

Status of TMD extractions

Andrea Signori

Hadronic physics with
lepton and hadron beams

Sept. 5th 2017



Outline of the talk

1) Transverse-Momentum-Dependent distributions (**TMDs**)

2) **formalism**

2) **extractions** : unpolarized

3) **extractions** : polarized (Sivers)

4) **gluon** TMDs

5) **spin-1** TMDs

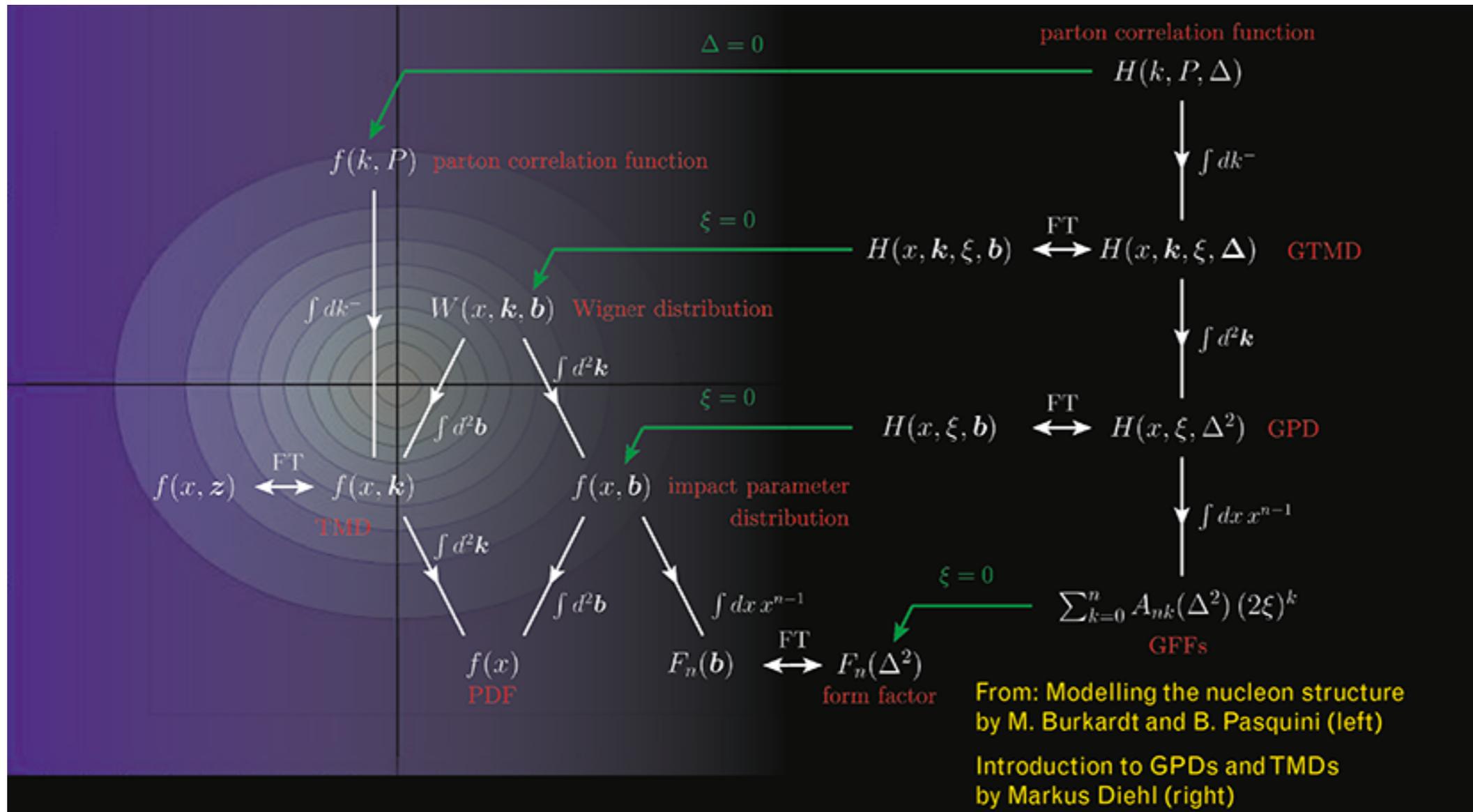
TMDs

References (intro and reviews) :

- “The 3D structure of the nucleon” **EPJ A (2016) 52**
- J.C. Collins “**Foundations of perturbative QCD**”
- P.J. Mulders’ **lecture notes**
- TMD collaboration **summer school**
- A. Bacchetta’s **lecture notes**

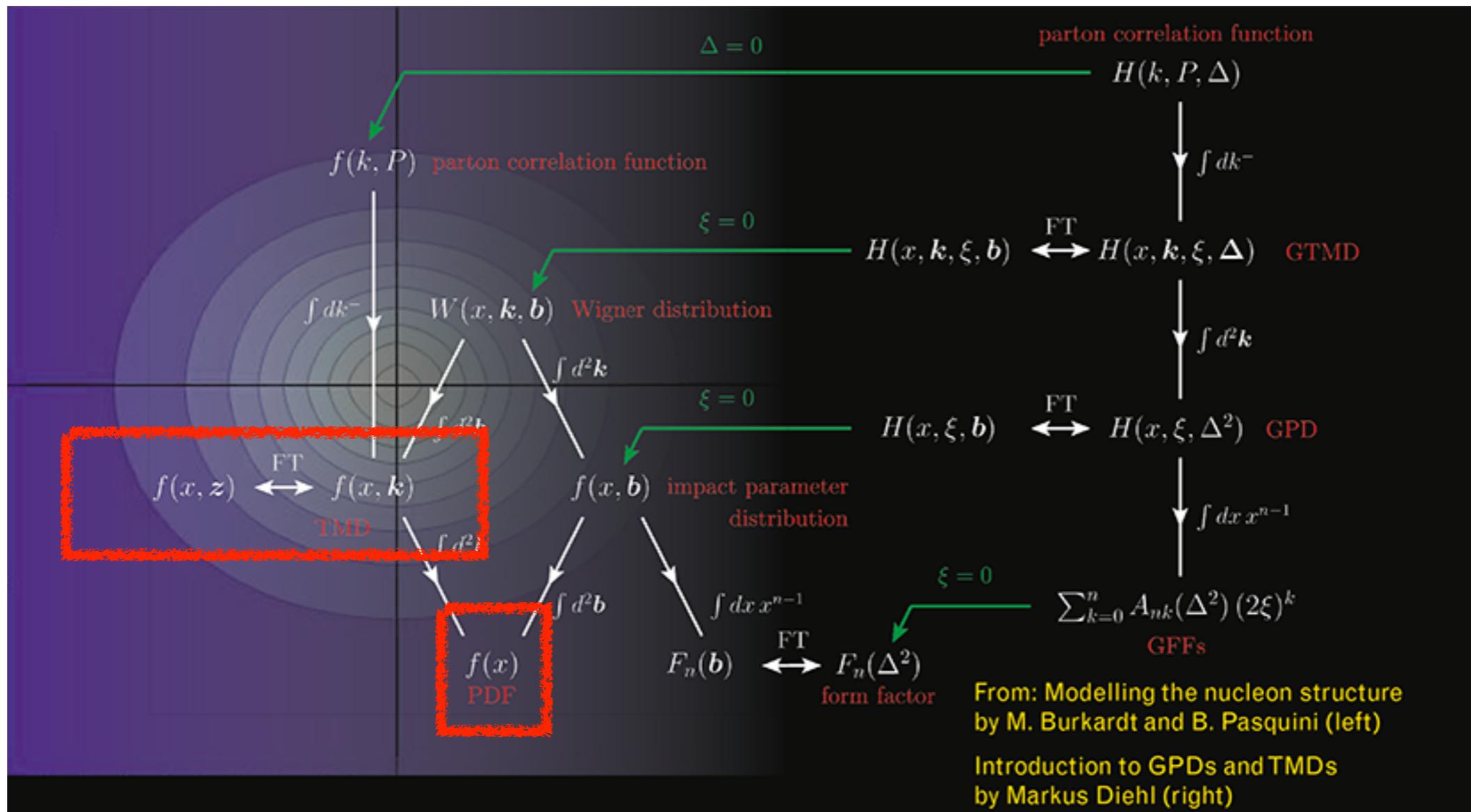
The hadronic landscape

Manifestation of hadron structure in scattering processes



The hadronic landscape

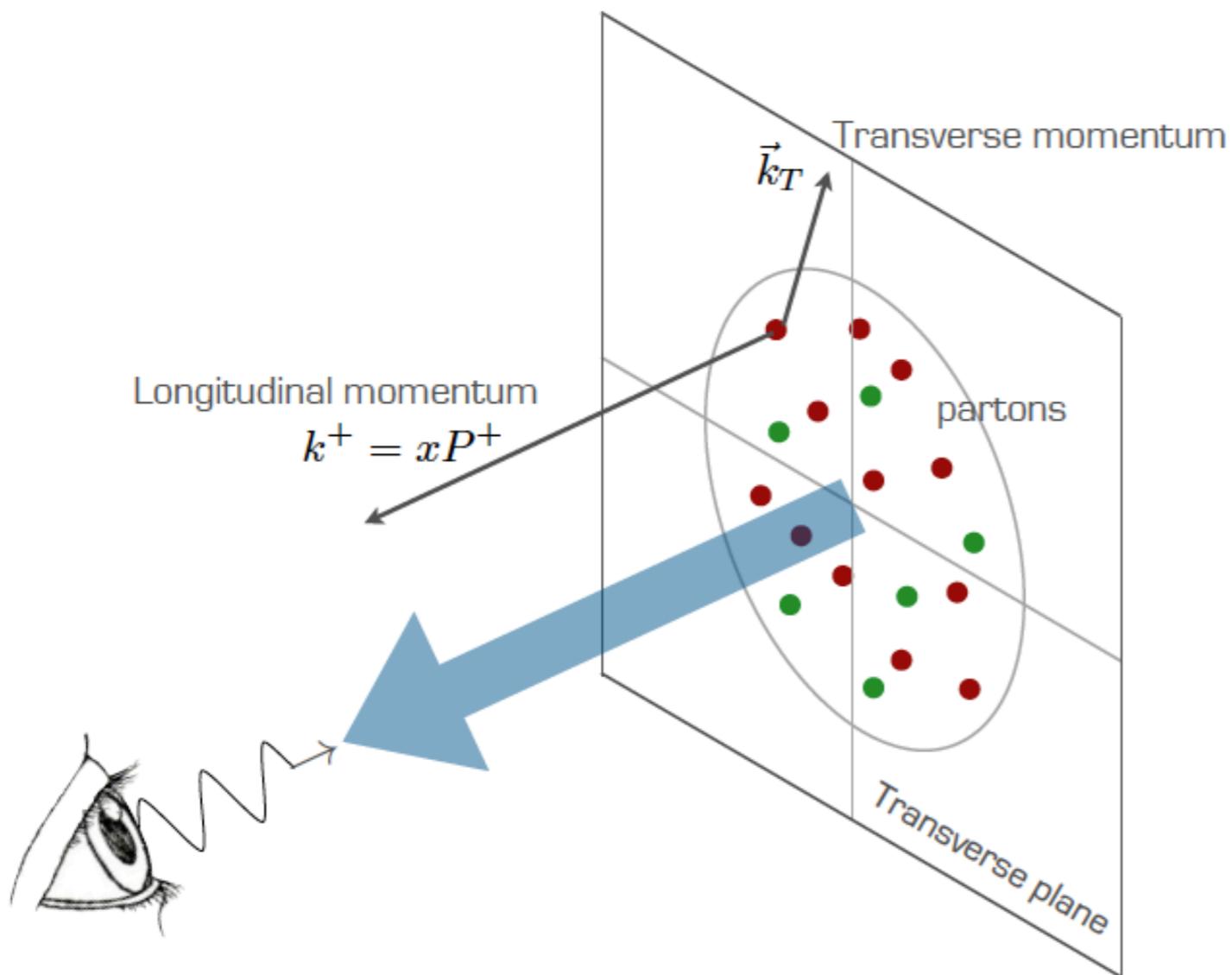
Manifestation of hadron structure in scattering processes



Nature is “smooth” : understand the link between TMDs & PDFs

quark TMD PDFs

$$\Phi_{ij}(k, P; S) \sim \text{F.T.} \langle PS | \bar{\psi}_j(0) U_{[0,\xi]} \psi_i(\xi) | PS \rangle_{LF}$$



extraction of a **quark**
not collinear with the proton

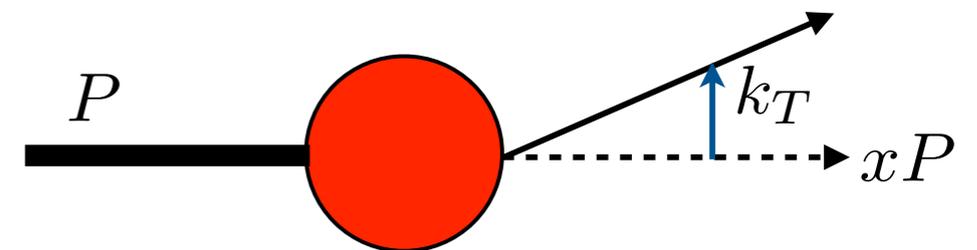
quark TMD PDFs

$$\Phi_{ij}(k, P; S_i, S_j) \sim \text{F.T.} \langle PS | \bar{\psi}_j(0) U_{[0,\xi]} \psi_i(\xi) | PS' \rangle_{LF}$$

Quarks	γ^+	$\gamma^+ \gamma^5$	$i\sigma^{i+} \gamma^5$
U	f_1		h_1^\perp
L		g_1	h_{1L}^\perp
T	f_{1T}^\perp	g_{1T}	h_1, h_{1T}^\perp

Sivers TMD PDF

unpolarized TMD PDF



extraction of a **quark**
not collinear with the proton

encode all the possible
spin-spin and **spin-orbit**
correlation

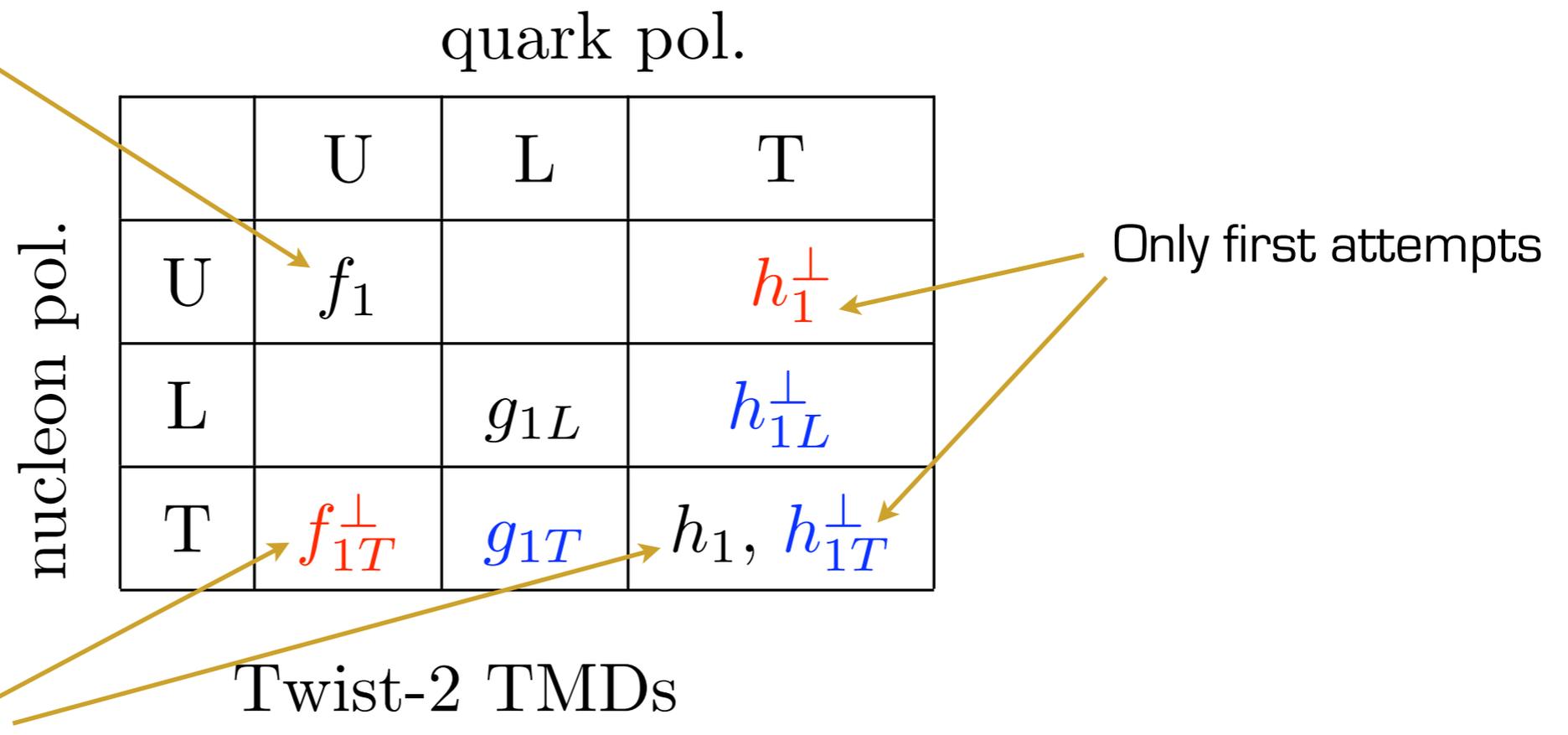
between the proton
and its constituents

bold : also collinear

red : time-reversal odd (universality properties)

Status of TMD phenomenology

Theory, data, fits : we are in a position to start validating the formalism

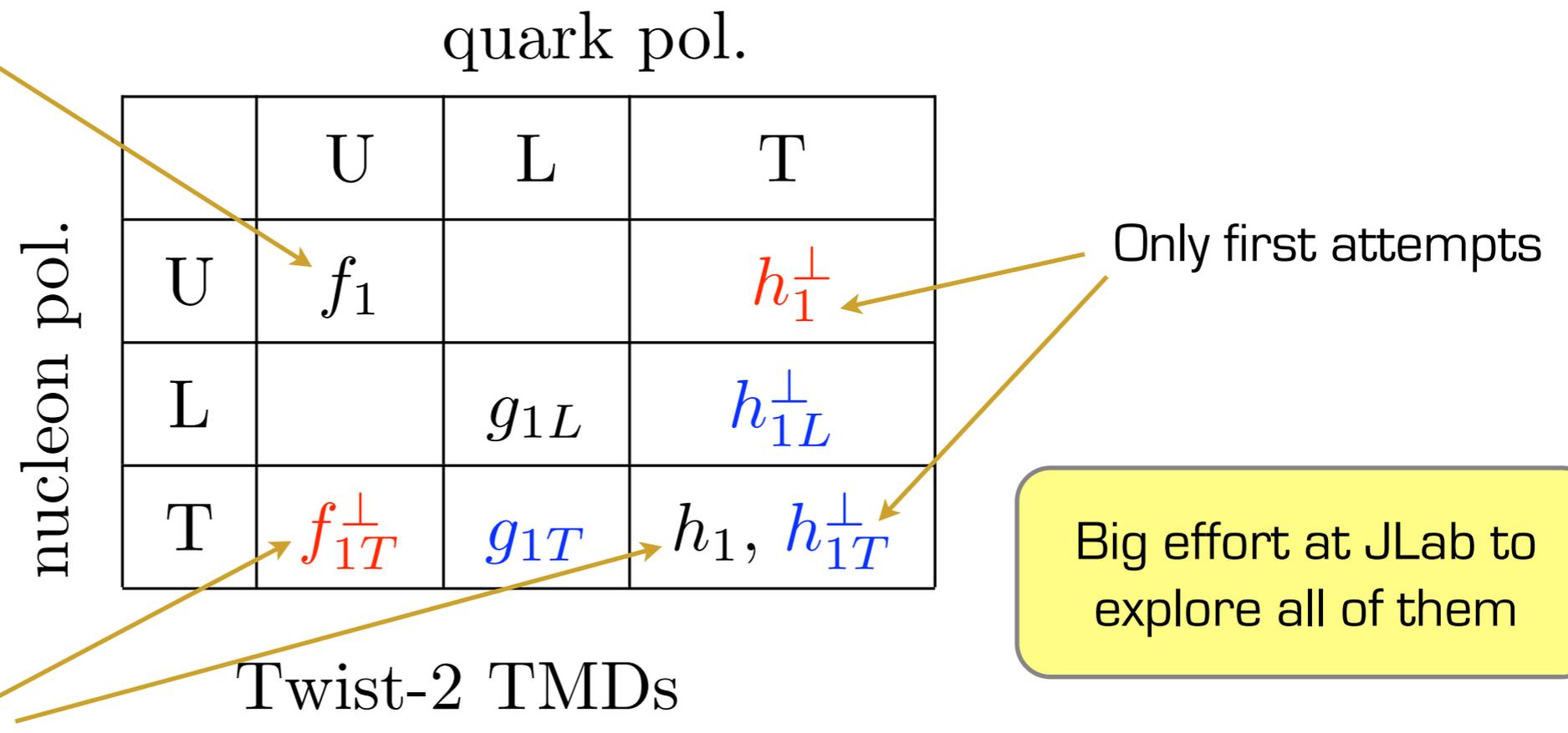


see, e.g, Bacchetta, Radici, *arXiv:1107.5755*
 Anselmino, Boglione, Melis, *PRD86 (12)*
 Echevarria, Idilbi, Kang, Vitev, *PRD 89 (14)*
 Anselmino, Boglione, D'Alesio, Murgia, Prokudin, *arXiv:1612.06413*
 Anselmino et al., *PRD87 (13)*
 Kang et al. *arXiv:1505.05589*

Lu, Ma, Schmidt, *arXiv:0912.2031*
 Lefky, Prokudin *arXiv:1411.0580*
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The frontier

Nucleon tomography in momentum space:

to understand how hadrons are built in terms of the elementary degrees of freedom of QCD

High-energy phenomenology:

to improve our understanding of high-energy scattering experiments and their potential to explore BSM physics

A selection of open questions (formalism) :

- 1) How well do we understand collinear and TMD **factorization** ?
- 2) How (well) can we **match** collinear and TMD factorization ?
- 3) can we quantify **factorization breaking** effects ?
- 4) how can we investigate gluon TMDs ?

...

The frontier

Nucleon tomography in momentum space:

to understand how hadrons are built in terms of the elementary degrees of freedom of QCD

High-energy phenomenology:

to improve our understanding of high-energy scattering experiments and their potential to explore BSM physics

More open questions (phenomenology) :

- 1) what is the **functional form** of TMDs at low transverse momentum ?
- 2) what is its **kinematic** and **flavor** dependence ?
- 3) can we attempt a global fit of TMDs ?
- 4) can we test the generalized **universality** of TMDs ?
- 5) what's the impact of hadron structure on the determination of Standard Model parameters ?

TMD & collinear factorization

References:

- J.C. Collins “**Foundations of perturbative QCD**”
- SCET literature

Collinear and TMD factorization

Let's consider a process with
three separate scales:

(SIDIS, Drell-Yan, e+e- to hadrons,
pp to quarkonium, ...)

hadronic
mass scale

$$\Lambda_{\text{QCD}} \ll q_T \ll Q$$

hard scale

(related to the)
transverse momentum of the observed particle

The ratios

$$\Lambda_{\text{QCD}}/Q$$

$$\Lambda_{\text{QCD}}/q_T$$

$$q_T/Q$$

select the factorization theorem that we rely on.

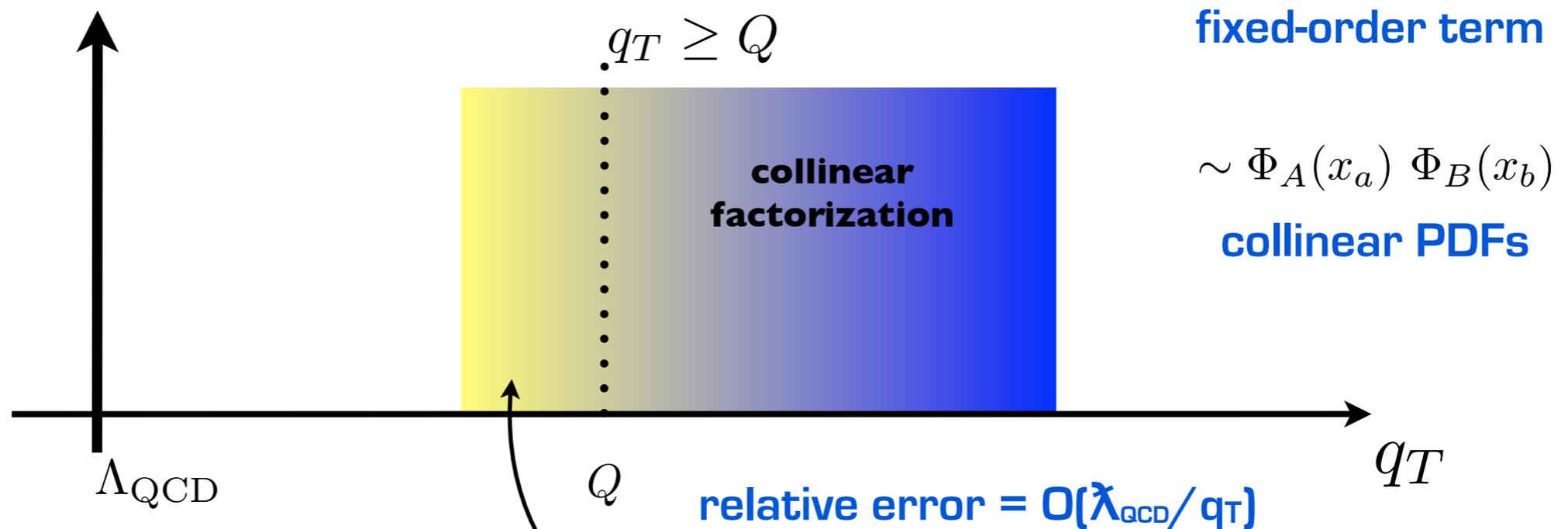
According to their **values** we can access **different**
“**projections**” of hadron structure

Collinear and TMD factorization

The key of phenomenology : emergence of TMD and collinear distributions from **factorization theorems**

fixed Q , variable q_T

$d\sigma/dq_T$



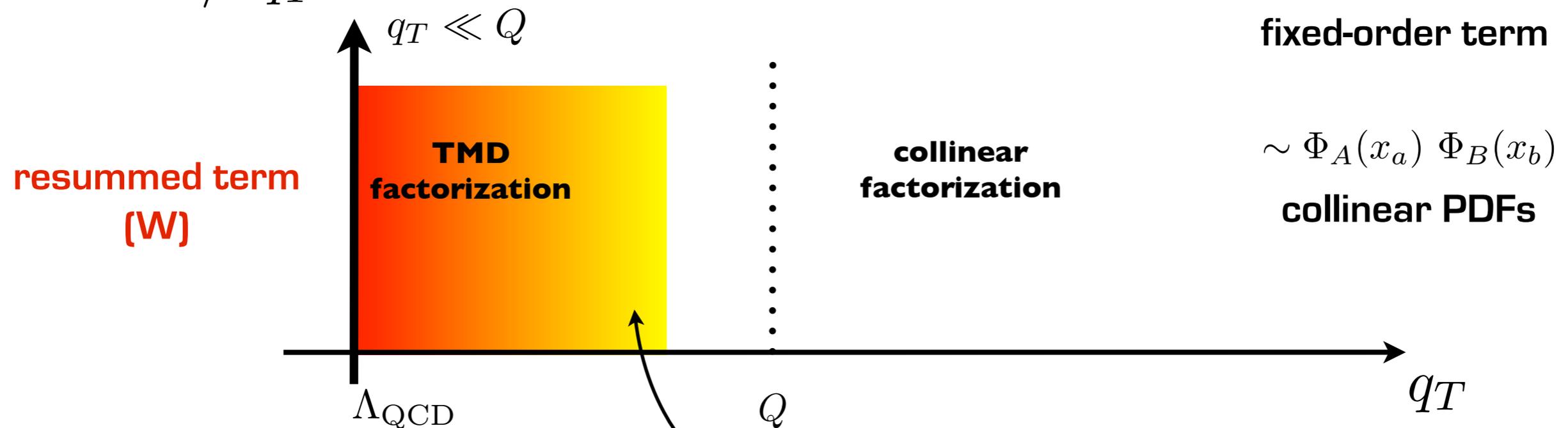
degraded description!

Collinear and TMD factorization

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fixed Q , variable q_T

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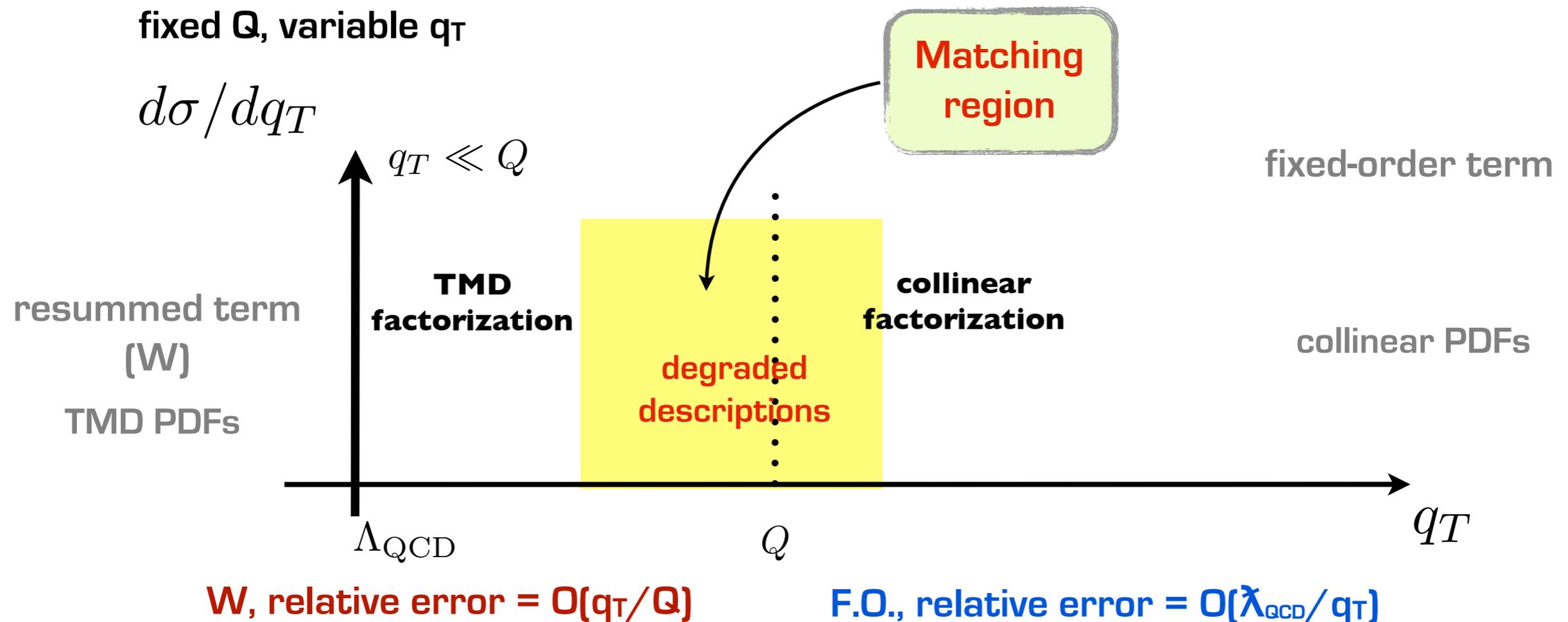
$$\Phi_A(x_a, \mathbf{k}_{Ta}) \Phi_B(x_b, \mathbf{k}_{Tb}) \sim \text{TMD PDFs}$$

relative error = $O(q_T/Q)$

degraded description!

Collinear and TMD factorization

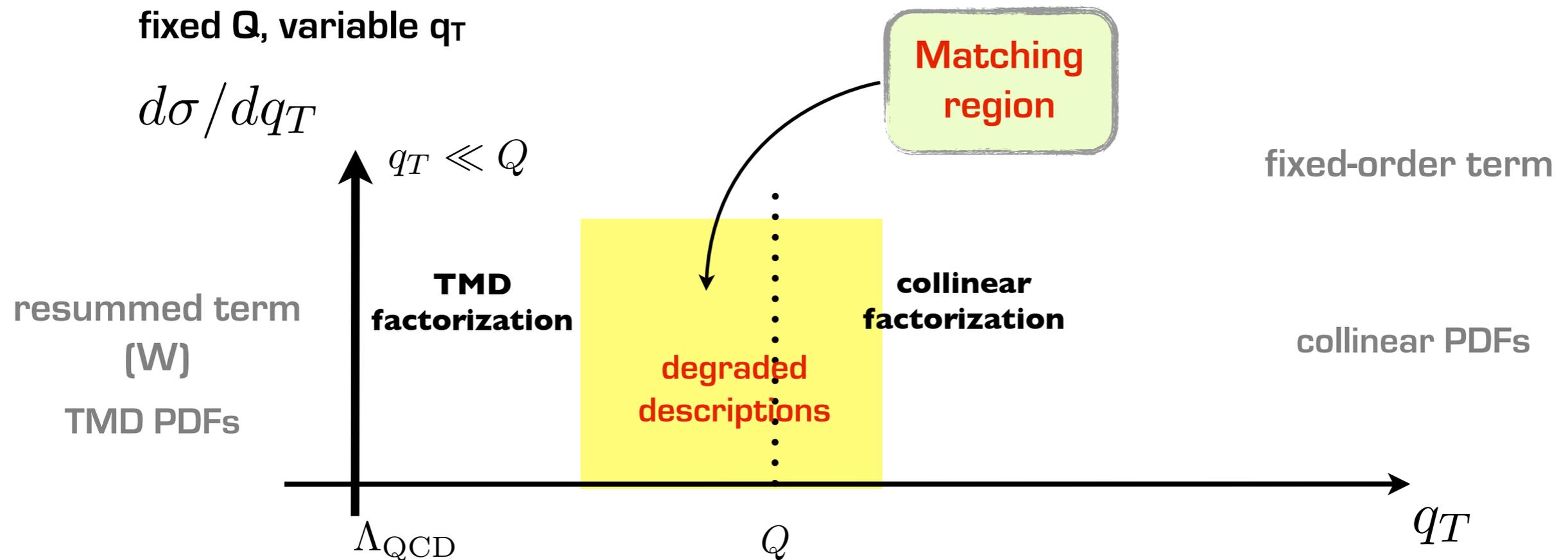
The key of phenomenology : emergence of TMD and collinear distributions from **factorization theorems**



We need a prescription to deal with the region where both descriptions are not good

Collinear and TMD factorization

The key of phenomenology : emergence of TMD and collinear distributions from **factorization theorems**



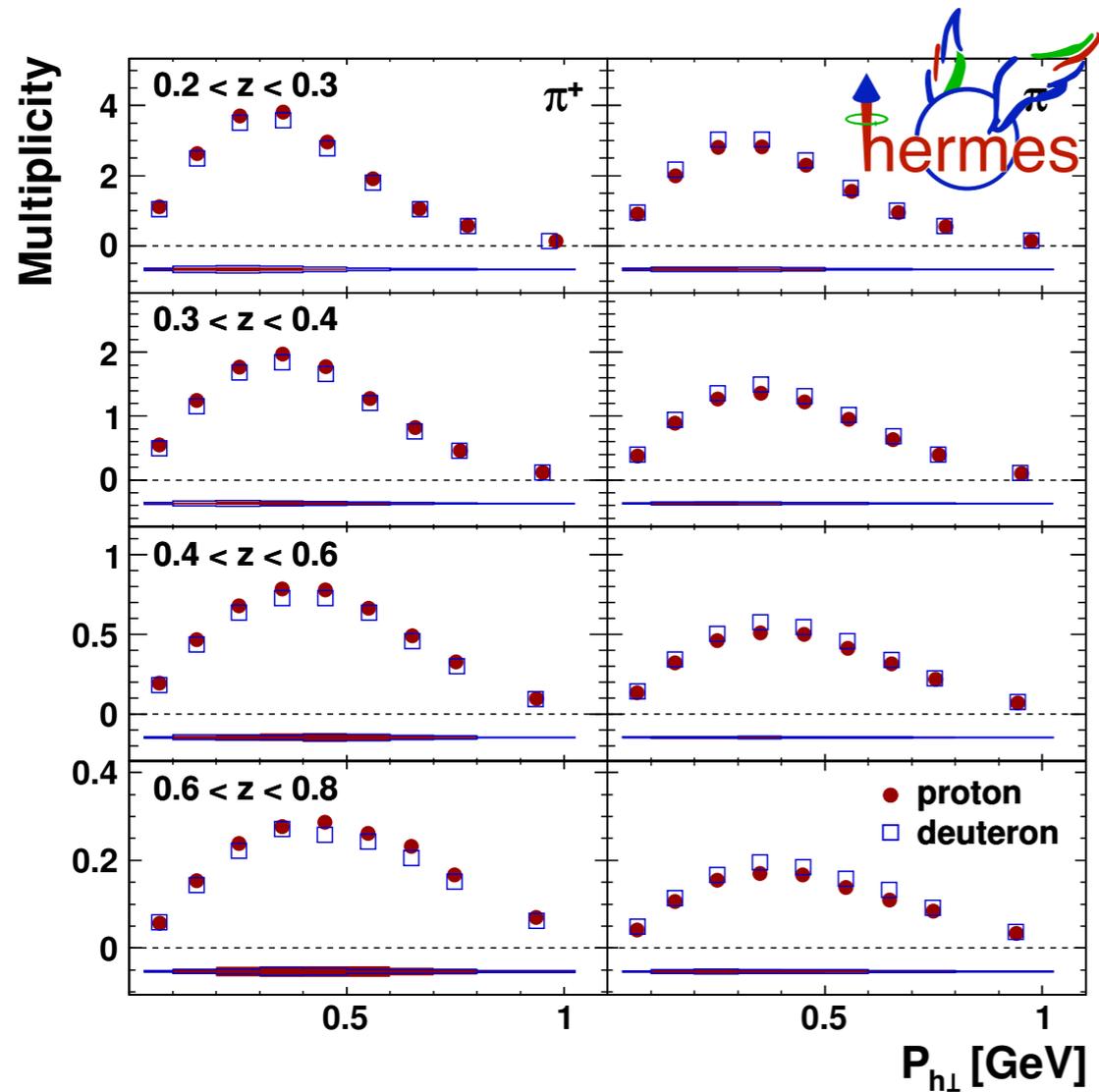
The extraction of the **nonperturbative part of TMDs** is affected by the description of the whole q_T range

Crucial, especially at **low Q** (e.g. JLab kinematics), where the **regions shrink**

polarization ?

Need of TMD evolution

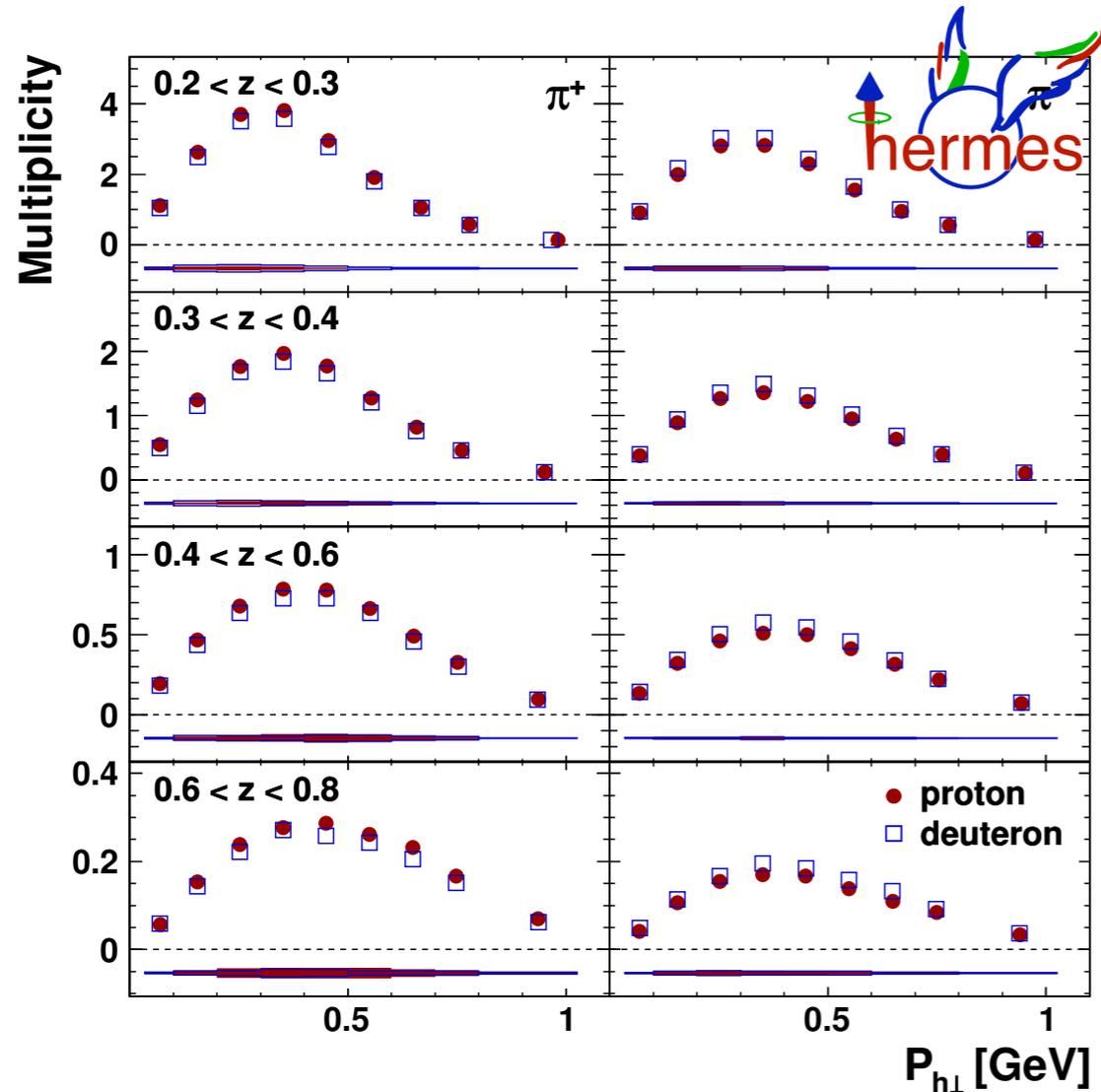
HERMES, $Q \approx 1.5$ GeV



Airapetian et al., PRD87 (2013)

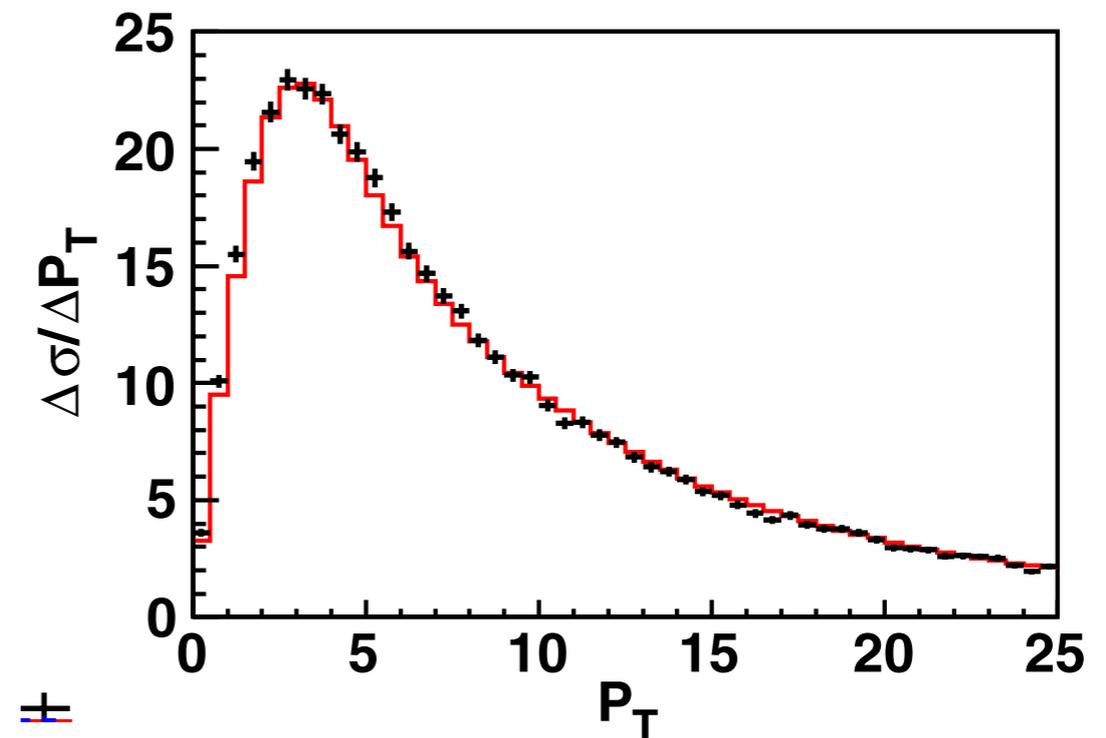
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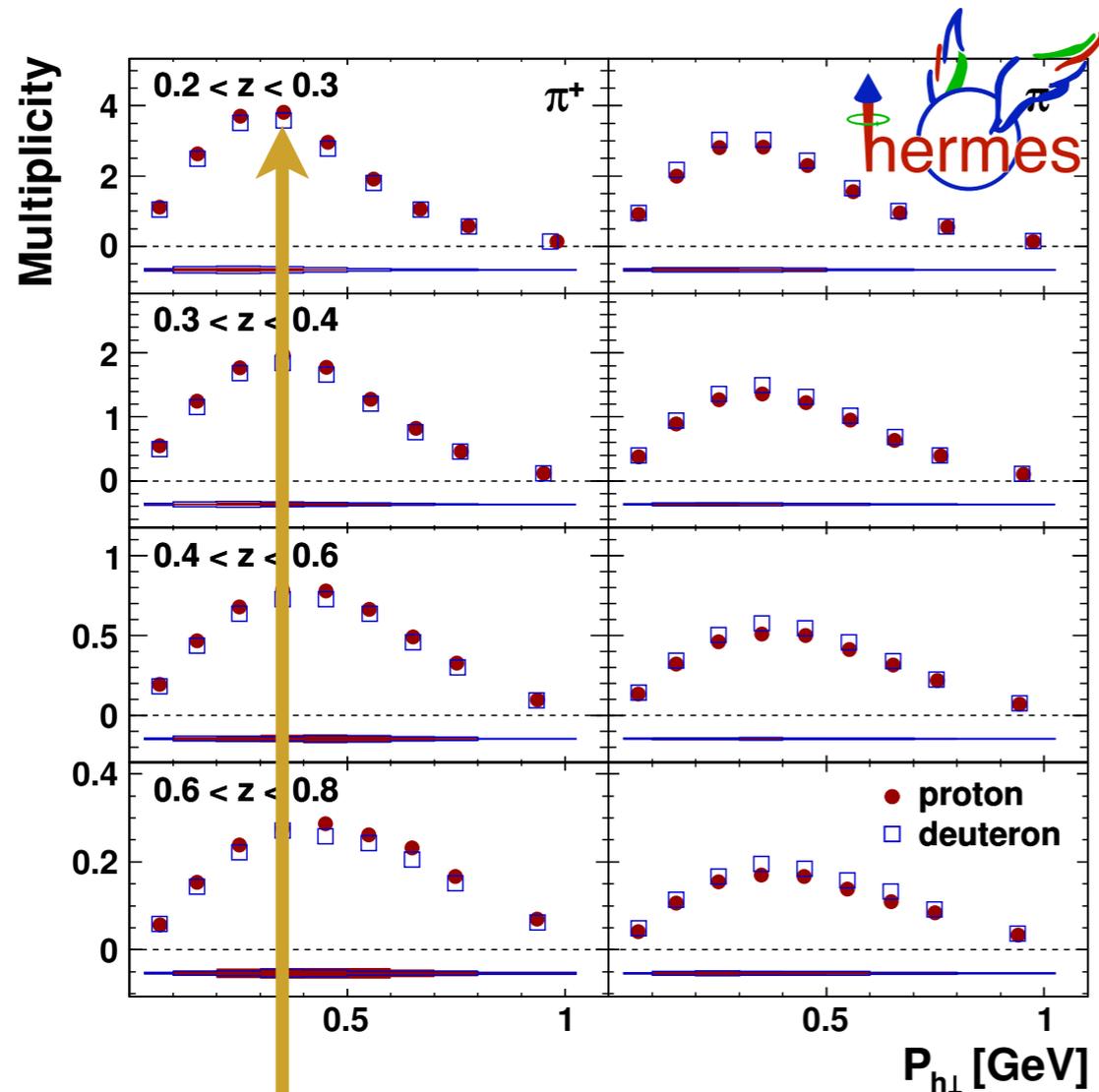
CDF, $Q \approx 91$ GeV



Aaltonen et al., PRD86 (2012)

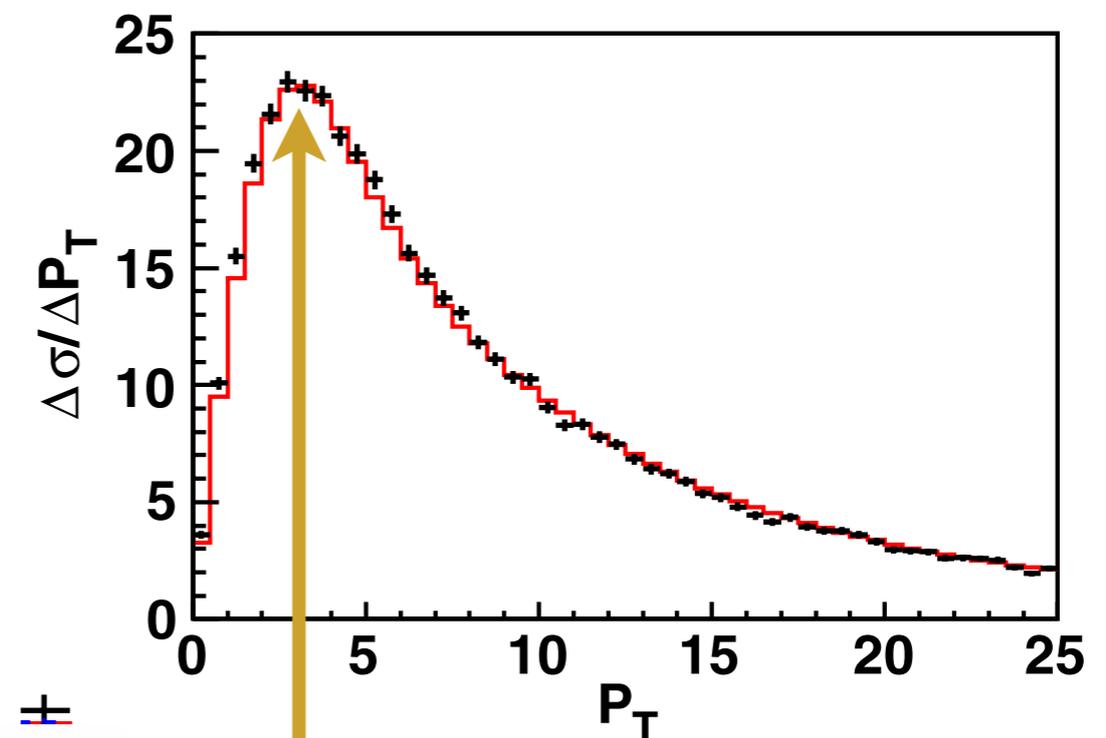
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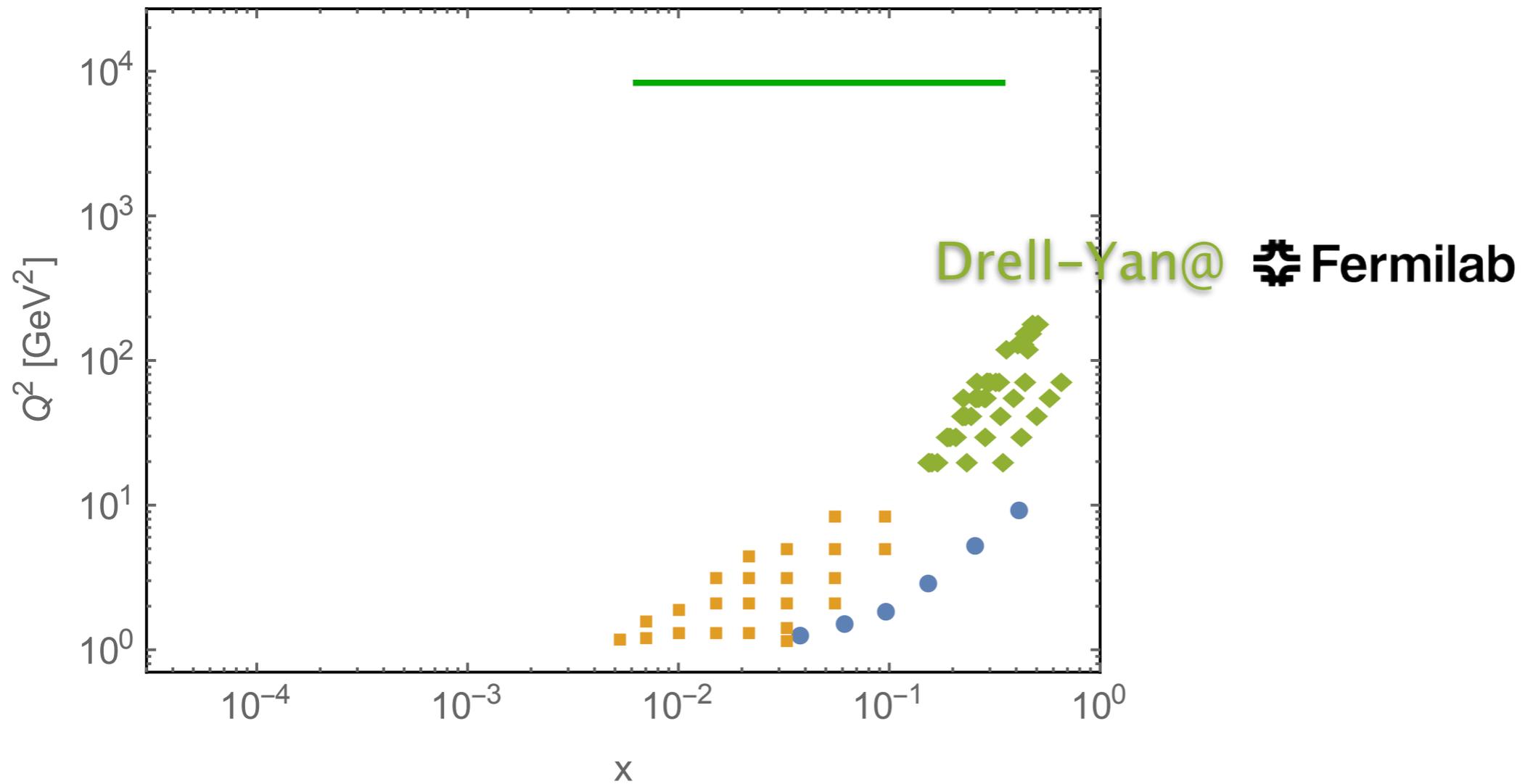
Width of TMDs changes of one order of magnitude:
we can explain this with TMD evolution

Extraction of unpolarized TMDs

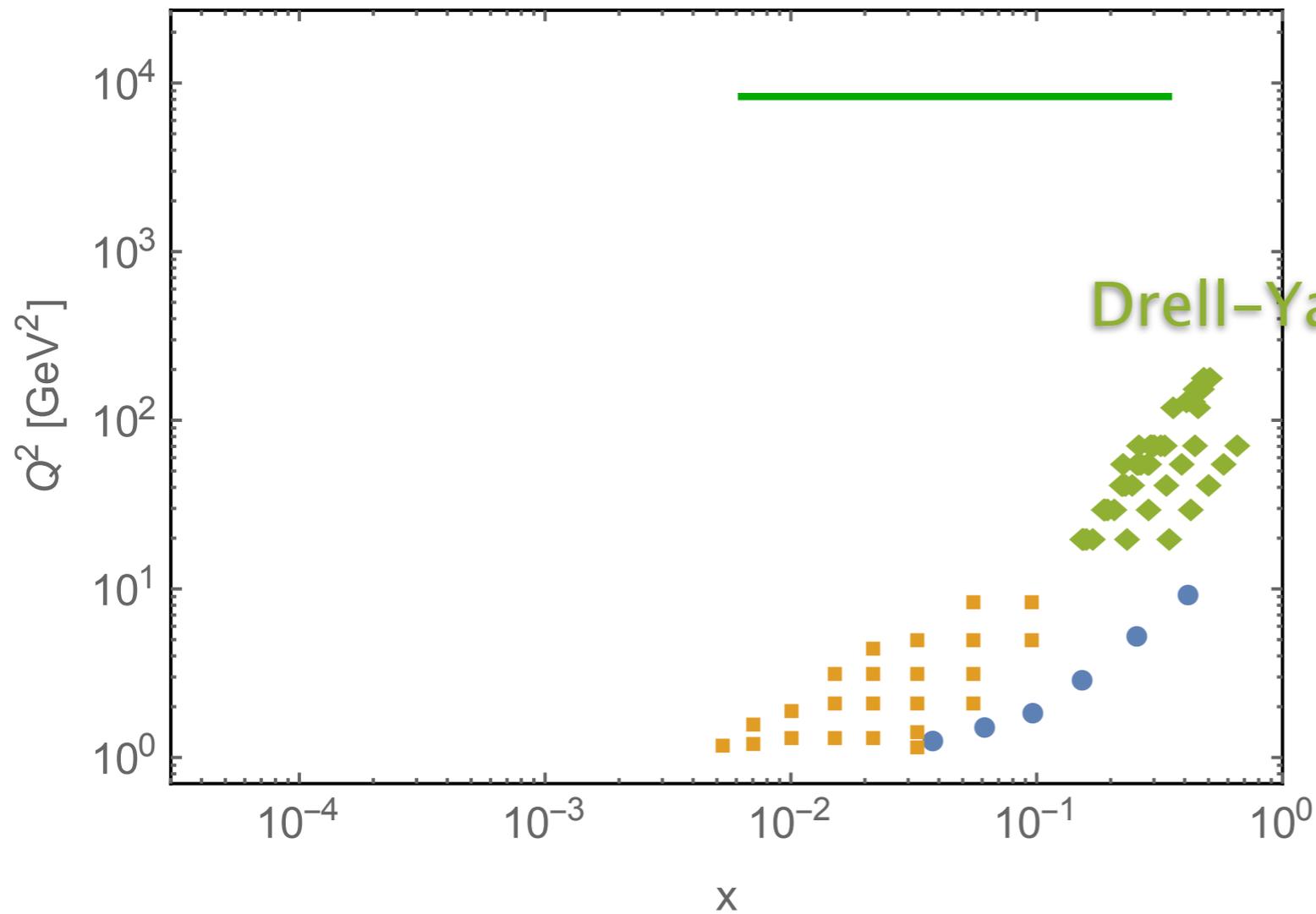
References :

- “The 3D structure of the nucleon” **EPJ A (2016) 52**
- Bacchetta et al. **JHEP 1706 (2017) 081**
- A. Signori , **PhD thesis**
- Angeles-Martinez et al. **arXiv:1507.05267**
- EIC white paper, JLab 12 GeV white paper, ...
- ...

Experimental measurements

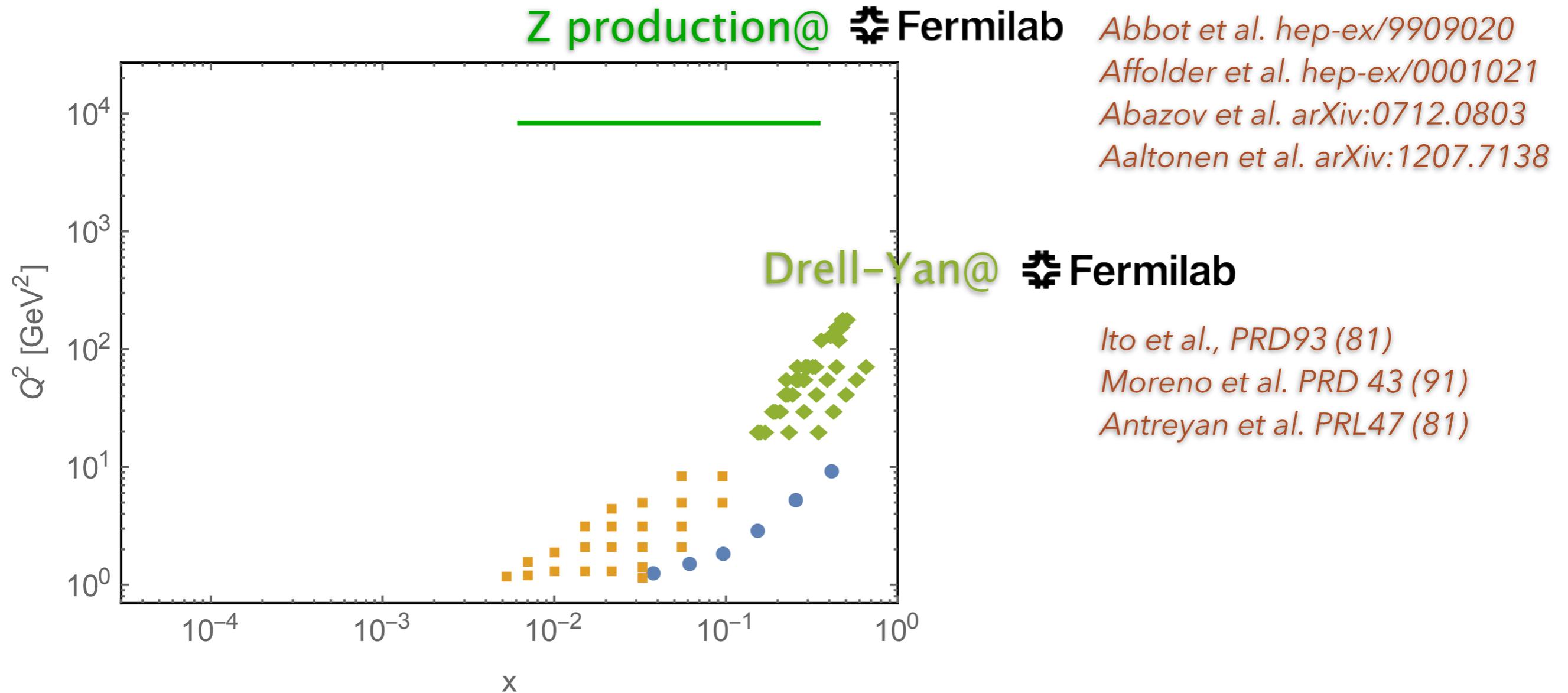


Experimental measurements

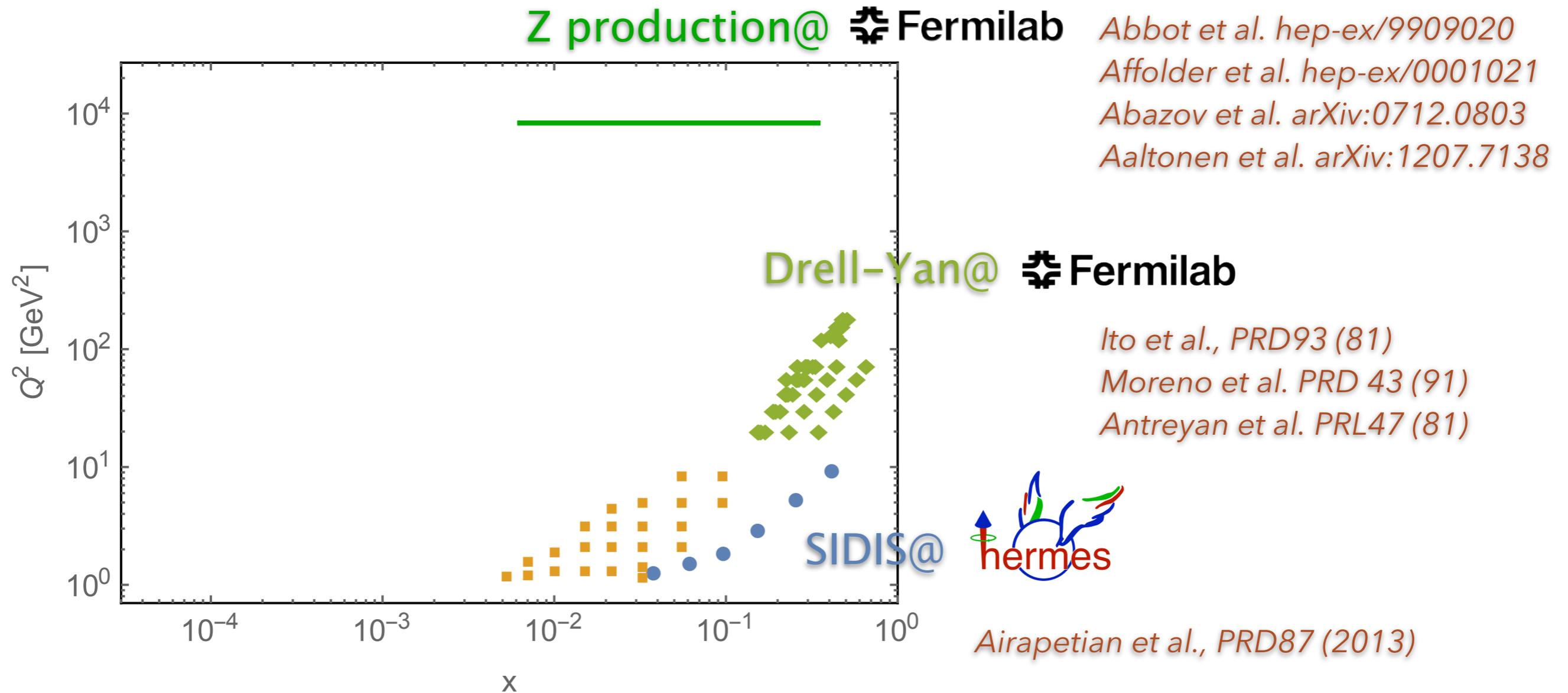


Ito et al., PRD93 (81)
Moreno et al. PRD 43 (91)
Antreyan et al. PRL47 (81)

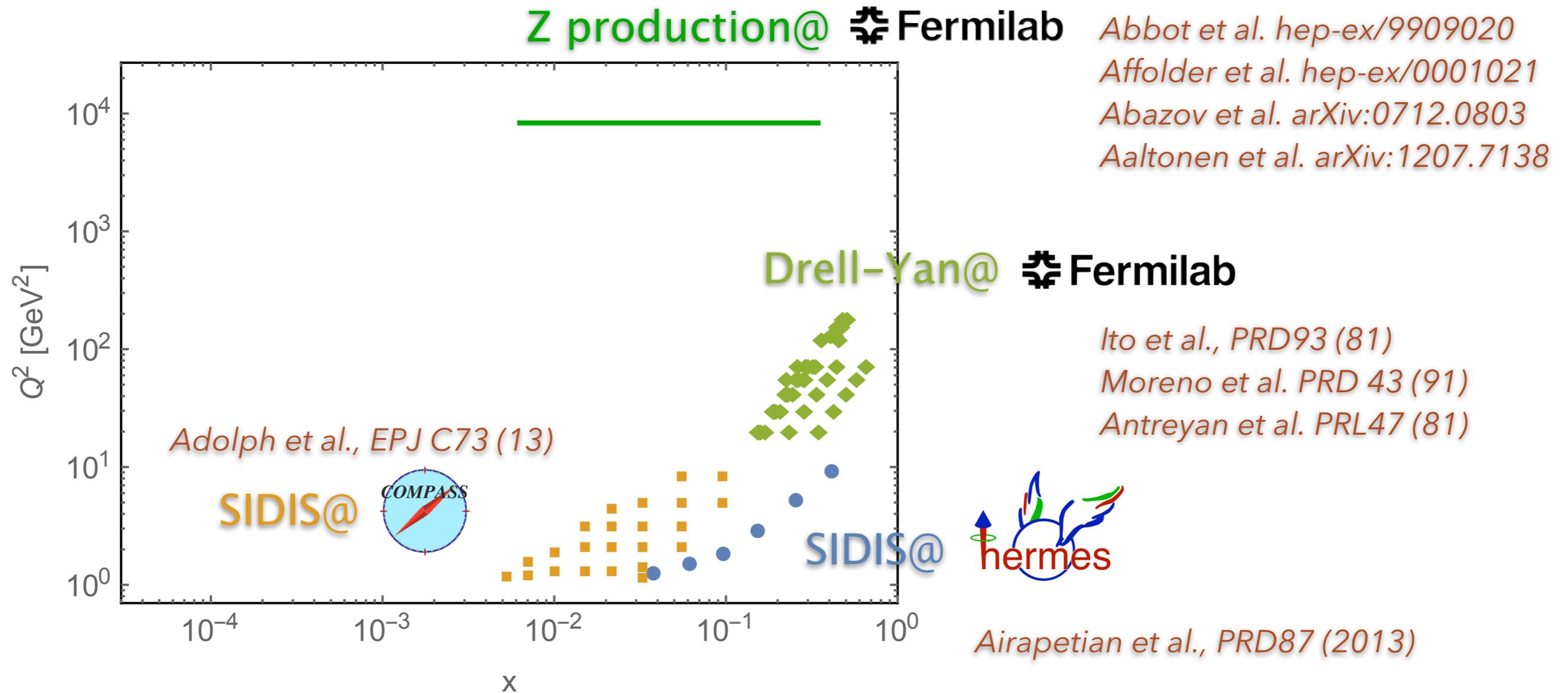
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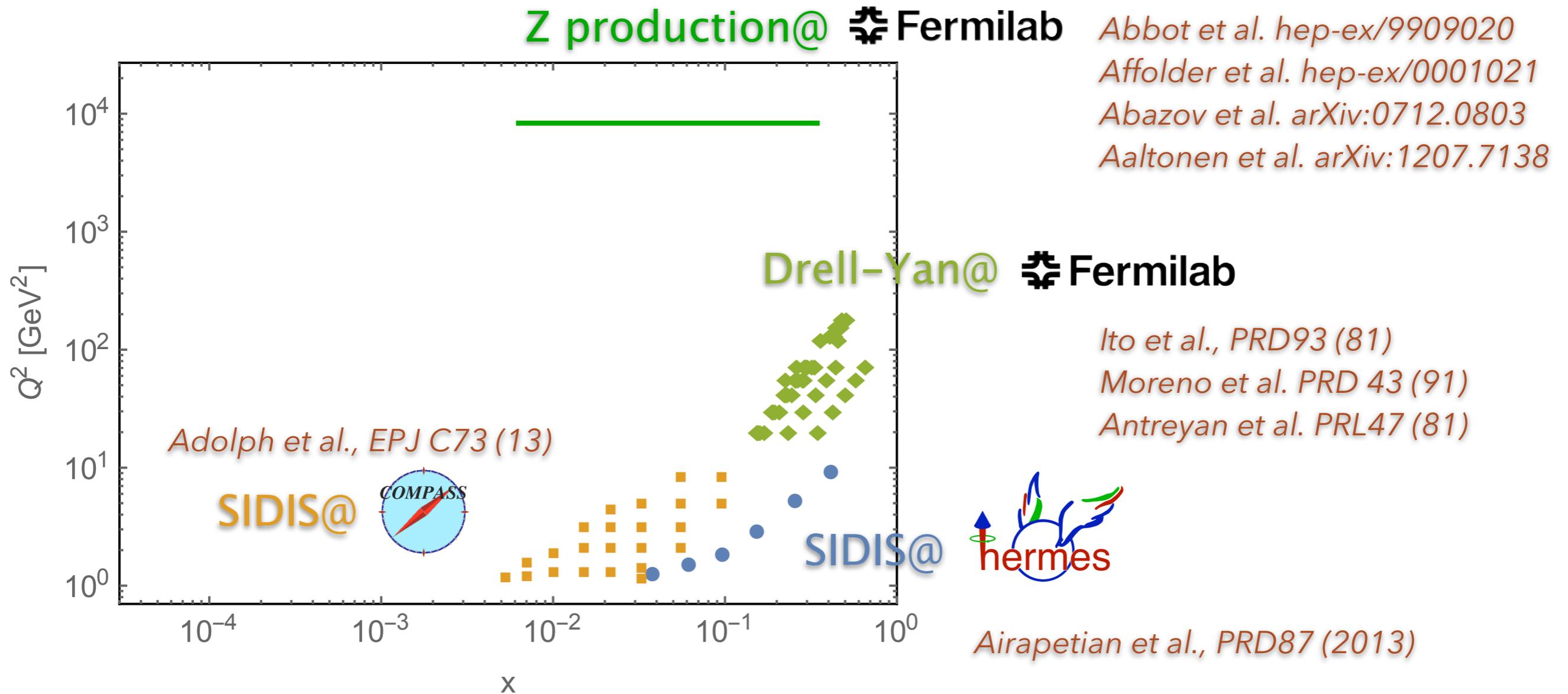
Experimental measurements



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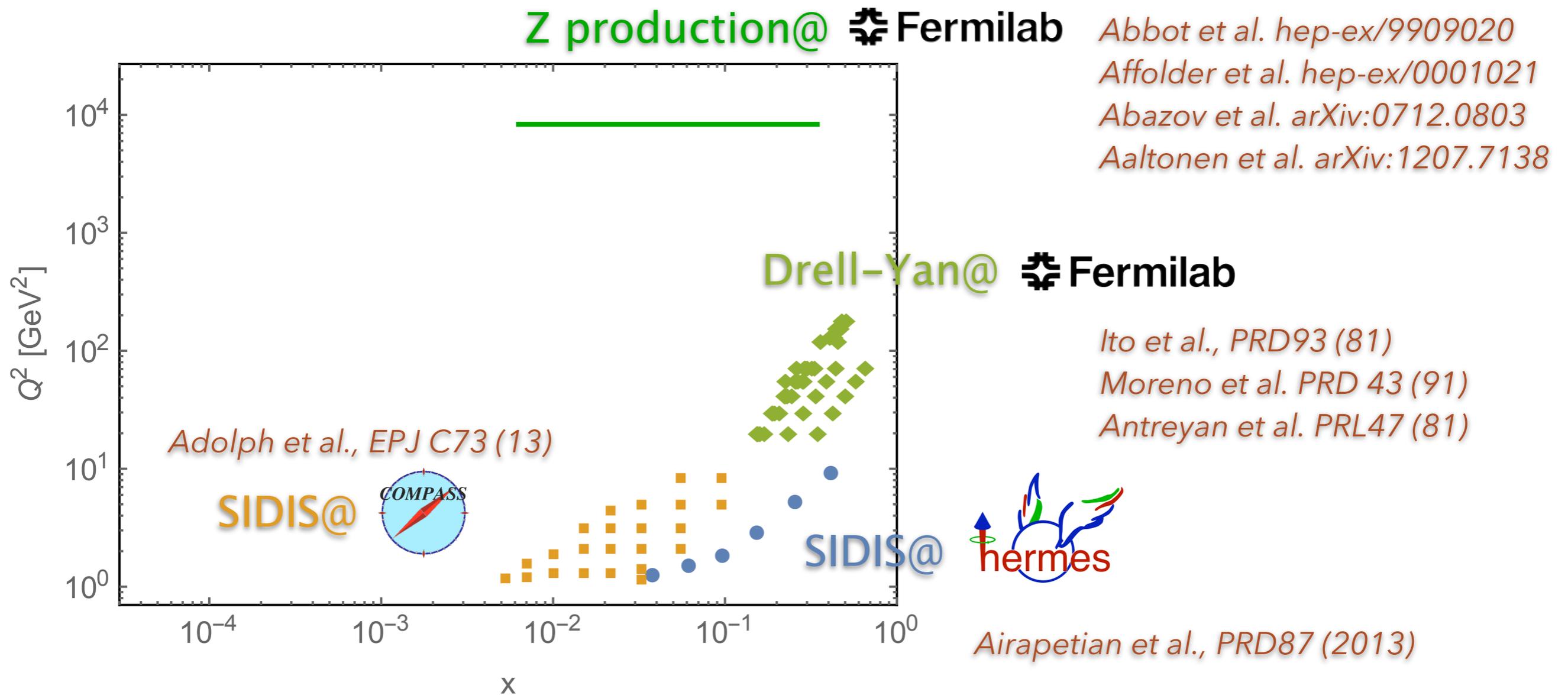


Experimental measurements



Electron-positron annihilation data are still **missing**
 (only some azimuthal asymmetries are available)

Experimental measurements

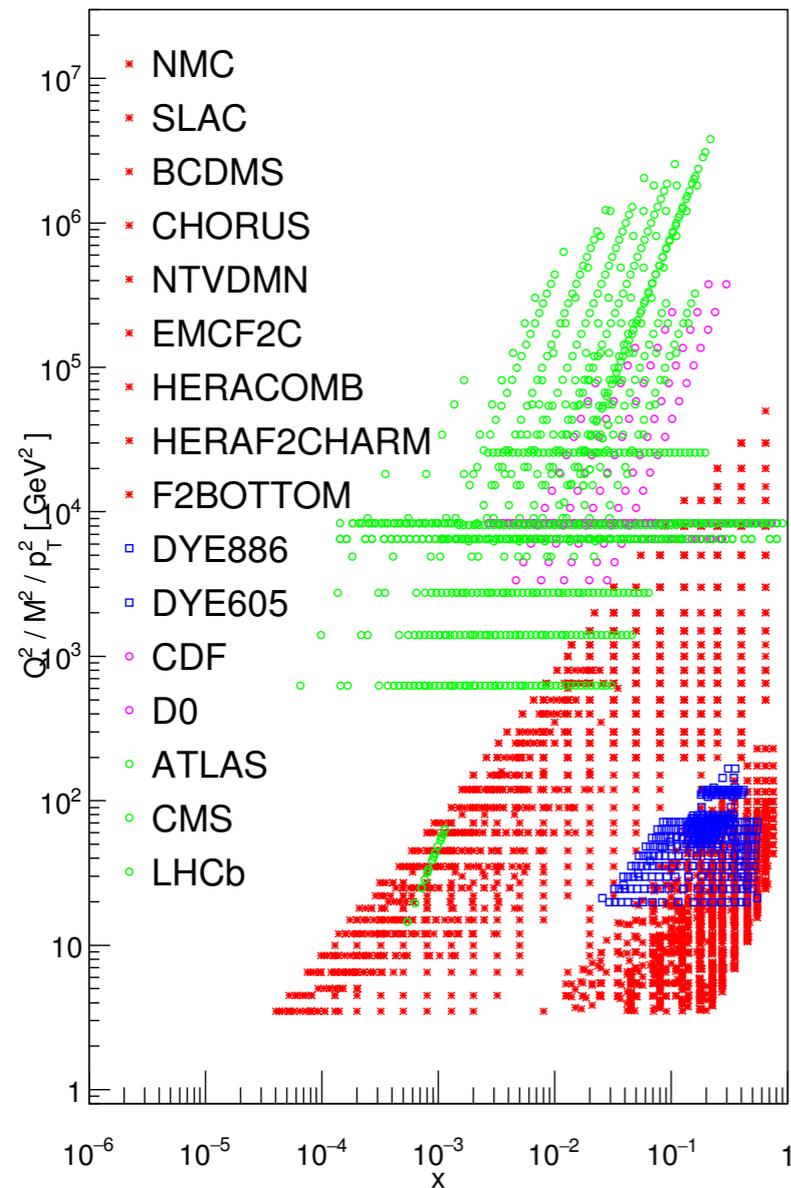


Electron-positron annihilation data are still **missing**
 (only some azimuthal asymmetries are available)

crucial for analyses
of TMD FFs !!

Comparison with collinear PDF fits

see talk by E. Nocera at POETIC2016

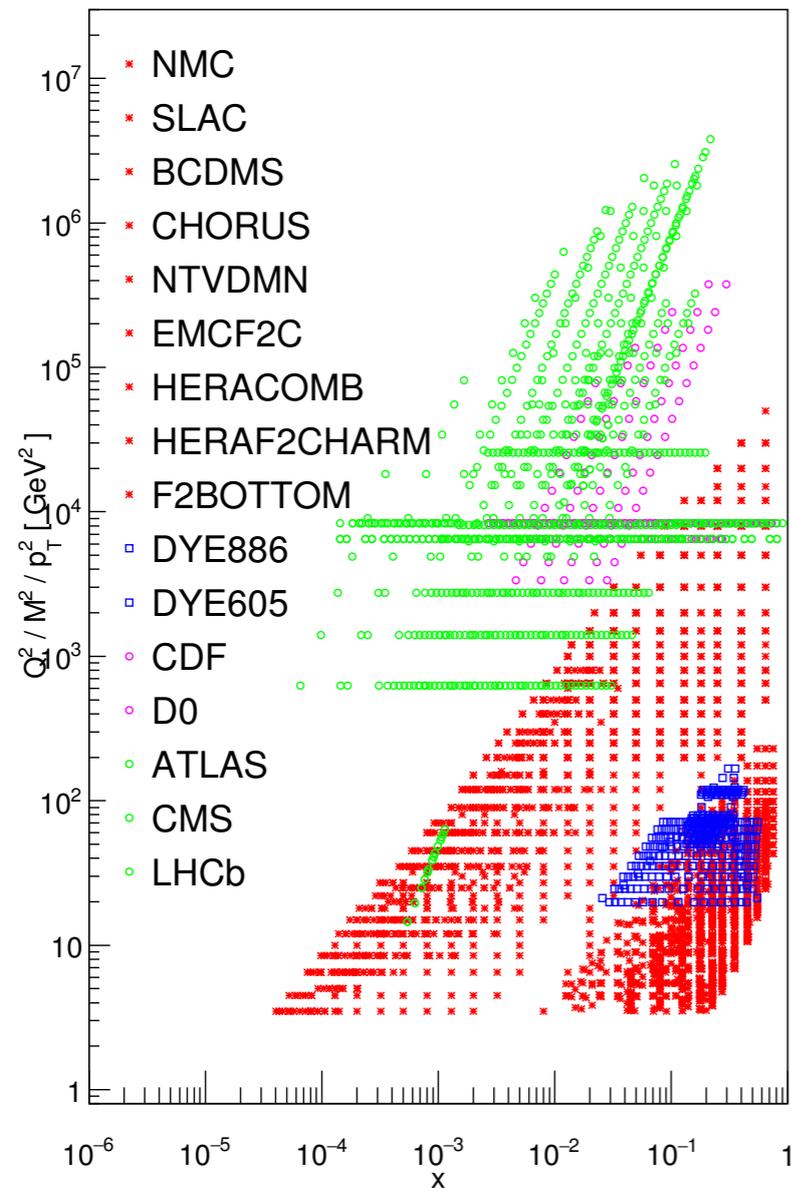


data sets available:

← collinear PDFs
vs
TMD PDFs →

Comparison with collinear PDF fits

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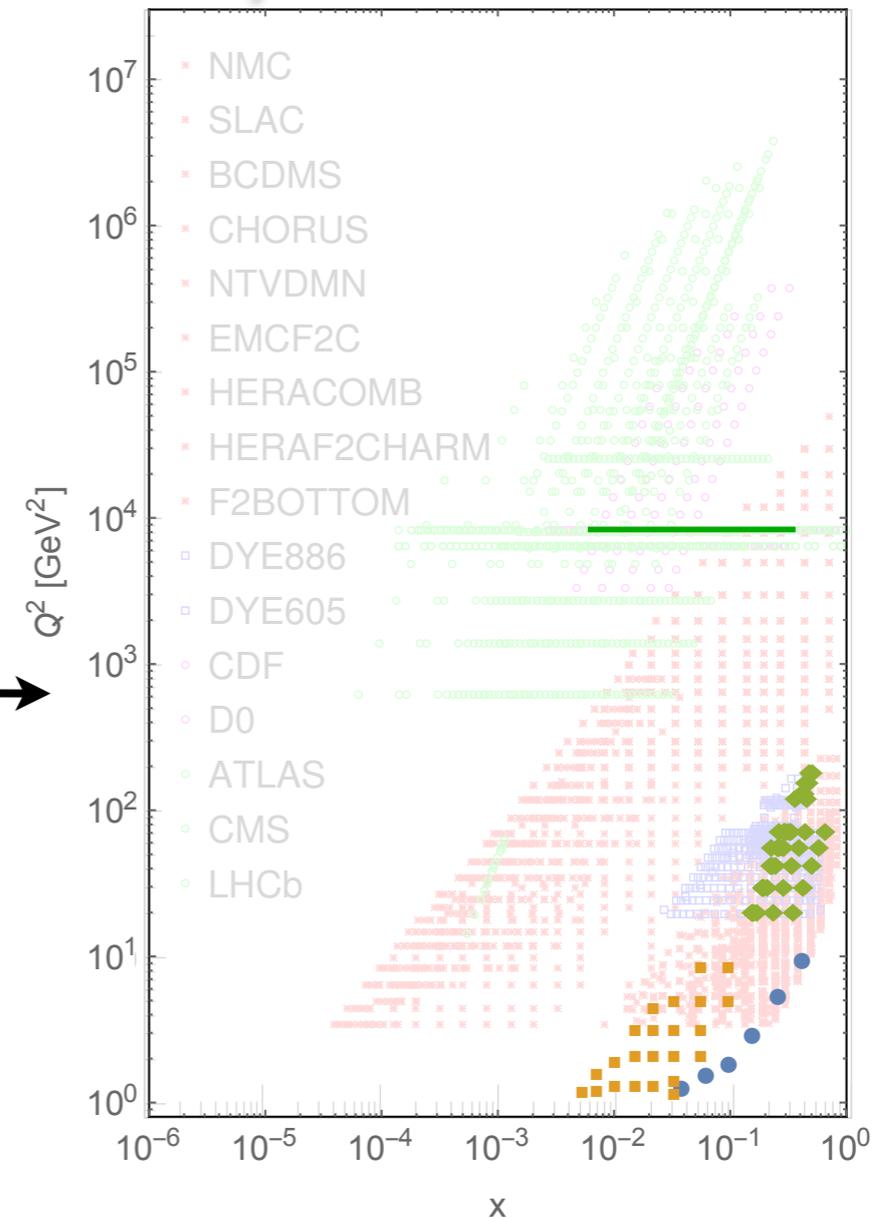


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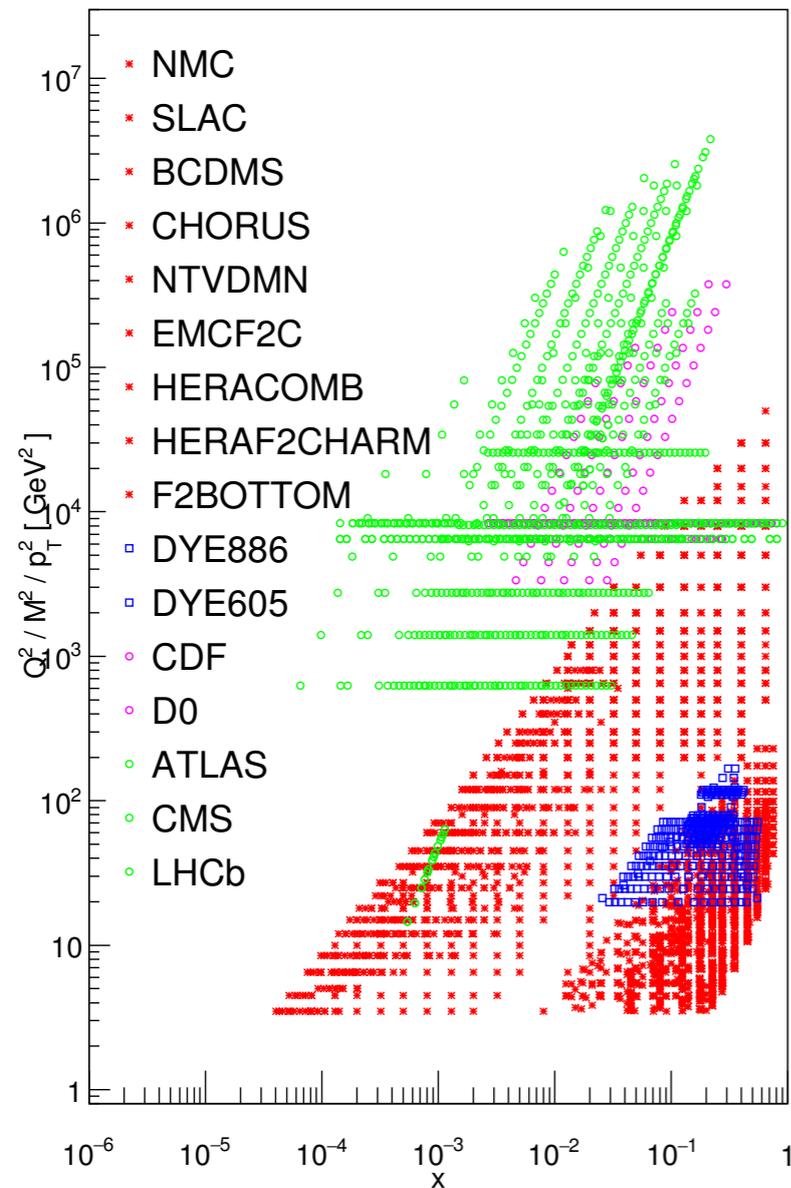
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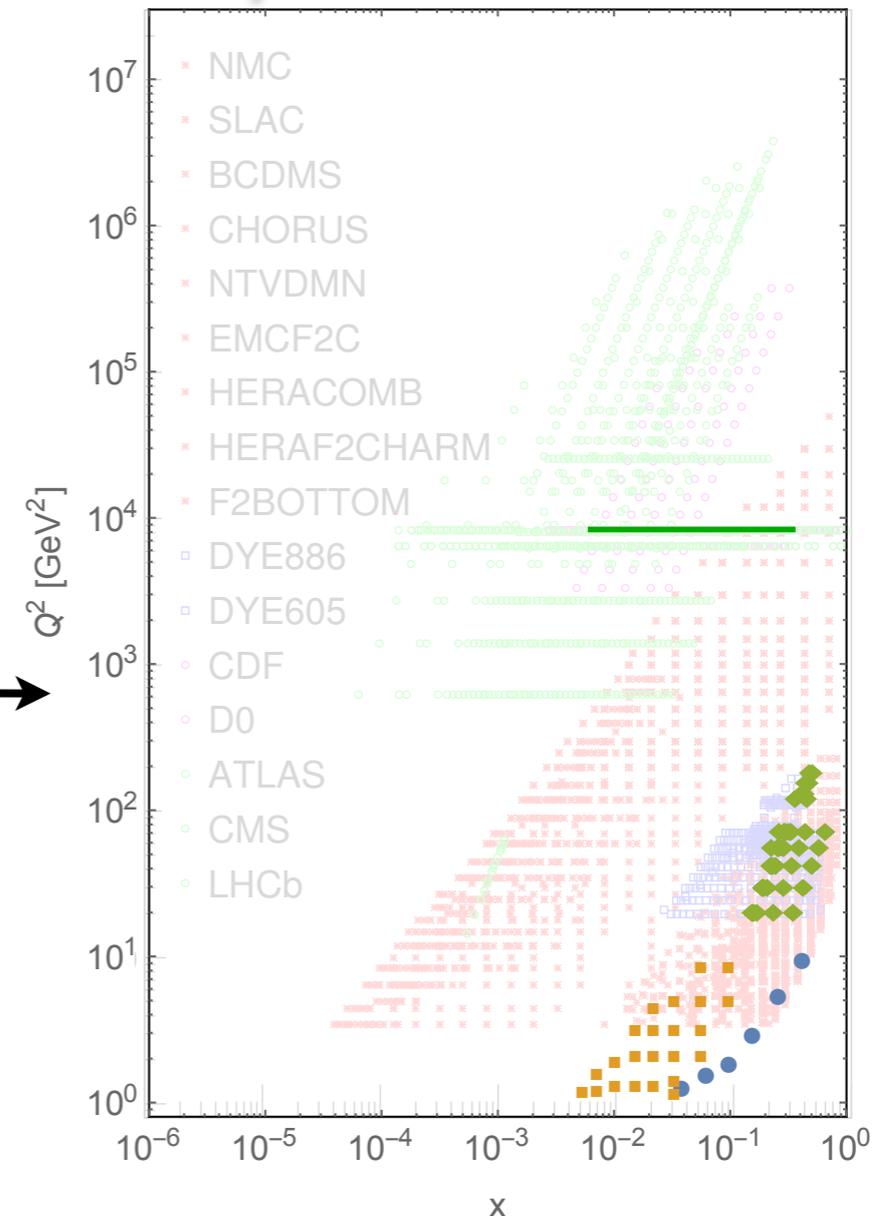


data sets available:

← collinear PDFs

vs

TMD PDFs →



On top of extending **data set**, many improvements are needed:
 higher **perturbative** orders, **matching** with high transverse momentum,
flavor dependence, flexible functional forms...

What do we know ?

(only a selection of results!)

	Framework	HERMES	COMPASS	DY	Z production	N of points
KN 2006 hep-ph/0506225	LO-NLL	✗	✗	✓	✓	98
Pavia 2013 (+Amsterdam, Bilbao) arXiv:1309.3507	No evo (QPM)	✓	✗	✗	✗	1538
Torino 2014 (+JLab) arXiv:1312.6261	No evo (QPM)	✓ (separately)	✓ (separately)	✗	✗	576 (H) 6284 (C)
DEMS 2014 arXiv:1407.3311	NLO-NNLL	✗	✗	✓	✓	223
EIKV 2014 arXiv:1401.5078	LO-NLL	1 (x, Q^2) bin	1 (x, Q^2) bin	✓	✓	500 (?)
Pavia/JLab 2017 arXiv:1703.10157	LO-NLL	✓	✓	✓	✓	8059
SV 2017 arXiv:1706.01473	NNLO- NNLL	✗	✗	✓	✓	309

[courtesy A. Bacchetta]

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Torino 2014 (+JLab) arXiv:1312.6261	No evo (QPM)	✓ (separately)	✓ (separately)	✗	✗	576 (H) 6284 (C)
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SV 2017 arXiv:1706.01473	NNLO- NNLL	✗	✗	✓	✓	309

[courtesy A. Bacchetta]

Features

	Framework	HERMES	COMPASS	DY	Z production	N of points
Pavia/JLab 2017 arXiv:1703.10157	LO-NLL	✓	✓	✓	✓	8059

PROs

almost a **global fit** of quark unpolarized TMDs

includes **TMD evolution**

replica (bootstrap)

fitting methodology

kinematic dependence

in intrinsic part of TMDs

intrinsic momentum: **beyond the Gaussian** assumption

CONs

no “pure” info on TMD FFs

accuracy of TMD evolution :
not the state of the art

only “low” transverse momentum
(no fixed order and Y-term)

flavor separation in
the transverse
plane : problematic

TMD PDFs at 1 GeV

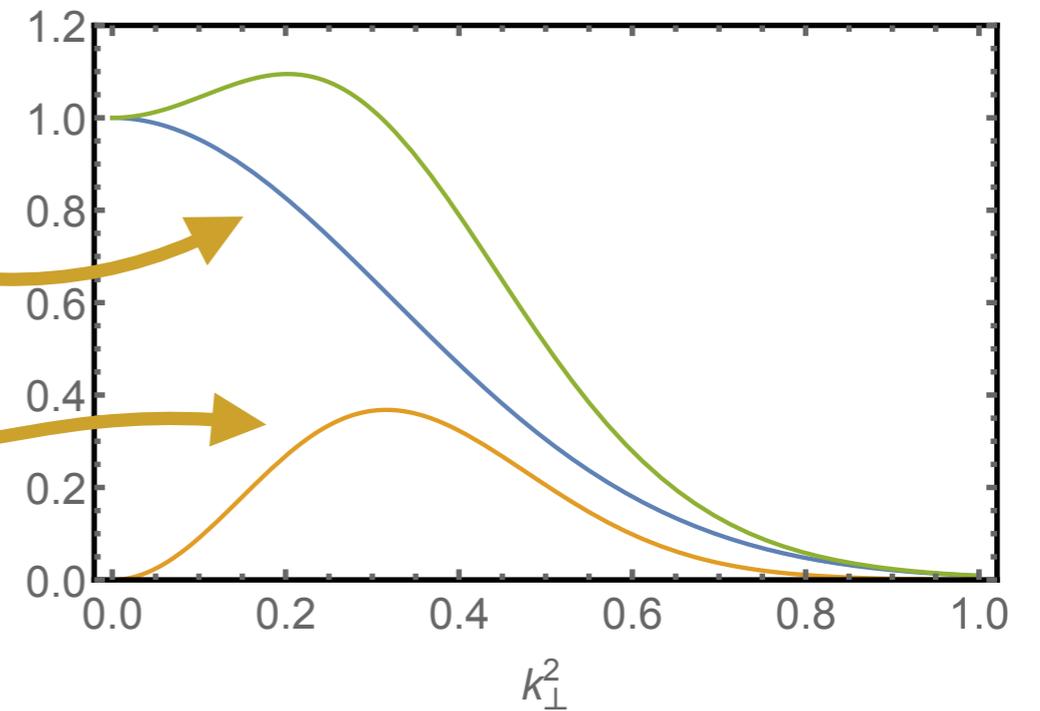
$$\tilde{f}_{1\text{NP}}^a(x, b_T^2) = \frac{1}{2\pi} e^{-g_{1a} \frac{b_T^2}{4}} \left(1 - \frac{\lambda g_{1a}^2}{1 + \lambda g_{1a}} \frac{b_T^2}{4} \right)$$

$$f_{1\text{NP}}^a(x, \mathbf{k}_\perp^2) = \frac{1}{\pi} \frac{\left(e^{-\frac{\mathbf{k}_\perp^2}{g_{1a}}} + \lambda \mathbf{k}_\perp^2 e^{-\frac{\mathbf{k}_\perp^2}{g_{1a}}} \right)}{g_{1a} + \lambda g_{1a}^2}$$

TMD PDFs at 1 GeV

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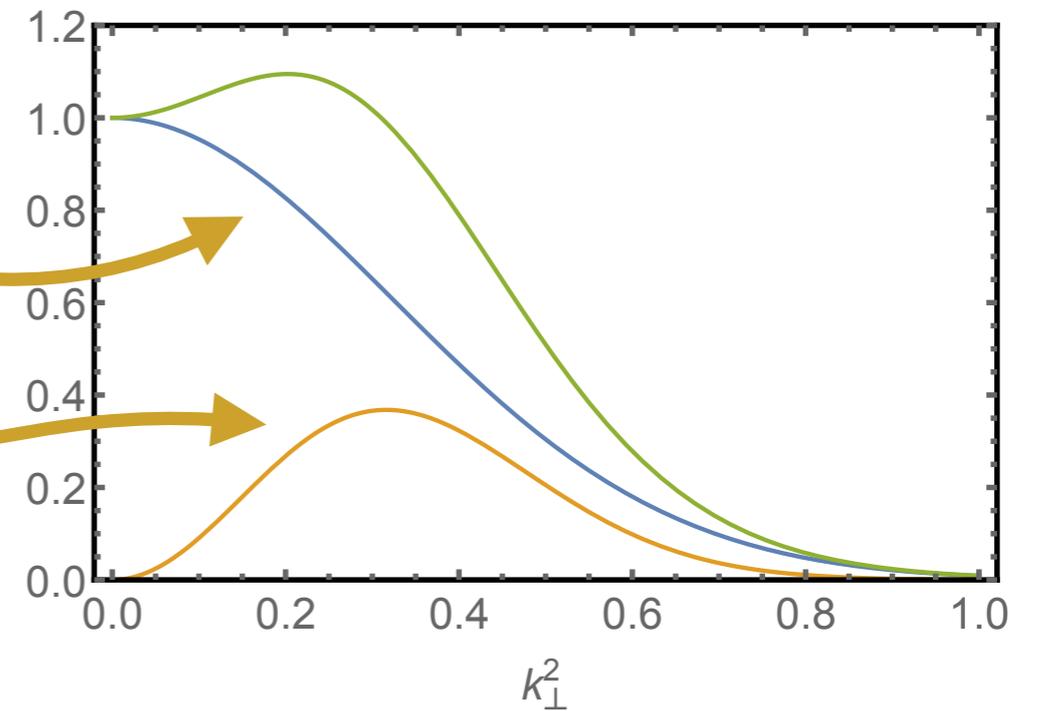
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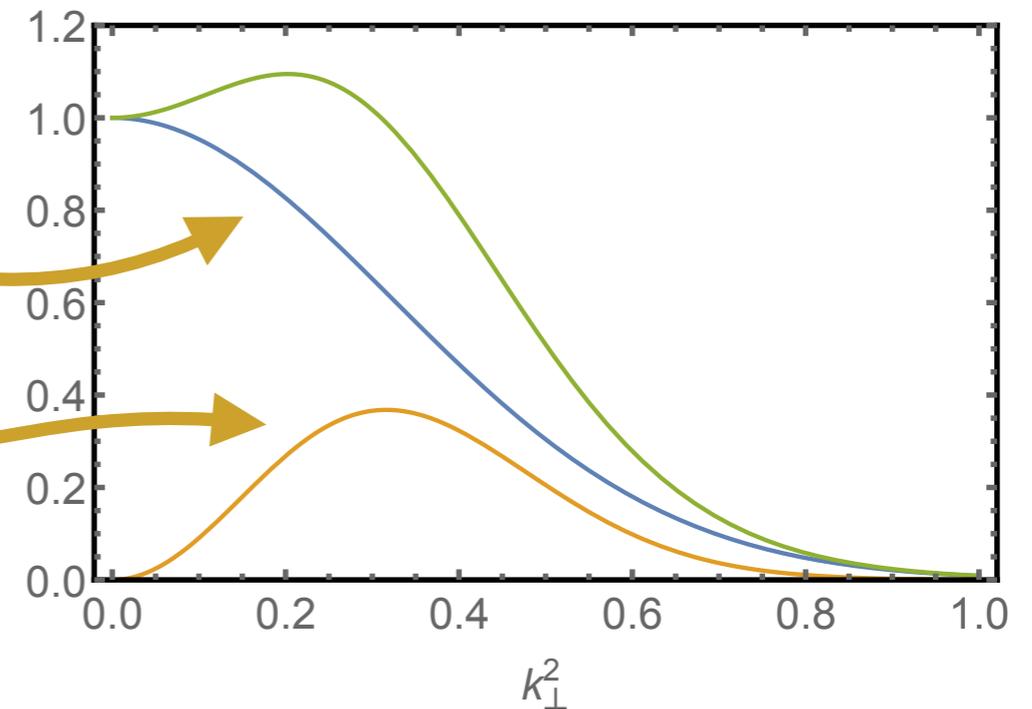
x-dependent width

$$g_1(x) = N_1 \frac{(1-x)^\alpha x^\sigma}{(1-\hat{x})^\alpha \hat{x}^\sigma}$$

TMD PDFs at 1 GeV

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x-dependent width

$$g_1(x) = N_1 \frac{(1-x)^\alpha x^\sigma}{(1-\hat{x})^\alpha \hat{x}^\sigma}$$

Fragmentation function is similar

Including TMD PDFs and FFs, in total: 11 free parameters

(4 for TMD PDFs, 6 for TMD FFs, 1 for TMD evolution)

Agreement data-theory

Flavor independent scenario

Flavor independent configuration | 11 parameters

Points	Parameters	χ^2	$\chi^2/\text{d.o.f.}$
8059	11	12629 ± 363	1.55 ± 0.05

	HERMES $p \rightarrow \pi^+$	HERMES $p \rightarrow \pi^-$	HERMES $p \rightarrow K^+$	HERMES $p \rightarrow K^-$
Points	190	190	189	187
χ^2/points	4.83	2.47	0.91	0.82

Hermes P/D into π^+ :
problems at low z

	HERMES $D \rightarrow \pi^+$	HERMES $D \rightarrow \pi^-$	HERMES $D \rightarrow K^+$	HERMES $D \rightarrow K^-$	COMPASS $D \rightarrow h^+$	COMPASS $D \rightarrow h^-$
Points	190	190	189	189	3125	3127
χ^2/points	3.46	2.00	1.31	2.54	1.11	1.61

	E288 [200]	E288 [300]	E288 [400]	E605
Points	45	45	78	35
χ^2/points	0.99	0.84	0.32	1.12

Hermes kaons better than pions:
larger uncertainties from FFs

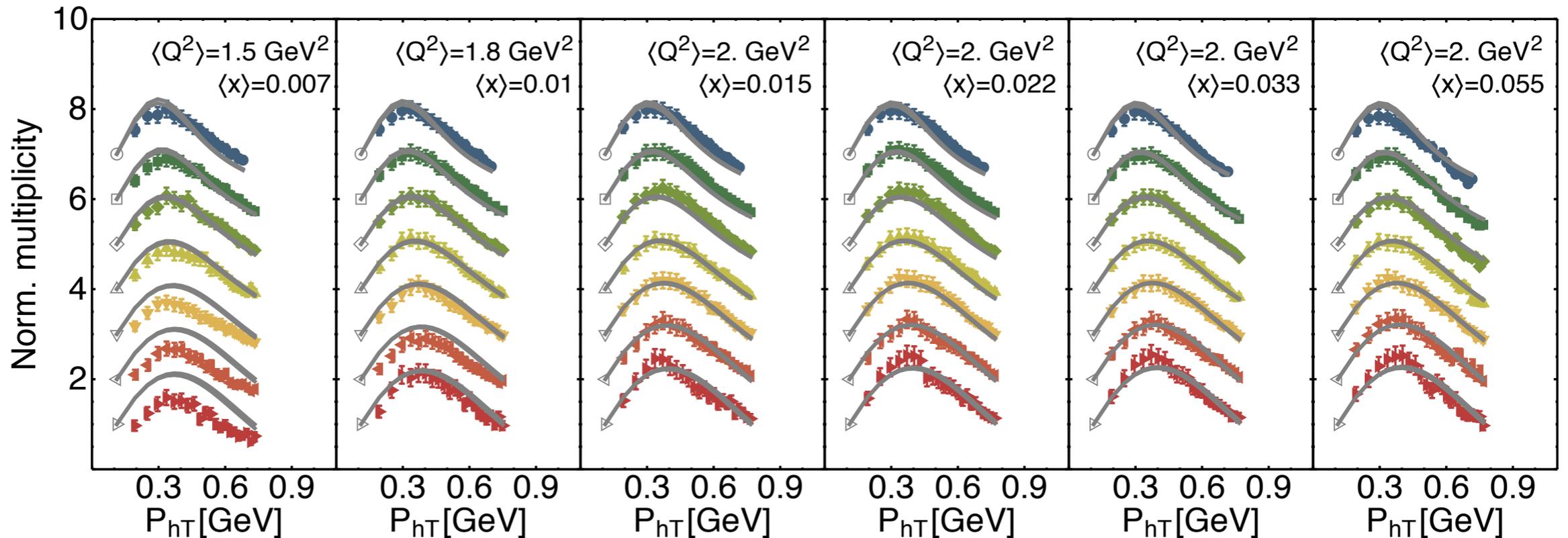
	CDF Run I	D0 Run I	CDF Run II	D0 Run II
Points	31	14	37	8
χ^2/points	1.36	1.11	2.00	1.73

Compass : better agreement due to
#points and normalization

COMPASS, selected bins



- $\langle z \rangle = 0.23$ (offset=6)
- $\langle z \rangle = 0.28$ (offset=5)
- ◆ $\langle z \rangle = 0.33$ (offset=4)
- ▲ $\langle z \rangle = 0.38$ (offset=3)
- ▼ $\langle z \rangle = 0.45$ (offset=2)
- ▲ $\langle z \rangle = 0.55$ (offset=1)
- ▼ $\langle z \rangle = 0.65$ (offset=0)

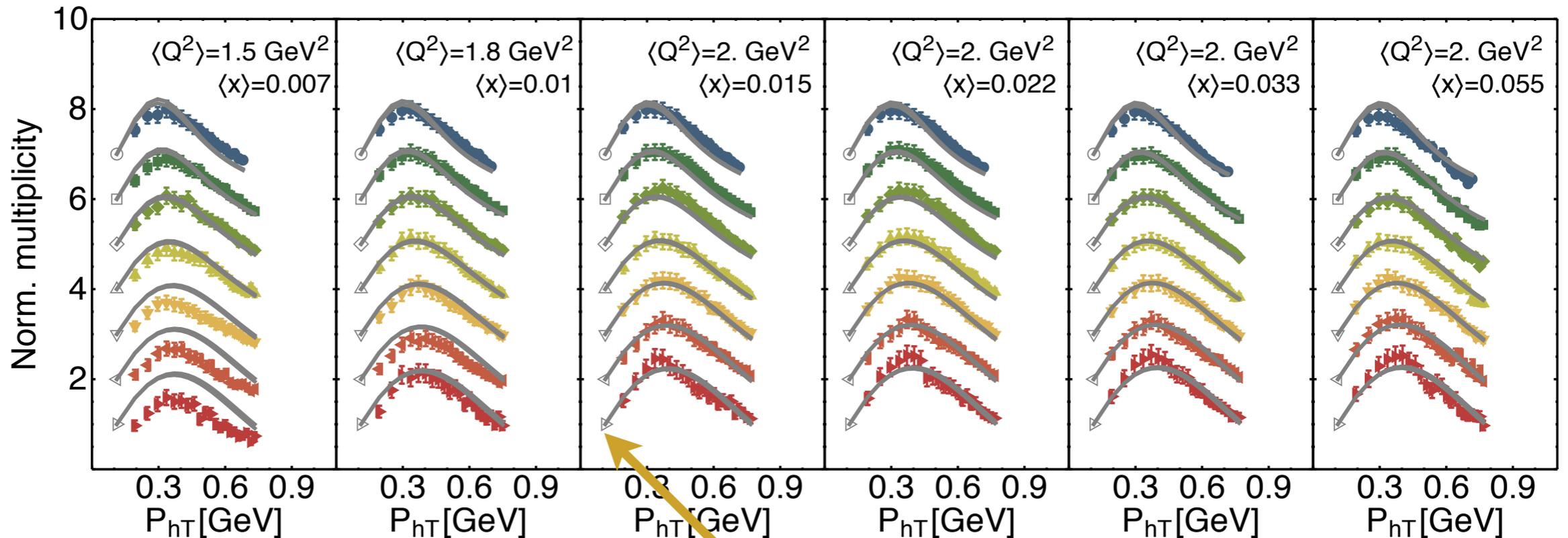


Deuteron h^- $\chi^2/\text{dof} = 1.58$

COMPASS, selected bins



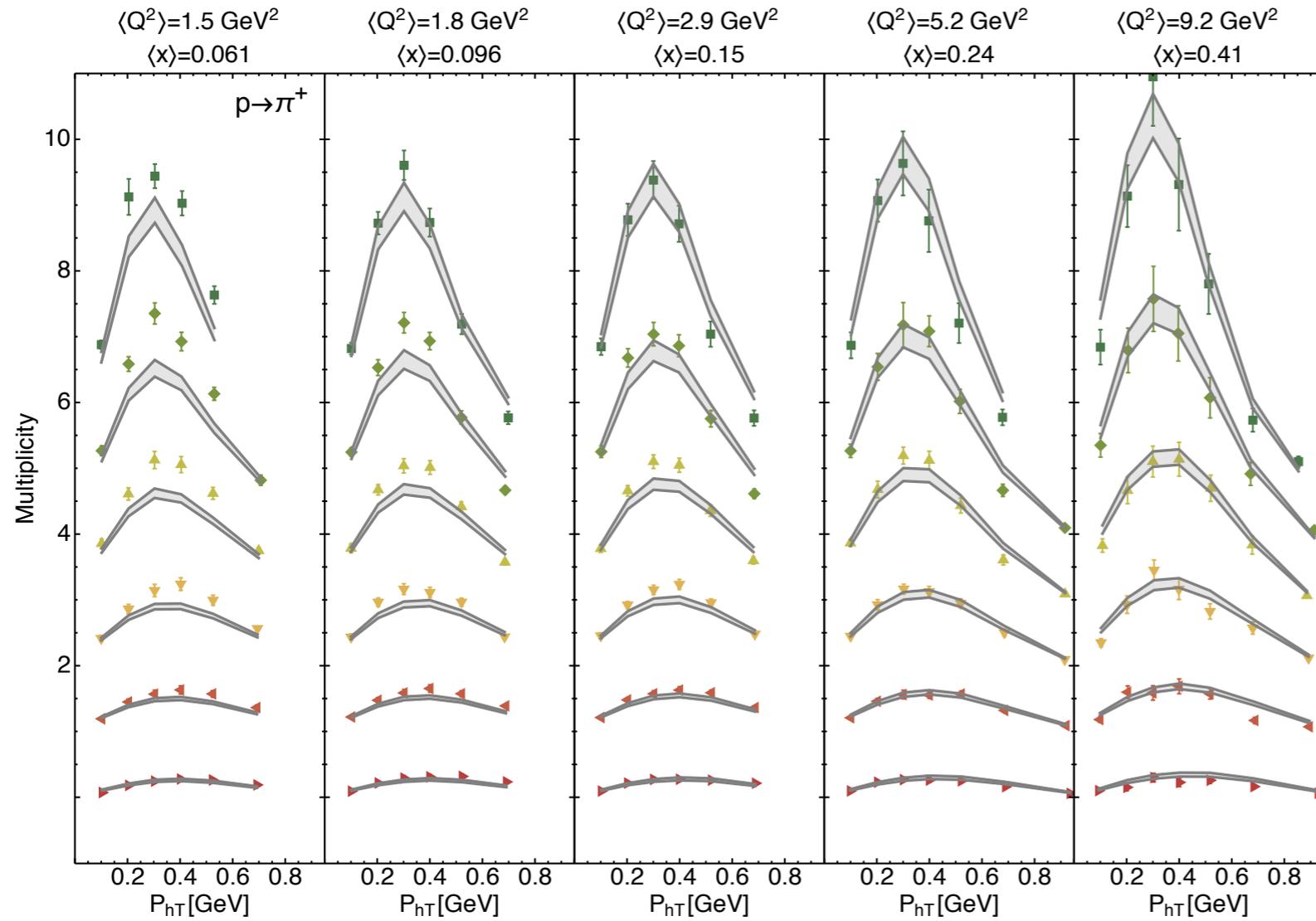
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- ▼ $\langle z \rangle = 0.65$ (offset=0)



First points are not fitted, but used as normalization

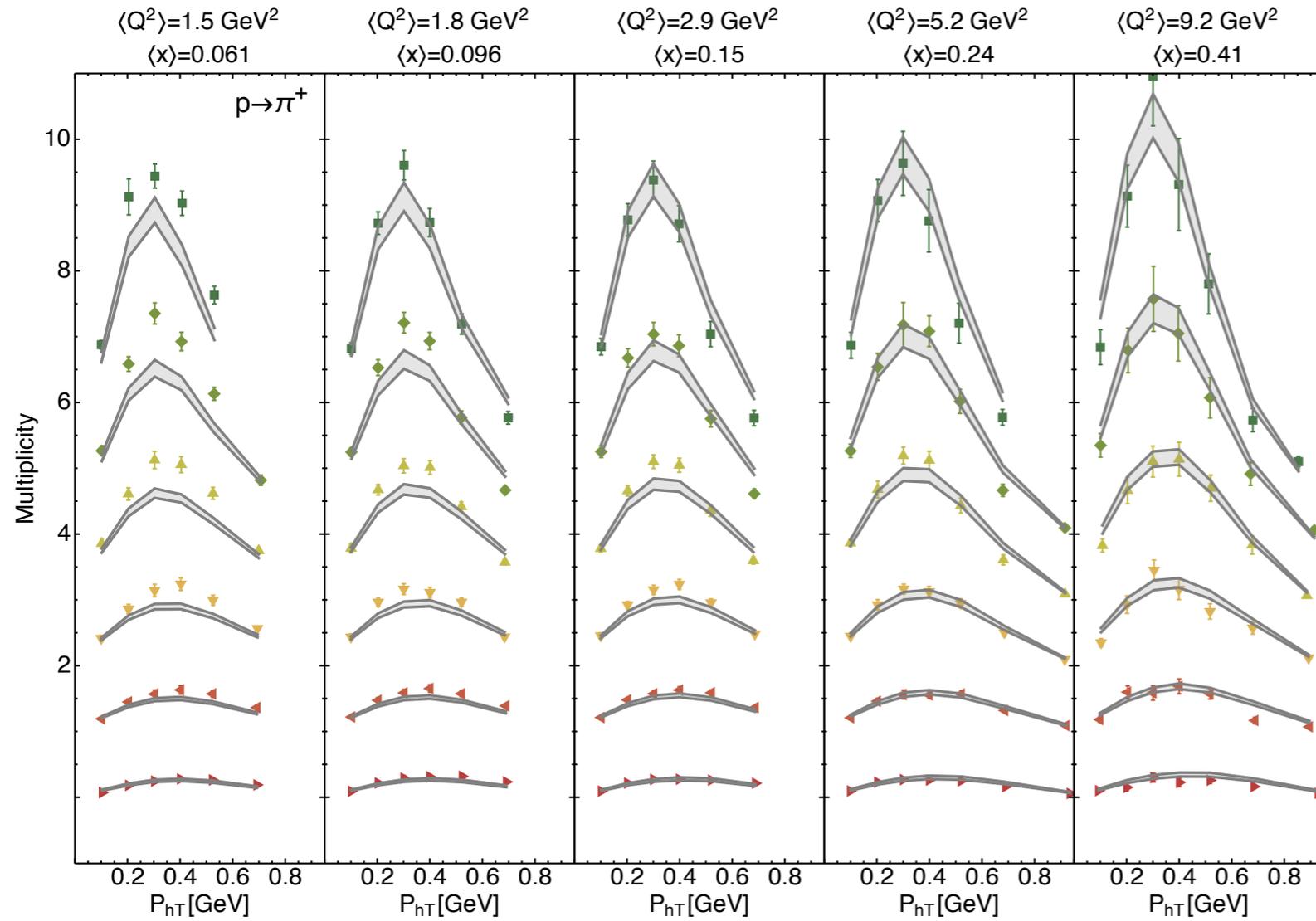
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HERMES, selected bins



Contributions to chi2 mainly from **normalization**, not shape
(also in Z-boson production)

HERMES, selected bins

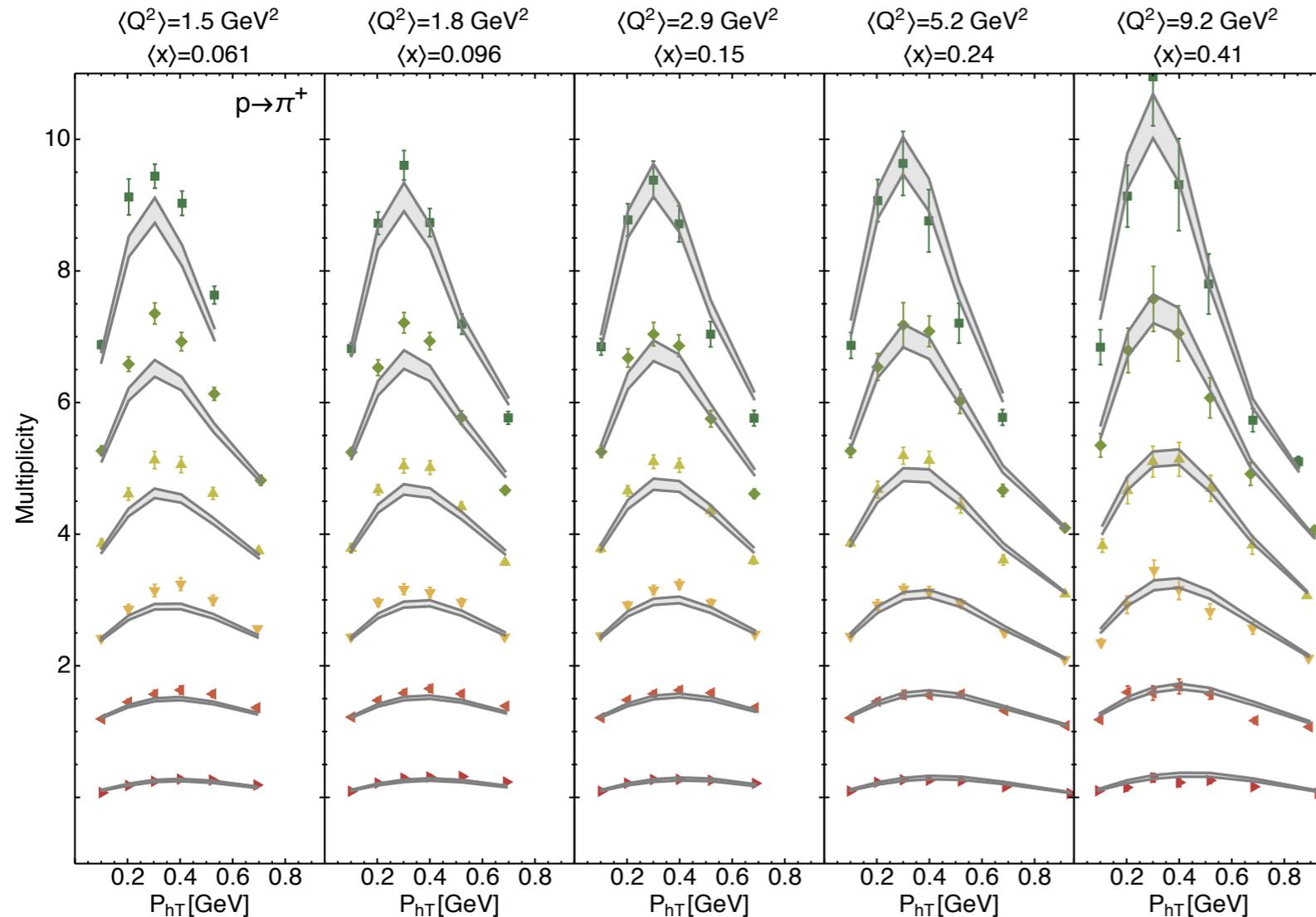


$$\chi^2 / \text{dof} = 4.80$$

The worst of all channels...

Contributions to chi2 mainly from **normalization**, not shape
(also in Z-boson production)

HERMES, selected bins



$$\chi^2/\text{dof} = 4.80$$

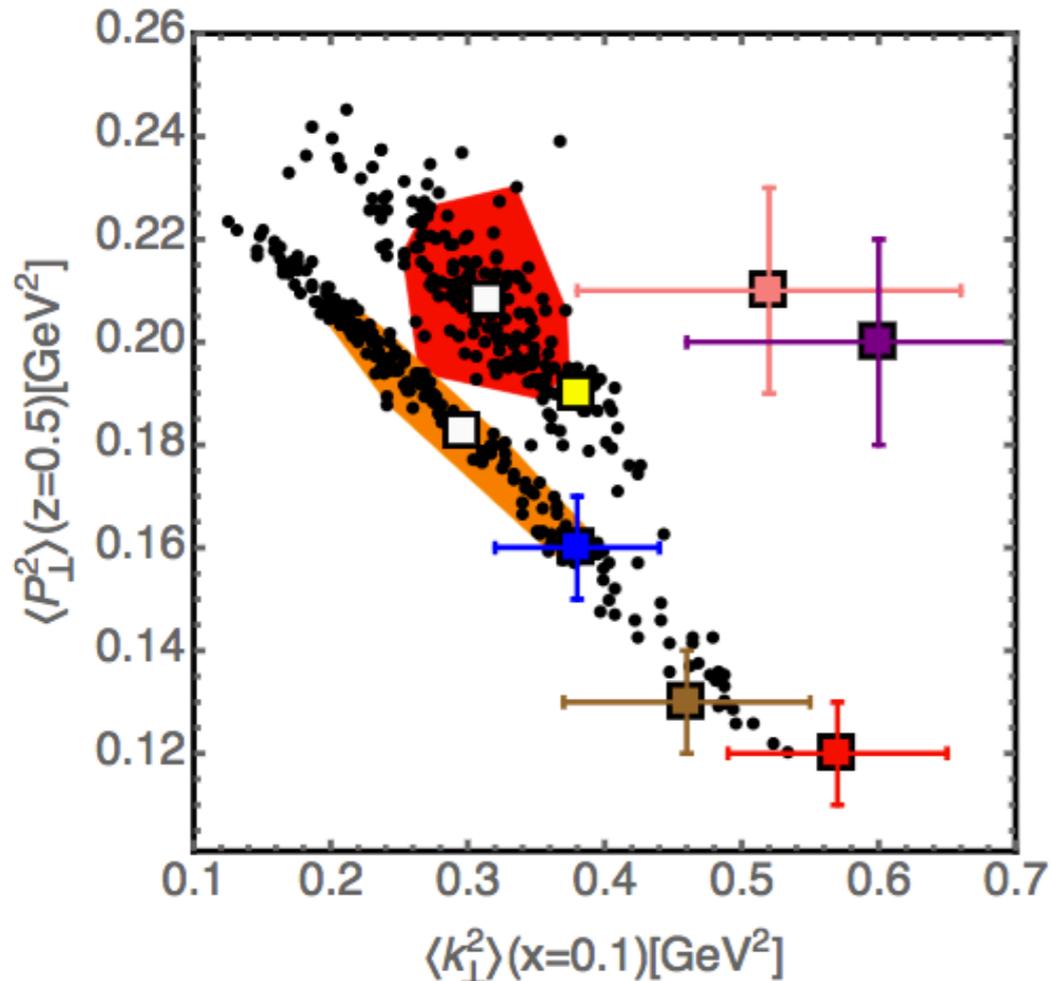
The worst of all channels...

However **normalizing** the theory curves to the first bin, without changing the parameters of the fit, χ^2/dof becomes good

Contributions to chi2 mainly from **normalization**, not shape
(also in Z-boson production)

Best-fit values

Flavor independent scenario



- Bacchetta, Delcarro, Pisano, Radici, Signori,
- Signori, Bacchetta, Radici, Schnell arXiv:1309.3507
- Schweitzer, Teckentrup, Metz, arXiv:1003.2190
- Anselmino et al. arXiv:1312.6261 [HERMES]
- Anselmino et al. arXiv:1312.6261 [HERMES, high z]
- Anselmino et al. arXiv:1312.6261 [COMPASS, norm.]
- Anselmino et al. arXiv:1312.6261 [COMPASS, high z, norm.]
- Echevarria, Idilbi, Kang, Vitev arXiv:1401.5078 ($Q = 1.5$ GeV)

Red/orange regions : **68% CL** from replica method

Inclusion of **DY/Z** diminishes the correlation

Inclusion of **Compass** increases the $\langle P_{\perp}^2 \rangle$
and reduces its spread

e+e- would further reduce the correlation

Caveat for comparisons :

NP effects (as the intrinsic momentum) always depend on the accuracy of the perturbative part ;

determined as observed - calculable

Polarized TMDs (Sivers)

References :

- "The 3D structure of the nucleon" **EPJ A (2016) 52**
- STAR **arXiv:1511.06003**
- Compass: **arxiv:1704.00488**
- ...

Process dependence

Gauge invariance and T-reversal invariance generate a **sign change** between the **Sivers** TMD PDF in **Drell-Yan** and **Semi-Inclusive DIS**

Process dependence

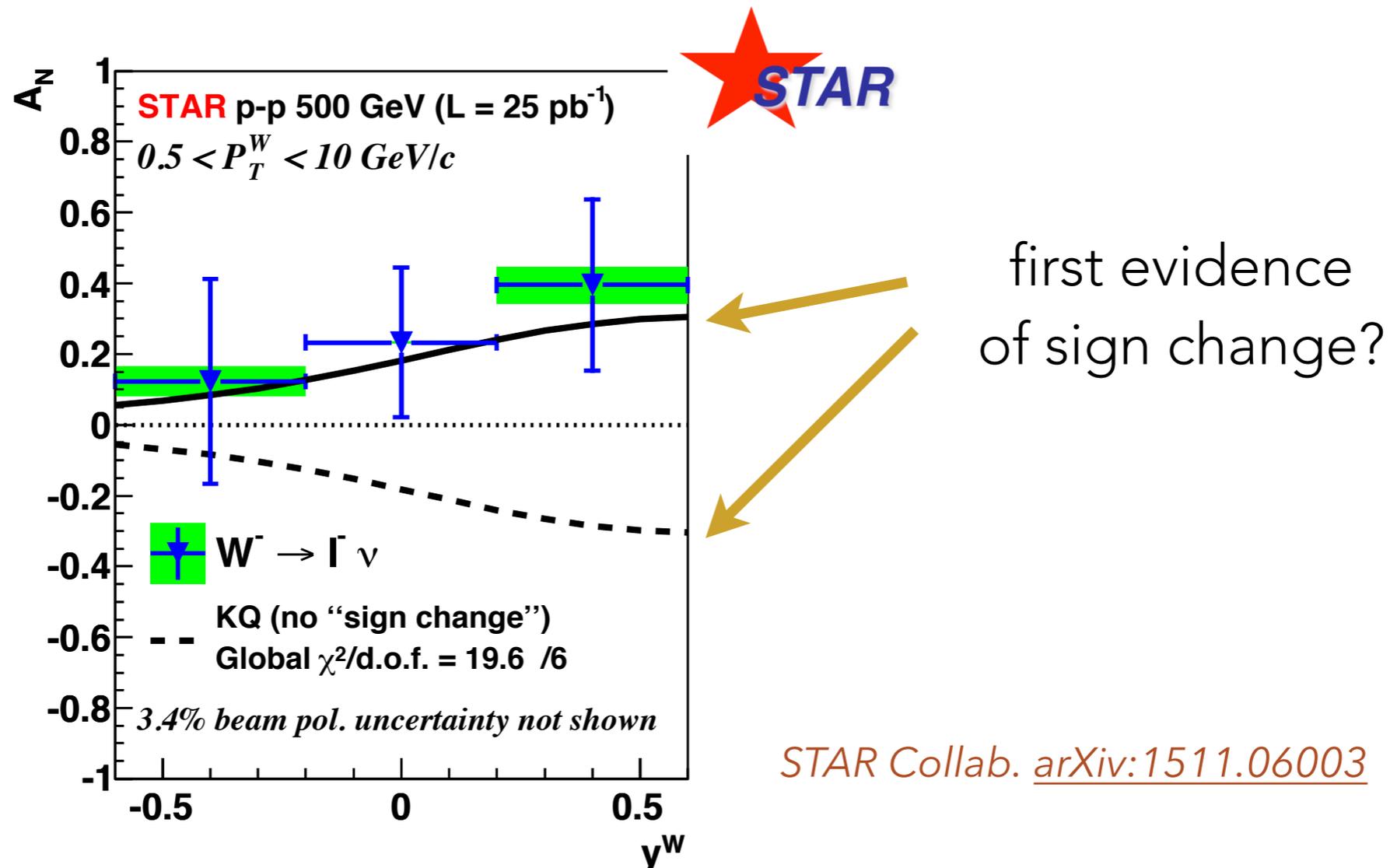
Gauge invariance and T-reversal invariance generate a **sign change** between the **Sivers** TMD PDF in **Drell-Yan** and **Semi-Inclusive DIS**

Collins, PLB 536 (02)

Process dependence

Gauge invariance and T-reversal invariance generate a **sign change** between the **Sivers** TMD PDF in **Drell-Yan** and **Semi-Inclusive DIS**

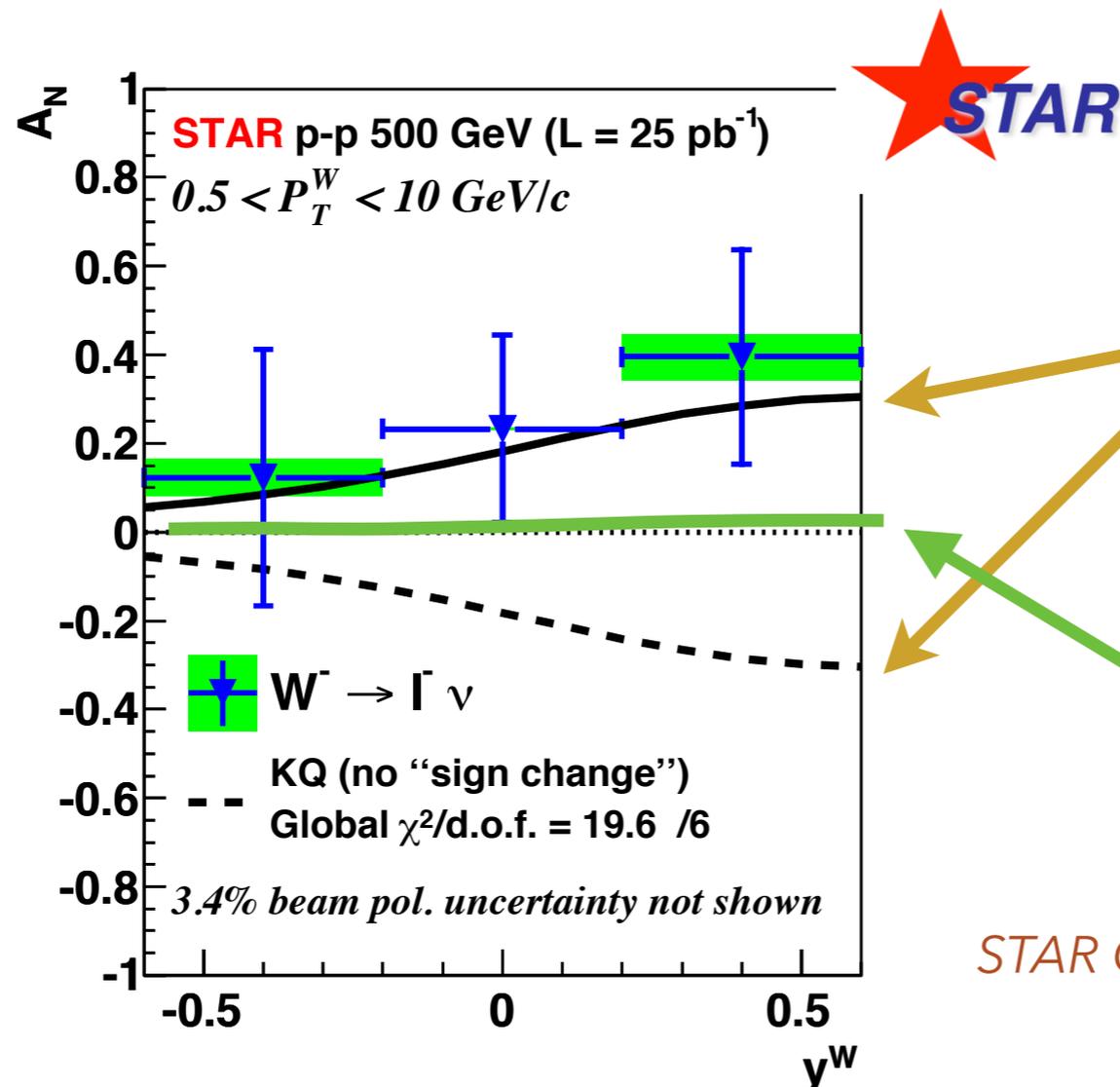
Collins, PLB 536 (02)



Process dependence

Gauge invariance and T-reversal invariance generate a **sign change** between the **Sivers** TMD PDF in **Drell-Yan** and **Semi-Inclusive DIS**

Collins, PLB 536 (02)



first evidence
of sign change?

prediction with TMD
evolution equations

STAR Collab. [arXiv:1511.06003](https://arxiv.org/abs/1511.06003)

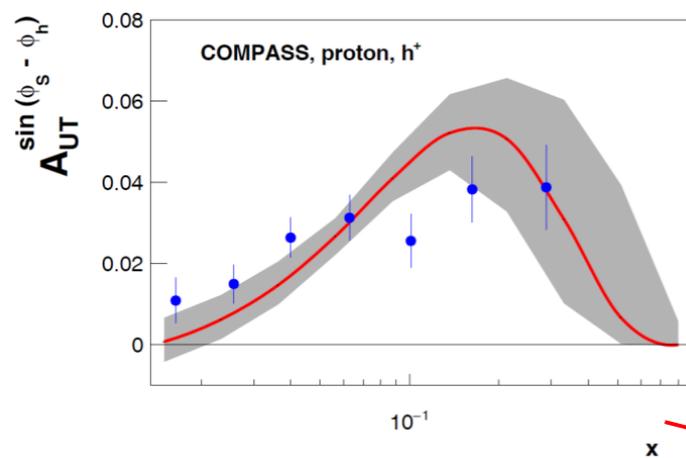
Process dependence

Sivers asymmetry in Semi-Inclusive DIS



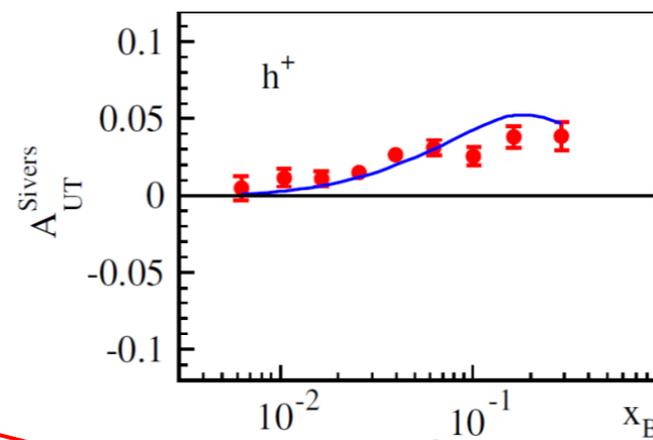
DGLAP (2016)

M. Anselmino et al., [arXiv:1612.06413](https://arxiv.org/abs/1612.06413)



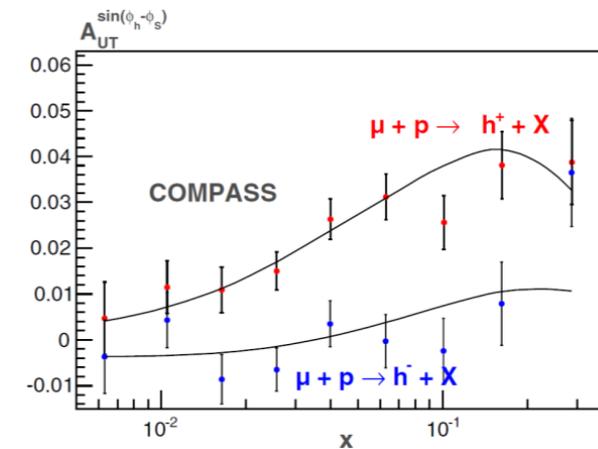
TMD-1 (2014)

M. G. Echevarria et al. [PRD89,074013](https://arxiv.org/abs/1407.0740)



TMD-2 (2013)

P. Sun, F. Yuan, [PRD88, 114012](https://arxiv.org/abs/1303.1140)

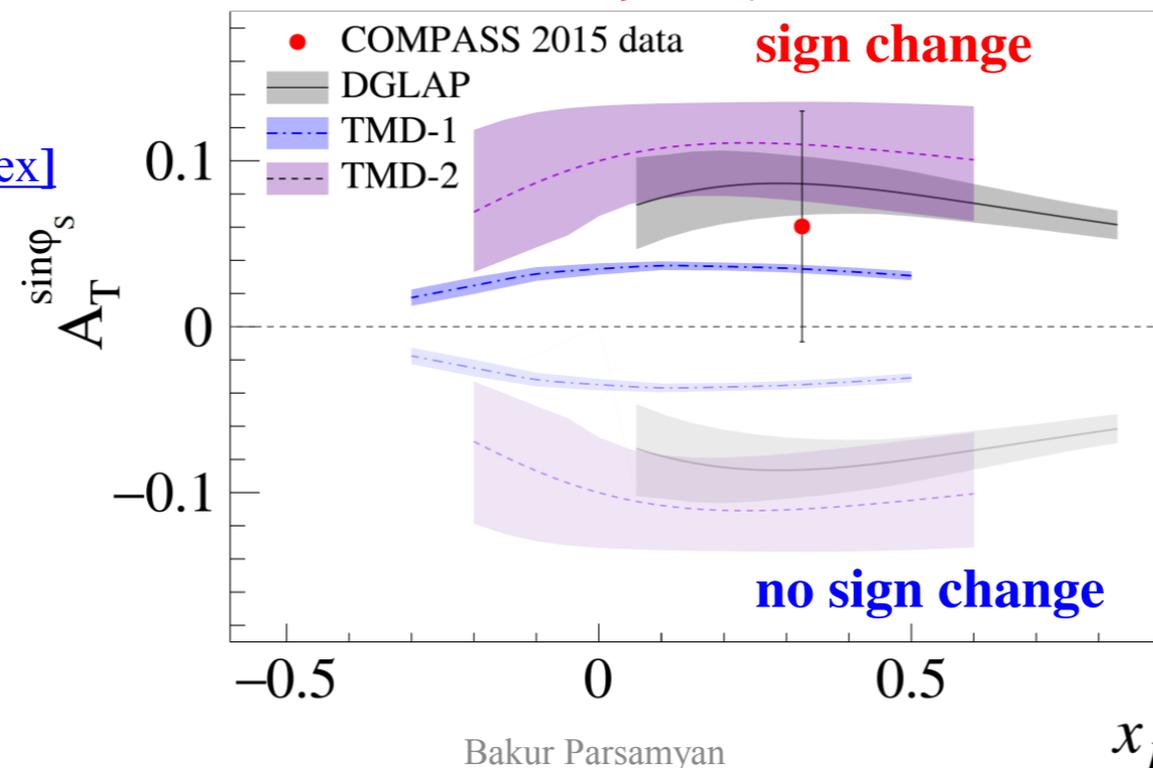


New! 03 April 2017

COMPASS

[CERN-EP-2017-059](https://arxiv.org/abs/1704.00488)

[arXiv:1704.00488\[hep-ex\]](https://arxiv.org/abs/1704.00488)



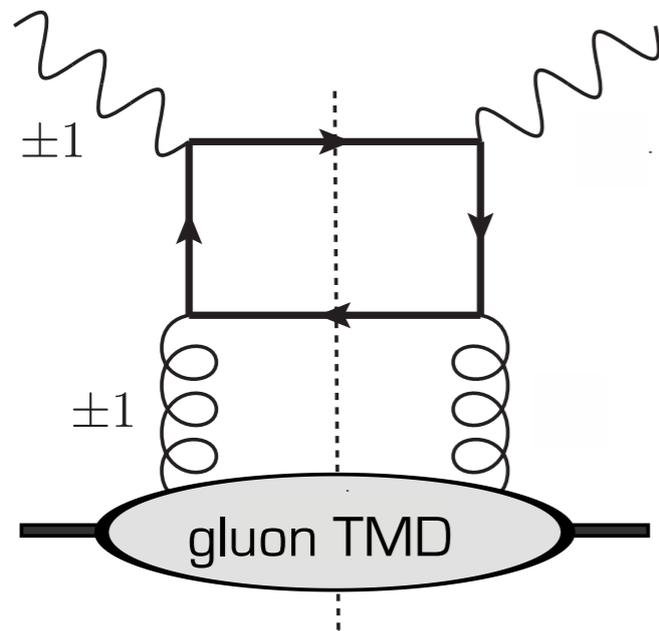
Sivers asymmetry in Drell-Yan

courtesy B. Parsamyan

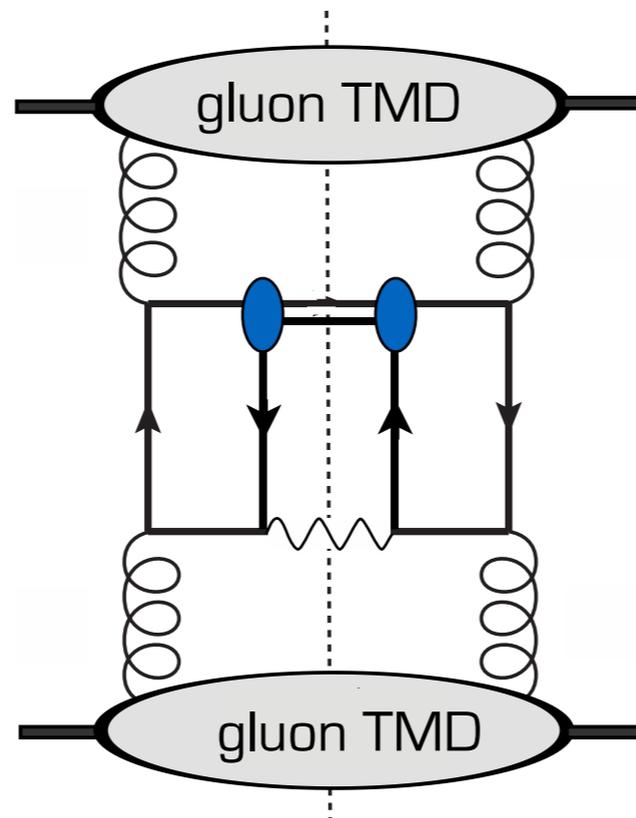
Gluon TMDs

Gluon TMDs

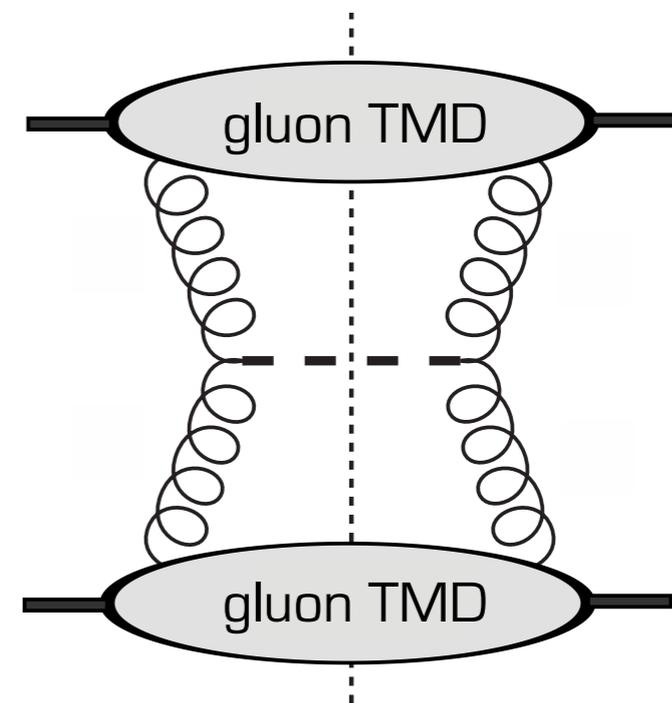
$$e p \rightarrow e \text{ jet jet } X$$



$$p p \rightarrow J/\psi \gamma X$$



$$p p \rightarrow \eta_c X$$



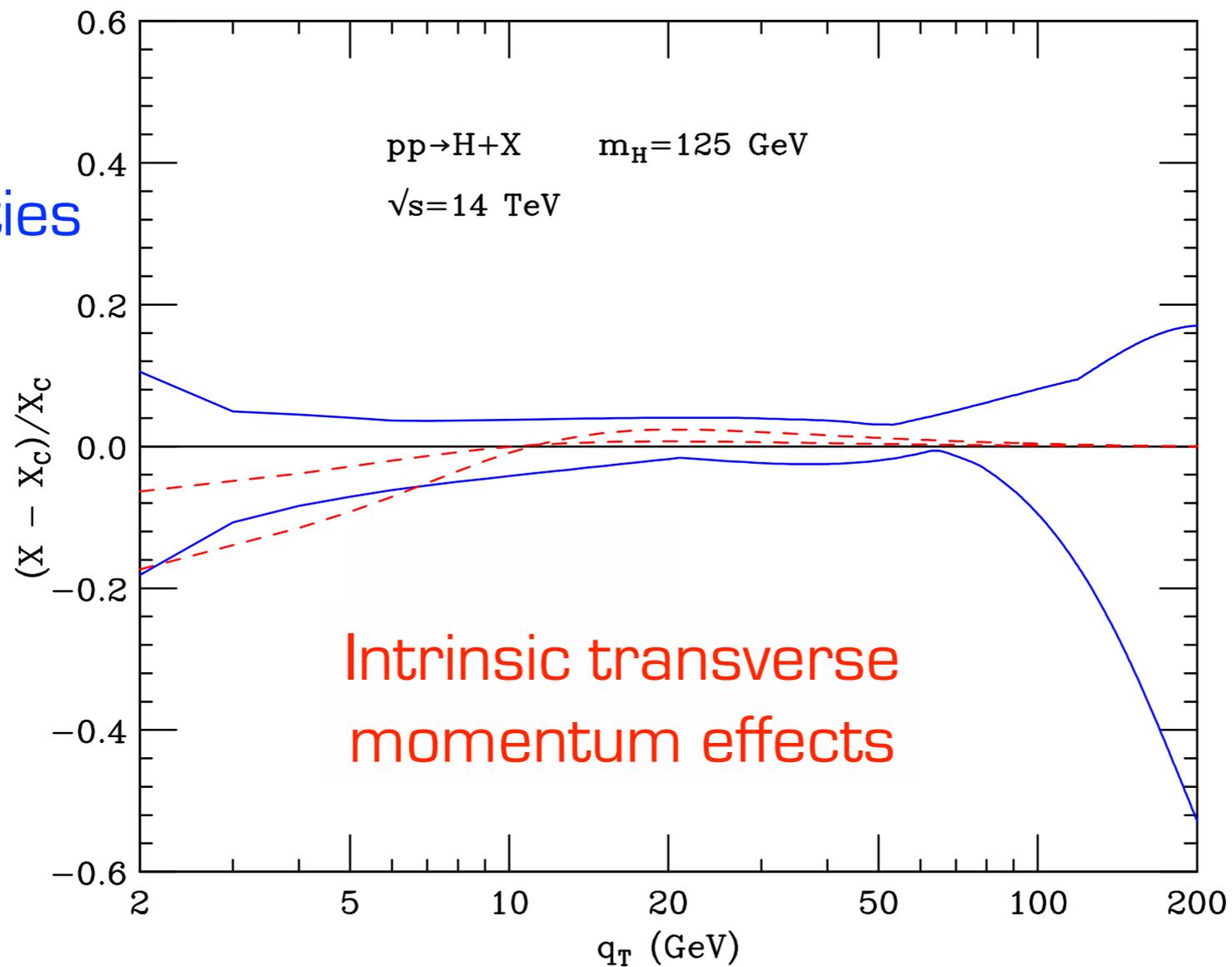
see, e.g.,

- Boer, den Dunnen, Pisano, Schlegel, Vogelsang, PRL 108 (12)
- den Dunnen, Lansberg, Pisano, Schlegel, PRL 112 (14)
- AS: PhD thesis , arXiv:1602.03405
- AFTER@LHC working group: arXiv:1702.01546 , arXiv:1610.05228 ,
- Echevarria et al. arXiv:1502.05354
- ...

Higgs transverse momentum

G. Ferrera, talk at REF 2014, Antwerp, <https://indico.cern.ch/event/330428/>

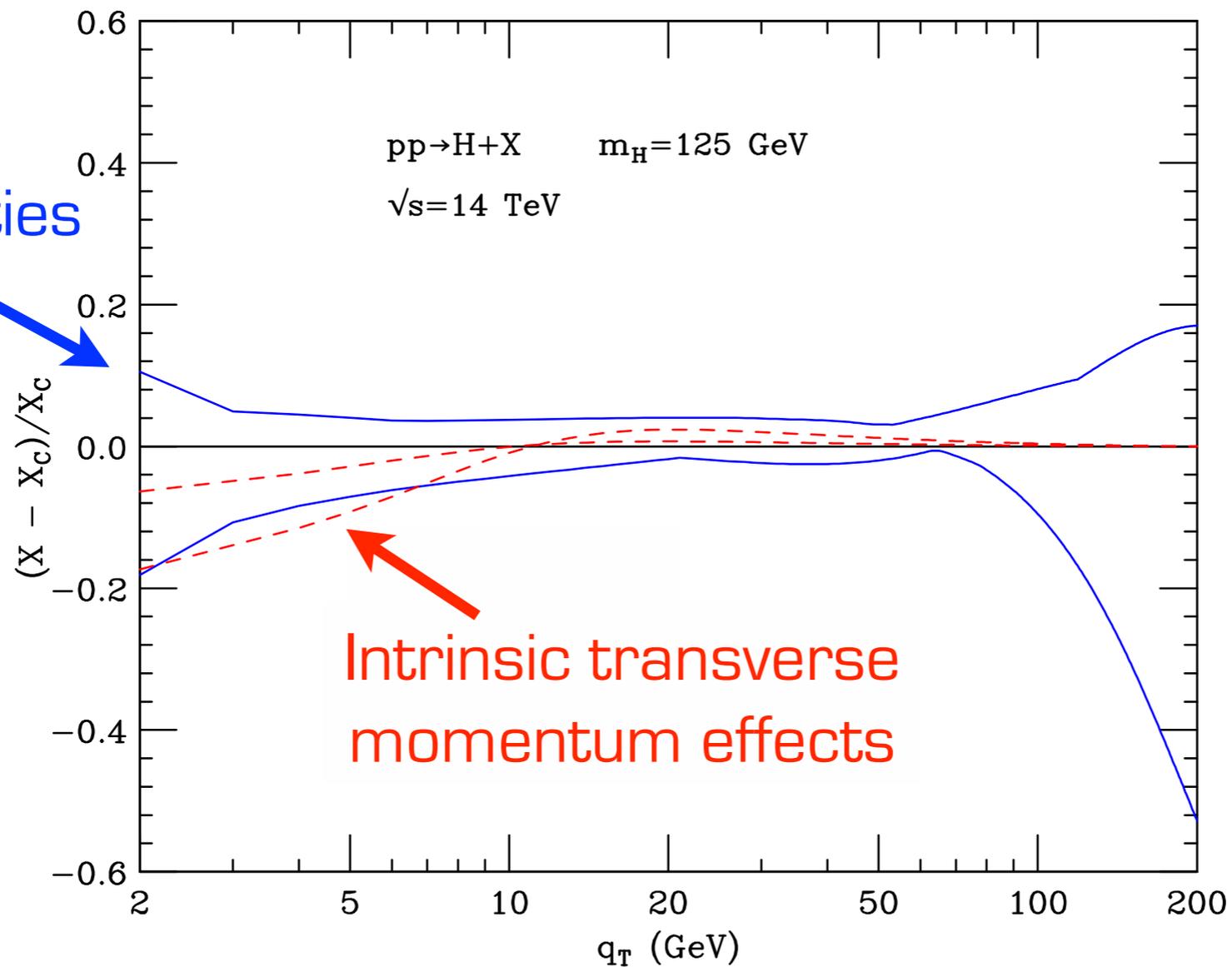
PDF uncertainties



Higgs transverse momentum

G. Ferrera, talk at REF 2014, Antwerp, <https://indico.cern.ch/event/330428/>

PDF uncertainties



Spin 1 TMDs

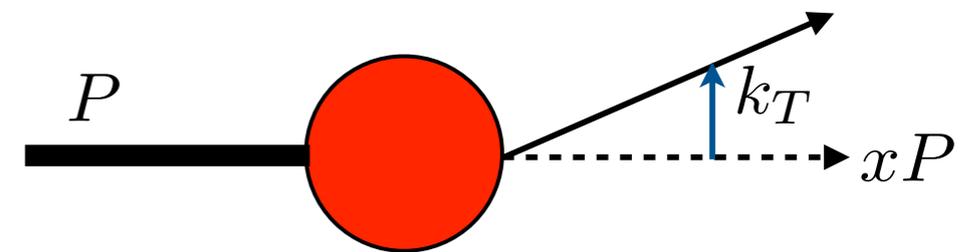
References :

- quark TMDs : Phys.Rev. D62 (2000) 114004
- gluon TMDs : JHEP 1610 (2016) 013
- ...

quark TMD PDFs

$$\Phi_{ij}(k, P; S, T) \sim \text{F.T.} \langle PST | \bar{\psi}_j(0) U_{[0,\xi]} \psi_i(\xi) | PST \rangle_{LF}$$

Quarks	γ^+	$\gamma^+ \gamma^5$	$i\sigma^{i+} \gamma^5$
U	f_1		h_1^\perp
L		g_1	h_{1L}^\perp
T	f_{1T}^\perp	g_{1T}	h_1, h_{1T}^\perp
LL	f_{1LL}		h_{1LL}^\perp
LT	f_{1LT}	g_{1LT}	h_{1LT}, h_{1LT}^\perp
TT	f_{1TT}	g_{1TT}	h_{1TT}, h_{1TT}^\perp



extraction of a **quark**
not collinear with the proton

a similar scheme holds for
 TMD FFs and gluons

bold : also collinear

red : time-reversal odd (universality properties)

quark TMD PDFs

recent investigations of the T-even
TMDs in the context of **DSE**
[arXiv:1707.03787](https://arxiv.org/abs/1707.03787)

Quarks	γ^+	$\gamma^+\gamma^5$	$i\sigma^{i+}\gamma^5$
U	f_1		h_1^\perp
L		g_1	h_{1L}^\perp
T	f_{1T}^\perp	g_{1T}	h_1, h_{1T}^\perp
LL	f_{1LL}		h_{1LL}^\perp
LT	f_{1LT}	g_{1LT}	h_{1LT}, h_{1LT}^\perp
TT	f_{1TT}	g_{1TT}	h_{1TT}, h_{1TT}^\perp

Collinear, related to $b_1(x)$
(under scrutiny at JLab)

Collinear & T-odd : should be zero!
(to be investigated)

bold : also collinear

red : time-reversal odd (universality properties)

Conclusions : a path to move forward

- 1) Phenomenology of TMDs is well underway ...
- 2) ... but there are a lot of theoretical challenges to be addressed: definition of kinematic regions in SIDIS, matching, perturbative accuracy, a better understanding of hadronization, context for gluon TMDs , ...
- 3) we definitely need more data, at the moment especially for $e+e-$
- 4) Working with some approximations, we are getting closer to a global fit analysis of TMDs
- 5) polarized structure functions unexplored from the point of view of QCD, but we have guidance from parton model studies (see JLab activities)

Backup

Beware of different notations...

Amsterdam

Seattle ([arXiv:1108.1713](https://arxiv.org/abs/1108.1713))

p

k momentum of parton in distribution function

\mathbf{p}_T

\mathbf{k}_\perp parton transverse momentum in distribution function

k

p momentum of fragmenting parton

\mathbf{k}_T

\mathbf{p}_\perp trans. momentum of fragmenting parton w.r.t. final hadron

\mathbf{K}_T

\mathbf{P}_\perp trans. momentum of final hadron w.r.t. fragmenting parton

$\mathbf{P}_{h\perp}$

\mathbf{P}_{hT} transverse momentum of final hadron w.r.t. virtual photon

TMDs and their evolution

FT of TMDs :

$$\tilde{F}_i(x, b_T; Q, Q^2) = \tilde{F}_i(x, b_T, \mu_{\hat{b}}, \mu_{\hat{b}}^2) \times \exp \left\{ \int_{\mu_{\hat{b}}}^Q \frac{d\mu}{\mu} \gamma_F[\alpha_s(\mu), Q^2/\mu^2] \right\} \left(\frac{Q^2}{\mu_{\hat{b}}^2} \right) \left[K(\hat{b}_T; \mu_{\hat{b}}) - g_K(\bar{b}_T; \{\lambda\}) \right]$$

Sudakov form factor : perturbative and **nonperturbative** contributions

(input) TMD distribution : Wilson coefficients and **intrinsic part** **Collinear distribution!**

$$\tilde{F}_i(x, b_T; \mu_{\hat{b}}, \mu_{\hat{b}}^2) = \sum_{j=q, \bar{q}, g} C_{i/j}(x, \hat{b}_T; \mu_{\hat{b}}, \mu_{\hat{b}}^2) \otimes f_j(x; \mu_{\hat{b}}) \tilde{F}_{i, NP}(x, \bar{b}_T; \{\lambda\})$$

Nonperturbative parts defined in a “negative” way : **observed-calculable**

TMDs and their evolution

Distribution for intrinsic transverse momentum
(and its FT):

$$\tilde{F}_{i,NP}(x, \bar{b}_T; \{\lambda\})$$

a Gaussian ?

Soft gluon emission

$$g_K(\bar{b}_T; \{\lambda\})$$

TMDs and their evolution

Distribution for intrinsic transverse momentum
(and its FT):

$$\tilde{F}_{i,NP}(x, \bar{b}_T; \{\lambda\})$$

a Gaussian ?

Soft gluon emission

$$g_K(\bar{b}_T; \{\lambda\})$$

Separation of b_T regions

$$\hat{b}_T(b_T; b_{\min}, b_{\max}) \begin{cases} \nearrow b_{\max}, & b_T \rightarrow +\infty \\ \sim b_T, & b_{\min} \ll b_T \ll b_{\max} \\ \searrow b_{\min}, & b_T \rightarrow 0 \end{cases}$$

High b_T limit : avoid Landau pole

Low b_T limit : recover fixed order expression

Models - evolution and b_T regions

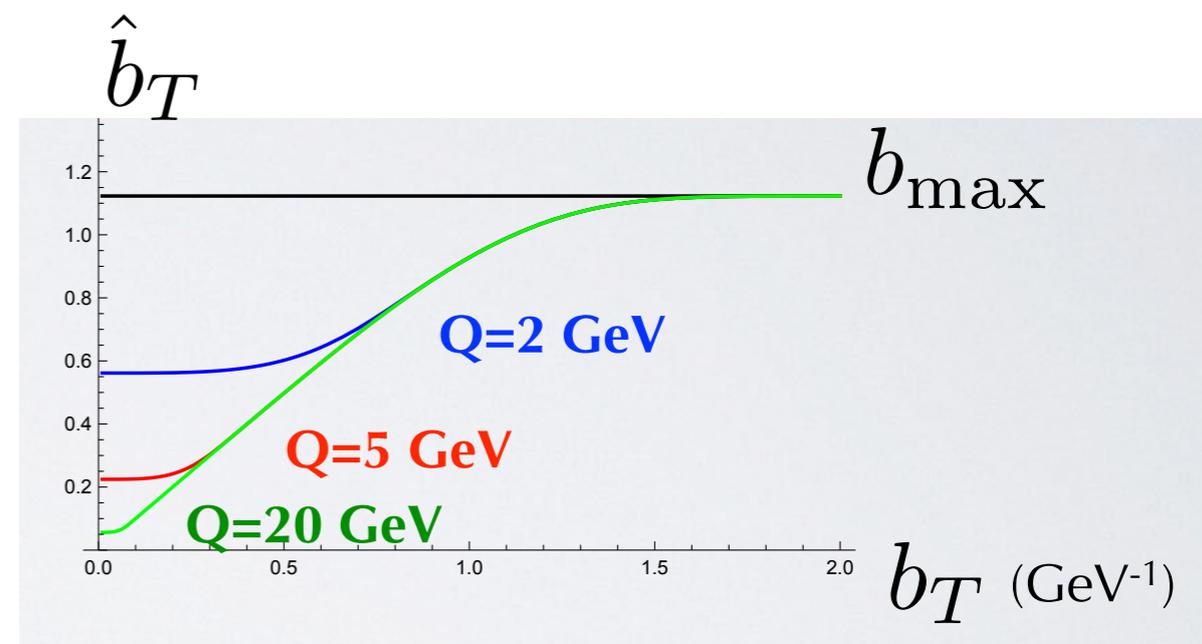
$$g_K(b_T; g_2) = -g_2 \frac{b_T^2}{2}$$

$$\hat{b}(b_T; b_{\min}, b_{\max}) = b_{\max} \left(\frac{1 - e^{-b_T^4/b_{\max}^4}}{1 - e^{-b_T^4/b_{\min}^4}} \right) \begin{array}{l} \nearrow b_{\max}, \quad b_T \rightarrow +\infty \\ \searrow b_{\min}, \quad b_T \rightarrow 0 \end{array}$$

$$b_{\max} = 2e^{-\gamma E}$$

$$b_{\min} = 2e^{-\gamma E} / Q$$

These choices guarantee that for $Q=1$ GeV the TMD coincides with the NP model



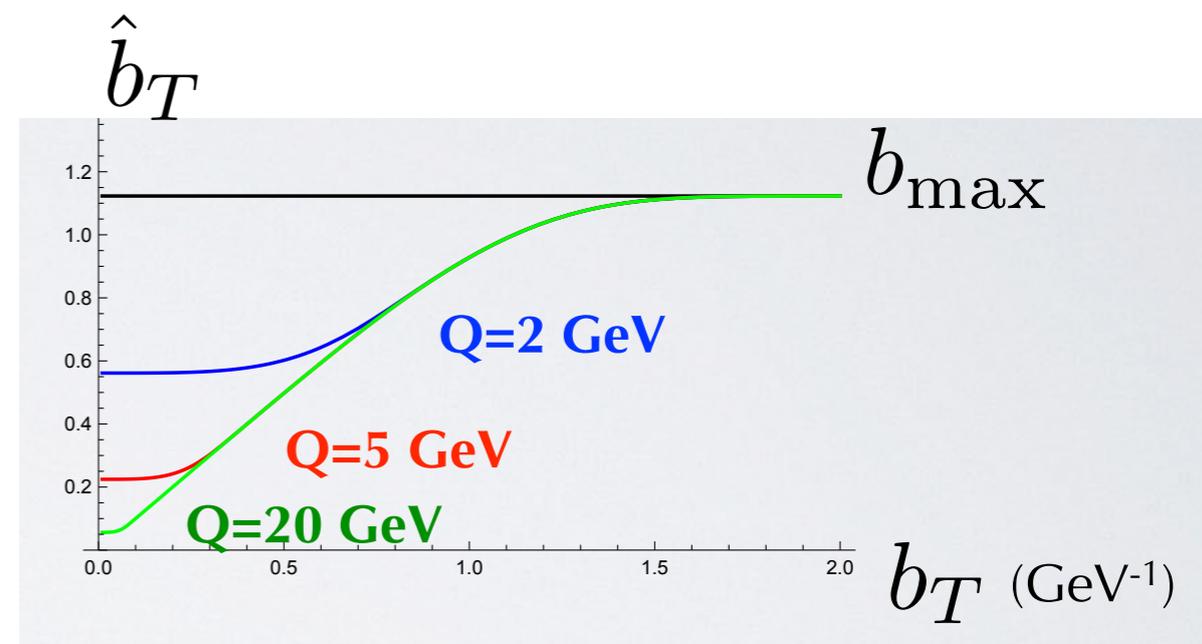
Models - evolution and b_T regions

$$g_K(b_T; g_2) = -g_2 \frac{b_T^2}{2}$$

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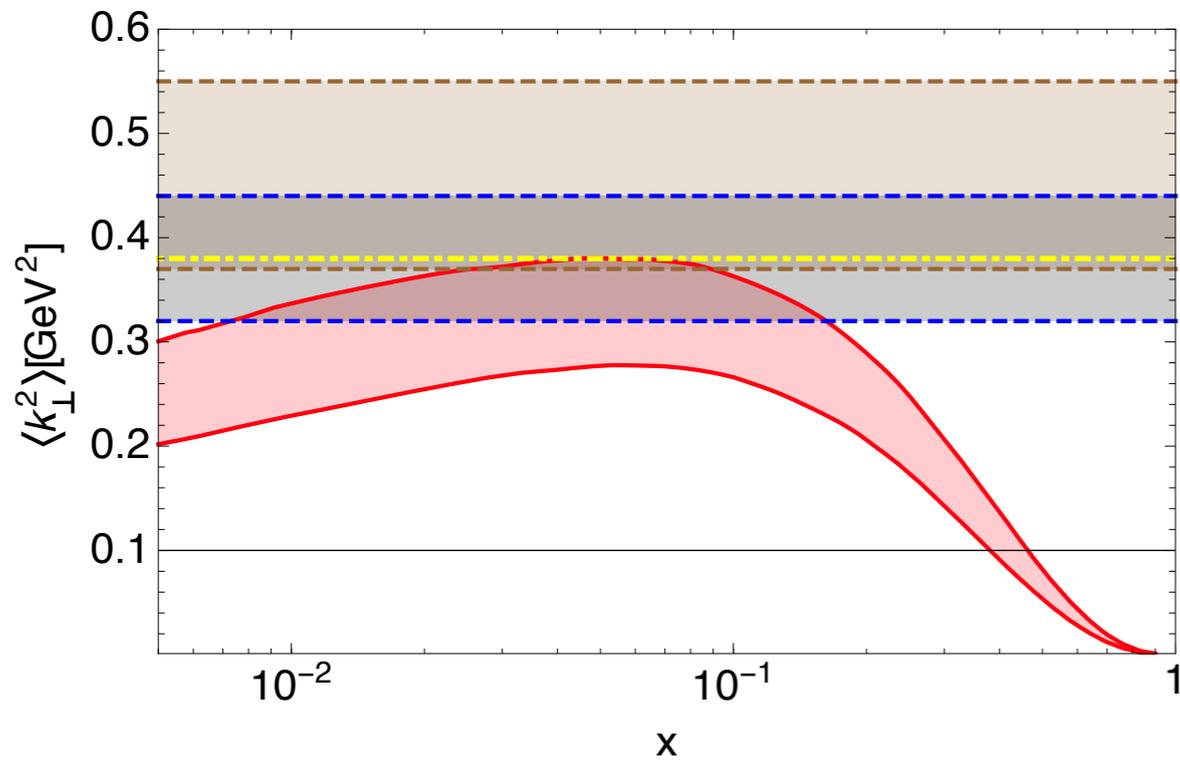
$$b_{\min} \sim 1/Q, \quad \mu_{\hat{b}} < Q$$

The phenomenological importance of b_{\min} is a signal that -especially in SIDIS data at **low Q**- we are exiting the proper TMD region and approaching the region of collinear factorization

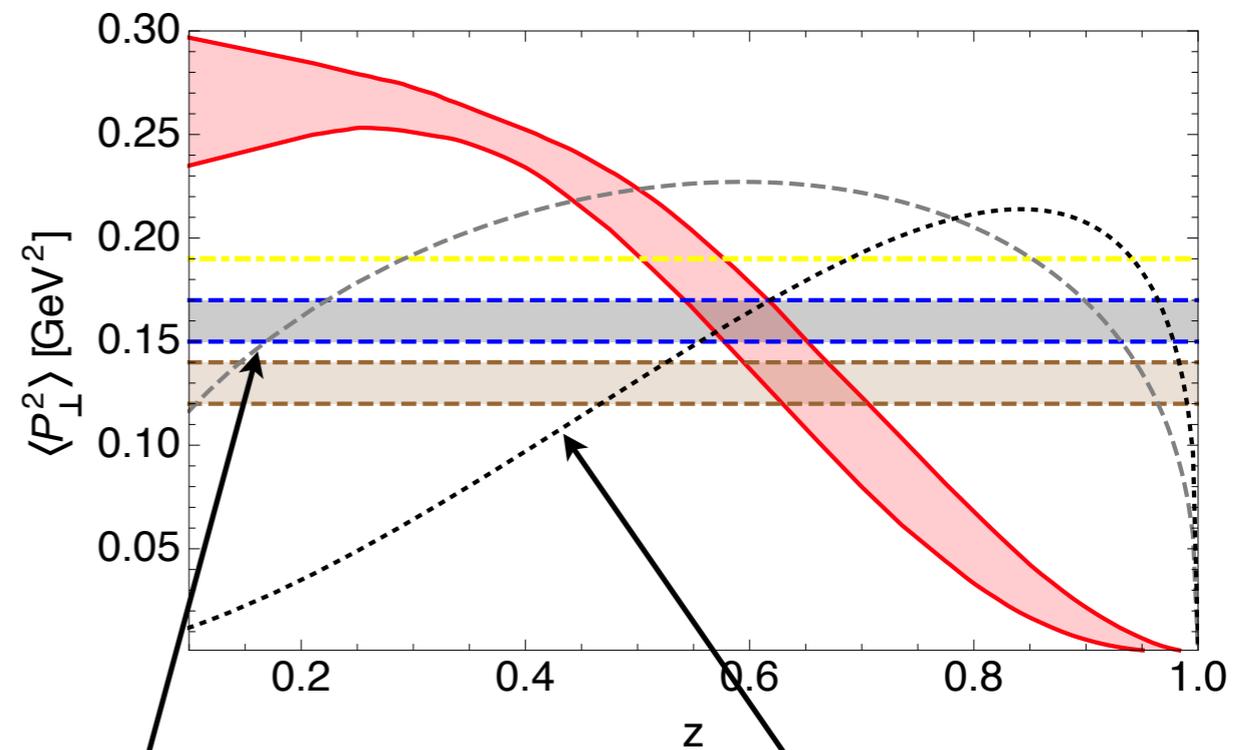


Kinematic dependence

Comparison with other extractions :



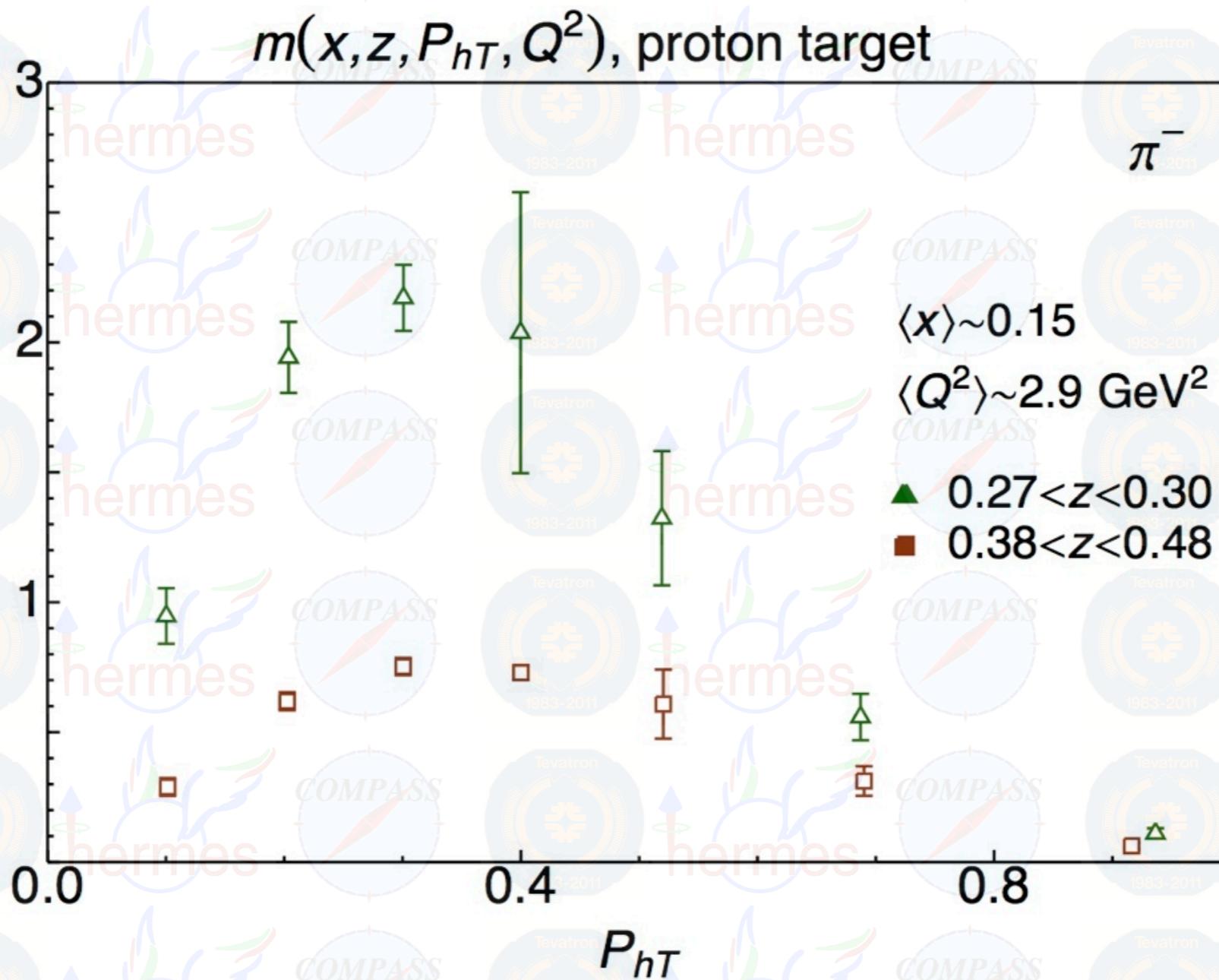
Color code : same as previous slide



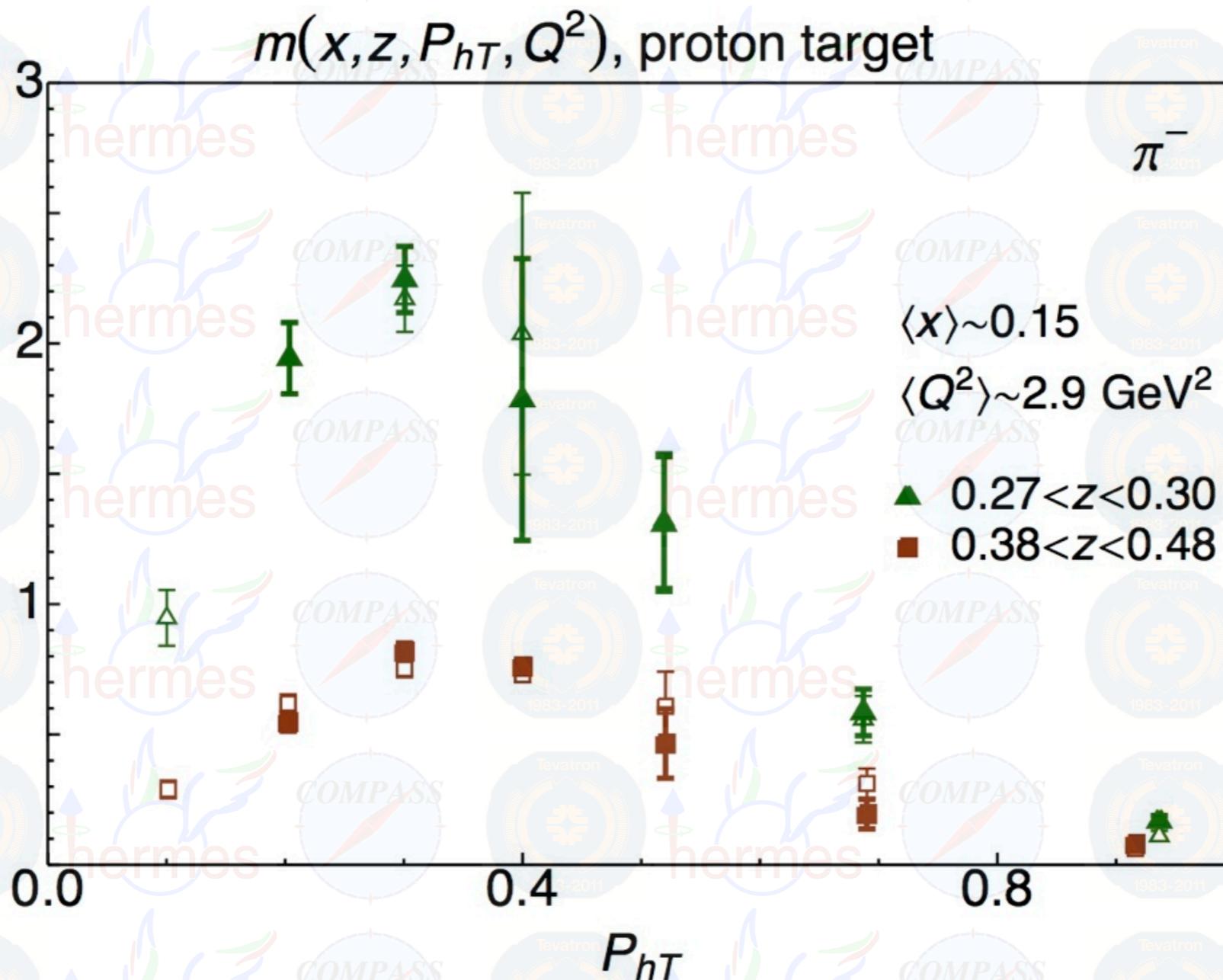
GMC trans

Anselmino et al.
hep-ph/9901442

The replica method

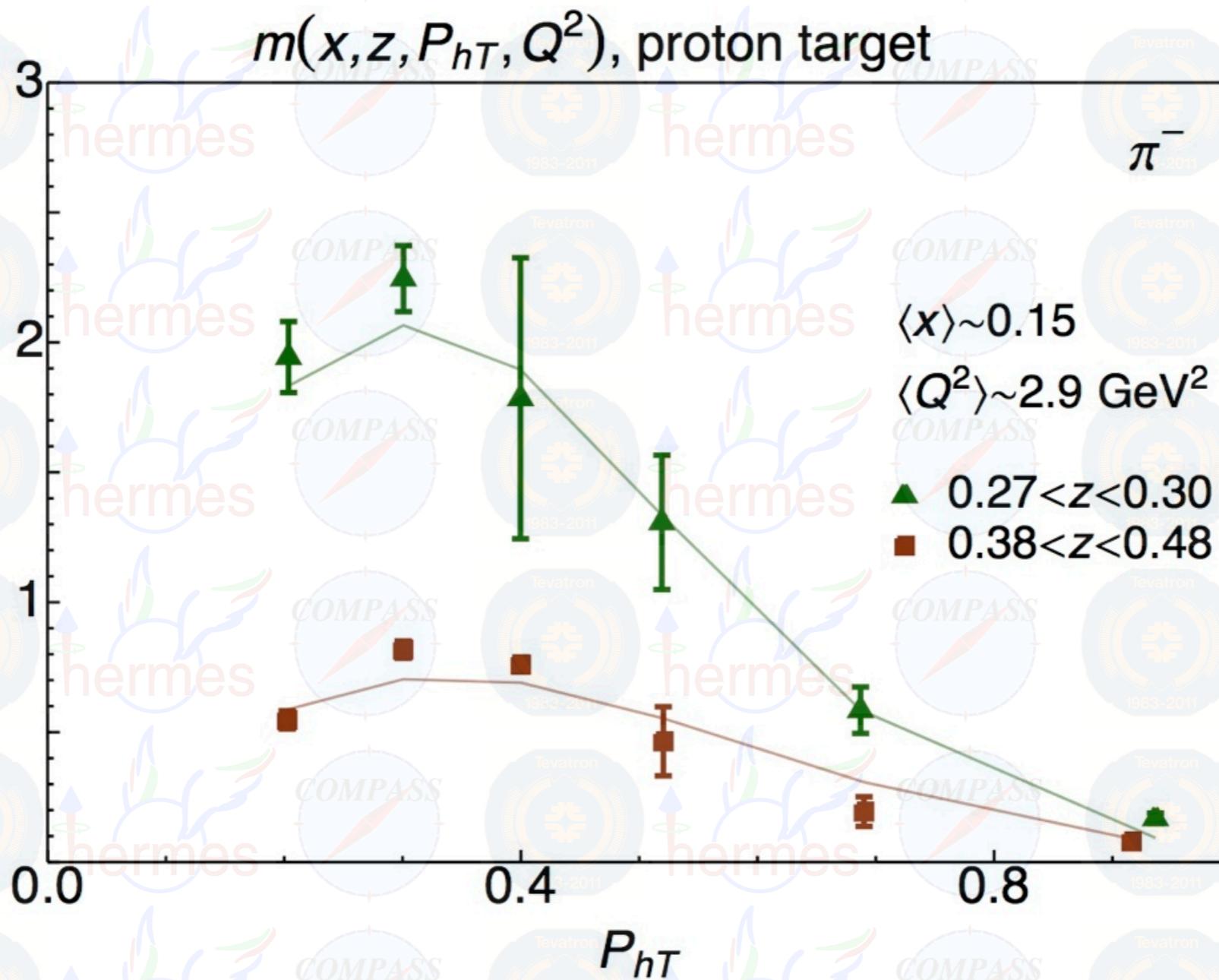


The replica method



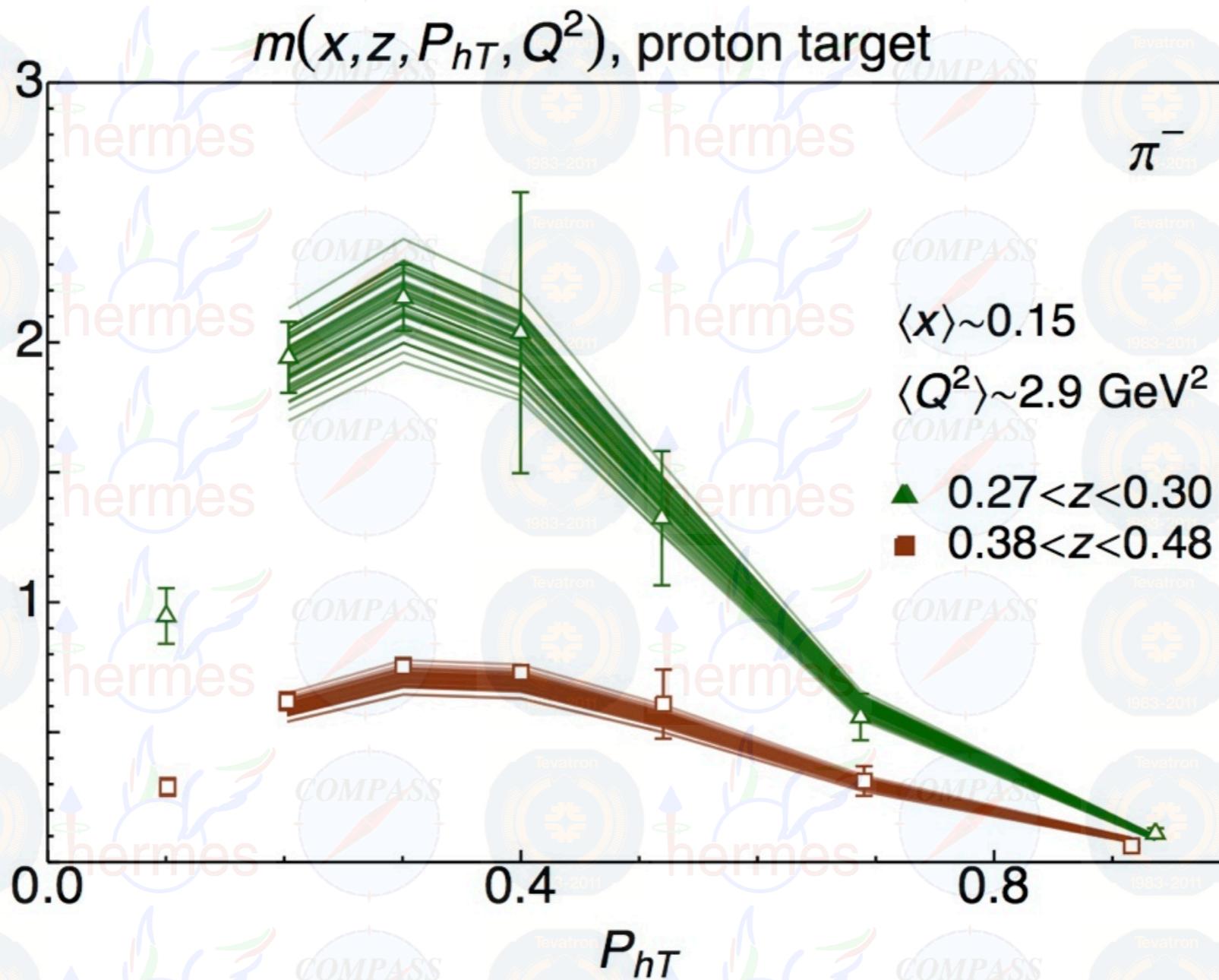
Replica of the original data with Gaussian noise

The replica method



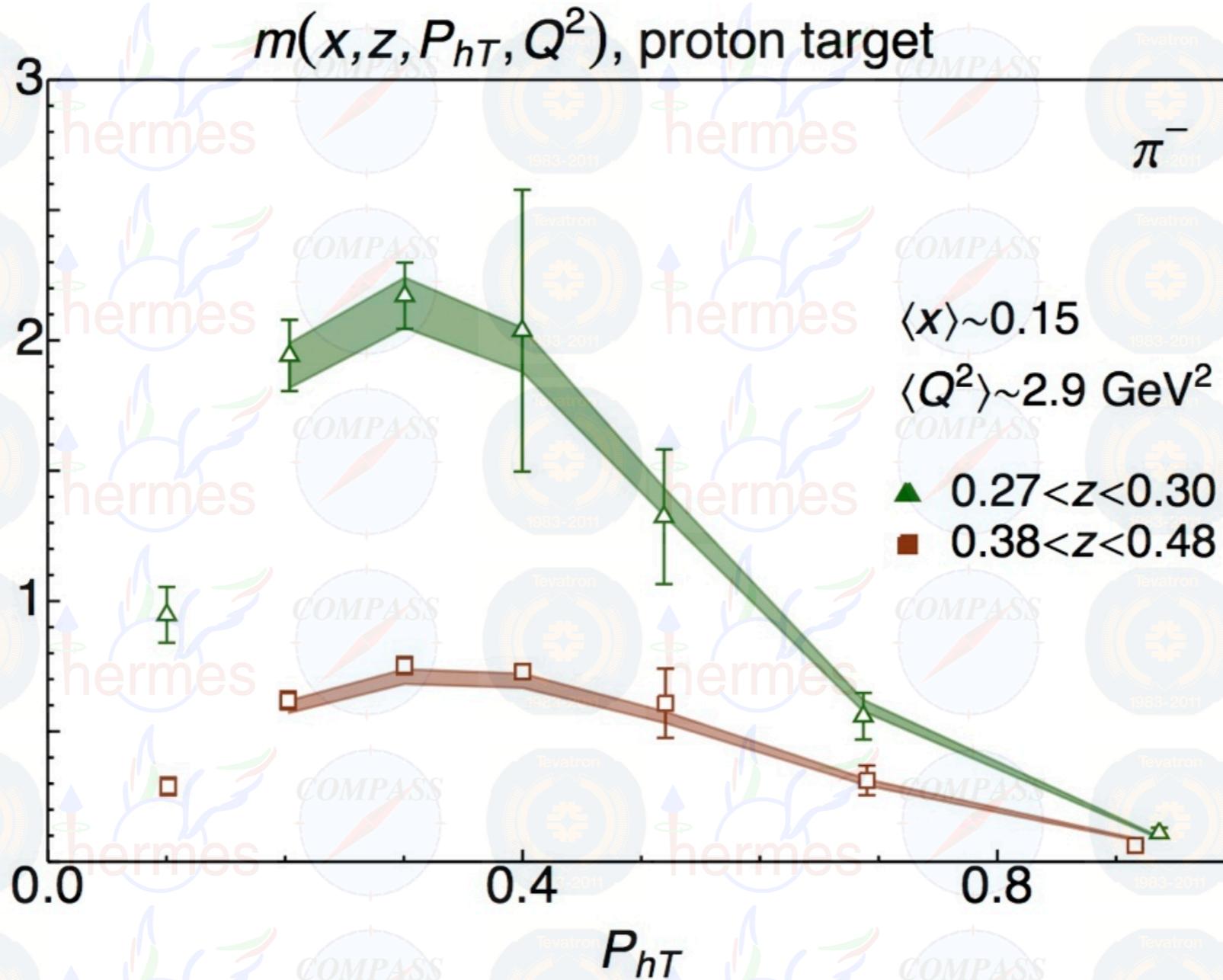
Fit of the replicated data

The replica method



Repeat the generation and the fit N times

The replica method



Obtain distributions of best values -
calculate **68% CL bands**

Data sets and selections

SIDIS

	HERMES $p \rightarrow \pi^+$	HERMES $p \rightarrow \pi^-$	HERMES $p \rightarrow K^+$	HERMES $p \rightarrow K^-$
Reference	[61]			
Cuts	$Q^2 > 1.4 \text{ GeV}^2$ $0.2 < z < 0.7$ $P_{hT} < \text{Min}[0.2 Q, 0.7 Qz] + 0.5 \text{ GeV}$			
Points	190	190	189	187
Max. Q^2	9.2 GeV ²			
x range	0.06 < x < 0.4			

TMD factorization ($P_{hT}/z \ll Q^2$)

avoid target fragmentation (low z)
and exclusive contributions (high z)

In order to avoid the problems
with the normalization in COMPASS data
(see Compass coll., Erratum)

	HERMES $D \rightarrow \pi^+$	HERMES $D \rightarrow \pi^-$	HERMES $D \rightarrow K^+$	HERMES $D \rightarrow K^-$	COMPASS $D \rightarrow h^+$	COMPASS $D \rightarrow h^-$
Reference	[61]				[62]	
Cuts	$Q^2 > 1.4 \text{ GeV}^2$ $0.2 < z < 0.7$ $P_{hT} < \text{Min}[0.2 Q, 0.7 Qz] + 0.5 \text{ GeV}$					
Points	190	190	189	189	3125	3127
Max. Q^2	9.2 GeV ²				10 GeV ²	
x range	0.06 < x < 0.4				0.006 < x < 0.12	
Notes					Observable: $m_{\text{norm}}(x, z, \mathbf{P}_{hT}^2, Q^2)$, eq. (38)	



Data sets and selections

	E288 200	E288 300	E288 400	E605
Reference	[65]	[65]	[65]	[66]
Cuts	$q_T < 0.2 Q + 0.5 \text{ GeV}$			
Points	45	45	78	35
\sqrt{s}	19.4 GeV	23.8 GeV	27.4 GeV	38.8 GeV
Q range	4-9 GeV	4-9 GeV	5-9, 11-14 GeV	7-9, 10.5-18 GeV
Kin. var.	$y=0.4$	$y=0.21$	$y=0.03$	$-0.1 < x_F < 0.2$

TMD factorization ($q_T \ll Q^2$)

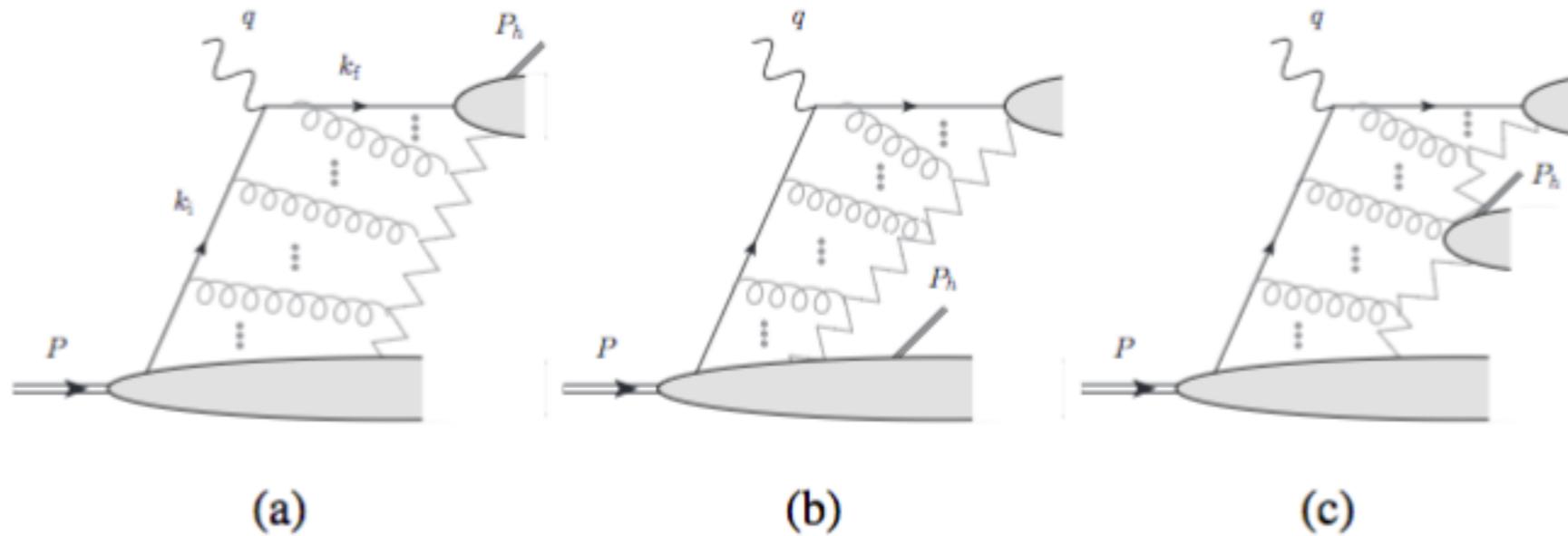
Drell-Yan

	CDF Run I	D0 Run I	CDF Run II	D0 Run II
Reference	[67]	[68]	[69]	[70]
Cuts	$q_T < 0.2 Q + 0.5 \text{ GeV} = 18.7 \text{ GeV}$			
Points	31	14	37	8
\sqrt{s}	1.8 TeV	1.8 TeV	1.96 TeV	1.96 TeV
Normalization	1.114	0.992	1.049	1.048

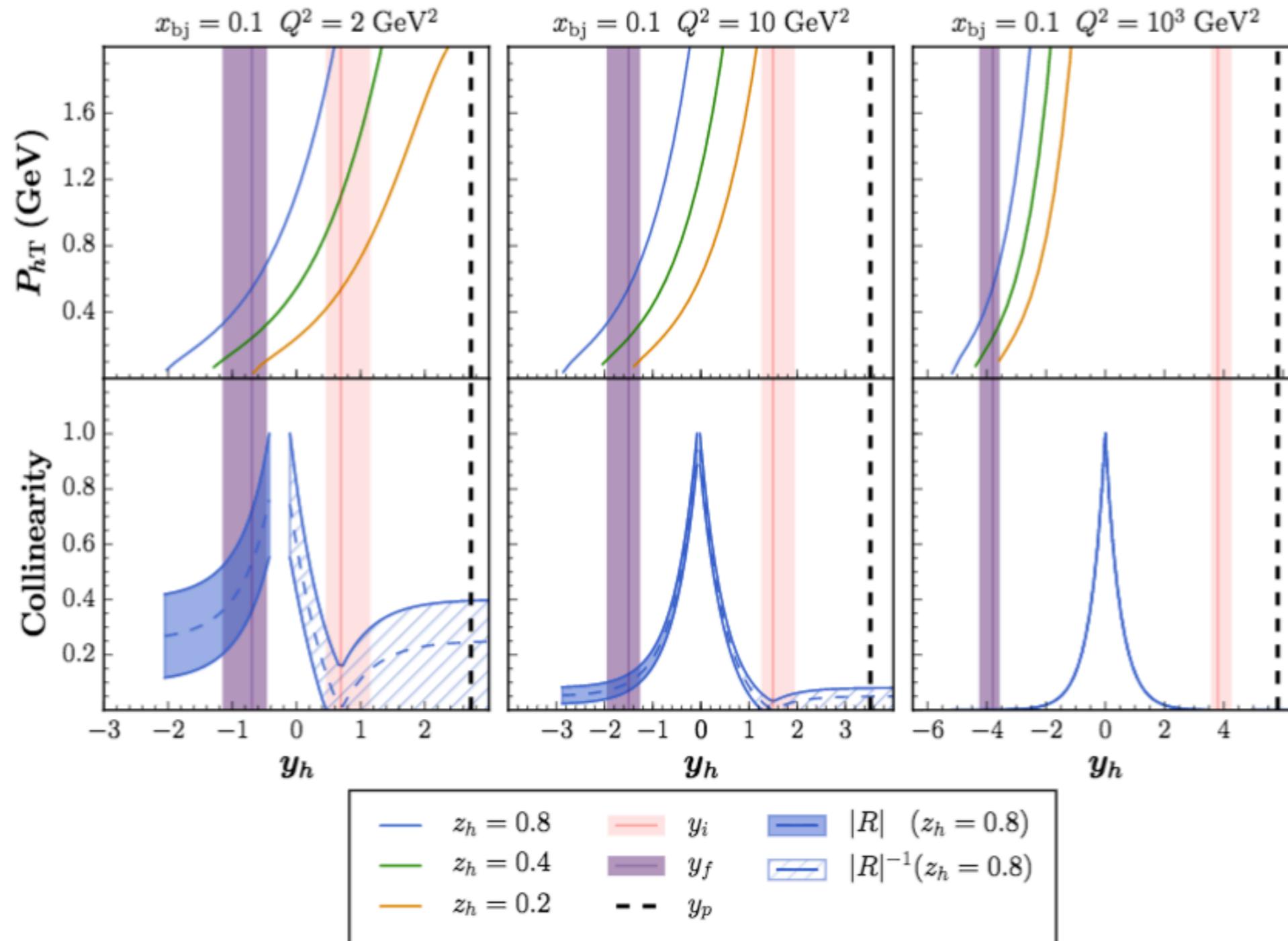
Z

normalization :
 fixed from DEMS fit,
 different from exp.
 (not really relevant for TMD
 parametrizations)

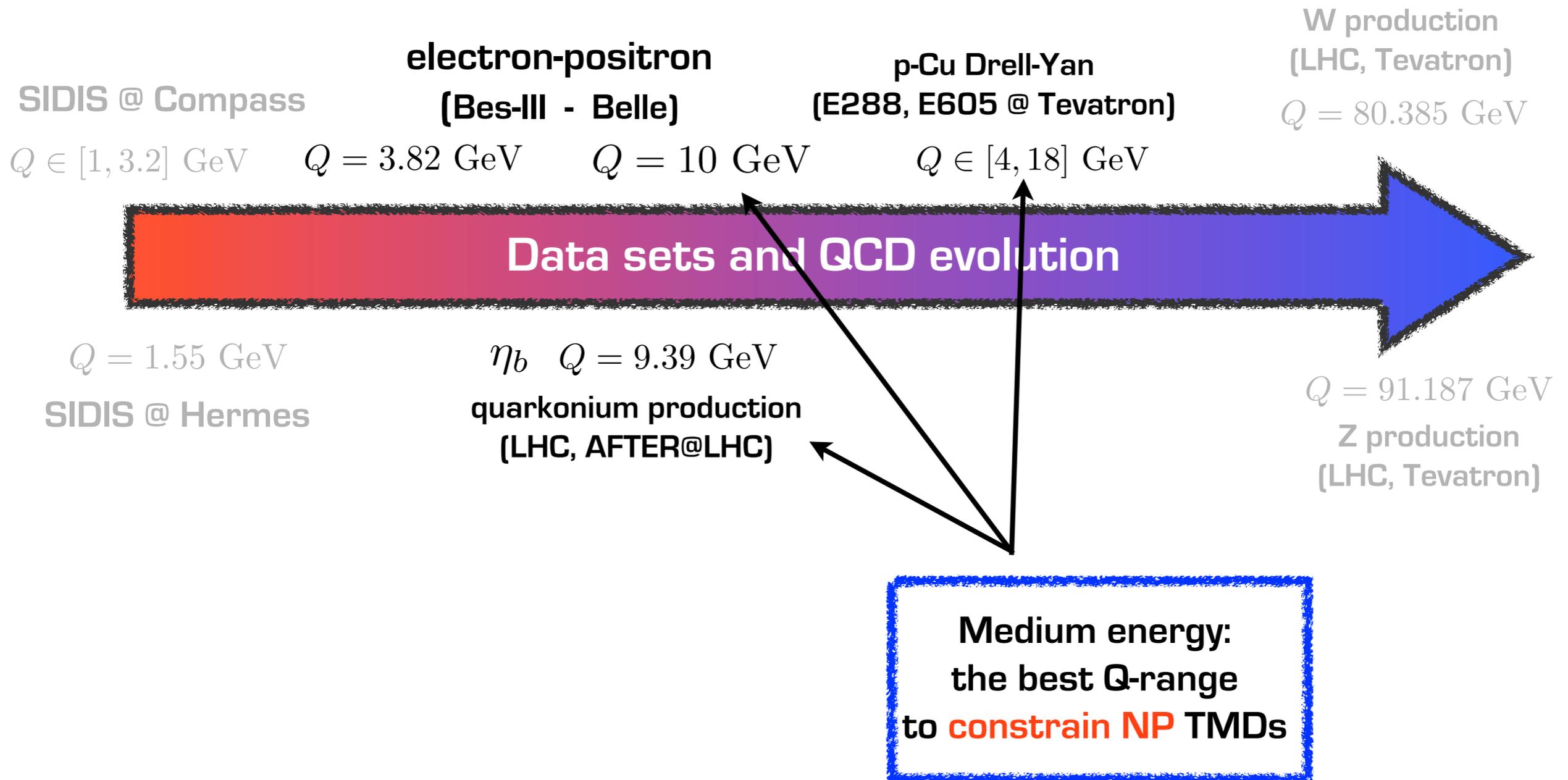
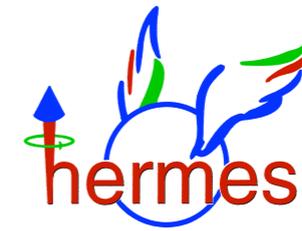
Target *vs* current *vs* central regions



Target *vs* current *vs* central regions



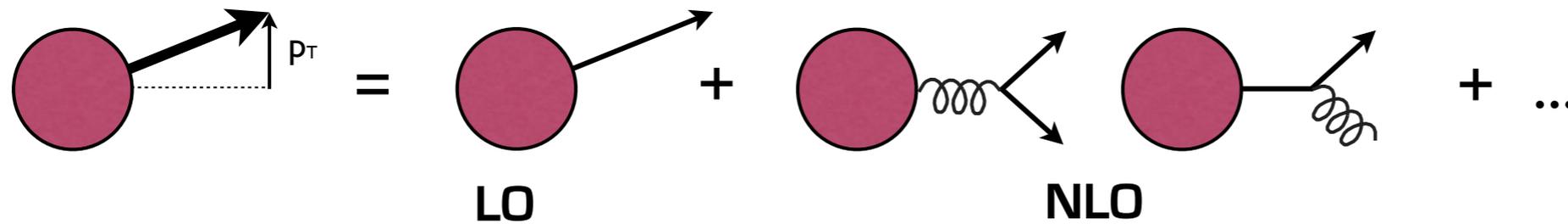
Evolution at work



Perturbative accuracy

Overview of the terminology

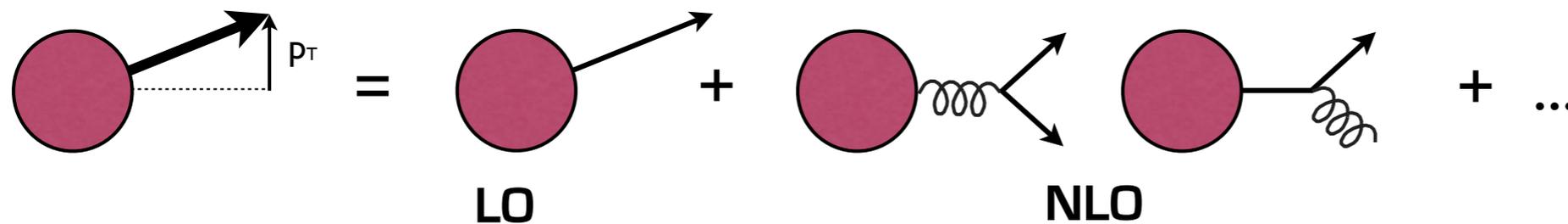
$C_{i/j}$ **Wilson coefficients** : expansion of the TMD distribution on a basis of collinear PDFs



Perturbative accuracy

Overview of the terminology

$C_{i/j}$ **Wilson coefficients** : expansion of the TMD distribution on a basis of collinear PDFs



Anomalous dimension of the TMD and **logarithmic expansion**

$$\gamma_F[\alpha_s(\mu), \zeta/\mu^2] \sim \underbrace{\alpha_s L}_{\text{LL}} + \underbrace{(\alpha_s + \alpha_s^2 L)}_{\text{NLL}} + \underbrace{(\alpha_s^2 + \alpha_s^3 L)}_{\text{NNLL}} + \dots$$

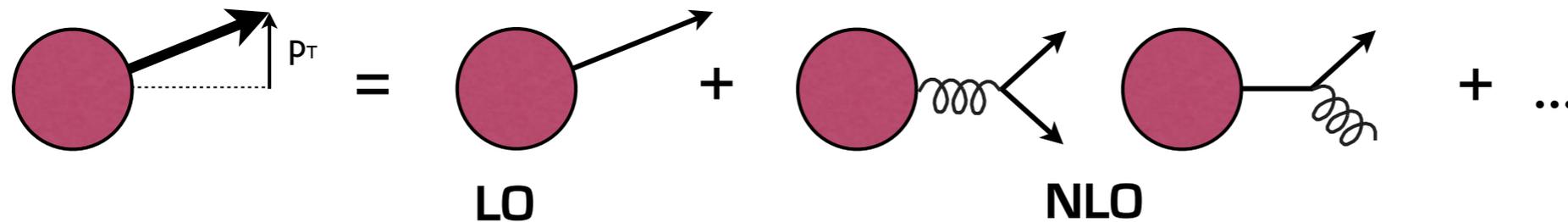
$$\sim 1 + \alpha_s + \alpha_s^2 + \dots$$

$$L = \ln \frac{Q^2}{\mu}, \quad \alpha_s L \sim 1$$

Perturbative accuracy

Overview of the terminology

$C_{i/j}$ **Wilson coefficients** : expansion of the TMD distribution on a basis of collinear PDFs



Anomalous dimension of the TMD and **logarithmic expansion**

$$\gamma_F[\alpha_s(\mu), \zeta/\mu^2] \sim \underbrace{\alpha_s L}_{\text{LL}} + \underbrace{(\alpha_s + \alpha_s^2 L)}_{\text{NLL}} + \underbrace{(\alpha_s^2 + \alpha_s^3 L)}_{\text{NNLL}} + \dots$$

$$\sim 1 + \alpha_s + \alpha_s^2 + \dots$$

$$L = \ln \frac{Q^2}{\mu}, \quad \alpha_s L \sim 1$$

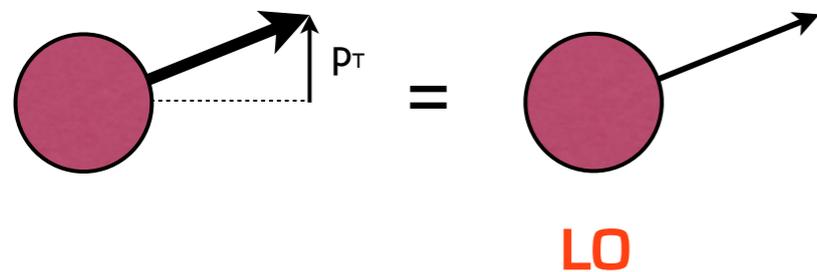
Collins-Soper kernel : a power series in the coupling

$$K(b_T; \mu_b) \sim 1 + \alpha_s + \alpha_s^2 \dots$$

accuracy chosen consistently
with Wilson coefficients
and anomalous dimension

Perturbative accuracy

$C_{i/j}$ **Wilson coefficients** : expansion of the TMD distribution on a basis of collinear PDFs



Anomalous dimension of the TMD and **logarithmic expansion**

$$\mu_{\hat{b}} = 2e^{-\gamma_E} / \bar{b}_*$$

$$\begin{aligned} \gamma_F[\alpha_s(\mu), \zeta/\mu^2] &\sim \underbrace{\alpha_s L}_{\text{LL}} + \underbrace{(\alpha_s + \alpha_s^2 L)}_{\text{NLL}} + \dots \\ &\sim 1 + \alpha_s + \dots \end{aligned}$$

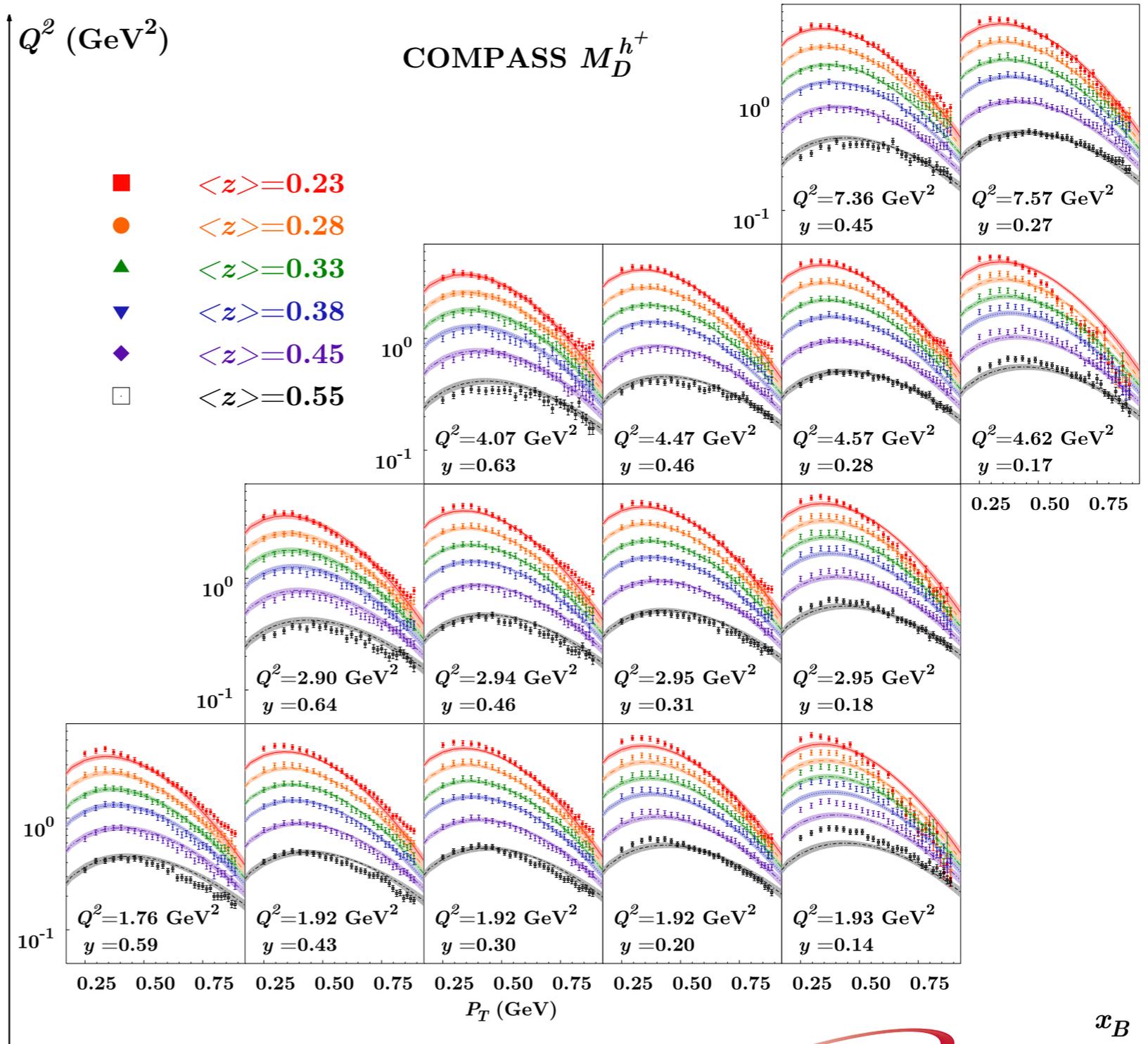
$C_{i/j}$	γ_{nc}	Γ_{cusp}	K	accuracy
0	0	0	0	QPM
0	0	1	0	LO-LL
0	1	2	1	LO-NLL
0	2	3	2	LO-NNLL
1	1	2	1	NLO-NLL
1	2	3	2	NLO-NNLL
2	2	3	2	NNLO-NNLL

Collins-Soper kernel : a power series in the coupling

$$K(b_T; \mu_b) \sim 1 + \alpha_s + \dots$$

Torino / JLab 2014

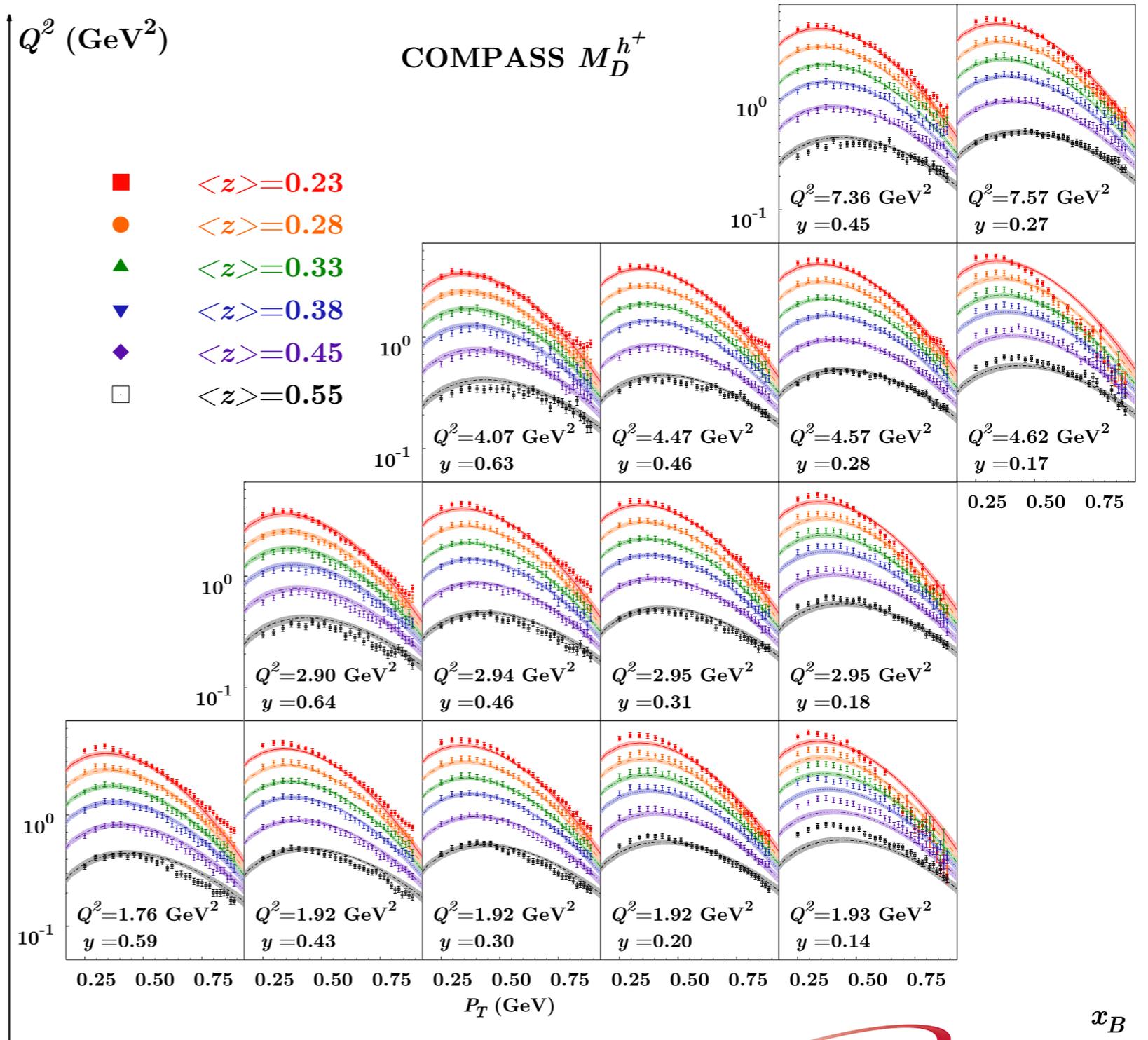
COMPASS $M_D^{h^+}$



Torino / JLab 2014

COMPASS $M_D^{h^+}$

$\chi^2/\text{dof} = 3.79$
with ad-hoc
normalization

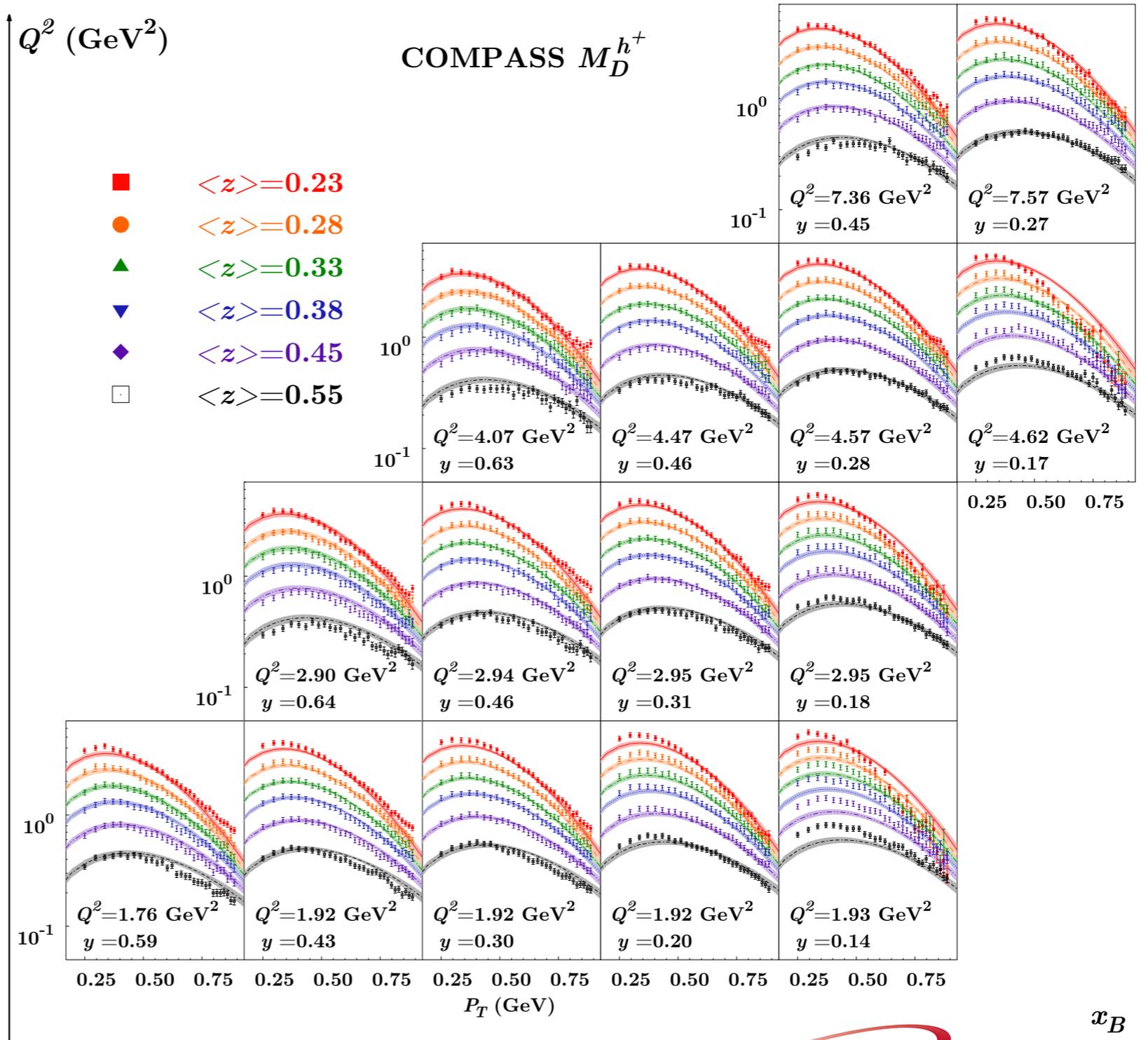


Torino / JLab 2014

COMPASS $M_D^{h^+}$

$\chi^2/\text{dof} = 3.79$
with ad-hoc
normalization

see Compass coll.
Erratum



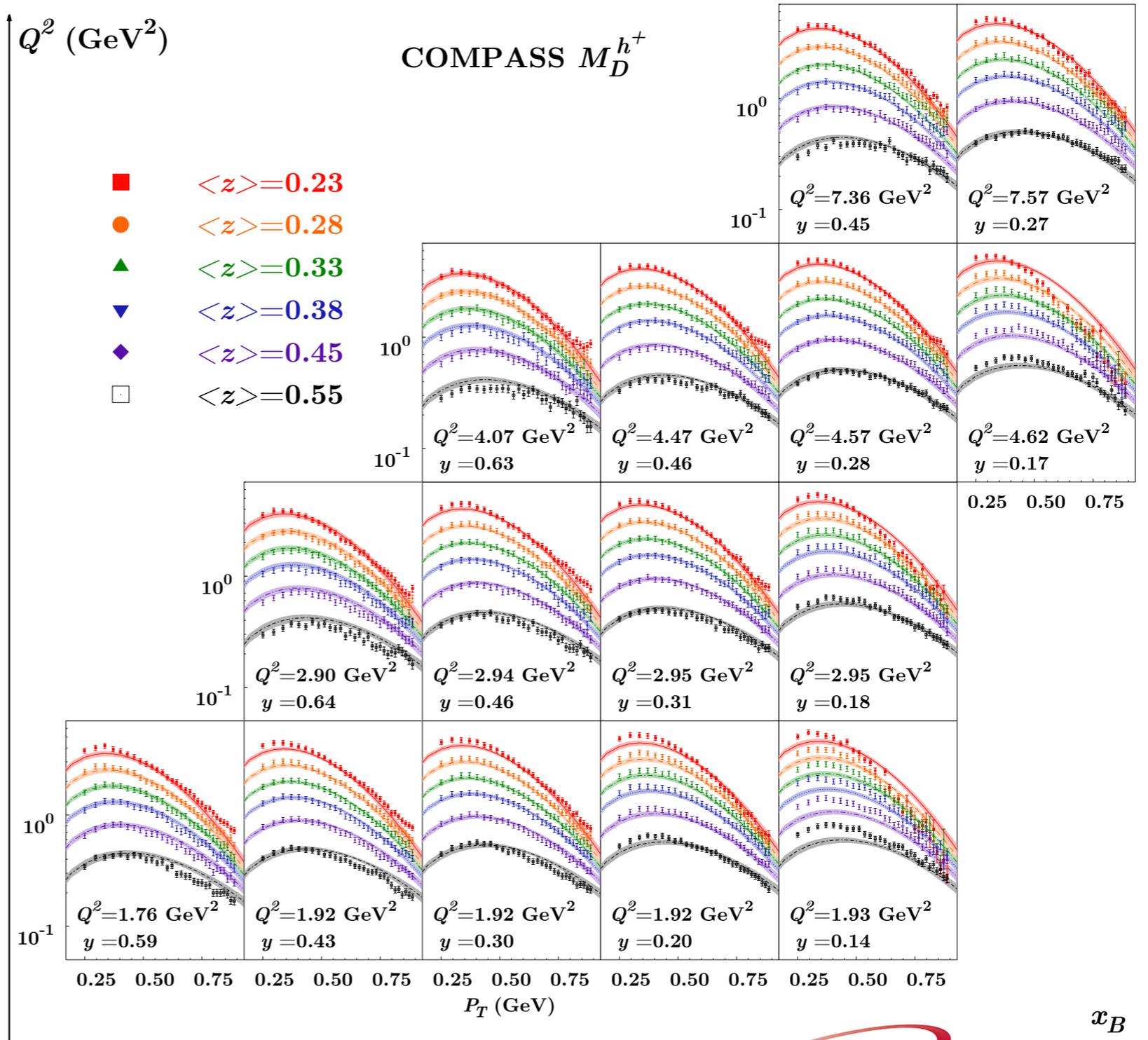
Torino / JLab 2014

COMPASS $M_D^{h^+}$

simple Gaussian ansatz

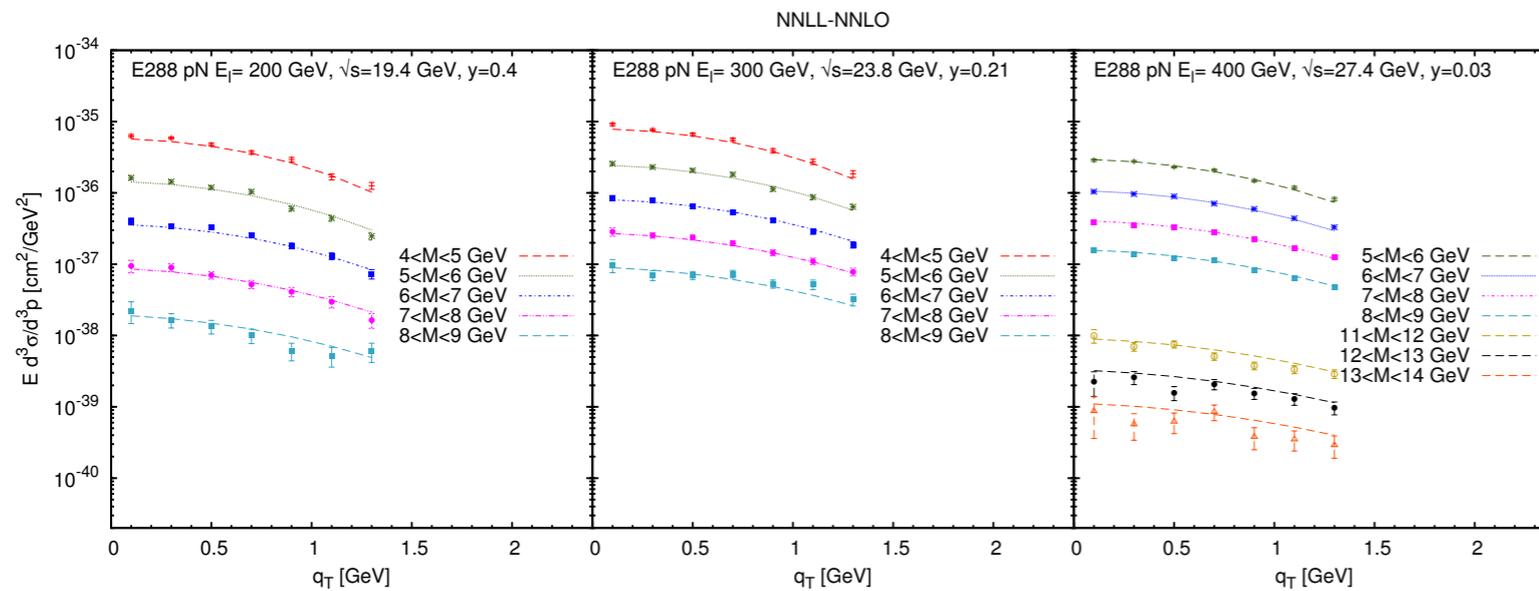
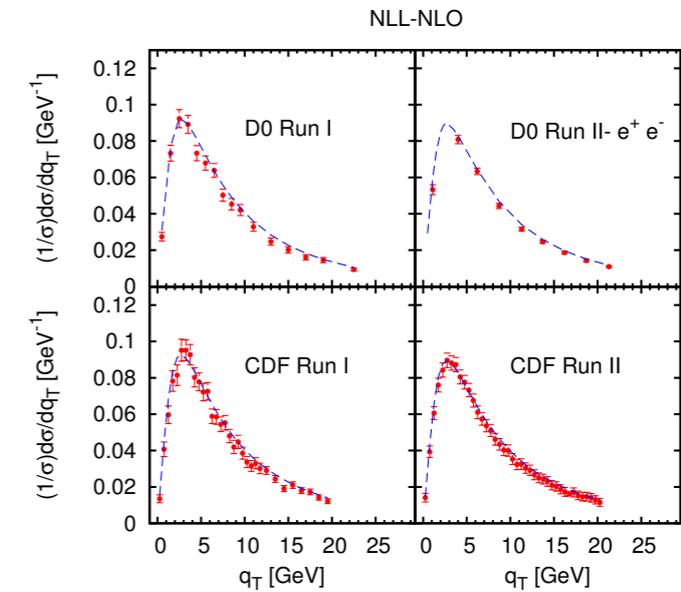
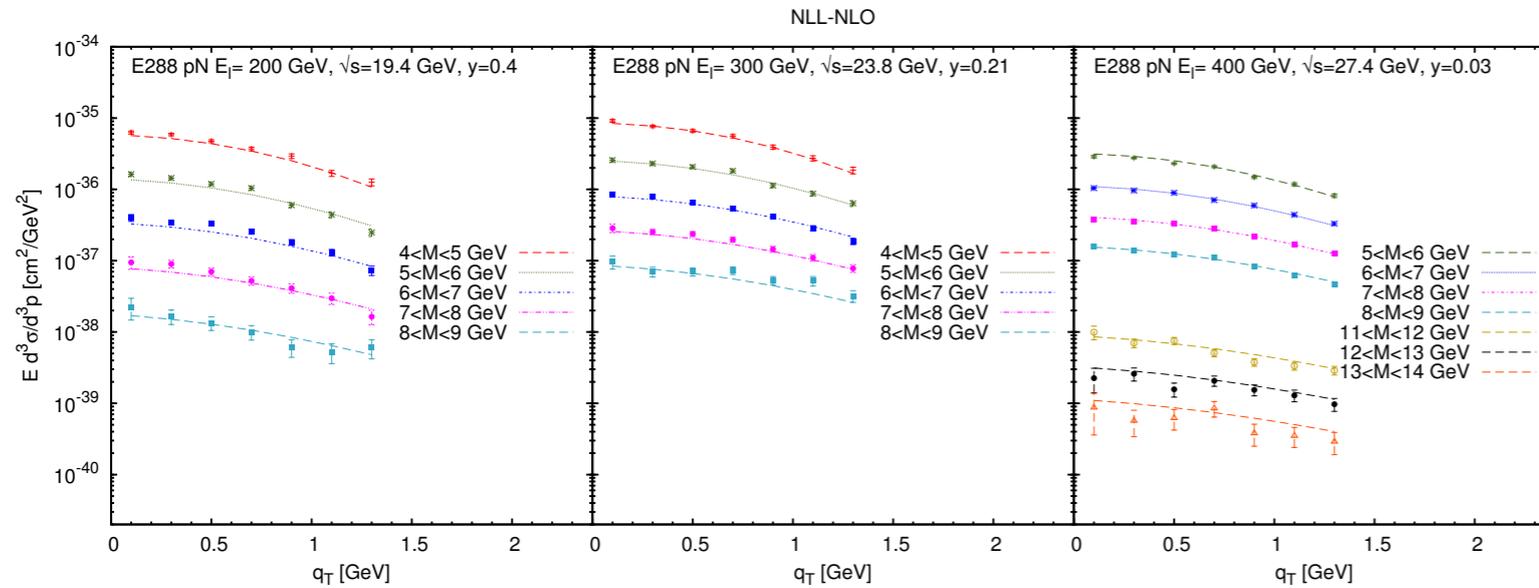
$\chi^2/\text{dof} = 3.79$
with ad-hoc
normalization

see Compass coll.
Erratum



DEMS 2014

D'Alesio, Echevarria, Melis, Scimemi, JHEP 1411 [14]



NLO-NNLL analysis
with evaluation of
theoretical uncertainties

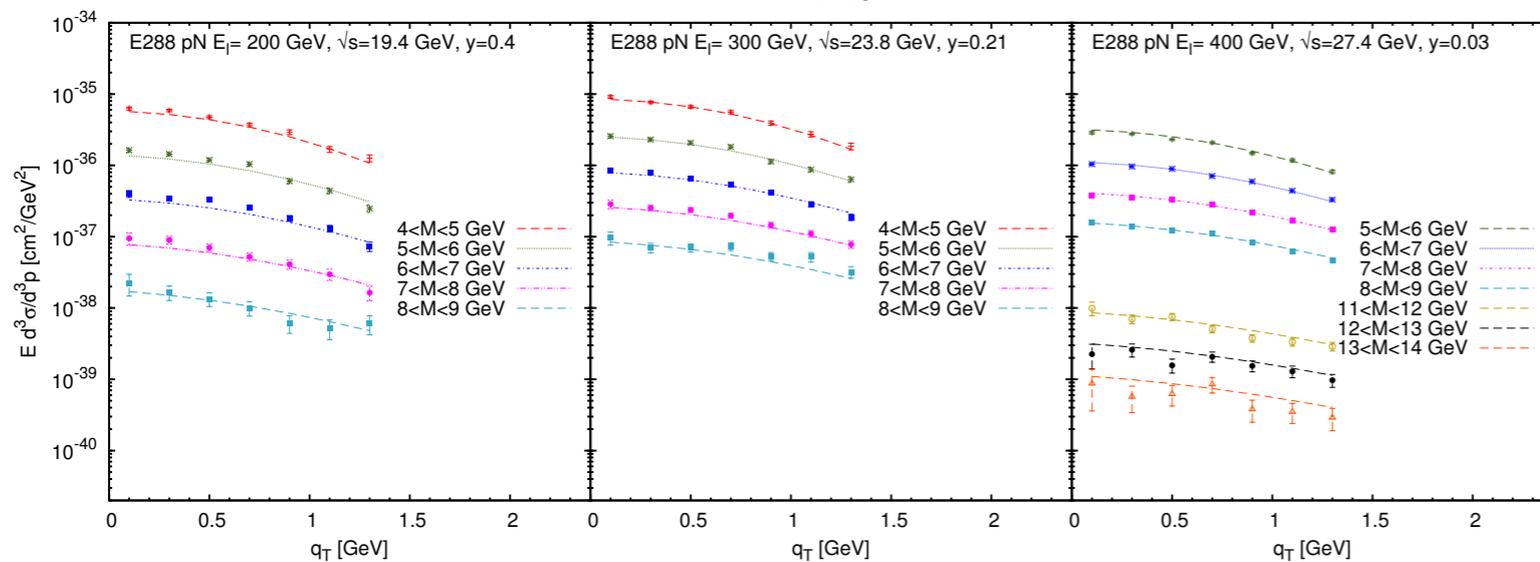
very good

DEMS 2014

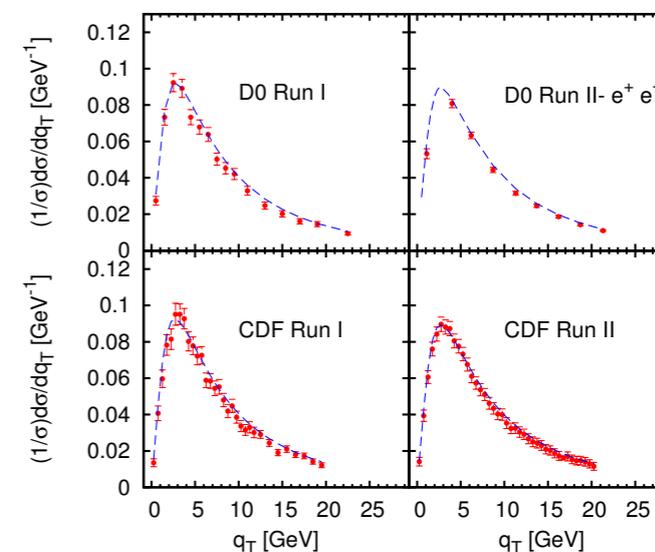
D'Alesio, Echevarria, Melis, Scimemi, JHEP 1411 [14]

$$\chi^2/\text{dof} = 0.81$$

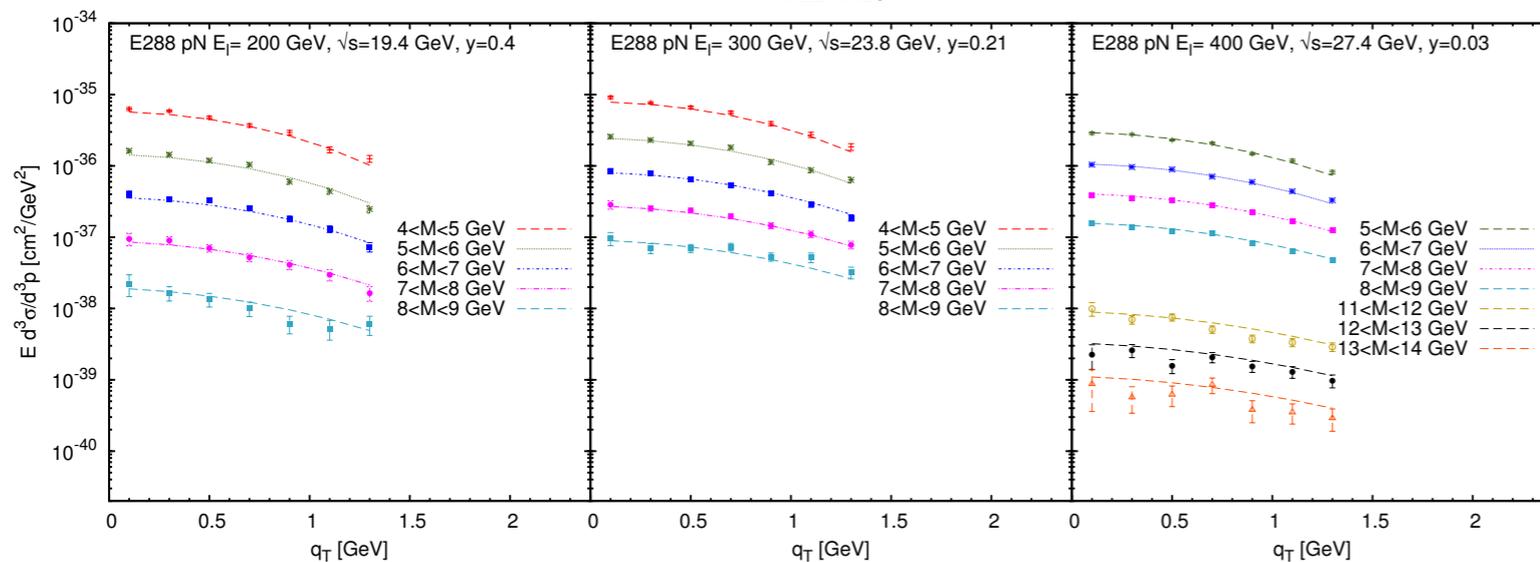
NLL-NLO



NLL-NLO



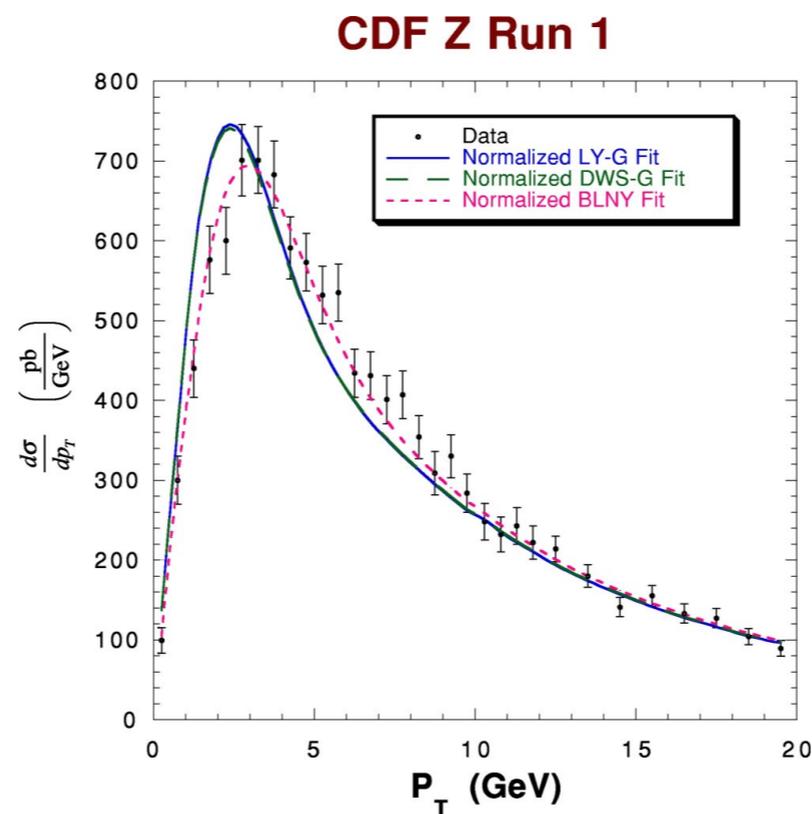
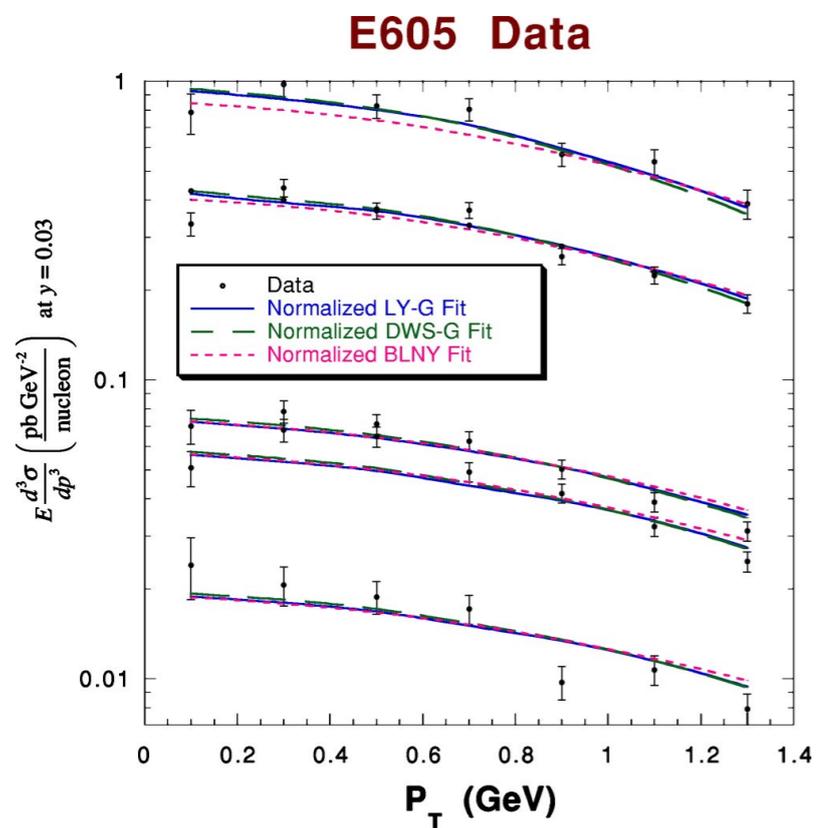
NNLL-NNLO



NLO-NNLL analysis
with evaluation of
theoretical uncertainties

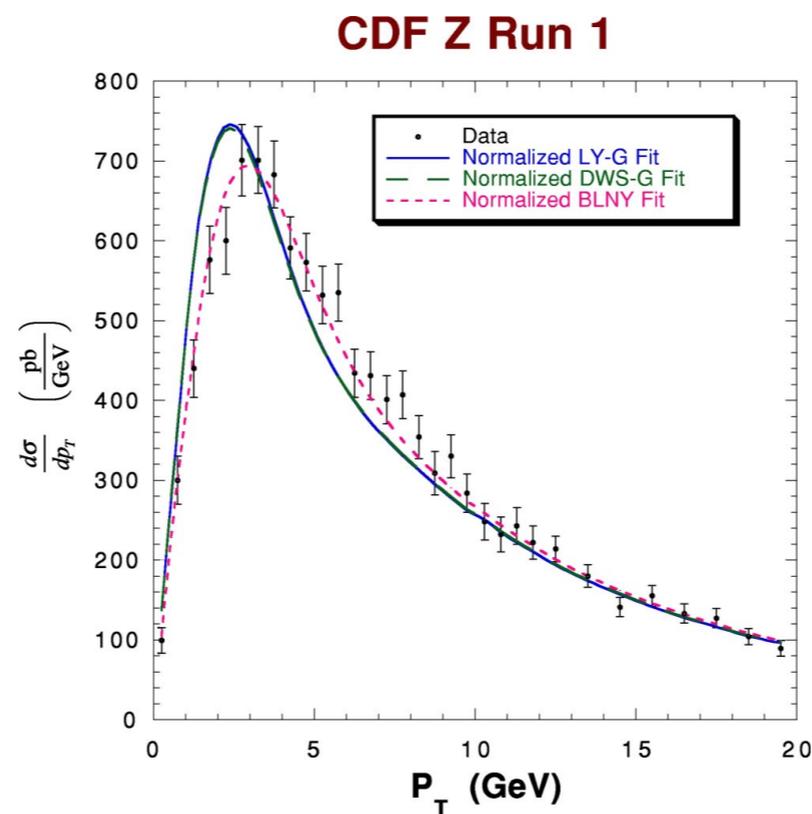
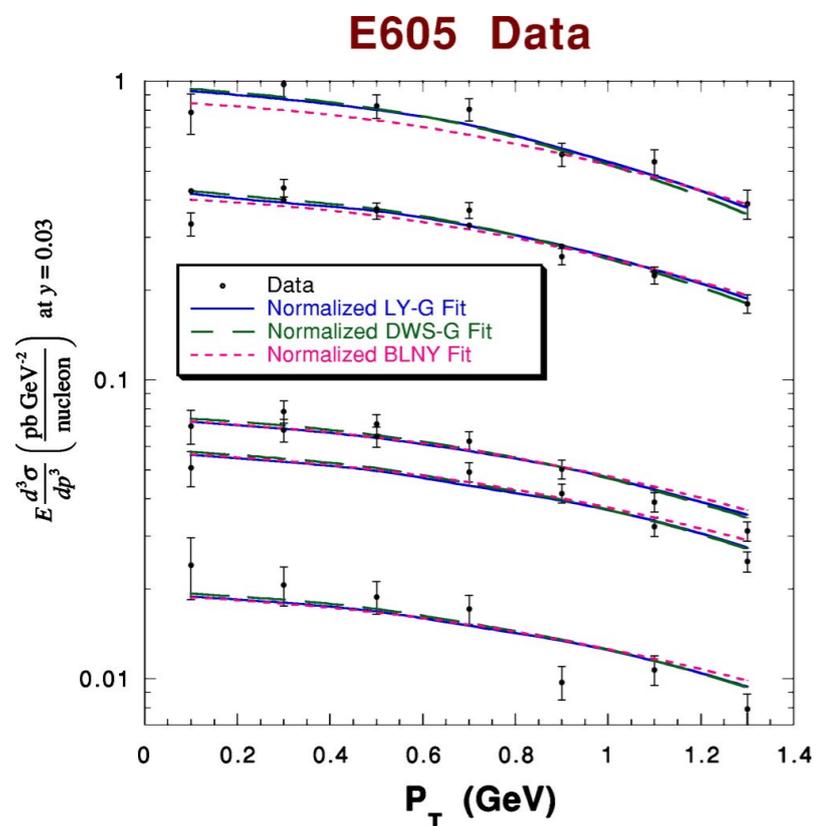
very good

KN 2006



≈ 100 data points
 $Q^2 > 4 \text{ GeV}$

KN 2006



≈ 100 data points
 $Q^2 > 4 \text{ GeV}^2$

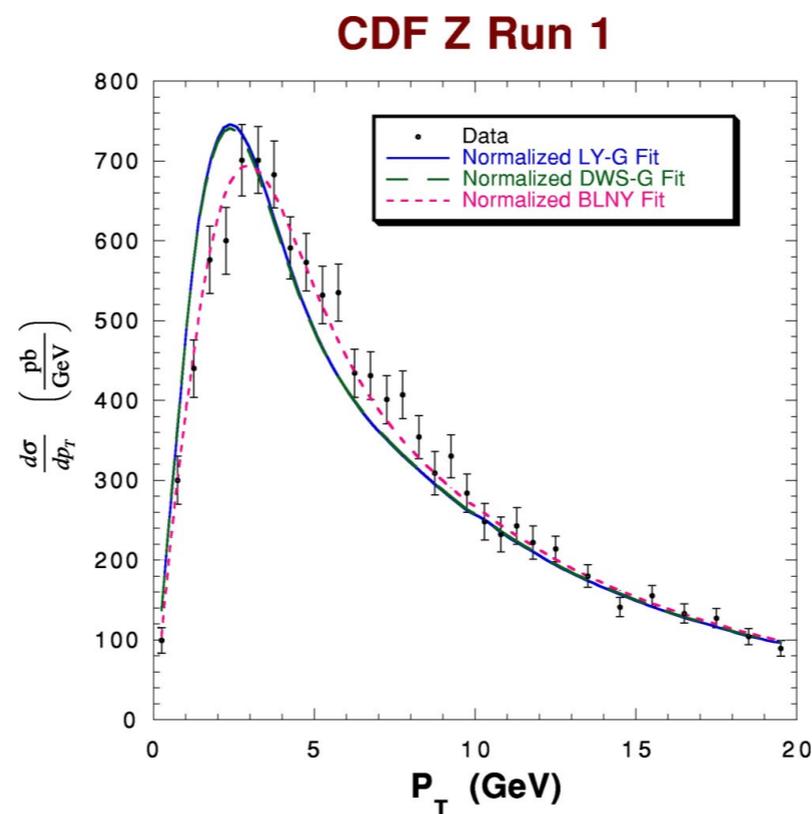
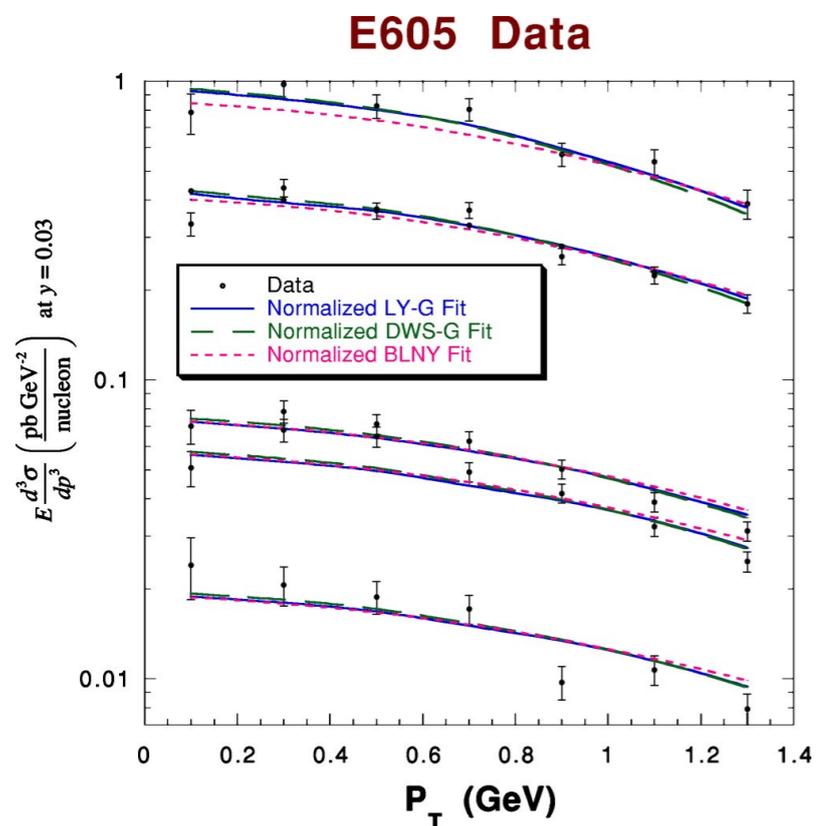
$$Q_0 = 3.2 \text{ GeV}$$

$$b_{\text{max}} = 0.5 \text{ GeV}^{-1}$$

$$\frac{1}{\langle b_T^2 \rangle} = \frac{1}{2} \left(0.21 + 0.68 \log \left(\frac{Q}{2Q_0} \right) - 0.25 \log(10x) \right)$$

Brock, Landry, Nadolsky, Yuan, PRD67 (03)

KN 2006



≈ 100 data points
 $Q^2 > 4 \text{ GeV}^2$

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Brock, Landry, Nadolsky, Yuan, PRD67 [03]

$$\frac{1}{\langle b_T^2 \rangle} = \frac{1}{2} \left(0.20 + 0.184 \log \left(\frac{Q}{2Q_0} \right) - 0.026 \log(10x) \right)$$

$$b_{\text{max}} = 1.5 \text{ GeV}^{-1}$$

EIKV 2014

Parametrizations for intrinsic momenta
and soft gluon emission :

$$F_{NP}(b_T, Q)^{\text{pdf}} = \exp \left[-b_T^2 \left(g_1^{\text{pdf}} + \frac{g_2}{2} \ln(Q/Q_0) \right) \right]$$

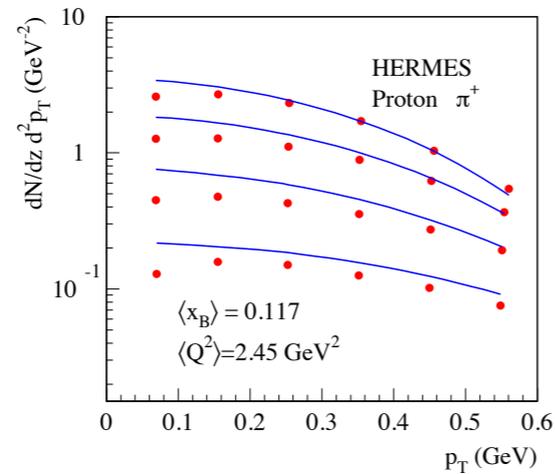
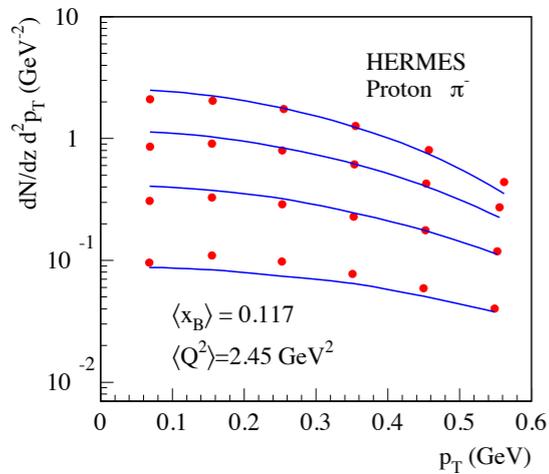
$$F_{NP}(b_T, Q)^{\text{ff}} = \exp \left[-b_T^2 \left(g_1^{\text{ff}} + \frac{g_2}{2} \ln(Q/Q_0) \right) \right]$$

Pros and Cons :

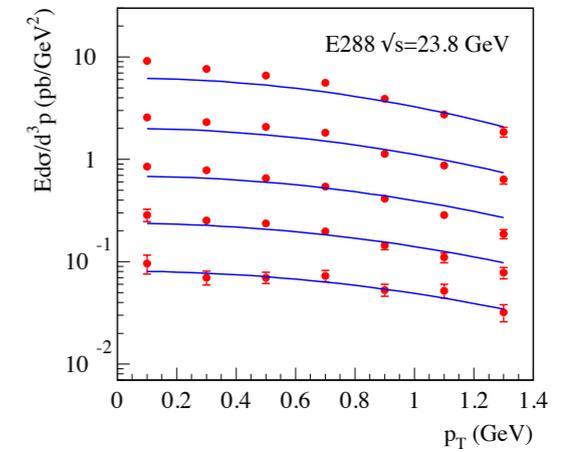
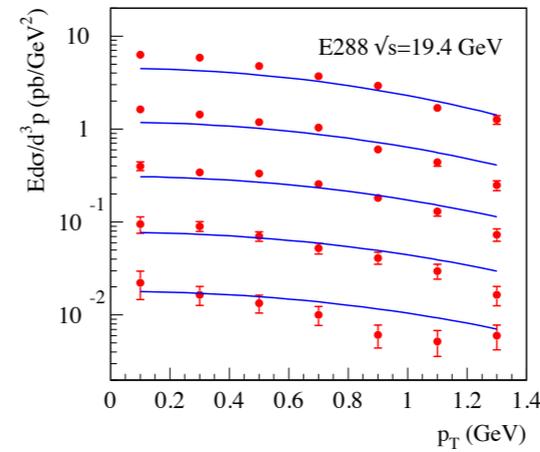
- 1) a global analysis of SIDIS and DY/Z/W data
- 2) TMD evolution at LO-NLL
- 3) multidimensionality not exploited
- 4) chi-square not provided
- 5) can't be considered as a "complete" fit**

EIKV 2014

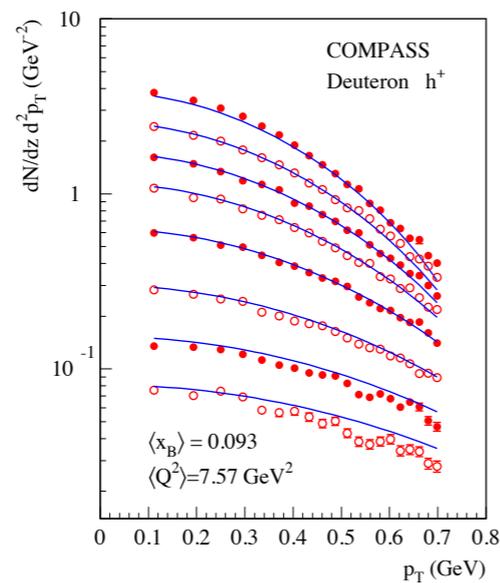
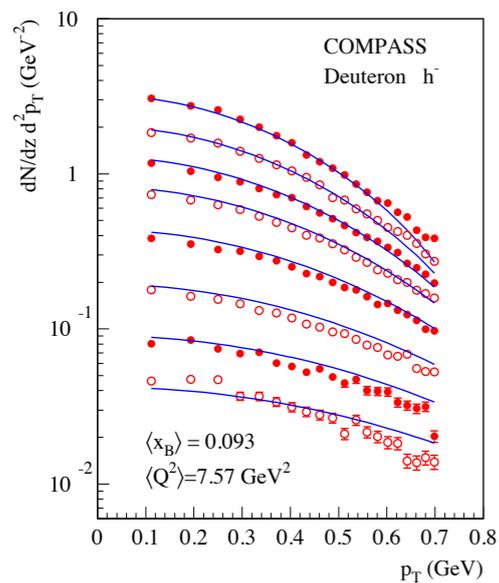
SIDIS



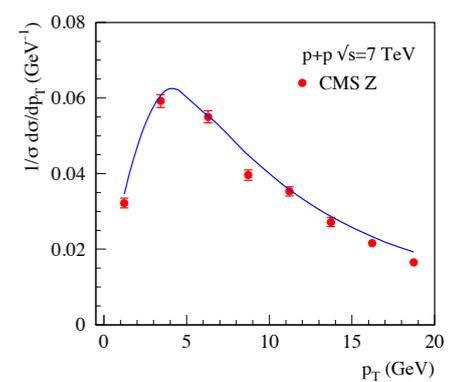
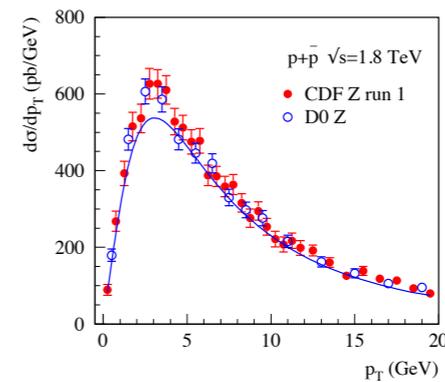
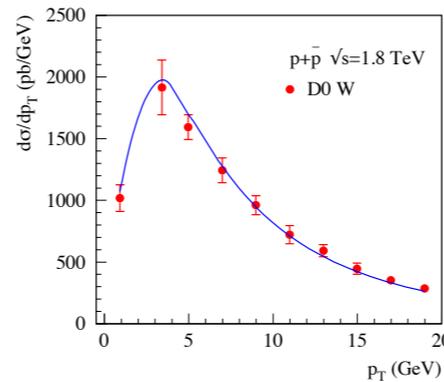
DRELL-YAN



SIDIS



W AND Z PRODUCTION



$$b_{\text{max}} = 1.5 \text{ GeV}^{-1}$$

$$g_2 = 0.16$$

Echevarria et al. [arXiv:1401.5078](https://arxiv.org/abs/1401.5078)



Other studies

CSS formalism on DY/Z/W data:

- 1) Davies-Webber-Stirling [DOI: [10.1016/0550-3213\(85\)90402-X](https://doi.org/10.1016/0550-3213(85)90402-X)]
- 2) Ladinsky-Yuan [DOI: [10.1103/PhysRevD.50.R4239](https://doi.org/10.1103/PhysRevD.50.R4239)]
- 3) BLNY [DOI: [10.1103/PhysRevD.63.013004](https://doi.org/10.1103/PhysRevD.63.013004)]
- 4) Hirai, Kawamura, Tanaka [DOI: [10.3204/DESY-PROC-2012-02/136](https://doi.org/10.3204/DESY-PROC-2012-02/136)] - complex-b prescription

...

combined SIDIS/DY/W/Z :

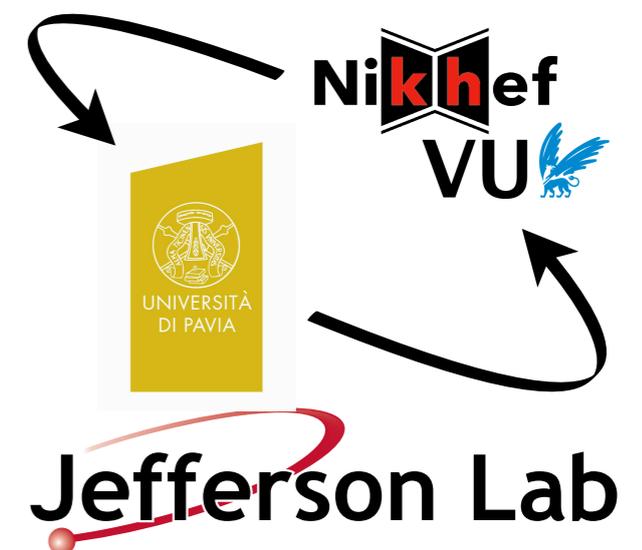
- 5) Sun, Yuan [[arXiv:1308.5003](https://arxiv.org/abs/1308.5003)]
- 6) Isaacson, Sun, Yuan, Yuan [[arXiv:1406.3073](https://arxiv.org/abs/1406.3073)]

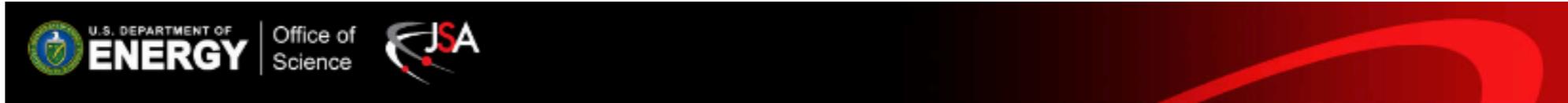
...

... and the next challenges

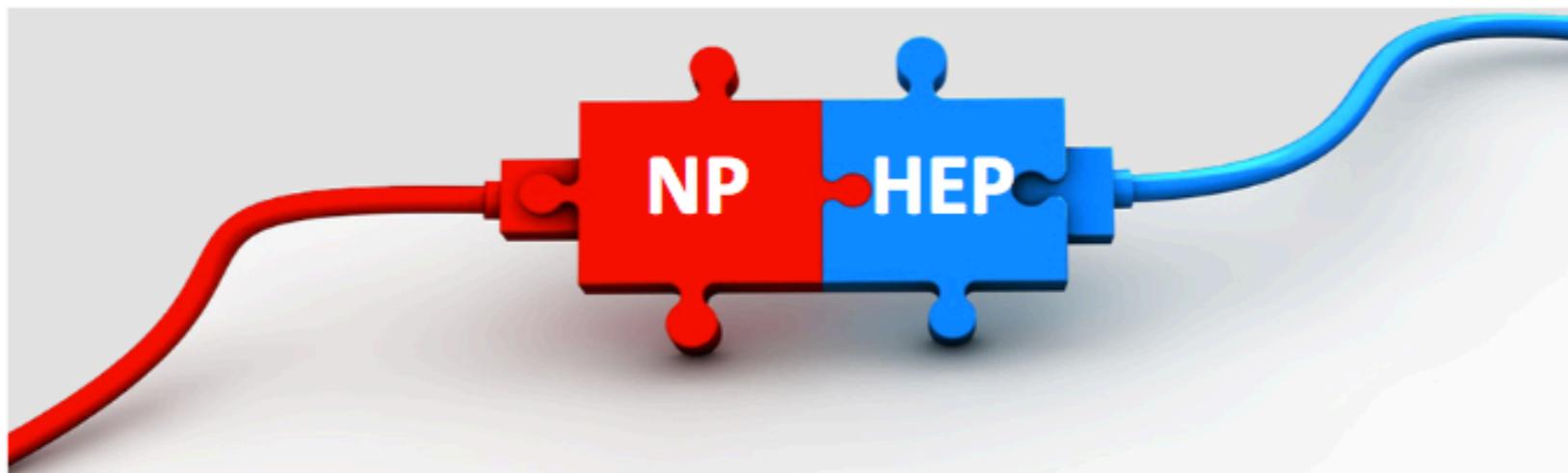
The goal is not only to fit data,
but to answer fundamental questions in QCD in the best possible way

- 11) identification of the current fragmentation region in SIDIS ?
 - 12) rise the accuracy of transverse momentum resummation
 - 13) match TMD and collinear factorization : fixed-order description of the high transverse momentum region and its matching to the low transverse momentum one
 - 14) order the hadronic tensor in terms of definite rank
-
- 15) include electron-positron annihilation, LHC and JLab data
 - 16) address the flavor decomposition in transverse momentum
 - 17) address the polarized structure functions
 - 18) Monte Carlo generators and TMDs
 - 19) what about spin 1 targets ?
 - 20) ...





Mapping the hadronization description in the Pythia MCEG to the correlation functions of TMD factorization



see the talk by M. Diefenthaler

