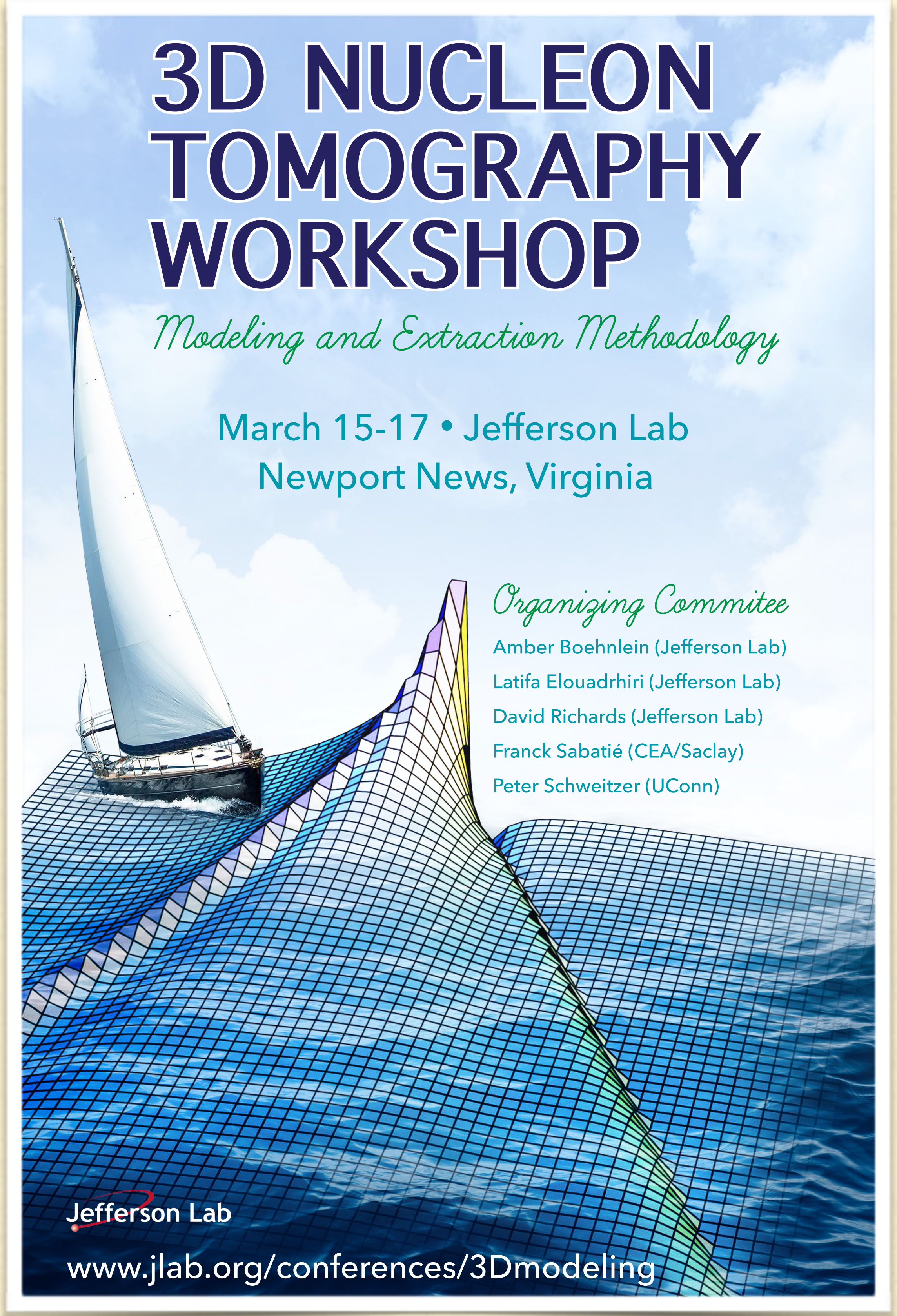
Kostas Orginos (W&M/JLab)
March 15-17, 2017 (JLab)

3D Nucleon Tomography from LQCD



INTRODUCTION

- Goal: Compute properties of hadrons from first principles
  - Parton distribution functions (PDFs) and Generalized Parton distributions (GPDs)
  - Transverse Momentum Dependent densities (TMDs)
  - Form Factos …
- Lattice QCD is a first principles method
  - For many years calculations focused on Mellin moments
  - Can be obtained from local matrix elements of the proton in Euclidean space
    - Breaking of rotational symmetry –> power divergences
    - only first few moments can be computed
- Recently direct calculations of PDFs in Lattice QCD are proposed

X. Ji, Phys.Rev.Lett. 110, (2013)

Y.-Q. Ma J.-W. Qiu (2014) 1404.6860

- First lattice Calculations already available

H.-W. Lin, J.-W. Chen, S. D. Cohen, and X. Ji, Phys.Rev. D91, 054510 (2015)

C. Alexandrou, et al, Phys. Rev. D92, 014502 (2015)

PDFS: DEFINITION

Light-cone PDFs:

$$f^{(0)}(\xi) = \int_{-\infty}^{\infty} \frac{\mathrm{d}\omega^-}{4\pi} e^{-i\xi P^+\omega^-} \left\langle P \left| T\, \overline{\psi}(0,\omega^-,\mathbf{0}_\mathrm{T}) W(\omega^-,0)\gamma^+ \frac{\lambda^a}{2}\psi(0) \right| P \right\rangle_\mathrm{C} .$$

$$W(\omega^-,0) = \mathcal{P}\exp\left[-ig_0 \int_0^{\omega^-} \mathrm{d}y^-\, A_\alpha^+(0,y^-,\mathbf{0}_\mathrm{T})T_\alpha\right]$$

$$\langle P'|P\rangle = (2\pi)^3 2P^+ \delta\left(P^+ - P'^+\right)\delta^{(2)}\left(\mathbf{P}_\mathrm{T} - \mathbf{P}'_\mathrm{T}\right)$$

Moments:

$$a_0^{(n)} = \int_0^1 \mathrm{d}\xi\, \xi^{n-1}\left[f^{(0)}(\xi) + (-1)^n \overline{f}^{(0)}(\xi)\right] = \int_{-1}^1 \mathrm{d}\xi\, \xi^{n-1} f(\xi)$$

Local matrix elements:

$$\left\langle P|\mathcal{O}_0^{\{\mu_1\ldots\mu_n\}}|P\right\rangle = 2a_0^{(n)}\left(P^{\mu_1}\cdots P^{\mu_n} - \text{traces}\right)$$

$$\mathcal{O}_0^{\{\mu_1\cdots\mu_n\}} = i^{n-1}\overline{\psi}(0)\gamma^{\{\mu_1}D^{\mu_2}\cdots D^{\mu_n\}}\frac{\lambda^a}{2}\psi(0) - \text{traces}$$

GPDS: DEFINITION

GPDs:

$$\bar{u}(P')\left(\gamma^+ H(x,\xi,t) + i\frac{\sigma^{+k}\Delta_k}{2m}E(x,\xi,t)\right) = \int_{-\infty}^{\infty}\frac{\mathrm{d}\omega^-}{4\pi}e^{-i\xi P^+\omega^-}\left\langle P'\left|T\,\overline{\psi}(0,\omega^-,\mathbf{0}_\mathrm{T})W(\omega^-,0)\gamma^+\frac{\lambda^a}{2}\psi(0)\right|P\right\rangle_\mathrm{C}$$

$$W(\omega^-,0) = \mathcal{P}\exp\left[-ig_0\int_0^{\omega^-}\mathrm{d}y^- A_\alpha^+(0,y^-,\mathbf{0}_\mathrm{T})T_\alpha\right]$$

$$\langle P'|P\rangle = (2\pi)^3 2P^+\delta\left(P^+ - P'^+\right)\delta^{(2)}\left(\mathbf{P}_\mathrm{T} - \mathbf{P}'_\mathrm{T}\right)$$

$$\Delta = P' - P$$

$$t = \Delta^2$$

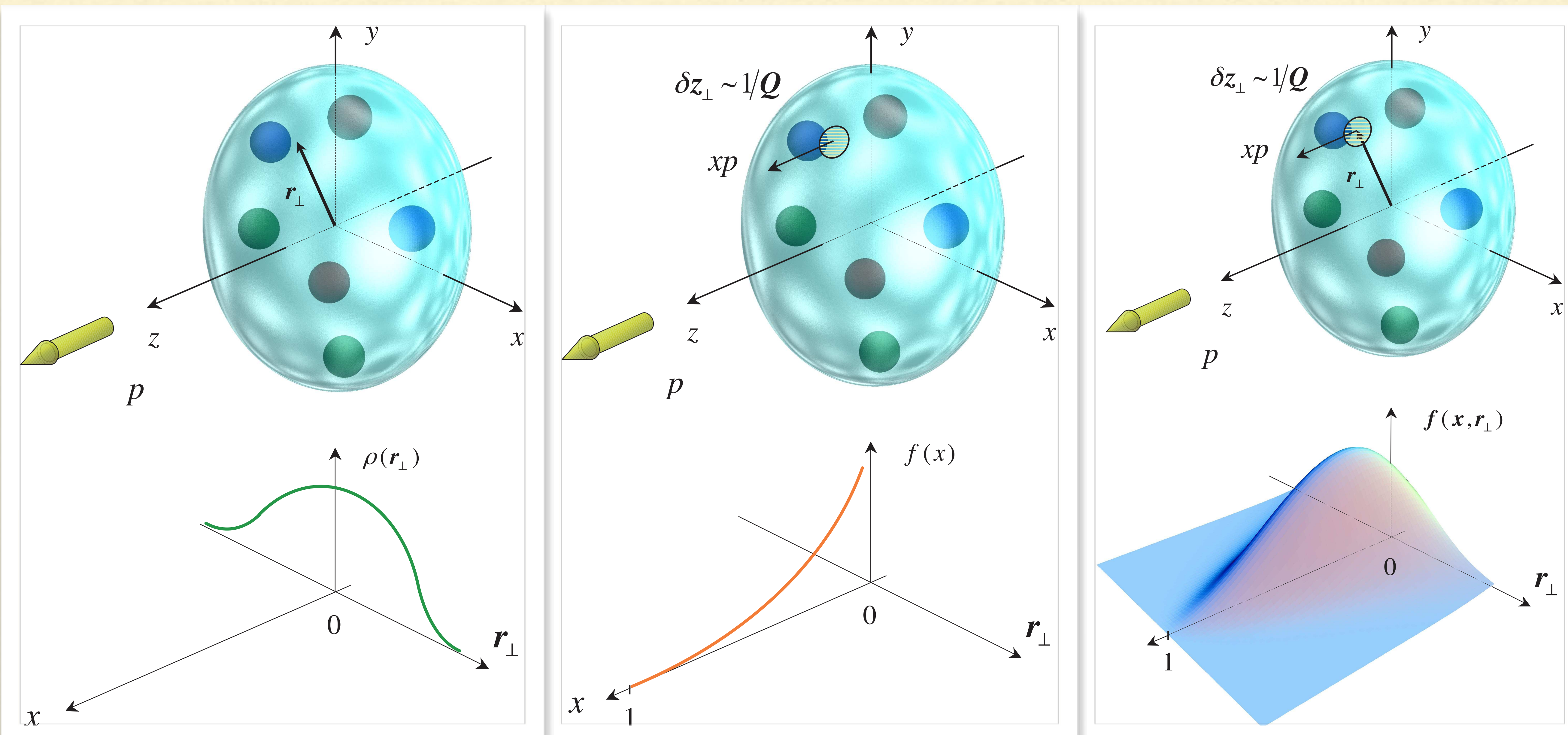
Moments:

$$\int_{-1}^{1} dx x^{n-1}\begin{bmatrix} H(x,\xi,t) \\ E(x,\xi,t)\end{bmatrix} = \sum_{k=0}^{[(n-1)/2]}(2\xi)^{2k}\begin{bmatrix} A_{n,2k}(t) \\ B_{n,2k}(t)\end{bmatrix} \pm \delta_{n,\mathrm{even}}(2\xi)^n C_n(t).$$

Matrix elements of twist-2 operators

$$\mathcal{O}_0^{\{\mu_1\cdots\mu_n\}} = i^{n-1}\overline{\psi}(0)\gamma^{\{\mu_1}D^{\mu_2}\cdots D^{\mu_n\}}\frac{\lambda^a}{2}\psi(0) - \text{traces}$$

X. Ji, D. Muller, A. Radyushkin (1994-1997)



Form Factors

Parton Distribution functions

Generalized Parton Distribution functions

Mellin moments are local matrix elements

Can be evaluated in Euclidean space

Lattice QCD calculations are possible

Challenges:

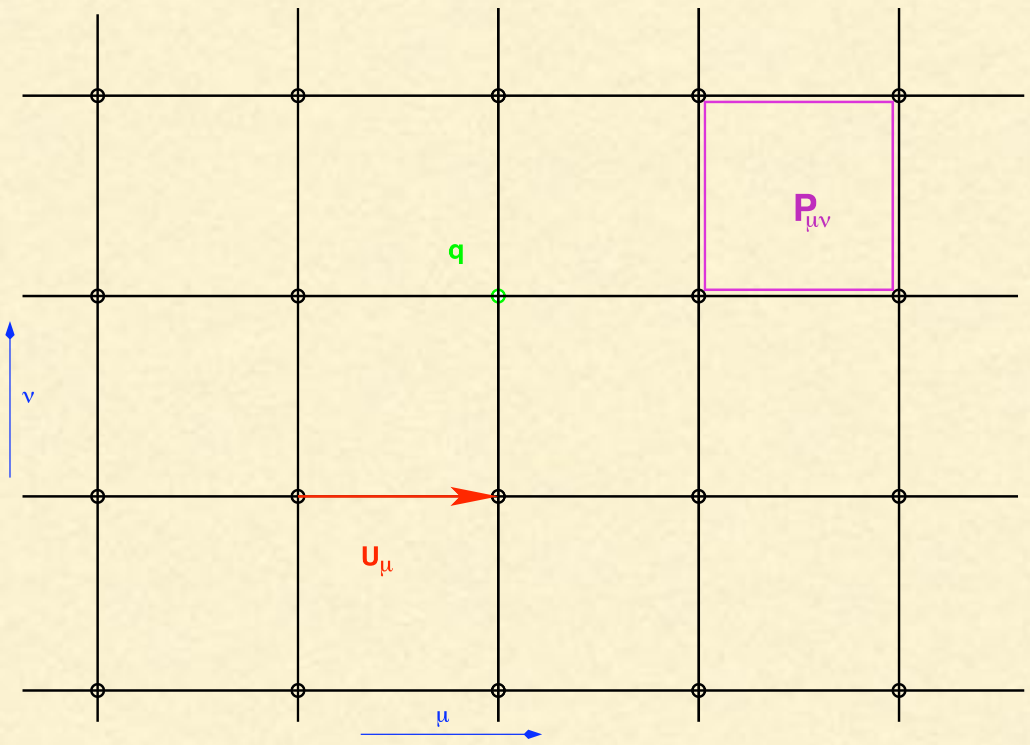
- Renormalization and power divergent mixing
- Lattice breaks O(4) symmetry
- Only few moments can be computed

LATTICE QCD

In continuous Euclidian space: $\mathcal{Z} = \int \mathcal{D}q\mathcal{D}\bar{q}\mathcal{D}A_\mu \, e^{-S[\bar{q},q,A_\mu]}$

$$\langle \mathcal{O} \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}q\mathcal{D}\bar{q}\mathcal{D}A_\mu \, \mathcal{O}(\bar{q}, q, A_\mu) \, e^{-S[\bar{q},q,A_\mu]}$$

Lattice regulator:



Gauge sector:

$$U_\mu(x) = e^{-iaA_\mu(x+\frac{\hat{\mu}}{2})}$$

Fermion sector:

$$S_f = \bar{\Psi} D \Psi$$

Ψ is now a vector whose components leave on the sites of the lattice

D is the Dirac matrix which is large and sparse

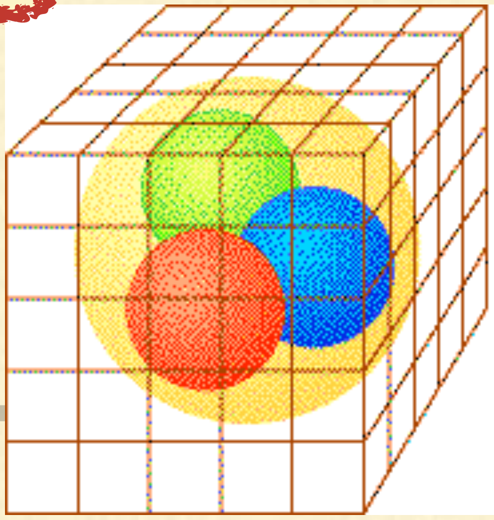
MONTE CARLO INTEGRATION

$$\langle \mathcal{O} \rangle = \frac{1}{\mathcal{Z}} \int \prod_{\mu,x} dU_\mu(x) \;\; \mathcal{O}[U, D(U)^{-1}] \;\; \det\left(D(U)^\dagger D(U)\right)^{n_f/2} \;\; e^{-S_g(U)}$$

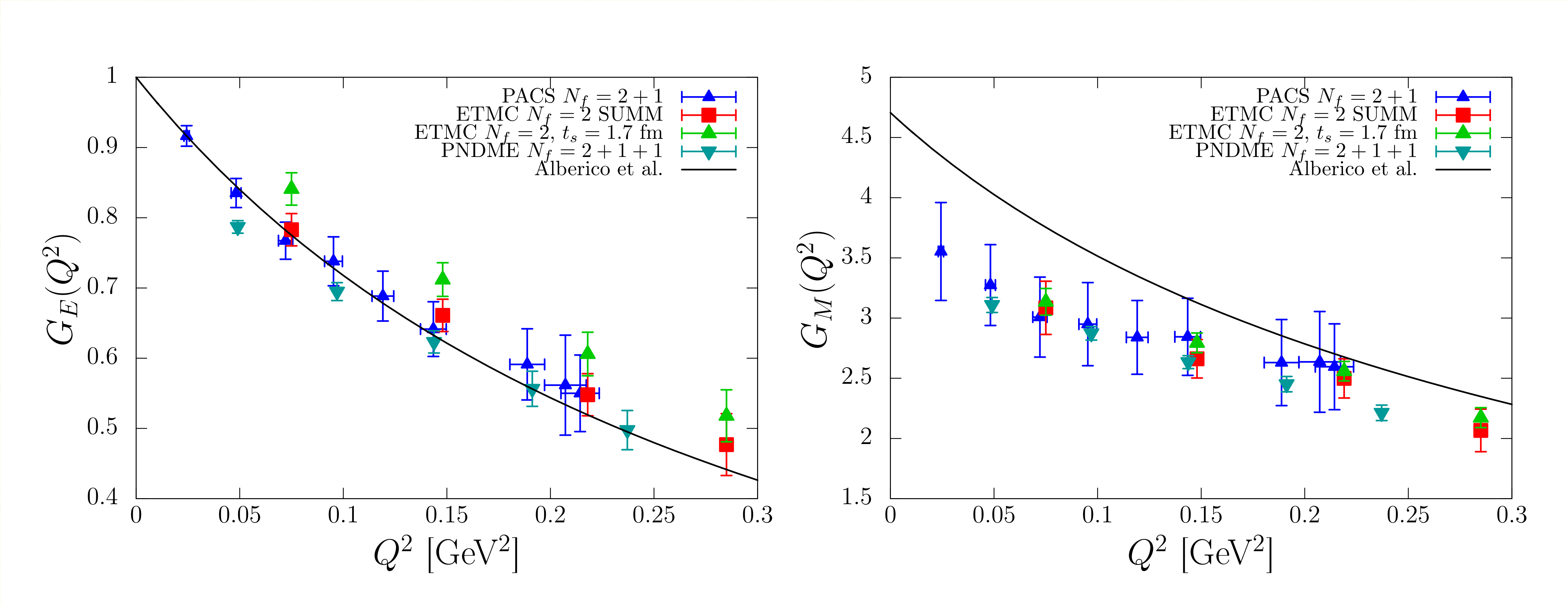
Monte Carlo Evaluation

$$\langle \mathcal{O} \rangle = \frac{1}{N} \sum_{i=1}^{N} \mathcal{O}(U_i)$$

Statistical error $\dfrac{1}{\sqrt{N}}$



Form Factors

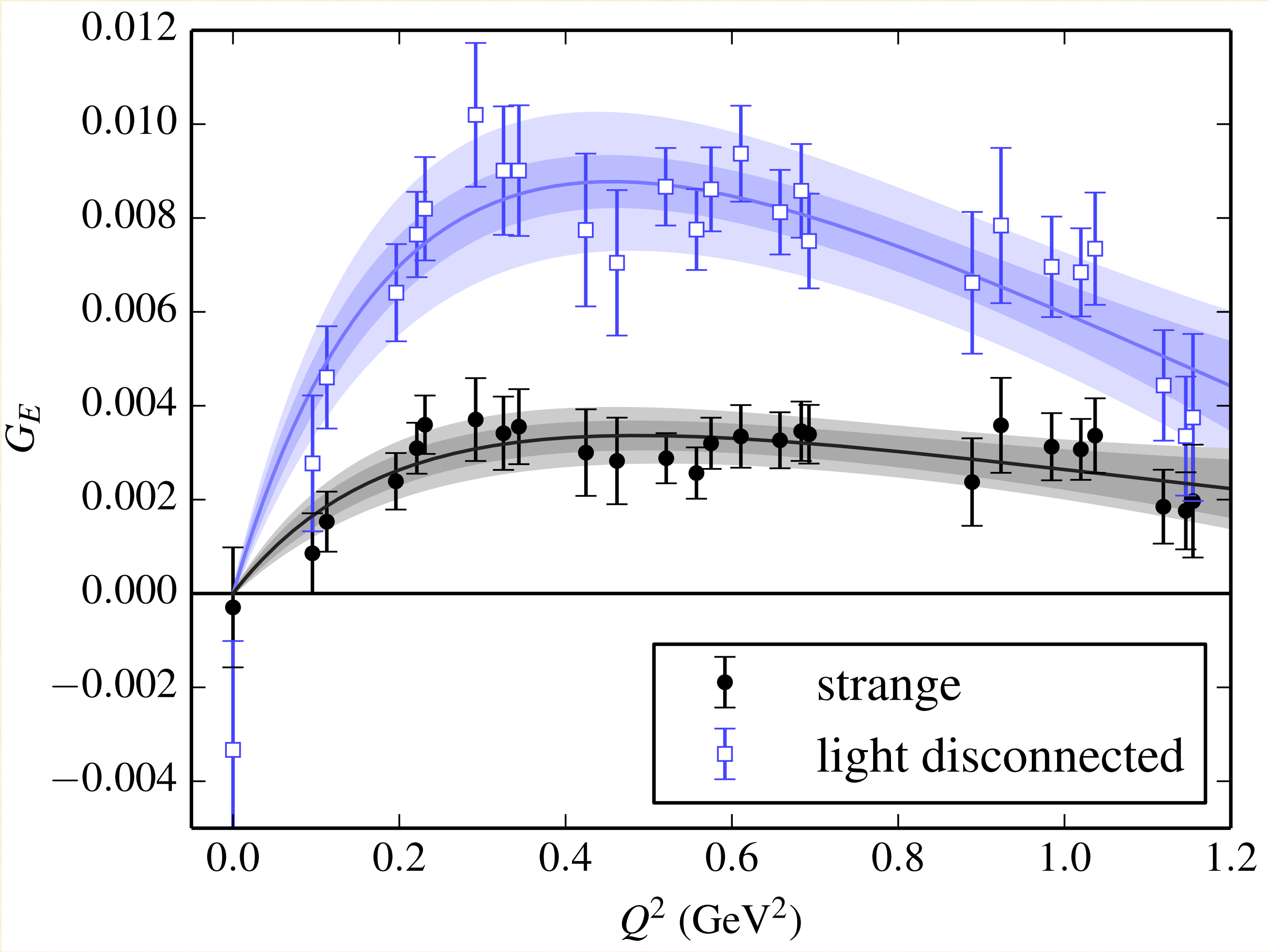


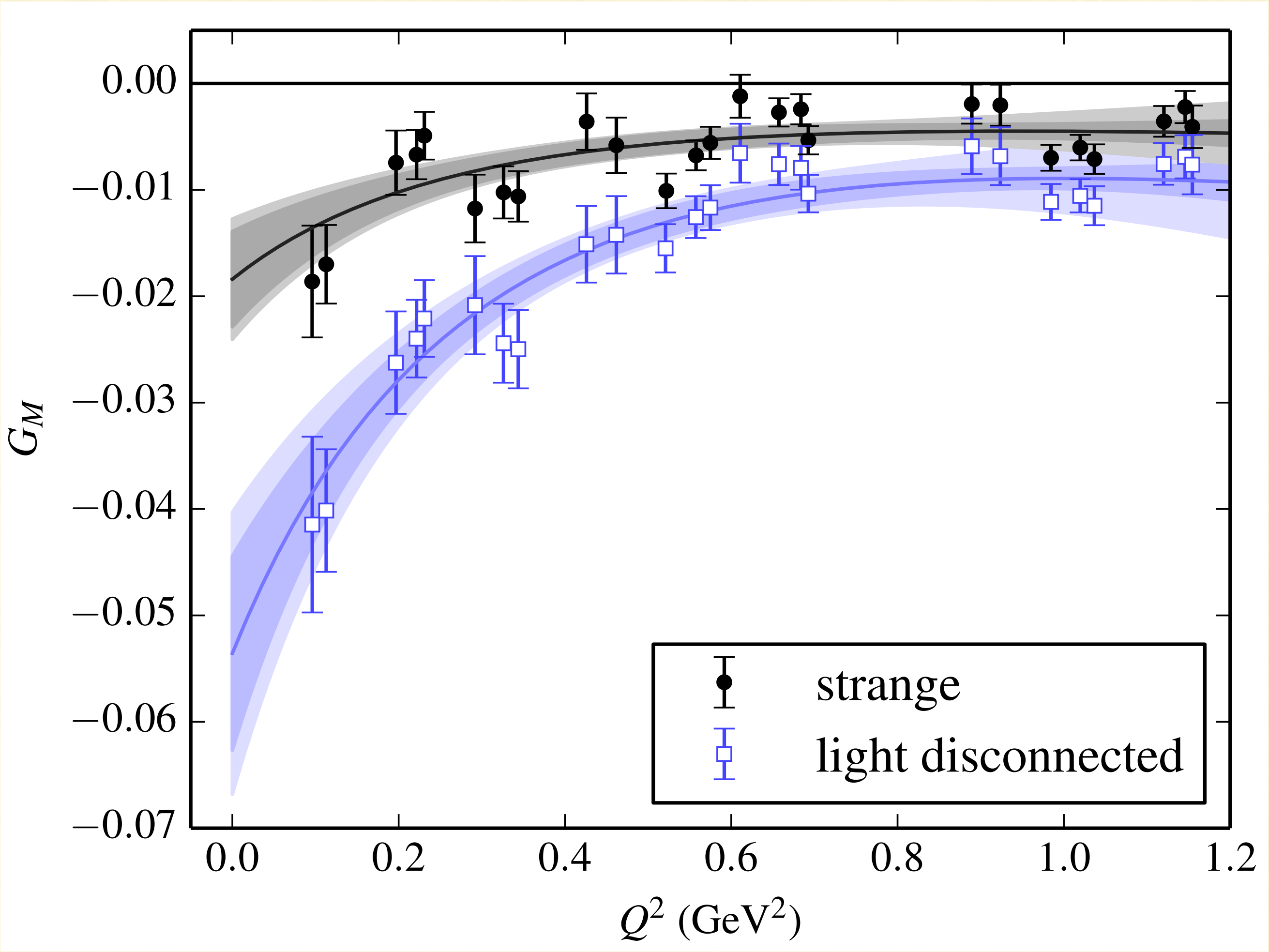
PACS: N_f=2+1 m_π = 145 MeV 8.1 fm box

ETMC: N_f=2+1 m_π = 131 MeV 4.5 fm box

PNDME: mixed action m_π = 138 MeV 5.6 fm box

Strange quark contribution to nucleon form factors

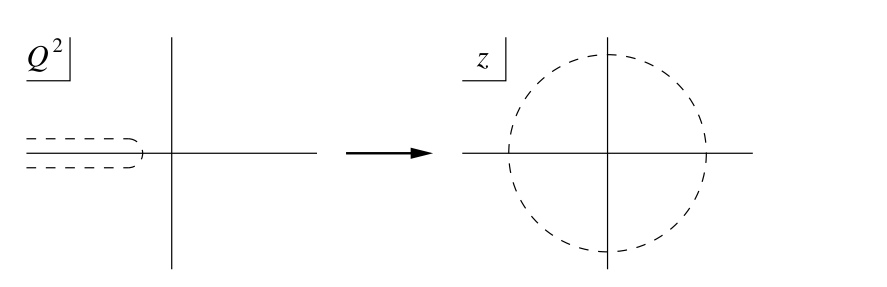




dynamical 2 + 1 flavors of Clover fermions

32^3 x 96 lattice of dimensions $(3.6\ \text{fm})^3$ x (10.9 fm)

a=0.115fm, pion mass 317 MeV



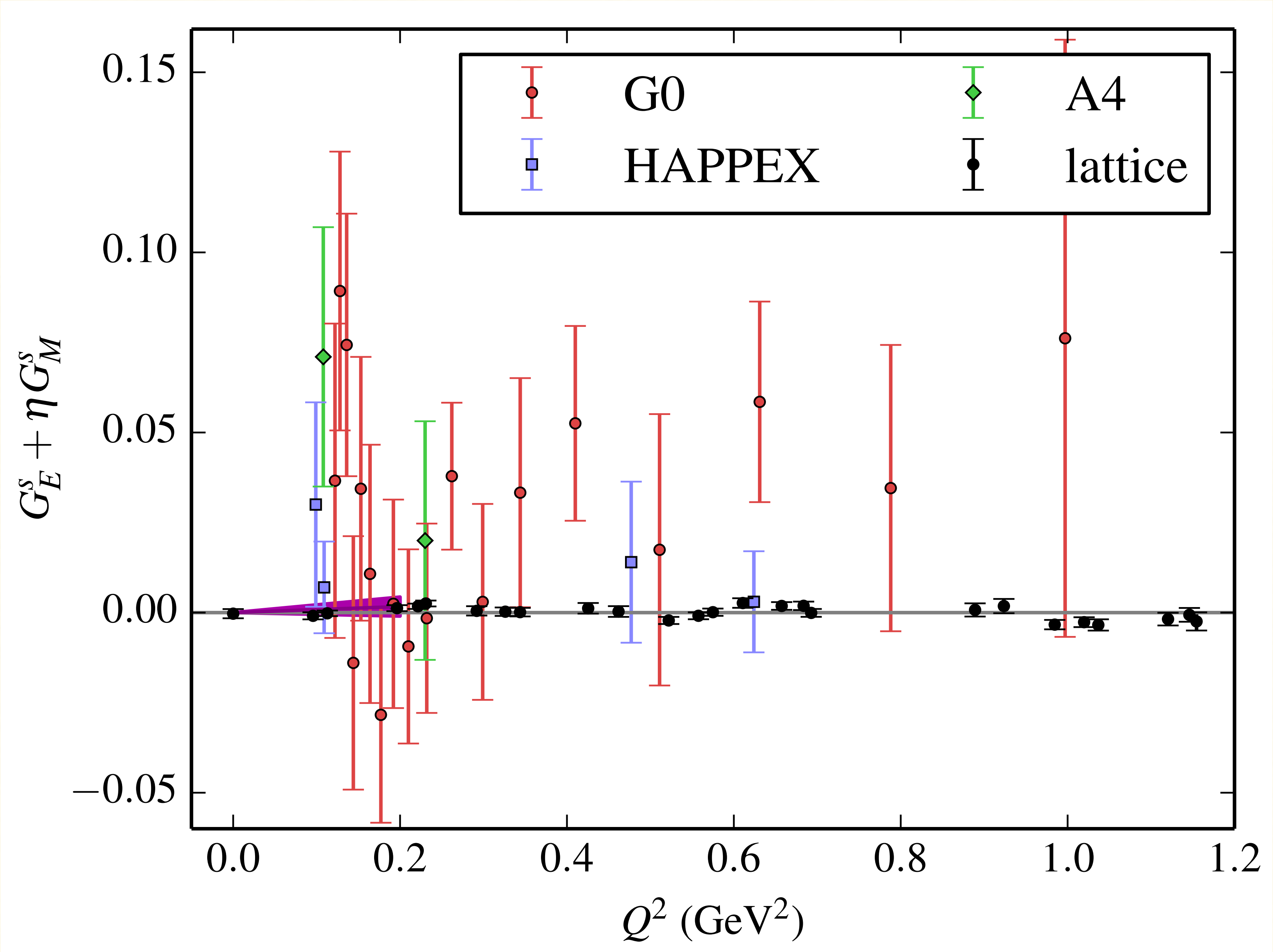
R. J. Hill and G. Paz, Phys. Rev. D **84** (2011) 073006

z-expansion fit:

$$G(Q^2) = \sum_k^{k_{\max}} a_k z^k, \quad z = \frac{\sqrt{t_{\text{cut}} + Q^2} - \sqrt{t_{\text{cut}}}}{\sqrt{t_{\text{cut}} + Q^2} + \sqrt{t_{\text{cut}}}},$$

J. Green *et al.* Phys.Rev. D92 (2015) no.3, 031501

Comparison with experiments

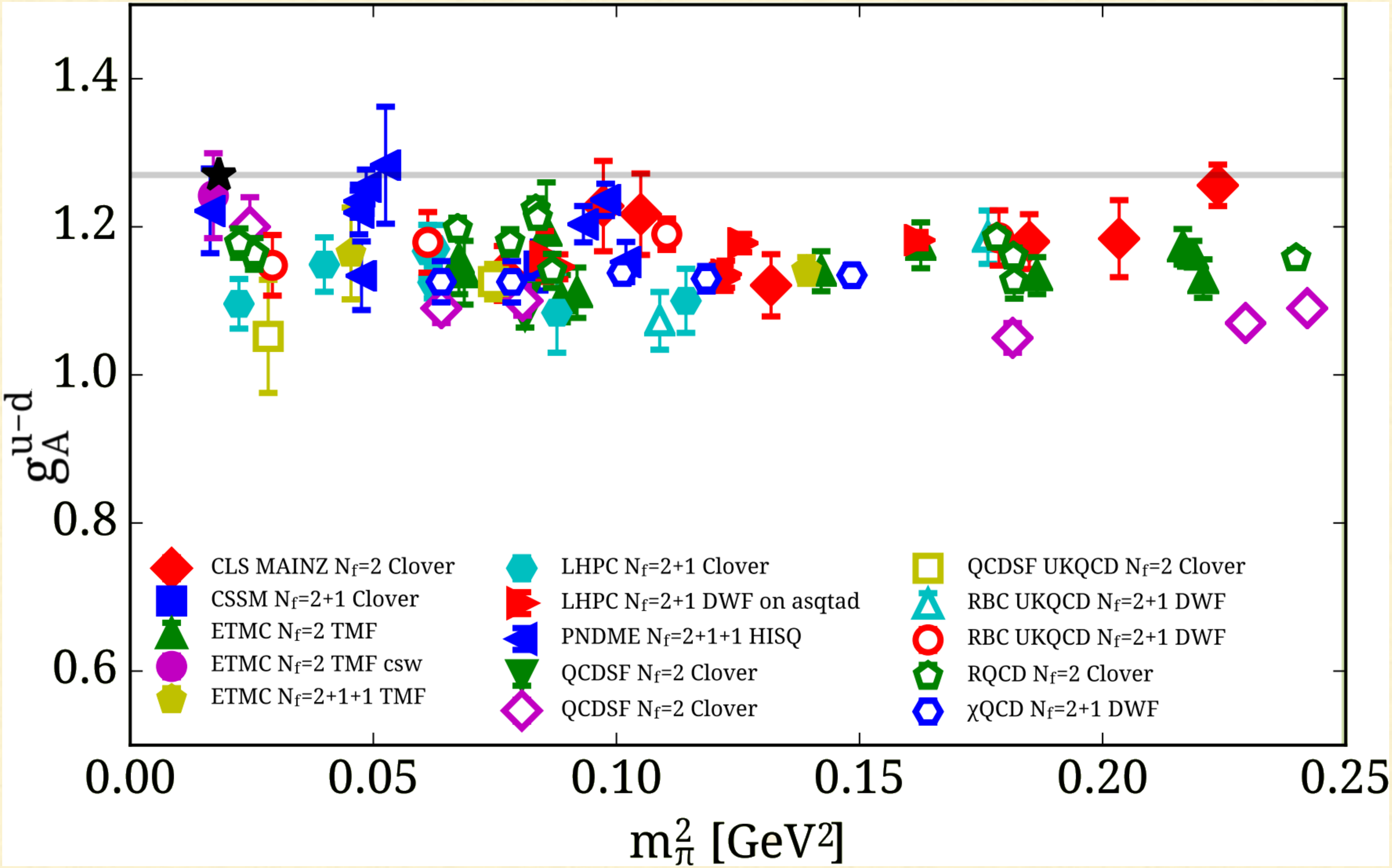


Experiment: forward-angle parity-violating elastic e-p scattering

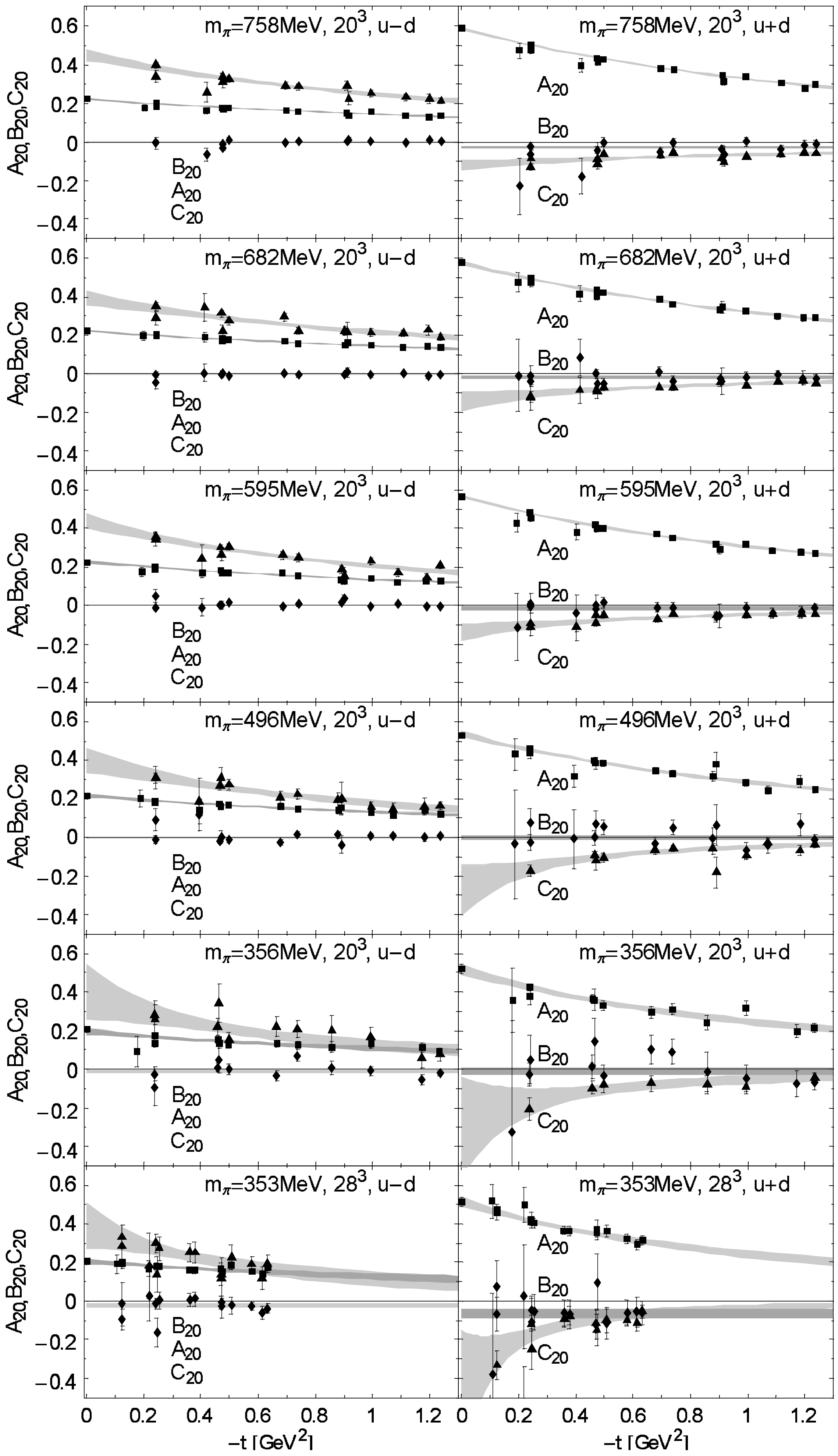
$$G_E^s + \eta G_M^s \qquad \eta = AQ^2,\ A = 0.94$$

Prediction: very hard for such experiments to measure a non-zero result

Axial Charge



From M. Constantinou: arXiv:1701.02855

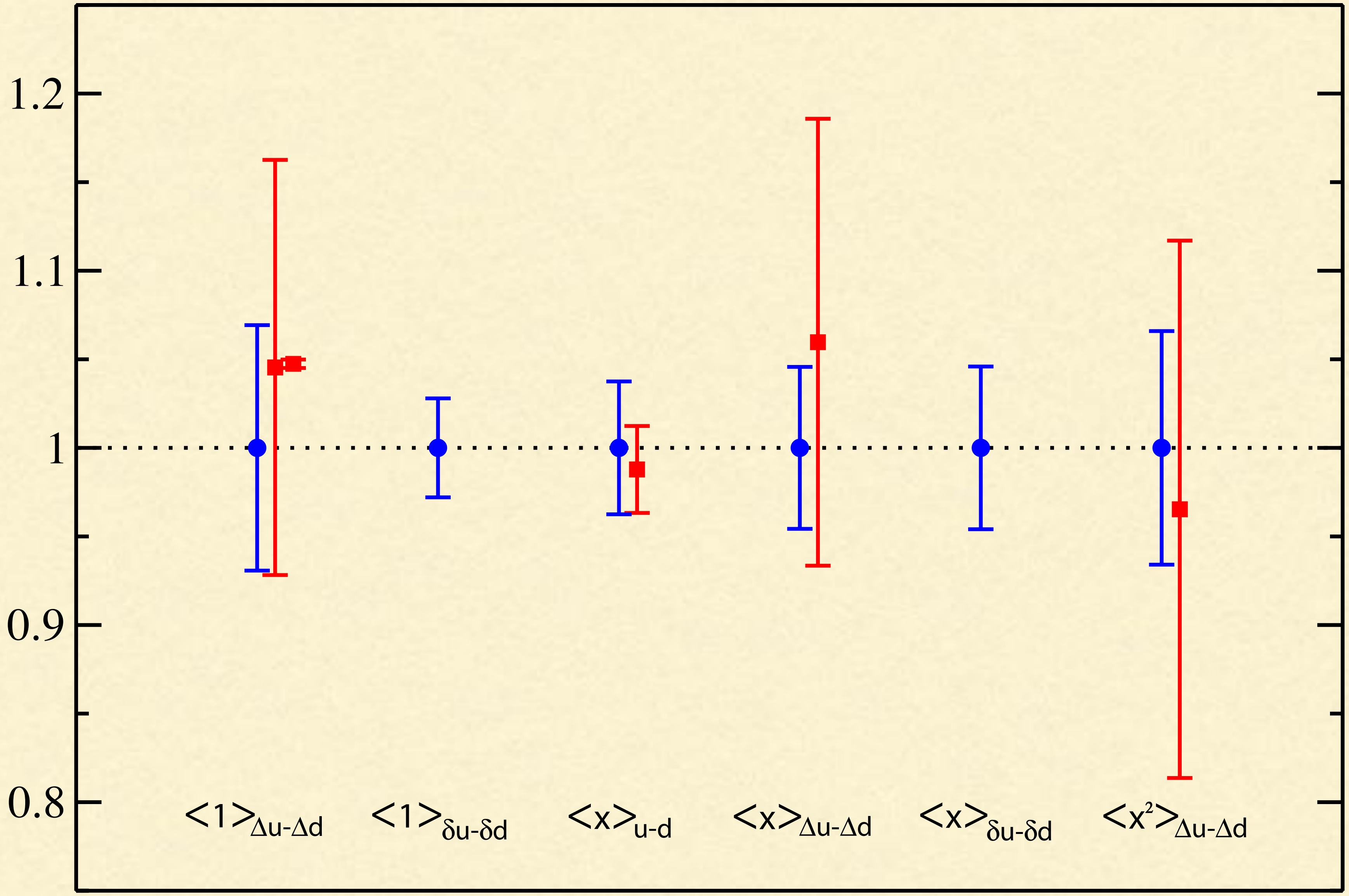


Moments of GPDs

LHPC: arXiv:0705.4295

Phys.Rev.D77:094502,2008

Moments of PDFs (Lattice vs Experiment)



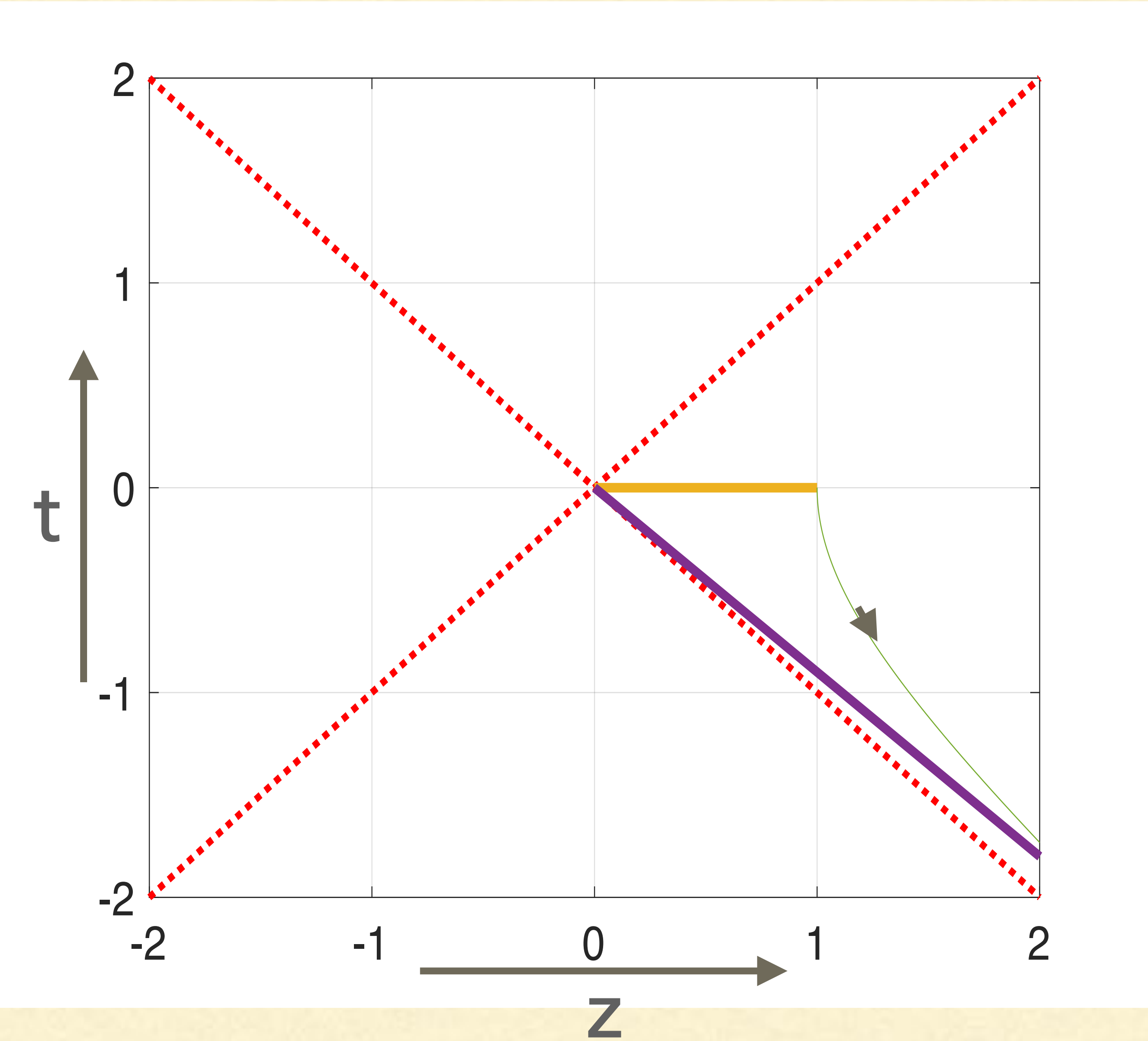
LHPC: 2007

Can Lattice QCD go beyond moments?

Lattice QCD can only compute time local matrix elements

Euclidean space

QPDFS: MAIN IDEA



$$\lim_{P_z \to \infty} q^{(0)}(x, P_z) = f(x)$$

X. Ji, Phys.Rev.Lett. 110, (2013)

Euclidean space time local matrix element is equal to the same matrix element in Minkowski space

A more general point of view:

Y.-Q. Ma J.-W. Qiu (2014) 1404.6860

$$\sigma(x, a, P_z) \xrightarrow{a \to 0} \tilde{\sigma}(x, \tilde{\mu}^2, P_z)$$

Minkowski space factorization:

$$\widetilde{\sigma}(x, \widetilde{\mu}^2, P_z) = \sum_{\alpha=\{q,\overline{q},g\}} H_\alpha\left(x, \frac{\widetilde{\mu}}{P_z}, \frac{\widetilde{\mu}}{\mu}\right) \otimes f_\alpha(x, \mu^2) + \mathcal{O}\left(\frac{\Lambda^2_{\mathrm{QCD}}}{\widetilde{\mu}^2}\right)$$

H_α computable in perturbation theory

Related ideas see:

K-F Liu Phys.Rev. D62 (2000) 074501

Detmold and Lin Phys.Rev.D73:014501,2006

$$q\left(x, P_z\right) = \int_{-1}^{1} \frac{d\xi}{\xi}\, \widetilde{Z}\left(\frac{x}{\xi}, \frac{\mu}{P_z}\right) f(\xi, \mu) + \mathcal{O}(\Lambda_{\mathrm{QCD}}/P_z, M_N/P_z)$$

The matching kernel can be computed in perturbation theory

X. Xiong, X. Ji, J. H. Zhang, Y. Zhao, Phys. Rev. D 90, no. 1, 014051 (2014)
T. Ishikawa et al. arXiv:1609.02018 (2016)

- Practical calculations require a regulator (Lattice)
- Continuum limit has to be taken
  - renormalization
- Momentum has to be large compared to hadronic scales to suppress higher twist effects
- Practical issue with LQCD calculations at large momentum … signal to noise ratio

QUASI-PDFS

$$h^{(s)}\left(\frac{z}{\sqrt{\tau}}, \sqrt{\tau}P_z, \sqrt{\tau}\Lambda_{\rm QCD}, \sqrt{\tau}M_{\rm N}\right) = \frac{1}{2P_z}\left\langle P_z \left| \overline{\chi}(z;\tau)\mathcal{W}(0,z;\tau)\gamma_z\frac{\lambda^a}{2}\chi(0;\tau)\right| P_z\right\rangle_{\rm C}$$

τ is the a regulator scale

χ quark field

$\mathcal{W}$ is the regulated gauge link

$$q^{(s)}\left(\xi, \sqrt{\tau}P_z, \sqrt{\tau}\Lambda_{\rm QCD}, \sqrt{\tau}M_{\rm N}\right) = \int_{-\infty}^{\infty}\frac{{\rm d}z}{2\pi}e^{i\xi z P_z}P_z\, h^{(s)}(\sqrt{\tau}z, \sqrt{\tau}P_z, \sqrt{\tau}\Lambda_{\rm QCD}, \sqrt{\tau}M_{\rm N}),$$

At fixed flow time the quasi-PDF is finite in the continuum limit

Ji 2013

Using the previous definitions we have

$$\left(\frac{i}{P_z}\frac{\partial}{\partial z}\right)^{n-1} h^{(s)}\left(\frac{z}{\sqrt{\tau}}, \sqrt{\tau}P_z, \sqrt{\tau}\Lambda_{\mathrm{QCD}}, \sqrt{\tau}M_{\mathrm{N}}\right) = \int_{-\infty}^{\infty} \mathrm{d}\xi\, \xi^{n-1} e^{-i\xi z P_z} q^{(s)}\left(\xi, \sqrt{\tau}P_z, \sqrt{\tau}\Lambda_{\mathrm{QCD}}, \sqrt{\tau}M_{\mathrm{N}}\right)$$

By introducing the moments

$$b_n^{(s)}\left(\sqrt{\tau}P_z, \frac{\Lambda_{\mathrm{QCD}}}{P_z}, \frac{M_{\mathrm{N}}}{P_z}\right) = \int_{-\infty}^{\infty} \mathrm{d}\xi\, \xi^{n-1} q^{(s)}\left(\xi, \sqrt{\tau}P_z, \sqrt{\tau}\Lambda_{\mathrm{QCD}}, \sqrt{\tau}M_{\mathrm{N}}\right)$$

Taking the limit of z going to 0 we obtain:

$$b_n^{(s)}\left(\sqrt{\tau}P_z, \frac{\Lambda_{\rm QCD}}{P_z}, \frac{M_{\rm N}}{P_z}\right) = \frac{c_n^{(s)}(\sqrt{\tau}P_z)}{2P_z^n}\left\langle P_z \left| \left[\overline{\chi}(z;\tau)\gamma_z(i\overleftarrow{D}_z)^{(n-1)}\frac{\lambda^a}{2}\chi(0;\tau)\right]_{z=0}\right| P_z\right\rangle_{\rm C}.$$

i.e. the moments of the quasi-PDF are related to local matrix elements of the smeared fields

These matrix elements are not twist-2. Higher twist effects enter as corrections that scale as powers of

$$\frac{\Lambda_{\rm QCD}}{P_z}, \frac{M_N}{P_z}$$

after removing M_N/P_z effects

[H.-W. Lin, et. al Phys.Rev. D91, 054510 (2015)]

$$b_n^{(s)}\left(\sqrt{\tau}P_z, \sqrt{\tau}\Lambda_{\rm QCD}\right) = c^{(s)}(\sqrt{\tau}P_z)b_n^{(s,{\rm twist-2})}\left(\sqrt{\tau}\Lambda_{\rm QCD}\right) + \mathcal{O}\left(\frac{\Lambda^2_{\rm QCD}}{P_z^2}\right)$$

Introducing a kernel function such that:

$$C_n^{(0)}(\sqrt{\tau}\mu, \sqrt{\tau}P_z) = \int_{-\infty}^{\infty} dx\, x^{n-1}\widetilde{Z}(x, \sqrt{\tau}\mu, \sqrt{\tau}P_z)$$

We can undo the Melin transform:

$$q^{(s)}\left(x, \sqrt{\tau}\Lambda_{\rm QCD}, \sqrt{\tau}P_z\right) = \int_{-1}^{1} \frac{d\xi}{\xi}\,\widetilde{Z}\left(\frac{x}{\xi}, \sqrt{\tau}\mu, \sqrt{\tau}P_z\right) f(\xi,\mu) + \mathcal{O}(\sqrt{\tau}\Lambda_{\rm QCD})$$

Therefore regulated quasi-PDFs are related to PDFs if

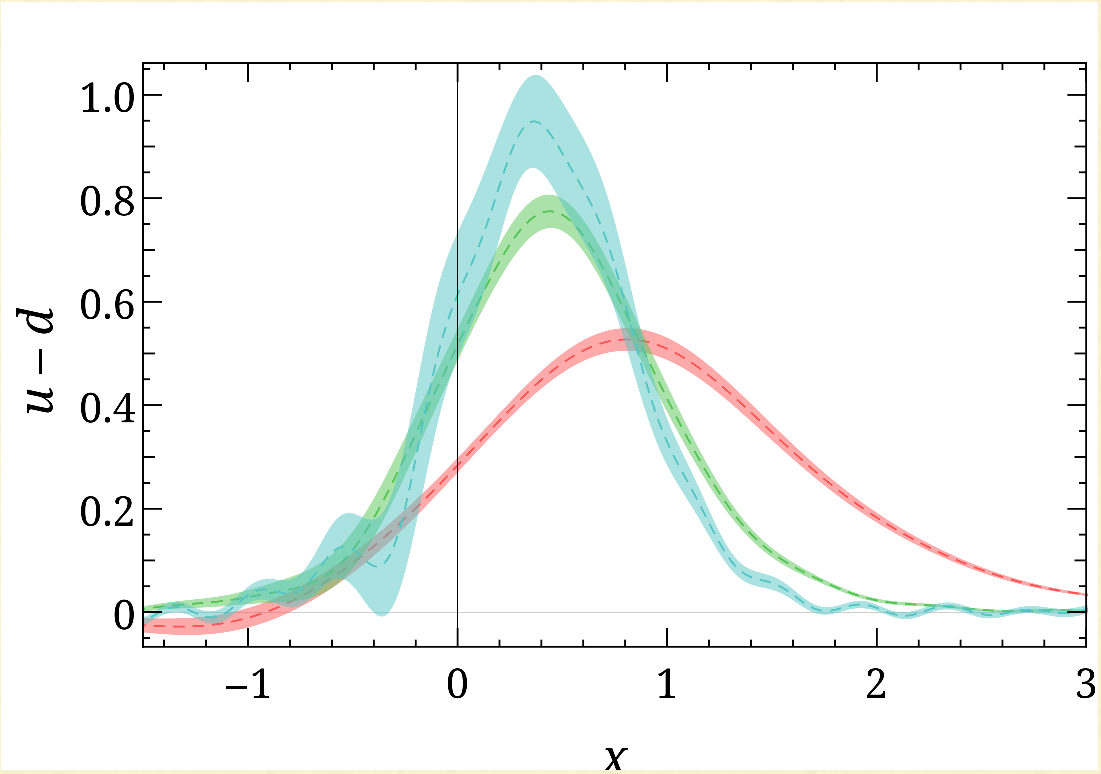
$$\Lambda_{\rm QCD}, M_N \ll P_z \ll \tau^{-1/2}.$$

Monahan and KO: arXiv:1612.01584

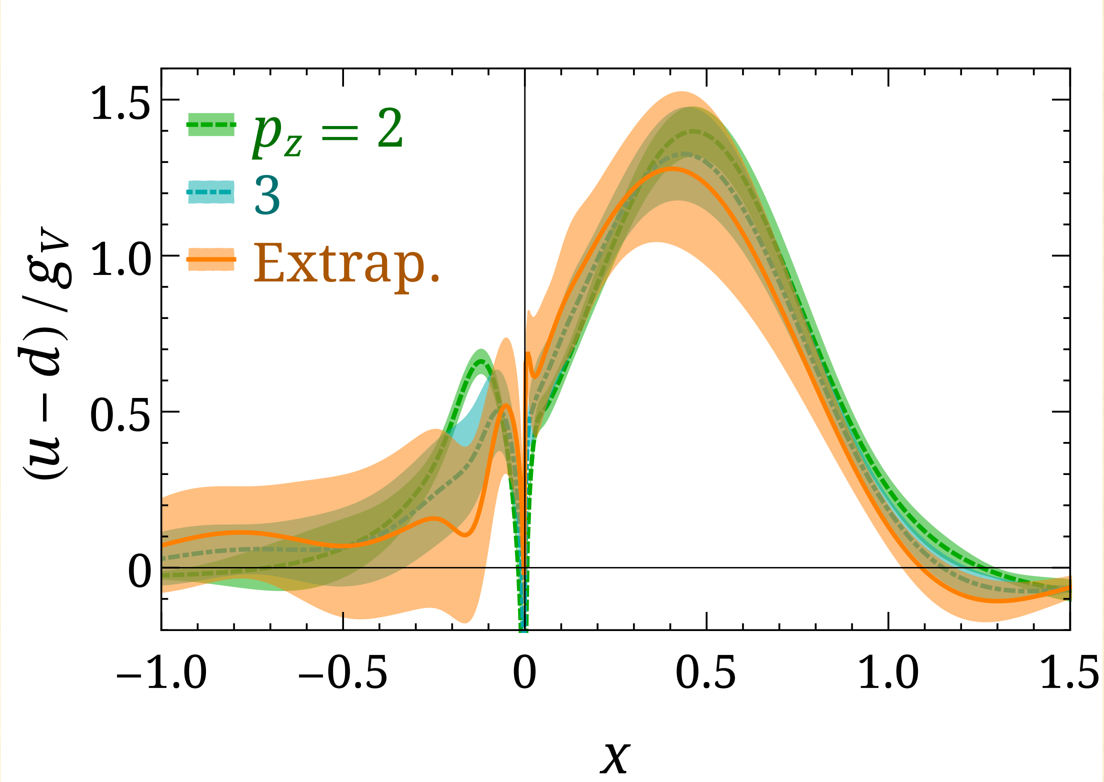
PROCEDURE OUTLINE

- Compute equal time matrix elements in Euclidean space using Lattice QCD at sufficiently large momentum in order to suppress higher twist effects
- Take the continuum limit (renormalization)
- Equal time: Minkowski – Euclidean equivalence
- Perform the matching Kernel calculation in the continuum

First Lattice results (Chen et. al)



Convergence with momentum extrapolation



Including the 1-loop matching kernel

Plots taken from: Chen et al. arXiv:1603.06664

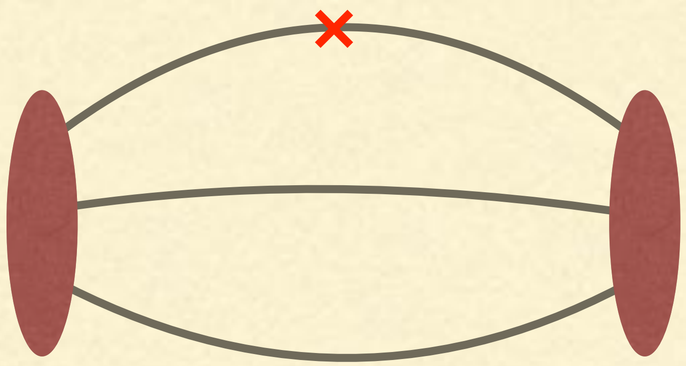
Similar results have been achieved by Alexandrou et. al (ETMC)

- Along these lines one can compute:
  - TMDs (see Engelhardt et. al.)
  - GPDs
  - Distribution amplitudes
  - Gluonic PDFs
  - …..

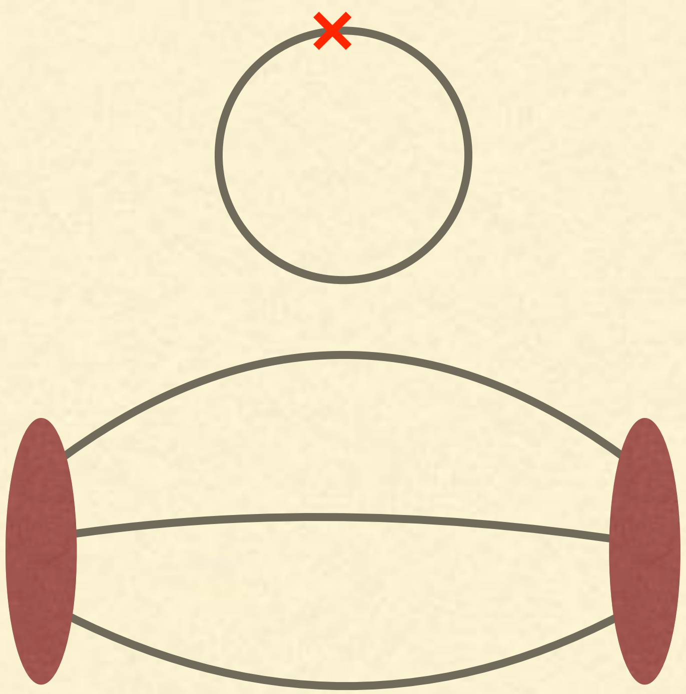
CONCLUSIONS

- Lattice QCD calculations have made a lot of progress and in some cases precision results are being obtained
    - Physical quark masses, large volumes, large scale calculations
- Quasi-PDFs provide a novel way to study hadron structure in Lattice QCD
- Lattice calculations from several groups are on the way
- Several ideas for dealing with the continuum limit are now developing
- Promising new ideas: Stay tuned!

NUCLEON FORM FACTOR



Connected



Disconnected

Strange quark : disconnected only