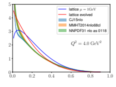
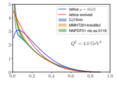


QuasiPDFs and Exploration of Pseudo-PDFs

A.V. Radyushkin (ODU/Jlab)
in collaboration with
K.Orginos, J. Karpie, S. Zafeiropoulos (W&M/Jlab)

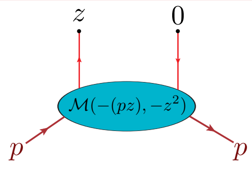
Workshop “Hadronic Physics with Lepton and Hadron Beams”
Jefferson Lab
September 8, 2017



Parton Densities and Matrix element

- Experimentally, one works with hadrons
- Theoretically, we work with quarks

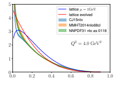
Hadron to Parton Transition



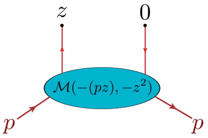
- Can be described in momentum or coordinate space
- Concept of PDFs does not rely on spin complications

$$\langle p|\phi(0)\phi(z)|p\rangle = \mathcal{M}(-(pz), -z^2)$$

- Lorentz: $\langle p|\phi(0)\phi(z)|p\rangle$ depends on z through (pz) and z^2



Ioffe-time distributions and Pseudo-PDFs

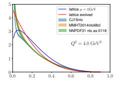


- $(pz) \equiv -\nu$ is Ioffe time [$(pz) = Mz^0$ in rest frame $p = \{M, 0, 0, 0\}$]
- $\mathcal{M}(\nu, -z^2)$ is Ioffe-time distribution
- Pseudo-PDF $\mathcal{P}(x, -z^2)$: Fourier transform with respect to (pz)

$$\mathcal{M}(-(pz), -z^2) = \int_{-1}^{1} dx\, e^{-ix(pz)}\, \mathcal{P}(x, -z^2)$$

- Inverse transformation

$$\mathcal{P}(x, -z^2) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\nu\, e^{-ix\nu}\, \mathcal{M}(\nu, -z^2)$$



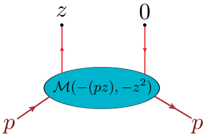
Collinear Parton Distributions and TMDs

Quasi- & PseudoPDFs

Parton Distributions
Matrix element
Pseudo-PDF
Collinear PDF
TMD
Quasi-PDF
Evolution

Lattice Results
Rest-frame density
Reduced ITD
Real Part
Imaginary Part
Antiquarks
Evolution

Summary



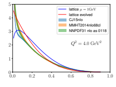
- Take light-like $z = z_-$: collinear parton distribution $f(x) = \mathcal{P}(x, 0)$

$$\mathcal{M}(-p_+ z_-, 0) = \int_{-1}^{1} dx\, f(x)\, e^{-ixp_+ z_-}$$

- Usual interpretation: parton carries fraction x of hadron p_+
- In QCD $\mathcal{M}(\nu, z^2)$ has $\sim \ln z^2$ singularities reflecting evolution
- Treat target momentum p as longitudinal $p = (E, \mathbf{0}_\perp, P)$
- Take z with z_- and $z_\perp = \{z_1, z_2\}$ components ($z_+ = 0$), then $(pz) = p_+ z_- \equiv -\nu$; define TMD

$$\mathcal{M}(\nu, z_\perp^2) = \int_{-1}^{1} dx\; e^{ix\nu} \int_{-\infty}^{\infty} d^2 k_\perp\; e^{-i(k_\perp z_\perp)} \mathcal{F}(x, k_\perp^2)$$

- Parton carries xp_+ and has transverse momentum $k_\perp$



Quasi-PDFs

- Take $z = \{0, 0, 0, z_3\}$, define Quasi-PDF (Ji, 2013)

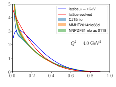
$$\langle p|\phi(0)\phi(z_3)|p\rangle \equiv \mathcal{M}(\underbrace{Pz_3}_{\nu}, \underbrace{z_3^2}_{\nu^2/P^2}) = \int_{-\infty}^{\infty} dy\, e^{iyPz_3}\, Q(y, P)$$

- Parton carries fraction yP of hadron 3-momentum P
- Inverse Fourier transformation: both arguments of $\mathcal{M}$ are involved

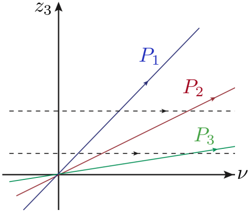
$$Q(y, P) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\nu\; e^{-iy\nu}\, \mathcal{M}(\nu, \nu^2/P^2)$$

- $Q(y, P)$ tends to $f(y)$ in $P \to \infty$ limit, as far as $\mathcal{M}(\nu, \nu^2/P^2) \to \mathcal{M}(\nu, 0)$
- Compare with pseudo-PDFs: only the first argument of $\mathcal{M}$ is integrated

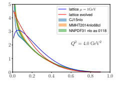
$$\mathcal{P}(x, z_3^2) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\nu\, e^{-ix\nu}\, \mathcal{M}(\nu, z_3^2)$$



Quasi-PDFs vs Pseudo-PDFs



- Quasi-PDFs $Q(y, P)$: integration of $\mathcal{M}(\nu, z_3^2)$ over $z_3 = \nu/P$ lines
- Profile: decrease of ITD with ν and extra fast decrease with z_3
- Origin of z_3-decrease 1)hadron finite-size effects
  2) link renormalization factor $Z(z_3)$
- Pseudo-PDFs: integration of $\mathcal{M}(\nu, z_3^2)$ over z_3 =const lines
- Dividing by ν-independent factor $D(z_3^2)$ one can reduce z_3-dependence without changing PDF (if $D(0) = 1$)



Relations between quasi-PDFs and TMDs

- z_3-dependence has the same origin as $k_\perp$ dependence of TMDs
- Quasi-PDFs can be obtained from TMDs (A.R., 2016)

$$Q(y,P)/P = \int_{-1}^{1} dx \int_{-\infty}^{\infty} dk_1 \mathcal{F}(x, k_1^2 + (y-x)^2 P^2)$$

- Or from pseudo-PDFs

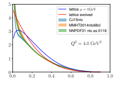
$$Q(y,P) = \frac{P}{2\pi} \int_{-1}^{1} dx \int_{-\infty}^{\infty} dz_3 \; e^{i(x-y)(Pz_3)} \, \mathcal{P}(x, z_3^2) \qquad (1)$$

- Try factorized model

$$\mathcal{P}^{\rm soft}(x, z_3^2) = f(x) I(z_3^2)$$

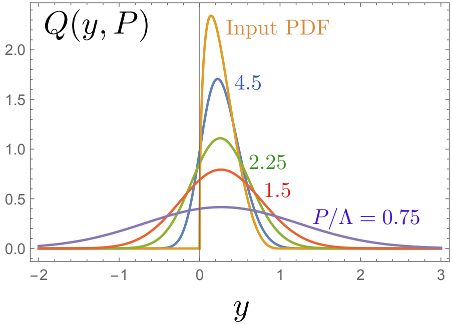
- Popular idea: Gaussian dependence $I(z_3^2) = e^{-z_3^2 \Lambda^2/4}$

$$Q_G^{\rm fact}(y,P) = \frac{P}{\Lambda\sqrt{\pi}} \int_{-1}^{1} dx \, f(x) \, e^{-(y-x)P^2/\Lambda^2}$$

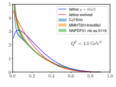


Numerical results for Gaussian model

- Take PDF $f(x) = u_v(x) - d_v(x) = \frac{315}{32}\sqrt{x}(1-x)^3\theta(0 \le x \le 1)$ obtained by pseudo-PDF method (Orginos et al. 2017)



- Curves for $P/\Lambda = 0.75, 1.5, 2.25$ are close to qPDFs obtained by Lin et al (2016), upper momentum $P = 1.3$ GeV, effective $\Lambda \approx 600$ MeV
- Need $P \sim 4.5\,\Lambda \approx 2.7$ GeV to get reasonably close to input PDF
- Note a lot of dirt for negative y, even for $P/\Lambda = 4.5$



Reduced Ioffe-time distribution

- Quasi-PDFs $Q(y, P)$: have x-convolution structure even if $\mathcal{M}(\nu, z_3^2)$ factorizes, i.e., $\mathcal{M}(\nu, z_3^2) = \mathcal{M}(\nu, 0)\mathcal{M}(0, z_3^2)$
- Divide out rest-frame function, take reduced ITD

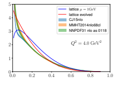
$$\mathfrak{M}(\nu, z_3^2) \equiv \frac{\mathcal{M}(\nu, z_3^2)}{\mathcal{M}(0, z_3^2)}$$

- Bonus: $Z(z_3^2)$-factor due to gauge link cancels in ratio
- LO Evolution equation (Braun et al. 1994) for Ioffe-time distribution

$$\frac{d}{d \ln z_3^2} \mathfrak{M}(\nu, z_3^2) = \frac{\alpha_s}{2\pi} C_F \int_0^1 du\, B(u) \mathfrak{M}(u\nu, z_3^2)$$

- Nonsinglet evolution kernel

$$B(u) = \left[\frac{1+u^2}{1-u}\right]_+$$

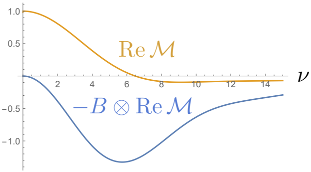


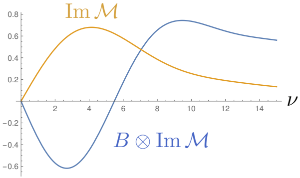
Structure of Ioffe-time distributions

- ITD has real and imaginary parts even if $f(x)$ is real

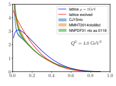
$$\mathfrak{M}(\nu) = \int_{-1}^{1} dx\, f(x)\, \cos(x\nu) + i \int_{-1}^{1} dx\, f(x)\, \sin(x\nu)$$

- $\text{Re}\, \mathcal{M}(\nu, z_3^2)$ is even function of ν, $\text{Im}\, \mathcal{M}(\nu, z_3^2)$ is odd function of ν
- $\Rightarrow \text{Im}\, \mathcal{M}(0, z_3^2) = 0$ and $\mathcal{M}(0, z_3^2)$ is real
- Take valence PDF $q(x) = \frac{315}{32}\sqrt{x}(1-x)^3 \theta(0 \leq x \leq 1)$



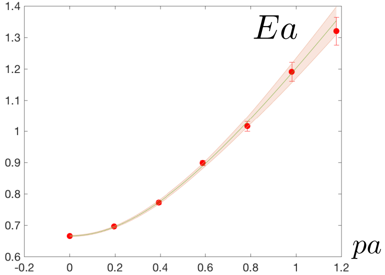


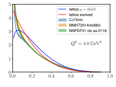
- No perturbative evolution for $\mathcal{M}(0, z_3^2)$ [vector current is conserved]
- Evolution of real part leads to its decrease when z_3^2 increases
- Evolution of imaginary part for $\nu \lesssim 5.5$ leads to its increase when z_3^2 increases



Lattice setup

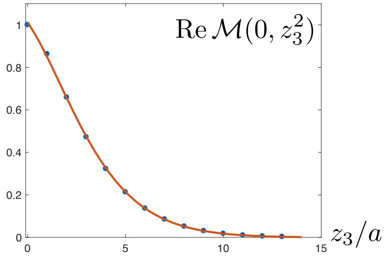
- Lattice QCD calculations in quenched approximation
- $\beta = 6.0$ on $32^3 \times 64$ lattices, lattice spacing $a = 0.093$ fm
- Clover fermion action with coefficients by the Alpha collaboration
- Total of 500 configurations separated by 1000 updates each
- Consisting of four over-relaxation and one heatbath sweeps
- Pion mass $601(1)$ MeV and nucleon mass $1411(4)$MeV
- Six lattice momenta $p_i\ (2\pi/L)$, with 2.5 GeV maximal momentum
- Dispersion relation satisfied



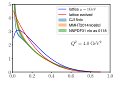


Rest-frame density and Z factor

- Rest-frame density $\mathcal{M}(0, z_3^2)$ is produced by data at $P = 0$
- Results for imaginary part are compatible with zero, as required

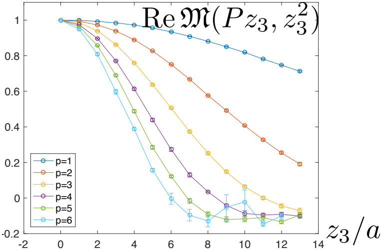


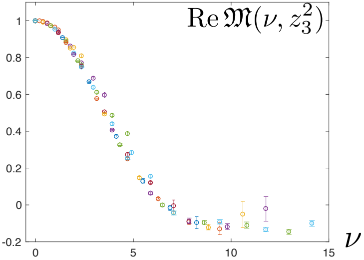
- Visible linear component in $|z_3|$ for small and middle values of $|z_3|$
- Linear exponential factor $Z(z_3^2) \sim e^{-c|z_3|/a}$ is expected
- Generated by straight-line gauge link renormalization



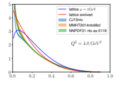
Reduced Ioffe-time distributions

- Real part of the ratio $\mathcal{M}(Pz_3, z_3^2)/\mathcal{M}(0, z_3^2)$ as a function of z_3
- Taken at six values of P $\Rightarrow$ curves have Gaussian-like shape
- $\Rightarrow$ $Z(z_3^2)$ link factor cancels in the ratio





- Same data, as functions of $\nu = Pz_3$ and z_3^2
- Data practically fall on the same curve
- Factorization of x- and $k_\perp$-dependence for soft TMD $\mathcal{F}(x, k_\perp^2)$



Fitting Real Part

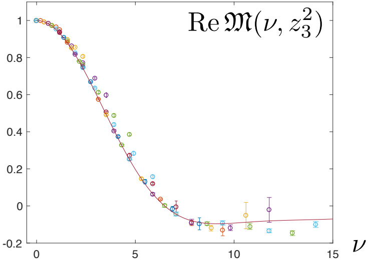
- Real part of ITD is obtained from cosine Fourier transform

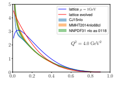
$$\mathrm{Re}\,\mathfrak{M}(\nu) \equiv \int_0^1 dx\, \cos(\nu x)\, q_v(x)$$

of $q_v(x) = q(x) - \bar{q}(x)$, difference of quark and antiquark densities

- Our data for real part are well described by function

$$q_v(x) = \frac{315}{32}\sqrt{x}(1-x)^3$$



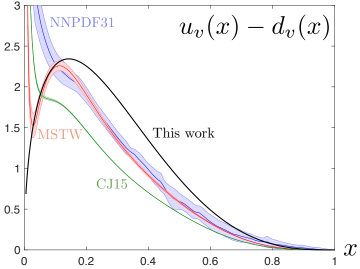


Valence PDF

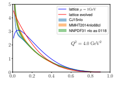
- Our data for real part are well described by function

$$q_v(x) = \frac{315}{32}\sqrt{x}(1-x)^3$$

- Took cosine FT $\mathfrak{M}(\nu; a, b)$ of normalized $x^a(1-x)^b$ functions
- Values of a, b were fitted from our data for real part



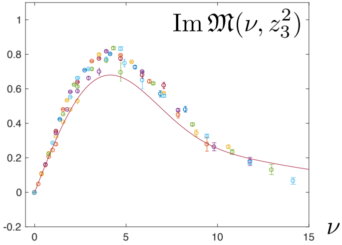
- Comparison with global fits: NNPDF31, MSTW and CJ15 for 1 GeV
- Our curve corresponds to “low normalization point”

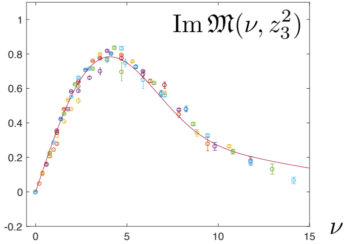


Fitting Imaginary Part

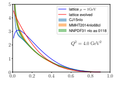
- Sine Fourier transform is built from function $q_+(x) = q(x) + \bar{q}(x) = q_v(x) + 2\bar{q}(x)$

$$\text{Im}\, \mathfrak{M}_I(\nu) \equiv \int_0^1 dx\, \sin(\nu x)\, q_+(x)$$





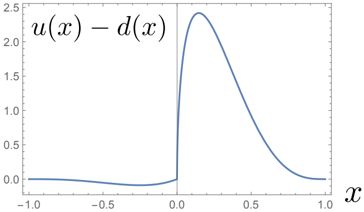
- Left: antiquark contribution is neglected, i.e., $q_+(x) = q_v(x)$
- Right: antiquark contribution $\bar{u}(x) - \bar{d}(x) = 0.07\,\left[20\,x(1-x)^3\right]$

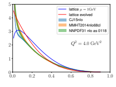


Antiquarks

- Antiquark contribution $\bar{u}(x) - \bar{d}(x) = 0.07\left[20\,x(1-x)^3\right]$ is positive
- In quenched approximation, antiquarks come from "connected diagrams"
- Must follow flavor content of the proton; i.e., $\bar{u}/\bar{d} \sim 2$ and $\bar{u} > \bar{d}$
- Combined distribution

$$q(x) = u(x) - d(x) = [q_v(x) + \bar{q}(x)]\,\theta(x > 0) - \bar{q}(-x)\,\theta(x < 0)$$



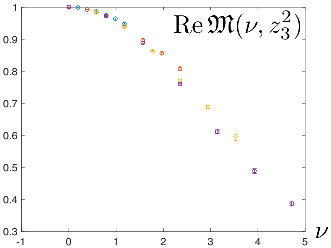


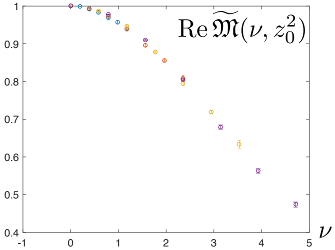
Evolution of Real Part

- Some residual z_3-dependence in the data
- Especially visible with several data points at the same ν
- Check if data corresponding to small z_3' and z_3 may be related by

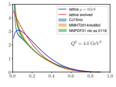
$$\mathfrak{M}(\nu, {z'}_3^2) = \mathfrak{M}(\nu, z_3^2) - \frac{2}{3}\frac{\alpha_s}{\pi} \ln({z_3'}^2/z_3^2) B \otimes \mathfrak{M}(\nu, z_3^2)$$

- Fix z_3' at value $z_0 = 2a$ corresponding to $\overline{\text{MS}}$ scale $\mu_0 = 1$ GeV



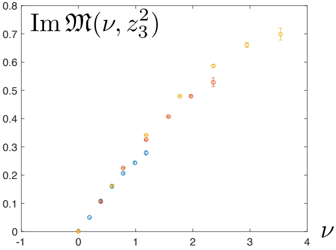


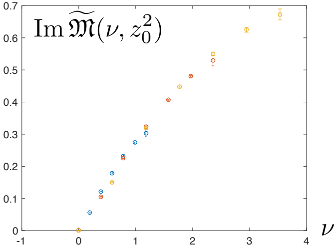
- Evolution of real part leads to its decrease when z_3^2 increases



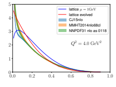
Evolution of Imaginary Part

- Evolution of imaginary part leads to its increase when z_3^2 increases



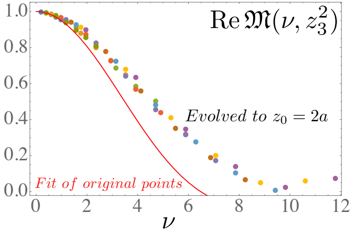


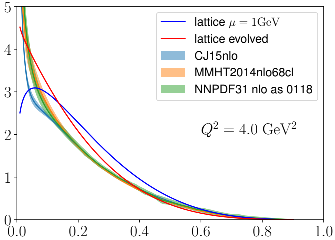
- z_3^2-dependence of $\mathfrak{M}\,(\nu, z_3^2)$ at fixed ν is compatible with logarithmic evolution at small z_3^2



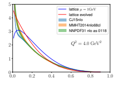
Evolution of all points for Real Part

- Using $\alpha_s/\pi = 0.1$, evolve all points to value $z_0 = 2a$ corresponding to $\overline{\text{MS}}$ scale $\mu_0 = 1$ GeV



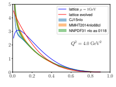


- Points evolved to $z_0 = 2a$ give $q(x) = 7.15(1-x)^{3.55}x^{0.22}$
- Further evolved to $Q^2 = 4$ GeV^2 by Nobuo Sato



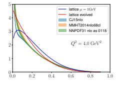
Summary

- Complicated convolution nature of quasi-PDFs necessitates $p_3 \gtrsim$ 3 GeV to wipe out primordial effects
- Use pseudo-PDFs $\mathcal{P}(x, z_3^2)$ related by Fourier transform to Ioffe-time distributions $\mathcal{M}(\nu, z_3^2)$
- Pseudo-PDFs have same $-1 \leq x \leq 1$ support as PDFs
- Their z_3^2-dependence for small z_3^2 is governed by a usual evolution equation
- Reduced ITD excludes z_3^2-dependence coming from gauge link self-energy corrections
- Lattice calculation confirms cancellation



Summary, cont.

- Using ratio $\mathcal{M}(\nu, z_3^2)/\mathcal{M}(0, z_3^2)$ of Ioffe-time distributions one also divides out z_3^2-dependence of primordial rest-frame distribution
- On the lattice: essentially complete cancellation of z_3-dependence for fixed ν
- Equivalent to factorization of x and $k_\perp$ dependence of TMD $\mathcal{F}(x, k_\perp^2)$
- Residual z_3-dependence agrees with perturbative evolution for small z_3
- Encouraging agreement of evolved PDF with global fits
- Further directions:
  smaller lattice spacing,
  dynamical fermions,
  application to pion distribution amplitude



QCD case

- Matrix element in QCD

$$\mathcal{M}^{\alpha}(z,p) \equiv \langle p|\bar{\psi}(0)\,\gamma^{\alpha}\,\hat{E}(0,z;A)\psi(z)|p\rangle$$

- with standard $0 \to z$ straight-line gauge link $\hat{E}(0,z;A)$
- Decompose into p^{α} and z^{α} parts

$$\mathcal{M}^{\alpha}(z,p) = 2p^{\alpha}\mathcal{M}_p(-(zp),-z^2) + z^{\alpha}\mathcal{M}_z(-(zp),-z^2)$$

- For TMD: take $z = (z_-, z_\perp)$ and $\alpha = + \Rightarrow z^{\alpha}$-part drops out
- TMD $\mathcal{F}(x,k_\perp^2)$ is related to $\mathcal{M}_p(\nu, z_\perp^2)$ by scalar formula
- For quasi- PDF: take time component of $\mathcal{M}^{\alpha}(z=z_3,p)$ and define

$$\mathcal{M}^{0}(z_3,p) = 2p^0 \int_{-1}^{1} dy\, Q(y,P)\; e^{iyPz_3}$$

- $\Rightarrow$ Quasi-PDF $Q(y,P)$ is related to TMD $\mathcal{F}(x,k_\perp^2)$ by scalar formula