Form factors on the lattice

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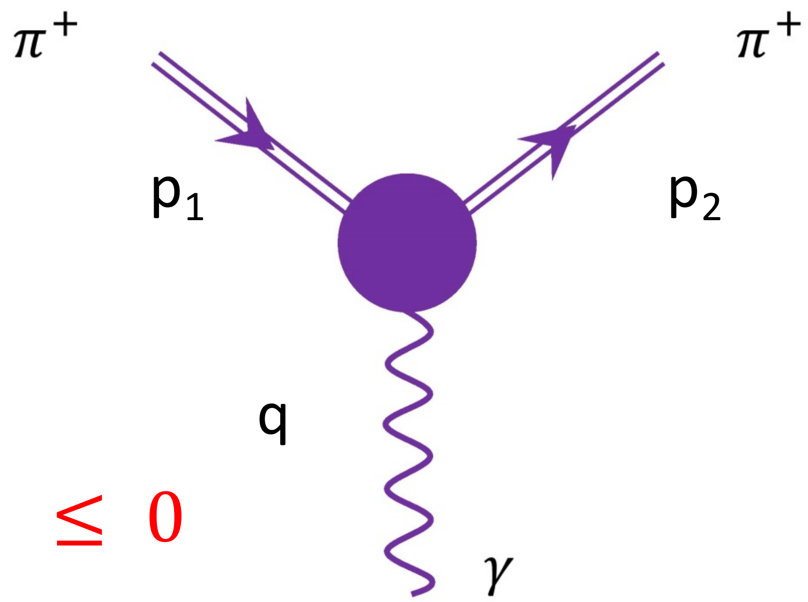
Jefferson Lab

Hadronic Physics with Leptonic and Hadronic Beams,
Newport News, USA
8th Sept, 2017.

Pion electromagnetic form factor

Simplest hadron



Space like "q":

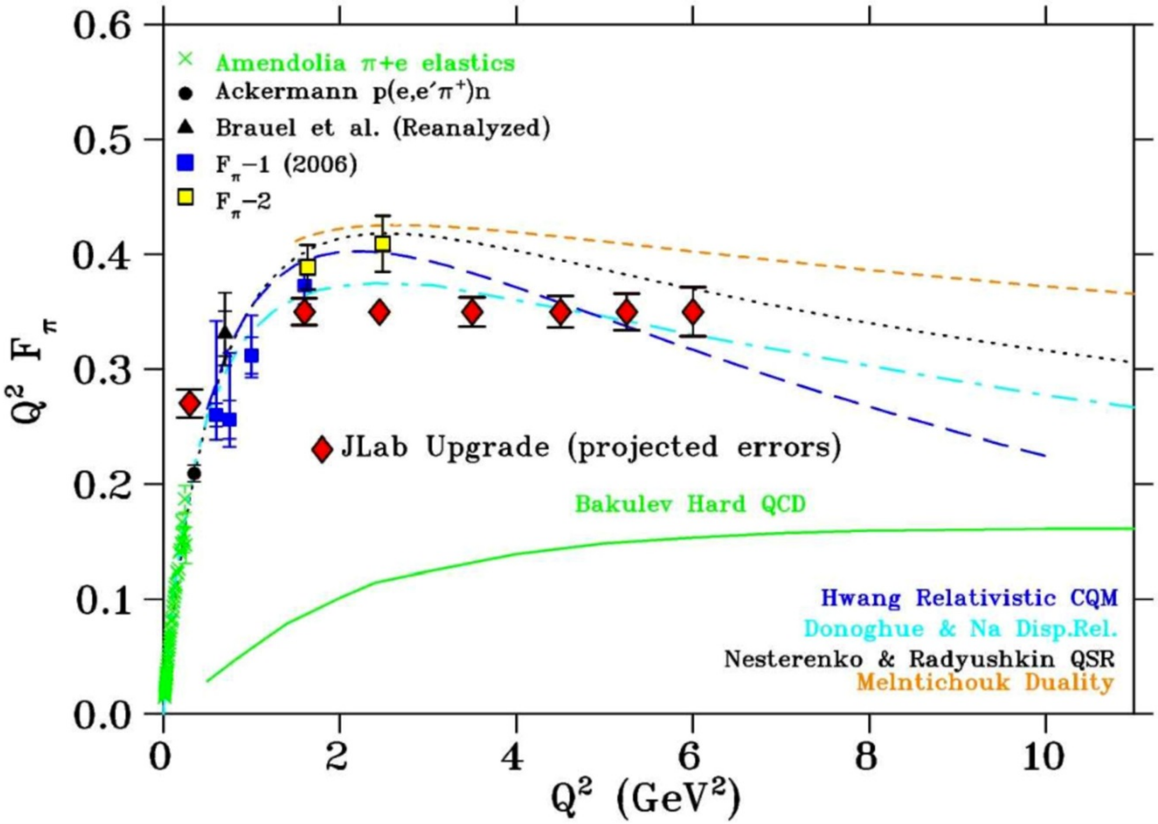
$$q^2 = (p_2 - p_1)^2 \leq 0$$

$$Q^2 = -q^2$$

$$\langle \pi^+(\vec{p}\,')| j^\mu | \pi^+(\vec{p}) \rangle = (p + p')^\mu \, F_\pi(Q^2)$$

(in units of 'e')

Interplay between hard and soft scales



G. Huber and D. Gaskell

arXiv:1705.05849

Hard tail ($Q^2 \to \infty$) from pQCD:

$$F_\pi(Q^2) \to \frac{16\pi\alpha_s(Q^2) f_\pi^2}{Q^2}$$

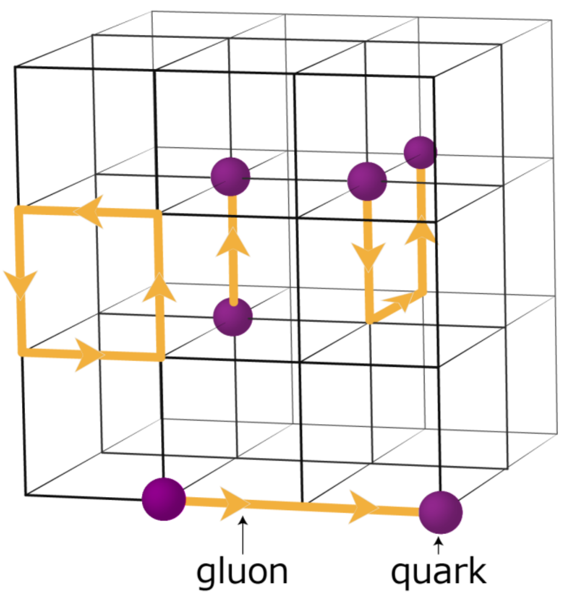
G. P. Lepage, S.J.Brodsky, Phys. Lett. 87B(1979)359

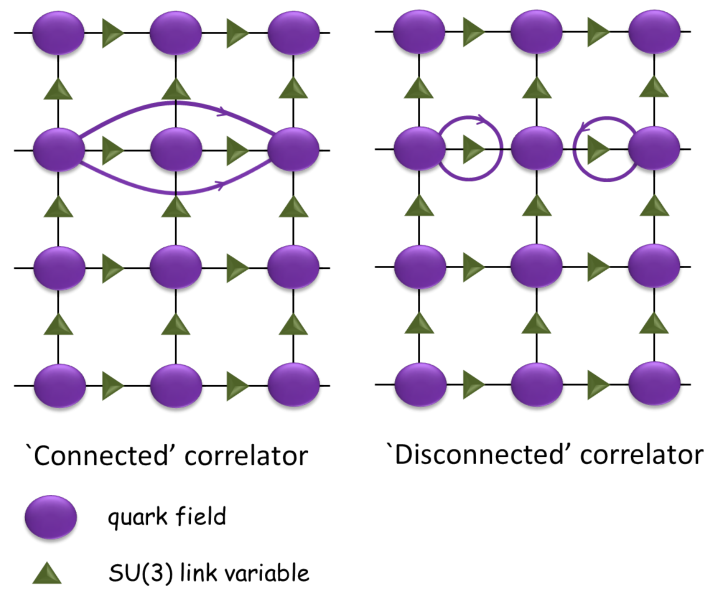
Soft part ($Q^2 < 1$ GeV²): vector meson dominance with $F_\pi(0) = 1$, data fits well

Need better understanding of the transition to the asymptotic region

Lattice recipe for meson correlators

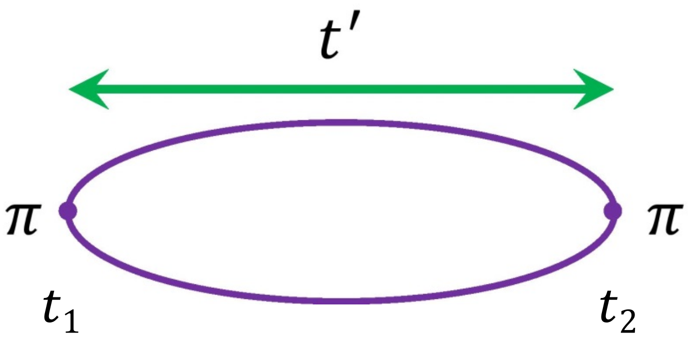
- Expectation values of observables : $\int DUD\psi D\bar{\psi} exp(-\int L_{QCD} d^4x)$
- 4-D space-time lattice
- Gauge configurations : gluons + sea quarks





- Discretise : $L_q \equiv \bar{\psi}(\gamma_\mu D^\mu + m)\psi \rightarrow \bar{\psi}(\gamma.\Delta + ma)\psi$
- Inversion of Dirac matrix : propagator
- 2-point, 3-point correlation functions : extract meson properties
- Corrections for lattice artifacts

Two-point correlator construction: JLab way



$$C_{ij}(t) = \langle 0|\mathcal{O}_i(t)\mathcal{O}_j^\dagger(0)|0\rangle$$

- Basis of operators

$$\mathcal{O} \sim \bar{\psi}\Gamma\overleftrightarrow{D}\cdots\overleftrightarrow{D}\psi$$

- Optimized operator for state $|n>$

$$\Omega_\mathfrak{n}^\dagger = \sum_i w_i^{(\mathfrak{n})}\mathcal{O}_i^\dagger$$

in a variational sense by solving generalized eigenvalue problem-

$$C(t)\,v^{(\mathfrak{n})} = \lambda_\mathfrak{n}(t)\,C(t_0)\,v^{(\mathfrak{n})}$$

- Diagonalize the correlation matrix – eigenvalues

$$\lambda_n(t) = \exp[-E_n(t-t_0)]$$

Two-point correlator construction : JLab way

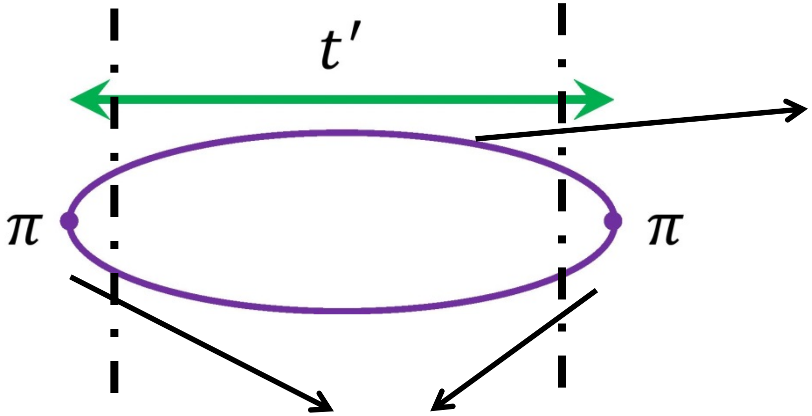
Correlator Construction: smearing of quark fields - 'distillation' with

$$\Box_{\vec{x}\vec{y}}(t) = \sum_{n=1}^{N_D} \xi_{\vec{x}}^{(n)}(t)\, \xi_{\vec{y}}^{(n)\dagger}(t)$$

→ Low lying hadron states

Meson creation operator :

$$\mathcal{O}^\dagger(\vec{p}) = \bar{\psi}_{\vec{x}} \Box_{\vec{x}\vec{y}}\, e^{-i\vec{p}\cdot\vec{y}}\, \mathbf{\Gamma}_{\vec{y}\vec{z}} \Box_{\vec{z}\vec{w}} \psi_{\vec{w}}$$



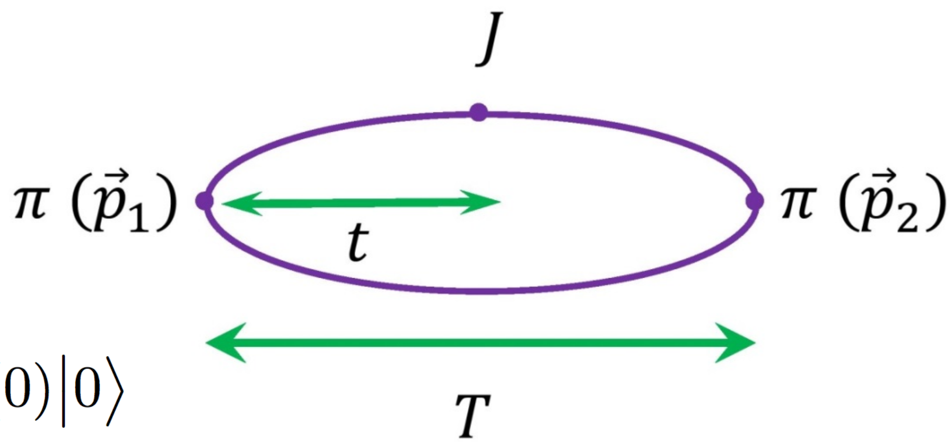
Perambulators by inverting the Dirac matrix

+

Operator construction with momentum projection

Form factor calculation

Need three-point correlator



$$C_{\mathrm{f}\mu\mathrm{i}}(\Delta t, t) = \langle 0 | \mathcal{O}_\mathrm{f}(\Delta t)\, j_\mu(t)\, \mathcal{O}_\mathrm{i}^\dagger(0) | 0 \rangle$$

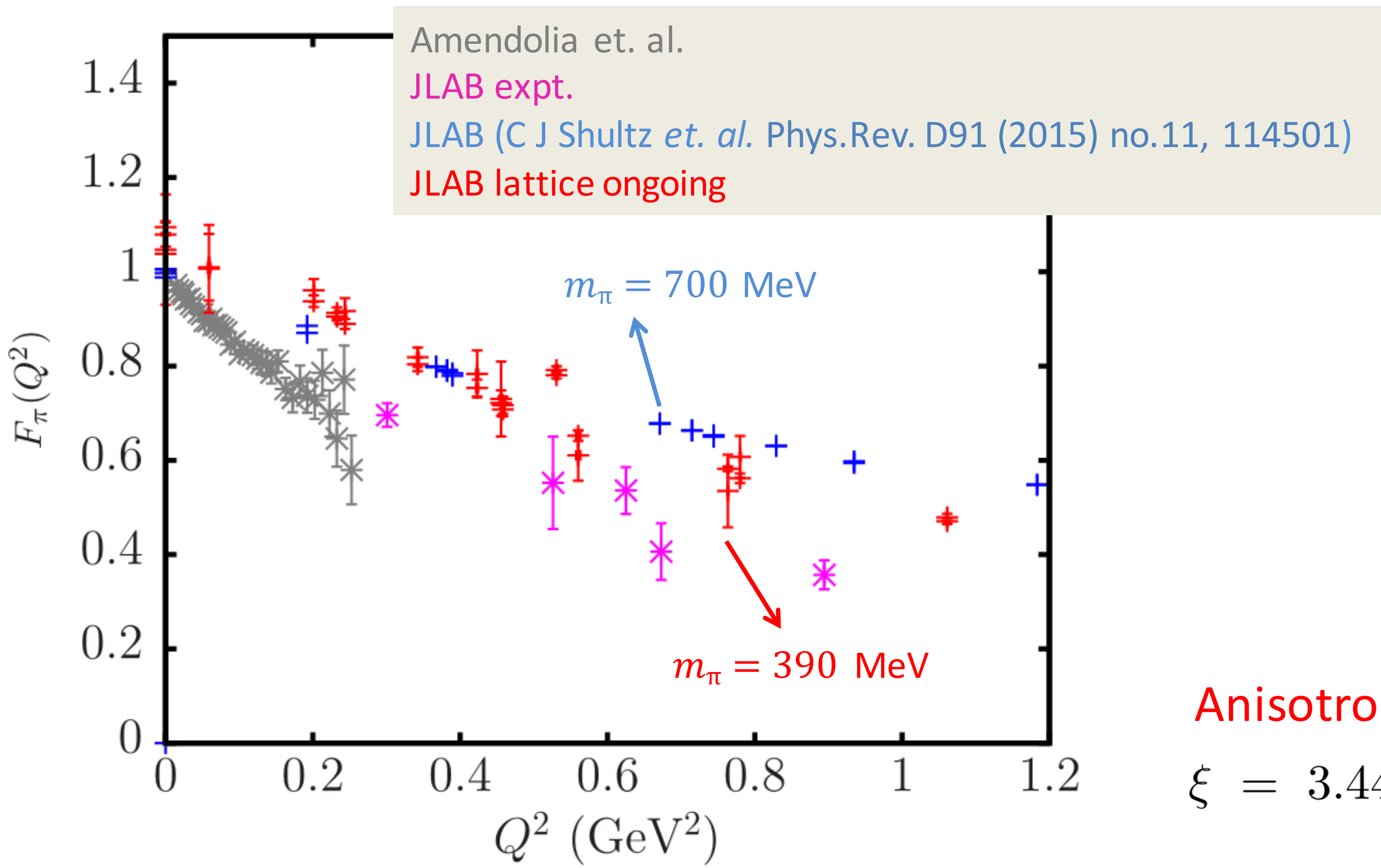
⇓

$$Z_V < \pi^+(p_2) | J^\mu{}_\pi(0) | \pi^+(p_1) > = e(p_1 + p_2)^\mu F_\pi(q^2)$$

Z_V → Clover discretised fermion action

Z_V calculated using $F_\pi(q^2 = 0) = 1$

Pion electromagnetic form factor: up to $Q^2 = 1$ GeV^2



$(L/a_s)^3 \times (T/a_t) = 20^3 \times 128$

Anisotropy

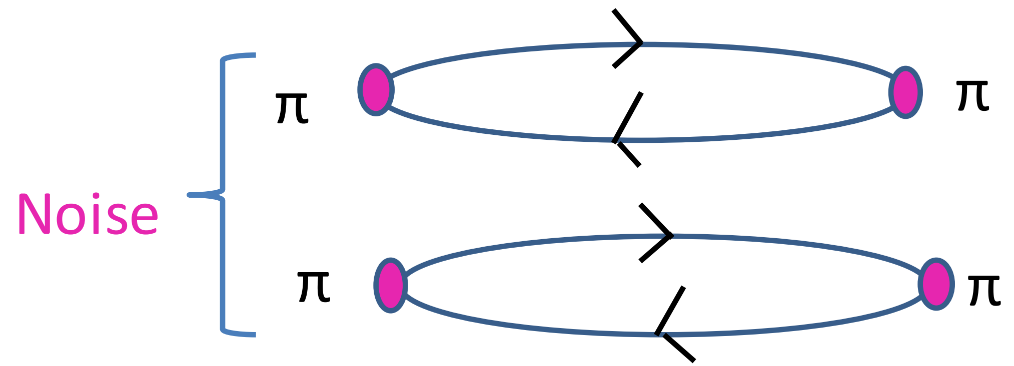
$\xi = 3.44$

$a_s = 0.12$ fm, $\frac{a_s}{a_t} = 3.44$

Towards higher Q^2

More difficult on lattice for higher momenta

Signal-to-noise ratio:



2-point correlators :

$$\exp[-(E_\pi(p) - 2m_\pi)t]$$

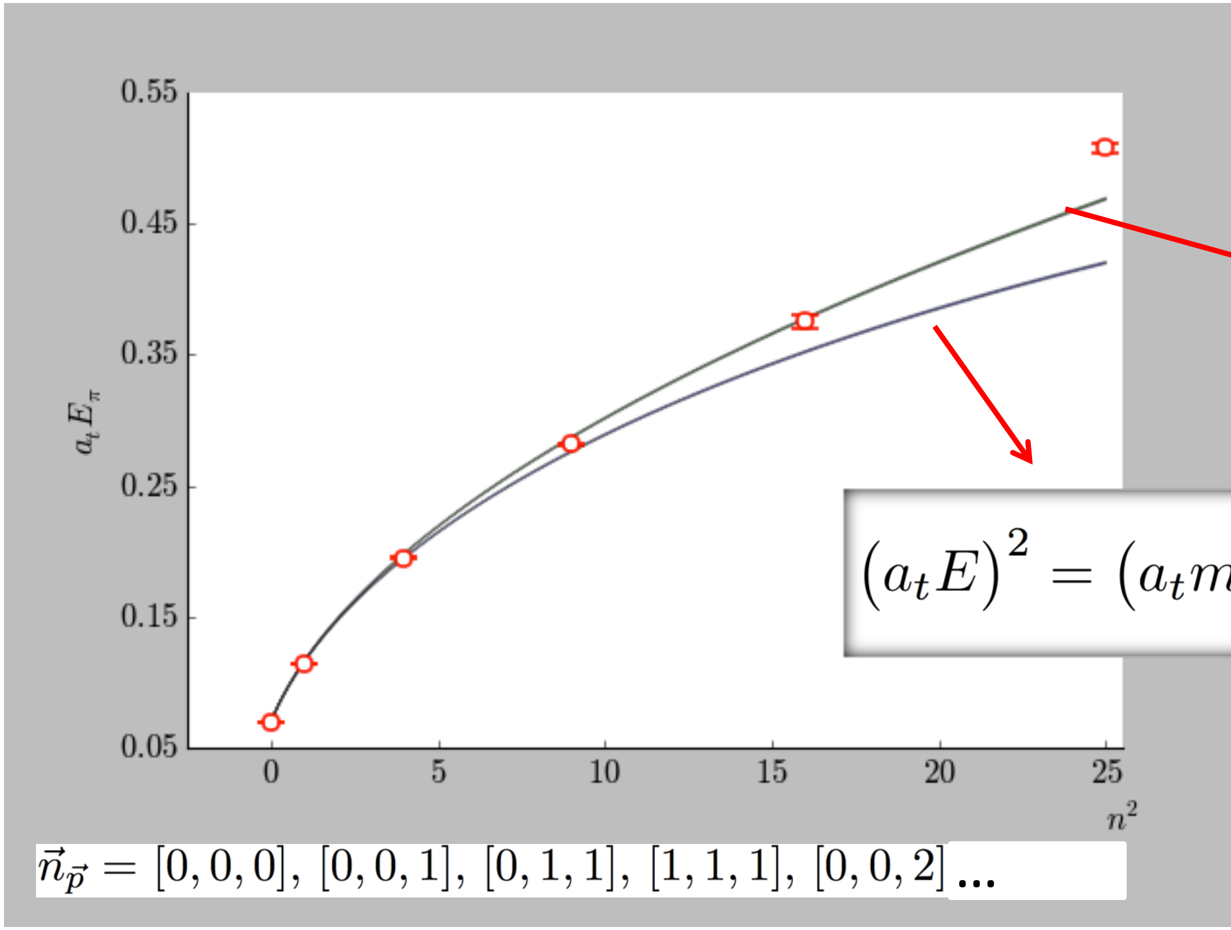
3-point correlators :

$$\exp[-(E_\pi(p_i) + E_\pi(p_f) - 2m_\pi)t/2]$$

in the middle of the plateau

Minimize energies for a given Q^2 to get better signal

Towards higher Q^2



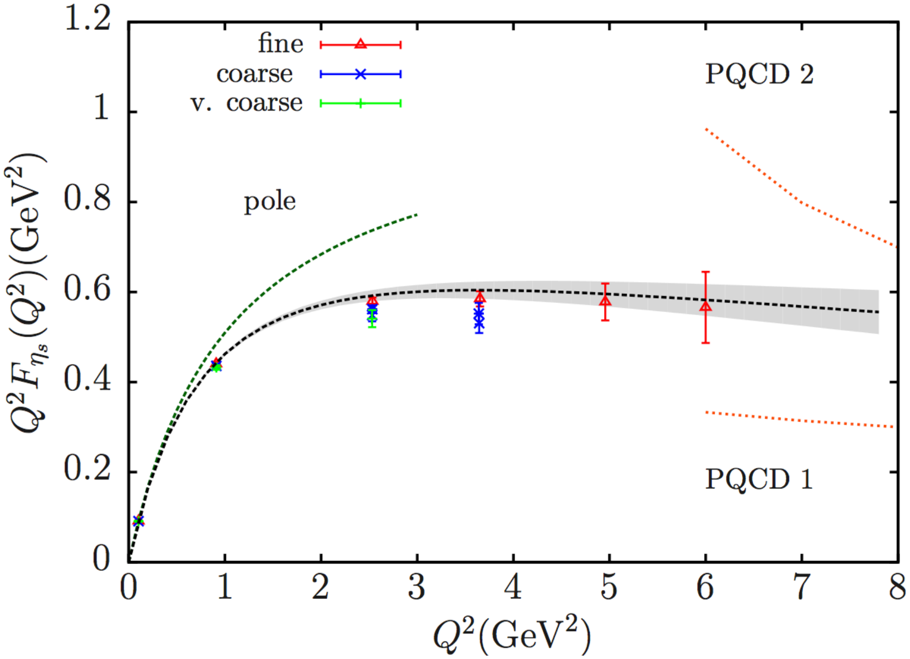
Dispersion relation:

$$E^2 = m^2 + p^2$$

$$\left(a_t E\right)^2 = \left(a_t m\right)^2 + \left(\frac{2\pi}{\xi(L/a_s)}\right)^2 |\vec{n}_{\vec{p}}|^2$$

Achieve maximum Q^2 by using Breit frame : $\overrightarrow{P_f} = -\overrightarrow{P_i}$

Work ongoing – reached up to 4.0 GeV^2 with 260 MeV pion

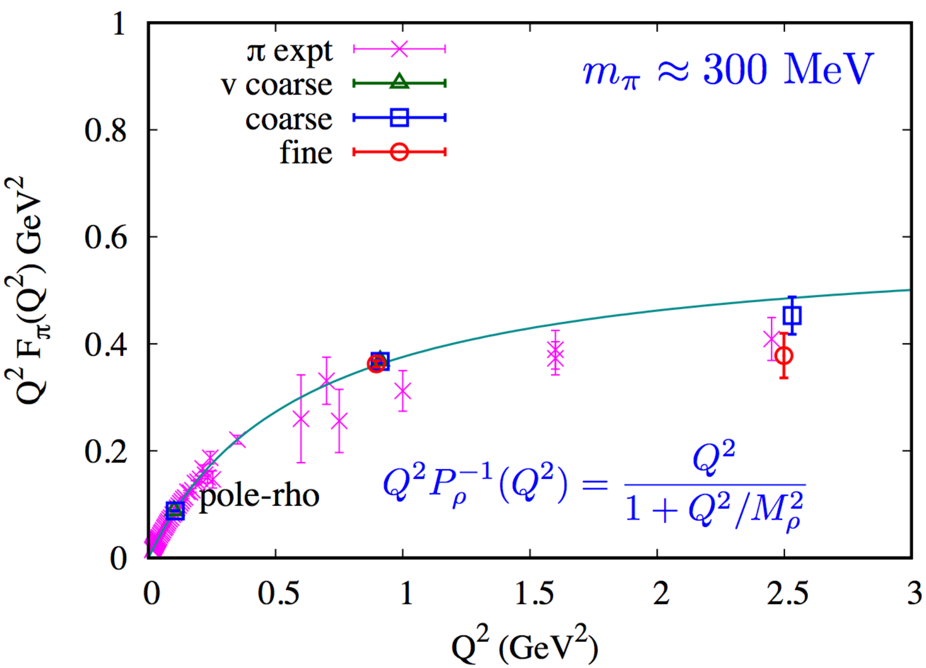


HPQCD
Phys.Rev. D96, 054501 (2017)

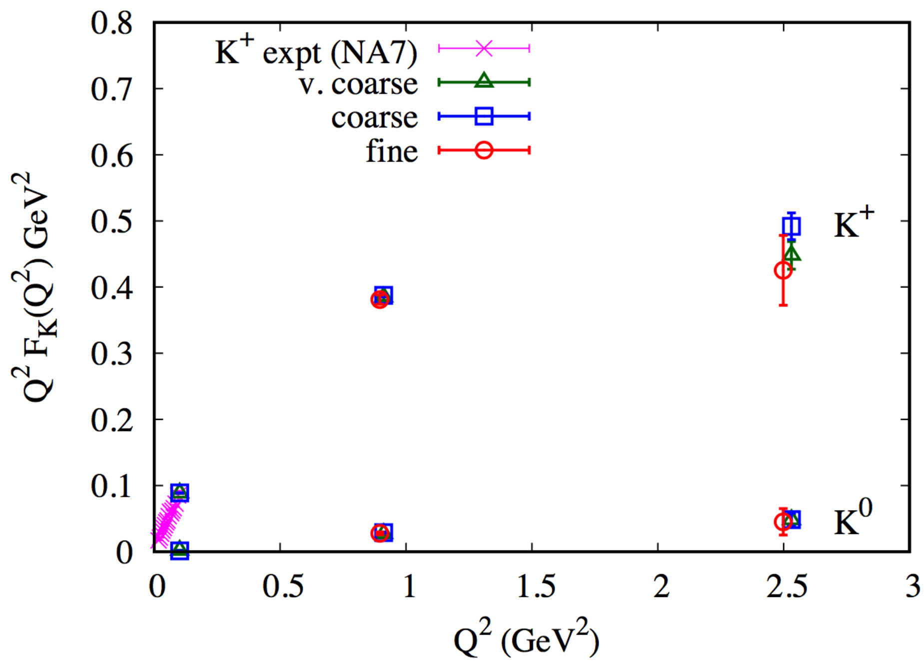
$$\langle r^2 \rangle_V^{(\pi)} = 0.403(18)(6)\,\mathrm{fm}^2$$

HPQCD- J. Koponen Lattice 2017

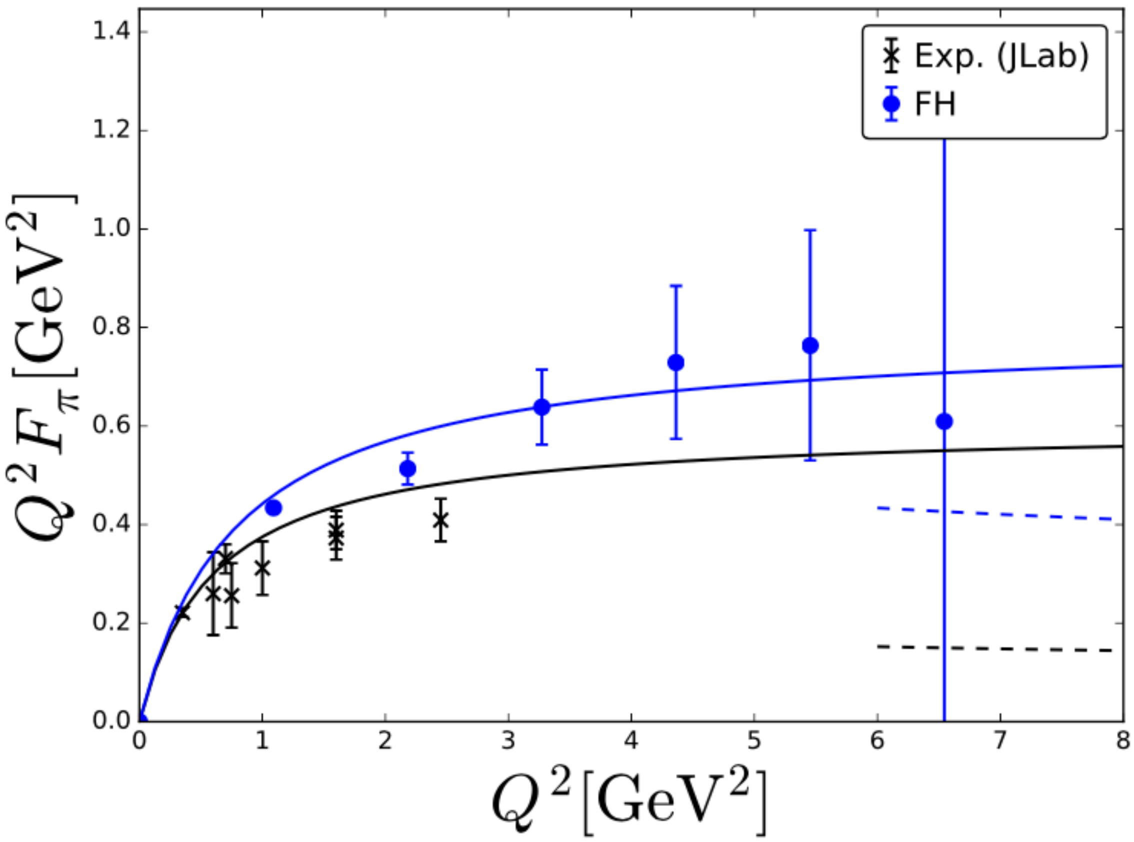
PRELIMINARY



PRELIMINARY



A. J. Chambers *et. al*. QCDSF/UKQCD/CSSM Collaborations, arXiv:1702.01513



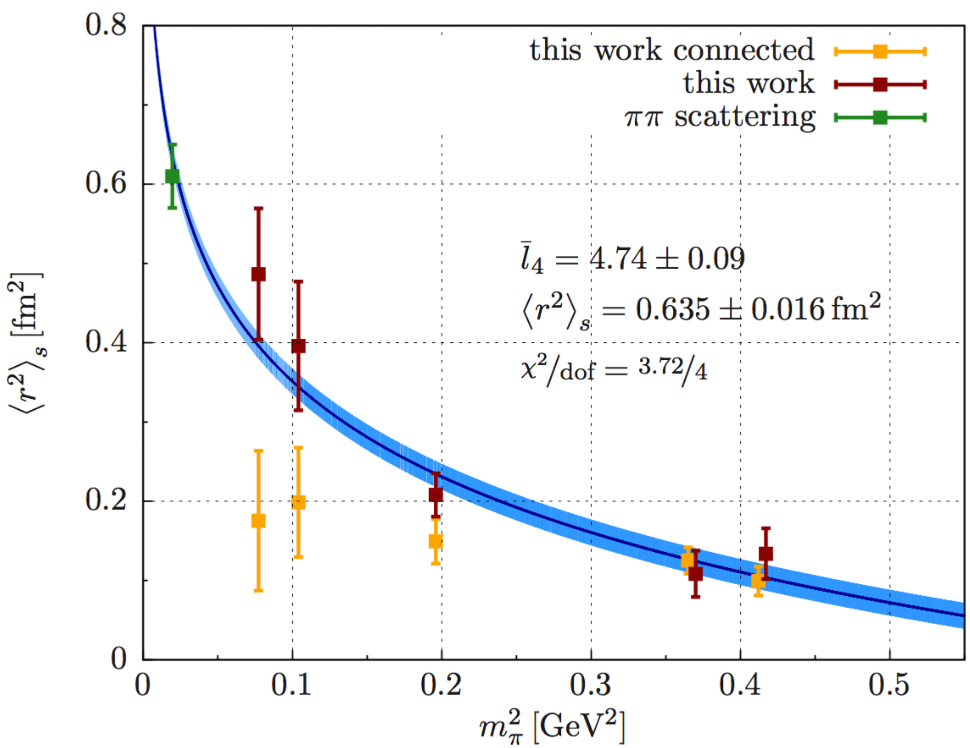
Pion mass = 450 MeV

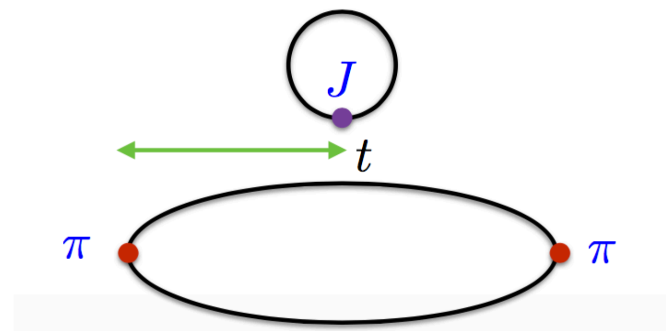
Using Feynman-Hellmann methods

Pion scalar form factor

$$F_S^\pi(Q^2) \equiv \langle \pi^+(p_f) | \, m_d \bar{d}d + m_u \bar{u}u \, | \pi^+(p_i) \rangle , \qquad Q^2 = -q^2 = -(p_f - p_i)^2$$

Scalar charge radius: $$\langle r^2 \rangle_S^\pi = - \frac{6}{F_S^\pi(0)} \frac{\partial F_S^\pi(Q^2)}{\partial Q^2} \Big|_{Q^2=0}$$

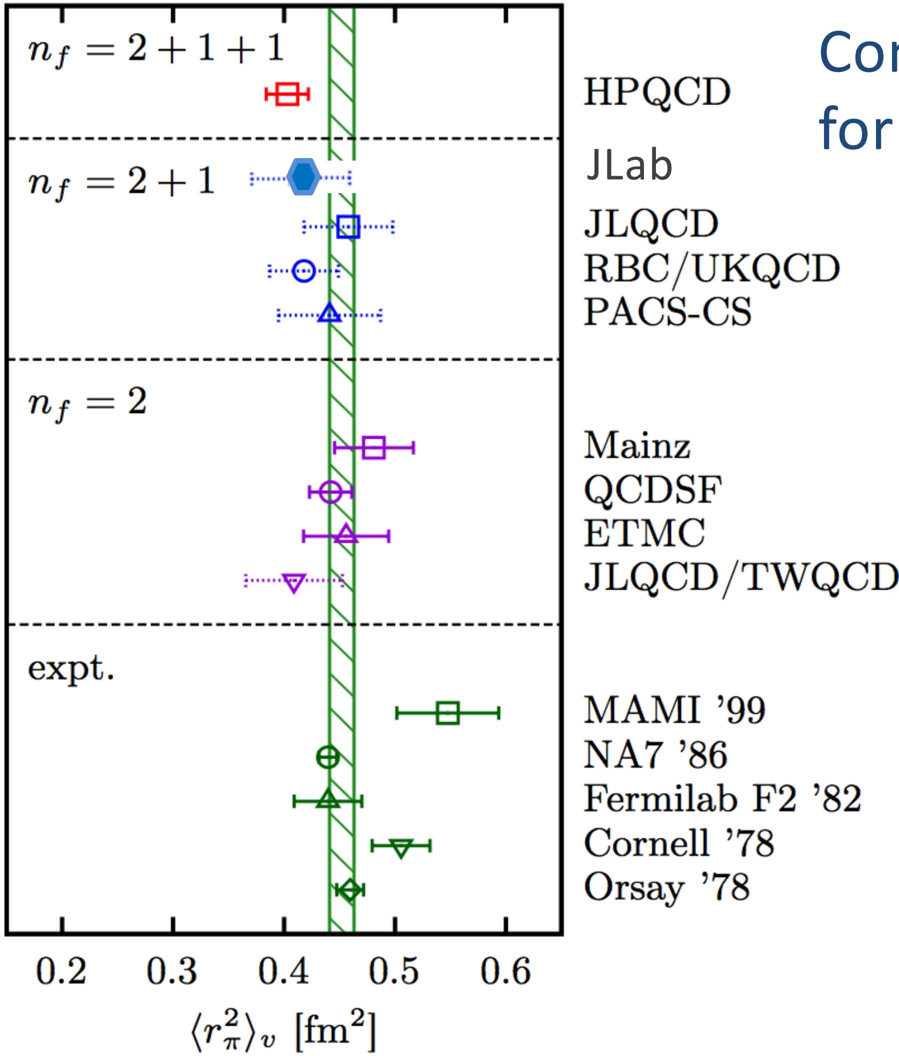


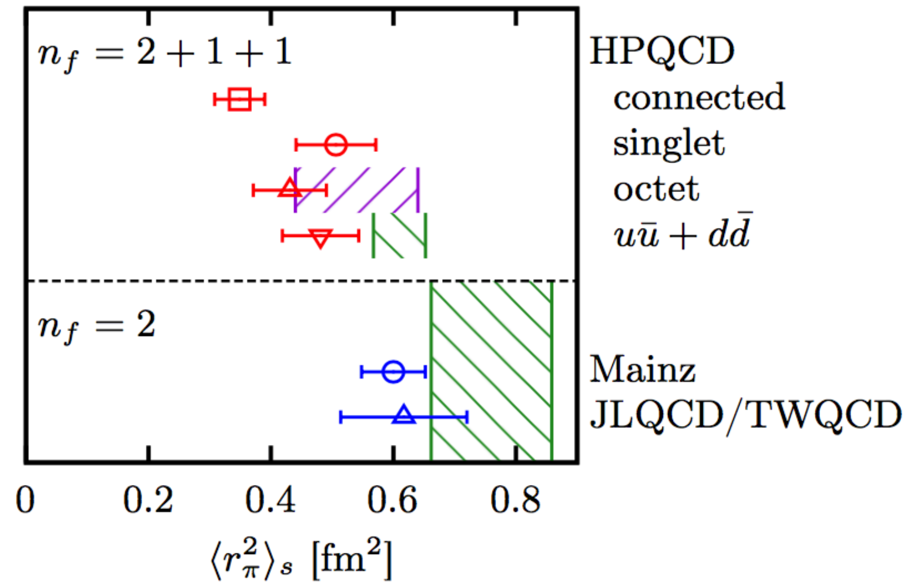


$$\langle r^2 \rangle_S^\pi = 0.635 \pm 0.016 \text{ fm}^2$$

V. Gulpers et. al.
Phys.Rev. D89 (2014) no.9, 094503

Comparison of different lattice results for pion vector and scalar charge radius





HPQCD, J. Koponen *et. al.*
Phys.Rev. D93, 054503

Nucleon electromagnetic form factor

$$\langle N(p', s') \,|\, \bar{q}(0)\gamma_\mu q(0) \,|\, N(p, s) \rangle =$$
$$\bar{u}(p', s') \left[\gamma_\mu F_{1q}(Q^2) + \frac{\sigma_{\mu\nu} q_\nu}{2M_N} F_{2q}(Q^2)\right] u(p, s)$$

Dirac FF

Pauli FF

$$Q^2 = -q^2 = -(p' - p)^2$$

Sachs form factors -

$$G_{Eq} = F_{1q} - \frac{Q^2}{(2M)^2} F_{2q}$$
$$G_{Mq} = F_{1q} + F_{2q}$$

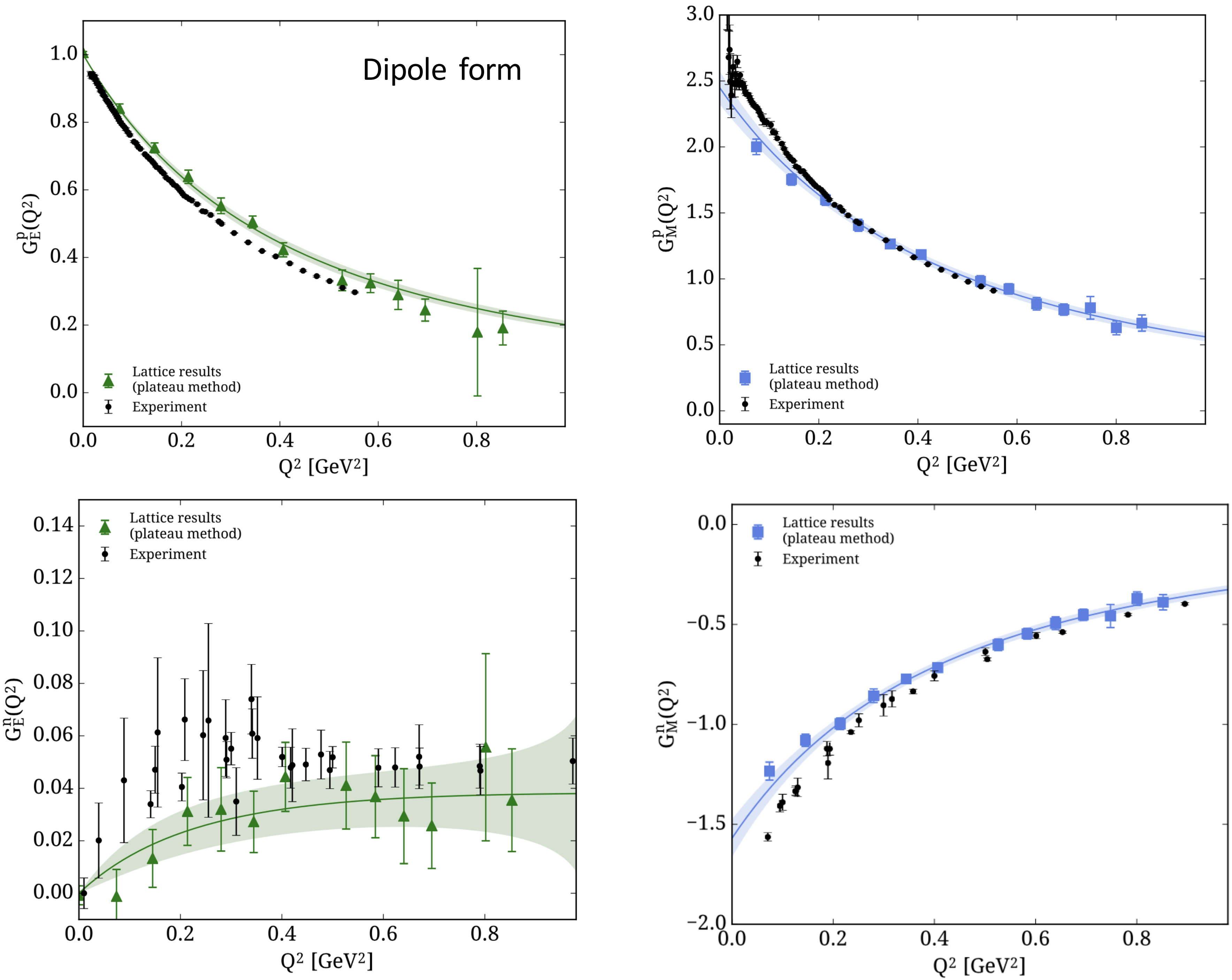
Calculated respectively from temporal and spatial component of currents

C Alexandrou *et. al.* , PhysRevD.96.034503 (First lattice calculation with physical pion; disconnected contributions included)

$$R_\mu(\Gamma;\vec{q};t_s;t_{\rm ins}) = \frac{G_\mu(\Gamma;\vec{q};t_s;t_{\rm ins})}{G(\vec{0};t_s)} \times \left[\frac{G(\vec{0};t_s)G(\vec{q};t_s-t_{\rm ins})G(\vec{0};t_{\rm ins})}{G(\vec{q};t_s)G(\vec{0};t_s-t_{\rm ins})G(\vec{q};t_{\rm ins})}\right]^{\frac{1}{2}}$$

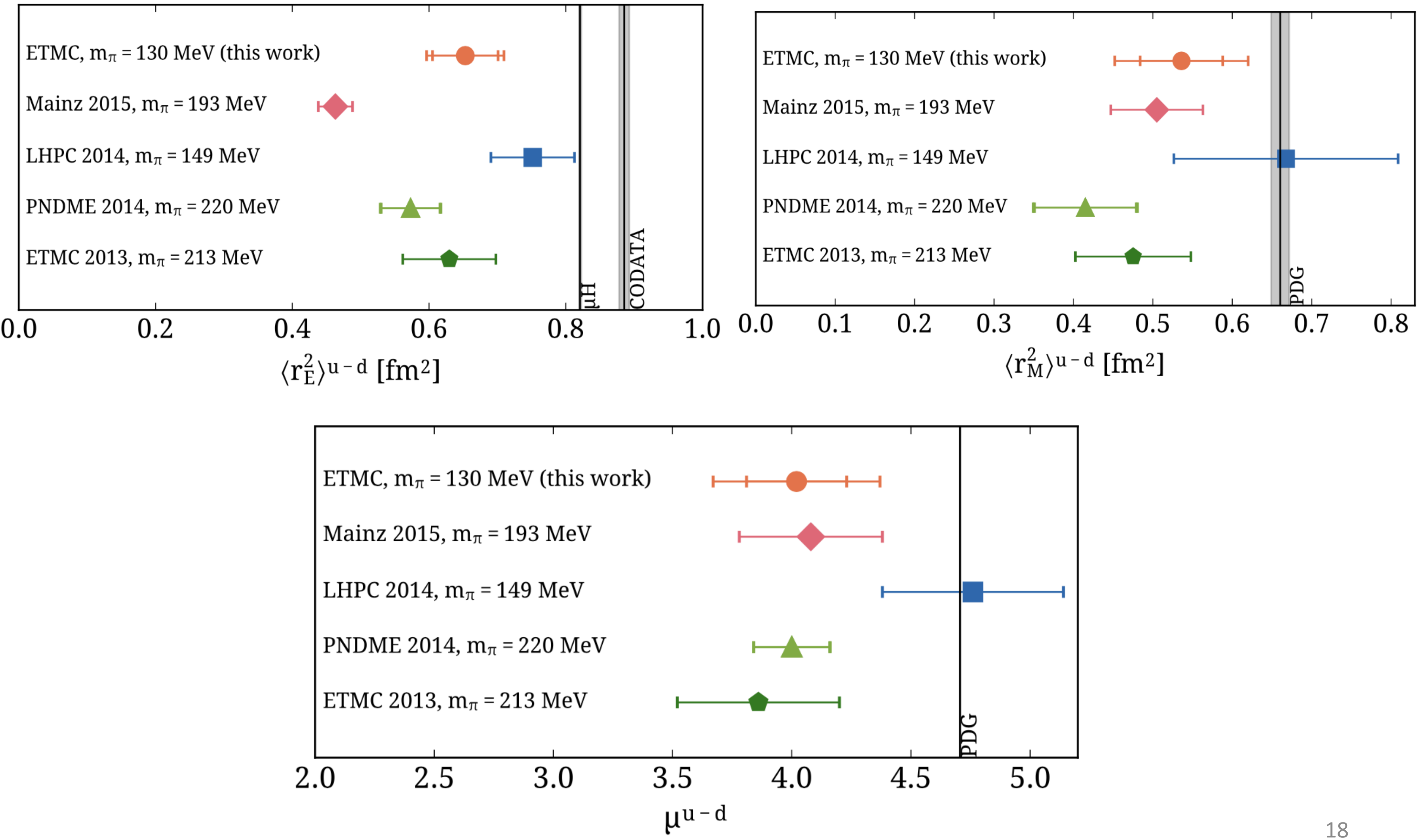
$$G(\vec{p};t) = c_0(\vec{p})e^{-E(\vec{p})t}[1+c_1(\vec{p})e^{-\Delta E_1(\vec{p})t}+\mathcal{O}(e^{-\Delta E_2(\vec{p})t})]$$

$$G_\mu(\Gamma;\vec{q};t_s,t_{\rm ins}) = a^\mu_{00}(\Gamma;\vec{q})e^{-m(t_s-t_{\rm ins})}e^{-E(\vec{q})t_{\rm ins}} \times \Big[1+a^\mu_{01}(\Gamma;\vec{q})e^{-\Delta E_1(\vec{q})t_{\rm ins}} + a^\mu_{10}(\Gamma;\vec{q})e^{-\Delta m_1(t_s-t_{\rm ins})} + a^\mu_{11}(\Gamma;\vec{q})e^{-\Delta m_1(t_s-t_{\rm ins})}e^{-\Delta E_1(\vec{q})t_{\rm ins}} + \cdots\Big]$$

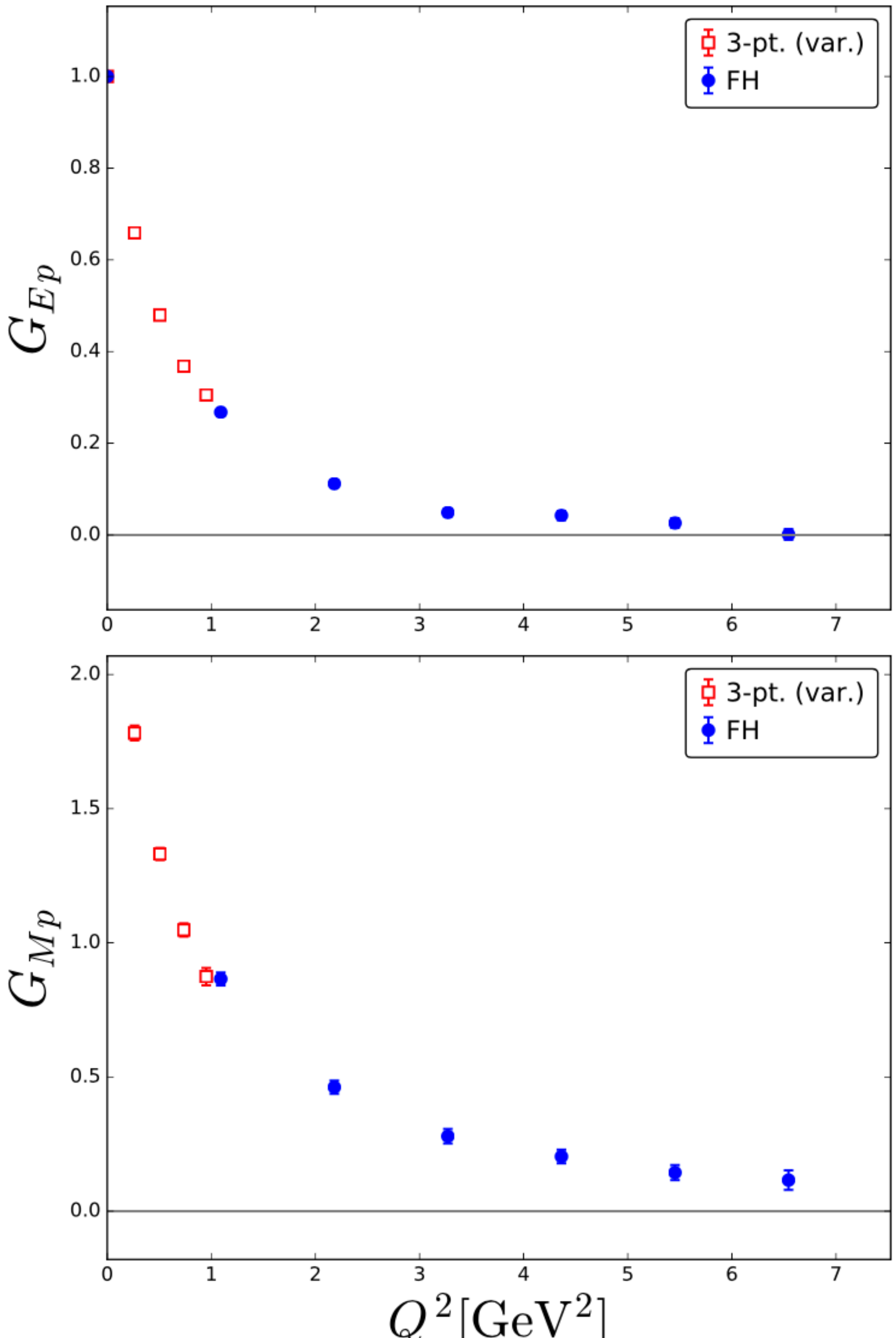


Dipole form
Lattice results
(plateau method)
Experiment
$G_E^p(Q^2)$
$G_M^p(Q^2)$
$G_E^n(Q^2)$
$G_M^n(Q^2)$
Q^2 [GeV^2]

Comparison among different lattice results for nucleon charge radii and magnetic moment



Sachs form factors at high Q^2



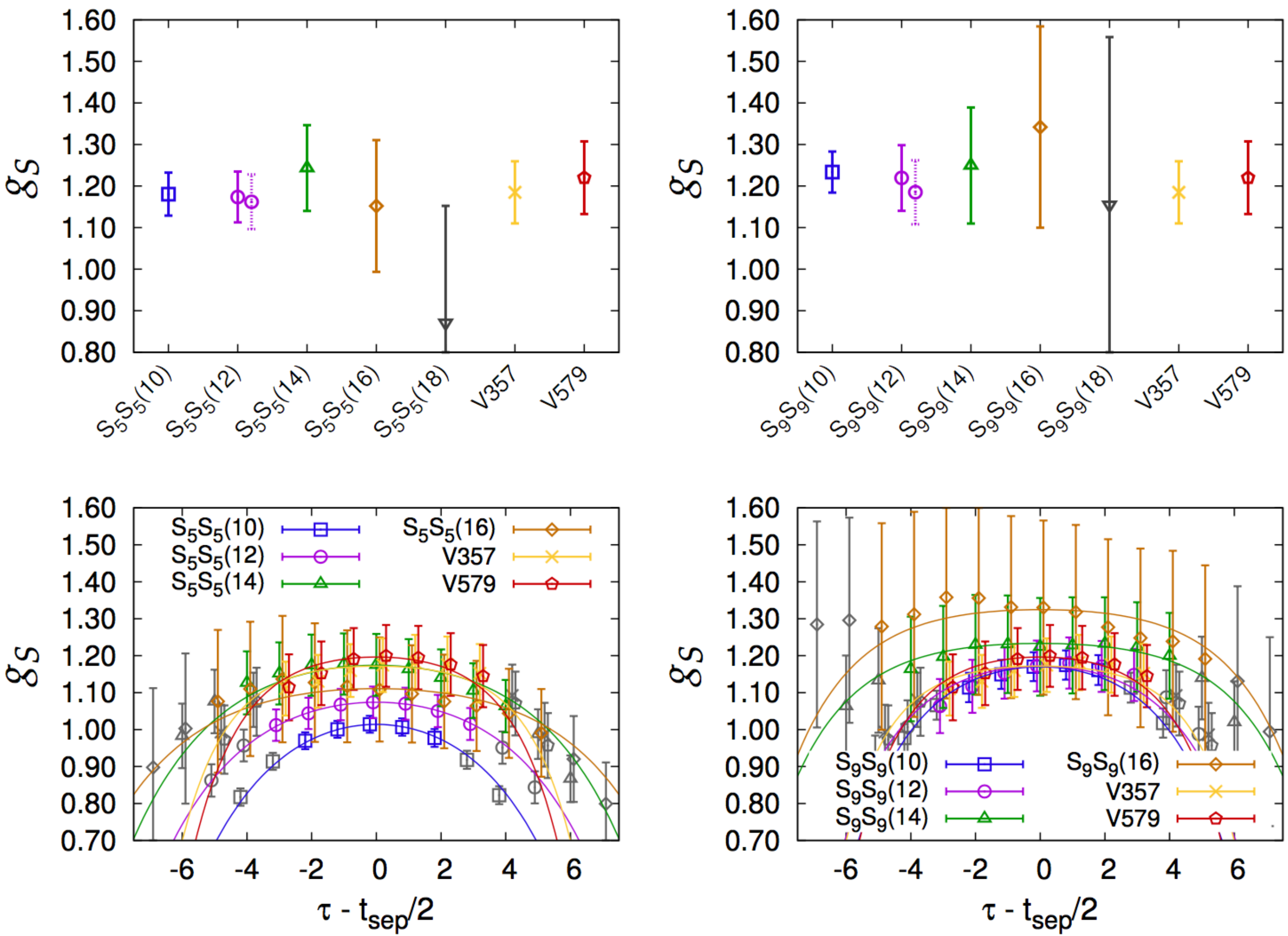
A. J. Chambers *et. al*. QCDSF/UKQCD/CSSM Collaborations, arXiv:1702.01513

- Use of Feynman-Hellmann theorem
- At 490 MeV pion mass

Isovector charges of nucleon

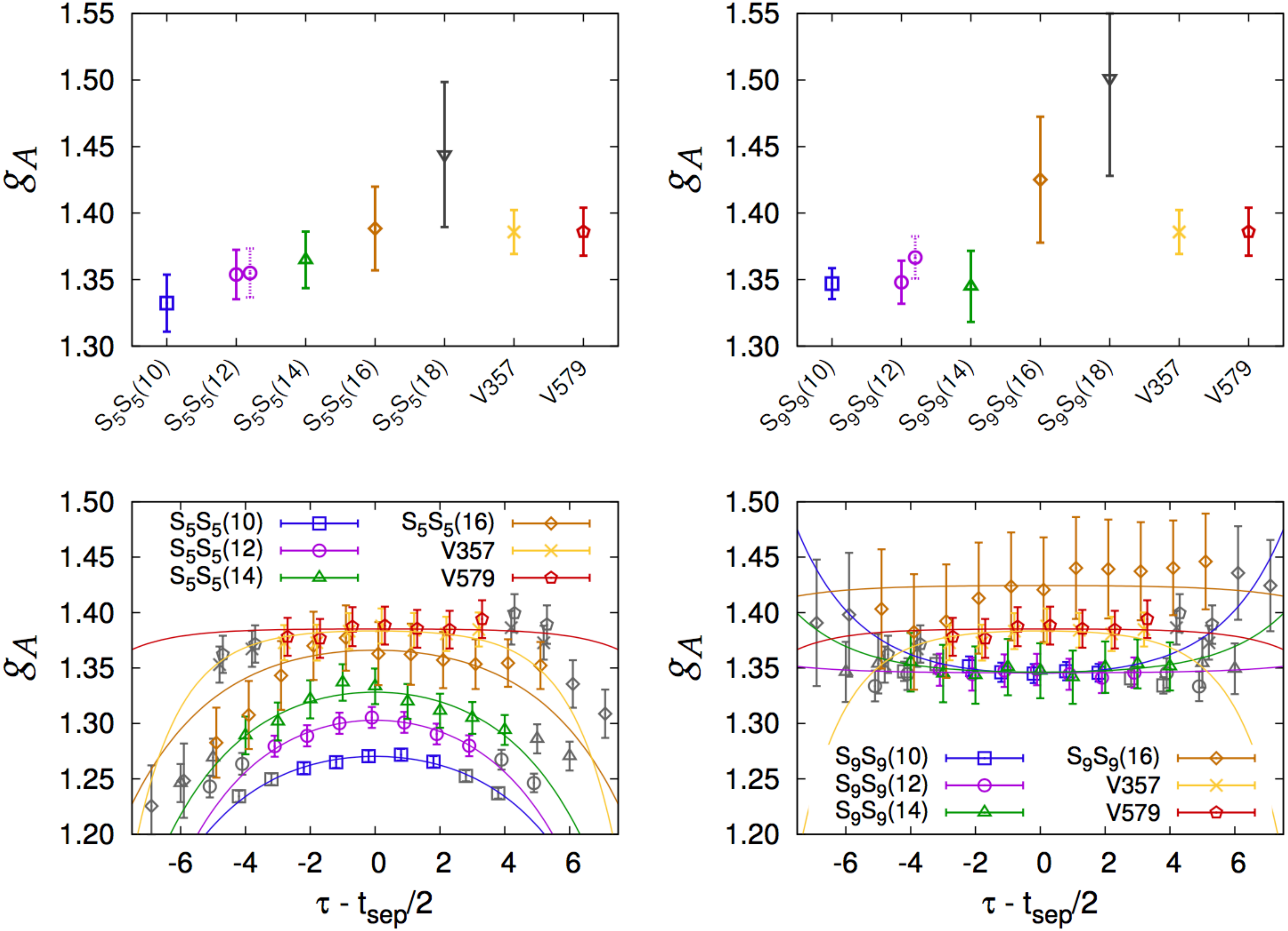
Boram Yoon *et. al.* Phys. Rev. D.93.114506 (Nucleon Matrix Elements (NME) Collaboration)
[JLab participation: David Richards, Kostas Orginos, Frank Winter]

$$\langle N(p,s)|\mathcal{O}^q_\Gamma|N(p,s)\rangle = g^q_\Gamma \bar{u}_s(p)\Gamma u_s(p)$$

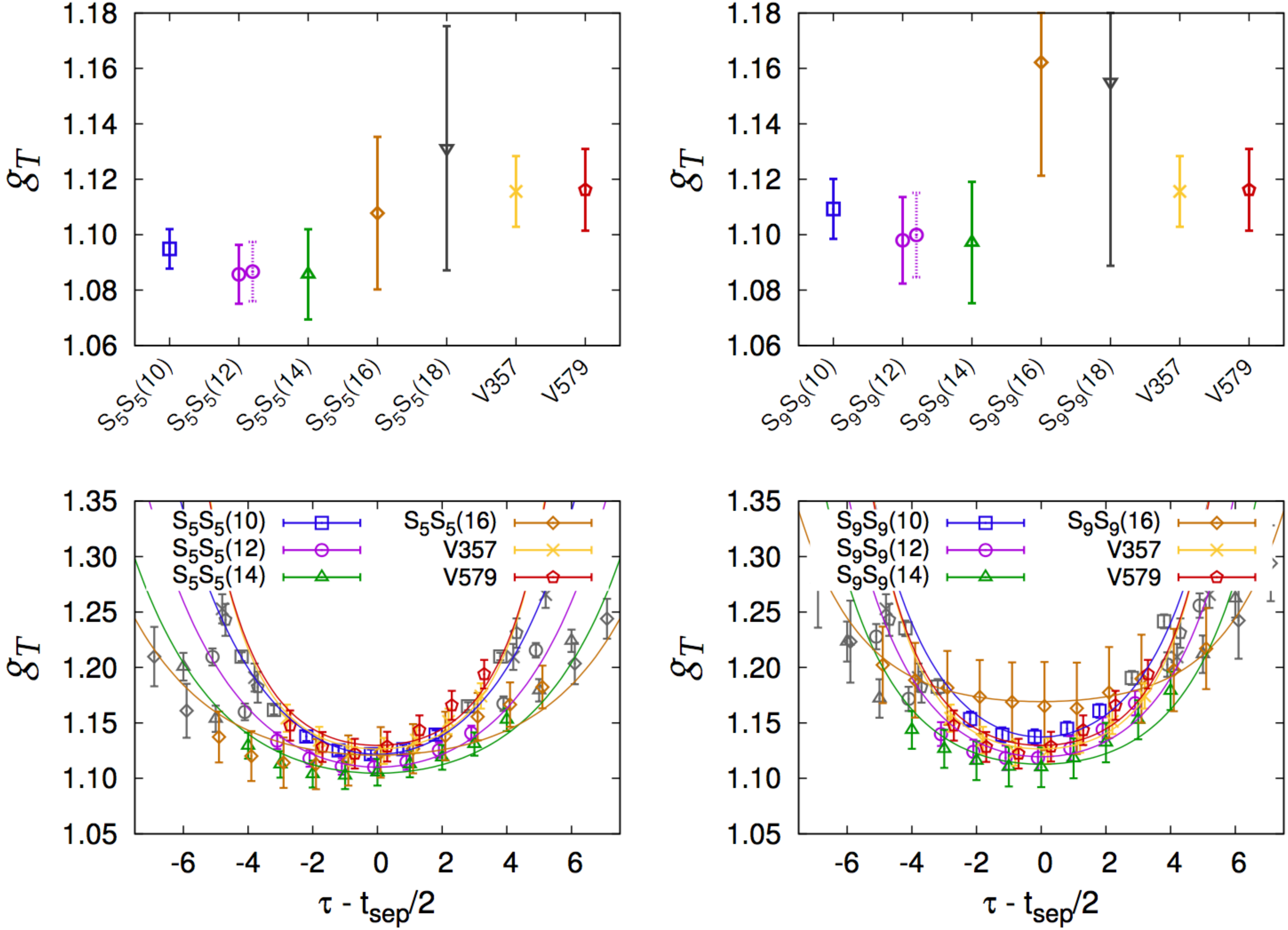


- Variational method
- AMA,
- RI-MOM

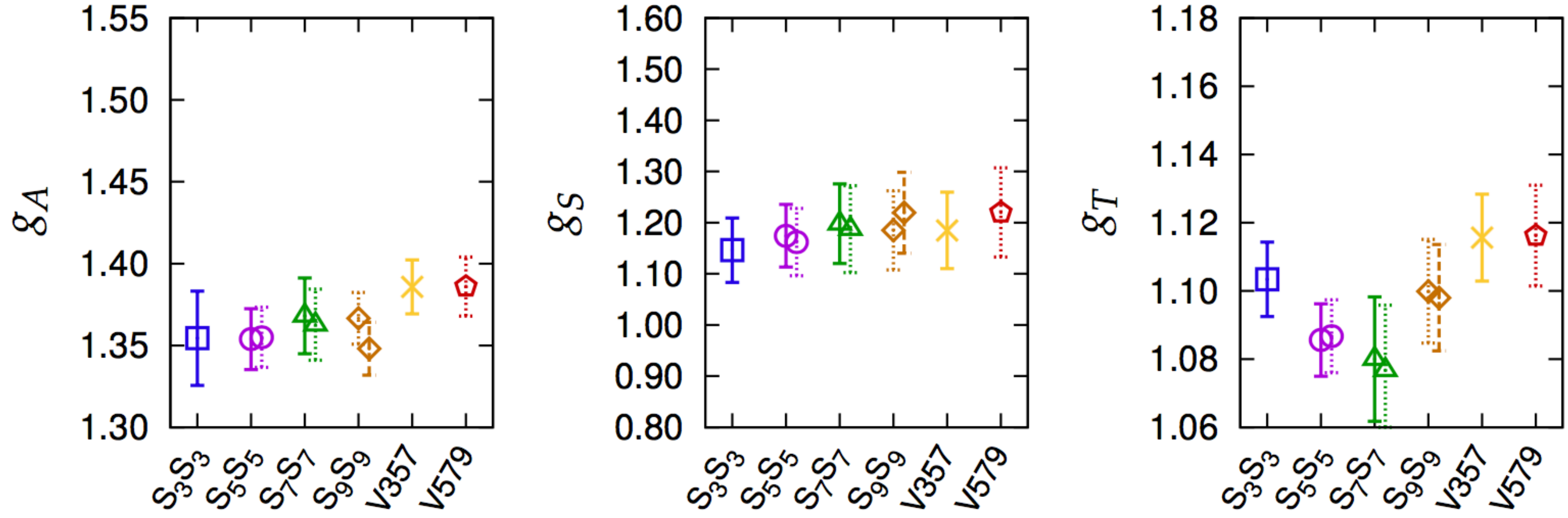
At pion mass = 312 MeV



g_A
1.55
1.50
1.45
1.40
1.35
1.30
$S_5S_5(10)$
$S_5S_5(12)$
$S_5S_5(14)$
$S_5S_5(16)$
$S_5S_5(18)$
V357
V579
$S_9S_9(10)$
$S_9S_9(12)$
$S_9S_9(14)$
$S_9S_9(16)$
$S_9S_9(18)$
1.25
1.20
-6
-4
-2
0
2
4
6
τ - $t_{sep}/2$



g_T
$S_5S_5(10)$
$S_5S_5(12)$
$S_5S_5(14)$
$S_5S_5(16)$
$S_5S_5(18)$
V357
V579
$S_9S_9(10)$
$S_9S_9(12)$
$S_9S_9(14)$
$S_9S_9(16)$
$S_9S_9(18)$
τ - $t_{sep}/2$



Consistent among different smearings,
and 2-state fit and variational fit

Another calculation of nucleon axial charge

Evan Berkowitz *et. al.*, arXiv:1704.01114

Using Feynman-Hellmann theorem:

$$\frac{\partial E_n}{\partial \lambda} = \langle n | H_\lambda | n \rangle$$

$$H = H_0 + \lambda H_\lambda$$

$$S_\lambda = \lambda \int d^4 x j(x)$$

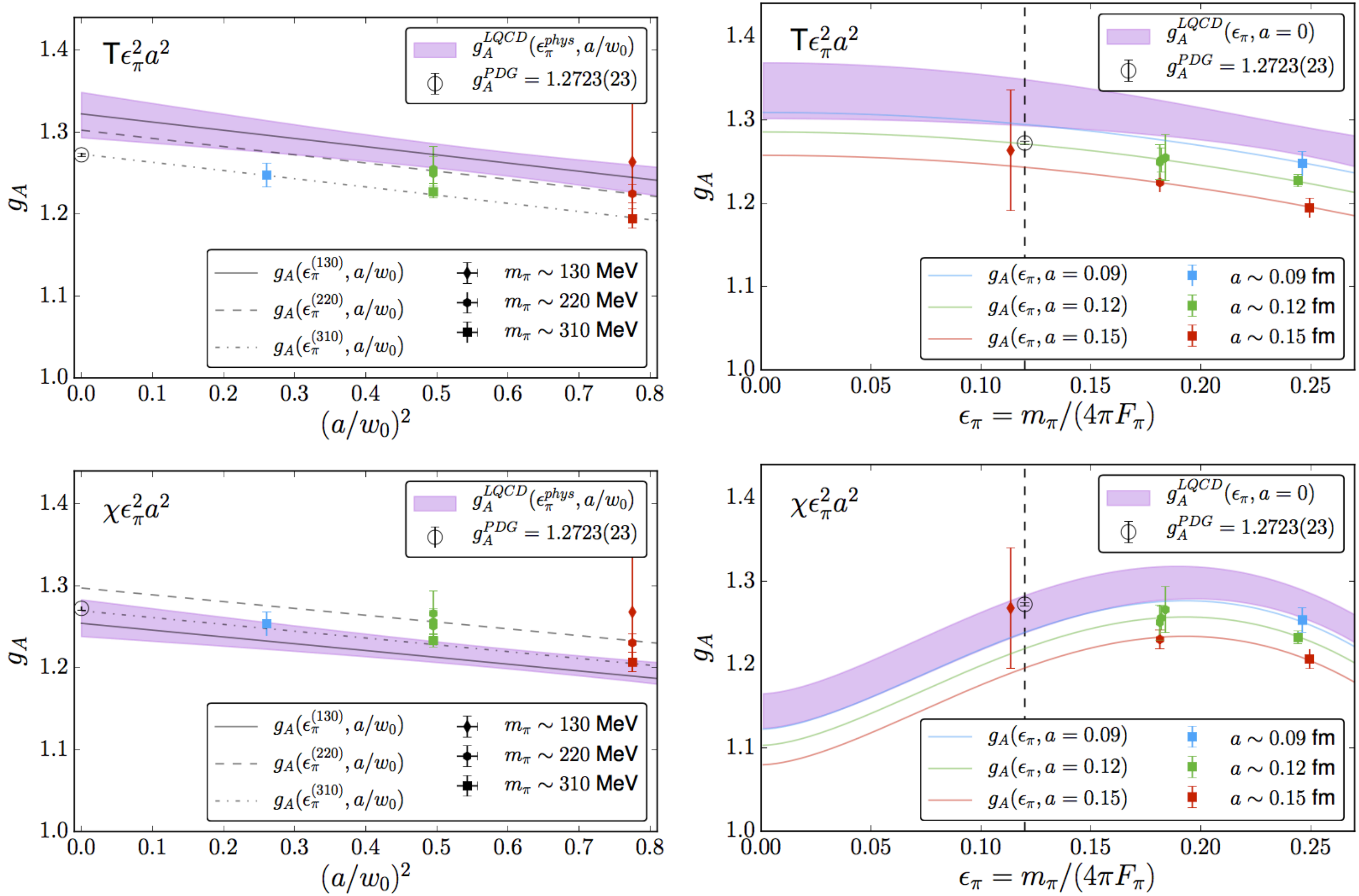
$$M_\lambda^{eff}(t, \tau) = \frac{1}{\tau} \ln \left(\frac{C_\lambda(t)}{C_\lambda(t+\tau)} \right)$$

$$\left.\frac{\partial M^{eff}(t,\tau)}{\partial\lambda}\right|_{\lambda=0} = \frac{1}{\tau}\left[\frac{\partial_\lambda C_\lambda(t)}{C_\lambda(t)} - \frac{\partial_\lambda C_\lambda(t+\tau)}{C_\lambda(t+\tau)}\right]\Bigg|_{\lambda=0}$$

$$-\left.\frac{\partial C_\lambda(t)}{\partial\lambda}\right|_{\lambda=0} = -C(t)\int dt'\langle\Omega|J(t')|\Omega\rangle + \int dt'\langle\Omega|T\{N(t)J(t')N^\dagger(0)\}|\Omega\rangle$$

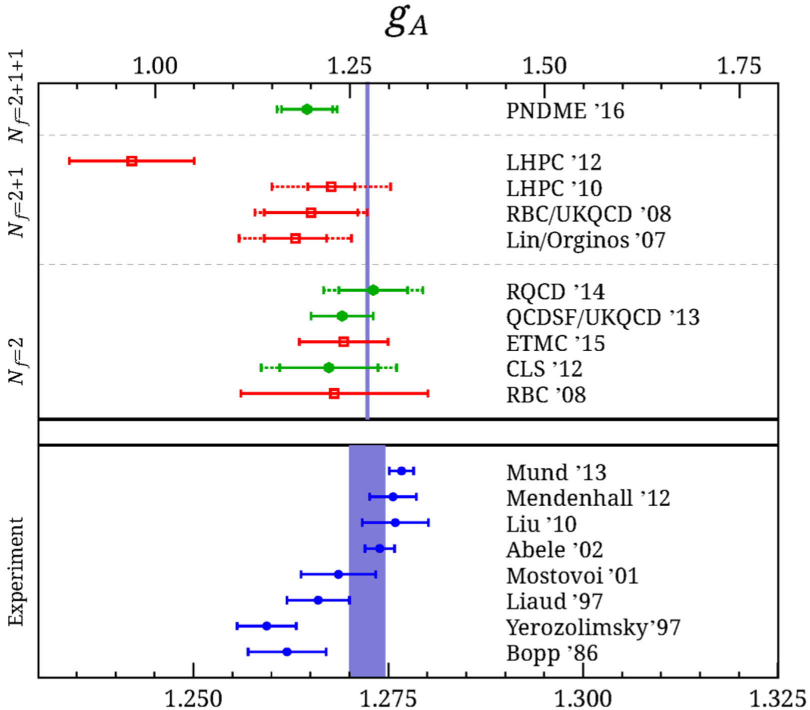
$$C(t) = \sum_n z_n z_n^\dagger e^{-E_n t}$$

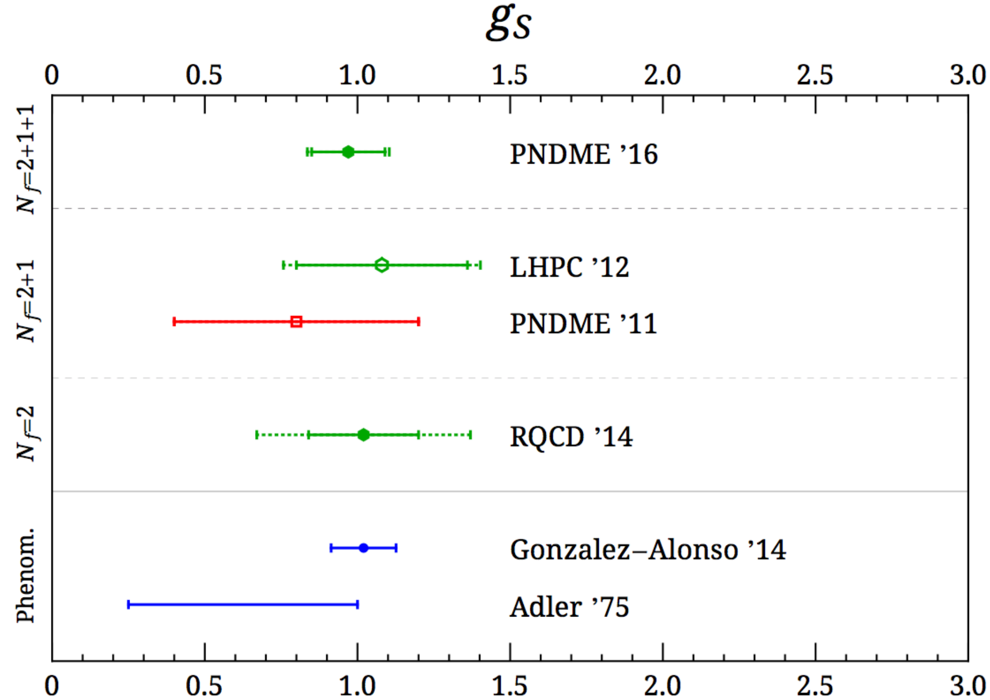
$$G_J(t) \equiv \frac{1}{\tau}\left[\frac{N_J(t+\tau)}{C(t+\tau)} - \frac{N_J(t)}{C(t)}\right]$$

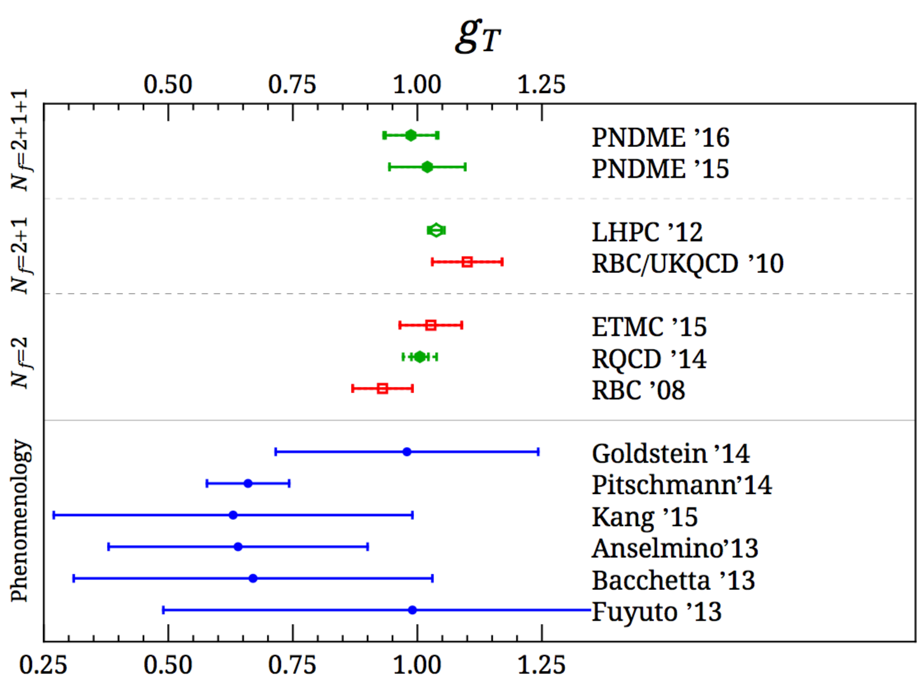


$T\epsilon_\pi^2 a^2$
$g_A^{LQCD}(\epsilon_\pi^{phys}, a/w_0)$
$g_A^{PDG} = 1.2723(23)$
$g_A(\epsilon_\pi^{(130)}, a/w_0)$
$g_A(\epsilon_\pi^{(220)}, a/w_0)$
$g_A(\epsilon_\pi^{(310)}, a/w_0)$
$m_\pi \sim 130$ MeV
$m_\pi \sim 220$ MeV
$m_\pi \sim 310$ MeV
g_A
$(a/w_0)^2$
$T\epsilon_\pi^2 a^2$
$g_A^{LQCD}(\epsilon_\pi, a=0)$
$g_A^{PDG} = 1.2723(23)$
$g_A(\epsilon_\pi, a=0.09)$
$g_A(\epsilon_\pi, a=0.12)$
$g_A(\epsilon_\pi, a=0.15)$
$a \sim 0.09$ fm
$a \sim 0.12$ fm
$a \sim 0.15$ fm
g_A
$\epsilon_\pi = m_\pi/(4\pi F_\pi)$
$\chi\epsilon_\pi^2 a^2$
$g_A^{LQCD}(\epsilon_\pi^{phys}, a/w_0)$
$g_A^{PDG} = 1.2723(23)$
$g_A(\epsilon_\pi^{(130)}, a/w_0)$
$g_A(\epsilon_\pi^{(220)}, a/w_0)$
$g_A(\epsilon_\pi^{(310)}, a/w_0)$
$m_\pi \sim 130$ MeV
$m_\pi \sim 220$ MeV
$m_\pi \sim 310$ MeV
g_A
$(a/w_0)^2$
$\chi\epsilon_\pi^2 a^2$
$g_A^{LQCD}(\epsilon_\pi, a=0)$
$g_A^{PDG} = 1.2723(23)$
$g_A(\epsilon_\pi, a=0.09)$
$g_A(\epsilon_\pi, a=0.12)$
$g_A(\epsilon_\pi, a=0.15)$
$a \sim 0.09$ fm
$a \sim 0.12$ fm
$a \sim 0.15$ fm
g_A
$\epsilon_\pi = m_\pi/(4\pi F_\pi)$

$$g_A = 1.278(21)(26)$$



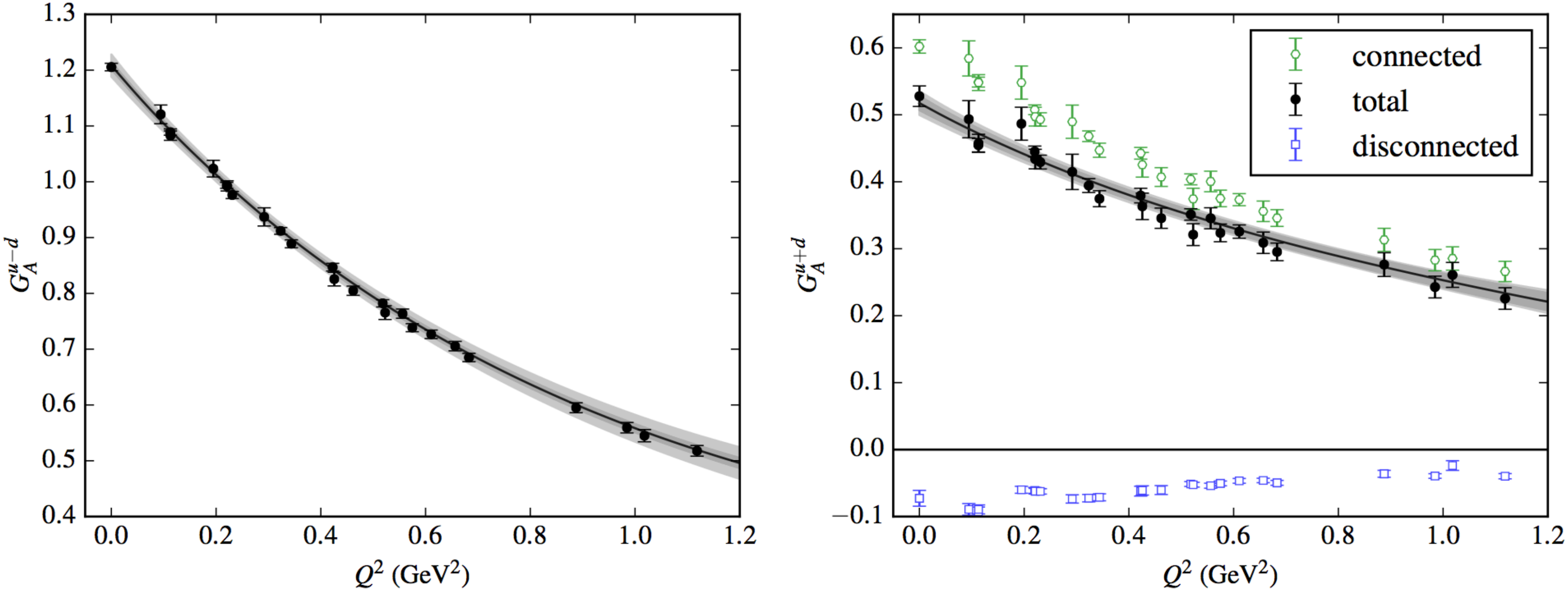


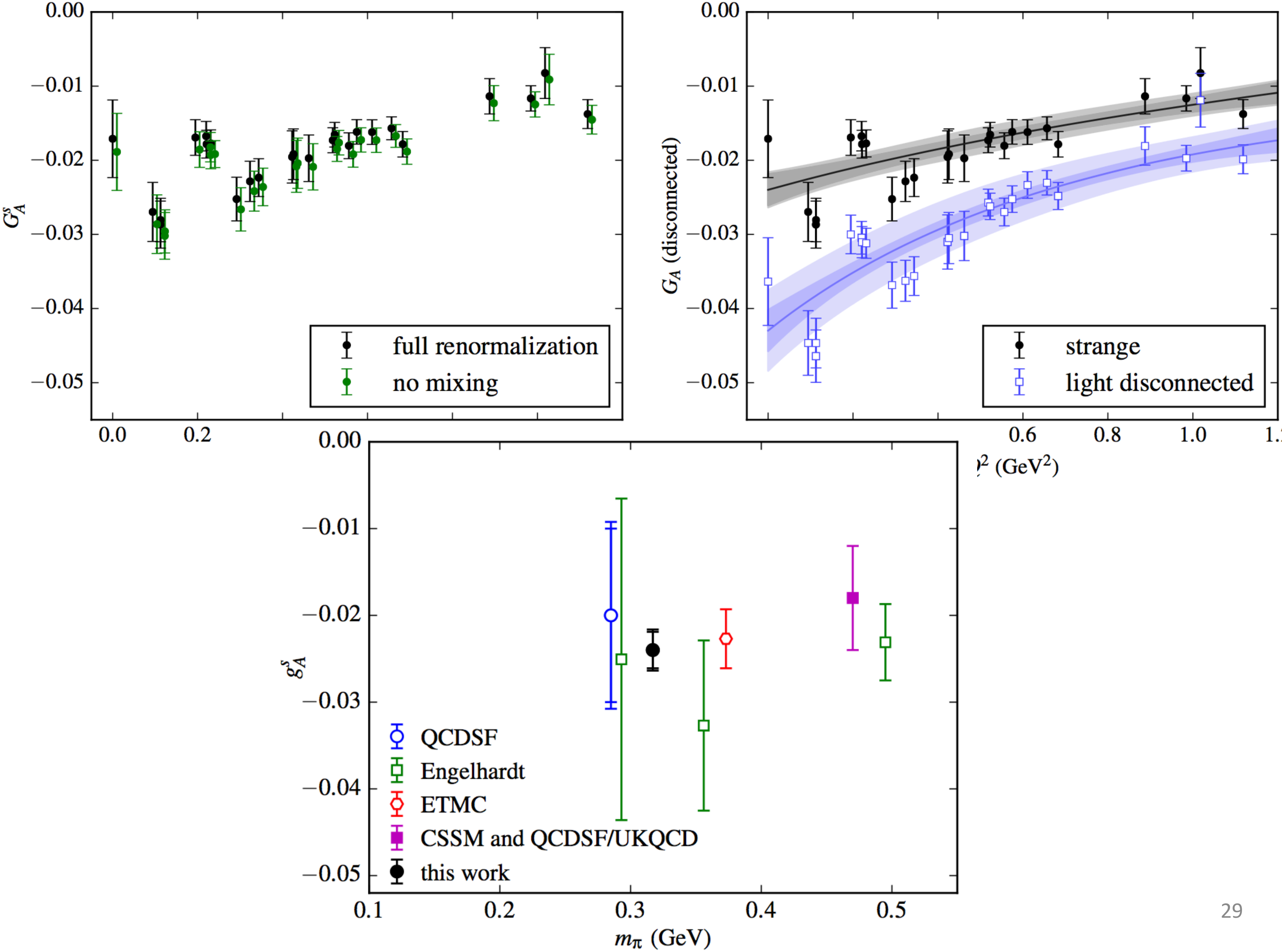


Tanmoy Bhattacharya *et. al*,
Phys.Rev. D94 (2016) no.5, 054508

Up, down, and strange nucleon axial form factors

Jeremy Green *et. al*. *Phys. Rev. D* 95, 114502 (2017)

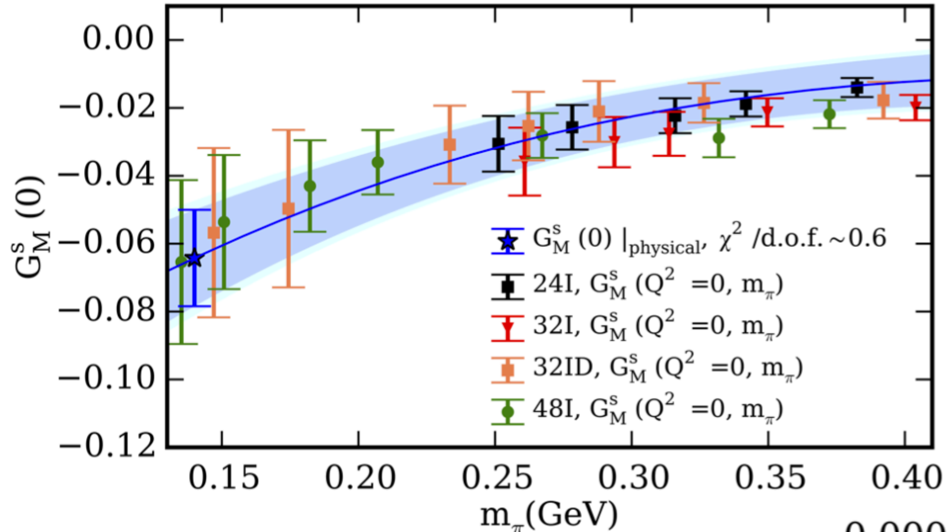


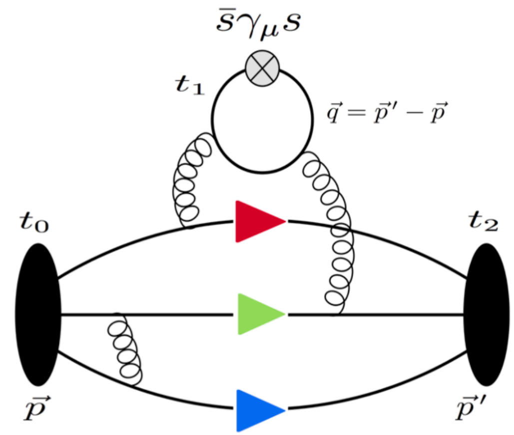

0.00
−0.01
−0.02
−0.03
−0.04
−0.05
G_A^s
0.0
0.2
full renormalization
no mixing
G_A (disconnected)
strange
light disconnected
0.6
0.8
1.0
1.2
Q^2 (GeV^2)
g_A^s
QCDSF
Engelhardt
ETMC
CSSM and QCDSF/UKQCD
this work
0.1
0.2
0.3
0.4
0.5
m_π (GeV)

Strange quark magnetic moment of the nucleon

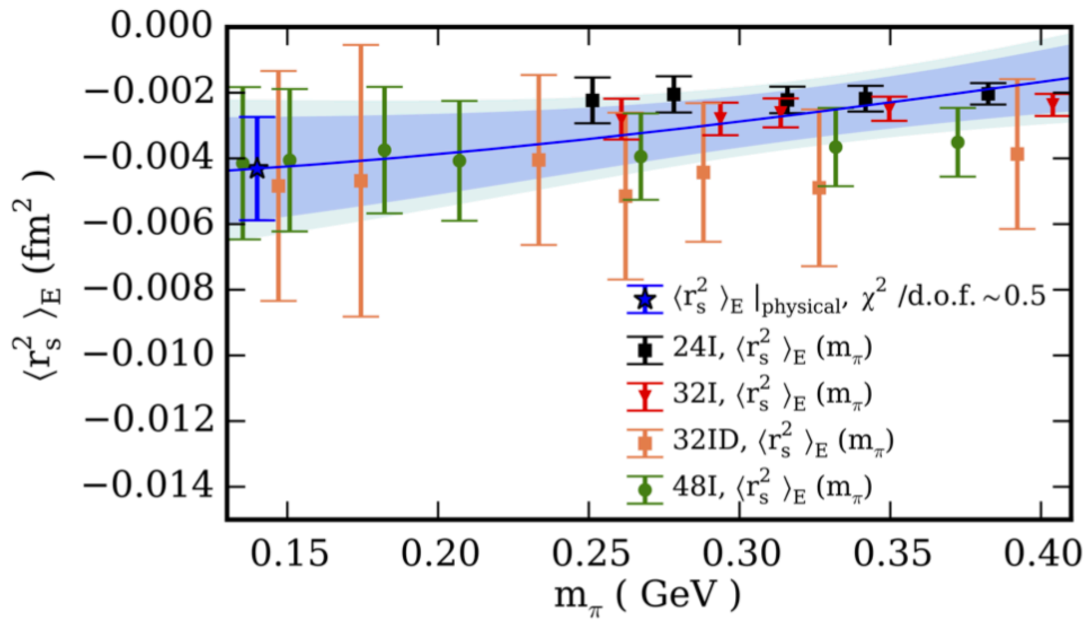
Raza Sufian, Phys. Rev. Lett.118.042001

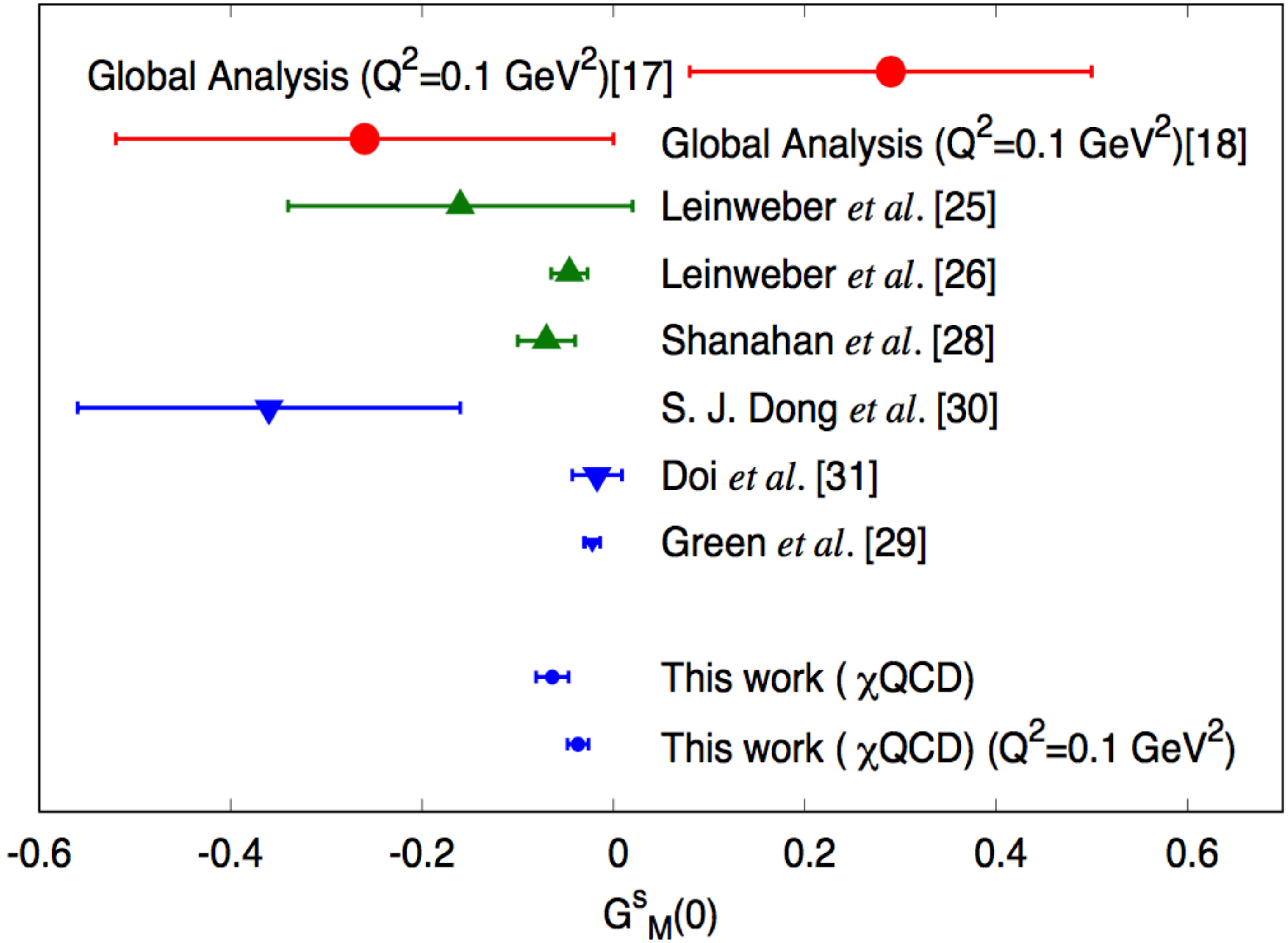
(at physical pion mass with Domain-wall fermions)





- Ratio 3pt/2pt method
- Z-expansion





Strange quark magnetic moment

More calculations:

- Nasreen Hasan *et. al.,* arXiv:1611.01383 – Nucleon Dirac and Pauli form factor
- S. Capitani *et. al.,* arXiv:1705.06186 - Iso-vector axial form factors of the nucleon in two-flavour lattice QCD
- C. Alexandrou *et. al.*, *arXiv:*1705.06186 - The nucleon axial form factors using lattice QCD simulations with a physical value of the pion mass
- Chris Bouchard *et. al.,* Phys. Rev. D96, 014504 - On the Feynman-Hellmann theorem in quantum field theory and the calculation of matrix elements
- Chris Bouchard *et. al*. –PoS(Lattice2016),160 - Matrix elements from moments of correlation functions

Some more calculations -

J Liang *et. al.*, - Phys. Rev. D.96.034519 - Lattice Calculation of Nucleon Isovector Axial Charge with Improved Currents

Raza Sufian *et. al*., arXiv:1705.05849 **-** Sea Quarks Contribution to the Nucleon Magnetic Moment and Charge Radius at the Physical Point

Tanmoy Bhattacharya et. al, Phys. Rev. D.92.094511 - Isovector and Isoscalar Tensor Charges of the Nucleon from Lattice QCD

Outlook

Immediate goals (JLab form factor program):

- Pion form factor at $Q^2 \geq 6$ GeV^2
- Extend to more ensembles with lighter pion masses , multiple volumes, multiple lattice spacing
- Take care of lattice artefacts
- Nucleon axial charge using distillation

Next:

- Distribution amplitude,
- TMDs, GPDs ….