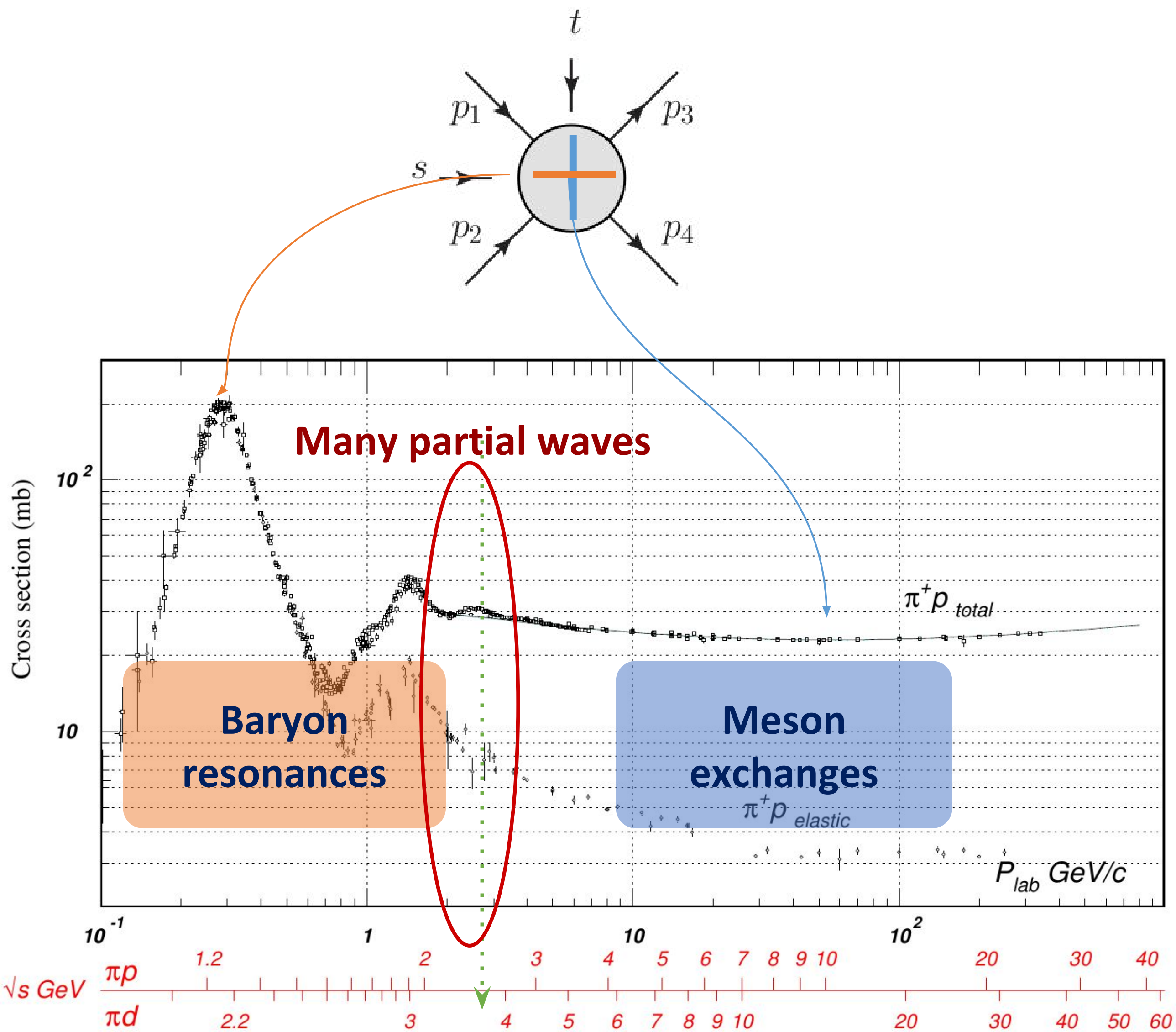
Finite-Energy Sum Rules:
Going high to solve low-energy issues

Jannes Nys
JPAC Collaboration

Overview



Connect low- and high-energy dynamics.

Dispersion relations - FESR

$A_i(\nu, t)$

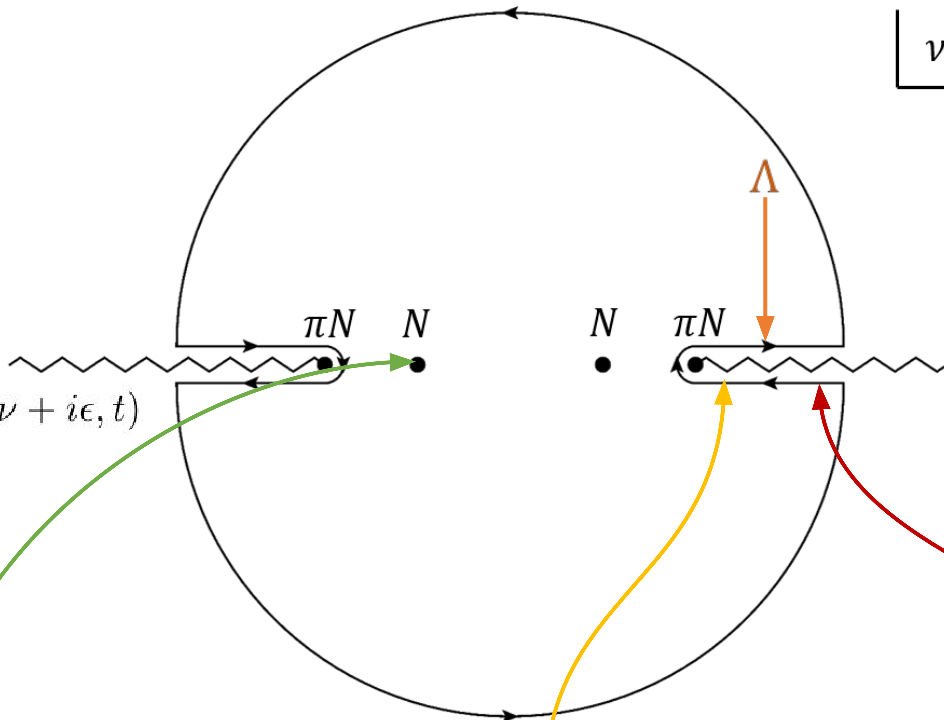
ν

Fixed t

Crossing variable: $\nu = \frac{s-u}{4\,M}$

s-channel: $+\,\nu$

u-channel: $-\,\nu$



$$A_i^\sigma(-\nu - i\epsilon, t) = \xi_i A_i^\sigma(\nu + i\epsilon, t)$$

$$-\beta_i(t)\frac{\tau + e^{-i\pi\alpha(t)}}{\sin \pi\alpha(t)}(r_i\nu)^{\alpha(t)-1}$$

$$\pi B_i^\sigma(t)\left(\frac{\nu_M}{\Lambda}\right)^k + \int_{\nu_\pi}^{\Lambda} \mathrm{Im}\ A_i^\sigma(\nu', t)\left(\frac{\nu'}{\Lambda}\right)^k d\nu' = \beta_i^\sigma(t)\frac{(\Lambda/\nu_0)^{\alpha(t)}}{\alpha(t)+k}\nu_0$$

Nucleon pole — Low-energy model — Regge pole

Analyticity results in Finite-Energy Sum Rules.

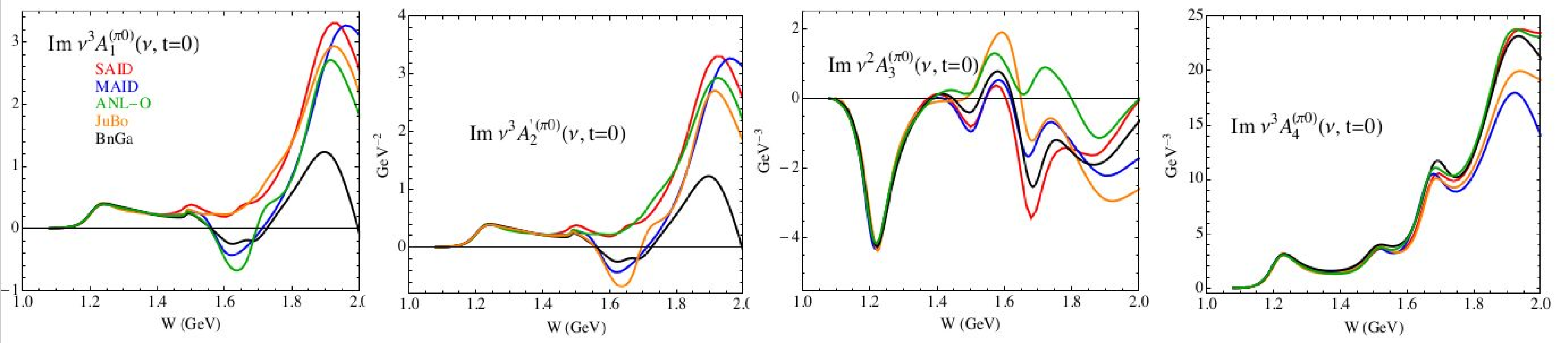
Low energies

$$\int_{\nu_\pi}^{\Lambda} \mathrm{Im}\ A_i^\sigma(\nu', t) \left(\frac{\nu'}{\Lambda}\right)^k d\nu'$$

Low energy models

- BnGa, Julich-Bonn, ANL-Osaka, SAID, MAID,...

$\gamma N \to \pi^0 N$

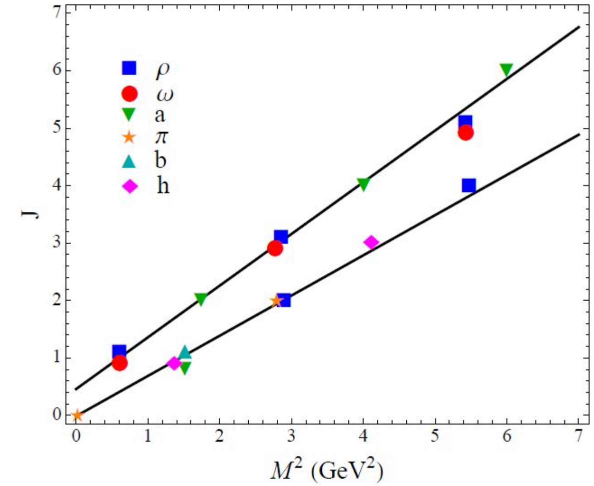


High energies

Regge pole model

$$A_{i,R}(\nu,t) = -\beta_i(t)\frac{\tau + e^{-i\pi\alpha(t)}}{\sin\pi\alpha(t)}(r_i\nu)^{\alpha(t)-1}$$

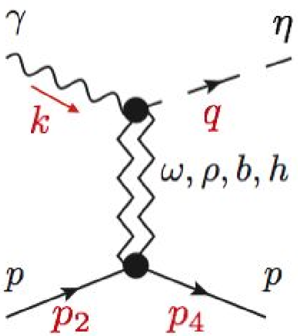
$J(m^2)$



Dominant: vector exchanges

A_i	I^G	J^{PC}	η	Leading exchanges
A_1	$0^-, 1^+$	$(1,3,5,...)^{--}$	$+1$	$\rho(770), \omega(782)$
A'_2	$0^-, 1^+$	$(1,3,5,...)^{+-}$	-1	$h_1(1170), b_1(1235)$
A_3	$0^-, 1^+$	$(2,4,...)^{--}$	-1	$\rho_2(??), \omega_2(??)$
A_4	$0^-, 1^+$	$(1,3,5,...)^{--}$	$+1$	$\rho(770), \omega(782)$

$A'_2 = A_1 + tA_2$

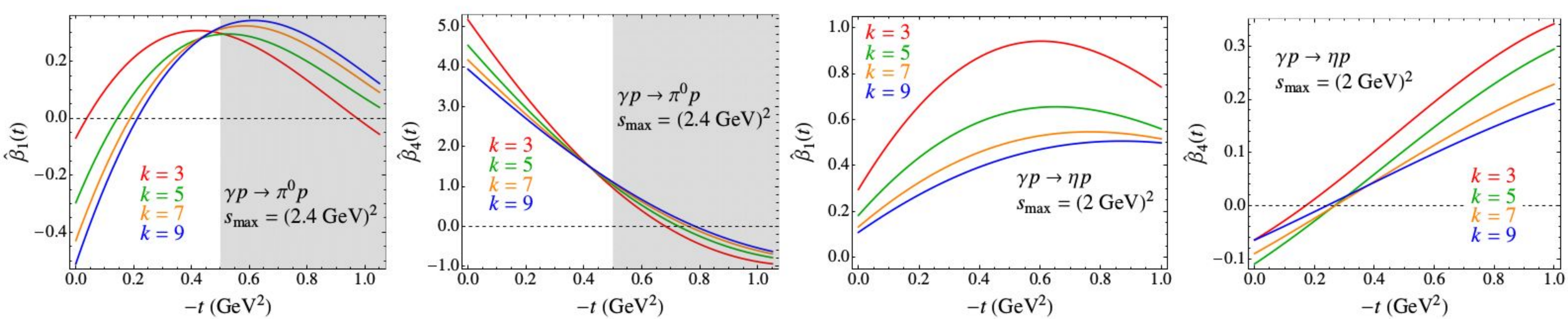


$$\gamma p \to \eta p\,, \qquad A = (\omega + h + \omega_2) + (\rho + b + \rho_2)$$
$$\gamma n \to \eta n\,, \qquad A = (\omega + h + \omega_2) - (\rho + b + \rho_2)$$

Sensitivity to k

$$\pi B_i^\sigma(t)\left(\frac{\nu_M}{\Lambda}\right)^k + \int_{\nu_\pi}^{\Lambda} \mathrm{Im}\; A_i^\sigma(\nu',t)\left(\frac{\nu'}{\Lambda}\right)^k d\nu' = \beta_i^\sigma(t)\frac{(\Lambda/\nu_0)^{\alpha(t)}}{\alpha(t)+k}\nu_0$$

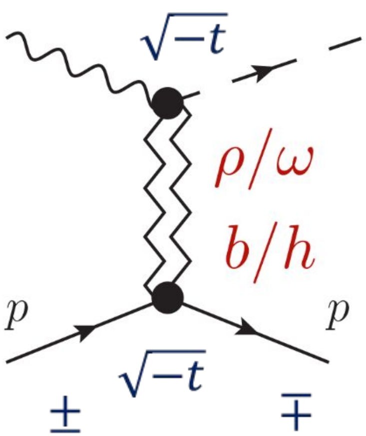
$$\widehat{\beta}_i(t) = \frac{\alpha(t)+k}{\Lambda^{\alpha(t)+k}} \int_0^{\Lambda} \mathrm{Im}\, A_i^{\mathrm{PWA}}(\nu,t)\,\nu^k\,\mathrm{d}\nu$$



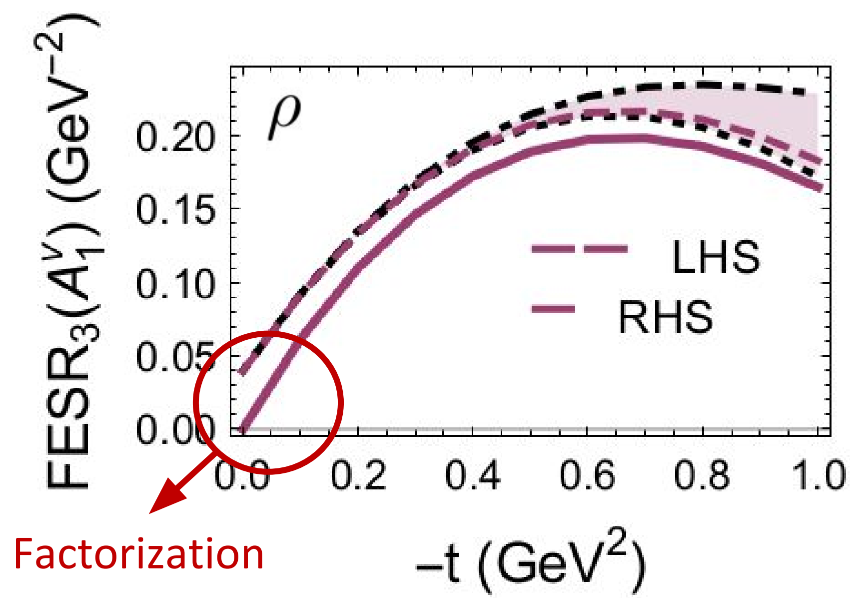
Matching: natural exchanges

$$\pi B_i^\sigma(t)\left(\frac{\nu_M}{\Lambda}\right)^k + \int_{\nu_\pi}^{\Lambda} \mathrm{Im}\, A_i^\sigma(\nu',t)\left(\frac{\nu'}{\Lambda}\right)^k d\nu' = \beta_i^\sigma(t)\frac{(\Lambda/\nu_0)^{\alpha(t)}}{\alpha(t)+k}\nu_0$$

Nucleon pole — Low-energy model — Regge pole

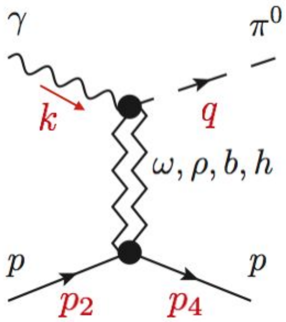


$$\text{ang. mom. : } A_1 \sim 1$$
$$\text{single pole : } A_1 \sim t$$

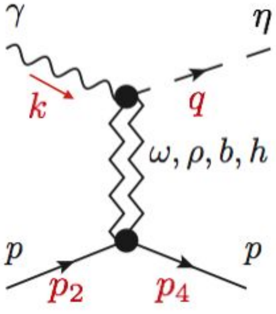


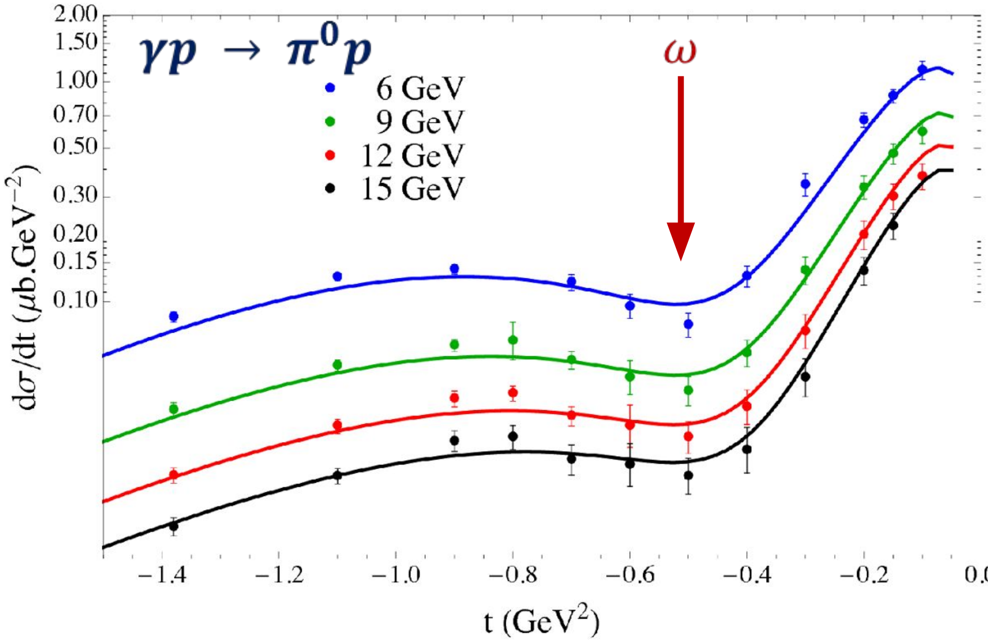
$$F_3 = 2\,M_N A_1 - t A_4$$

Data

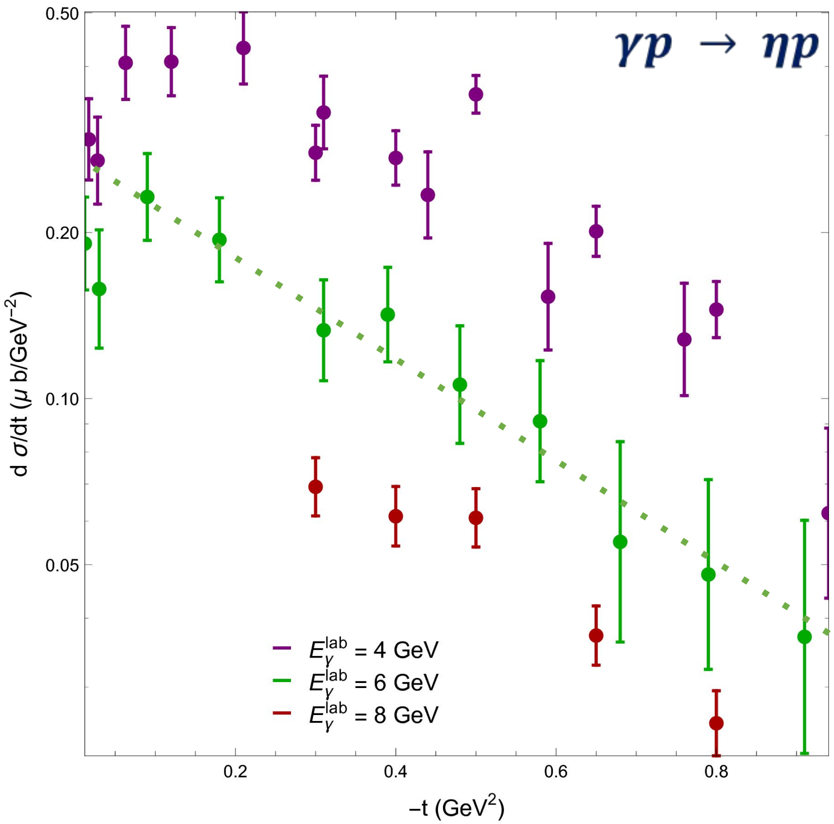


$$A(\eta) = \sqrt{3}\mathcal{A}\left[A_\rho(\pi^0) + A_b(\pi^0) + \frac{1}{9}(A_\omega(\pi^0) + A_h(\pi^0))\right]$$



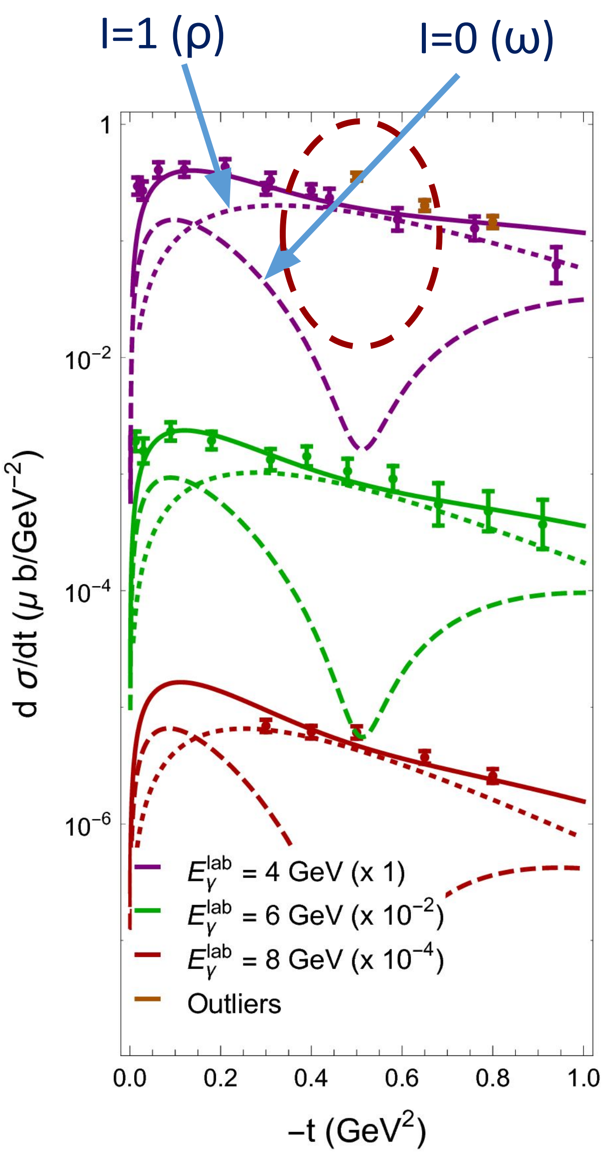


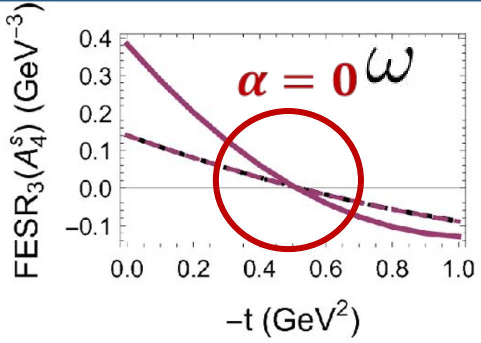
[V. Mathieu]

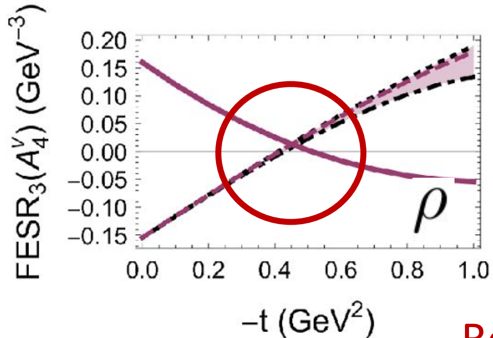


[Data: Dewire 1971, Braunschweig 1970]

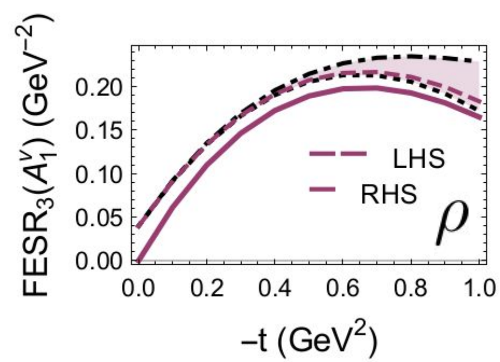
Results





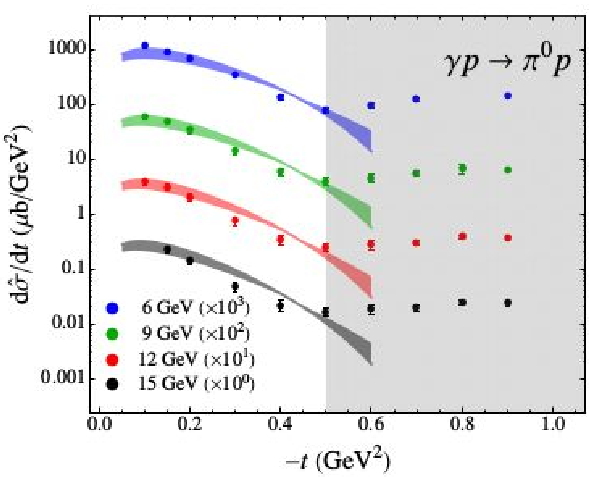


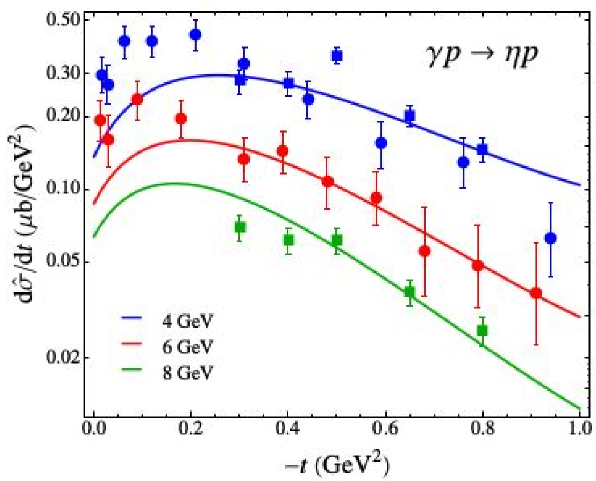
Remove spin 0 component (NWSZ)



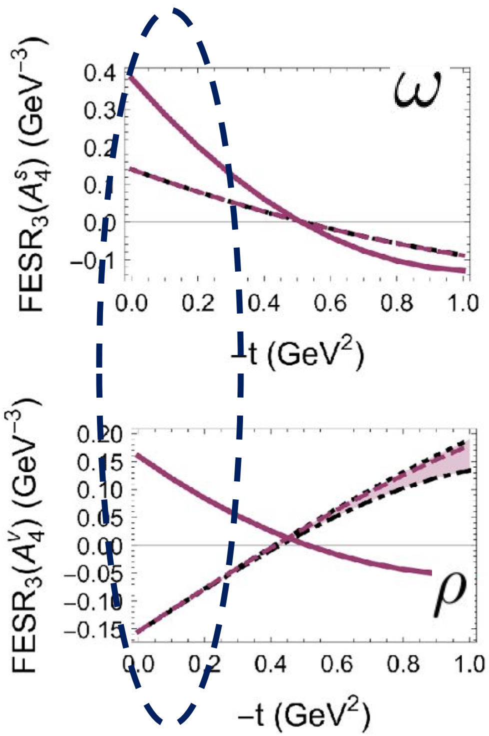
Results

$$\widehat{\beta}_i(t) = \frac{\alpha(t)+k}{\Lambda^{\alpha(t)+k}} \int_0^{\Lambda} \mathrm{Im}\, A_i^{\mathrm{PWA}}(\nu, t)\, \nu^k \, \mathrm{d}\nu$$

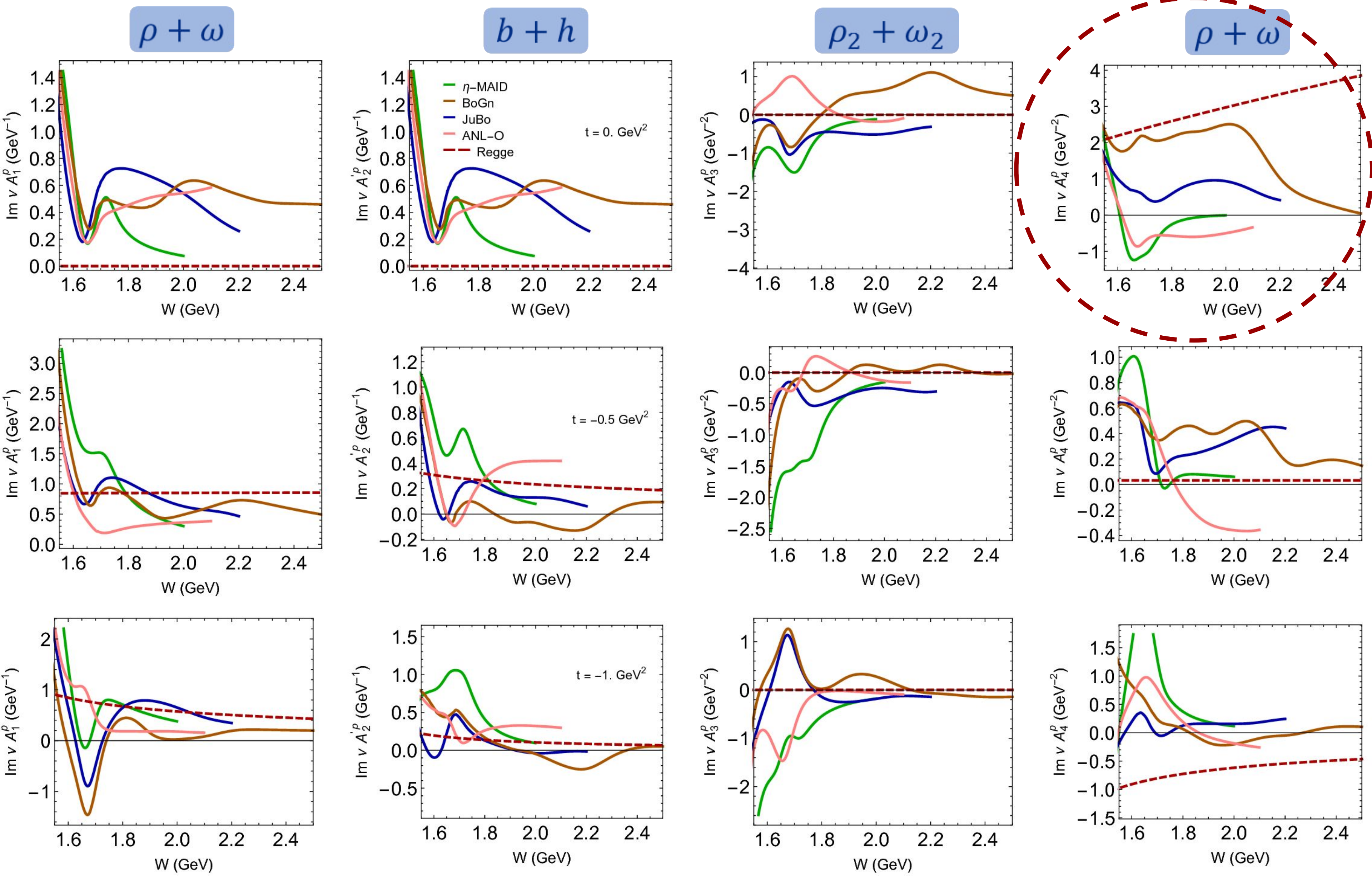




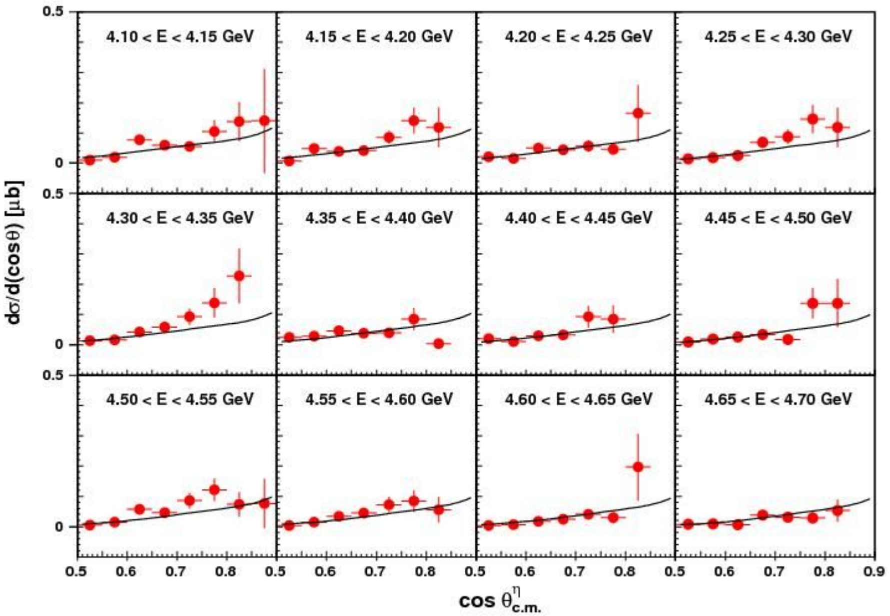
[V. Mathieu, J.N. *et al.* (JPAC) 1708.07779 (2017)]



Comparison for $\gamma p \rightarrow \eta p$



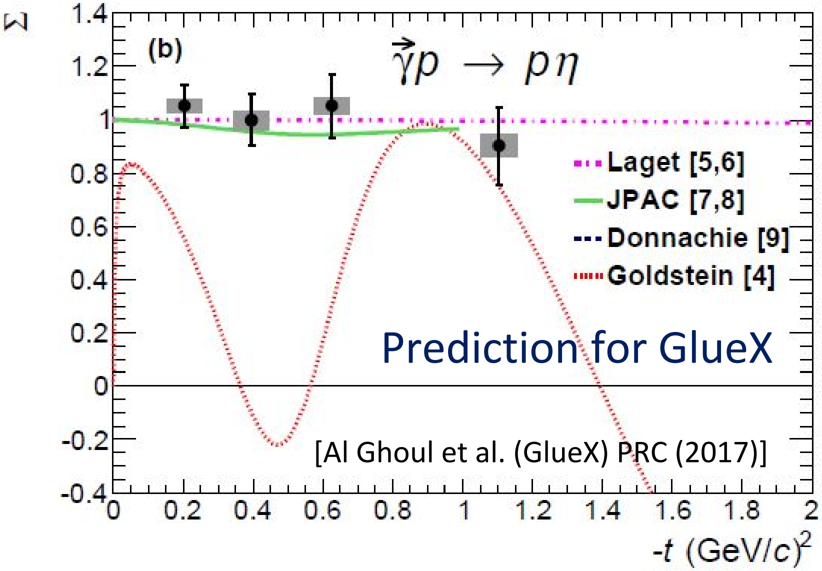
Predictions for GlueX & CLAS



Preliminary (transition region)
[Courtesy of Zulkaida Akbar (**CLAS**)]

Natural dominant: $\Sigma = +1$

Unnatural dominant: $\Sigma = -1$



Fill up the dip with natural contribution: ρ

η'/η beam asymmetry

$$\Sigma^{(\prime)} = \frac{d\sigma_\perp^{(\prime)} - d\sigma_\parallel^{(\prime)}}{d\sigma_\perp^{(\prime)} + d\sigma_\parallel^{(\prime)}}$$ for $\eta^{(\prime)}$

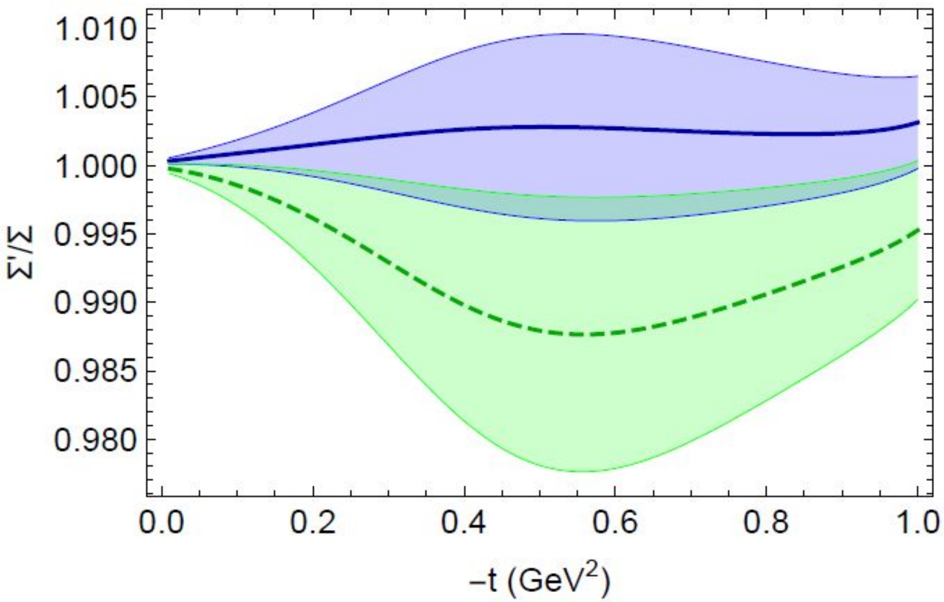
$$\frac{\Sigma'}{\Sigma} = 1 + \frac{1-\Sigma^2}{\Sigma} \cdot \frac{k_V - k_A}{(1+\Sigma)k_V + (1-\Sigma)k_A}$$

$$k_V = \frac{d\sigma'_\perp}{d\sigma_\perp}, \qquad k_A = \frac{d\sigma'_\parallel}{d\sigma_\parallel}.$$

Quark model predictions:

$$k_V = k_A = \tan^2\varphi$$

V.Mathieu, J.N. *et al.* (JPAC) [arXiv:1704.07684]



Dominant exchanges: ρ, ω
Variations: b, h radiative decays

Sizable deviation from 1:

- Non-negligible contributions from **hidden strangeness**
- Signicant deviation from the quark model description

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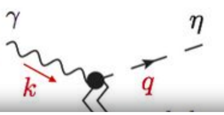
This project is supported by NSF

$\gamma p \to \eta p$



We present the model published in [Nys16] .
The differential cross section for $\gamma p \to \eta p$ is computed with Regge amplitudes in the domain $E_\gamma \geq 4$ GeV and $0 \leq -t \leq 1$ (in GeV^2).
We use the CGLN invariant amplitudes A_i defined in [Chew57a].
See the section Formalism for the definition of the variables.
The model and its context is detailed in [Nys16] . We report here only th
features of the model.

Formalism

The differential cross section is a function of 2 kinematic variables. The f
laboratory frame E_γ (in GeV) or the total energy squared s (in GeV^2). T
scattering angle in the rest frame $\cos\theta$ or the momentum transfered squar
The momenta of the particles are k (photon), q (eta), p_2 (target) and p_4 (r
μ and the proton mass is M_N. The Mandelstam variables, $s = (k + p_2)^2$,
are related through $s + t + u = 2M_N^2 + \mu^2$. Furthermore, we intro

- **Publication:** [Nys16]
- **C/C++ observables:** C-code main, Input file, C-code source, C-code header, Eta-MAID 2001 multip
- **C/C++ minimal script to calculate the amplitudes:** C-code zip
- **Data:** Dewire , Braunschweig
- **Contact person:** Jannes Nys
- **Last update:** November 2016

Step-by-step introduction to calculating the model amplitudes of the high-energy model.

[hide] [show]

Run the code

Choose the beam energy in the lab frame E_γ, the other variable (t or $\cos\theta$) and its minimal, maximal, and increment values.
If you choose t (cos) only the min, max and step values of t ($\cos\theta$) are read.
Only physical t-values are calculated. Hence, for example $t = 0$ will be set to $t(\cos\theta = +1)$. Below $W = 2$ GeV, we use the Eta-MAID 2001 model using the lowest $l \leq 5$ multipoles. Above $W = 2$ GeV, the Regge model is evaluated. There is no smooth transition.

E_γ in GeV 9

t cos

t in GeV^2 (min max step) -1 0 0.01

$\cos\theta$ (min max step) 0.85 1 0.01

Start reset

November 2016:

- The $\gamma p \to \eta p$ page is online.

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Summary

[J.N., et al. (JPAC) PRD (2017)]
[V. Mathieu, J.N. et al. (JPAC) 1708.07779 (2017)]

- **Finite-Energy Sum Rules** relate high and low energy regimes
  - Low-energy models provide detailed predictions for high-energy data
  - Information at the amplitude level
  - Ultimately: combined fit of low- and high-energy data

[V. Mathieu, J.N., et al. (JPAC) 1704.07684]

- η/η' beam asymmetry
  - Source of information about *b* and *h* radiative decays
  - Sensitive to hidden strangeness

Backup

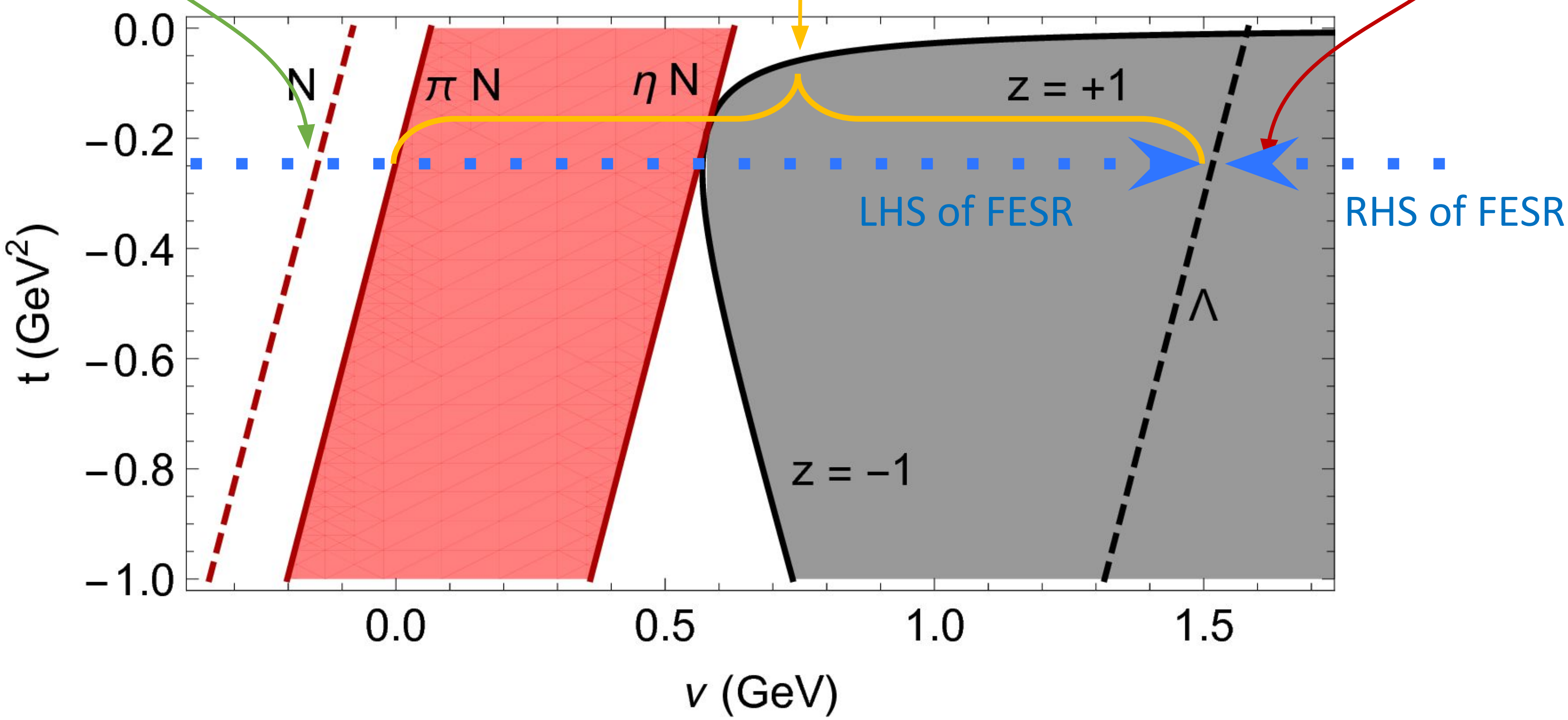
Dispersion relations

$$\pi B_i^\sigma(t)\left(\frac{\nu_M}{\Lambda}\right)^k + \int_{\nu_\pi}^{\Lambda} \text{Im } A_i^\sigma(\nu',t)\left(\frac{\nu'}{\Lambda}\right)^k d\nu' = \beta_i^\sigma(t)\,\frac{(\Lambda/\nu_0)^{\alpha(t)}}{\alpha(t)+k}\,\nu_0$$

Nucleon pole

Low-energy model

Regge pole



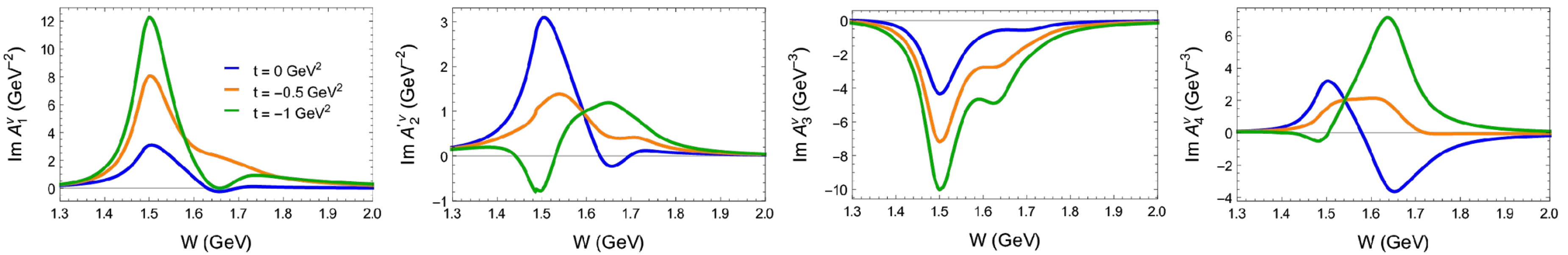
Low energies

$$\int_{\nu_\pi}^{\Lambda} \text{Im } A_i^\sigma(\nu', t) \left(\frac{\nu'}{\Lambda}\right)^k d\nu'$$

Low energy models

- BnGa, Julich-Bonn, ANL-Osaka, SAID, **MAID**

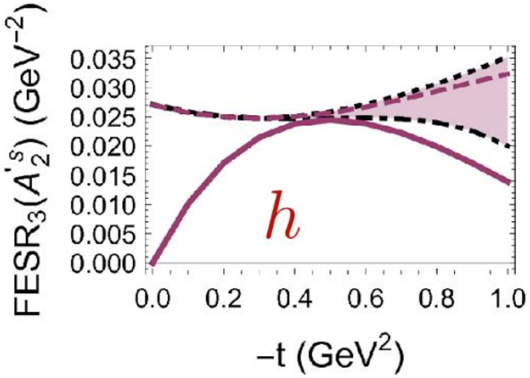
$$\gamma N \to \eta N$$

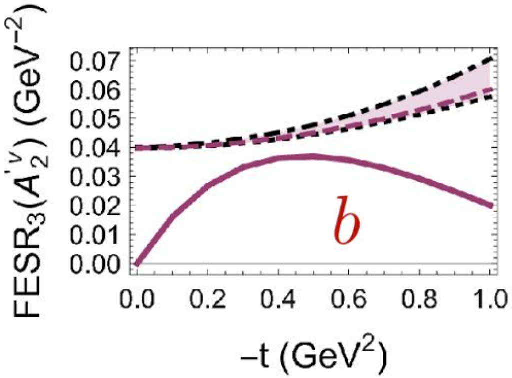


Matching: unnatural exchanges

$$\pi B_i^\sigma(t)\left(\frac{\nu_M}{\Lambda}\right)^k + \int_{\nu_\pi}^{\Lambda} \text{Im}\, A_i^\sigma(\nu',t)\left(\frac{\nu'}{\Lambda}\right)^k d\nu' = \beta_i^\sigma(t)\frac{(\Lambda/\nu_0)^{\alpha(t)}}{\alpha(t)+k}\nu_0$$

Nucleon pole — Low-energy model — Regge pole





Look for unnatural contributions in the beam asymmetry

Formalism

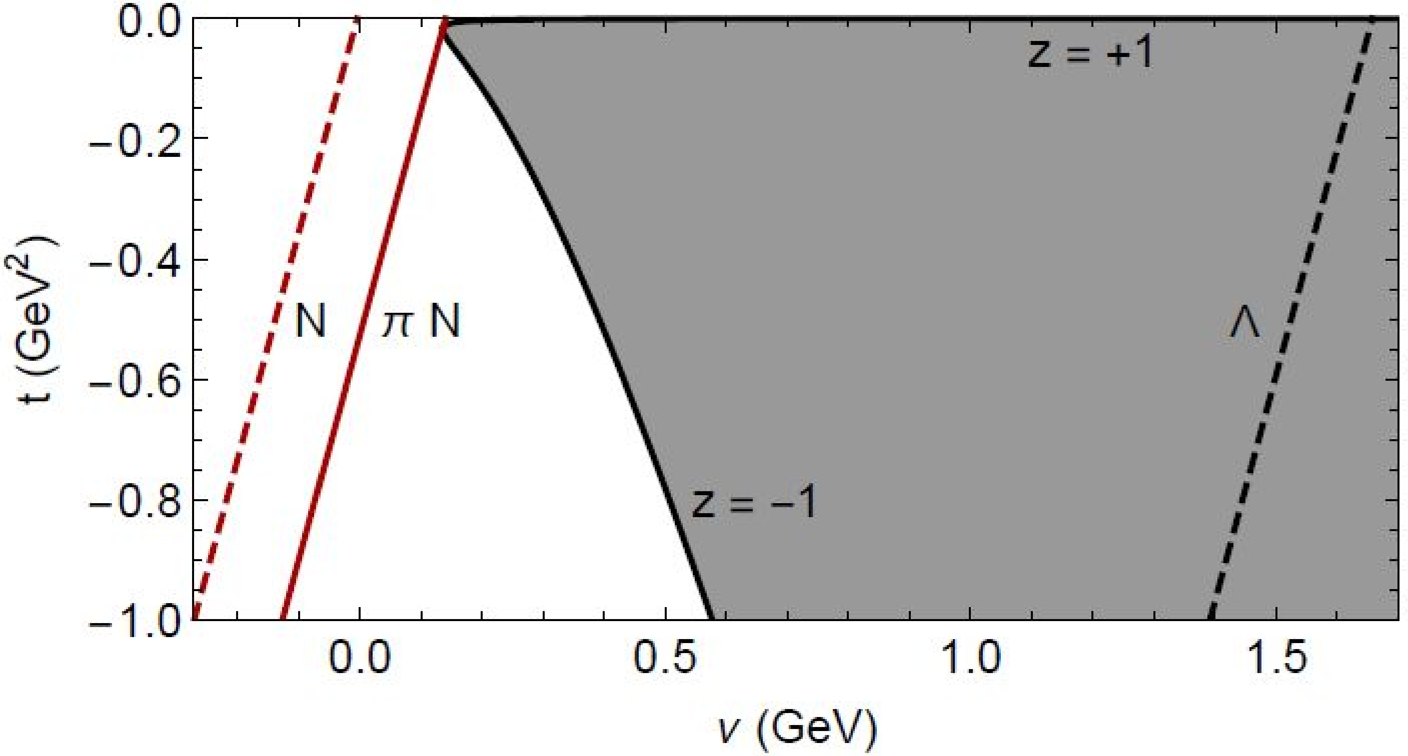
$$A_{\lambda';\lambda\,\lambda_\gamma}(s,t) = \overline{u}_{\lambda'}(p')\left(\sum_{k=1}^{4} A_k(s,t) M_k\right) u_\lambda(p)$$

- No kinematic singularities
- No kinematic zeros
- Discontinuities:
  - Unitarity cut
  - Nucleon pole

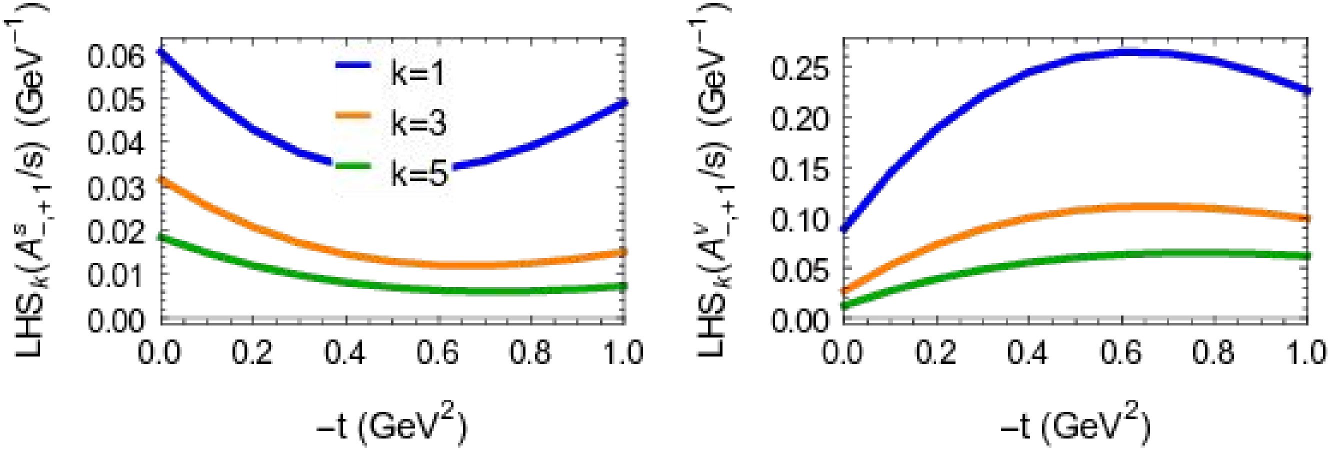
$$M_k \equiv M_k(s,t,\lambda_\gamma)$$

$$M_1 = \frac{1}{2}\gamma_5\gamma_\mu\gamma_\nu F^{\mu\nu}\,,$$
$$M_2 = 2\gamma_5 q_\mu P_\nu F^{\mu\nu}\,,$$
$$M_3 = \gamma_5\gamma_\mu q_\nu F^{\mu\nu}\,,$$
$$M_4 = \frac{i}{2}\epsilon_{\alpha\beta\mu\nu}\gamma^\alpha q^\beta F^{\mu\nu}\,.$$

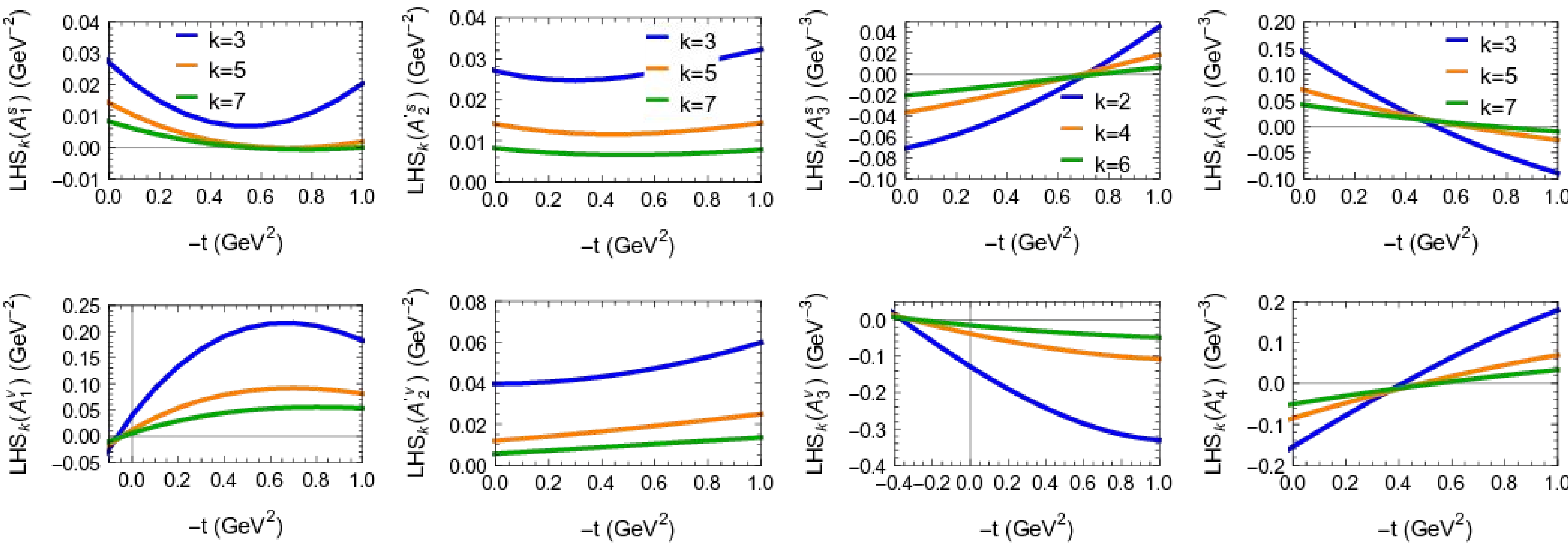
$$\alpha_{1,4}^{(\sigma)} \equiv \alpha_N(t) = 0.9(t - m_\rho^2) + 1$$
$$\alpha_{2,3}^{(\sigma)} \equiv \alpha_U(t) = 0.7(t - m_\pi^2) + 0$$



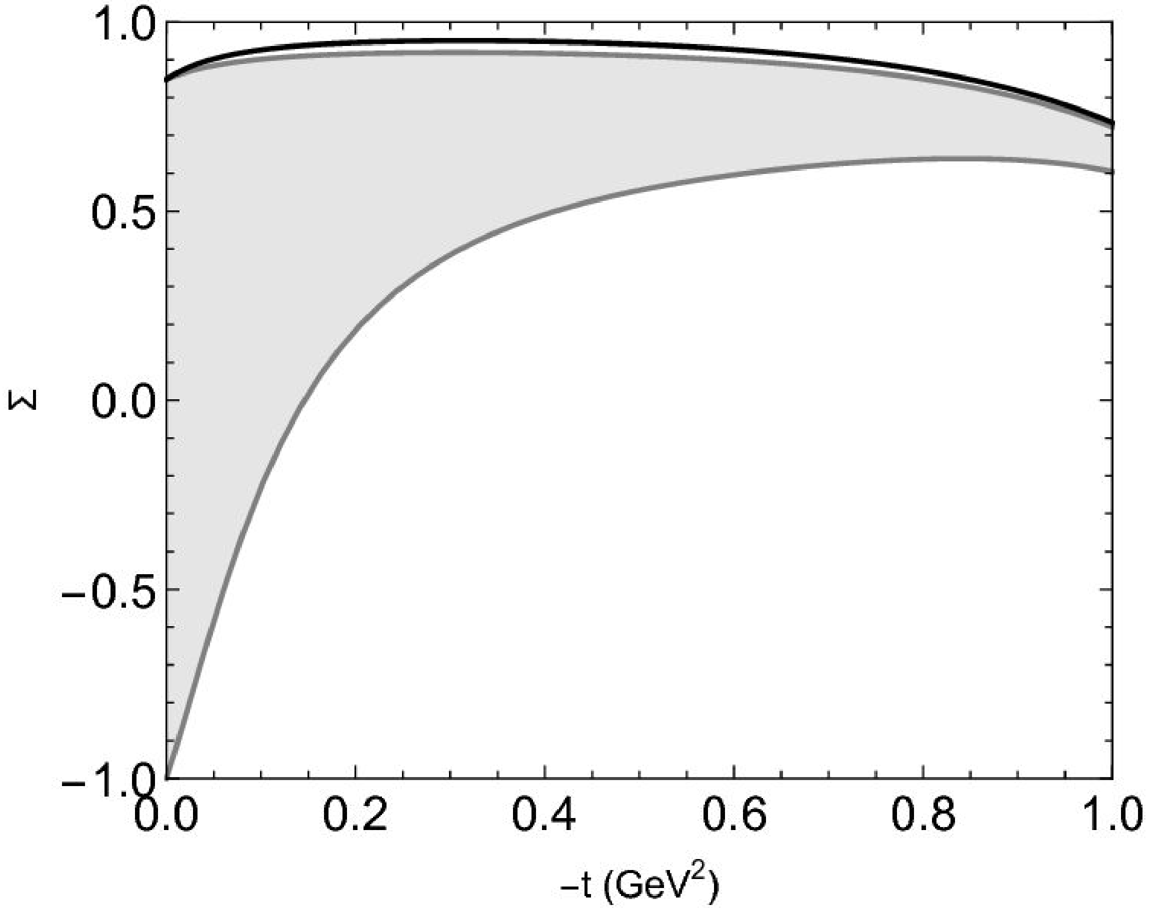
0.0
−0.2
−0.4
−0.6
−0.8
−1.0
t (GeV²)
0.0
0.5
1.0
1.5
ν (GeV)
N
π N
z = +1
z = −1
Λ



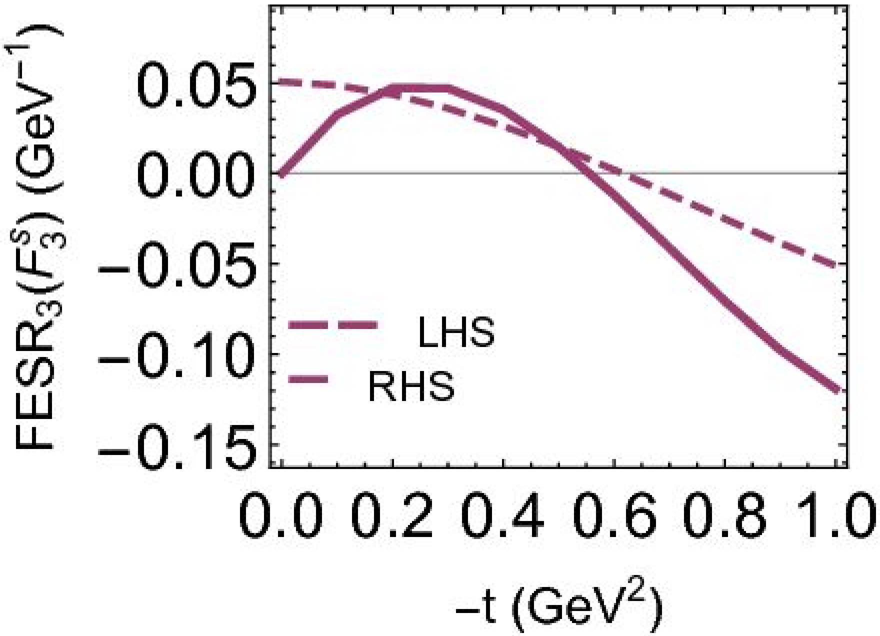
k=1
k=3
k=5
0.00
0.01
0.02
0.03
0.04
0.05
0.06
0.0
0.2
0.4
0.6
0.8
1.0
LHS$_k(A^s_{-,+1}/s)$ (GeV^{-1})
−t (GeV^2)
0.00
0.05
0.10
0.15
0.20
0.25
LHS$_k(A^v_{-,+1}/s)$ (GeV^{-1})
−t (GeV^2)

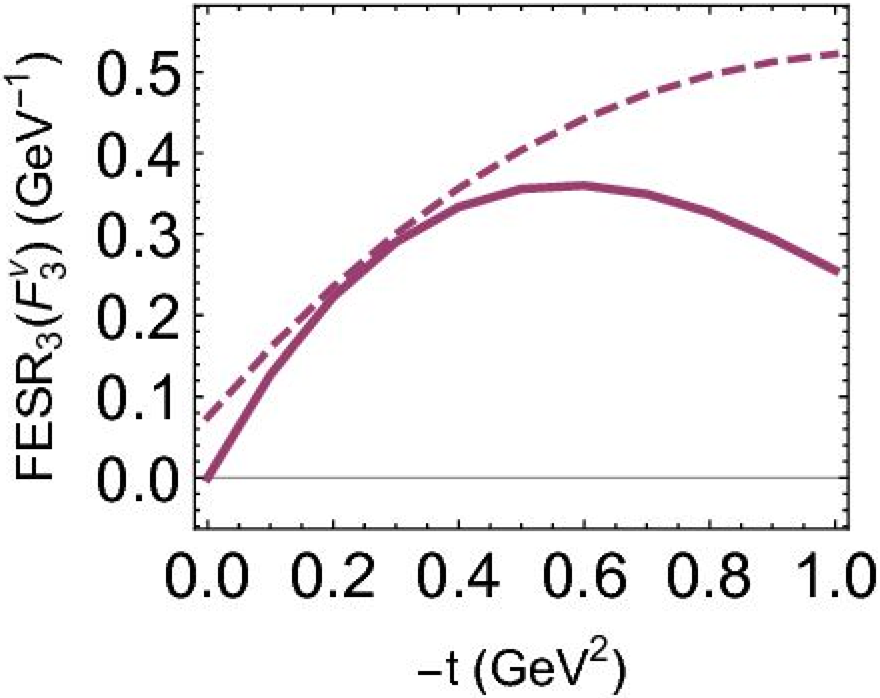


LHSk(A1s) (GeV−2)
k=3
k=5
k=7
−t (GeV2)
LHSk(A'2s) (GeV−2)
k=3
k=5
k=7
−t (GeV2)
LHSk(A3s) (GeV−3)
k=2
k=4
k=6
−t (GeV2)
LHSk(A4s) (GeV−3)
k=3
k=5
k=7
−t (GeV2)
LHSk(A1v) (GeV−2)
−t (GeV2)
LHSk(A'2v) (GeV−2)
−t (GeV2)
LHSk(A3v) (GeV−3)
−t (GeV2)
LHSk(A4v) (GeV−3)
−t (GeV2)

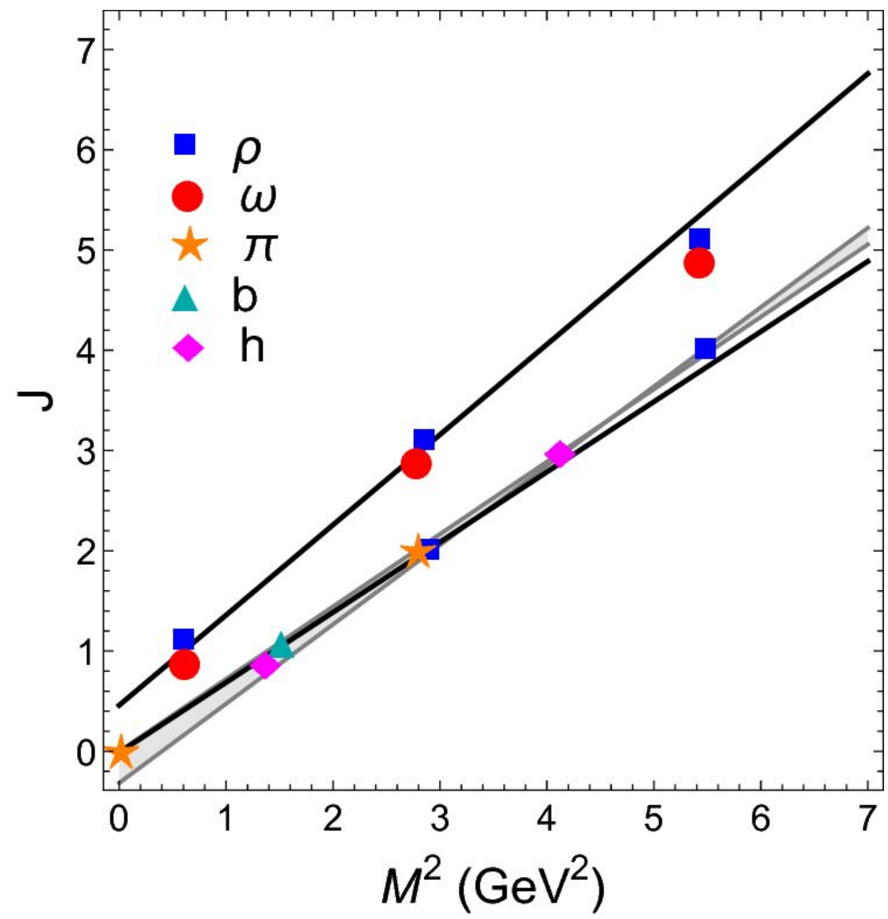


1.0
0.5
0.0
−0.5
−1.0
Σ
0.0
0.2
0.4
0.6
0.8
1.0
−t (GeV²)





$$F_3 = 2M_N A_1 - tA_4$$
$$F_4 = A_3\,.$$


ρ
ω
π
b
h
J
M^2 (GeV^2)
0
1
2
3
4
5
6
7

$$\frac{1}{\sqrt{2}s}\left(A_{+,+1}+A_{-,-1}\right)=\sqrt{-t}A_4 \tag{19}$$

$$\frac{1}{\sqrt{2}s}\left(A_{+,-1}-A_{-,+1}\right)=A_1 \tag{20}$$

$$\frac{1}{\sqrt{2}s}\left(A_{+,+1}-A_{-,-1}\right)=\sqrt{-t}A_3 \tag{21}$$

$$\frac{1}{\sqrt{2}s}\left(A_{+,-1}+A_{-,+1}\right)=-A_2'=-(A_1+tA_2) \tag{22}$$

Thus, at high energies the invariants A_3 and A_4 (A_1 and A_2') correspond to the s-channel nucleon-helicity non-flip (flip), respectively. Combining Eqs. (20) and (22) we obtain

$$A_{-,+1}=-\frac{s}{\sqrt{2}}\left(A_2'+A_1\right) . \tag{23}$$

$$A_{\mu_f,\mu_i\,\mu_\gamma} \underset{t\to 0}{\sim} (-t)^{n/2} , \tag{17}$$

where $n=|(\mu_\gamma-\mu_i)-(-\mu_f)|\geq 0$ is the net s-channel helicity flip. This is a weaker condition than the one imposed by angular-momentum conservation on factorizable Regge amplitudes,

$$A_{\mu_f,\mu_i\,\mu_\gamma} \underset{t\to 0}{\sim} (-t)^{(n+x)/2} , \tag{18}$$

where $n+x=|\mu_\gamma|+|\mu_i-\mu_f|\geq 1$. We summarize the