BeamBeam3D Simulations with Crab Cavities

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Accelerator Technology and Applied Physics

JLEIC Collaboration Meeting, April 5, 2017

ICFA Beam Dynamics Workshop on "Beam-Beam Effects in Circular Colliders"

September 27 – 29, 2017, Berkeley, CA

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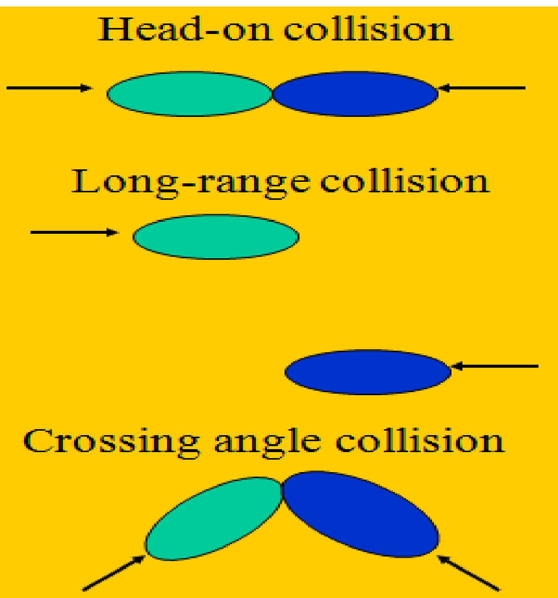
https://indico.physics.lbl.gov/indico/event/431/

Berkeley Lab Accelerator Simulation Toolkit

BeamBeam3D: A Parallel Colliding Beam Simulation Code

http://blast.lbl.gov



Some key features of the BeamBeam3D

- Multiple-slice model for finite bunch length
- New algorithm -- shifted Green function -- efficiently models long-range collisions
- Parallel particle-field based decomposition to achieve perfect load balance
- Lorentz boost to handle crossing angle
- Arbitrary closed-orbit separation
- Multiple bunches, multiple collision points
- Linear transfer matrix + one turn chromaticity
- Conducting wire, crab cavity, e-lens compensation model
- Feedback model
- Impedance model

Efficient Green's Function Method to the Poisson Equation for Beam-Beam Force Calculation (1)

$$\phi(r) = \int G(r, r')\rho(r')dr'$$

$$\phi(r_i) = h\sum_{i'=1}^{N} G(r_i - r_{i'})\rho(r_{i'})$$

$$G(x, y) = -\frac{1}{2}\log(x^2 + y^2)$$

Direct summation of the convolution scales as N^4 !!!!

N – grid number in each dimension

Efficient Green's Function Method to the Poisson Equation for Beam-Beam Force Calculation (2)

Hockney's Algorithm:- scales as $(2N)^2\log(2N)$

- Ref: Hockney and Easwood, ***Computer Simulation using Particles***, McGraw-Hill Book Company, New York, 1985.

$$\phi_c(r_i) = h\sum_{i'=1}^{2N} G_c(r_i - r_{i'})\rho_c(r_{i'})$$

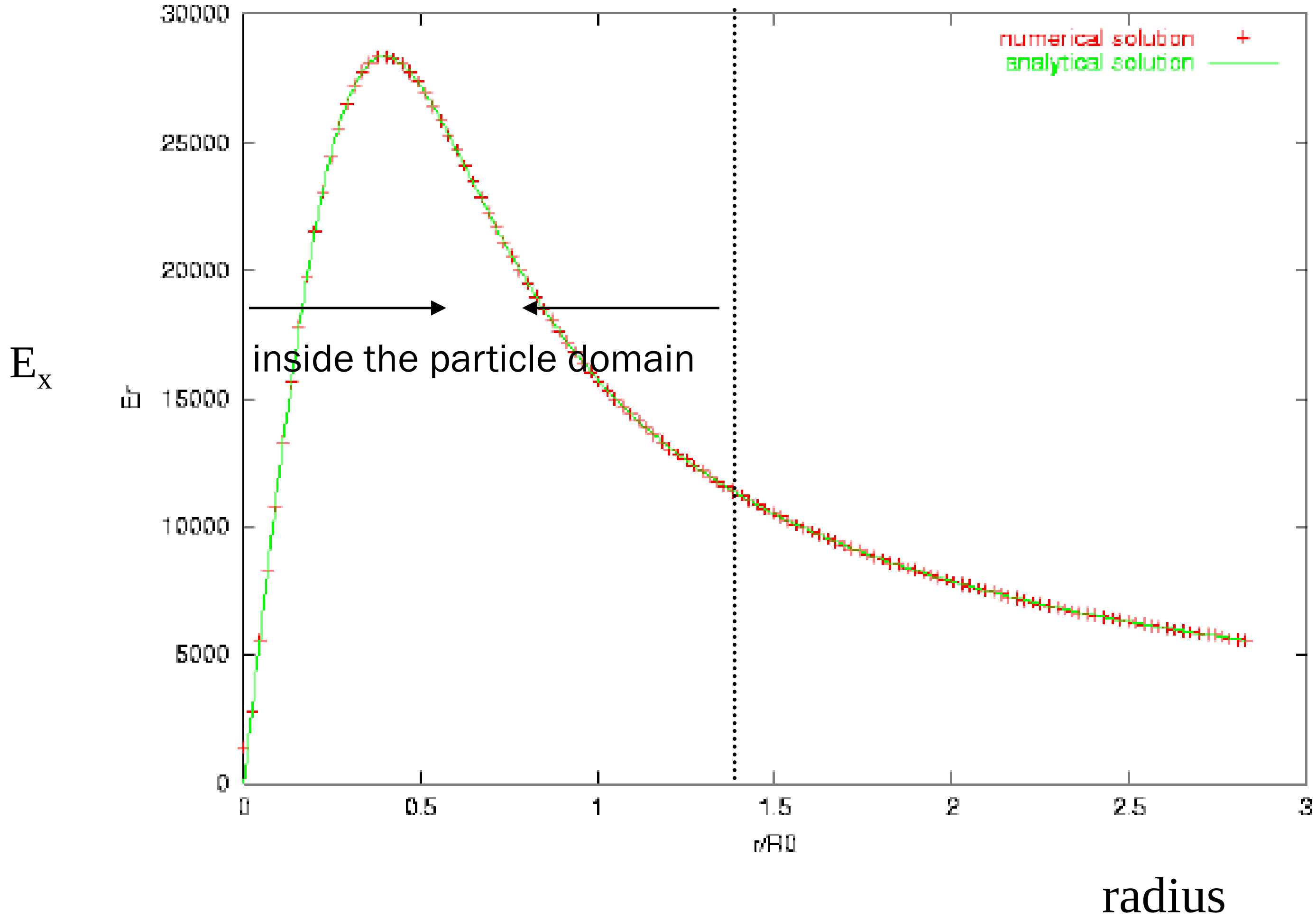
$$\phi(r_i) = \phi_c(r_i) \quad \text{for i} = 1, \text{N}$$

Shifted Green function Algorithm:

$$\phi_F(r) = \int G_s(r,r')\rho(r')dr'$$

$$G_s(r,r') = G(r + r_s, r')$$

Good Agreement between the Numerical Solution from the Shifted Green Function and the Analytical Solution

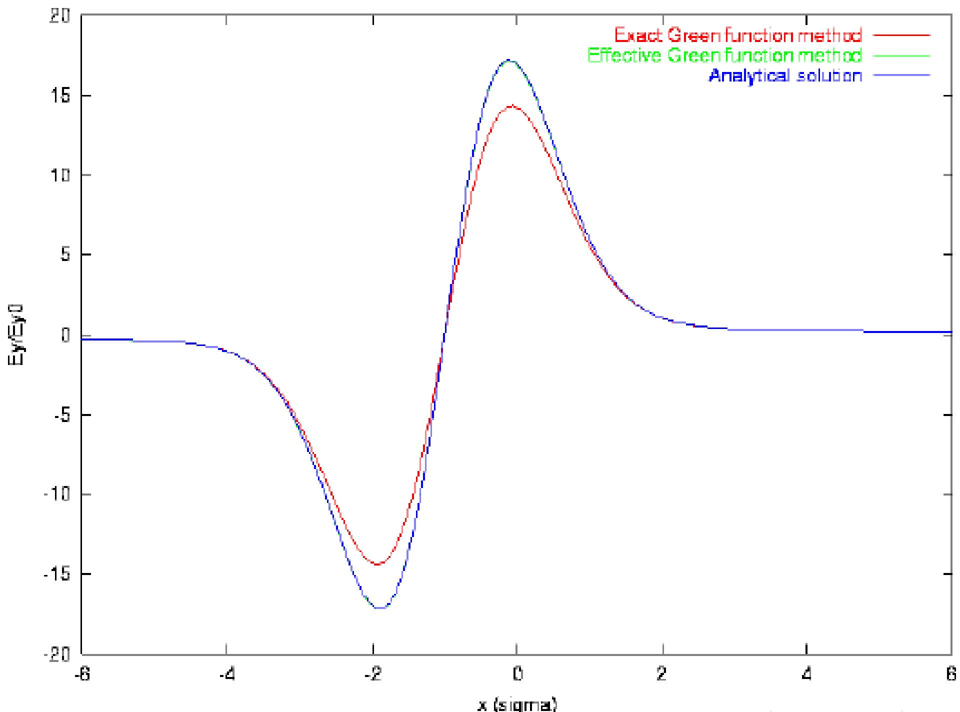


Efficient Green's Function Method to the Poisson Equation (3)
(Integrated Green function Algorithm for large aspect ratio)

$$\phi_c(r_i) = \sum_{i'=1}^{2N} G_i(r_i - r_{i'})\rho_c(r_{i'})$$

$$G_i(r, r') = \oint G_s(r, r')\, dr'$$

E_y



x (sigma)

A Fully Symplectic Space-Charge Model with Gridless Spectral Method

multi-particle Hamiltonian $H(\mathbf{r}_1, \mathbf{r}_2, \cdots, \mathbf{p}_1, \mathbf{p}_2, \cdots, s)$

$$H = \sum_i \mathbf{p}_i^2/2 + \frac{1}{2}\sum_i \sum_j q\phi(\mathbf{r}_i, \mathbf{r}_j) + \sum_i q\psi(\mathbf{r}_i)$$

space-charge Coulomb potential

external focusing/acceleration

$$\frac{d\mathbf{r}_i}{ds} = \frac{\partial H}{\partial \mathbf{p}_i}$$

$$\frac{d\mathbf{p}_i}{ds} = -\frac{\partial H}{\partial \mathbf{r}_i}$$

$$\frac{d\zeta}{ds} = -[H, \zeta]$$

A formal single step solution

$$\zeta(\tau) = \exp(-\tau(:H:))\zeta(0)$$

H = H₁+H₂

$$\zeta(\tau) = \exp(-\tau(:H_1: + :H_2:))\zeta(0)$$
$$= \exp(-\frac{1}{2}\tau :H_1:)\exp(-\tau :H_2:)\exp(-\frac{1}{2}\tau :H_1:)\zeta(0) + O(\tau^3)$$

$$\zeta(\tau) = \mathcal{M}(\tau)\zeta(0)$$
$$= \mathcal{M}_1(\tau/2)\mathcal{M}_2(\tau)\mathcal{M}_1(\tau/2)\zeta(0)$$

Symplectic Space-Charge Model Avoids Numerical Grid Heating

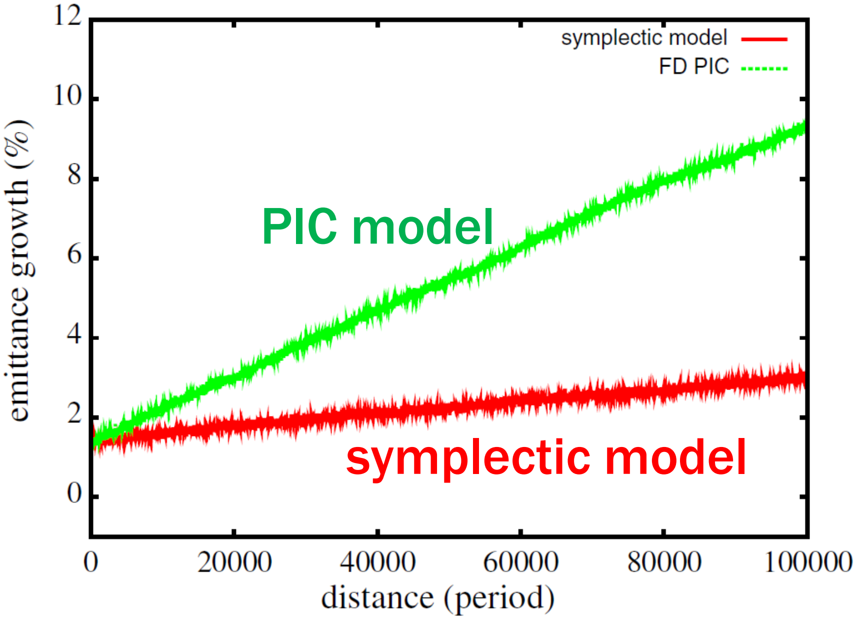
$$H_2 = \frac{1}{2\epsilon_0}\frac{4}{ab} w \sum_i \sum_j \sum_l \sum_m \frac{1}{\gamma_{lm}^2} \sin(\alpha_l x_j) \sin(\beta_m y_j) \sin(\alpha_l x_i) \sin(\beta_m y_i)$$

$\mathcal{M}_2$

$$p_{xi}(\tau) = p_{xi}(0) - \tau \frac{1}{\epsilon_0}\frac{4}{ab} w \sum_j \sum_l \sum_m \frac{\alpha_l}{\gamma_{lm}^2} \sin(\alpha_l x_j) \sin(\beta_m y_j) \cos(\alpha_l x_i) \sin(\beta_m y_i)$$

$$p_{yi}(\tau) = p_{yi}(0) - \tau \frac{1}{\epsilon_0}\frac{4}{ab} w \sum_j \sum_l \sum_m \frac{\beta_m}{\gamma_{lm}^2} \sin(\alpha_l x_j) \sin(\beta_m y_j) \sin(\alpha_l x_i) \cos(\beta_m y_i)$$

transverse emittance evolution

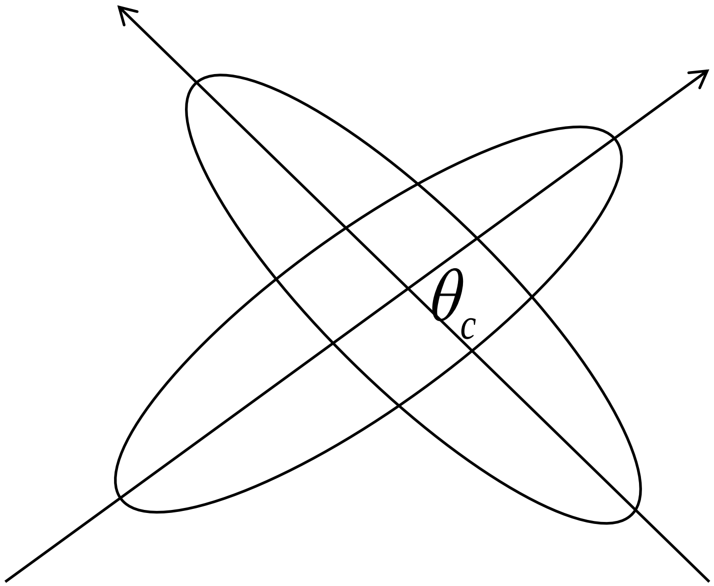


J. Qiang, "A Symplectic Multi-Particle Tracking Model for Self-Consistent Space-Charge Simulation," Phys. Rev. ST Accel. Beams 20, 014203 (2017).

Electron-Ion Collider Needs High Luminosity but Crossing Angle Collision Degrades Luminosity

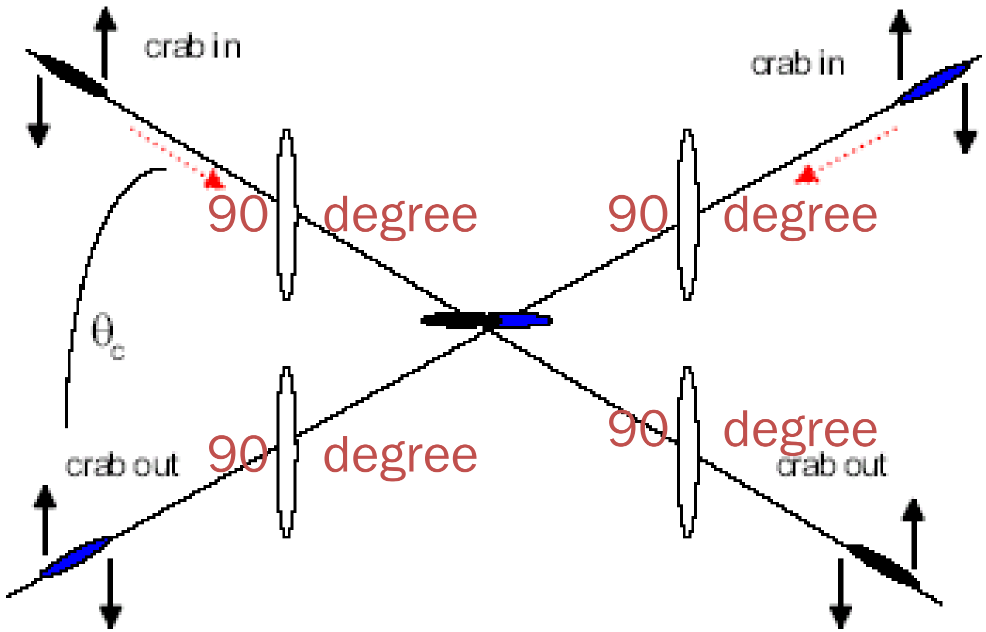
$$L_0 \approx f_b \left(\frac{4\pi\gamma_p\gamma_e}{r_p r_e} \right) (\xi_p \xi_e)$$

$$\xi_p = \frac{r_p \beta^*_p}{4\pi\gamma_p} \frac{N_e}{\sigma_e^2} \qquad \xi_e = \frac{r_e \beta^*_e}{4\pi\gamma_e} \frac{N_p}{\sigma_p^2}$$



$$L = L_0 \frac{1}{\sqrt{1+\Theta^2}}; \quad \Theta \equiv \frac{\tan(\theta_c/2)\sigma_z}{\sigma_x}$$

Crab Cavity Recovers the Geometric Luminosity Loss



$$RF\ voltage:$$

$$V = \frac{cE_s \tan(\frac{\theta_c}{2})}{\omega\sqrt{\beta_{x,crab}\beta_x^*}}$$

Thin Lens Approximation for Crab Cavity Deflection

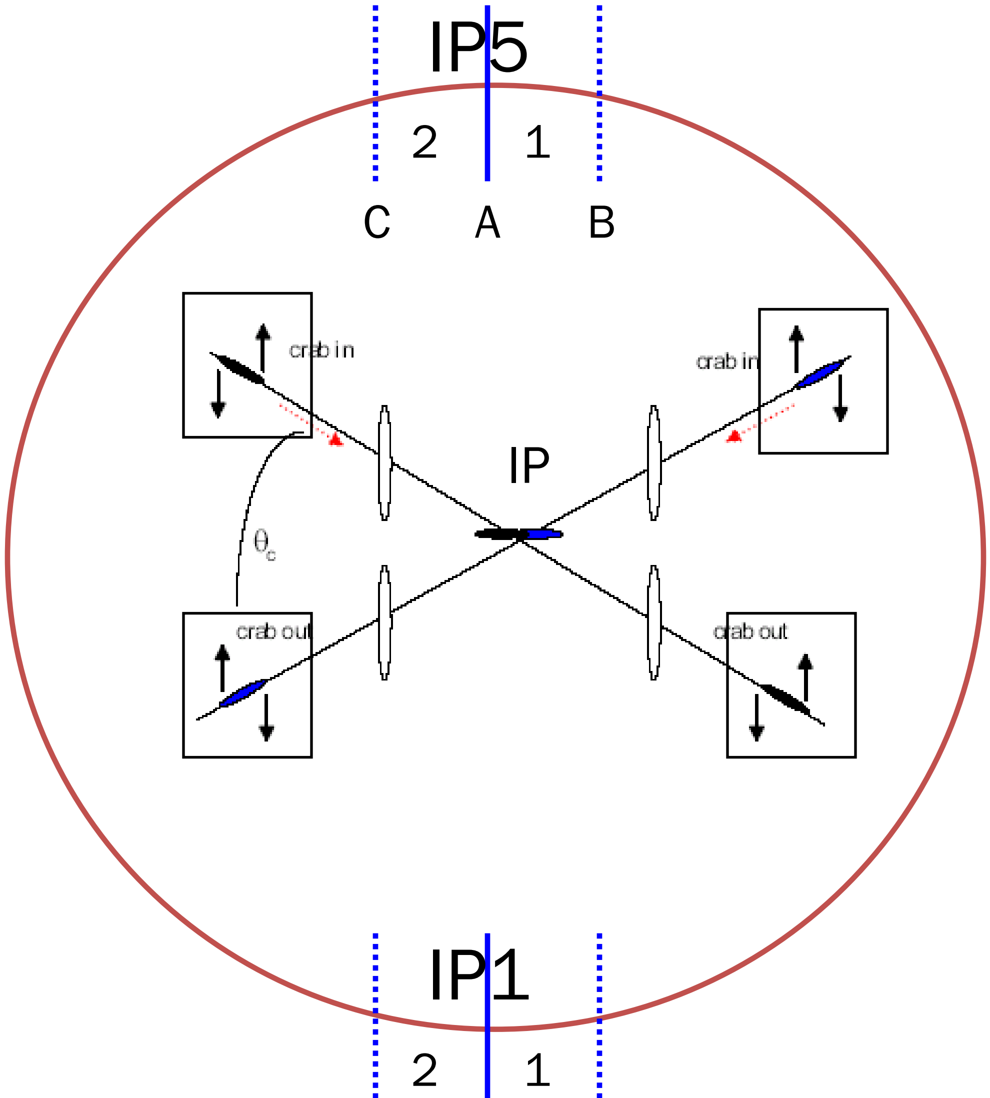
$$x^{n+1} = x^n$$

$$Px^{n+1} = Px^n + \frac{qV}{E_s}\sin(\omega z^n / c)$$

$$z^{n+1} = z^n$$

$$\delta E^{n+1} = \delta E^n + \frac{qV}{E_s}\cos(\omega z^n / c)x^n$$

An Application Example of LHC Upgrade Using Crab Cavities



crabcavity RF voltage :

$$V = \frac{cE_s \tan(\frac{\theta_c}{2})}{\omega\sqrt{\beta_{x,crab}\beta_x^*}}$$

One Turn Transfer Map with Beam-Beam and Crab Cavity

$$M = Mb\ M1\ Mc_1\ M1^{-1}\ M\ M2^{-1}\ Mc_2\ M2$$

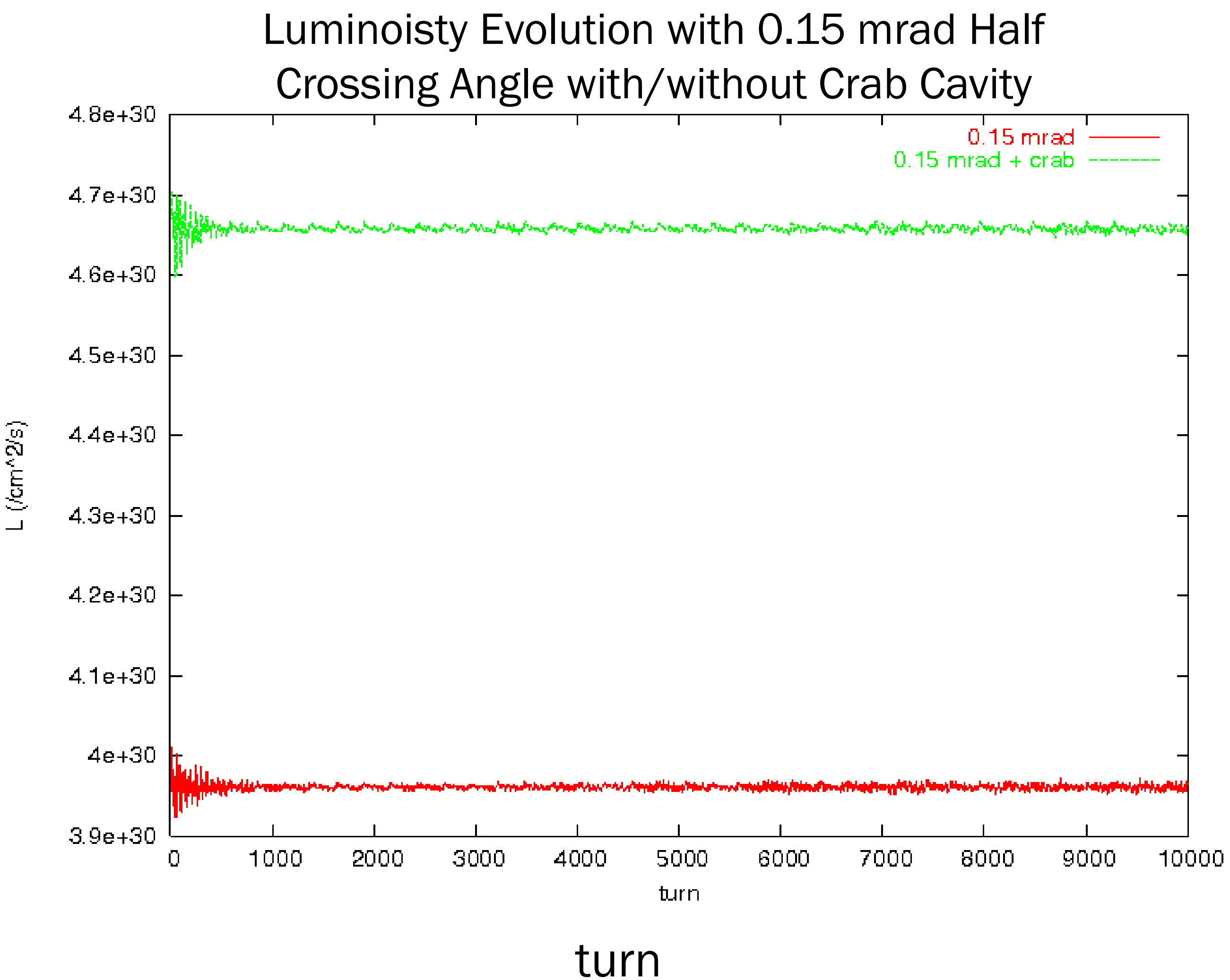
Mb: transfer map from head-on crossing angle beam-beam collision

$Mc_{1,2}$: transfer maps from crab cavity deflection

M1-2: transfer maps between crab cavity and collision point

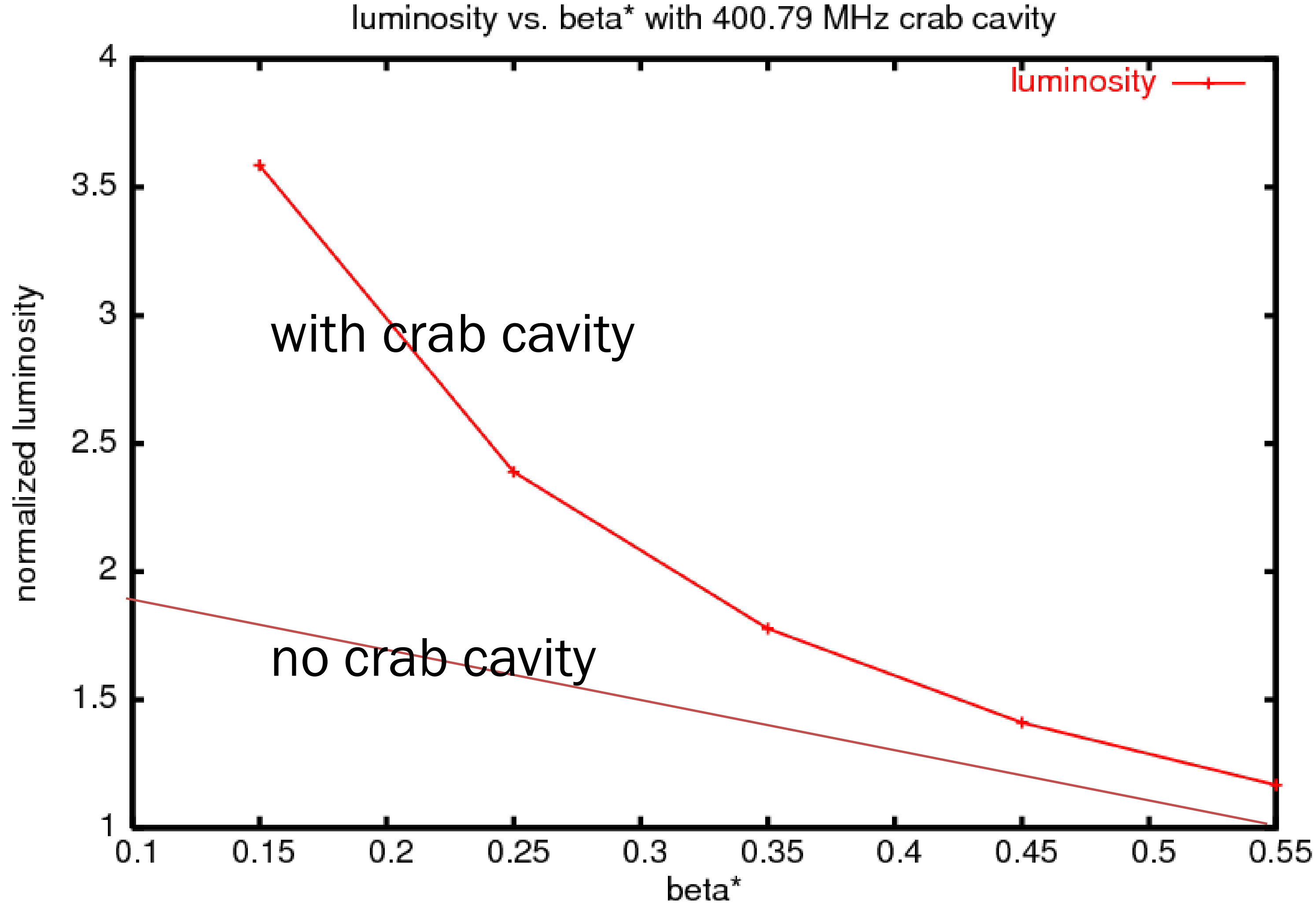
M: one turn transfer map of machine

Crab Cavity Helps Improve Luminosity (1)



Crab Cavity Helps Improve Luminosity (2)

Luminosity vs. Beta* for LHC Crab Cavity Compensation



RF Noise in the Crab Cavity Causes Emittance Growth and Luminosity Degradation

$$x_i \propto V_{cc} \sin(kz_i + \delta\varphi)$$

0th order error (phase error):

$$\delta X = -\frac{c}{\omega_{cc}} \tan\left(\frac{\theta}{2}\right) \delta\varphi$$

1st order error (voltage error):

$$\delta x_i \propto \delta V_{cc} \sin(kz_i) \approx \delta V_{cc} k z_i$$

white noise offset collision drives emittance growth

$$\frac{\delta\varepsilon}{\varepsilon} \approx \frac{K}{\left(1 + \frac{G}{2\pi|\xi|}\right)^2} \frac{\delta x^2}{\sigma_x^2}$$

$$\frac{\Delta L}{L} = 10.8\left(\xi \frac{\Delta x}{\sigma}\right)^2$$

G. Stupakov, SSC-560 (1991).
T. Sen and J. Ellison, PRL 77, 1051 (1996)
Y. Alexahin, NIM A391,73 (1996)
K. Ohmi, in Proc. Beam-Beam 2013 workshop.

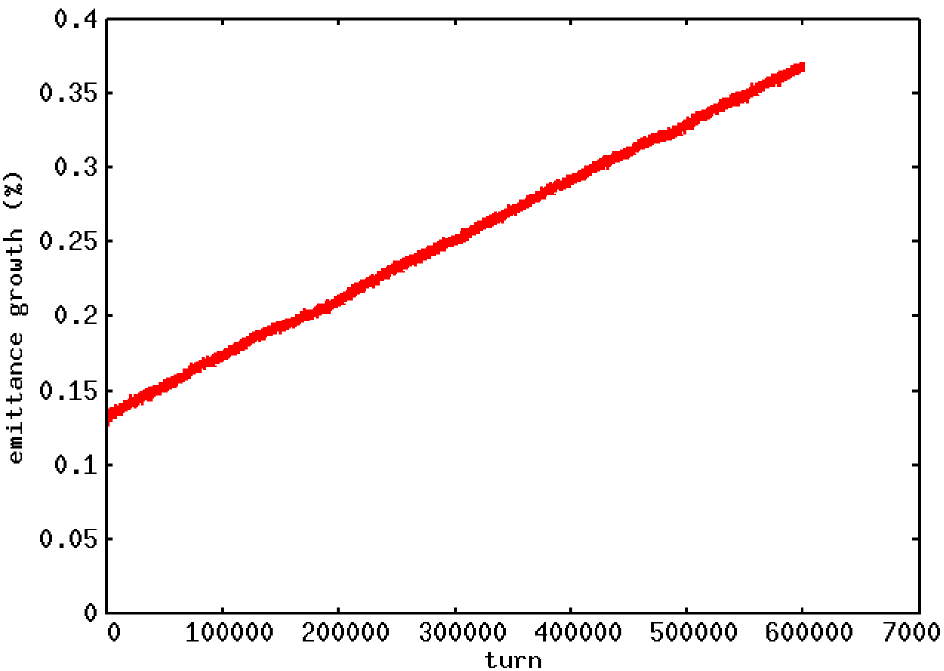
Some Physical Parameters Used in the Simulations

Physical parameters	
ε (norm.)	2.5 um
pick-up gain	0.05/0.05
Tunes Chromaticity	62.31/60.32 0 – 4
*β** Θ	15-60 cm 0.59 mrad
ξ_tot *N* IPs	0.011 - 0.022 1.1 - $2.2\text{x}10^{11}$ 2

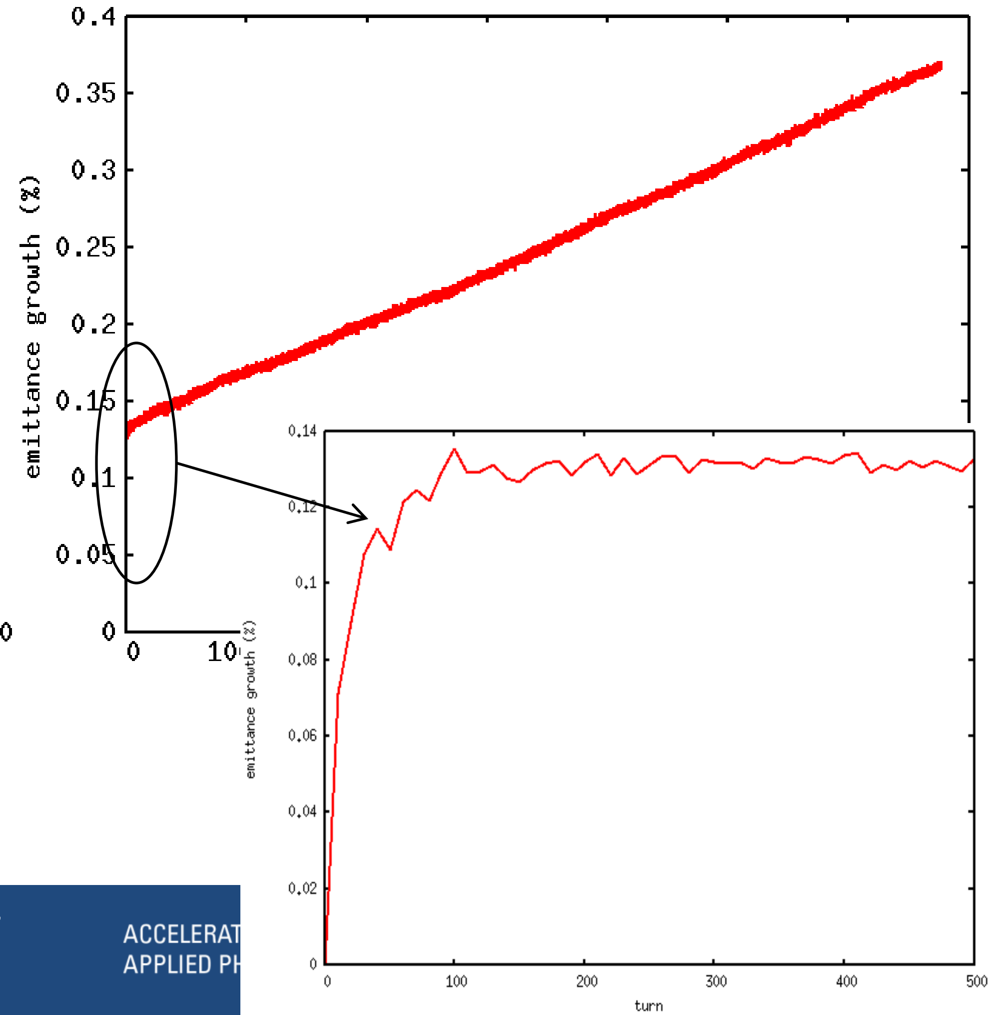
Emittance Blow-up due to Phase or Voltage White Noise

Np = 2.2 x 10^{11}, beta* = 0.49 m

emittance blow-up from phase error 8e-5



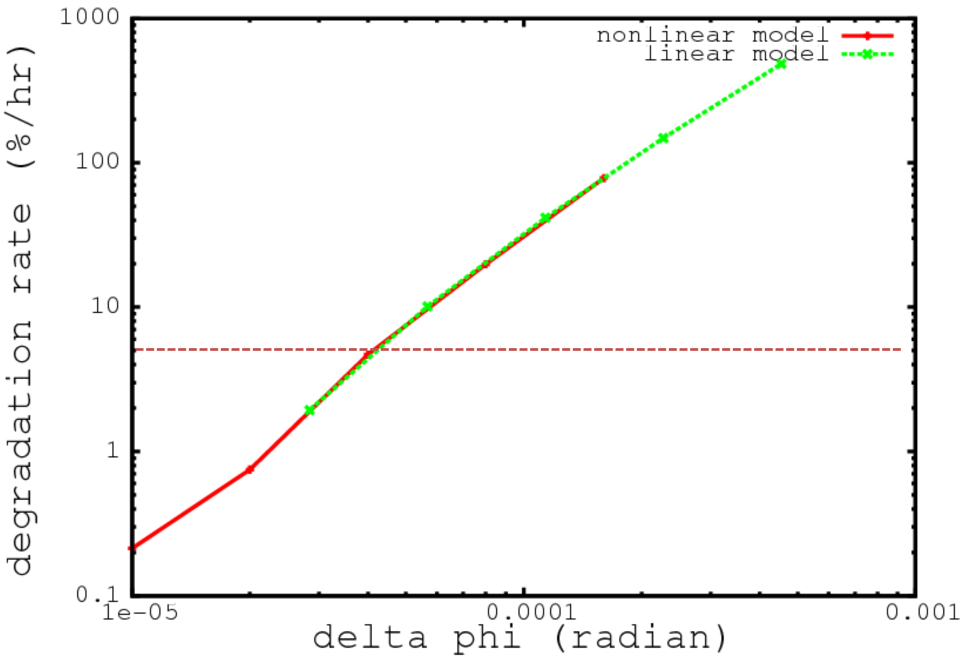
emittance blow-up from voltage error 5e-5.



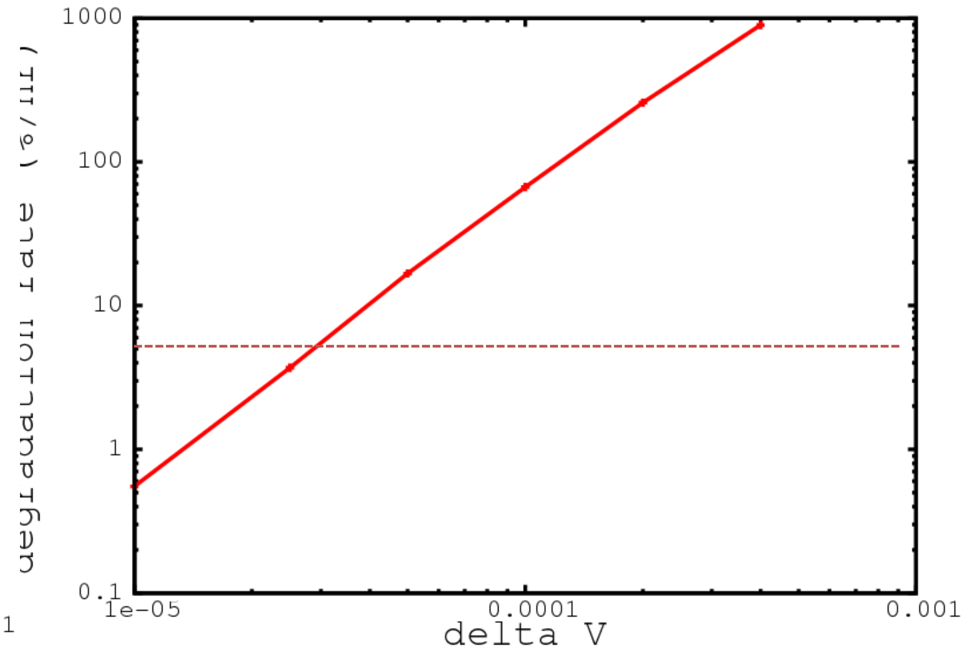
Luminosity Degradation due to Phase or Voltage White Noise

crab cavity white noise tolerance level ~10^{-5}

Lum. degradation rate vs. phase noise amp.



Lum. degradation rate vs. voltage noise amp

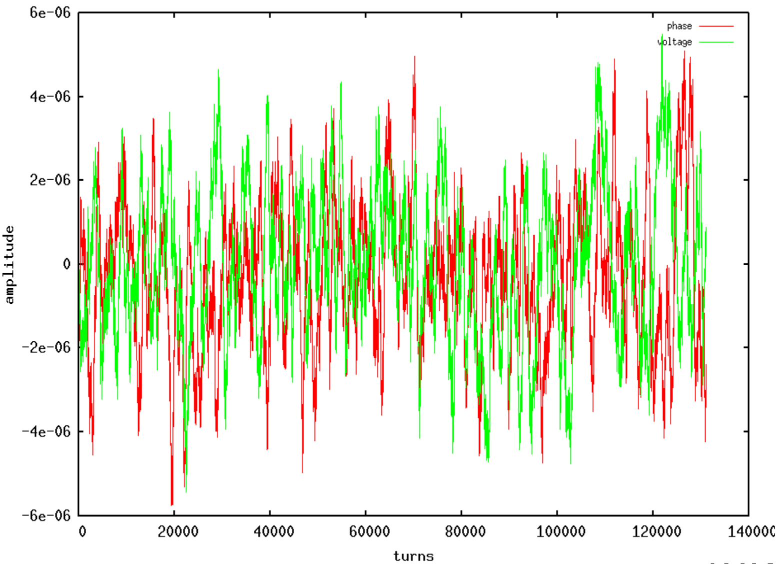


In order to have a good luminosity lifetime ~ 20 hours,
the white noise amplitude needs to be kept below the level of a few 10^{-5}.

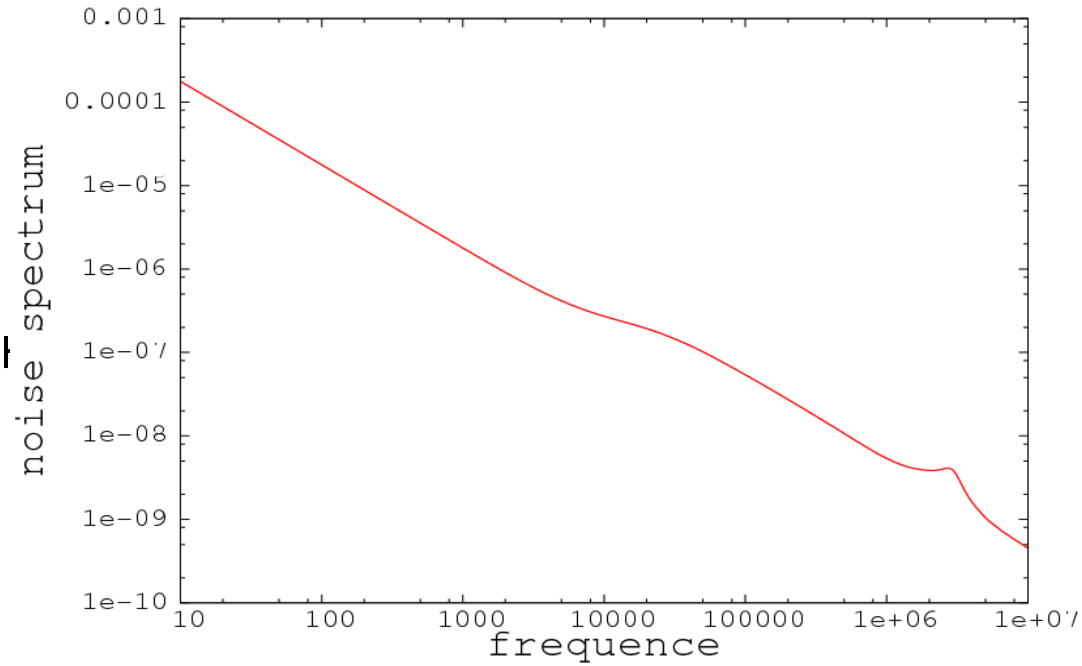
J. Qiang et al., in Proc. IPAC2015.

In Reality, the Noise in Crab Cavity Is not White Noise, but with Frequency Dependence

Phase and Amplitude Noise



Frequency-Dependent Crab Cavity Noise Power Spectrum



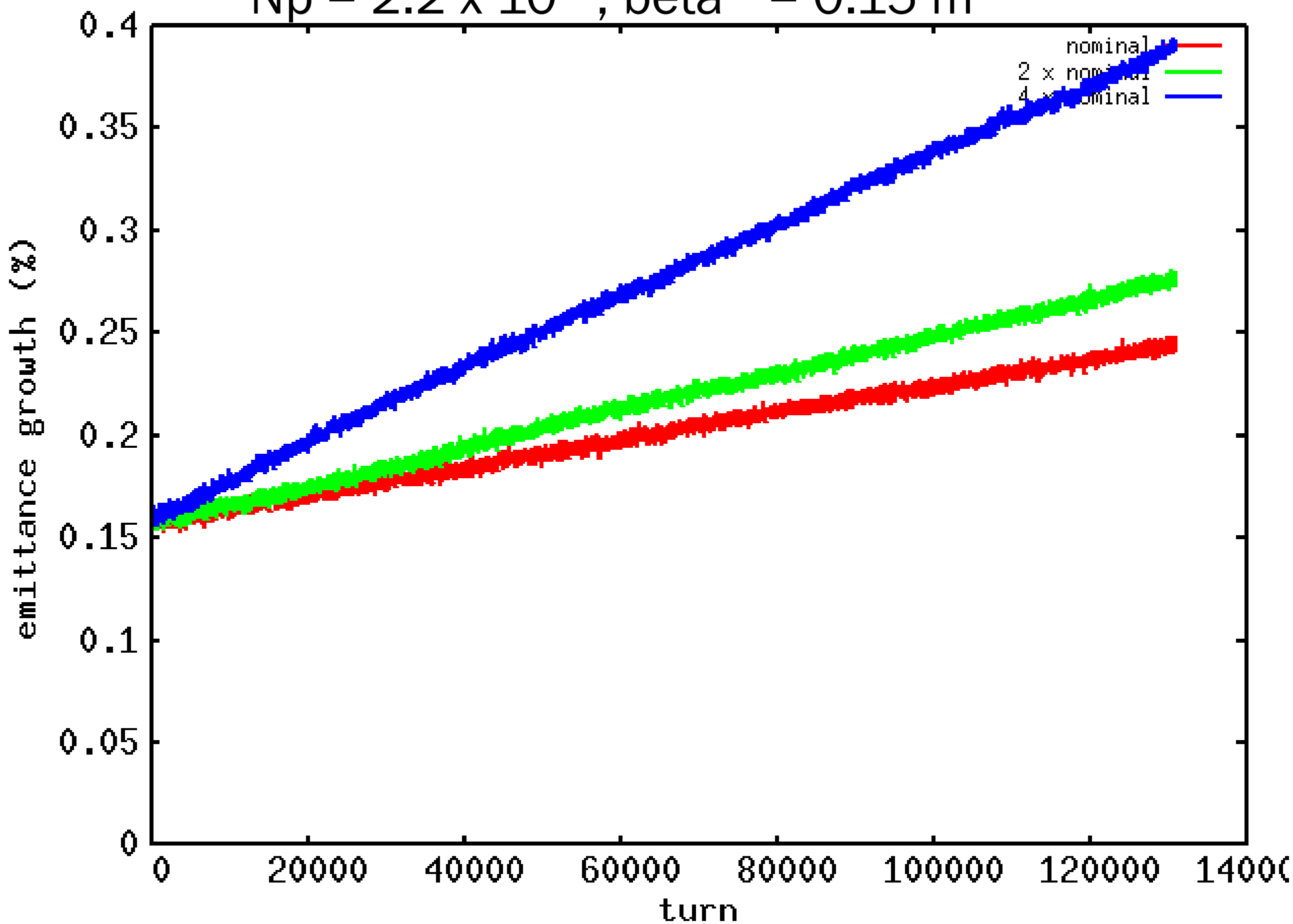
(dBc/H

frequency (Hz)

Frequency Dependent Noise also Causes Emittance Growth

RMS Emittance Evolution with Different Noise Amplitudes

Np = 2.2 x 10^{11}, beta* = 0.15 m

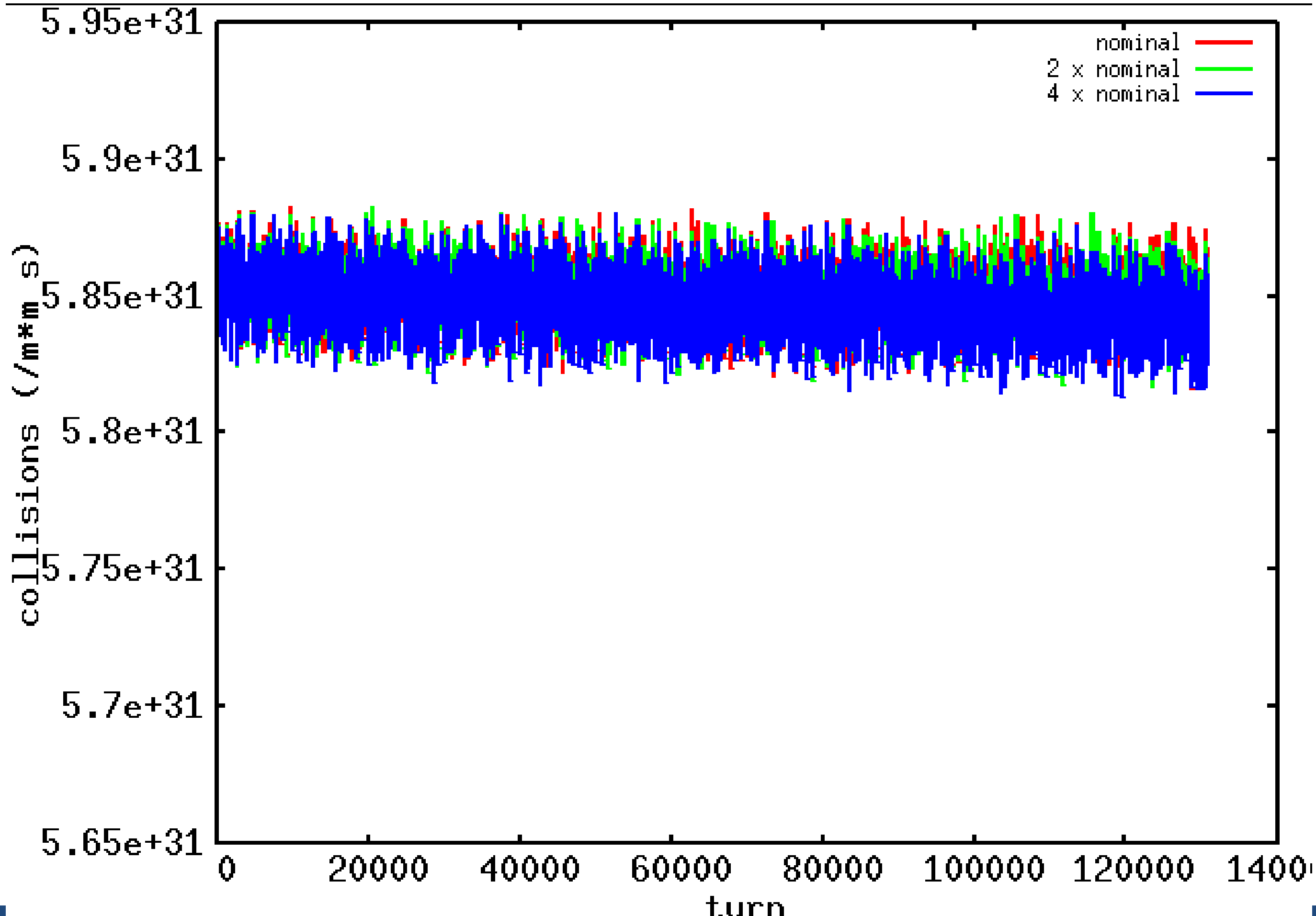


nominal noise amplitude = 3 x 10^{-4}

Frequency Dependent Noise also Causes Luminosity Degradation

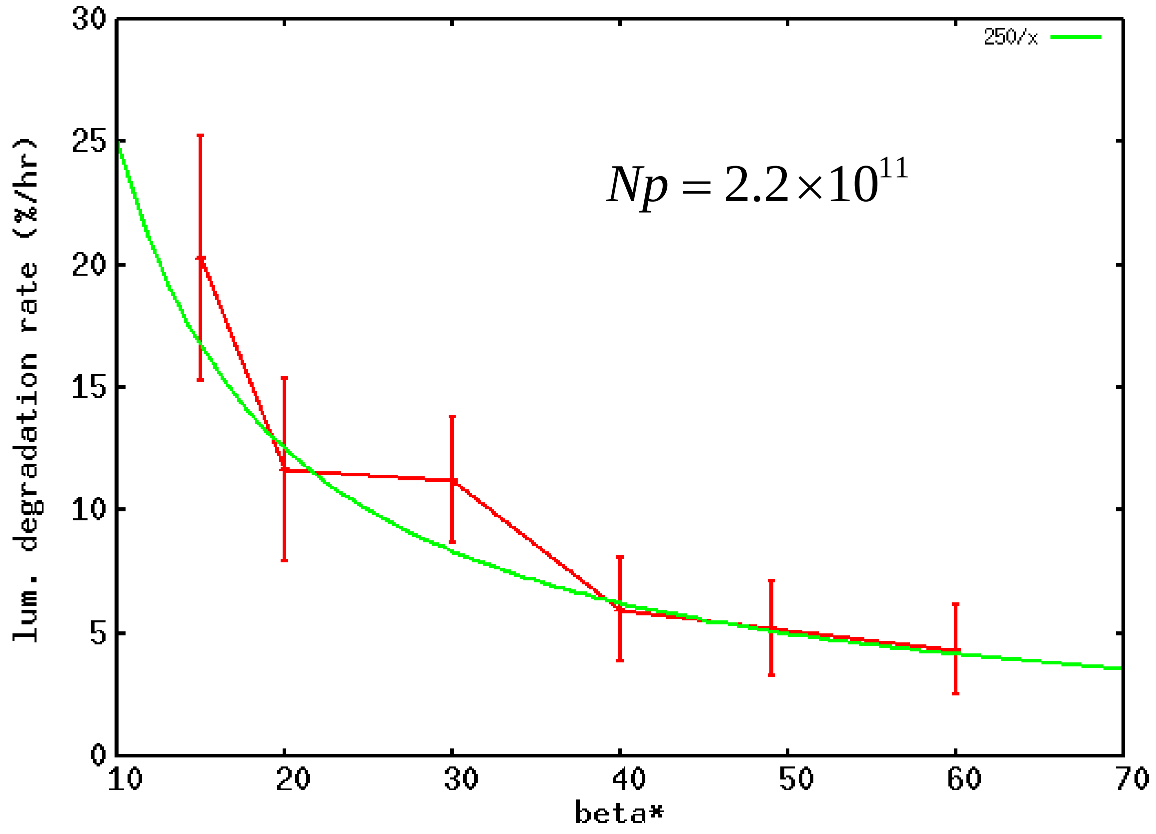
Peak Luminosity Evolution with Different Noise Amplitudes

Np = 2.2 x 10^{11}, beta* = 0.15 m



Noise Induced Luminosity Degradation Shows Strong Dependence on the Beta Function at IP

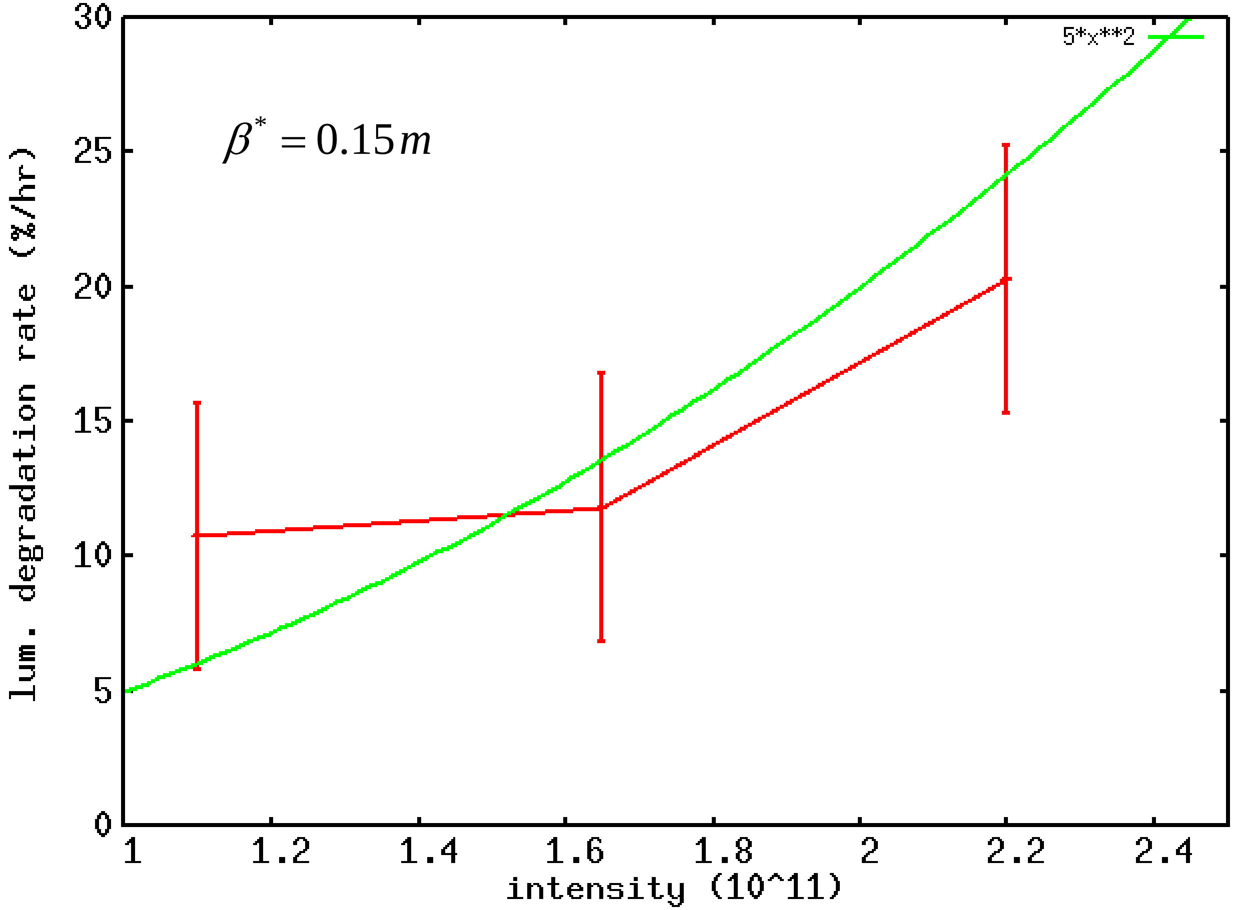
CC Noise Induced Lumi. Degradation with vs. beta*
(with nominal noise amplitude)



strong dependence on the beta function at IP

Noise Induced Luminosity Degradation Shows Weaker Dependence on the Bean Intensity

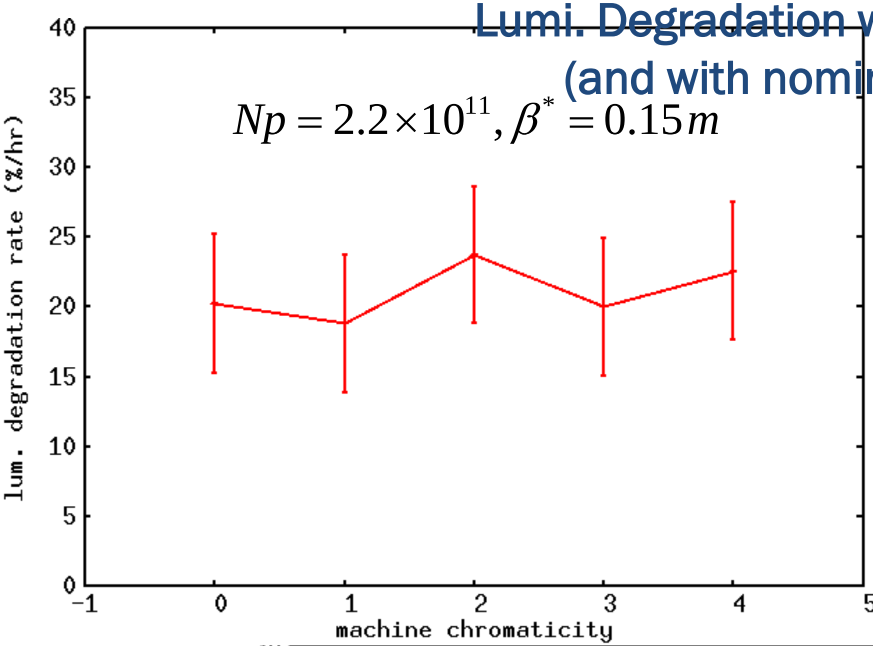
CC Noise Induced Lumi. Degradation with vs. Intensity
(with nominal noise amplitude)

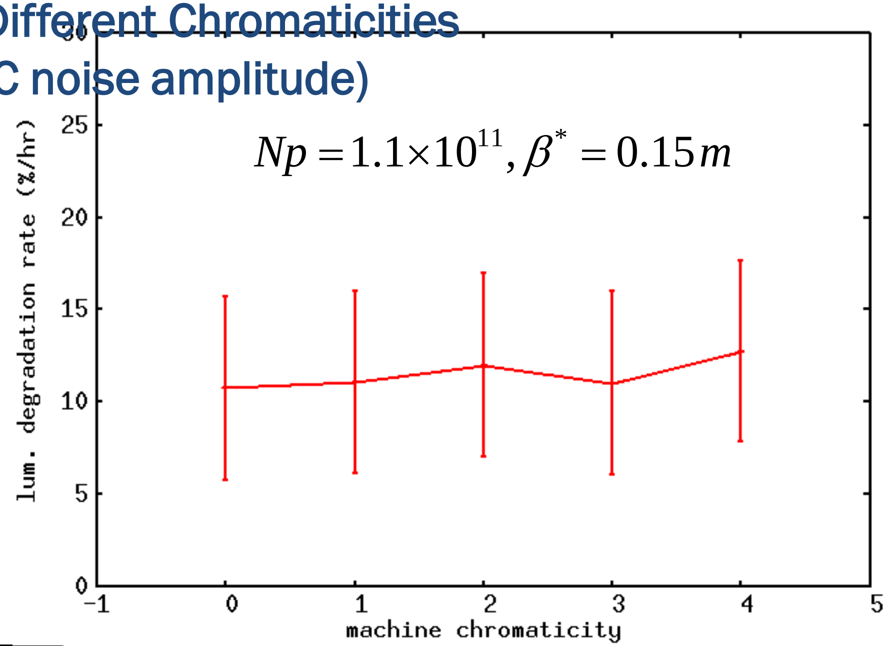


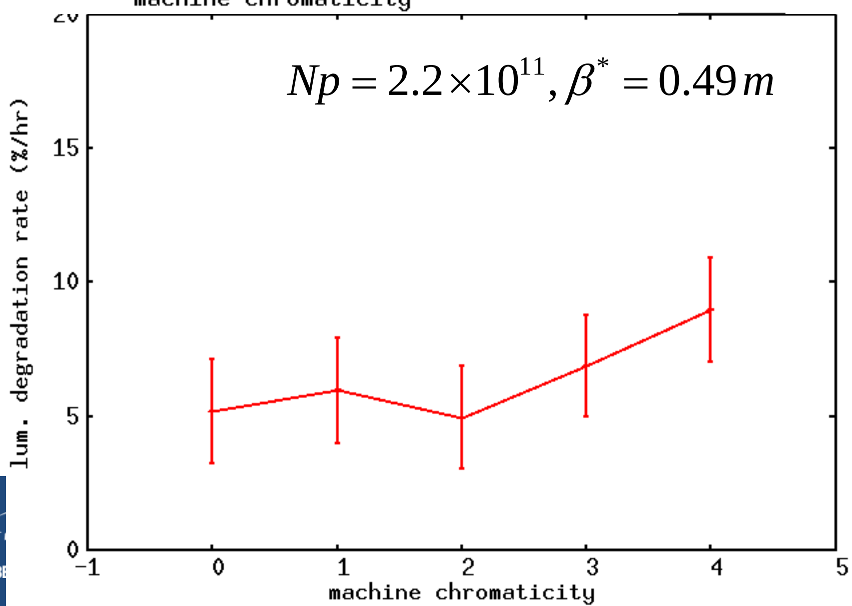
weaker dependence on the beam intensity

Noise Induced Luminosity Degradation Not Sensitive to Machine Chromaticity

Lumi. Degradation with Different Chromaticities (and with nominal CC noise amplitude)







not quite sensitive to machine linear chromaticity

Crab Cavity also Has RF Multipole Errors

Normal Quadrupole

$$\Delta x' = -b_2 x \cos\left(\frac{\omega z}{c} + \phi_s + \phi_{\text{RF,quad}}\right)$$
$$\Delta y' = b_2 y \cos\left(\frac{\omega z}{c} + \phi_s + \phi_{\text{RF,quad}}\right)$$
$$\Delta \delta = \frac{b_2}{2}\left(x^2 - y^2\right) \sin\left(\frac{\omega z}{c} + \phi_s + \phi_{\text{RF,quad}}\right)\frac{\omega}{c}$$

Normal Sextupole

$$\Delta x' = -b_3\left(x^2 - y^2\right) \cos\left(\frac{\omega z}{c} + \phi_s + \phi_{\text{RF,sext}}\right)$$
$$\Delta y' = 2 b_3 x y \cos\left(\frac{\omega z}{c} + \phi_s + \phi_{\text{RF,sext}}\right)$$
$$\Delta \delta = \frac{b_3}{3}\left(x^3 - 3 x y^2\right) \sin\left(\frac{\omega z}{c} + \phi_s + \phi_{\text{RF,sext}}\right)\frac{\omega}{c}$$

Normal Octupole

$$\Delta x' = -b_4\left(x^3 - 3 x y^2\right) \cos\left(\frac{\omega z}{c} + \phi_s + \phi_{\text{RF,oct}}\right)$$
$$\Delta y' = b_4\left(3x^2 y - y^3\right) \cos\left(\frac{\omega z}{c} + \phi_s + \phi_{\text{RF,oct}}\right)$$
$$\Delta \delta = \frac{b_4}{4}\left(x^4 - 6x^2 y^2 + y^4\right) \sin\left(\frac{\omega z}{c} + \phi_s + \phi_{\text{RF,oct}}\right)\frac{\omega}{c}$$

Skew Quadrupole

$$\Delta x' = -b_2 y \cos\left(\frac{\omega z}{c} + \phi_s + \phi_{\text{RF,quad}}\right)$$
$$\Delta y' = -b_2 x \cos\left(\frac{\omega z}{c} + \phi_s + \phi_{\text{RF,quad}}\right)$$
$$\Delta \delta = b_2 x y \sin\left(\frac{\omega z}{c} + \phi_s + \phi_{\text{RF,quad}}\right)\frac{\omega}{c}$$

Skew Sextupole

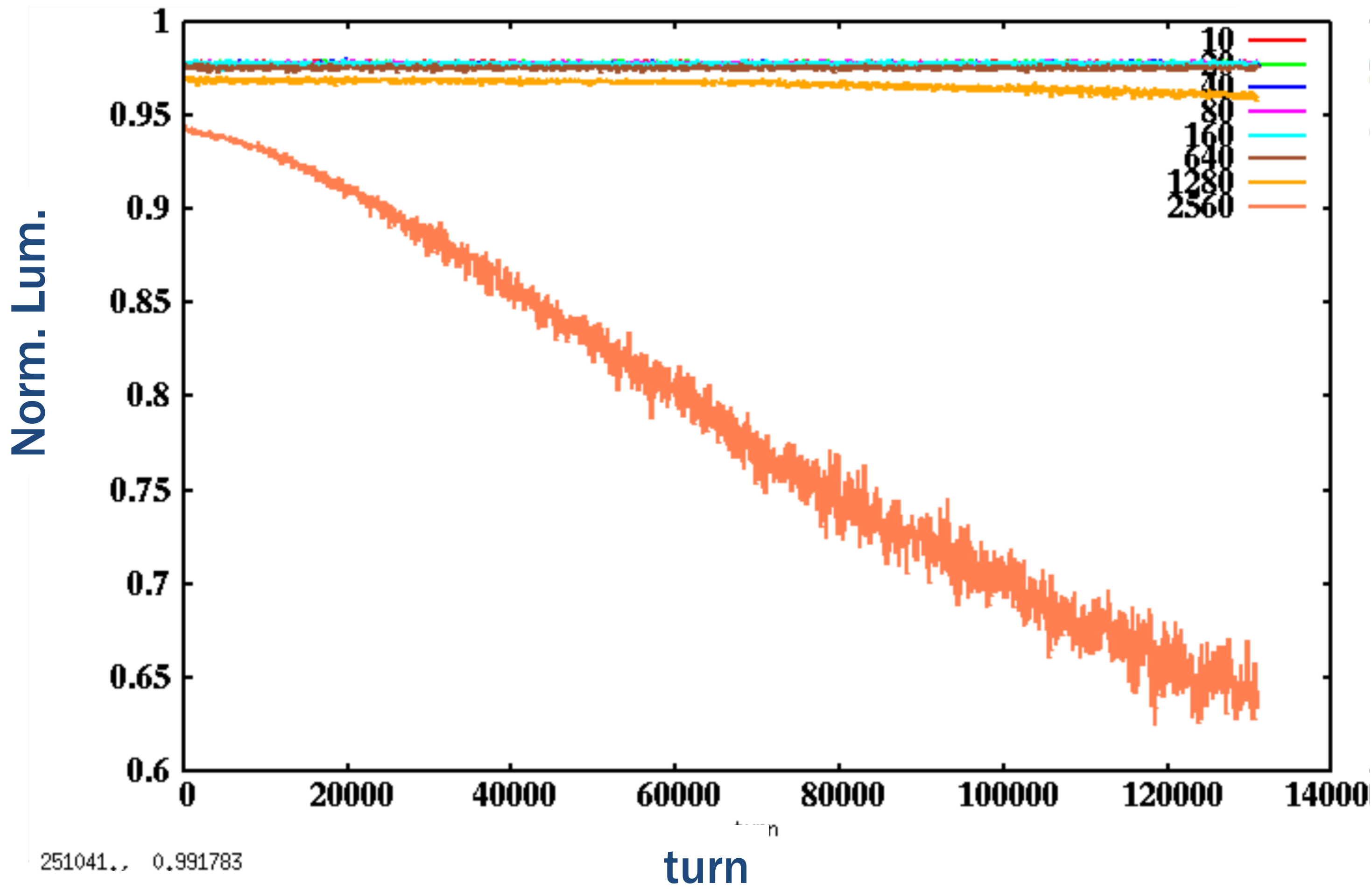
$$\Delta x' = -2 b_3 x y \cos\left(\frac{\omega z}{c} + \phi_s + \phi_{\text{RF,sext}}\right)$$
$$\Delta y' = b_3\left(y^2 - x^2\right) \cos\left(\frac{\omega z}{c} + \phi_s + \phi_{\text{RF,sext}}\right)$$
$$\Delta \delta = -\frac{b_3}{3}\left(y^3 - 3 y x^2\right) \sin\left(\frac{\omega z}{c} + \phi_s + \phi_{\text{RF,sext}}\right)\frac{\omega}{c}$$

Skew Octupole

$$\Delta x' = -b_4\left(y^3 + 3x^2 y\right) \cos\left(\frac{\omega z}{c} + \phi_s + \phi_{\text{RF,oct}}\right)$$
$$\Delta y' = -b_4\left(3y^2 x - x^3\right) \cos\left(\frac{\omega z}{c} + \phi_s + \phi_{\text{RF,oct}}\right)$$
$$\Delta \delta = b_4\left(x^3 y - y^3 x\right) \sin\left(\frac{\omega z}{c} + \phi_s + \phi_{\text{RF,oct}}\right)\frac{\omega}{c}$$

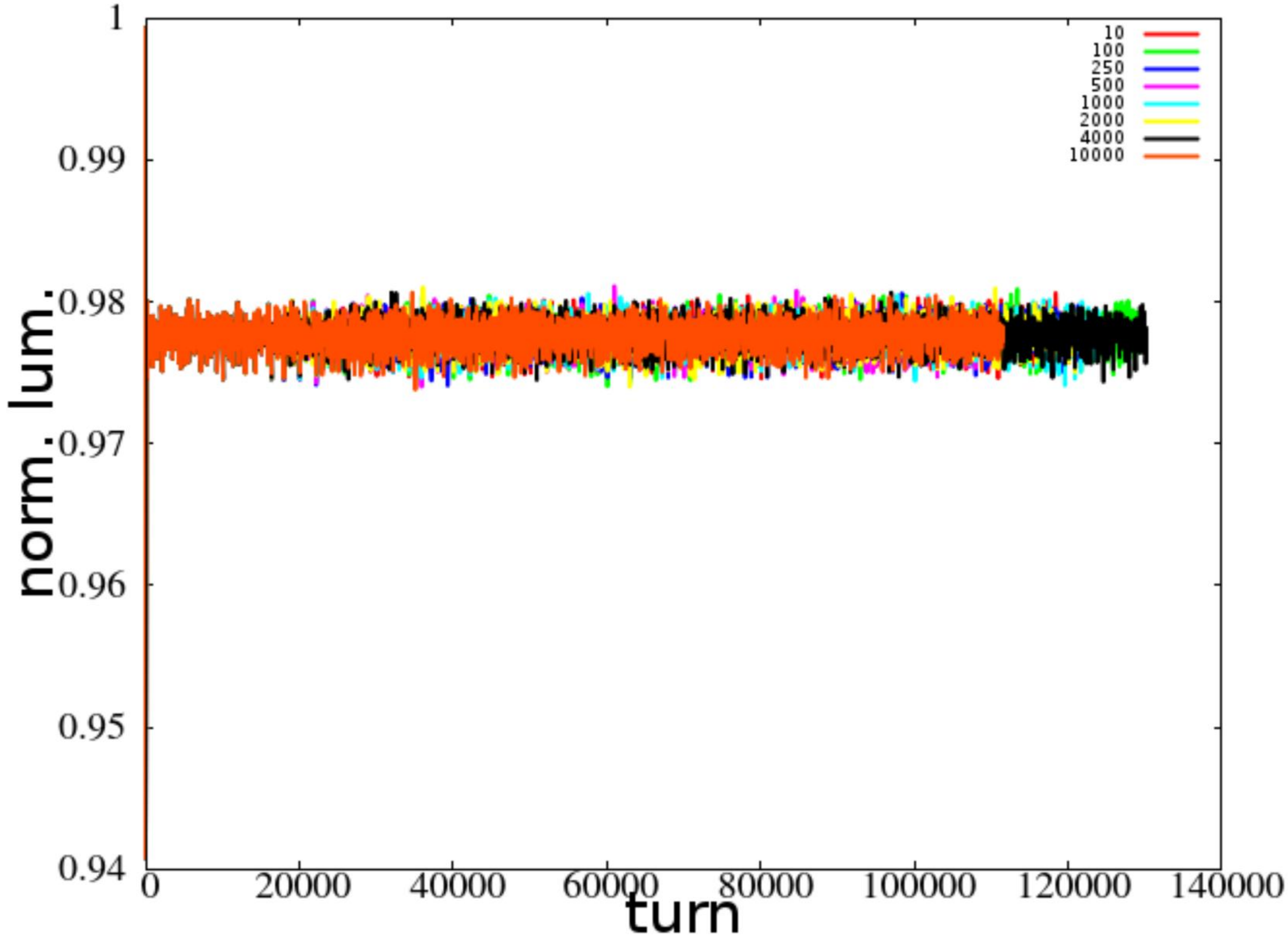
Large Sextupole Errors Cause Fast Luminosity Degradation

Luminosity evolution with different RF sextupole errors



Decapole Error Does Not Cause Fast Luminosity Degradation

Luminosity evolution with different RF decapole errors



Conclusions

- Crab cavities can be used as effective devices to compensate geometric luminosity loss.
- RF noise in the crab cavity needs to be minimized to avoid significant beam emittance growth and luminosity degradation.
- Low order RF multipole errors in the crab cavity also need to be controlled within a given tolerance level.

Thank You!