.26$ GeV~~
  - ~~CLEO-c data PLB727, 366 $J/\psi\,\pi^\pm, J/\psi\,\pi^0$ at at $E_{CM} = 4.17$ GeV~~

- Published data are not efficiency/acceptance corrected,
  → we are not able to give the absolute normalization of the amplitudes

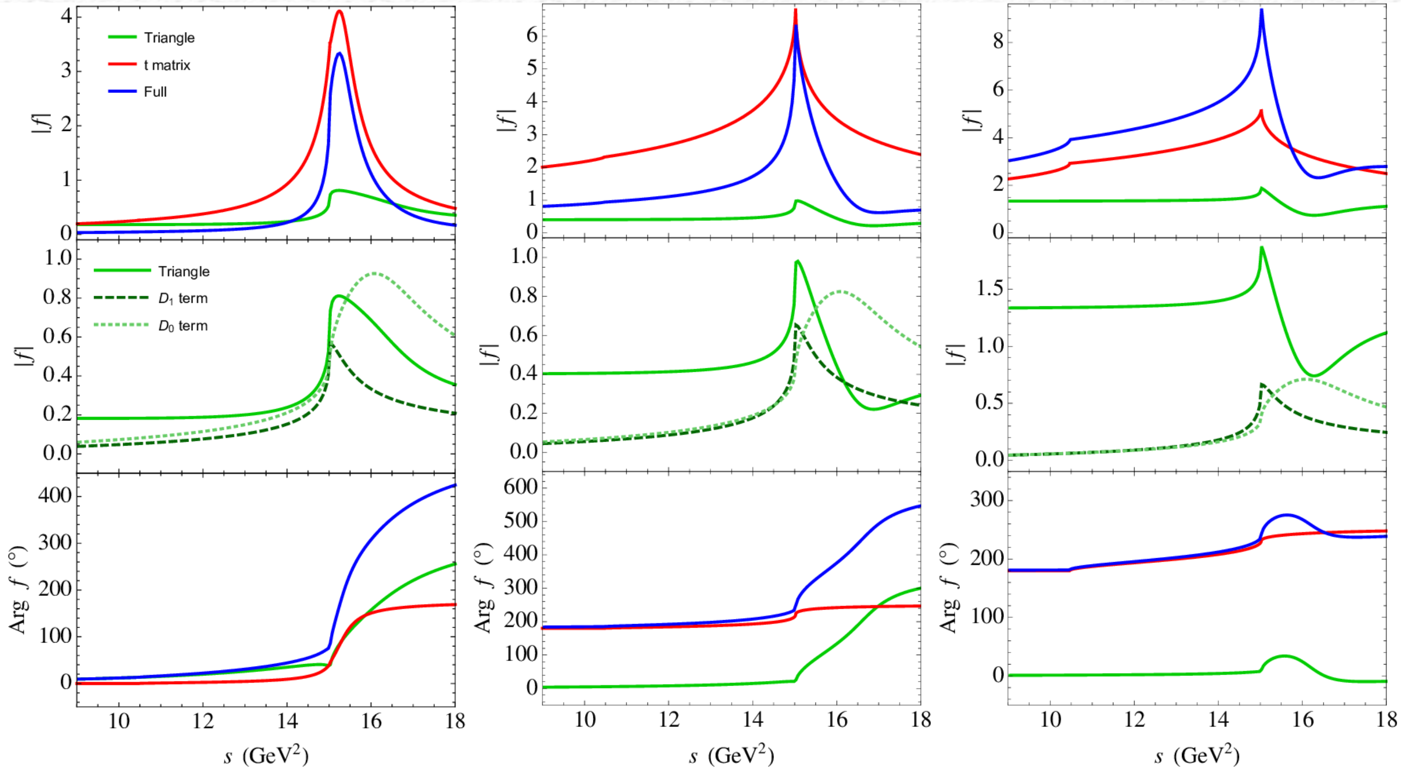
- No given dependence on E_{CM} is assumed – the couplings at different E_{CM} are independent parameters

Strategy

AP *et al.* (JPAC), PLB772, 200

- Reducible (incoherent) backgrounds are pretty flat and do not influence the analysis, except the peaking background in $\overline{D^0}D^{*0}$, $\overline{D^{*0}}D^0$ (subtracted)
- Some information about angular distributions has been published, but it's not constraining enough → we do not include in the fit
- Because of that, we approximate all the particles to be scalar – this affects the value of couplings, which are not normalized anyway – but not the position of singularities. This also limits the number of free parameters

Lineshapes at 4260



(a) III+tr. (b) IV+tr. (c) tr.

Figure 7: Interplay of scattering amplitude poles and triangle singularity to reconstruct the peak. We focus on the $J/\psi\,\pi$ channel, at $E_{CM} = 4.26$ GeV. The red curve is the t_{12} scattering amplitude, the green curve is the $c_1 + H(s, D_1) + +H(s, D_0)$ term in Eq. (9), and the blue curve is the product of the two. The upper plots show the magnitudes of these terms, the lower plots the phases. The middle row shows the contributions to the unitarized term due to the D_1 (dashed) and the D_0 (dotted). Only for D_1 the singularity is close enough to the physical region to generate a large peak. (a) The pole on the III sheet generates a narrow Breit-Wigner-like peak. The contribution of the triangle is not particularly relevant. (b) The sharp cusp in the scattering amplitude is due to the IV sheet pole close by; the triangle contributes to make the peak sharper. (c) The scattering amplitude has a small cusp due to the threshold factor, and the triangle is needed to make it sharp enough to fit the data.

Lineshapes at 4230

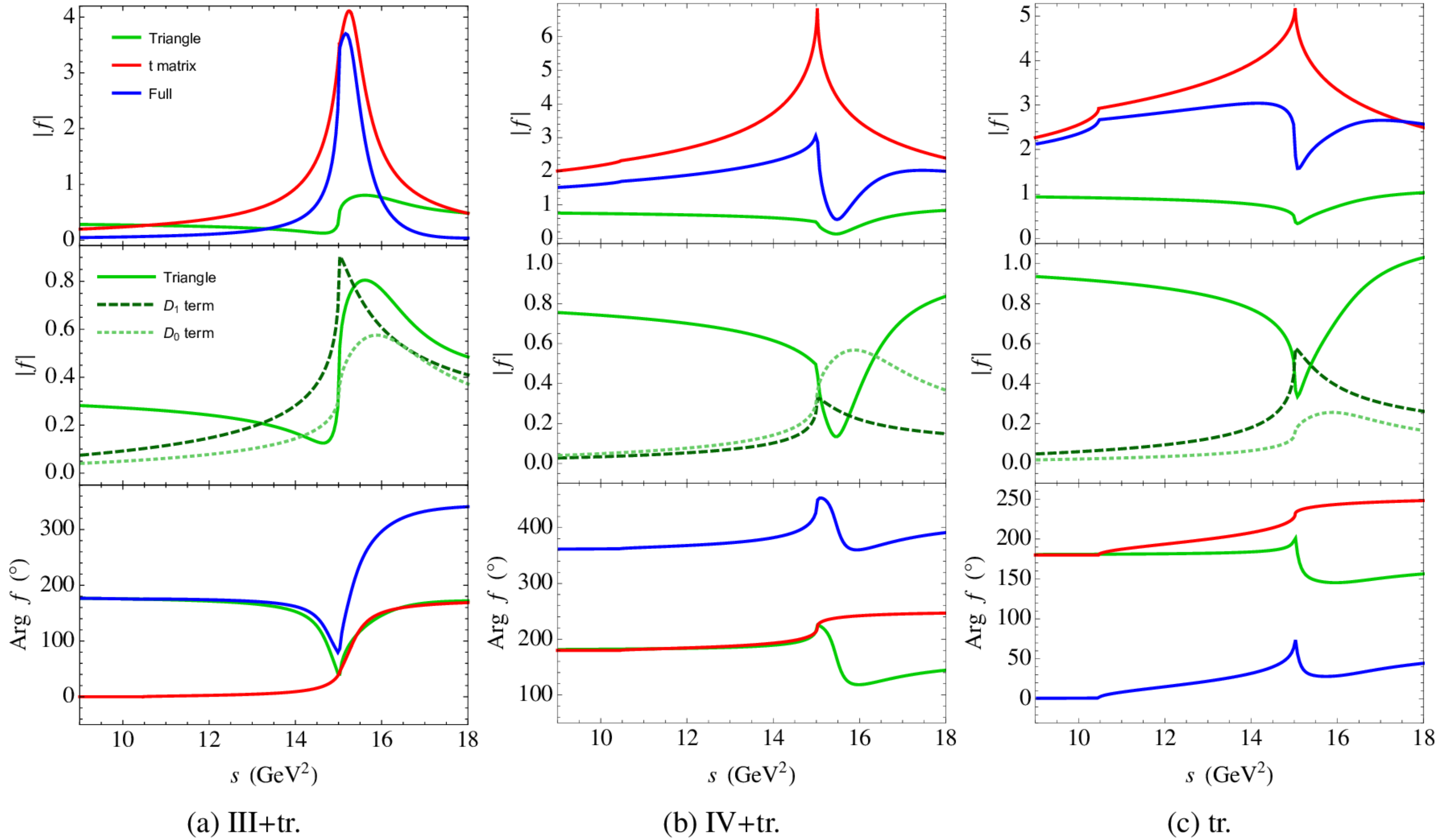
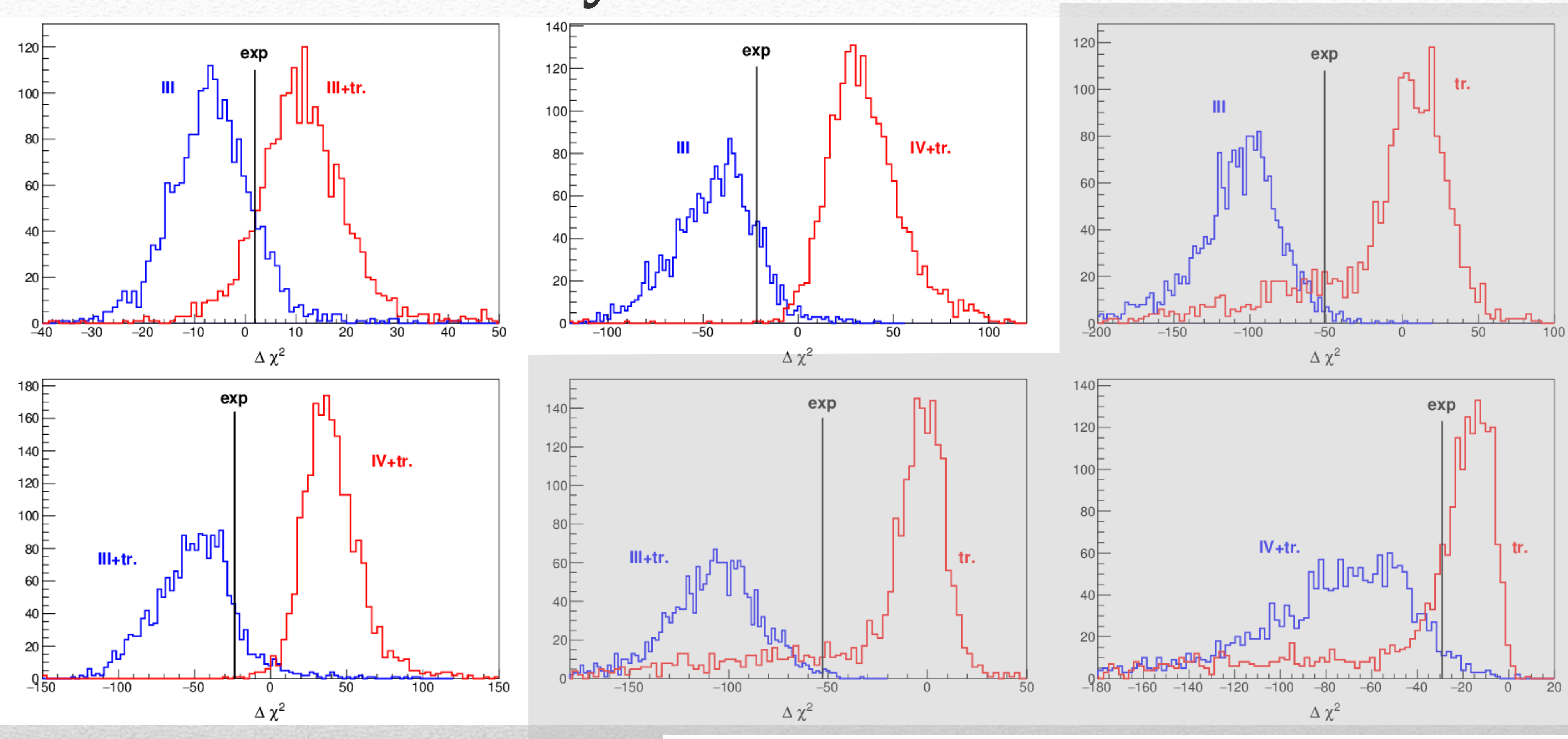


Figure 8: Same as Figure 7, but for $E_{CM} = 4.23$ GeV.

Statistical analysis



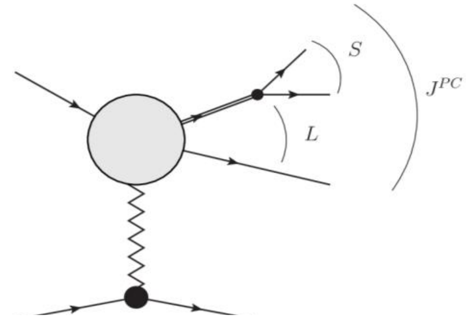
Toy experiments according to the different hypotheses, to estimate the relative rejection of various scenarios

Scenario	III+tr.	IV+tr.	tr.
III	1.5σ (1.5σ)	1.5σ (2.7σ)	"2.4σ" ("1.4σ")
III+tr.	–	1.5σ (3.1σ)	"2.6σ" ("1.3σ")
IV+tr.			"2.1σ" ("0.9σ")

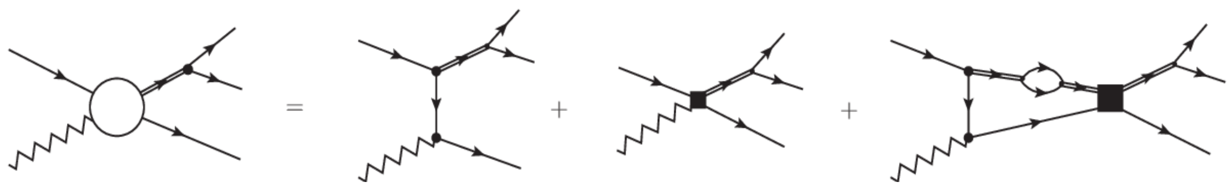
Not conclusive at this stage

PWA of 3π sytem

We start from 2^{-+}, long standing puzzle about $\pi_2(1670) - \pi_2(1880)$ interplay



$$F_{LS}(s) = b_{LS}(s) + h_L \bar{T}(s) c_{L'S'} + \frac{h_L \bar{T}(s)}{\pi} \int_{s_{\text{th}}}^{\infty} \frac{\rho(s') b_{L'S'}(s') h_L(s')}{s' - s - i0} \mathrm{d}s'$$



- The rescattering (Unitarisation) term has to be added to preserve usitarity.
- Shape of the background is fixed by projections of one-pion-exchange diagram
- Fit parameters are strengths of background for each channel, production constants c_{LS} and K-matrix parameters.

Details of one-pion-exchange amplitude calculations

- Pomeron trajectory $(s/s_0)^{\alpha(t)}$, $s_0 = 1\,\text{GeV}^2$, $\alpha(t) = 1$.
- Pion propagator is not "reggeized"
- Proton spin and structure is neglected
- Isobar decay amplitude is taken out, remaining isobar mass dependence is smeared out.

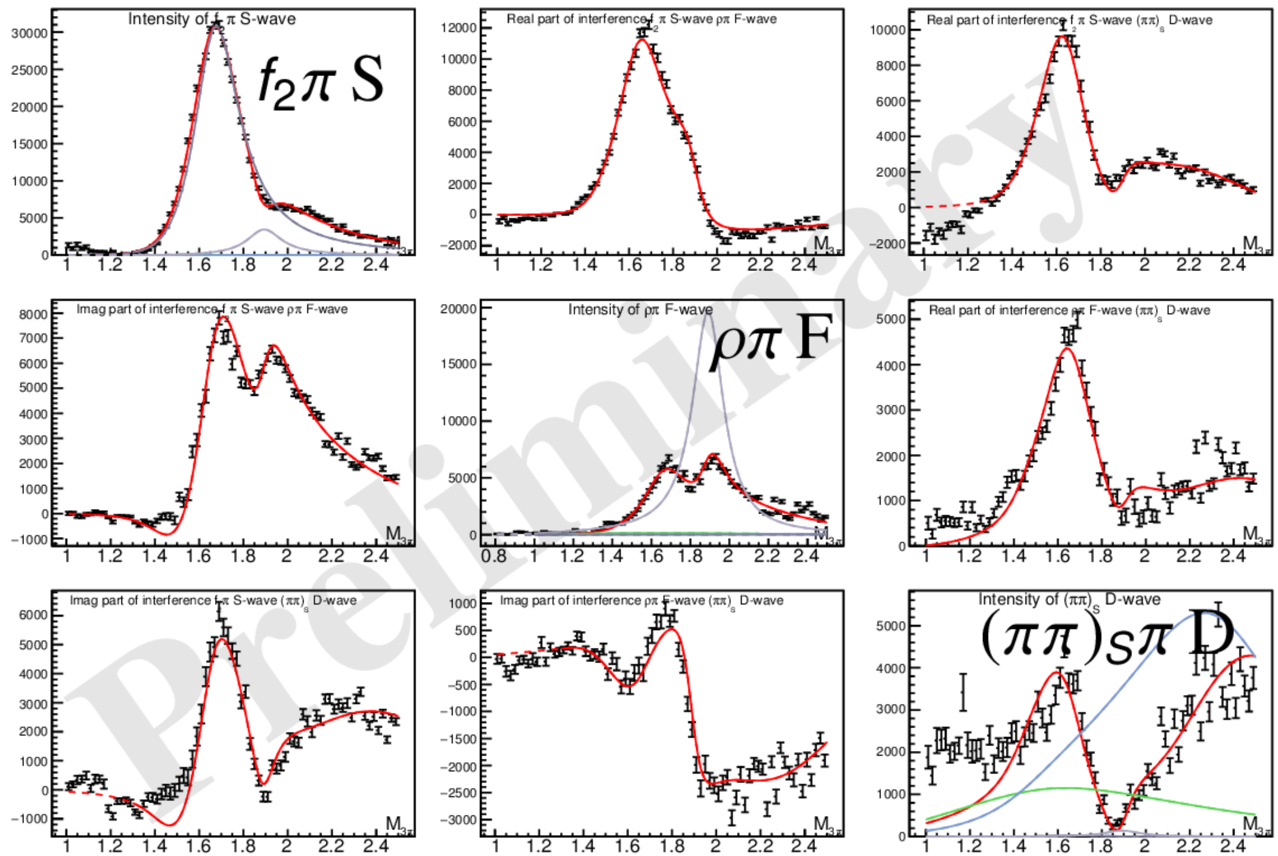
A. Jackura, M. Mikhasenko (JPAC), in progress

PWA of 3π sytem

Model-II, 3 waves fit

$0.12\,\text{GeV}^2 < t' < 0.26\,\text{GeV}^2$, 3 poles, unitarized background

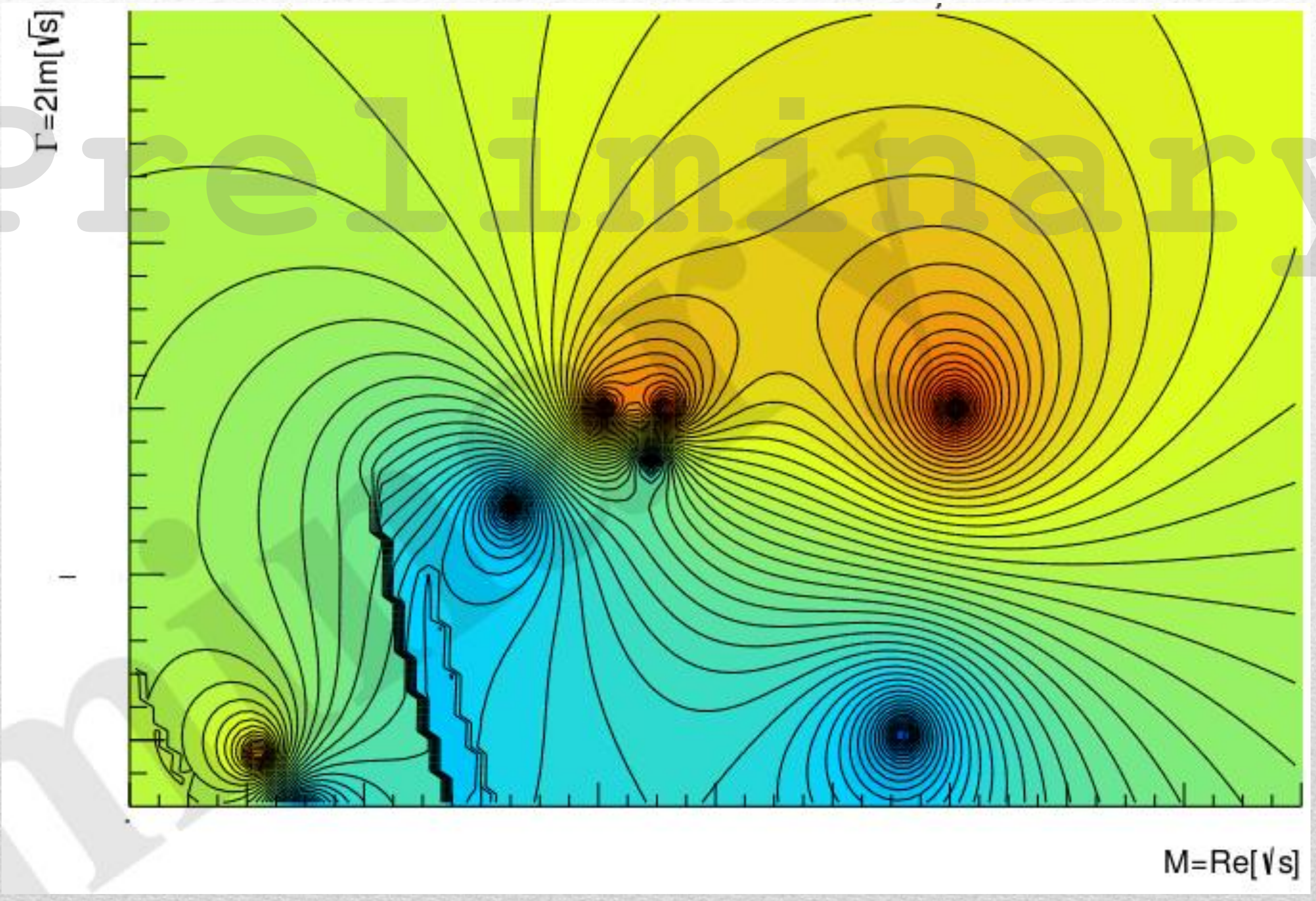
Spin-density matrix: Intensity, Real and Imaginary part of intergerences.



A. Jackur

PWA of 3π sytem

We start from 2^{-+}, long standing puzzle about $\pi_2(1670) - \pi_2(1880)$ interplay

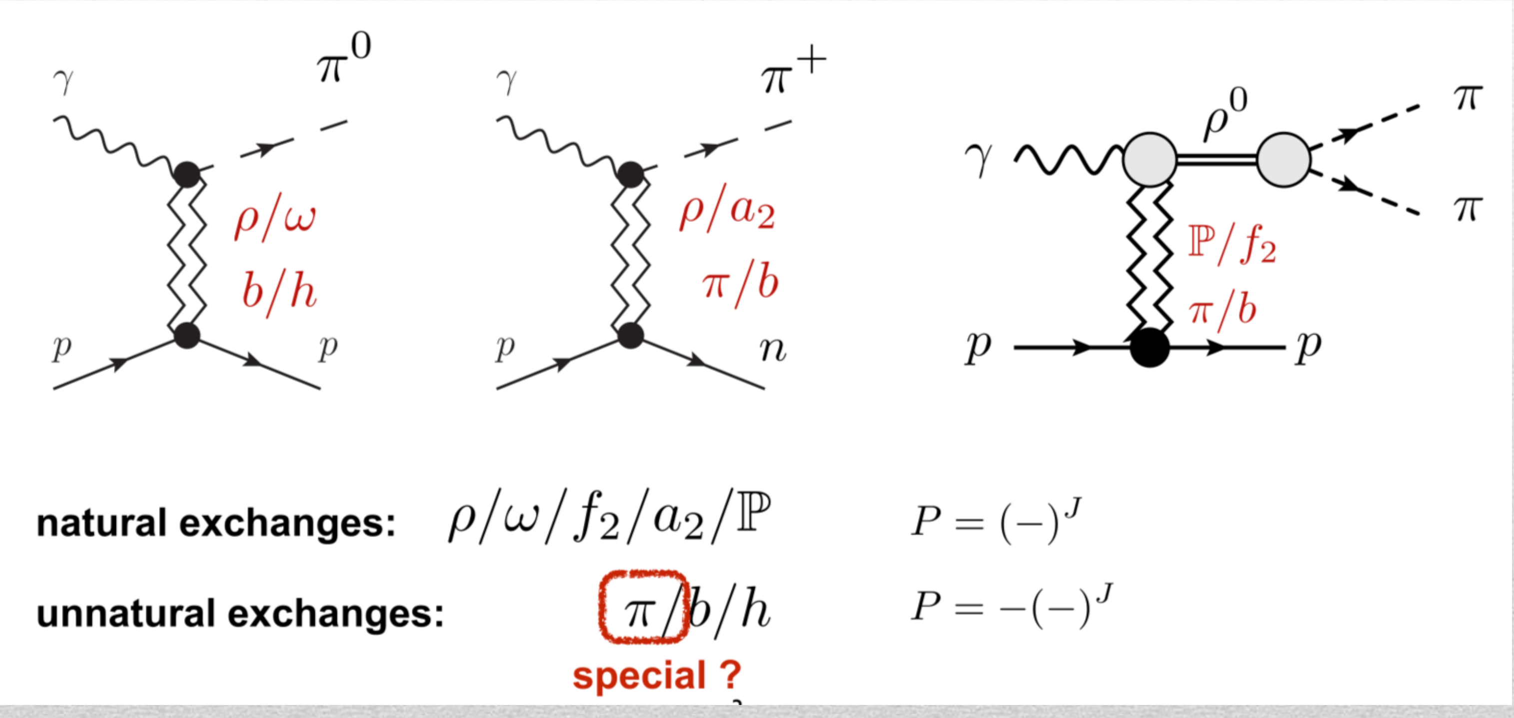


A. Jackura, M. Mikhasenko (JPAC), in progress

π, ρ photoproduction

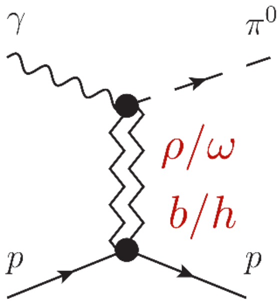
Test factorization on the simplest cases

1. Neutral pion photoproduction
2. Charged pion photoproduction
3. Rho meson photoproduction



$\gamma\, p \to \pi^0\, p$

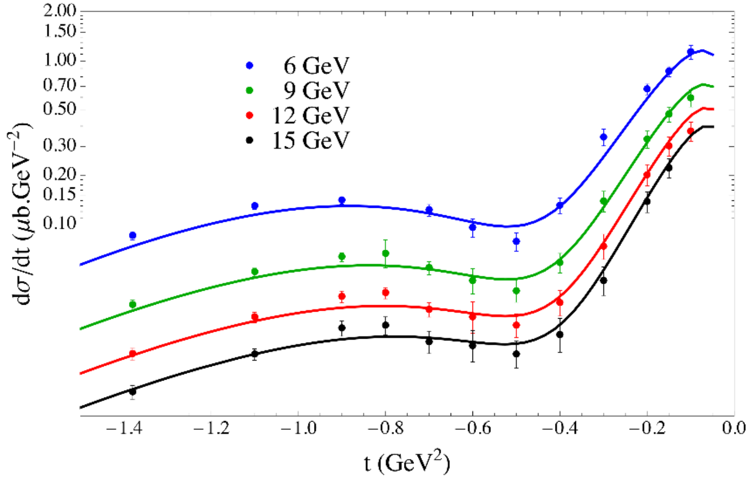
Model based on factorization
with parameters fitted

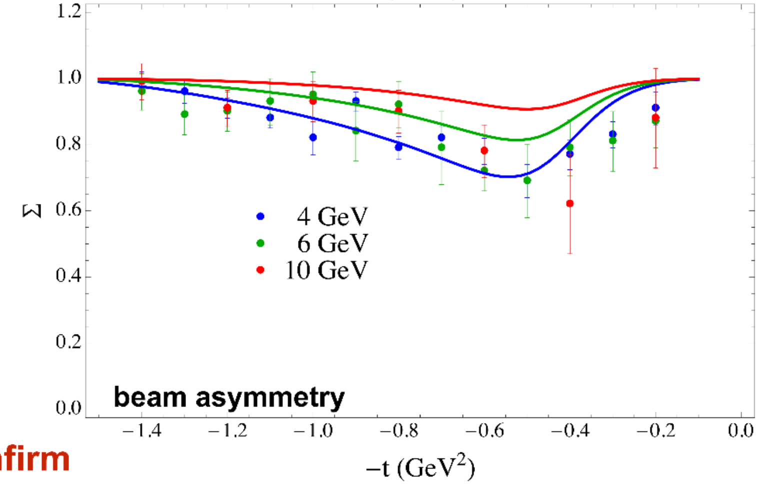


$$\Sigma = \frac{\sigma_\perp - \sigma_\parallel}{\sigma_\perp + \sigma_\parallel} = \frac{|\rho+\omega|^2 - |b+h|^2}{|\rho+\omega|^2 + |b+h|^2}$$

axial-vector exchanges strength decreases with energy

More precise data@JLAB could confirm



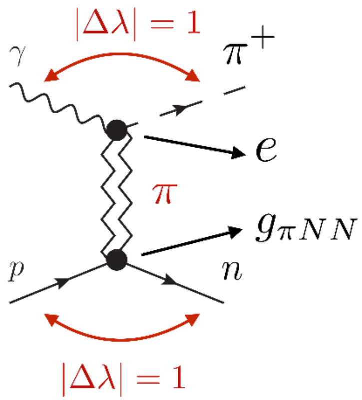


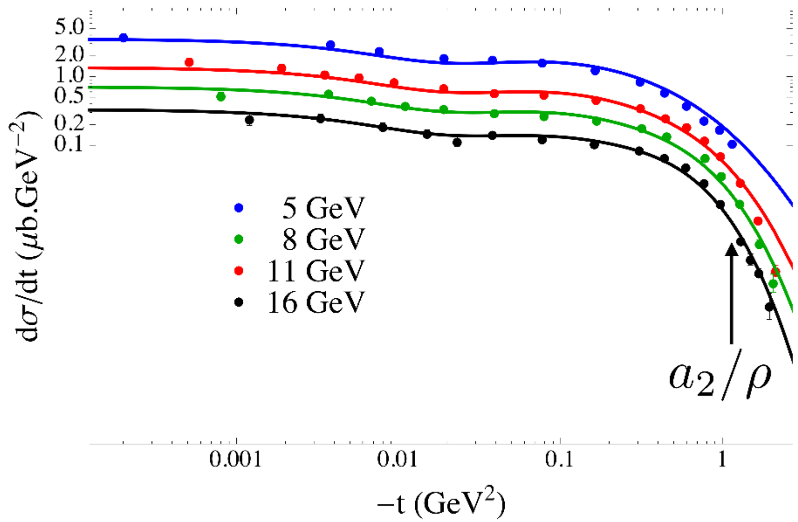
Mathieu *et al.* (JPAC), PRD92, 074013

$\gamma\, p \to \pi^+\, n$

Pion dominate very small |t| :

[Boyarski et al. 1968]





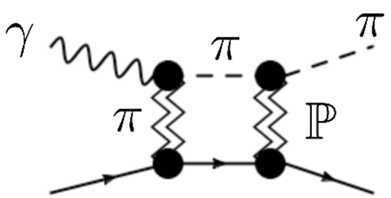
Factorization of Regge residues:
$(\lambda_\gamma, \lambda_\pi) = (1, 0)$ **and**

$$(\lambda_p, \lambda_n) = \left(-\frac{1}{2}, +\frac{1}{2}\right) \qquad A^{10}_{-\frac{1}{2}\frac{1}{2}} \propto \frac{-t}{m_\pi^2 - t}$$

$$(\lambda_p, \lambda_n) = \left(+\frac{1}{2}, -\frac{1}{2}\right) \qquad A^{10}_{\frac{1}{2}-\frac{1}{2}} \propto \frac{-t}{m_\pi^2 - t}$$

$$|(\lambda_\gamma - \lambda_p) - (\lambda_\pi - \lambda_{p'})| = 0$$

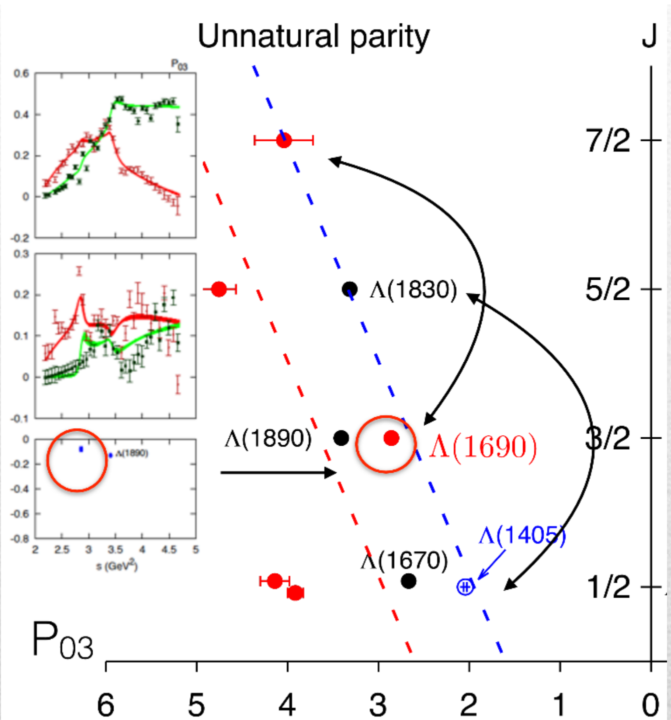
William's Poor man absorption: $\to \dfrac{-m_\pi^2}{m_\pi^2 - t}$

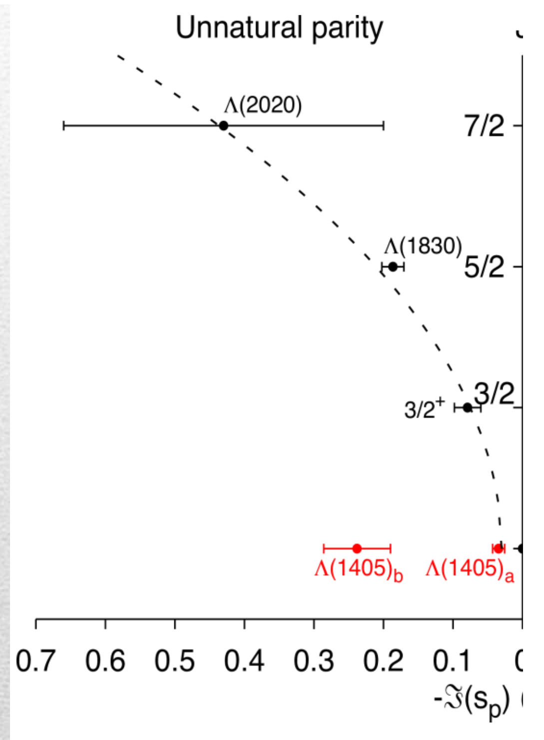


Mathieu (JPAC), in progress

KN scattering and the $\Lambda(1405)$

Coupled-channel K matrix model (up to 13 channels per partial wave), analyticity in angular momentum enforced, fit to KSU partial waves

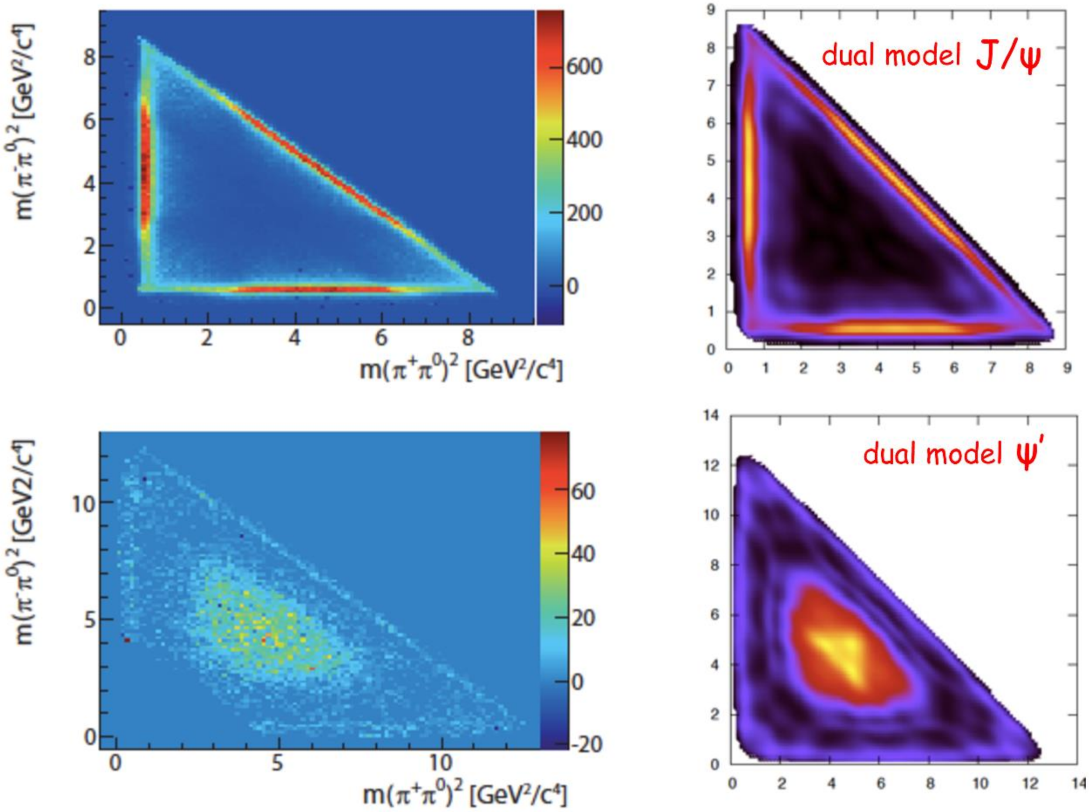




One of the $\Lambda(1405)$ poles is out of the trajectory
→ non 3-q state

Fernandez-Ramirez *et al*. (JPAC), PRD93, 034029
Fernandez-Ramirez *et al*. (JPAC), PRD93, 074015

$\psi^{(\prime)} \to \pi^+\pi^-\pi^0$ within dual models



$$A(s,t) = \frac{\Gamma(-J(s))\Gamma(-J(t))}{\Gamma(-J(s)-J(t))}$$

BESIII, Phys.Lett. B710 (2012) 594-599

Szczepaniak and Pennington, PLB737, 283