Computational Considerations for Spin Tracking in Electron-Ion Colliders

Dan T. Abell

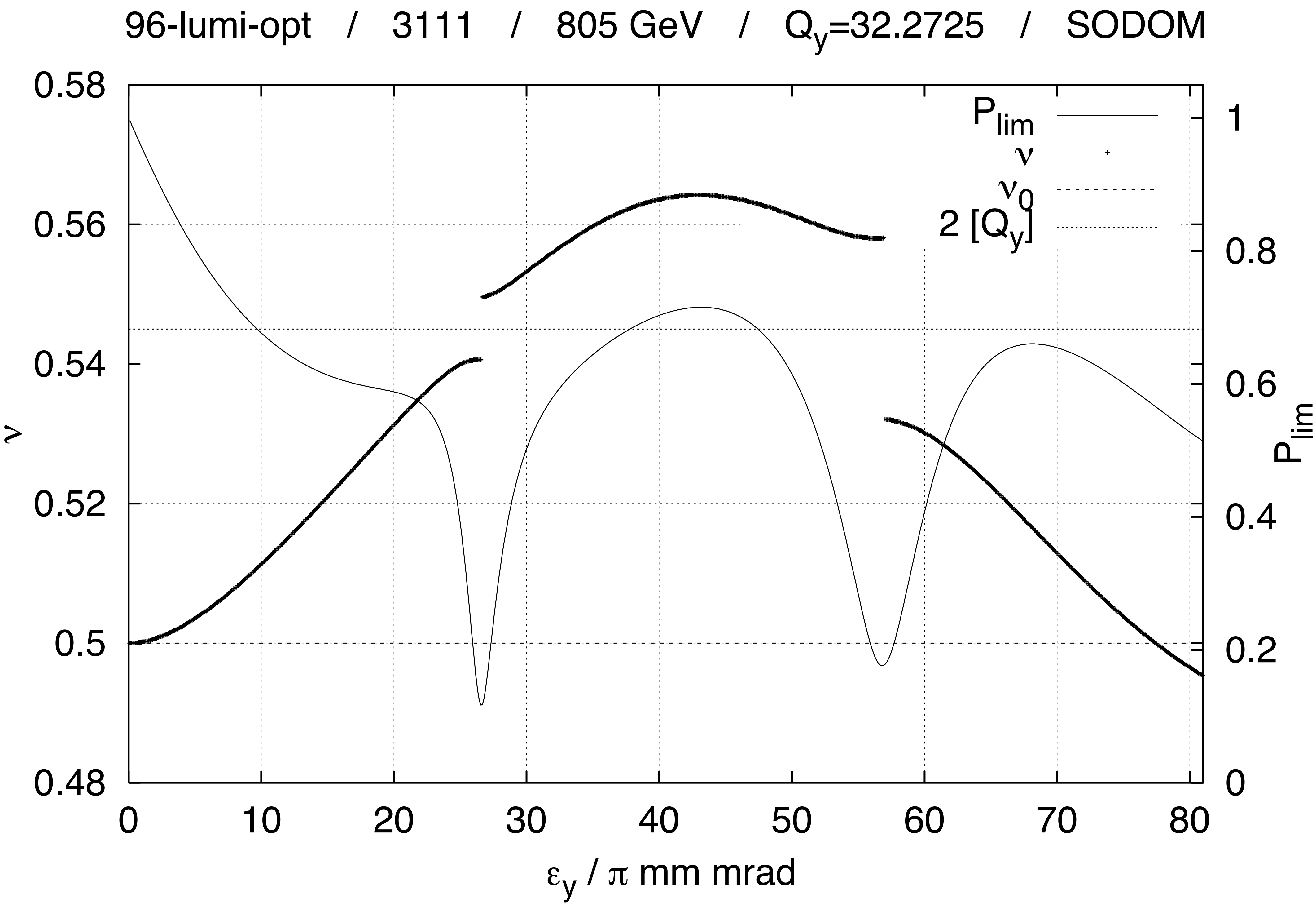
RadiaSoft, LLC, Boulder, CO

dabell@radiasoft.net

JLEIC Collaboration Meeting

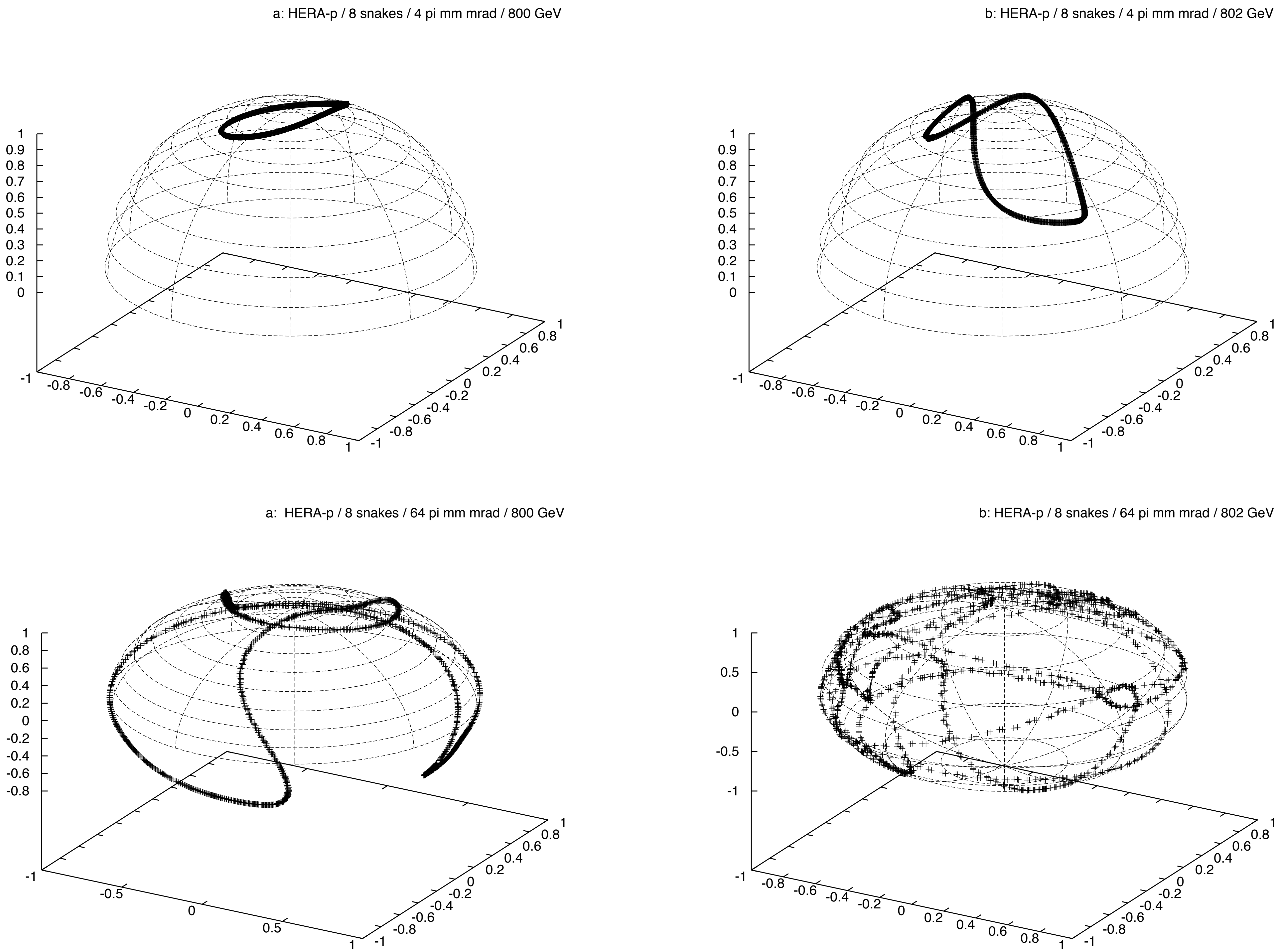
3–5 April 2017

The Goal of Spin Tracking is Insight



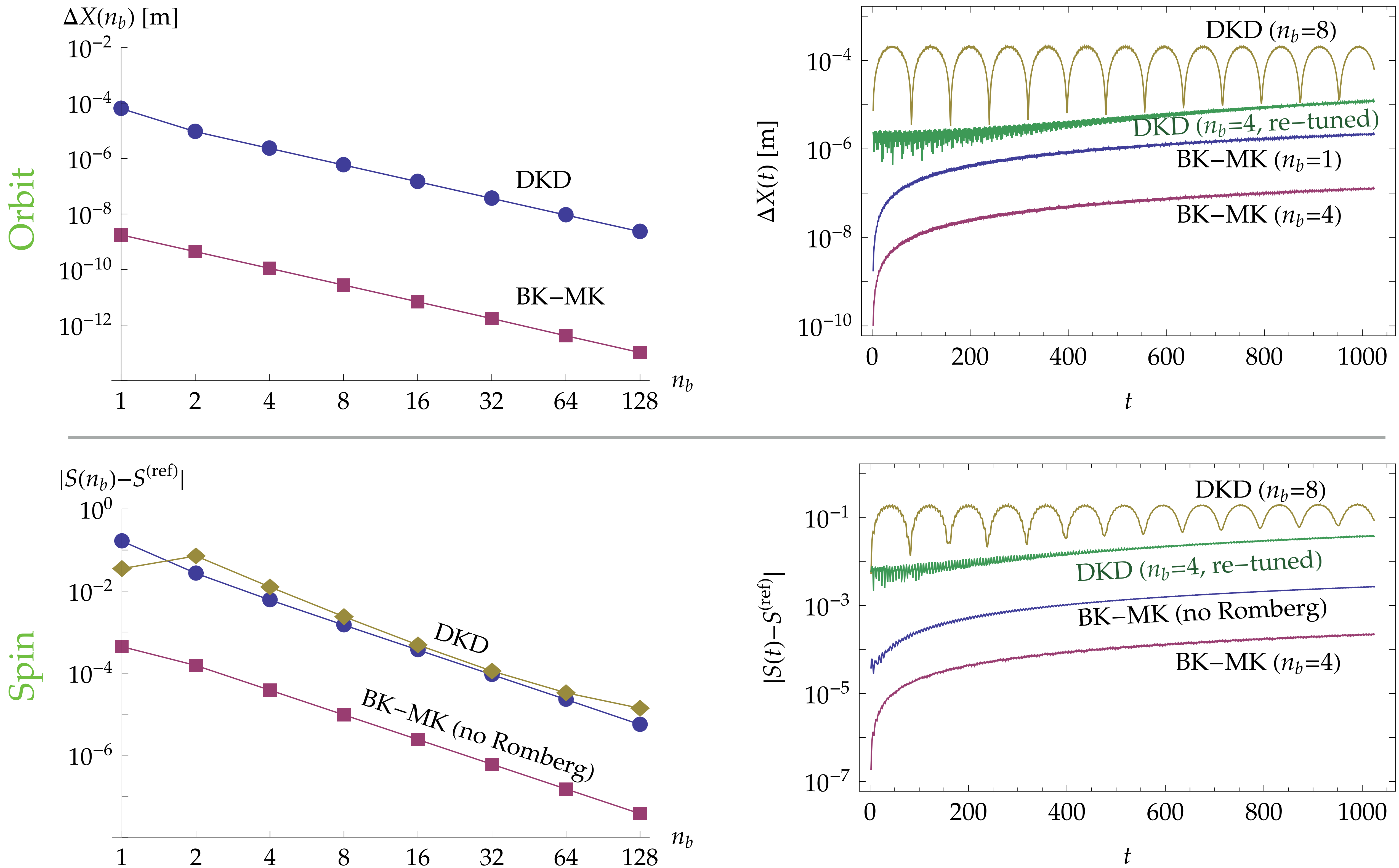
M. Vogt, *Bounds on the Maximum Attainable Equilibrium Polarization of Protons at High Energy in HERA*, PhD dissertation, Universität Hamburg, 2000.

The Goal of Spin Tracking is Insight



D.P. Barber, K. Heinemann, G.H. Hoffstaetter, and M. Vogt, *arXiv:physics/9901042*, Jan. 1999.

The Accuracy of Orbital Tracking Affects Spin Tracking



How You Integrate the Quadrupole Affects the Tune

Drift-Kick

$$H_Q = \underbrace{-\sqrt{1+\frac{2}{\beta_0}P_t+P_t^2-P_x^2-P_y^2}+\frac{1}{\beta_0}P_t}_{\text{drift}}+\underbrace{\frac{k_2}{2}(X^2-Y^2)}_{\text{kick}}$$

Matrix-Kick

$$= \underbrace{\frac{1}{2}(P_x^2+P_y^2)+\frac{k_2}{2}(X^2-Y^2)}_{\text{matrix}}\underbrace{-\sqrt{1+\frac{2}{\beta_0}P_t+P_t^2-P_x^2-P_y^2}+\frac{1}{\beta_0}P_t-\frac{1}{2}(P_x^2+P_y^2)}_{\text{``kick''}}$$

"Matrix"-Kick

$$= \underbrace{\frac{1}{2}\frac{P_x^2+P_y^2}{\sqrt{1+\frac{2}{\beta_0}P_t+P_t^2}}+\frac{k_2}{2}(X^2-Y^2)}_{\text{``matrix''}}\underbrace{-\sqrt{1+\frac{2}{\beta_0}P_t+P_t^2-P_x^2-P_y^2}+\frac{1}{\beta_0}P_t-\frac{1}{2}\frac{P_x^2+P_y^2}{\sqrt{1+\frac{2}{\beta_0}P_t+P_t^2}}}_{\text{``kick''}}$$

with $P_t = \dfrac{m\gamma c^2 - m\gamma_0 c^2}{p_0 c} = \dfrac{\gamma-\gamma_0}{\beta_0\gamma_0}$

Symplectic integration made easy: (*i*) Split the Hamiltonian into solvable parts. (*ii*) Construct a symmetric mapping of the parts. Voilà: a second-order symplectic map!

How You Integrate the Quadrupole Affects the Tune

Drift-Kick

$$H_Q = \underbrace{-\sqrt{1+\frac{2}{\beta_0}P_t+P_t^2-P_x^2-P_y^2}+\frac{1}{\beta_0}P_t}_{\text{drift}}+\underbrace{\frac{k_2}{2}(X^2-Y^2)}_{\text{kick}}$$

Matrix-Kick

$$= \underbrace{\frac{1}{2}(P_x^2+P_y^2)+\frac{k_2}{2}(X^2-Y^2)}_{\text{matrix}}\underbrace{-\sqrt{1+\frac{2}{\beta_0}P_t+P_t^2-P_x^2-P_y^2}+\frac{1}{\beta_0}P_t-\frac{1}{2}(P_x^2+P_y^2)}_{\text{“kick”}}$$

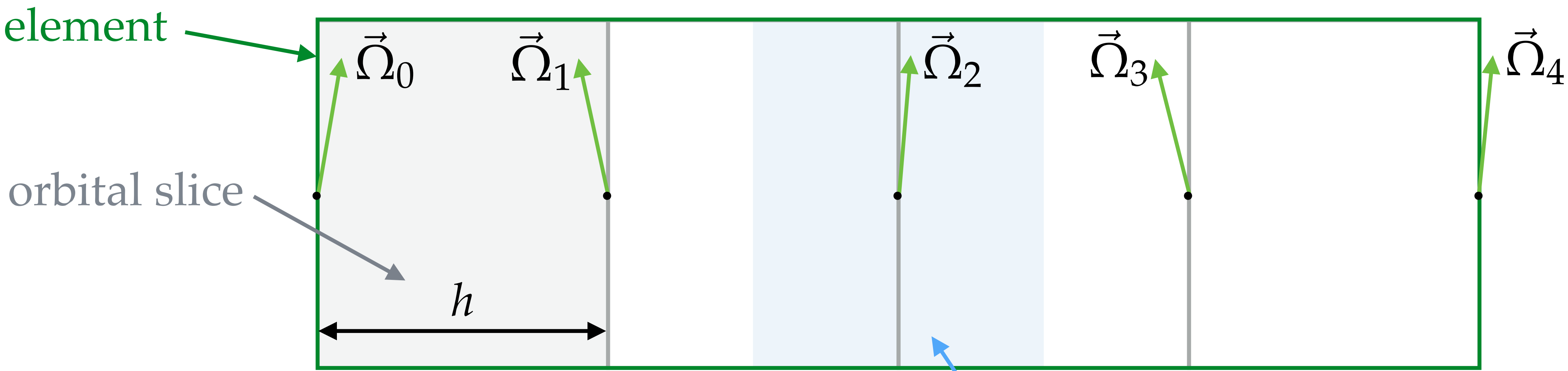
“Matrix”-Kick

$$= \underbrace{\frac{1}{2}\frac{P_x^2+P_y^2}{\sqrt{1+\frac{2}{\beta_0}P_t+P_t^2}}+\frac{k_2}{2}(X^2-Y^2)}_{\text{“matrix”}}\underbrace{-\sqrt{1+\frac{2}{\beta_0}P_t+P_t^2-P_x^2-P_y^2}+\frac{1}{\beta_0}P_t-\frac{1}{2}\frac{P_x^2+P_y^2}{\sqrt{1+\frac{2}{\beta_0}P_t+P_t^2}}}_{\text{“kick”}}$$

with $P_t = \dfrac{m\gamma c^2 - m\gamma_0 c^2}{p_0 c} = \dfrac{\gamma-\gamma_0}{\beta_0\gamma_0}$

This splitting does not require fitting the quad strengths.
Moreover, it yields correct tunes for off-energy particles.

Use Quaternions for Integrating the Spin Motion

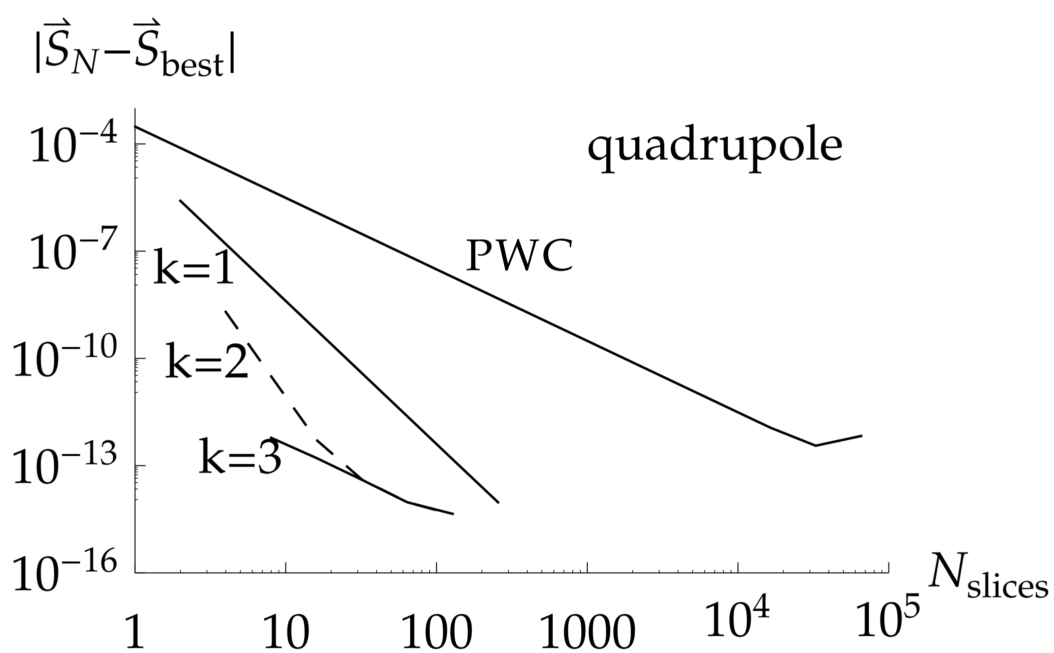


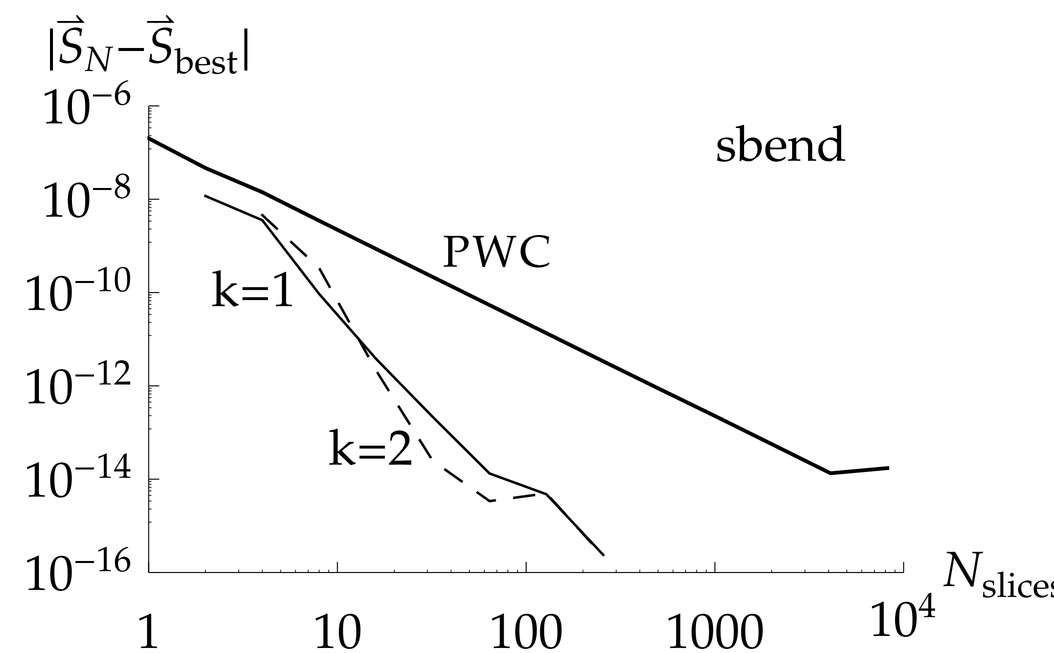
quaternion for crossing this element →

$$\mathcal{Q}(h) = q(\tfrac{1}{2}h\vec\Omega_4)\, q(h\vec\Omega_3)\, q(h\vec\Omega_2)\, q(h\vec\Omega_1)\, q(\tfrac{1}{2}h\vec\Omega_0)$$

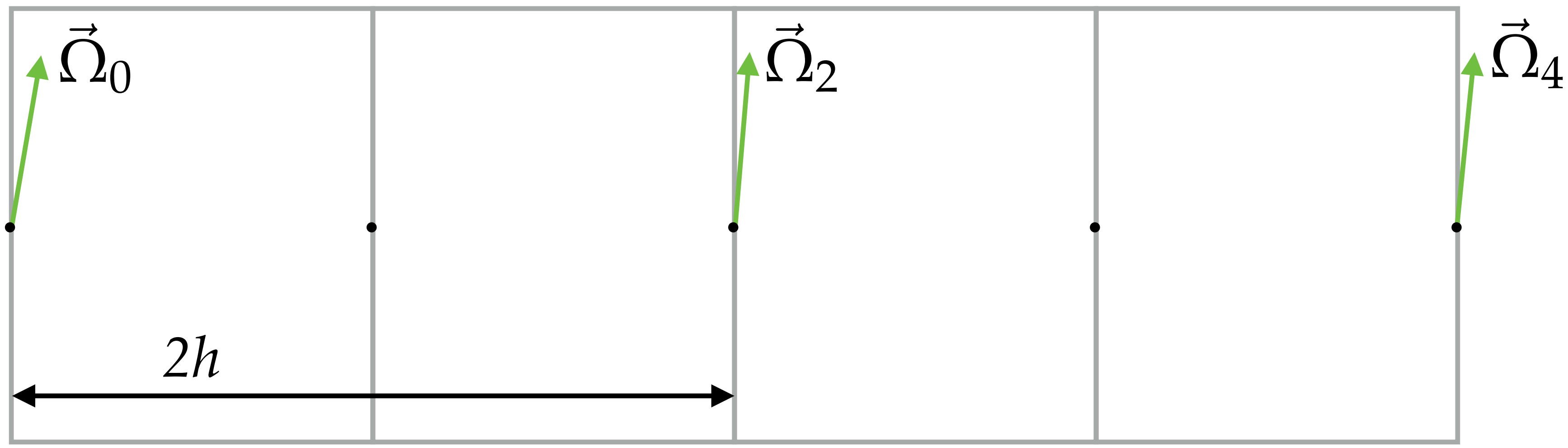
2nd-order convergence for the spin motion

Then use **Romberg quadratures** *to accelerate the spin convergence.*





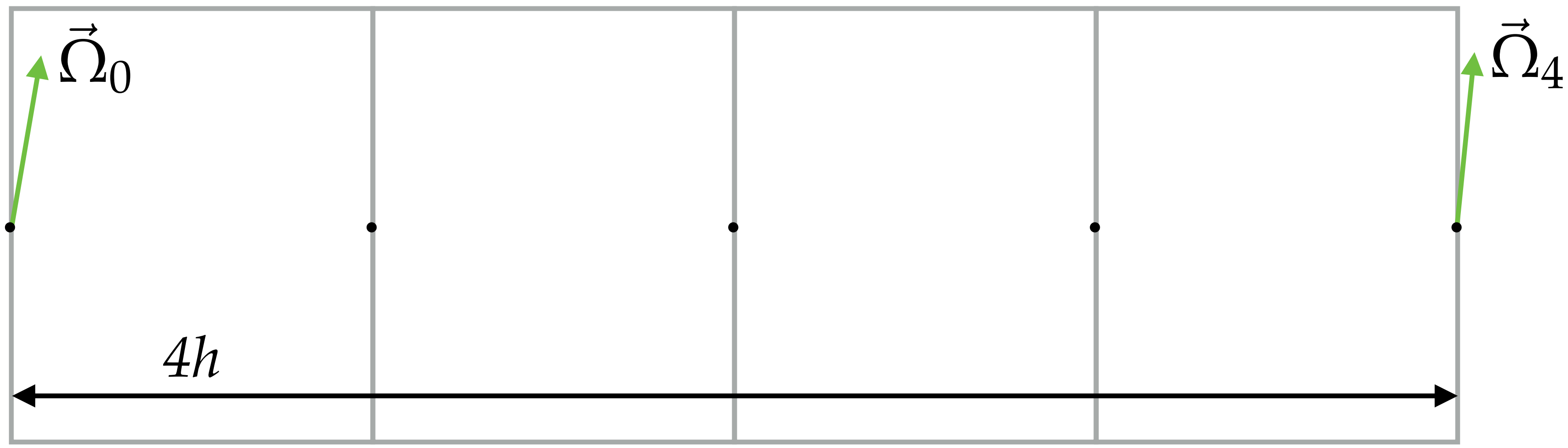
Use Quaternions for Integrating the Spin Motion (cont.)



$$\mathcal{Q}(h) = q(\tfrac{1}{2}h\vec{\Omega}_4)\, q(h\vec{\Omega}_3)\, q(h\vec{\Omega}_2)\, q(h\vec{\Omega}_1)\, q(\tfrac{1}{2}h\vec{\Omega}_0)$$

$$\mathcal{Q}(2h) = q(h\vec{\Omega}_4)\, q(2h\vec{\Omega}_2)\, q(h\vec{\Omega}_0)$$

Use Quaternions for Integrating the Spin Motion (cont.)

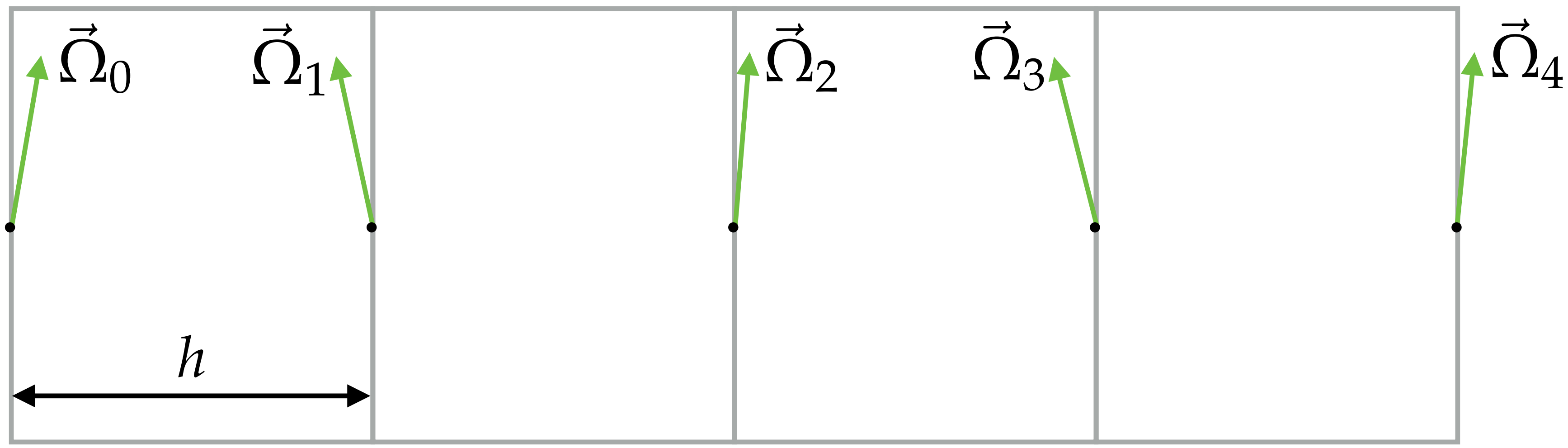


$$\mathcal{Q}(h) = q(\tfrac{1}{2}h\vec{\Omega}_4)\,q(h\vec{\Omega}_3)\,q(h\vec{\Omega}_2)\,q(h\vec{\Omega}_1)\,q(\tfrac{1}{2}h\vec{\Omega}_0)$$

$$\mathcal{Q}(2h) = q(h\vec{\Omega}_4)\,q(2h\vec{\Omega}_2)\,q(h\vec{\Omega}_0)$$

$$\mathcal{Q}(4h) = q(2h\vec{\Omega}_4)\,q(2h\vec{\Omega}_0)$$

Use Romberg Quadratures *to Accelerate the Convergence*



$$\mathcal{R}_{02} = \mathcal{Q}(h) = q(\tfrac{1}{2}h\vec{\Omega}_4)\, q(h\vec{\Omega}_3)\, q(h\vec{\Omega}_2)\, q(h\vec{\Omega}_1)\, q(\tfrac{1}{2}h\vec{\Omega}_0)$$

$$\mathcal{R}_{01} = \mathcal{Q}(2h) = q(h\vec{\Omega}_4)\, q(2h\vec{\Omega}_2)\, q(h\vec{\Omega}_0)$$

$$\mathcal{R}_{00} = \mathcal{Q}(4h) = q(2h\vec{\Omega}_4)\, q(2h\vec{\Omega}_0)$$

Romberg Tableau

$$\mathcal{R}_{00}$$

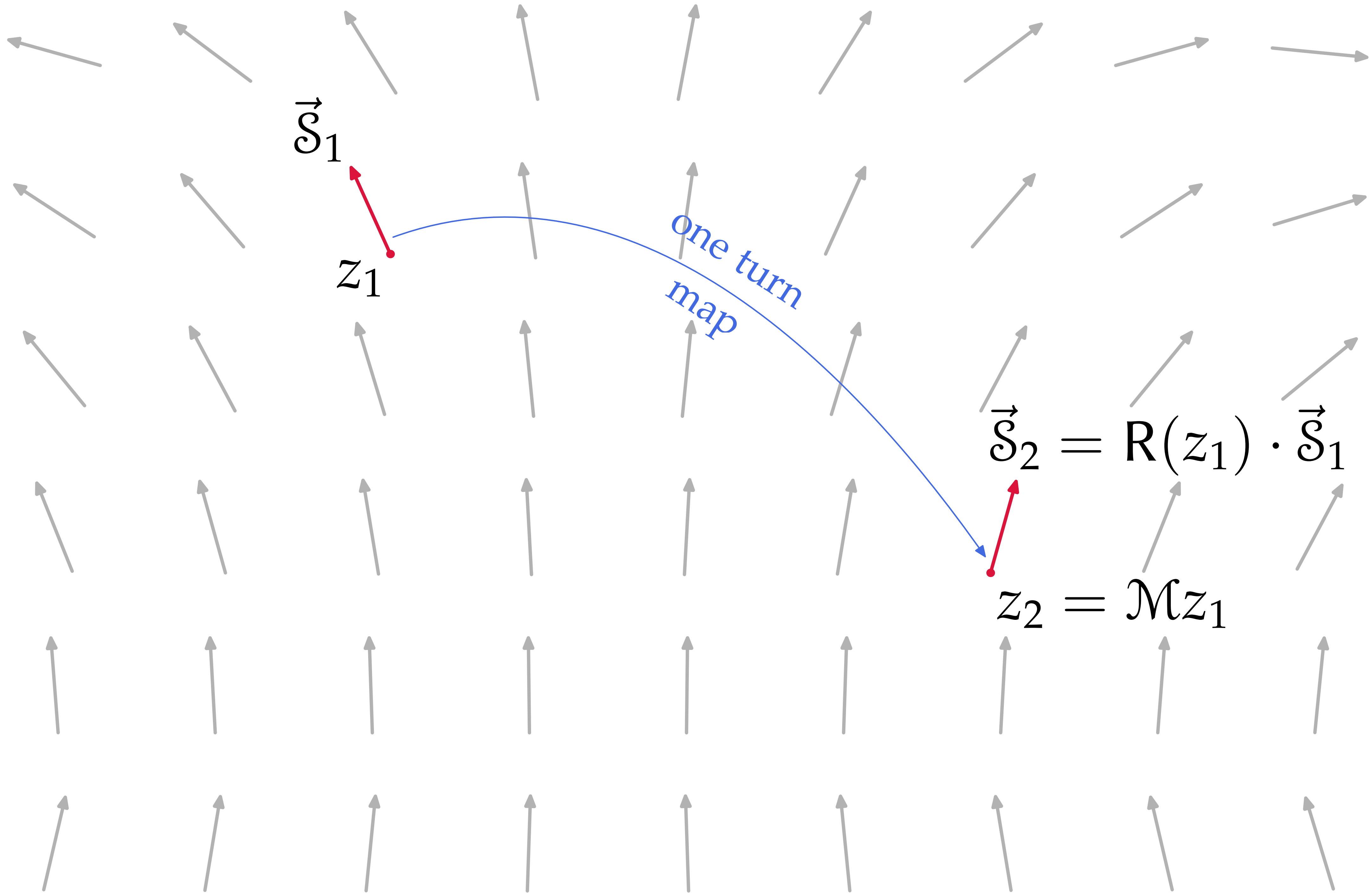
$$\mathcal{R}_{01} \qquad \mathcal{R}_{11} = \frac{4\mathcal{R}_{01} - \mathcal{R}_{00}}{4-1}$$

$$\mathcal{R}_{02} \qquad \mathcal{R}_{12} = \frac{4\mathcal{R}_{02} - \mathcal{R}_{01}}{4-1} \qquad \mathcal{R}_{22} = \frac{4^2\mathcal{R}_{12} - \mathcal{R}_{11}}{4^2-1}$$

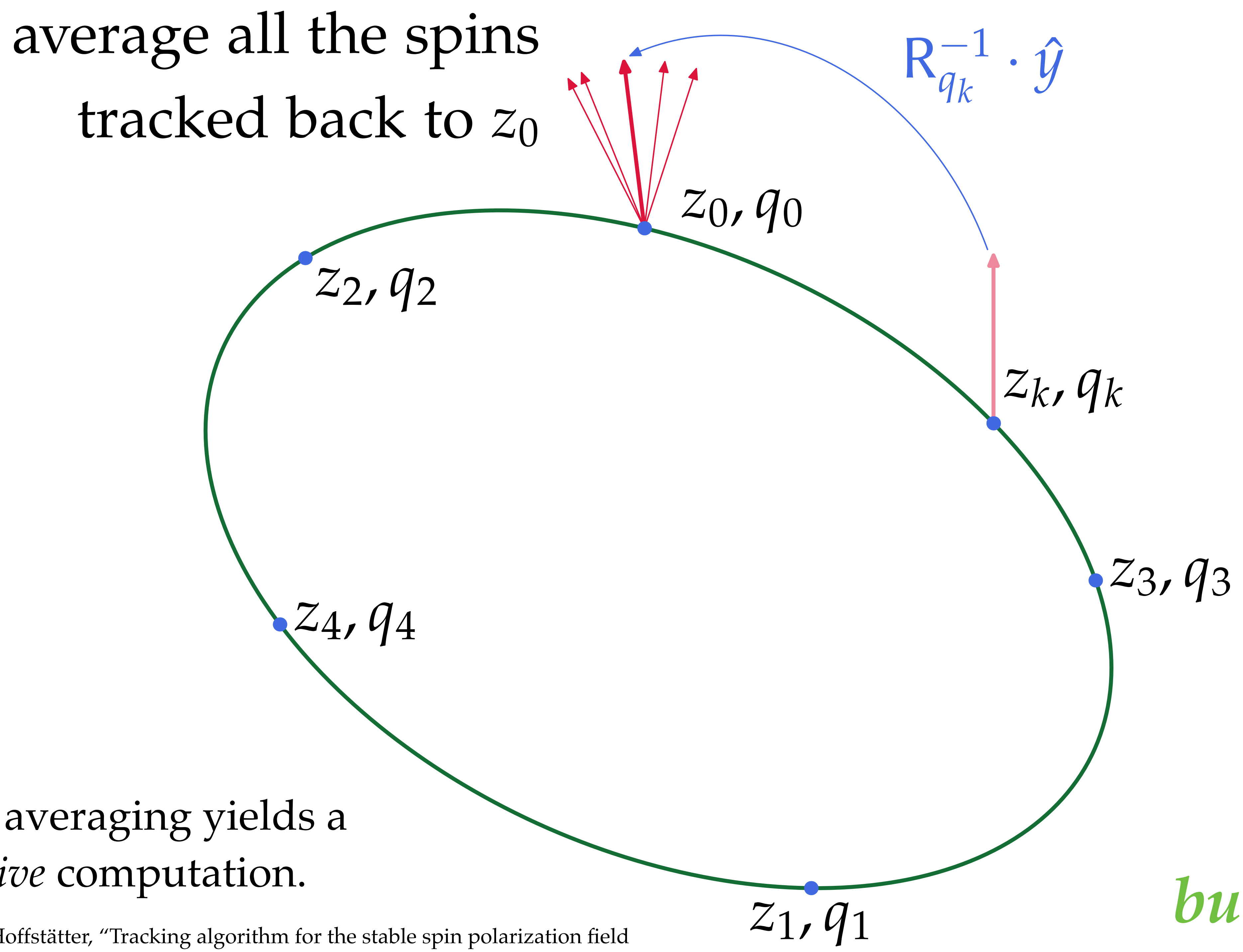
general term

$$\mathcal{R}_{j+1,k} = \frac{4^{j+1}\mathcal{R}_{jk} - \mathcal{R}_{j,k-1}}{4^{j+1}-1}$$

Invariant Spin Field Restricts Equilibrium Polarization



Use Stroboscopic (Ergodic) Averaging to Compute the ISF

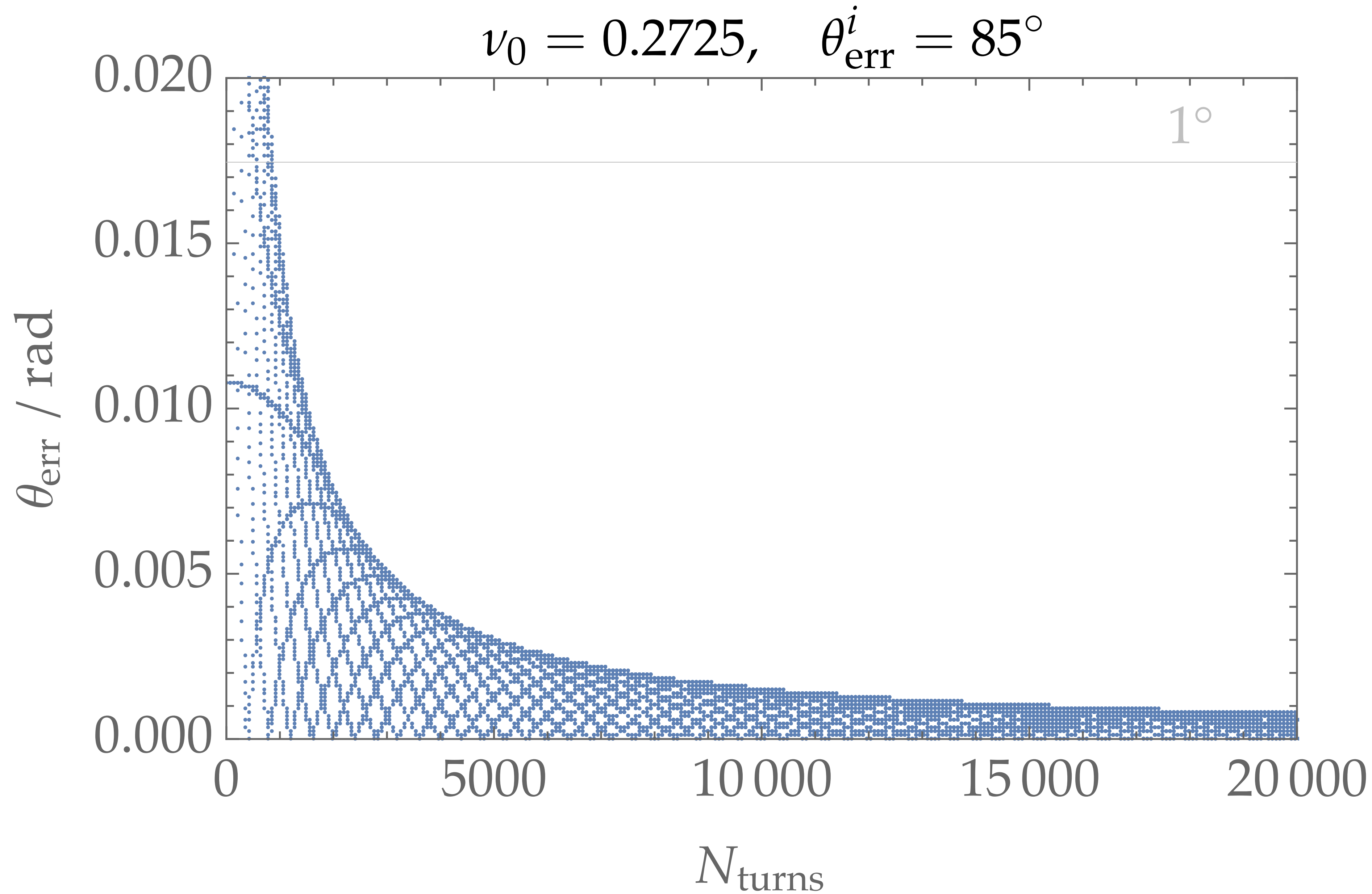


Stroboscopic averaging yields a *non-perturbative* computation.

but ...

K. Heinemann and G.H. Hoffstätter, "Tracking algorithm for the stable spin polarization field in storage rings using stroboscopic averaging". *Phys. Rev. E* 54(4):4240–4255, Oct. 1996.

Convergence can be Slow at Large Angles



It should be possible to speed this up dramatically by averaging over a small number of turns and then using that result as a starting point for further computation. See Dave Sagan's *BMad*.

Summary

The goal of spin tracking is insight.

The accuracy of orbital tracking affects the accuracy of spin tracking.

Avoid any scheme that requires you to refit the quad strengths.

Symplectic tracking is important even for electrons.

The invariant spin field tells us best we can achieve.
Look at the ISF first to understand where you want to end up.
Then investigate acceleration.

Thank you!

Many thanks also to

D.P. Barber, D. Meiser, V.H. Ranjbar, D.C. Sagan, and F. Méot.

Supported in part by the US Department of Energy, Office of Science, Office of Nuclear Physics, including Award No. DE-SC0017181.