GPD extraction — status and some technical points

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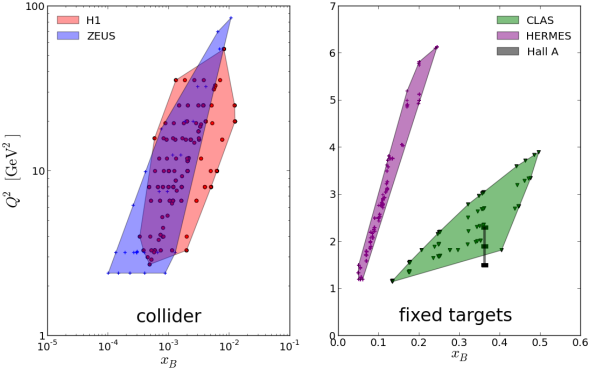
Outline

DVCS experimental coverage



- COMPASS II, JLAB@12 and EIC to fill in the gaps

Experimental coverage (fixed target)

Collab.	Year	Observables	Kinematics			No. of points	
			x_B	Q^2 [GeV^2]	$\|t\|$ [GeV^2]	total	indep.
HERMES	2001	$A_{LU}^{\sin\phi}$	0.11	2.6	0.27	1	1
CLAS	2001	$A_{LU}^{\sin\phi}$	0.19	1.25	0.19	1	1
CLAS	2006	$A_{UL}^{\sin\phi}$	0.2–0.4	1.82	0.15–0.44	6	3
HERMES	2006	$A_{C}^{\cos\phi}$	0.08–0.12	2.0–3.7	0.03–0.42	4	4
Hall A	2006	$\sigma(\phi)$, $\Delta\sigma(\phi)$	0.36	1.5–2.3	0.17–0.33	4×24+12×24	4×24+12×24
CLAS	2007	$A_{LU}(\phi)$	0.11–0.58	1.0–4.8	0.09–1.8	62×12	62×12
HERMES	2008	$A_{C}^{\cos(0,1)\phi}$, $A_{UT,DVCS}^{\sin(\phi-\phi_S)}$, $A_{UT,I}^{\sin(\phi-\phi_S)\cos(0,1)\phi}$, $A_{UT,I}^{\cos(\phi-\phi_S)\sin\phi}$	0.03–0.35	1–10	<0.7	*12*+12+12 12+12 *12*	*4*+4+4 4+4 *4*
CLAS	2008	$A_{LU}(\phi)$	0.12–0.48	1.0–2.8	0.1–0.8	66	33
HERMES	2009	$A_{LU,I}^{\sin(1,2)\phi}$, $A_{LU,DVCS}^{\sin\phi}$, $A_{C}^{\cos(0,1,2,3)\phi}$	0.05–0.24	1.2–5.75	<0.7	18+18+18 18+18+*18*+*18*	6+6+6 6+6+*6*+*6*
HERMES	2010	$A_{UL}^{\sin(1,2,3)\phi}$, $A_{LL}^{\cos(0,1,2)\phi}$	0.03–0.35	1–10	<0.7	12+12+*12* 12+*12*+12	4+4+*4* 4+*4*+4
HERMES	2011	$A_{LT,I}^{\cos(\phi-\phi_S)\cos(0,1,2)\phi}$, $A_{LT,I}^{\sin(\phi-\phi_S)\sin(1,2)\phi}$, $A_{LT,BH+DVCS}^{\cos(\phi-\phi_S)\cos(0,1)\phi}$, $A_{LT,BH+DVCS}^{\sin(\phi-\phi_S)\sin\phi}$	0.03–0.35	1–10	<0.7	*12*+*12*+*12* *12*+*12* *12*+12 12	*4*+*4*+*4* *4*+*4* *4*+4 4
HERMES	2012	$A_{LU,I}^{\sin(1,2)\phi}$, $A_{LU,DVCS}^{\sin\phi}$, $A_{C}^{\cos(0,1,2,3)\phi}$	0.03–0.35	1–10	<0.7	18+*18*+*18* 18+18+*18*+*18*	6+*6*+*6* 6+6+*6*+*6*
CLAS	2015	$A_{LU}(\phi)$, $A_{UL}(\phi)$, $A_{LL}(\phi)$	0.17–0.47	1.3–3.5	0.1–1.4	166+166+166	166+166+166
CLAS	2015	$\sigma(\phi)$, $\Delta\sigma(\phi)$	0.1–0.58	1–4.6	0.09–0.52	2640+2640	2640+2640
Hall A	2015	$\sigma(\phi)$, $\Delta\sigma(\phi)$	0.33–0.40	1.5–2.6	0.17–0.37	480+600	240+360

DVCS cross section

$$d\sigma \propto |\mathcal{T}|^2 = |\mathcal{T}_{\rm BH}|^2 + |\mathcal{T}_{\rm DVCS}|^2 + \mathcal{I} \, .$$

$$\mathcal{I} \quad \propto \quad \frac{-e_\ell}{\mathcal{P}_1(\phi)\mathcal{P}_2(\phi)} \left\{ c_0^{\mathcal{I}} + \sum_{n=1}^{3} \left[c_n^{\mathcal{I}} \cos(n\phi) + s_n^{\mathcal{I}} \sin(n\phi) \right] \right\},$$

$$|\mathcal{T}_{\rm DVCS}|^2 \quad \propto \quad \left\{ c_0^{\rm DVCS} + \sum_{n=1}^{2} \left[c_n^{\rm DVCS} \cos(n\phi) + s_n^{\rm DVCS} \sin(n\phi) \right] \right\},$$

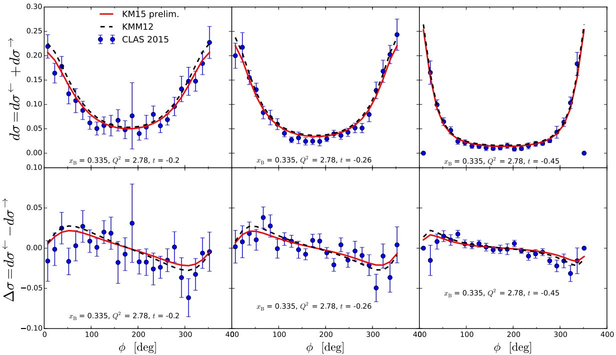
- Choosing polarizations (and charges) we focus on particular harmonics:

$$c_{1,\rm unpol.}^{\mathcal{I}} \propto \left[F_1 \, \mathfrak{Re}\, \mathcal{H} - \frac{t}{4M_p^2} F_2 \, \mathfrak{Re}\, \mathcal{E} + \frac{x_{\rm B}}{2 - x_{\rm B}} (F_1 + F_2) \, \mathfrak{Re}\, \widetilde{\mathcal{H}} \right]$$

[Belitsky, Müller et. al '01–'14]

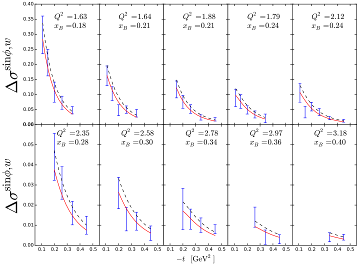
- $\mathcal{H}(x_{\rm B}, t, \mathcal{Q}^2)$, …— four Compton form factors (CFFs)

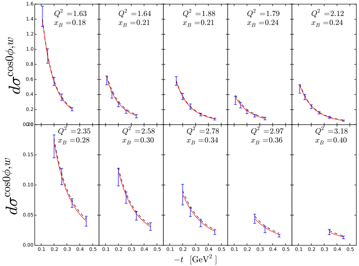
Case #1: CLAS cross-section (ϕ-space)

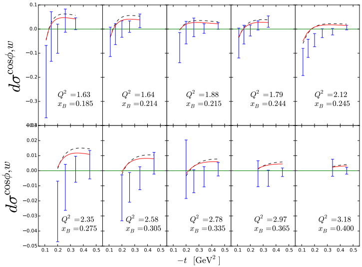


- KM15 model (global refit including this data):
  $\chi^2/\mathrm{npts} = 1032.0/1014$ for $d\sigma$ and $936.1/1012$ for $\Delta\sigma$
- looks excellent

Case #1: CLAS cross-section (n-space)







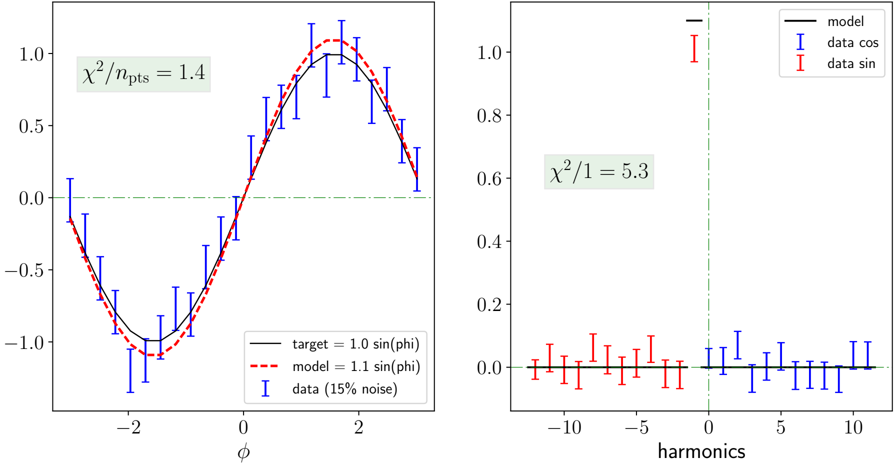
- $\chi^2/\mathrm{npts} = 62.2/48$ for $d\sigma^{\cos\phi,w}$

(O.K. but not so perfect as in ϕ-space)

In addition to $\chi^2/\mathrm{d.o.f.}$ use:

$$\text{pull} \equiv \frac{1}{\sqrt{N}} \sum_{i=1}^{N} \frac{f(x_i) - y_i}{\Delta y_i}$$

Case #2: Sine toy model



- ϕ-space view can be misleading

Extraction of harmonics

- General model for cross section or any other observable

$$f(\phi) = \sum_{k=0}^{c_{\max}} c_k \cos(k\phi) + \sum_{k=1}^{s_{\max}} s_k \sin(k\phi)$$

- $c_{\max} + s_{\max} + 1$ real parameters to be determined from N equidistantly measured values $f(\phi_{i=1,\ldots,N})$.
- least-squares optimization gives standard Fourier formulas, e.g., for $k \geq 1$,

$$c_k = \frac{2}{N} \sum_{i=1}^{N} f(\phi_i) cos(k\phi_i)$$

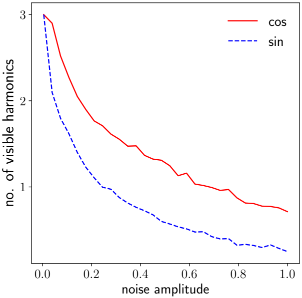
- c_k and their uncertainties are independent of $c_{\max}, s_{\max}$, thanks to orthogonality of trig functions (but only if you have measured full set of ϕ bins!)

How many harmonics are there?

- Case #3: Target function is

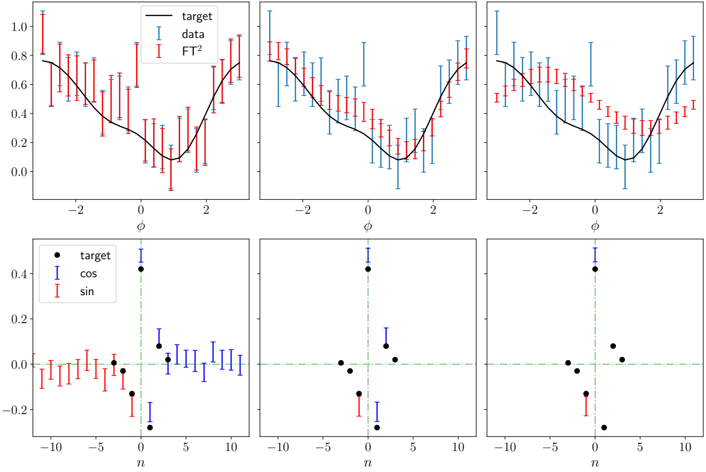
$$f(\phi) = 0.42 - 0.28\cos(\phi) + 0.08\cos(2\phi) + 0.02\cos(3\phi) - 0.13\sin(\phi) - 0.03\sin(2\phi) + 0.006\sin(3\phi)$$

and is used to generate data with variable noise
- Fitting Fourier series with increasing number of harmonics until $\chi^2/\mathrm{d.o.f.}$ w.r.t. target function starts to deteriorate



But we don't know target function in real life

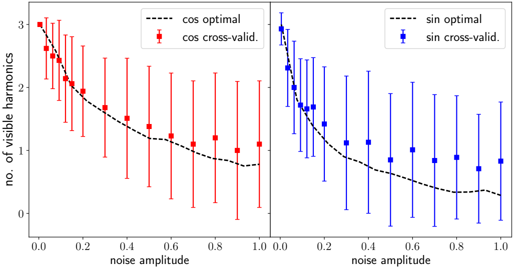
Bias-variance trade-off



Cross-validation procedure

To determine the optimal number of harmonics using only data:

- Divide data randomly in n subsets
- Leave one out and fit the model to remaining $n-1$ subsets
- Evaluate model (calculating χ^2) on the left-out set
- Repeat with leaving out other $n-1$ sets and average the error
- Repeat with increasing number of harmonics until description of left-out sets starts to deteriorate



Finaly, determine just optimal harmonics using all data.

Application to 2015 JLab data

- It is convenient to work with weighted harmonics

$$\sigma^{\sin n\phi, w} \equiv \frac{1}{\pi} \int_{-\pi}^{\pi} dw \, \sin(n\phi)\, \sigma(\phi) \, ,$$

with specialy weighted Fourier integral measure

$$dw \equiv \frac{2\pi \mathcal{P}_1(\phi)\mathcal{P}_2(\phi)}{\int_{-\pi}^{\pi} d\phi \, \mathcal{P}_1(\phi)\mathcal{P}_2(\phi)} d\phi \, ,$$

thus cancelling strongly oscillating factors $1/(\mathcal{P}_1(\phi)\mathcal{P}_2(\phi))$ in Bethe-Heitler and interference terms in $d\sigma$.

- Highest extractable harmonics:

	CLAS		Hall A	
	sine	cosine	sine	cosine
$\Delta\sigma^w$	1.6 ± 0.8	0.8 ± 1.1	1.4 ± 0.5	0.3 ± 0.8
$d\sigma^w$	0.7 ± 0.9	1.6 ± 1.2	1.2 ± 1.1	2.1 ± 0.8

Propagation of uncertainties to harmonics

- Consider three types of uncertainty:
  1. uncorrelated point-to-point uncertainty (absolute size ϵ)
  2. correlated normalization uncertainty (relative size ϵ)
  3. correlated modulated (ϕ-dependent) uncertainty (e.g., relative size $\epsilon\cos(\phi)$)
- Uncorrelated uncertainty: $\Delta c_k = \sqrt{2/N}\,\epsilon$
- Normalization uncertainty: $\Delta c_k / c_k = \epsilon$
- Correlated modulated uncertainty: more complicated, but for hierarchical case $c_0 \gg c_1 \gg \cdots$ one obtains

$$\frac{\Delta c_0}{c_0} = \frac{c_1}{2c_0}\epsilon, \qquad \frac{\Delta c_1}{c_1} = \frac{c_0}{c_1}\epsilon$$

i.e. we have enhancement of uncertainty for subleading harmonics!

$$(c_0 + c_1\cos\phi + \cdots)\times(1+\epsilon\cos\phi) = c_0\left(1 + \frac{c_1}{2c_0}\epsilon\right) + c_1\left(1 + \frac{c_0}{c_1}\epsilon\right)\cos\phi$$

Modulated correlated error in the wild

Hall A [M. Defurne et. al 2015] discussing systematic uncertainties:

tematic error from the parameter choice to be 1%.

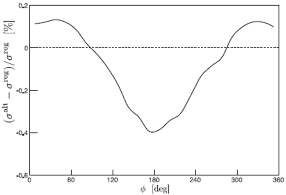
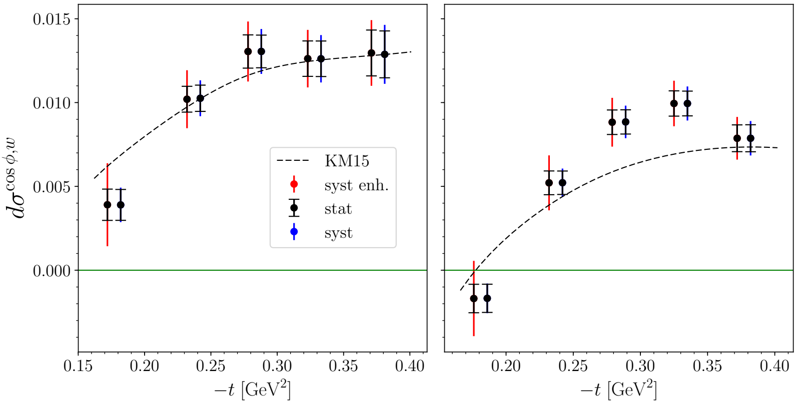


FIG. 20. Difference in % between the cross section extracted with the squared DVCS amplitude term and with the $\Re e\, \mathcal{C}^{\mathcal{I},V}$ term for $x_B = 0.37$, $Q^2 = 2.36$ GeV^2 and $-t = 0.33$ GeV^2. The ϕ-profile of the difference is a consequence of the small $\cos\phi$ and $\cos 2\phi$ dependences of the $\Re e\, \mathcal{C}^{\mathcal{I},V}$ kinematic coefficient. Both extractions give almost the same reduced χ^2/dof=0.94 (nominal) and 0.93 (alternate) for the entire Kin2 setting.

Systematic uncertainty	Value	Section
HRS acceptance cut	1%	IV A
Electron ID	0.5%	IV D
HRS multitrack	0.5%	IV D
Multi-cluster	0.4%	IV D
Corrected luminosity	1%	IV D
Fit parameters	1%	VI B
Radiative corrections	2%	V
Beam polarization	2%	III A 3
Total (helicity-independent)	2.8%	
Total (helicity-dependent)	3.4%	

TABLE V. Normalization systematic uncertainties in the extracted photon electroproduction cross sections. The systematic error coming from the fit parameter choice is not a normalization error per se, but we consider that 1% is an upper limit for this error on all kinematic bins. The helicity-dependent cross sections have an extra uncertainty stemming from the beam polarization measurement. The last column gives the section in which each systematic effect is discussed.

Hall A cos(ϕ) harmonics



(Syst added *linearly* on top of stat.)

Hybrid GPD models for global fits

- Sea quarks and gluons modelled using SO(3) partial wave expansion in conformal GPD moment space + $\mathcal{Q}^2$ evolution.
- Valence quarks — model CFFs directly (ignoring $\mathcal{Q}^2$ evolution):

$$\mathfrak{Im}\,\mathcal{H}(\xi,t)=\pi\left[\frac{4}{9}H^{u_{\rm val}}(\xi,\xi,t)+\frac{1}{9}H^{d_{\rm val}}(\xi,\xi,t)+\frac{2}{9}H^{\rm sea}(\xi,\xi,t)\right]$$

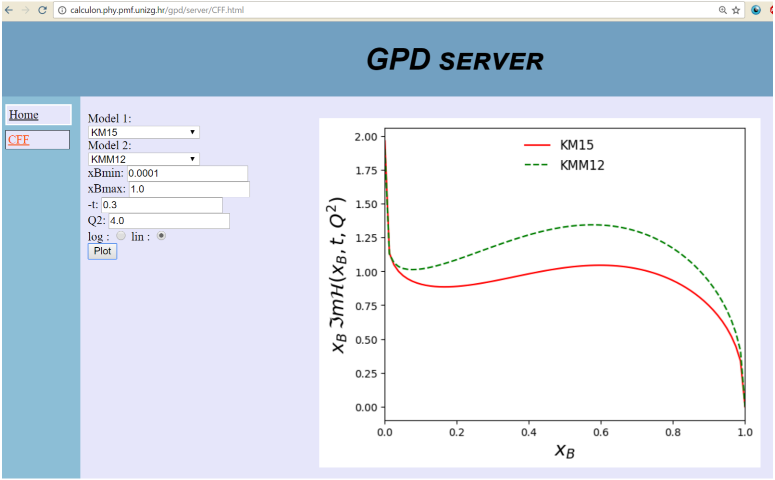
$$H(x,x,t)=n\,r\,2^{\alpha}\left(\frac{2x}{1+x}\right)^{-\alpha(t)}\left(\frac{1-x}{1+x}\right)^{b}\frac{1}{\left(1-\frac{1-x}{1+x}\frac{t}{M^2}\right)^{p}}\,.$$

- $\mathfrak{Re}\,\mathcal{H}$ determined by dispersion relations
- 15 free parameters in total for H, $\tilde{H}$, E, $\tilde{E}$.

Model	KM09a	KM09b	KM10	KM10a	KM10b	KMS11	KMM12	KM15
free params.	{3}+(3)+5	{3}+(3)+6	{3}+15	{3}+10	{3}+15	NNet	{3}+15	{3}+15
$\chi^2/\mathrm{d.o.f.}$	32.0/31	33.4/34	135.7/160	129.2/149	115.5/126	13.8/36	123.5/80	240./275
F_2	{85}	{85}	{85}	{85}	{85}		{85}	{85}
σ_{DVCS}	(45)	(45)	51	51	45		11	11
$d\sigma_{\mathrm{DVCS}}/dt$	(56)	(56)	56	56	56		24	24
$A_{LU}^{\sin\phi}$	12+12	12+12	12	16	12+12		4	13
$A_{LU,I}^{\sin\phi}$			18	18		18	6	6
$A_{C}^{\cos 0\phi}$							6	6
$A_{C}^{\cos\phi}$	12	12	18	18	12	18	6	6
$\Delta\sigma^{\sin\phi,w}$			12				12	63
$\sigma^{\cos 0\phi,w}$			4				4	58
$\sigma^{\cos\phi,w}$			4				4	58
$\sigma^{\cos\phi,w}/\sigma^{\cos 0\phi,w}$		4			4			
$A_{UL}^{\sin\phi}$							10	17
$A_{LL}^{\cos 0\phi}$							4	14
$A_{LL}^{\cos\phi}$								10
$A_{UT,I}^{\sin(\phi-\phi_S)\cos\phi}$							4	4

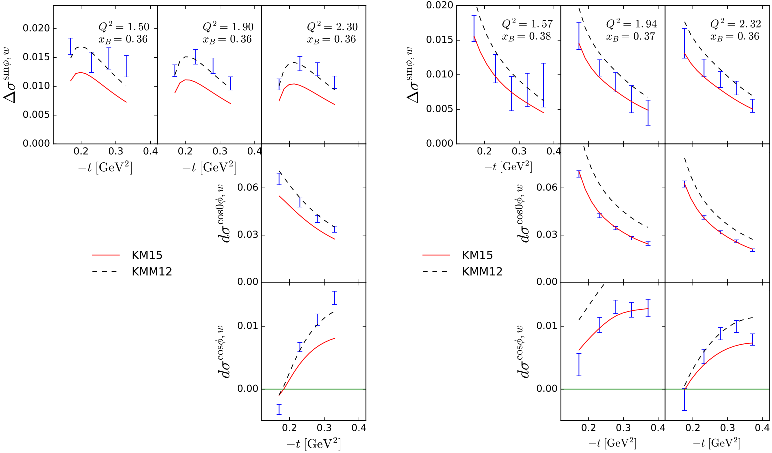
- [K.K., Müller, et al. '09–'15]
- These models are publicly available (google for "gpd page")

KM GPD server



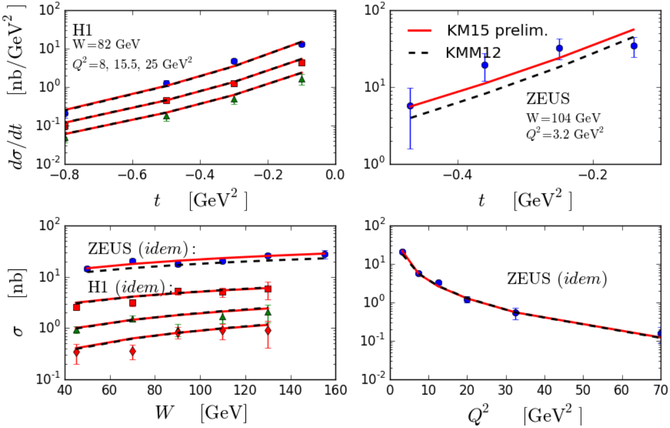
- (Only $\Im\mathrm{m}\,\mathcal{H}$ plots available; other CFFs and numerical values soon to come . . .)

2006 vs 2015 Hall A cross-sections

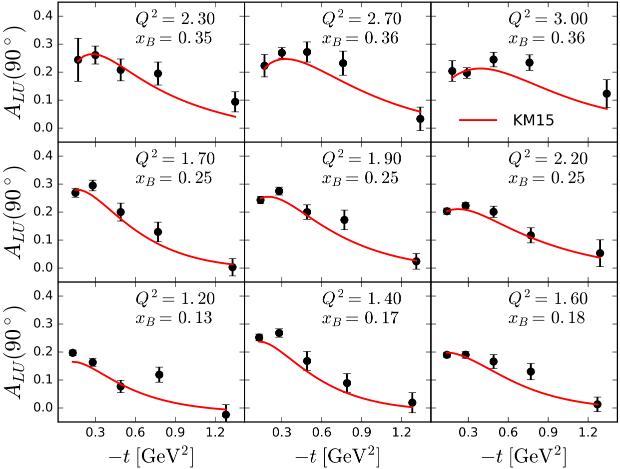


- Improvement of global $\chi^2/\mathrm{d.o.f.}$ 123.5/80 $\rightarrow$ 240./275

H1 (2007), ZEUS (2008)



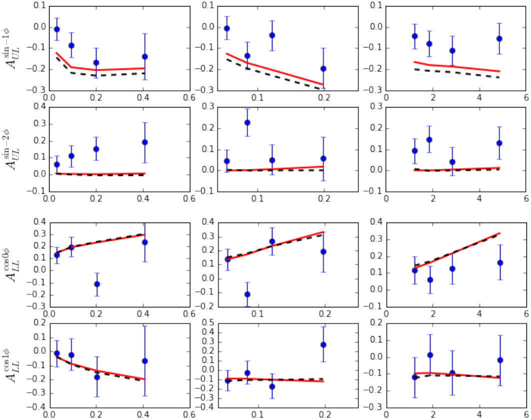
2007 CLAS beam spin asymmetry



- Only data with $|t| \leq 0.3\,\mathrm{GeV}^2$ used for fits.

Longitudinally polarized target — HERMES (2010)

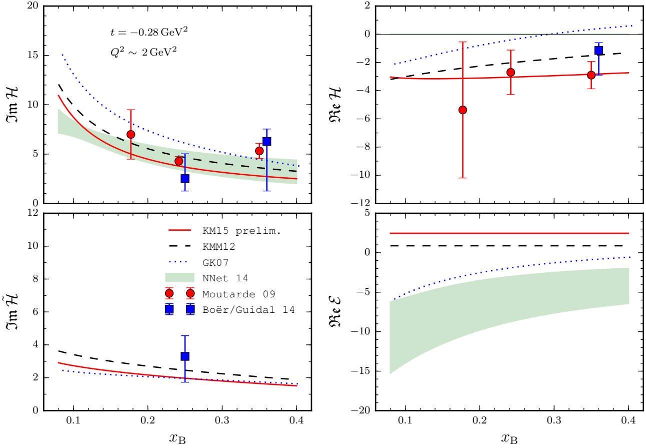
- Surprisingly large $\sin(2\phi)$ harmonic of A_{UL} cannot be described within this leading twist framework



$\chi^2/npts$ and pull values

Collaboration	Observable	$n_{\rm pts}$	KMM12		KM15	
			$\chi^2/n_{\rm pts}$	pull	$\chi^2/n_{\rm pts}$	pull
ZEUS	$\sigma_{\rm DVCS}$	11	0.49	-1.76	0.51	-1.74
ZEUS,H1	$d\sigma_{\rm DVCS}/dt$	24	0.97	0.85	1.04	1.37
HERMES	$A_{\rm C}^{\cos 0\phi}$	6	1.31	0.49	1.24	0.29
HERMES	$A_{\rm C}^{\cos\phi}$	6	0.24	-0.56	0.07	-0.20
HERMES	$A_{\rm LU,I}^{\sin\phi}$	6	2.08	-2.52	1.34	-1.28
CLAS	$A_{\rm LU}^{\sin\phi}$	4	1.28	2.09		
CLAS	$A_{\rm LU}^{\sin\phi}$	13			1.24	0.63
CLAS	$\Delta\sigma^{\sin\phi,w}$	48			0.41	-1.66
CLAS	$d\sigma^{\cos 0\phi,w}$	48			0.16	-0.21
CLAS	$d\sigma^{\cos\phi,w}$	48			1.16	6.36
Hall A	$\Delta\sigma^{\sin\phi,w}$	12	1.06	-2.55		
Hall A	$d\sigma^{\cos 0\phi,w}$	4	1.21	2.14		
Hall A	$d\sigma^{\cos\phi,w}$	4	3.49	-0.26		
Hall A	$\Delta\sigma^{\sin\phi,w}$	15			0.81	-2.84
Hall A	$d\sigma^{\cos 0\phi,w}$	10			0.40	0.92
Hall A	$d\sigma^{\cos\phi,w}$	10			2.52	-2.42
HERMES,CLAS	$A_{\rm UL}^{\sin\phi}$	10	1.90	-1.89	1.10	-1.94
HERMES	$A_{\rm LL}^{\cos 0\phi}$	4	3.44	2.17	3.19	1.99
HERMES	$A_{\rm UT,I}^{\sin(\phi-\phi_S)\cos\phi}$	4	0.90	0.61	0.90	0.71
CLAS	$A_{\rm UL}^{\sin\phi}$	10			0.76	0.38
CLAS	$A_{\rm LL}^{\cos 0\phi}$	10			0.50	-0.22
CLAS	$A_{\rm LL}^{\cos\phi}$	10			1.54	2.40

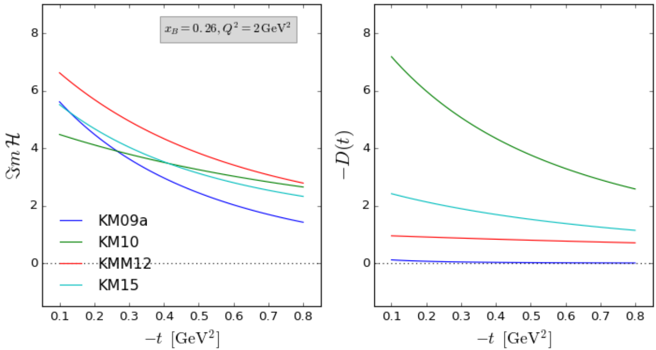
CFFs from various fits



(from [K.K., Moutarde and Liuti '16])

CFFs from various fits (II)

Comparing $\mathfrak{Im}\,\mathcal{H}$ and D-term for a typical JLab kinematics:



- We need access to ERBL region.

Going beyond first approximations

Published DVCS data is well described within leading order, leading twist and other approximations. Some available corrections:

- NLO evolution and NLO coefficient functions
- finite-t and target mass corrections [Braun et al. '14]
- twist-3 GPDs
- massive charm [Noritzsch '03]
- small-x resummation [Ivanov et al. '16]

can presently be absorbed in redefinition of GPDs (think "DVCS scheme" factorization), but have to be taken into account when working with more processes (striving towards universal GPDs) and with more precise future data.

Summary

- Global fits of all proton DVCS data using flexible hybrid models are in healthy shape (some tension for first $cos(\phi)$ harmonic of JLab cross sections).
- Harmonics are better suited for phenomenology than ϕ-space data, but some care must be exercised in their extraction.

The End