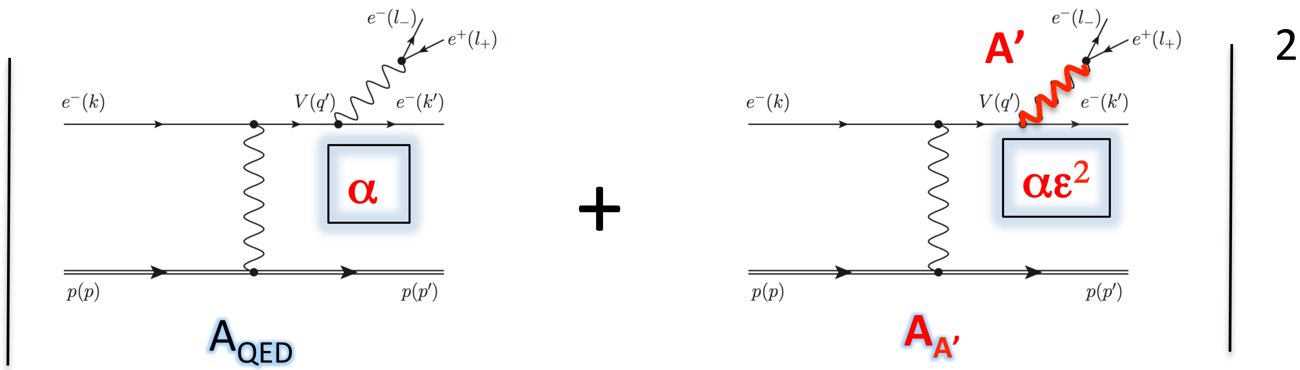
Notes about A’ and QED Background Interference

Valery Kubarovsky
Jlab

HPS Collaboration Meeting
May 2-3, 2017

A'-QED Background Interference



- A_{QED} pure real
- $A_{A'}$ has Imaginary and Real part

$$\sigma \sim \left|\alpha \mathcal{A}_{QED} + \alpha\epsilon^2 \mathcal{A}_{A'}\right|^2$$

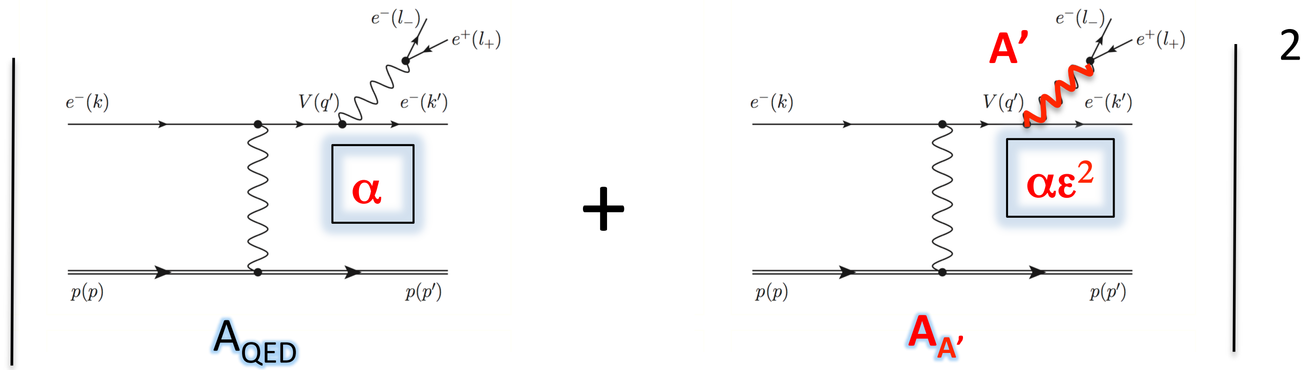
$$\sigma \sim \alpha^2 \left|\mathcal{A}_{QED} + \epsilon^2 \mathcal{A}_{A'}\right|^2$$

$$\sigma \sim \alpha^2 (|\mathcal{A}_{QED}|^2 + \epsilon^4 |\mathcal{A}_{A'}|^2 + 2\epsilon^2 \mathcal{A}_{QED} Re \mathcal{A}_{A'})$$

$$\epsilon^2 \sim 10^{-6}$$

$$\epsilon^4 \sim 10^{-12}$$

A'-QED Background Interference



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Do We Have a Bump?

$$\sigma \sim \left|\alpha \mathcal{A}_{QED} + \alpha\epsilon^2 \mathcal{A}_{A'}\right|^2$$

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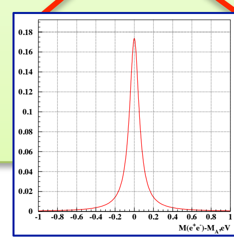
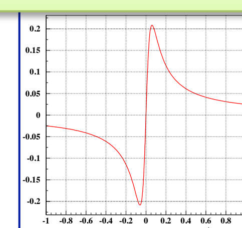


$$\sigma \sim \alpha^2 (|\mathcal{A}_{QED}|^2 + 2\epsilon^2 \mathcal{A}_{QED} Re \mathcal{A}_{A'})$$

Const



Breit-Wigner Distribution

$$\frac{k}{(E^2 - M^2)^2 + M^2\Gamma^2}$$

- E is the center-of-mass energy, $M(e^+e^-)$ in our case
- M is the mas of the resonance, A' mass
- Γ is the resonance width, related to its mean lifetime $\tau = 1/\Gamma$.
- k is normalization constant

- The form of the relativistic Breit-Wigner distribution arises from the *propagator* of an unstable particle, which has a denominator $q^2 - M^2 + iMG$. In the rest frame of the particle $q^2 = E_{CM} = M(e^+e^-)$.
- The propagator is proportional to the quantum-mechanical amplitude

$$\frac{\sqrt{k}}{(E^2 - M^2) + iM\Gamma}$$

A' Amplitude

M=50 MeV
$\varepsilon^2 = 10^{-6}$
$\Gamma = 0.1$ eV
$\Gamma/M = 2\text{x}10^{-9}$
$(E+M) \sim 2M$

$$\mathcal{A}_{A'} \sim \frac{1}{(E^2 - M^2) + i\Gamma M}$$

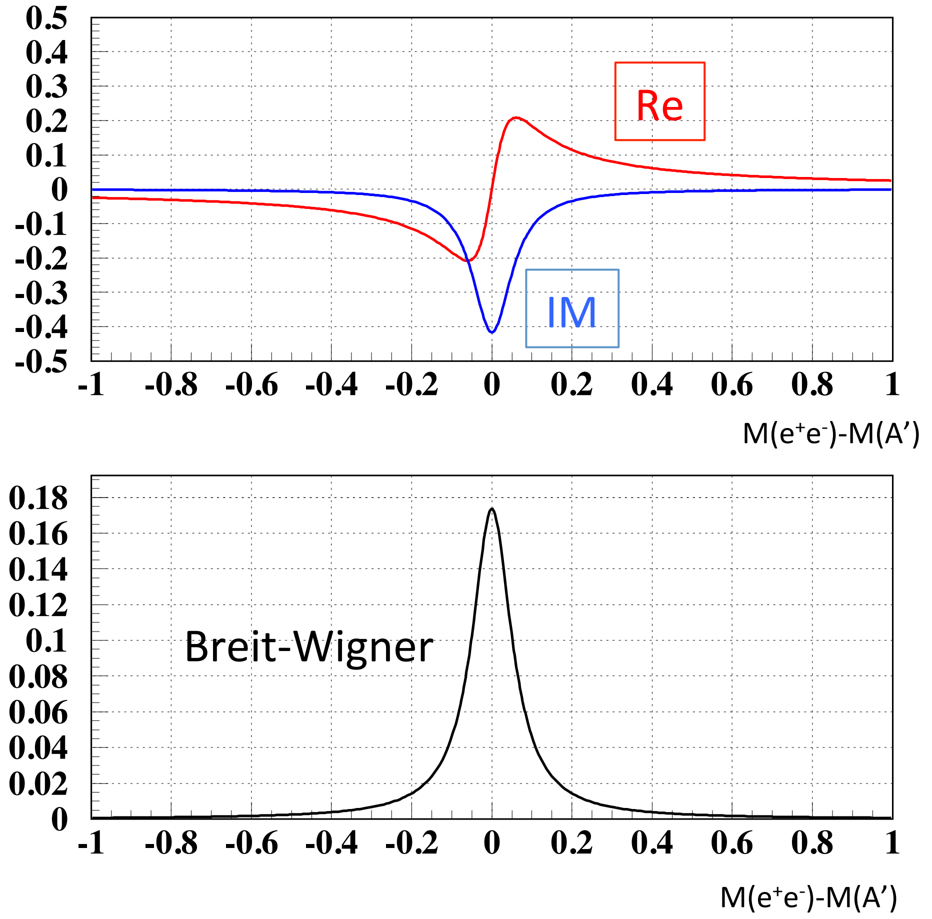
$$\mathcal{A}_{A'} \sim \frac{1}{2M(E - M) + i\Gamma M}$$

$$\mathcal{A}_{A'} \sim \frac{1}{2M} \frac{1}{(E - M) + i\Gamma/2}$$

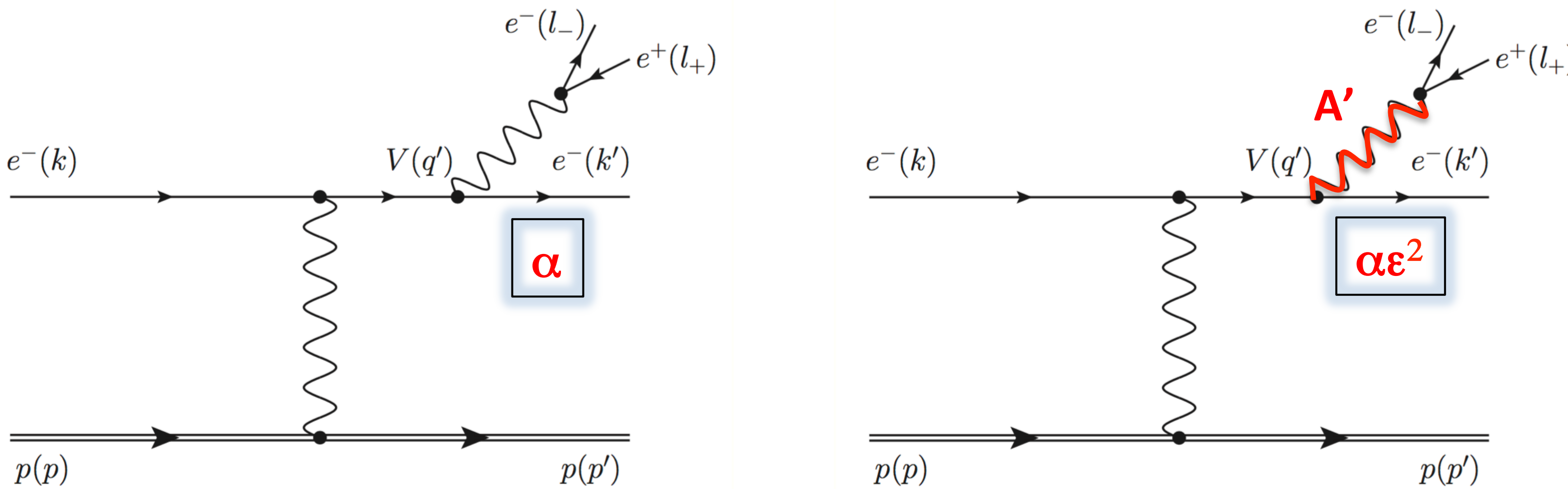
$$Re\mathcal{A}_{A'} = \frac{1}{2M} \frac{E - M}{(E - M)^2 + \Gamma^2/4}$$

$$Im\mathcal{A}_{A'} = -\frac{1}{2M} \frac{\Gamma/2}{(E - M)^2 + \Gamma^2/4}$$

RE, IM and Breit-Wigner



Virtual Photon and A'

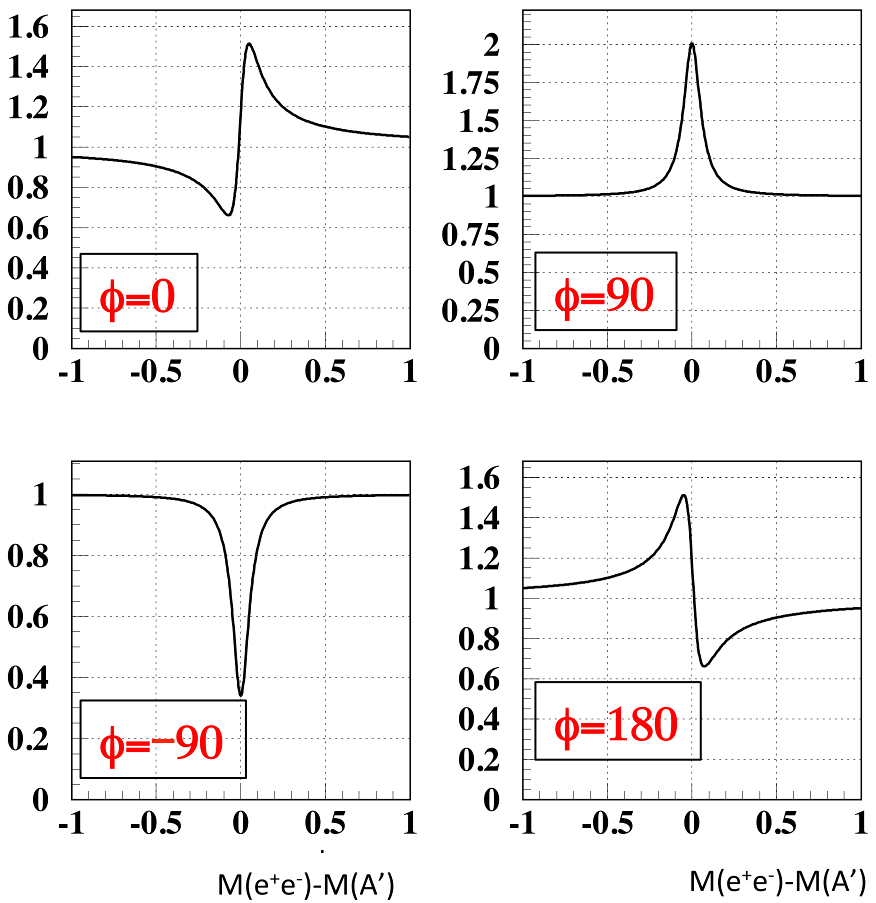


$$\left|\frac{\alpha}{q^2} + \frac{\alpha\epsilon^2}{(q^2 - M^2) + i\Gamma M}\right|^2$$

$$\left|\frac{\alpha}{M^2} + \frac{1}{2M}\frac{\alpha\epsilon^2}{(M_{e^+e^-} - M_{A'}) - i\Gamma/2}\right|^2$$

$$\frac{\alpha}{M^2}\left|1 + \frac{1}{2}\frac{M\epsilon^2}{(M_{e^+e^-} - M_{A'}) - i\Gamma/2}\right|^2$$

Examples of Interference



$$\left| 1 + \frac{e^{\phi} \epsilon^2}{(M_{e^+e^-} - M_{A'}) - i\Gamma/2} \right|^2$$

- ε^2 is arbitrary constant
- ϕ is arbitrary relative phase

But A' is different

$$\left|\frac{\alpha}{M^2} + \frac{1}{2M}\frac{\alpha\epsilon^2}{(M_{e^+e^-} - M_{A'}) - i\Gamma/2}\right|^2 =$$

$$\frac{\alpha}{M^2}\left|1 + \frac{1}{2}\frac{M\epsilon^2}{(M_{e^+e^-} - M_{A'}) - i\Gamma/2}\right|^2$$

$$\Gamma = M\epsilon^2\frac{\alpha}{3}$$

$$\left|1 + \frac{3}{2\alpha}\frac{\Gamma}{(M_{e^+e^-} - M_{A'}) - i\Gamma/2}\right|^2$$

But A' is different

$$\left|\frac{\alpha}{M^2} + \frac{1}{2M}\frac{\alpha\epsilon^2}{(M_{e^+e^-} - M_{A'}) - i\Gamma/2}\right|^2 =$$

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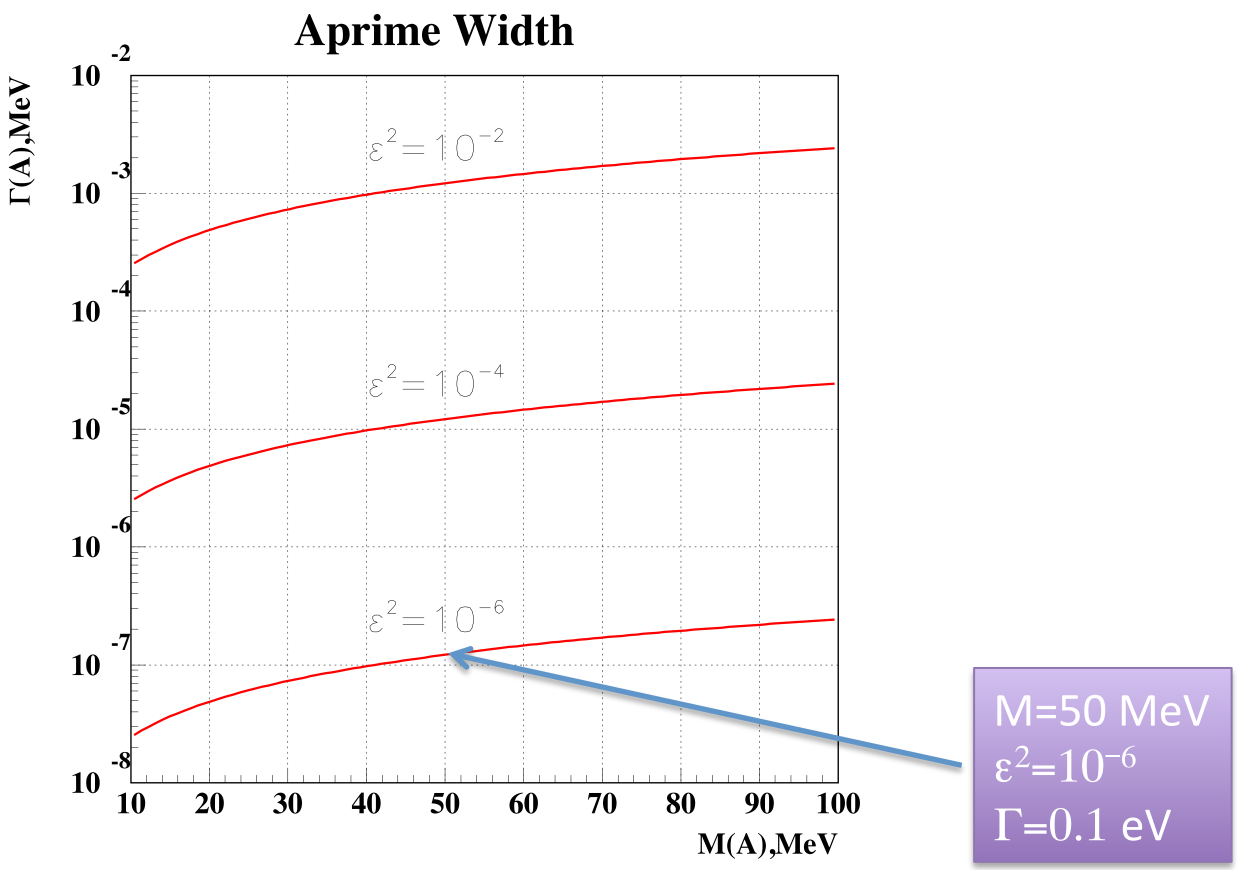
$$\Gamma = M\epsilon^2\frac{\alpha}{3}$$

$$\left|1 + \frac{3}{2\alpha}\frac{\Gamma}{(M_{e^+e^-} - M_{A'}) - i\Gamma/2}\right|^2$$

A’ Width



Aprime Width
Γ(A),MeV
M(A),MeV
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10
20
30
40
50
60
70
80
90
100
$\varepsilon^2=10^{-2}$
$\varepsilon^2=10^{-4}$
$\varepsilon^2=10^{-6}$
M=50 MeV
$\varepsilon^2=10^{-6}$
Γ=0.1 eV

A' amplitude at the pole

$$\left|1+\frac{3}{2\alpha}\frac{\Gamma}{(M_{e^+e^-}-M_{A'})-i\Gamma/2}\right|^2$$

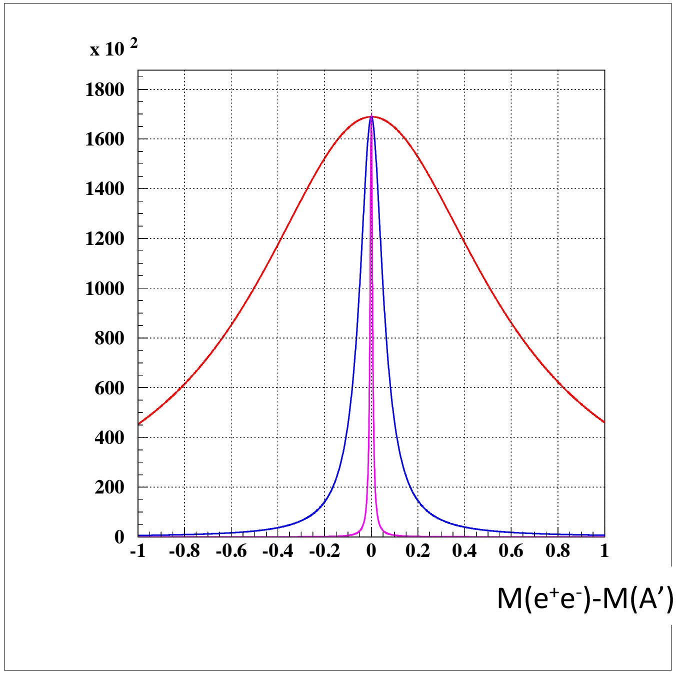
$$if\ M_{e^+e^-}=M_{A'}$$

$$\left|1-\frac{3}{\alpha}i\right|^2=\left|1-411i\right|^2$$

- A' amplitude at the pole doesn't depend on the mass, width or coupling constant
- A' amplitude is much more than electromagnetic amplitude
- The interference effects are negligible.

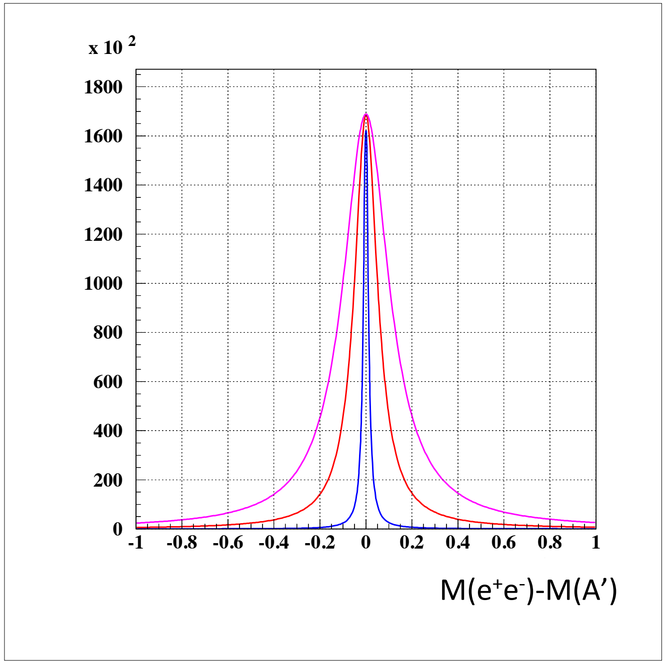
A' cross section normalized to the electromagnetic background

$\varepsilon = 10^{-5}, 10^{-6}, 10^{-7}$



M= 50 MeV

$M_{A'}$ =50, 10, 100 MeV



$\varepsilon^2 = 10^{-6}$

Ratio of Integrated Cross Sections

$$\sigma \sim \left| 1 + \frac{1}{2} \frac{M\epsilon^2}{(M_{e^+e^-} - M_{A'}) - i\Gamma/2} \right|^2$$

- Integral of the BW distribution

$$\sigma(A') \sim M_A^4 \epsilon^4 \int \frac{dM_{ee}}{(M_{ee}^2 - M_A^2)^2 + \Gamma^2 M_A^2} = M_A^4 \epsilon^4 \frac{3\pi}{2\alpha M_A^3 \epsilon^2} = \frac{3\pi\epsilon^2}{2\alpha} M_A$$

- Integral radiative QED

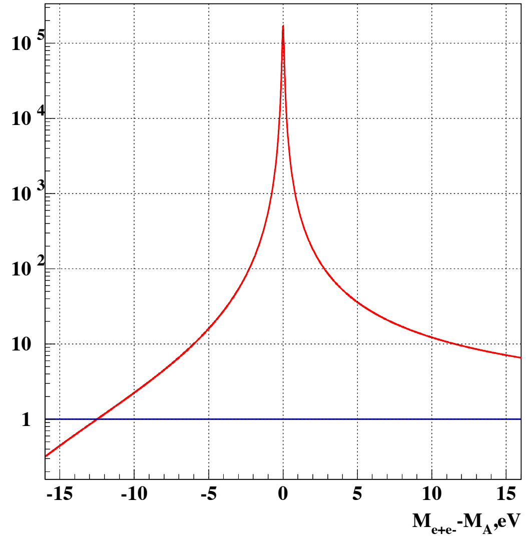
$$\sigma(rad) = \int_{\delta m} \frac{d\sigma(rad)}{dM_{ee}} \, dm_{ee} \sim \int_{\delta m} 1 \cdot dM_{ee} = \delta m$$

- Ratio A'/rad

$$\frac{\sigma(A')}{\sigma(rad)} = \frac{3\pi\epsilon^2}{2\alpha} \frac{M_A}{\delta m}$$

Conclusion

- A' cross section is much bigger than electromagnetic background at the pole
- The interference effects are negligible.



M=50 MeV
$\varepsilon^2=10^{-6}$
$\Gamma=0.1$ eV

Radiative Corrections

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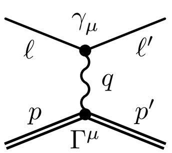
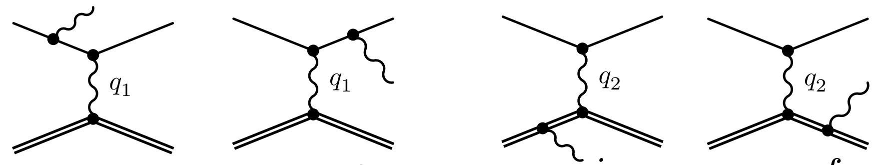
Outline

- Elastic scattering radiative corrections
  - eC elastic scattering
  - eW elastic scattering
- A’ and trident RC
  - Mainz A1
  - HPS
- Conclusion

RC for Elastic Electron Scattering



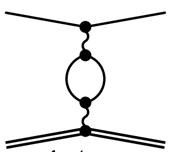
$\mathcal{M}_{Born}$



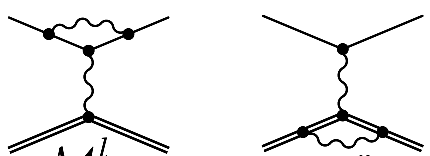
$\mathcal{M}^{li}_{Brem}$ $\mathcal{M}^{lf}_{Brem}$ $\mathcal{M}^{pi}_{Brem}$ $\mathcal{M}^{pf}_{Brem}$

Born term

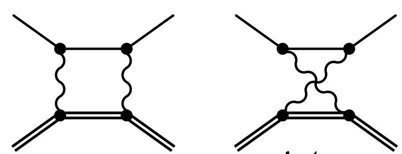
Bremsstrahlung process



$\mathcal{M}_{Vac}$



$\mathcal{M}^{l}_{Vert}$ $\mathcal{M}^{p}_{Vert}$



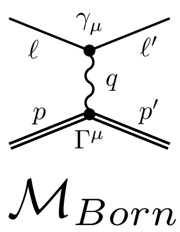
$\mathcal{M}_{Box}$ $\mathcal{M}_{xBox}$

Vacuum polarization

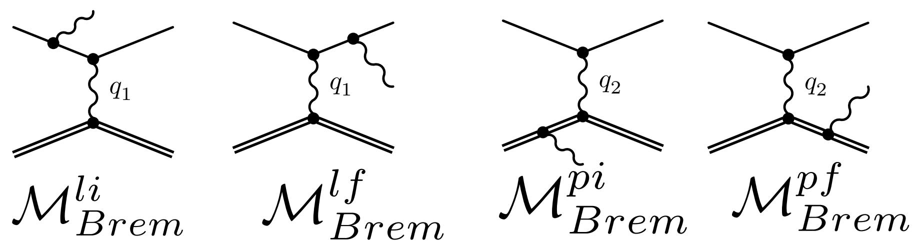
Vertex Correction

Two photon exchange

Elastic ep-scattering



Born term



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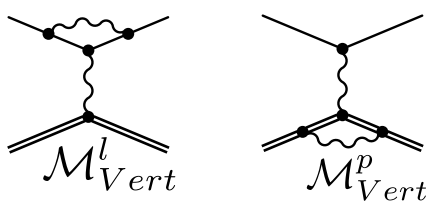

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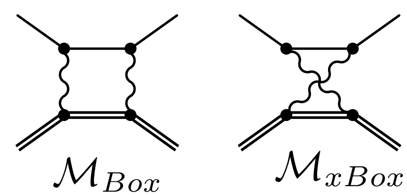

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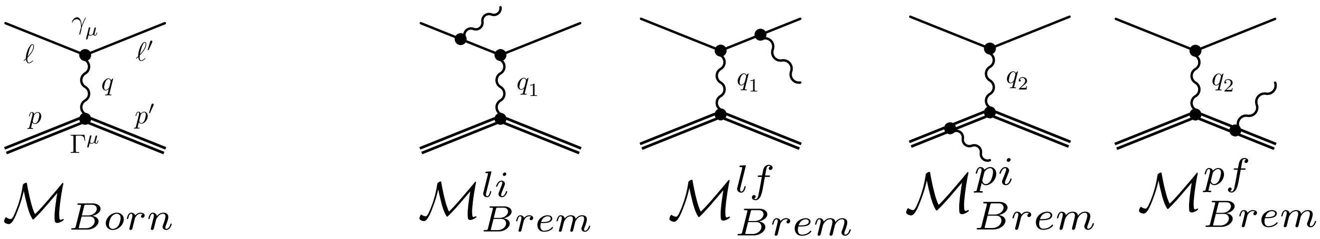
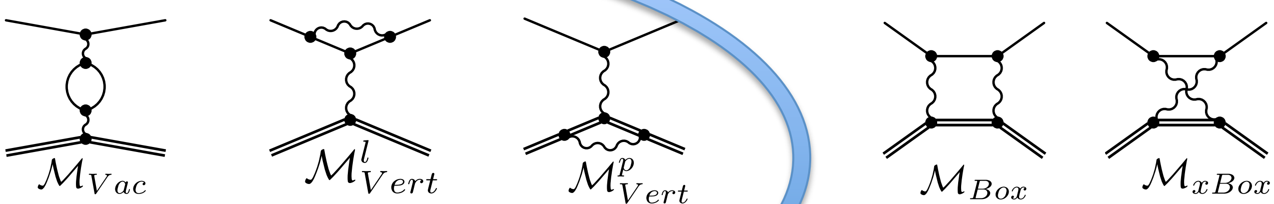


Born term

Bremsstrahlung process



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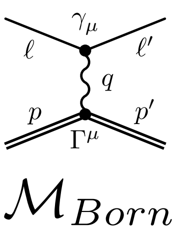
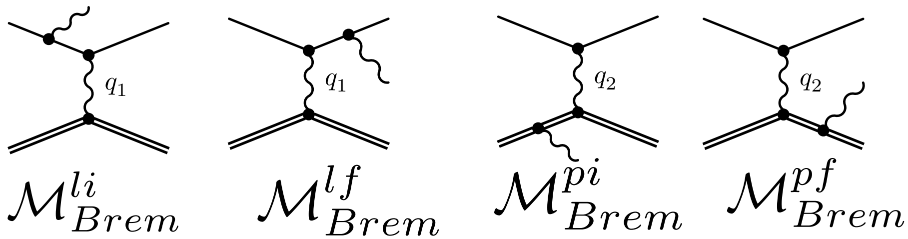
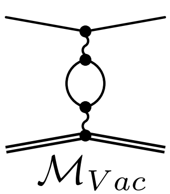
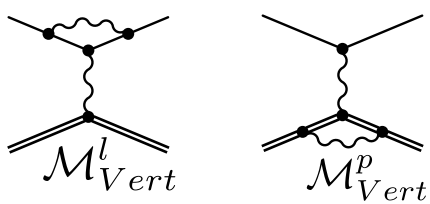


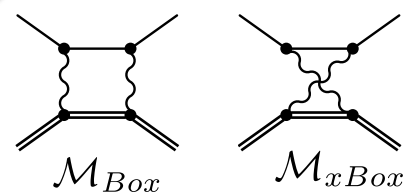


Born term

Bremsstrahlung process







Vacuum polarization

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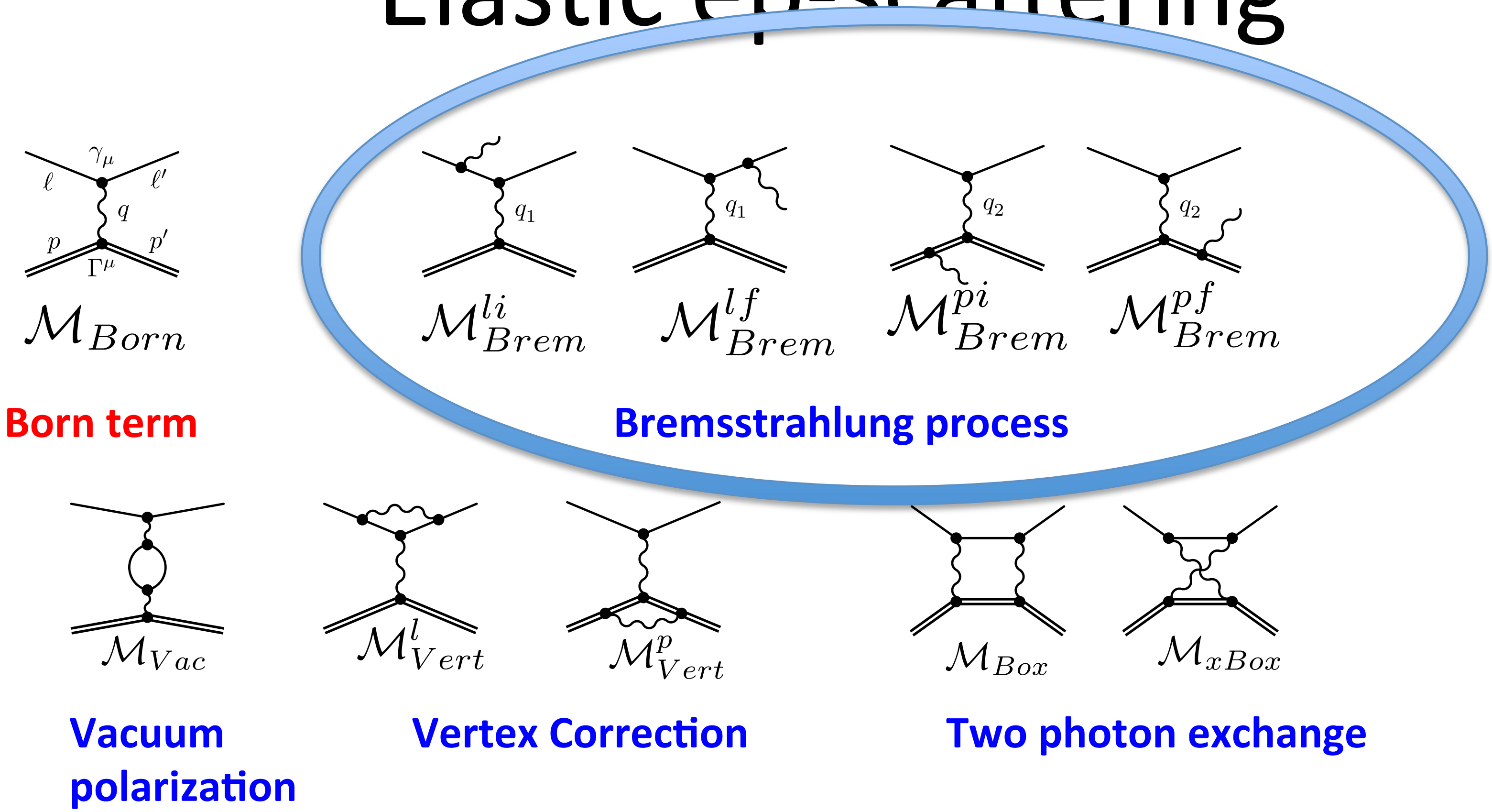


The cross section of e-p scattering

Taking into account the first-order QED radiative corrections, we can write the following schematic expression for the cross section of charged leptons scattering on protons

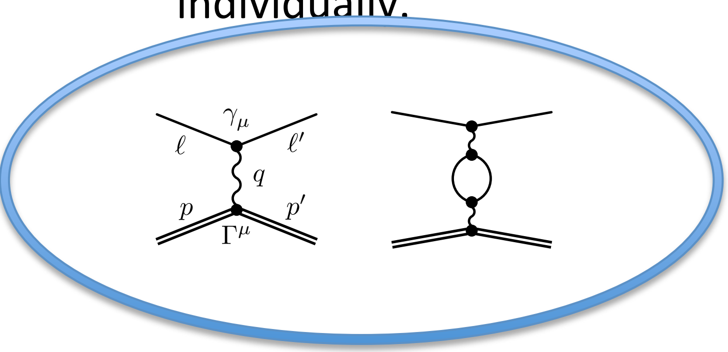
$$\sigma(\ell^{\pm}p) \propto |\mathcal{M}_{\rm Born}|^2 + 2Re\big[\mathcal{M}^{\dagger}_{\rm Born}\big(\mathcal{M}_{\rm vac} + \mathcal{M}^{\ell}_{\rm vert} + \mathcal{M}^{p}_{\rm vert}\big)\big]$$
$$+ 2Re\big[\mathcal{M}^{\dagger}_{\rm Born}\big(\mathcal{M}_{\rm box} + \mathcal{M}_{\rm xbox}\big)\big] + \big|\mathcal{M}^{\rm li}_{\rm brems} + \mathcal{M}^{\rm lf}_{\rm brems}\big|^2 + \big|\mathcal{M}^{\rm pi}_{\rm brems} + \mathcal{M}^{\rm pf}_{\rm brems}\big|^2$$
$$+ 2Re\big[\big(\mathcal{M}^{\rm li}_{\rm brems} + \mathcal{M}^{\rm lf}_{\rm brems}\big)^{\dagger}\big(\mathcal{M}^{\rm pi}_{\rm brems} + \mathcal{M}^{\rm pf}_{\rm brems}\big)\big] + \mathcal{O}(\alpha^4),$$

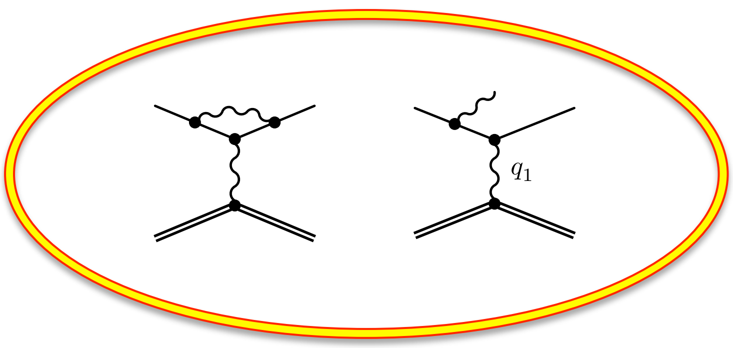
- All amplitudes, except Born and M_{vac}, contain *infrared-divergent terms* (tending to infinity in the limit of very soft photons). These are canceled out completely in the sum, which is therefore finite.
- There are the cancellations of the infrared divergences between
  - the lepton vertex correction and the lepton bremsstrahlung correction
  - between the proton vertex correction and the proton bremsstrahlung correction

Correction to the Born term

$$\frac{d\sigma_{\text{meas}}}{d\Omega_\ell} = (1+\delta)\,\frac{d\sigma_{\text{Born}}}{d\Omega_\ell}$$

$d\sigma_{meas}$ is the experimentally measured differential cross section.
δ includes both the corrections due to the emission of a real bremsstrahlung photons and the virtual-photon corrections.
Corrections of both these types are infrared-divergent, so that only the total correction **δ** can be defined uniquely. The only exception is the vacuum polarization correction **δ_{vac}**, which is finite and therefore can be determined individually.





General Comments

The size of the radiative corrections depends not only on the kinematics of elastic scattering, but also on the certain experimental conditions and cuts used to select elastic scattering events. Therefore, a realistic Monte Carlo simulation is required to carefully take into account the radiative corrections in the general case.

Single Arm Experiments

The situation is simpler in the particular case of single-arm (inclusive) experiments, when only the scattered lepton is detected. In such a case, the procedure to select elastic scattering events can be described by the single parameter **ΔE** *, **which is the maximum allowable energy loss of the scattered lepton due to inelastic processes.***

This parameter means that the energy of the lepton, detected at the certain angle θ_ℓ , should be in the range from $(E_\ell - \Delta E)$ to E_ℓ, where $E_\ell \approx ME\,[M + E\,(1 - \cos\theta_l)]$ is the elastic peak energy and M is the proton mass.

Virtual and Brems RC

$$\frac{d\sigma_{elast}}{d\Omega_l} + \frac{d\sigma_{brem}}{d\Omega_l}\Big|_{E_\gamma < \Delta E} = (1 + \delta_{vac} + \delta_{vert} + \delta_{brem} + \delta_{sm})\frac{d\sigma_{Born}}{d\Omega_l}$$

$$\delta_{virt} = \delta^e_{vac} + \delta_{vert}$$

$$\delta^e_{vac} = \frac{2\alpha}{3\pi}\left(\ln\frac{Q^2}{m_e^2} - \frac{5}{3}\right)$$

$$\delta_{vert} = \frac{\alpha}{\pi}\left(\frac{3}{2}\ln\frac{Q^2}{m_e^2} - 2\right)$$

=

$$\delta_{virt} = \frac{\alpha}{\pi}\left(\frac{13}{6}\ln\frac{Q^2}{m_e^2} - \frac{28}{9}\right)$$

$$\delta_{virt} \sim \ln\frac{Q^2}{m_e^2}$$

$$\delta_{brems} \sim \ln\frac{\Delta E}{E_e}$$

$$\delta_{brems} = -\frac{\alpha}{\pi}\left(\left(\ln\frac{E_e}{\Delta E} + \ln\frac{E_{e'}}{\Delta E}\right)\left(\ln\frac{Q^2}{m_e^2} - 1\right)\right)$$

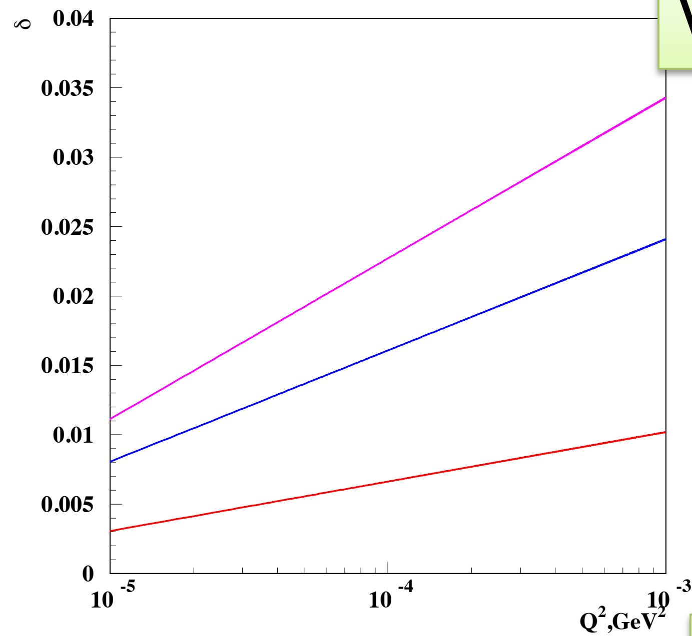
$$\delta_{sm} = \frac{\alpha}{\pi}\left(-\frac{\pi^2}{6} - \frac{1}{2}\ln^2\frac{E_e}{E_{e'}} - \int_0^{\cos^2\frac{\theta}{2}} dt\,\frac{\ln(1-t)}{t}\right)$$

Is residual of the cancellation of infrared divergent terms independent on ΔE

Only sum $\left(\delta_{vert} + \delta_{brem} + \delta_{sm}\right)$ makes sense.

I added δ_{sm} to δ_{brem} in my plots

Virtual corrections f(Q²)

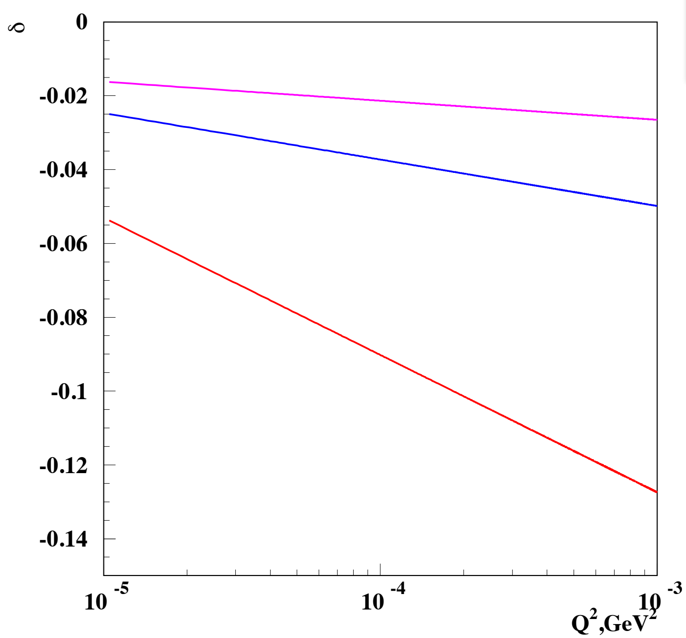


Vertex+Vacuum

Vertex corrections

Vacuum polarization

Brems corrections f(Q²,ΔE)

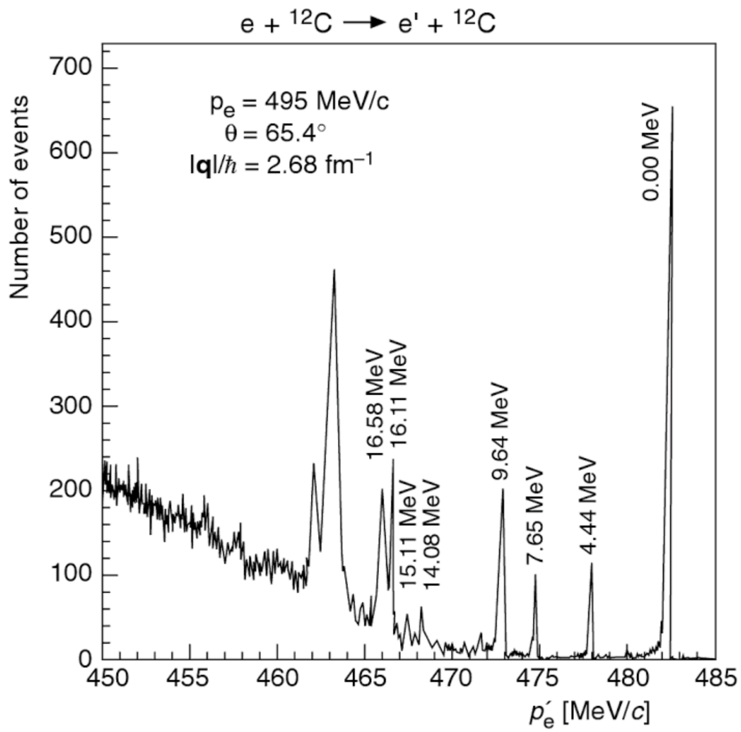


ΔE=0.2

ΔE=0.1

ΔE=0.01

Spectrum of electrons scattering off ^{12}C

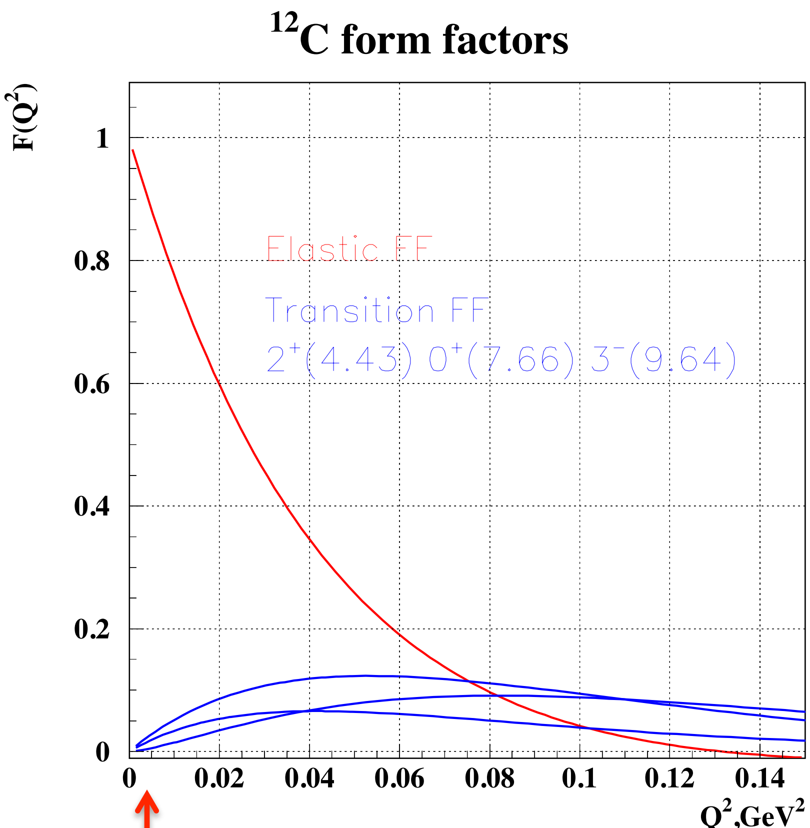


- E=495 MeV
- θ=65.4°
- Q^2=0.26 GeV^2
- High resolution magnetic spectrometer
- The excitation energy is given for each peak

The sharp peaks correspond to elastic scattering and to the excitation of discrete energy levels in the ^{12}C nucleus by inelastic scattering.

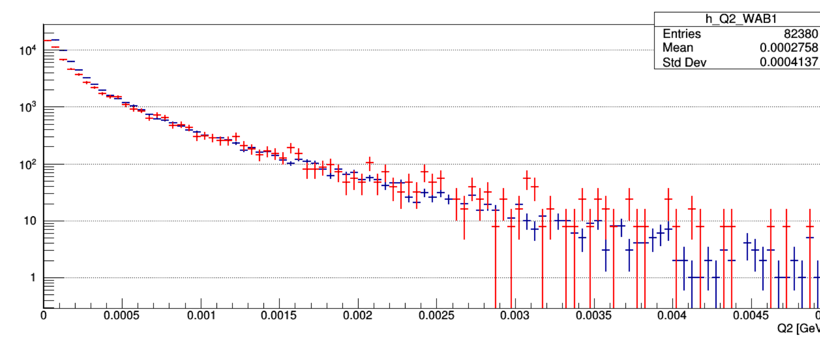
HPS can not separate elastic scattering from inelastic scattering

Transition Form Factors



HPS

$$\Delta E = E_W - M_w = \frac{Q^2}{2M_W} \sim \frac{2 \cdot 10^{-3}}{2M_W} \sim 5keV$$



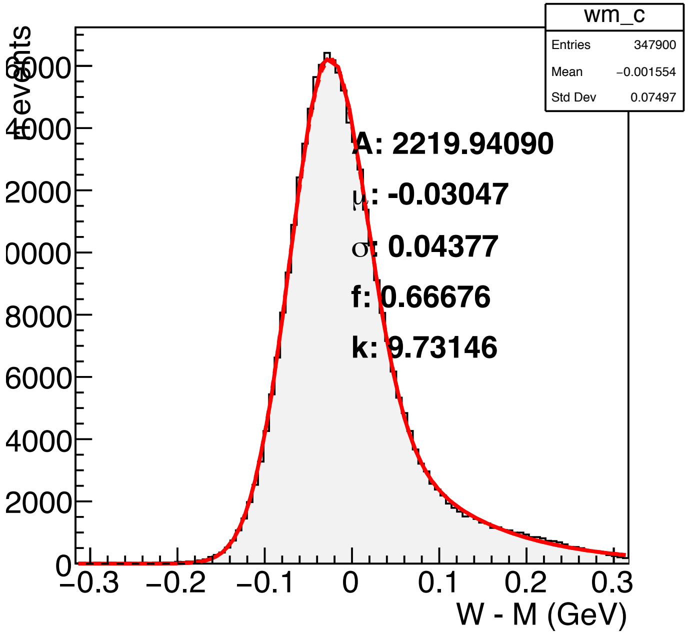
HPS trident: <Q^2>=2.7 10^{-4} GeV^2

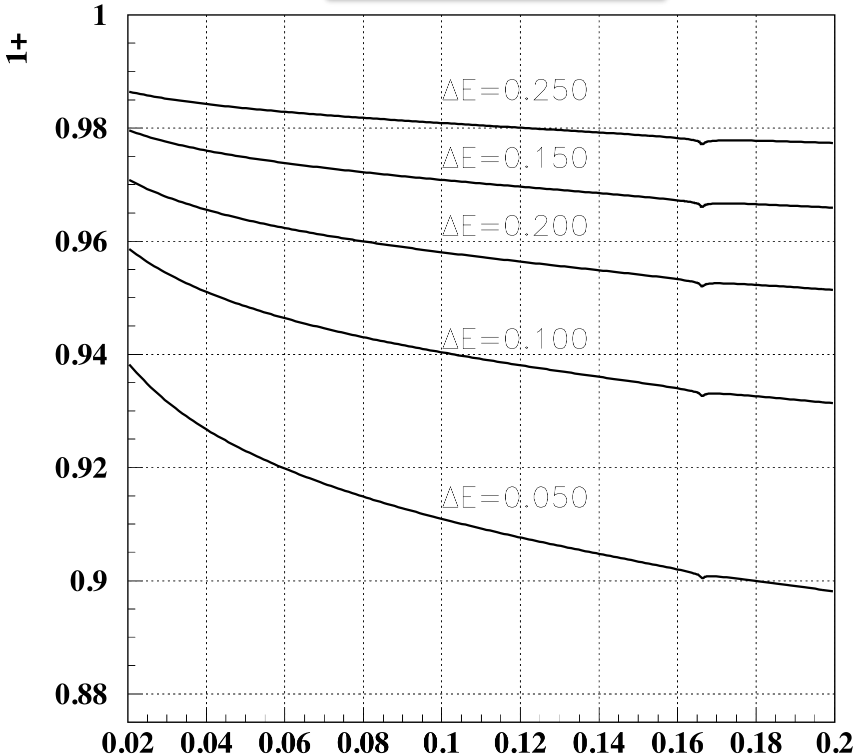
HPS elastic:: <Q^2>=2.0 10^{-3} GeV^2

eC elastic scattering

$$\frac{d\sigma_{\text{meas}}}{d\Omega_\ell} = (1+\delta)\,\frac{d\sigma_{\text{Born}}}{d\Omega_\ell}$$

RC=1+δ



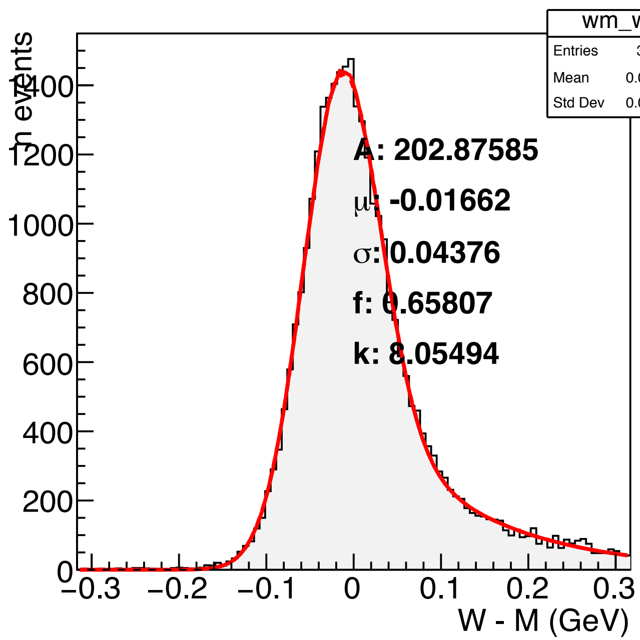


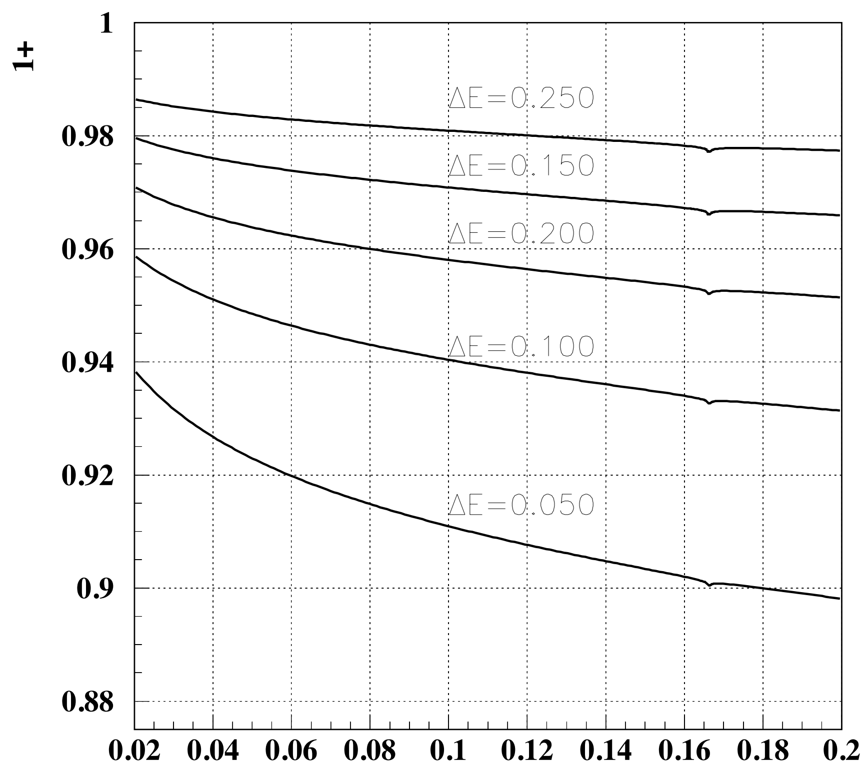
HPS resolution σ=44 MeV

eW elastic scattering

$$\frac{d\sigma_{\text{meas}}}{d\Omega_\ell} = (1+\delta)\,\frac{d\sigma_{\text{Born}}}{d\Omega_\ell}$$

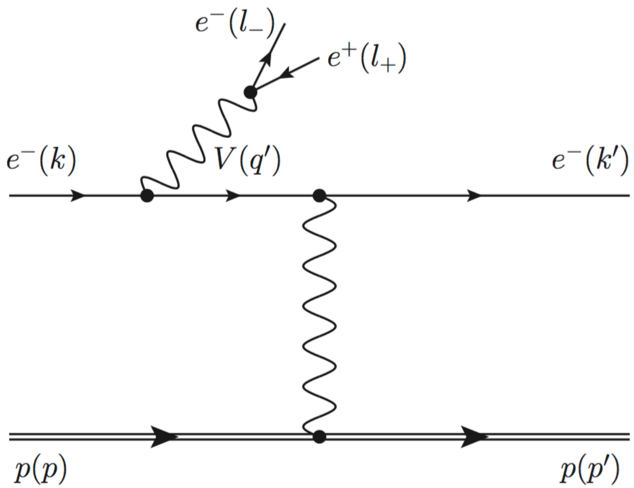
RC=1+δ

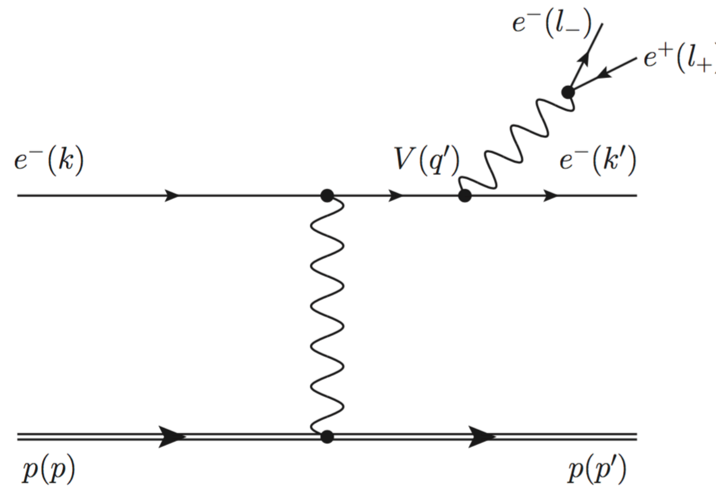


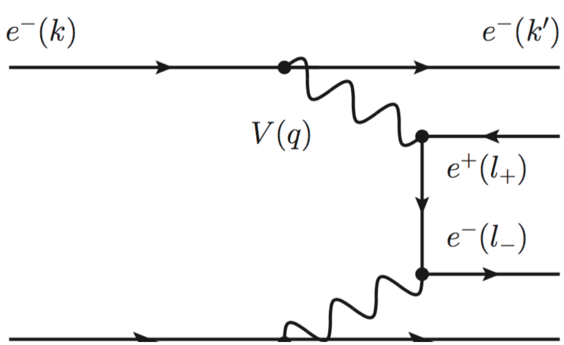


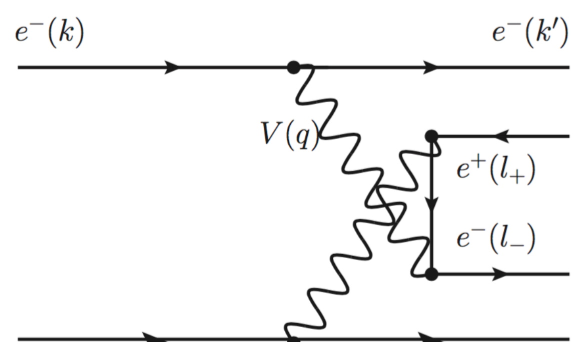
HPS resolution σ=44 MeV

Tridents Born term

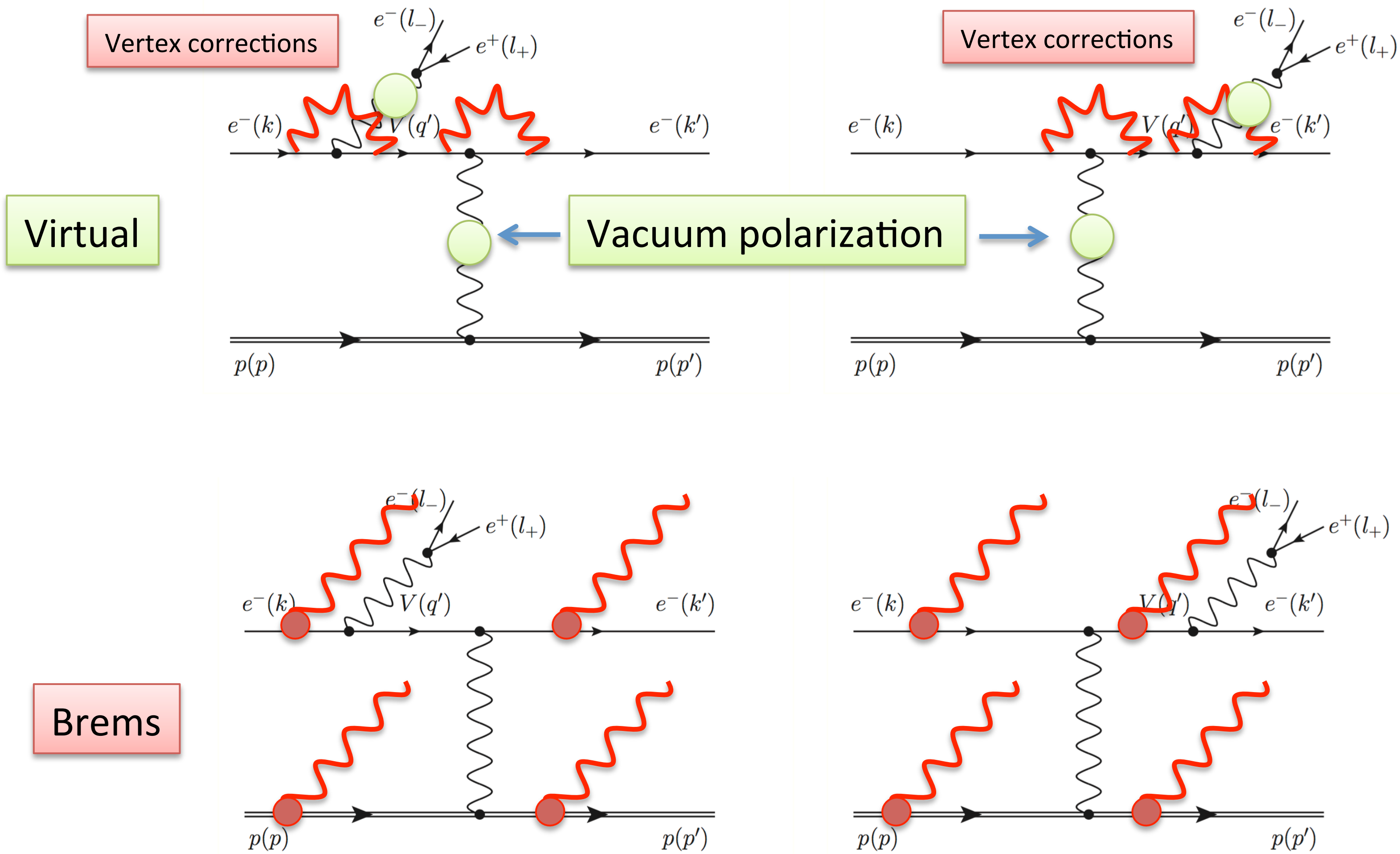








Virtual and Brems Rad. Corrections



THERE ARE NO THEORETICAL CALCULATION OF RC FOR TRIDENTS AT ALL!

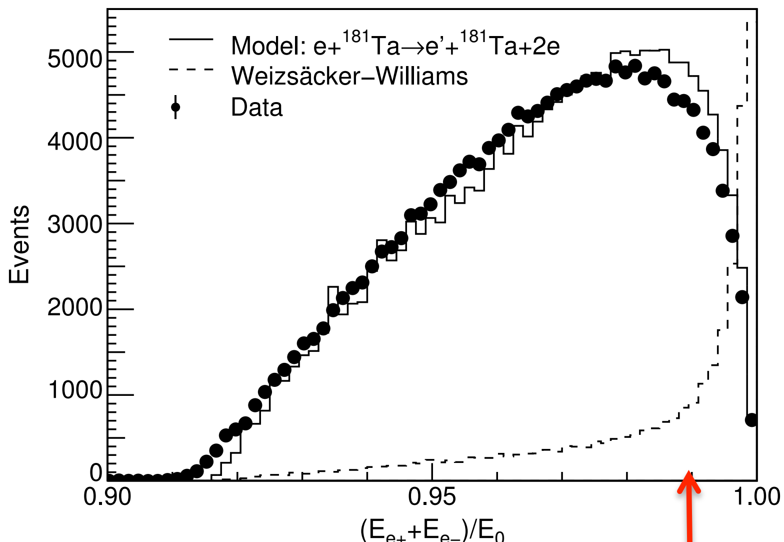
Mainz A1

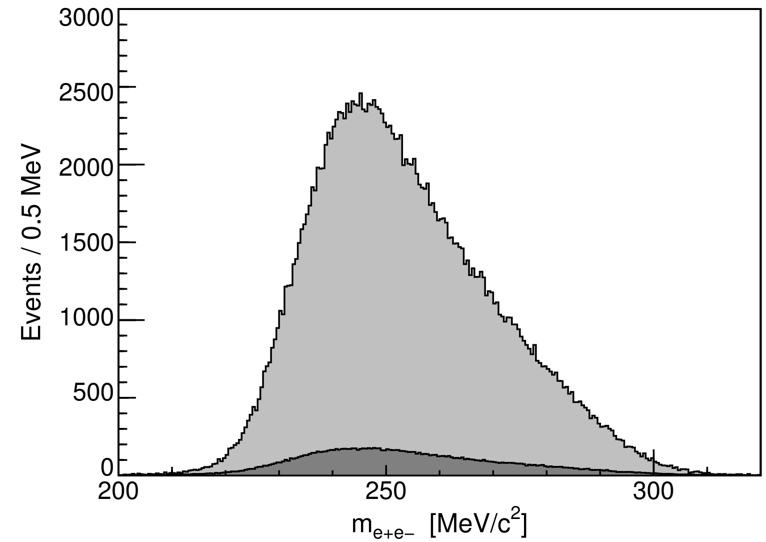
- Mainz A1 experiment used the elastic scattering radiative corrections (RC) for the trident production
- The corrections depend on Q^2 and ΔE_s.
- The minimum momentum transfer in Mainz (PRL 106, 251802, 2011) is around 0.001 GeV^2. So I made my estimation using this value of

  $Q^2 = 0.001\ GeV^2$
- A1 chose $\Delta E_s = 0.01\ E_{beam}$ for RC estimations.

A1 kinematics

$\langle M_{e+e-} \rangle$=250 MeV





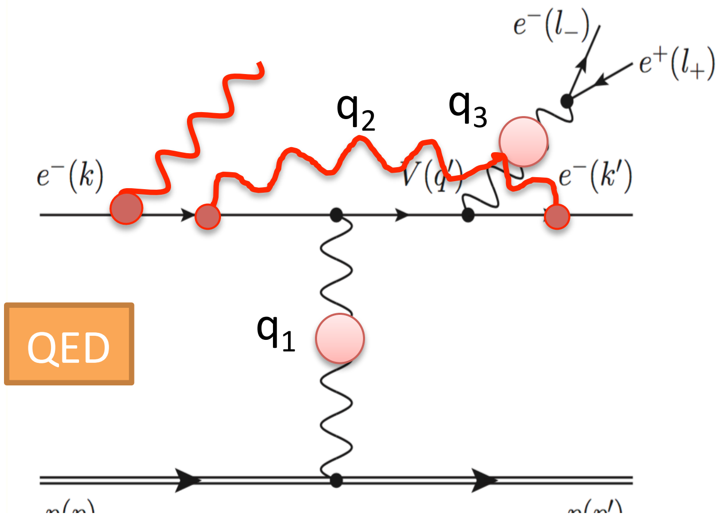
Mainz value for the calculation of RC

Q^2=0.001 GeV^2
ΔE=0.01*E_{beam}

TABLE I. Kinematic settings. The incident beam energy was $E_0 = 855$ MeV, and the settings are roughly centered around $E_{e^+} + E_{e^-} = E_0$ and $m_{\gamma'} = 250$ MeV$/c^2$.

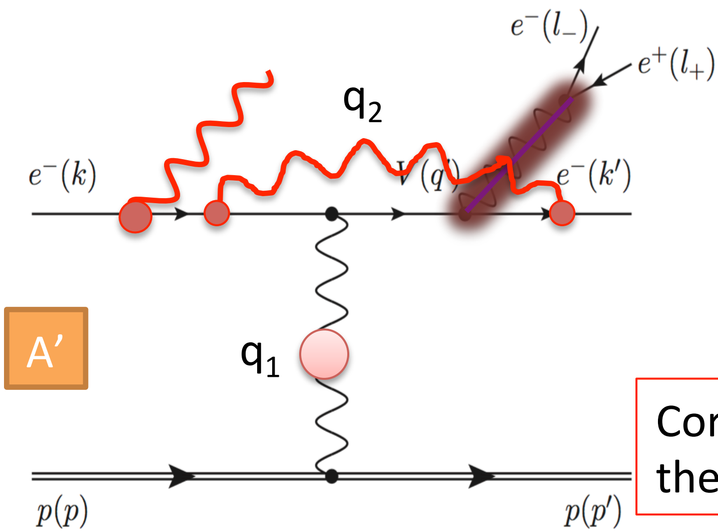
	Spec. A (e^+)			Spec. B (e^-)			
	p (MeV)	θ	$d\Omega$ (msr)	p (MeV)	θ	$d\Omega$ (msr)	Events
Setup 1	346.3	22.8°	21	507.9	15.2°	5.6	208×10^6
Setup 2	338.0	22.8°	21	469.9	15.2°	5.6	47×10^6

p1+p2	(p1+p2)/E0
854.2	0.999
807.9	0.944



$$\delta^{QED}_{vert+rad} = \delta_{vert+rad}(q_2^2)$$

$$\delta^{QED}_{vac} = \delta_{vac}(q_1^2) + \delta_{vac}(q_3^2)$$



$$\delta^{A'}_{vert+rad} = \delta_{vert+rad}(q_2^2)$$

$$\delta^{A'}_{vac} = \delta_{vac}(q_1^2)$$

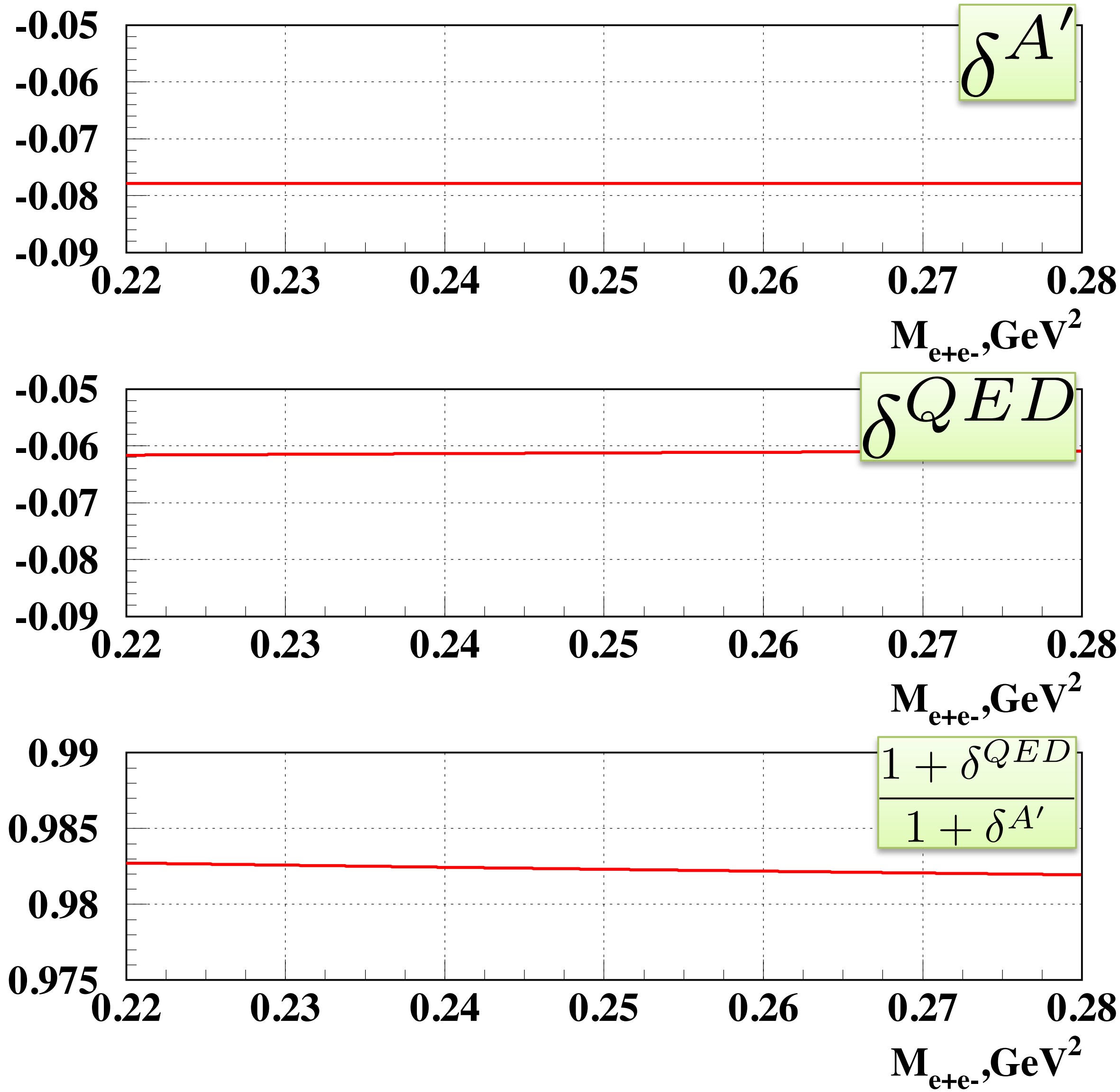
Correction to the upper limits:

$$\frac{\sigma^{A'}_{meas}}{1+\delta^{A'}} \frac{1+\delta^{QED}}{\sigma^{QED}_{meas}} = \frac{3\pi\epsilon^2}{2\alpha} \frac{M_{A'}}{\delta_m}$$

$$R = \frac{1+\delta^{QED}}{1+\delta^{A'}} = \frac{1+\delta_{vert+rad}(q_2^2)+\delta_{vac}(q_1^2)+\delta_{vac}(q_3^2)}{1+\delta_{vert+rad}(q_2^2)+\delta_{vac}(q_1^2)}$$

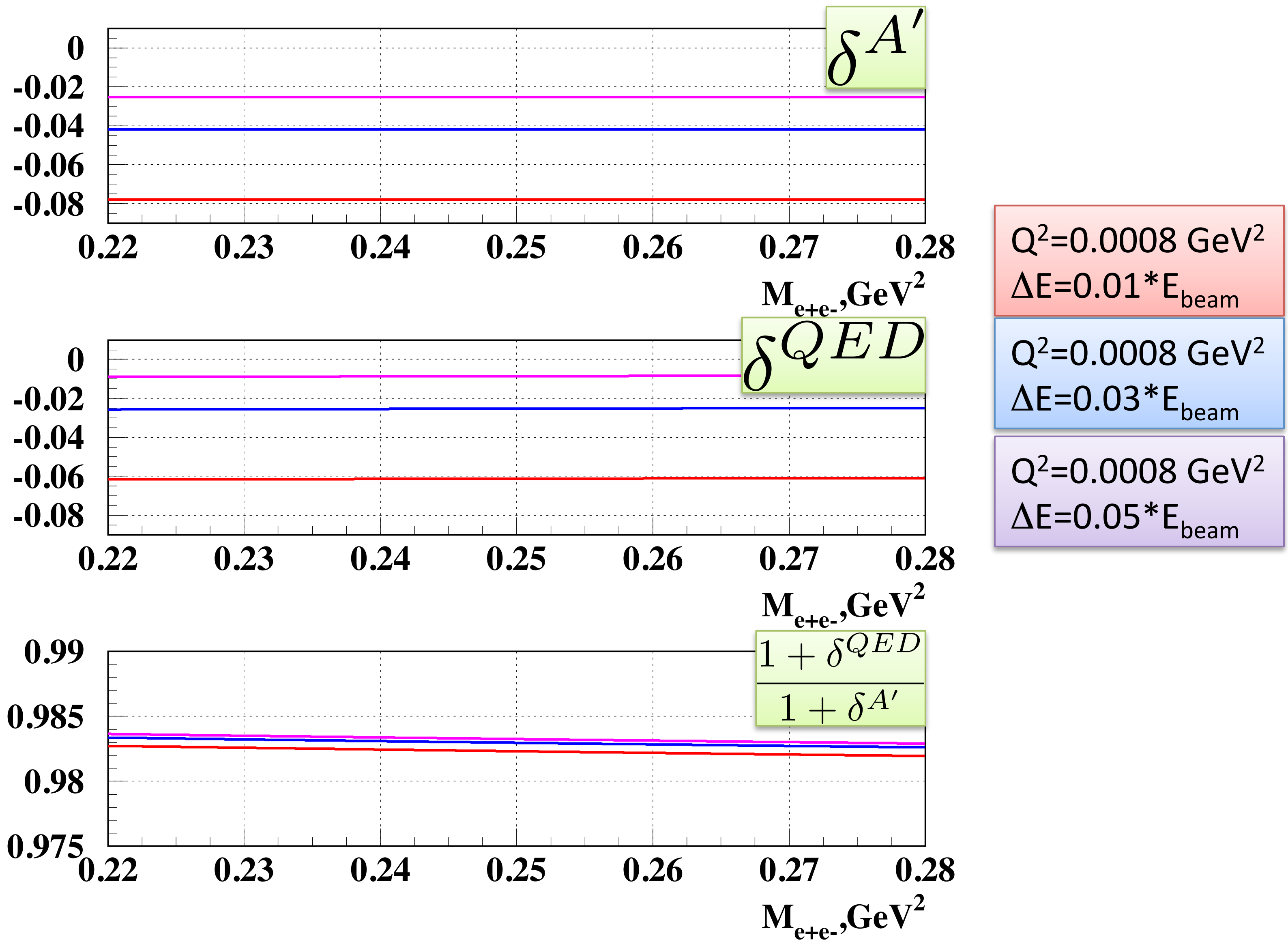
We don't need to consider the emission of photons from the final state e^+e^- state. Cancellation of large logarithm $\ln(M^2_{ee}/m^2_e)$ is the consequence of Kinoshita-Lee-Nauenberg theorem.

Mainz radiative corrections

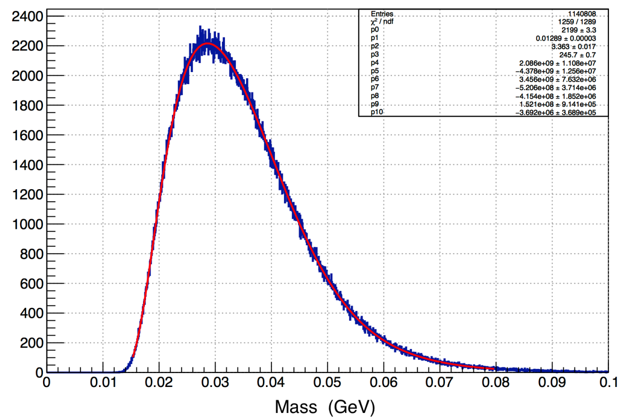


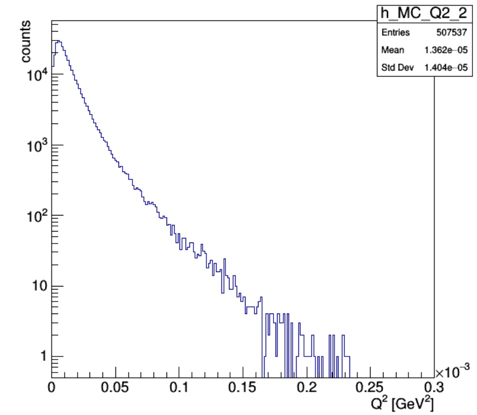
Q^2=0.0008 GeV^2
$\Delta E=0.01*E_{beam}$

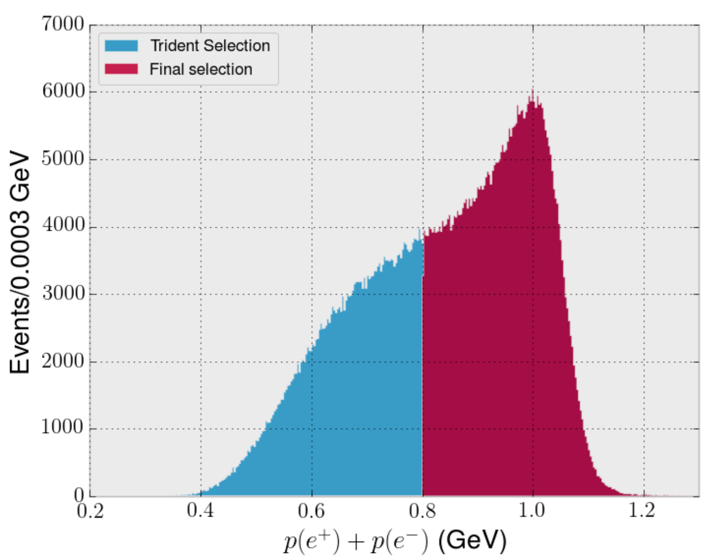
Mainz radiative corrections



HPS kinematics



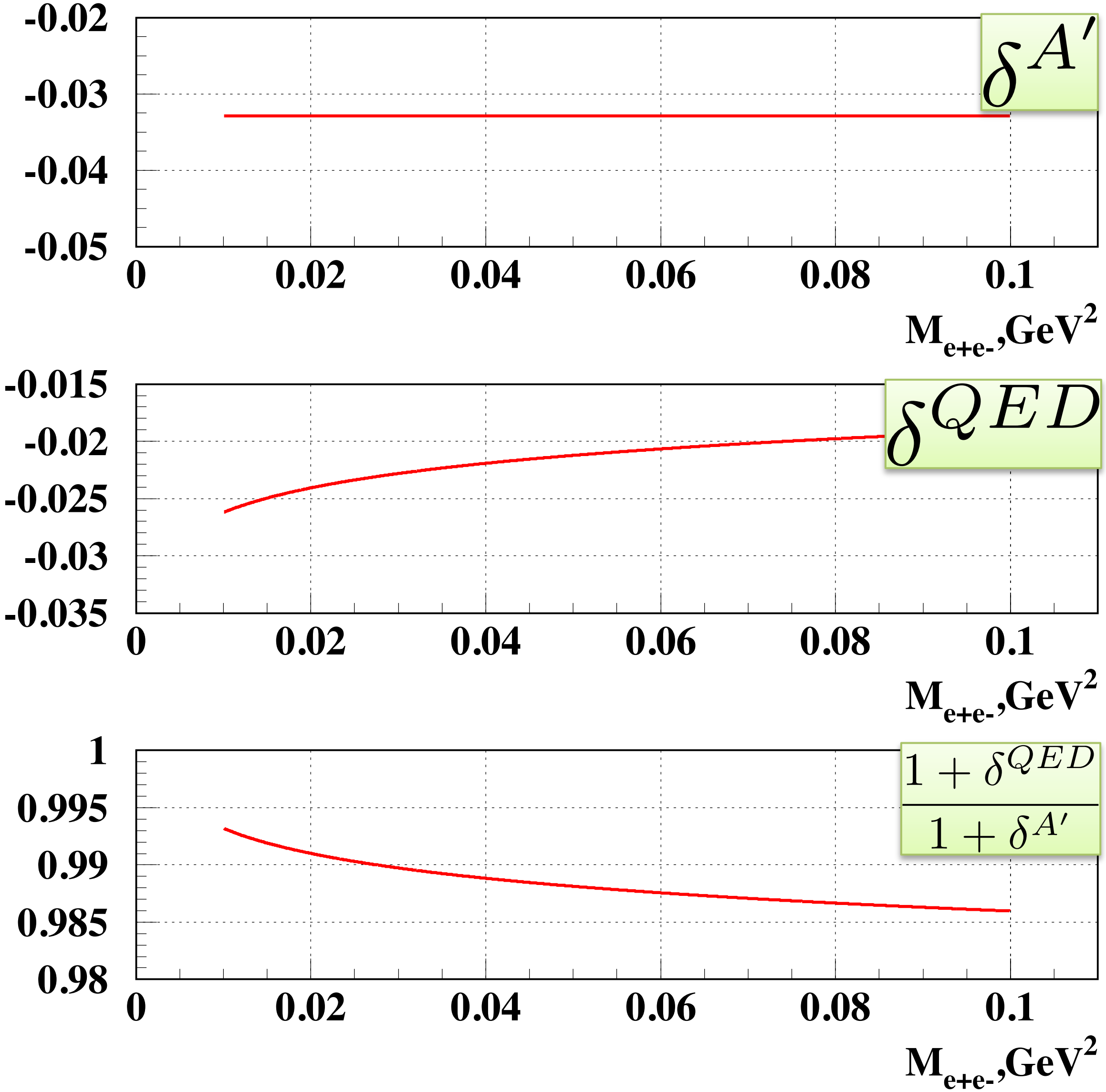




Q^2=1.3 10^{-5} GeV^2

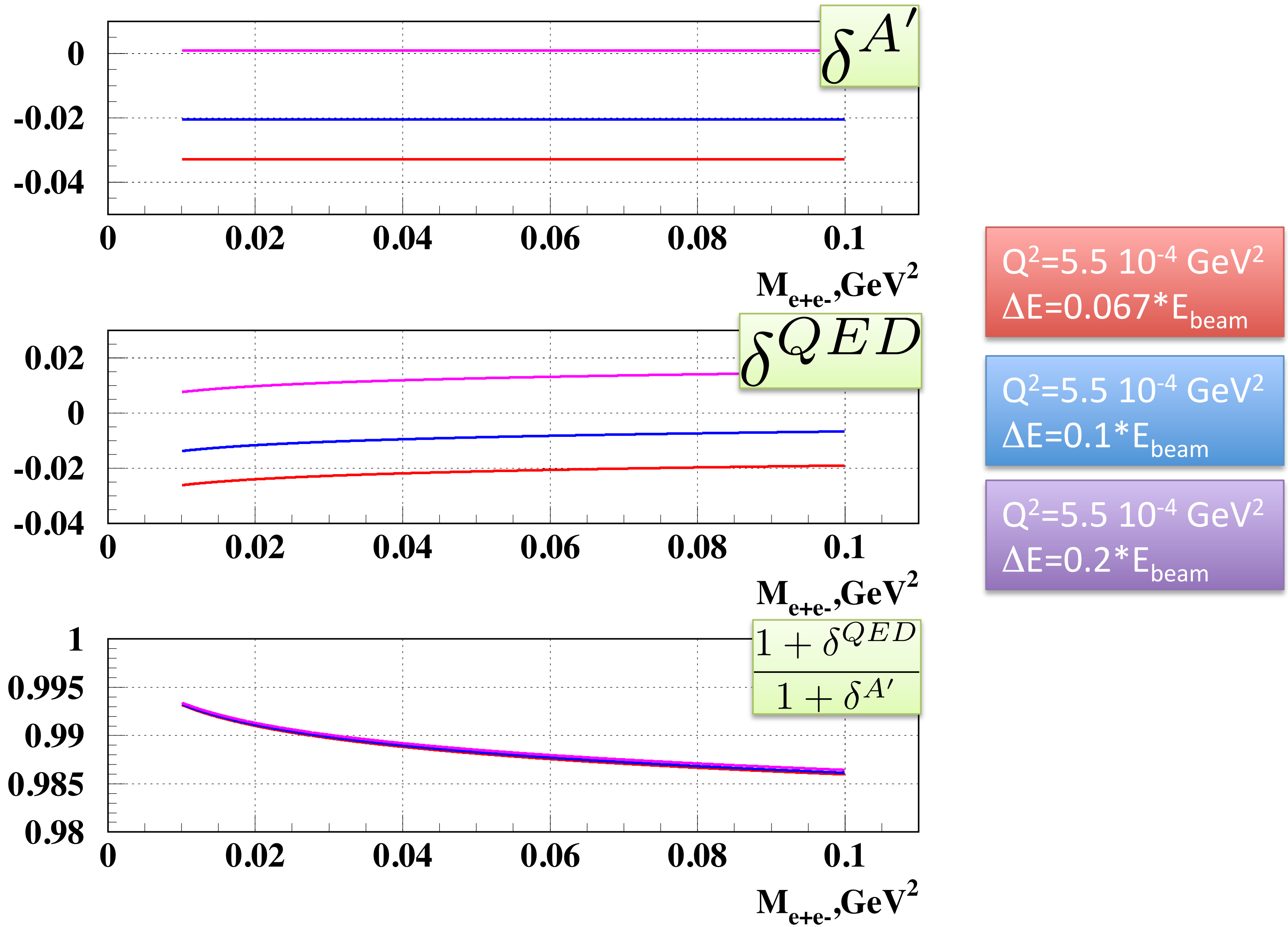
ΔE=0.067*E_{beam}

HPS radiative corrections



Q^2=5.5 10^{-4} GeV^2
ΔE=0.067*E_{beam}

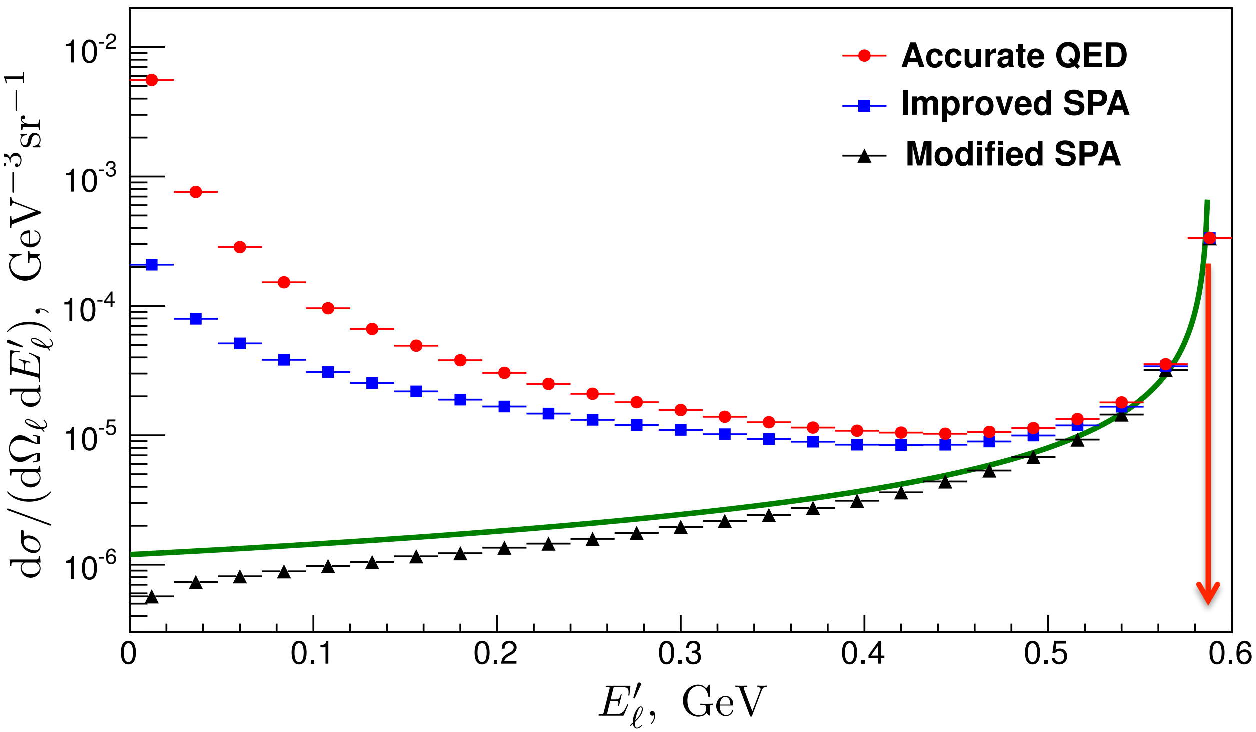
HPS radiative corrections



Conclusion

- RC for the eC and eW elastic scattering were calculated and can be used for the analysis
- There is no theoretical RC for the trident and/or A' production
- Mainz A1 experiment used simple elastic scattering RC. No RC corrections for the upper limits applied
- RC *were estimated* using a simple model a la elastic with taking into account one more vacuum loop.
- The trident RC corrections are in the range from -4% up to 2%
- The RC for upper limits are less than 2%.
- More serious progress in the RC may be achieved with the help of experts in the RC calculations.

Example: Single Arm Scattering



Elastic scattering
E_0 =1 GeV
$\theta_{e=}$=70^0

Elastic peak

- Improved soft-photon approximation is closer to the accurate calculation
- Modified soft-photon approximation is closer to the analytical prediction.
- All the predictions are in good agreement with each other in the vicinity of the elastic peak.

Coincidence Experiments

The situation is much more complicated in the case of coincidence (exclusive) experiments, when both the scattered lepton and recoil proton are detected. It is then impossible to describe all the experimental conditions and cuts by a single parameter, making a careful MC simulation necessary.