Jefferson Lab

3-D Nucleon Tomography Workshop
Jefferson Lab, March 16, 2017

Role of JAM* in 3-D nucleon structure

Wally Melnitchouk

* *Jefferson Lab Angular Momentum (JAM) Collaboration*

Nobuo Sato, Jake Ethier, Alberto Accardi

http://www.jlab.org/JAM

JAM

N. Sato et al., PRD ***93****, 074005 (2016)*
PRD ***94****, 114004 (2016)*

Outline

- What can PDF analysis ("1-D tomography") do for the study of 3-D nucleon structure?

- "JAM" global PDF analysis
  - ⟶ new Iterative Monte Carlo (IMC) methodology, with Bayesian determination of PDF errors

- Applications of IMC
  - ⟶ first MC extraction of twist-2 and 3 helicity PDFs ("JAM15")
  - ⟶ first MC analysis of fragmentation functions from e^+e^- ("JAM16")
  - ⟶ first simultaneous PDF/FF analysis of DIS, SIDIS and SIA for unambiguous flavor separation ("JAM17")

- Role of PDFs in TMD extraction
  - ⟶ challenges and opportunities…

Methodology

- Analysis of data requires estimating expectation values and variances of observables $\mathcal{O}$ (= PDFs, FFs)

$$E[\mathcal{O}] = \int d^n a \, \mathcal{P}(\vec{a}|\mathrm{data}) \, \mathcal{O}(\vec{a})$$

$$V[\mathcal{O}] = \int d^n a \, \mathcal{P}(\vec{a}|\mathrm{data}) \, \left[\mathcal{O}(\vec{a}) - E[\mathcal{O}]\right]^2$$

→ probability distribution

$$\mathcal{P}(\vec{a}|\mathrm{data}) \;\propto\; \mathcal{L}(\mathrm{data}|\vec{a}) \; \pi(\vec{a})$$

Bayes' theorem

priors

→ likelihood function

$$\mathcal{L}(\mathrm{data}|\vec{a}) \;\sim\; \exp\left[-\frac{1}{2}\,\chi^2(\vec{a}) \right]$$

$$\chi^2(\vec{a}) = \sum_i \left(\frac{\mathrm{data}_{\,i} - \mathrm{theory}_i(\vec{a})}{\delta(\mathrm{data})} \right)^2$$

Methodology

- Standard method for evaluating E, V is "maximum likelihood"

  ⟶ maximize probability distribution

$$\mathcal{P}(\vec{a}|\mathrm{data}) \;\rightarrow\; \vec{a}_0$$

  ⟶ if $\mathcal{O}$ linear in parameters, and if probability is symmetric in all parameters

$$E[\mathcal{O}(\vec{a})] = \mathcal{O}(\vec{a}_0)\,, \qquad V[\mathcal{O}(\vec{a})] \rightarrow \mathrm{Hessian}$$

- In practice, since in general $E[f(\vec{a})] \neq f(E[\vec{a}])$, maximum likelihood method will sometimes fail

  ⟶ need more versatile approach ($e.g.$ Monte Carlo)

$$E[\mathcal{O}] \;\approx\; \frac{1}{N}\sum_k \mathcal{O}(\vec{a}_k)\,, \qquad V[\mathcal{O}] \;\approx\; \frac{1}{N}\sum_k \left[\mathcal{O}(\vec{a}_k) - E[\mathcal{O}]\right]^2$$

Iterative Monte Carlo

- Can use traditional functional form for input distribution shape

$$xf(x) = N\, x^{a}\, (1-x)^{b}\, (1 + c\sqrt{x} + d\, x)$$

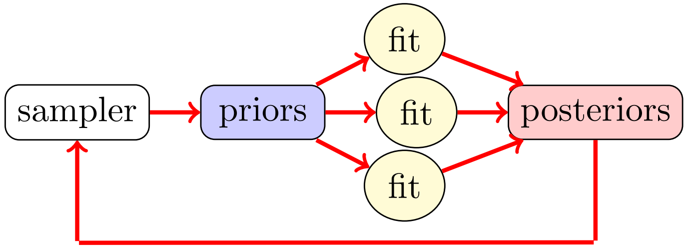
but sample significantly larger parameter space than possible in single-fit analyses

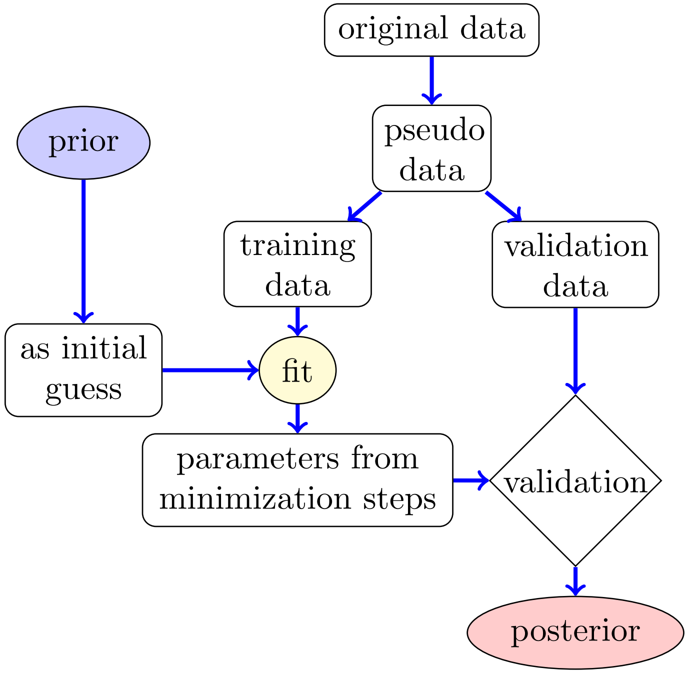
→ no assumptions on exponents

→ cross-validation to avoid overfitting

→ iterate until convergence criteria satisfied

→ unambiguous determination of PDF uncertainties





Inclusive DIS global analysis

- Maximally utilize high-precision, high-statistics spin data at lower (as well as higher) energies

  → ~ 15 experiments completed at JLab, with data straddling resonance & DIS regions

  → explore systematics of lowering kinematic cuts down to $Q^2 > 1\ \mathrm{GeV}^2, \quad W^2 > 3.5\ \mathrm{GeV}^2$

  → control of nuclear and finite-Q^2 corrections

  → fit experimental L & T asymmetries rather than derived structure functions

  → constrain (poorly-determined) PDFs at large x, and extract higher twist (twist-3) distributions

Inclusive DIS global analysis

- Inclusive DIS data constrain Δu^+ & Δd^+ distributions

  $\longrightarrow$ mostly insensitive to polarized strangeness and glue

- Assume g_1, g_2 can be described as sum of twist $\tau = 2$ and higher twist terms

$$g_1 = g_1^{\tau 2(\mathrm{TMC})} + g_1^{\tau 3(\mathrm{TMC})} + g_1^{\tau 4}$$

$$g_2 = g_2^{\tau 2(\mathrm{TMC})} + g_2^{\tau 3(\mathrm{TMC})}$$

includes OPE target mass corrections

- Structure function (moments) at leading twist τ (at NLO)

$$g_{1,\tau 2}^{(n)} = \frac{1}{2} \sum_q e_q^2 \left(\Delta C_{qq}^{(n)} \, \Delta q^{(n)} + \Delta C_g^{(n)} \, \Delta g^{(n)} \right)$$

$$g_{2,\tau 2}^{(n)} = -\frac{n-1}{n} g_{1,\tau 2}^{(n)}$$

Wandzura–Wilczek relation

Inclusive DIS global analysis

Higher twist corrections

→ twist-3 part of g_1 related to twist-3 part of g_2

$$g_1^{\tau 3} = (\rho^2 - 1)\left[g_2^{\tau 3} - 2\int_x^1 \frac{dy}{y}\, g_2^{\tau 3}\right]$$

→ twist-3 part of g_2 parametrized via twist-3 PDFs

$$D^{\tau 3}(x) = N x^a (1-x)^b (1 + c\, x)$$

NOT Q^2 SUPPRESSED!

– at parton level

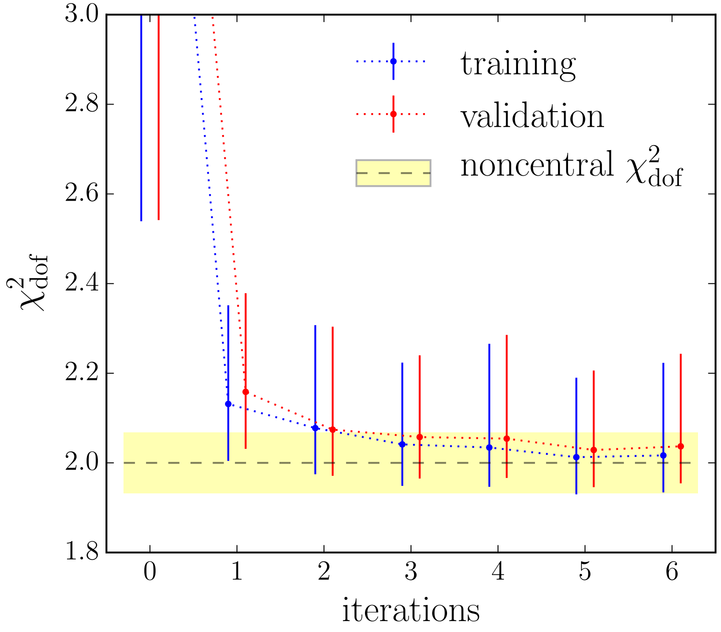
→ similar functional form also for twist-4 part

$$g_1^{\tau 4} = \frac{h(x)}{Q^2} = N' x^{a'} (1-x)^{b'} (1 + \gamma'\, x)\, \frac{1}{Q^2}$$

– at hadron level

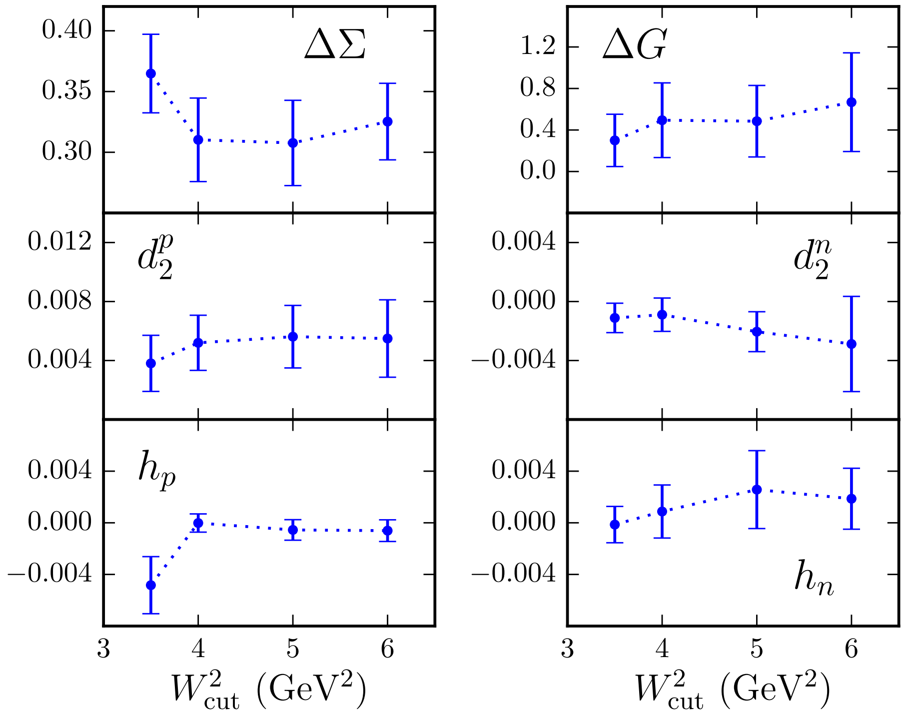
Inclusive DIS global analysis

Convergence criteria



→ convergence after $\sim$ 5–6 iterations

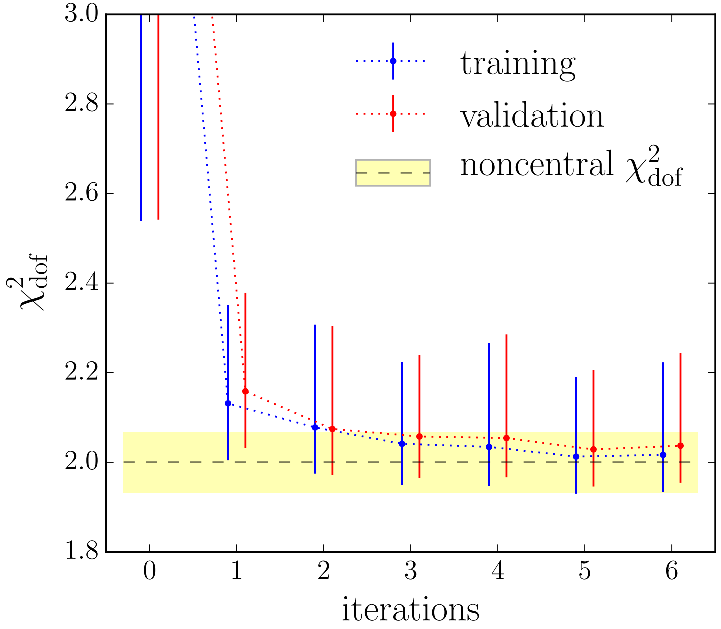
Sensitivity to kinematic cuts



→ stability for $W^2 > 4\ {\rm GeV}^2$

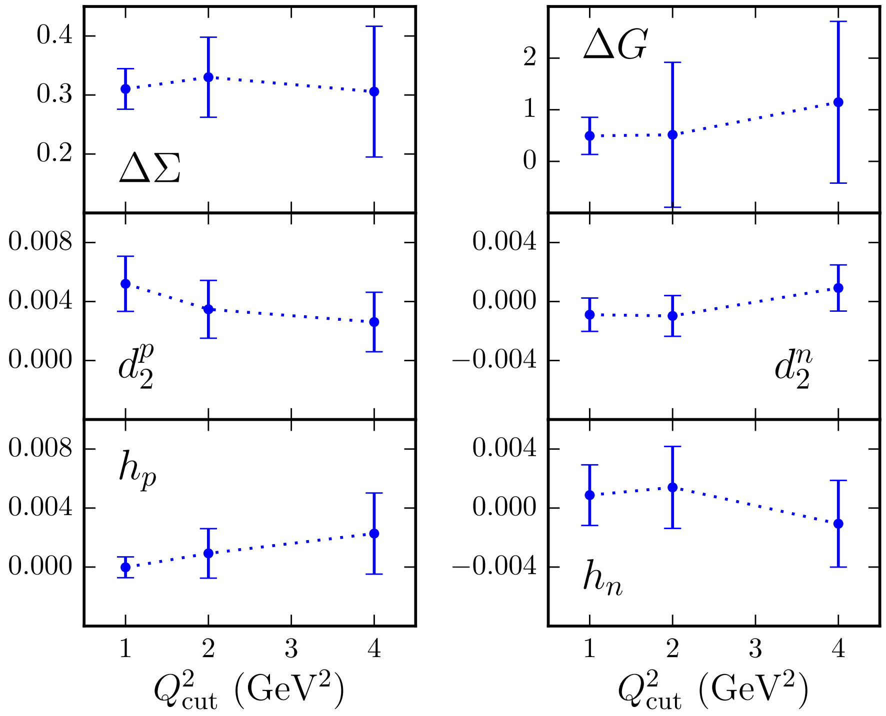
Inclusive DIS global analysis

🟥 Convergence criteria



→ convergence after ~ 5–6 iterations

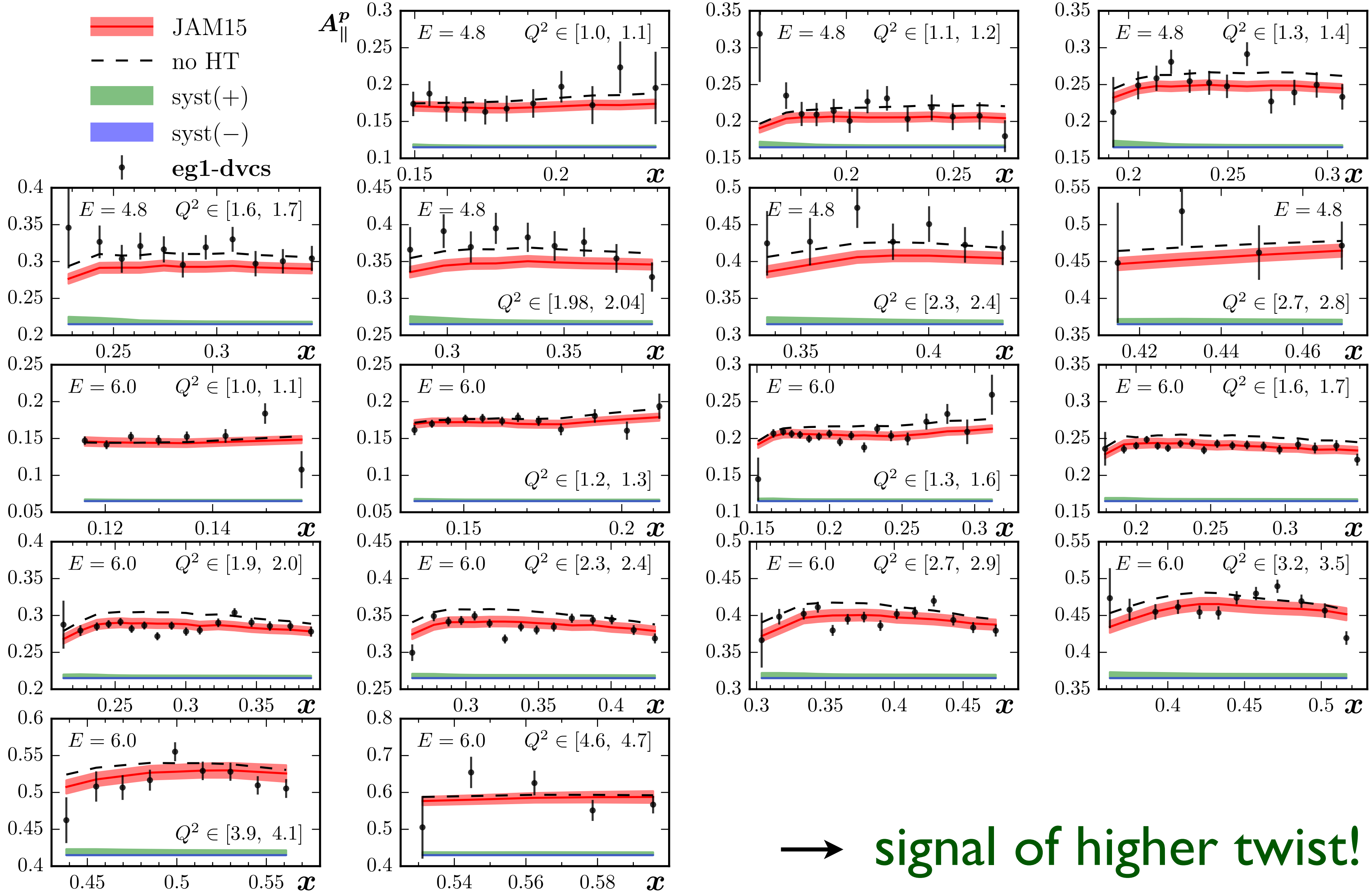
🟥 Sensitivity to kinematic cuts



→ stability for $W^2 > 4$ GeV^2 and $Q^2 > 1$ GeV^2

Inclusive DIS global analysis

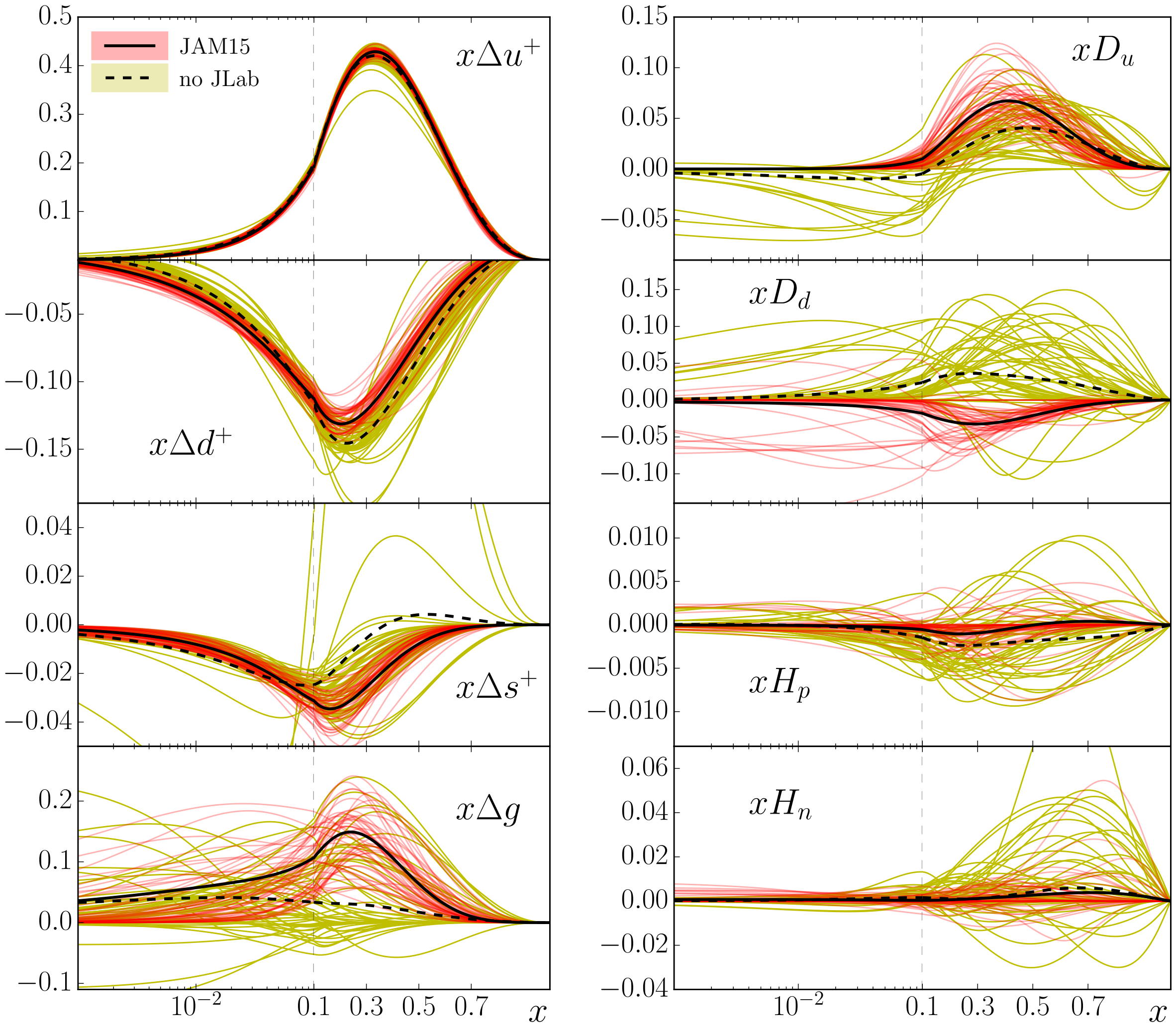
- **JLab eg1-dvcs (CLAS) data**



→ signal of higher twist!

Impact of JLab data

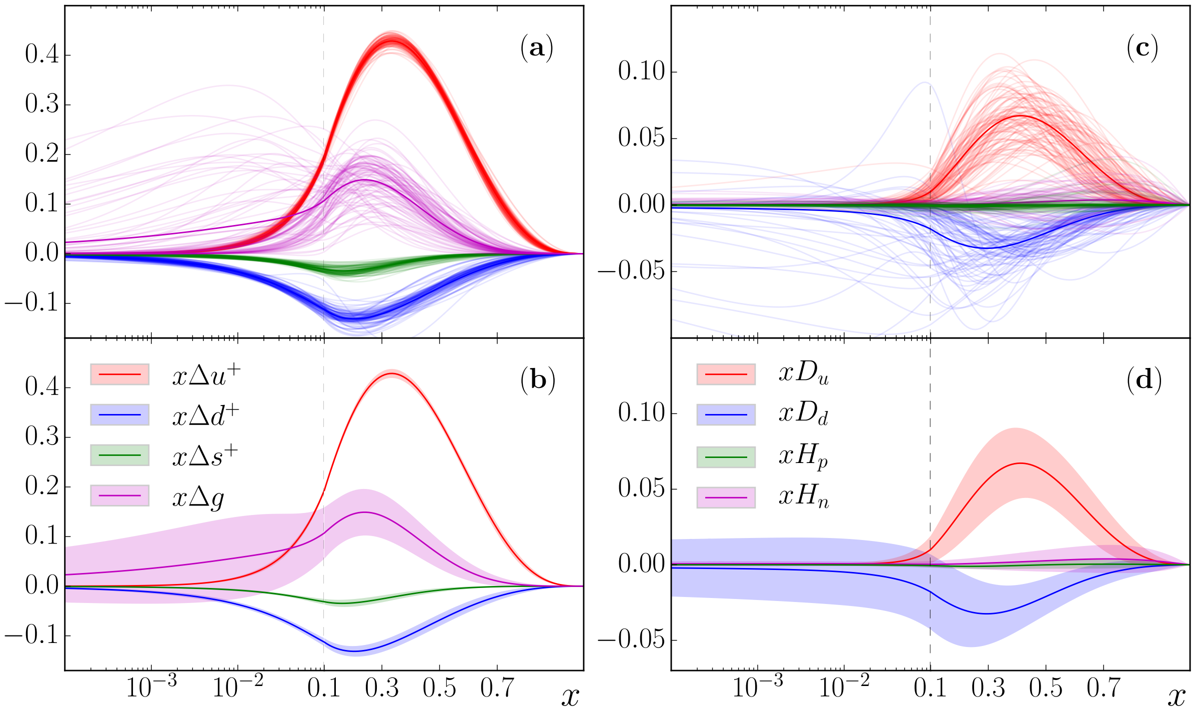
Experiment	Reference	Observable	Target	Number of points	$\chi^2_{\rm dof}$
EMC	[69]	A_1	p	10	0.40
SMC	[70]	A_1	p	12	0.47
SMC	[70]	A_1	d	12	1.62
SMC	[71]	A_1	p	8	1.26
SMC	[71]	A_1	d	8	0.57
COMPASS	[72]	A_1	p	15	0.92
COMPASS	[73]	A_1	d	15	0.67
COMPASS	[39]	A_1	p	51	0.76
SLAC E80/E130	[74]	$A_\parallel$	p	22	0.59
SLAC E142	[75]	A_1	^{3}He	8	0.49
SLAC E142	[75]	A_2	^{3}He	8	0.60
SLAC E143	[76]	$A_\parallel$	p	81	0.80
SLAC E143	[76]	$A_\parallel$	d	81	1.12
SLAC E143	[76]	$A_\perp$	p	48	0.89
SLAC E143	[76]	$A_\perp$	d	48	0.91
SLAC E154	[77]	$A_\parallel$	^{3}He	18	0.51
SLAC E154	[77]	$A_\perp$	^{3}He	18	0.97
SLAC E155	[78]	$A_\parallel$	p	71	1.20
SLAC E155	[79]	$A_\parallel$	d	71	1.05
SLAC E155	[80]	$A_\perp$	p	65	0.99
SLAC E155	[80]	$A_\perp$	d	65	1.52
SLAC E155x	[81]	$\tilde{A}_\perp$	p	116	1.27
SLAC E155x	[81]	$\tilde{A}_\perp$	d	115	0.83
HERMES	[82]	A_1	"n"	9	0.25
HERMES	[83]	$A_\parallel$	p	35	0.47
HERMES	[83]	$A_\parallel$	d	35	0.94
HERMES	[84]	A_2	p	19	0.93
JLab E99-117	[85]	$A_\parallel$	^{3}He	3	0.27
JLab E99-117	[85]	$A_\perp$	^{3}He	3	1.58
JLab E06-014	[17]	$A_\parallel$	^{3}He	14	2.12
JLab E06-014	[18]	$A_\perp$	^{3}He	14	1.06
JLab eg1-dvcs	[15]	$A_\parallel$	p	195	1.52
JLab eg1-dvcs	[15]	$A_\parallel$	d	114	0.94
JLab eg1b	[14]	$A_\parallel$	p	890	1.11
JLab eg1b	[16]	$A_\parallel$	d	218	1.02
Total				2515	1.07



→ reduced uncertainty in Δs^+, Δg

→ nonzero twist-3 contributions

Impact of JLab data



→ twist-3 PDFs large!

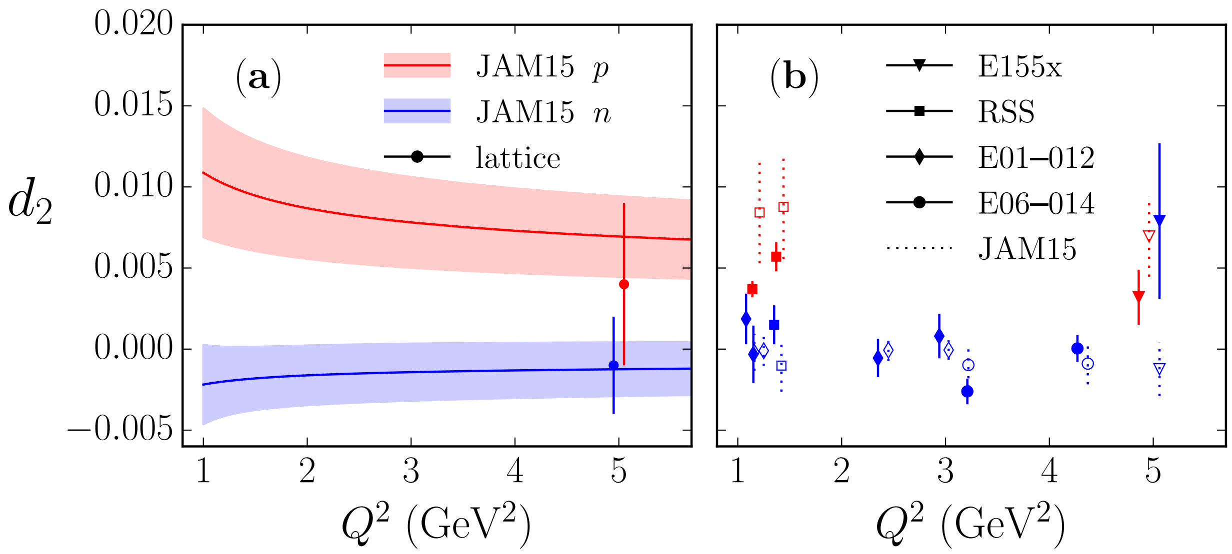
→ same sign as twist-2

→ twist-4 term negligible

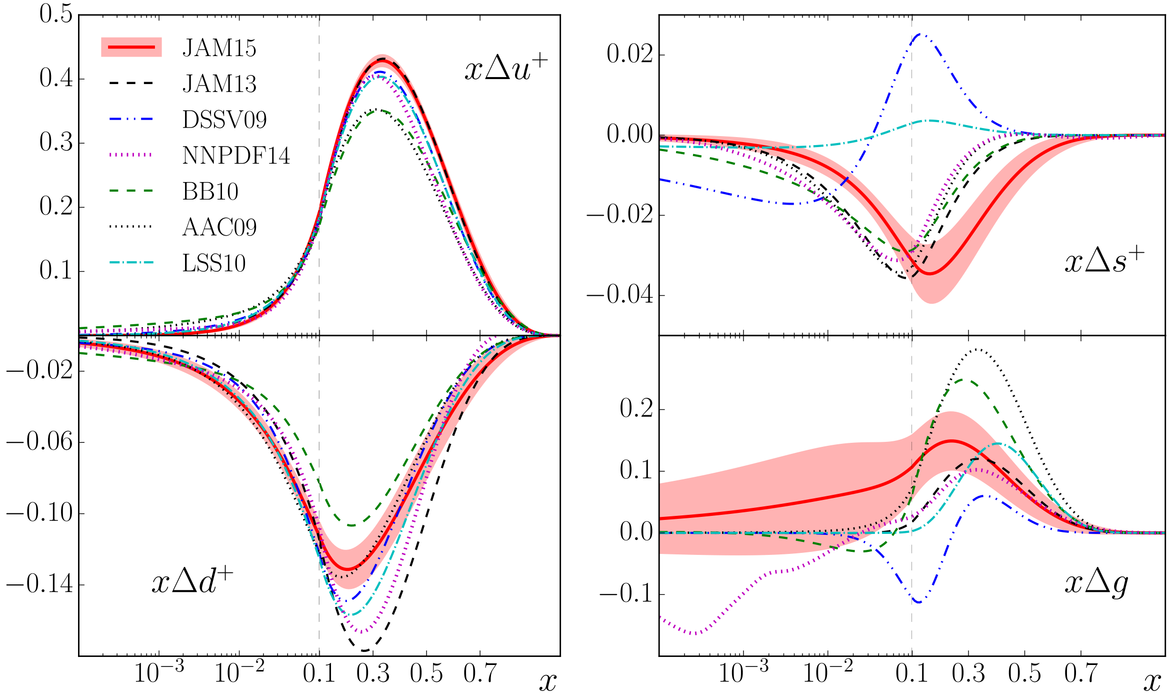
→ matrix element

$$d_2 = 2g_1^{(3)} + 3g_2^{(3)}$$

related to
"*color polarizability*"
or "*transverse force*"
acting on quarks



Comparison with other analyses



Moment	Truncated	Full
Δu^+	0.82 ± 0.01	0.83 ± 0.01
Δd^+	-0.42 ± 0.01	-0.44 ± 0.01
Δs^+	-0.10 ± 0.01	-0.10 ± 0.01
$\Delta\Sigma$	0.31 ± 0.03	0.28 ± 0.04
ΔG	0.5 ± 0.4	1 ± 15
d_2^p	0.005 ± 0.002	0.005 ± 0.002
d_2^n	-0.001 ± 0.001	-0.001 ± 0.001
h_p	-0.000 ± 0.001	0.000 ± 0.001
h_n	0.001 ± 0.002	0.001 ± 0.003

→ *u* and *d* polarization similar to earlier results

→ *s*-quark polarization *negative*

→ gluon polarization similar to recent DSSV fits
— moment unconstrained

Polarization of quark sea?

- Inclusive DIS data cannot distinguish between q and $\bar{q}$

  → semi-inclusive DIS sensitive to Δq & $\Delta\bar{q}$

$$\sim \sum_q e_q^2 \left[\Delta q(x)\, D_q^h(z) + \Delta\bar{q}(x)\, D_{\bar{q}}^h(z)\right]$$

  → but need fragmentation functions!

- Global analysis of DIS + SIDIS data gives different *sign* for strange quark polarization for different fragmentation functions!

  → $\Delta s > 0$ for "DSS" parametrization *de Florian et al., PRD75, 094009 (2007)*

  $\Delta s < 0$ for "HKNS" parametrization *Hirai et al., PRD75, 114010 (2007)*

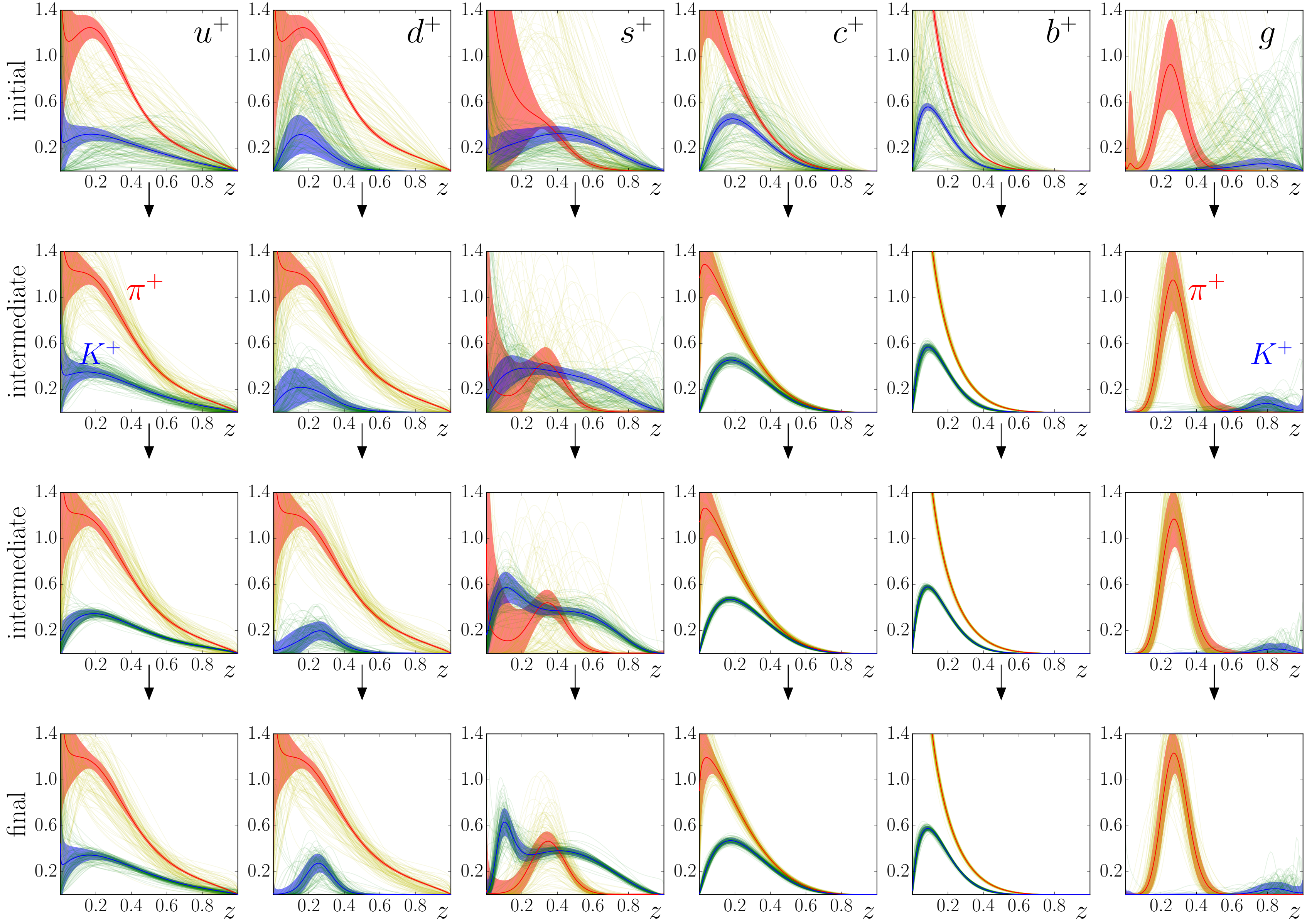
  → need to understand origin of differences in fragmentation!

IMC analysis of fragmentation functions

- Analyze single-inclusive e^+e^- annihilation data for pion & kaon production from DESY, CERN, SLAC & KEK from $Q \sim 10$ GeV to Z-boson pole

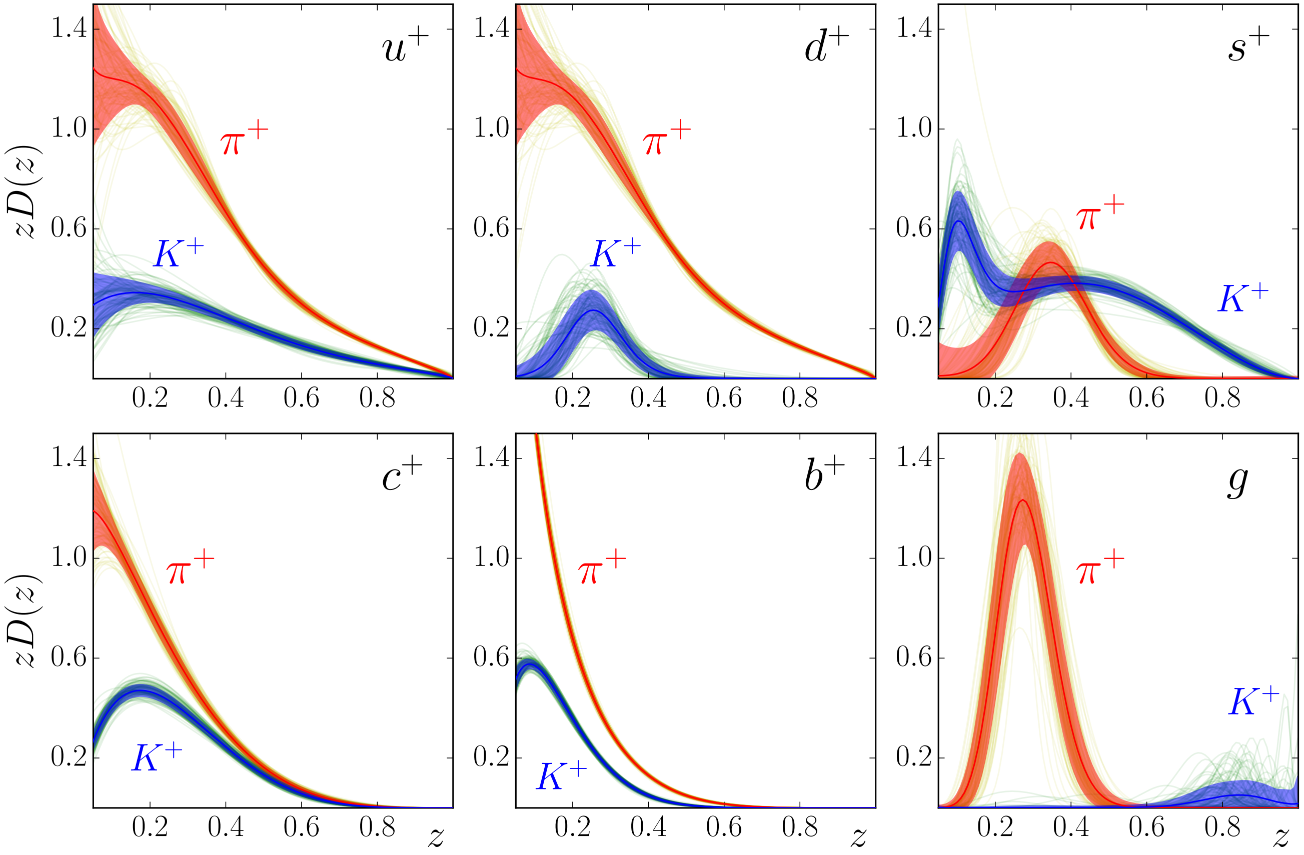
				Pions			Kaons		
Experiment	Ref.	Observable	Q (GeV)	$N_{\rm dat}$	norm.	χ^2	$N_{\rm dat}$	norm.	χ^2
ARGUS	[26]	Inclusive	9.98	35	1.024 (1.058)	51.1 (55.8)	15	1.007	8.5
Belle	[38,39]	Inclusive	10.52	78	0.900 (0.919)	37.6 (21.7)	78	0.988	10.9
BABAR	[40]	Inclusive	10.54	39	0.993 (0.948)	31.6 (70.7)	30	0.992	4.9
TASSO	[23–25]	Inclusive	12-44	29	(*)	37.0 (38.8)	18	(*)	14.3
TPC	[27–29]	Inclusive	29.00	18	1	36.3 (57.8)	16	1	47.8
		uds tag	29.00	6	1	3.7 (4.6)			
		b tag	29.00	6	1	8.7 (8.6)			
		c tag	29.00	6	1	3.3 (3.0)			
HRS	[30]	Inclusive	29.00	2	1	4.2 (6.2)	3	1	0.3
TOPAZ	[37]	Inclusive	58.00	4	1	4.8 (6.3)	3	1	0.9
OPAL	[32,33]	Inclusive	91.20	22	1	33.3 (37.2)	10	1	6.3
		u tag	91.20	5	1.203 (1.203)	6.6 (8.1)	5	1.185	2.1
		d tag	91.20	5	1.204 (1.203)	6.1 (7.6)	5	1.075	0.6
		s tag	91.20	5	1.126 (1.200)	14.4 (11.0)	5	1.173	1.5
		c tag	91.20	5	1.174 (1.323)	10.7 (6.1)	5	1.169	13.2
		b tag	91.20	5	1.218 (1.209)	34.2 (36.6)	4	1.177	10.9
ALEPH	[34]	Inclusive	91.20	22	0.987 (0.989)	15.6 (20.4)	18	1.008	6.1
DELPHI	[35,36]	Inclusive	91.20	17	1	21.0 (20.2)	27	1	3.9
		uds tag	91.20	17	1	13.3 (13.4)	17	1	22.5
		b tag	91.20	17	1	41.9 (42.9)	17	1	9.1
SLD	[31]	Inclusive	91.28	29	1.002 (1.004)	27.3 (36.3)	29	0.994	14.3
		uds tag	91.28	29	1.003 (1.004)	51.7 (55.6)	29	0.994	42.6
		c tag	91.28	29	0.998 (1.001)	30.2 (40.4)	29	1.000	31.7
		b tag	91.28	29	1.005 (1.005)	74.6 (61.9)	28	0.992	134.1
Total:				459		599.3 (671.2)	391		395.0
					$\chi^2/N_{\rm dat} = 1.31$ (1.46)				$\chi^2/N_{\rm dat} = 1.01$

IMC analysis of fragmentation functions



→ convergence after ~ 20 iterations

IMC analysis of fragmentation functions



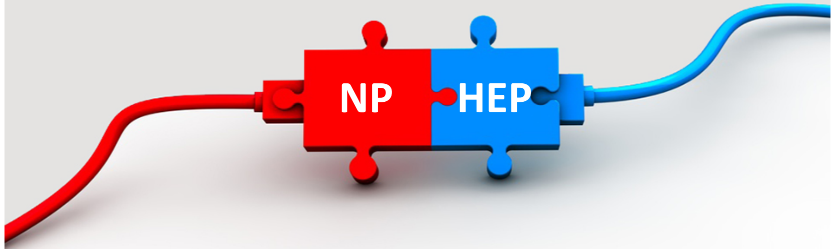
- → favored FFs well constrained; unfavored not as well…
- → nontrivial shape of $s \to K$ fragmentation — impact on Δs^+ extraction?
- → very hard $g \to K$ fragmentation??

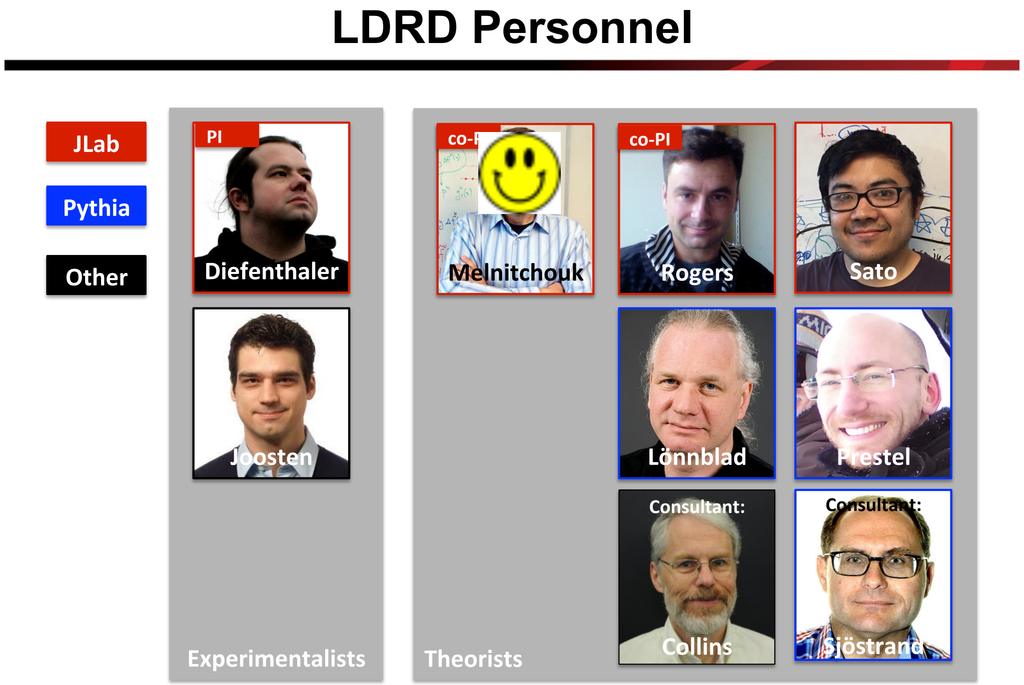
Synergy with event generators

- Can one obtain further insights into shapes and magnitudes of FFs from MC event generators, $e.g.$ Pythia?

→ JLab LDRD

Phenomenological Study of Hadronization in Nuclear and High-Energy Physics Experiments





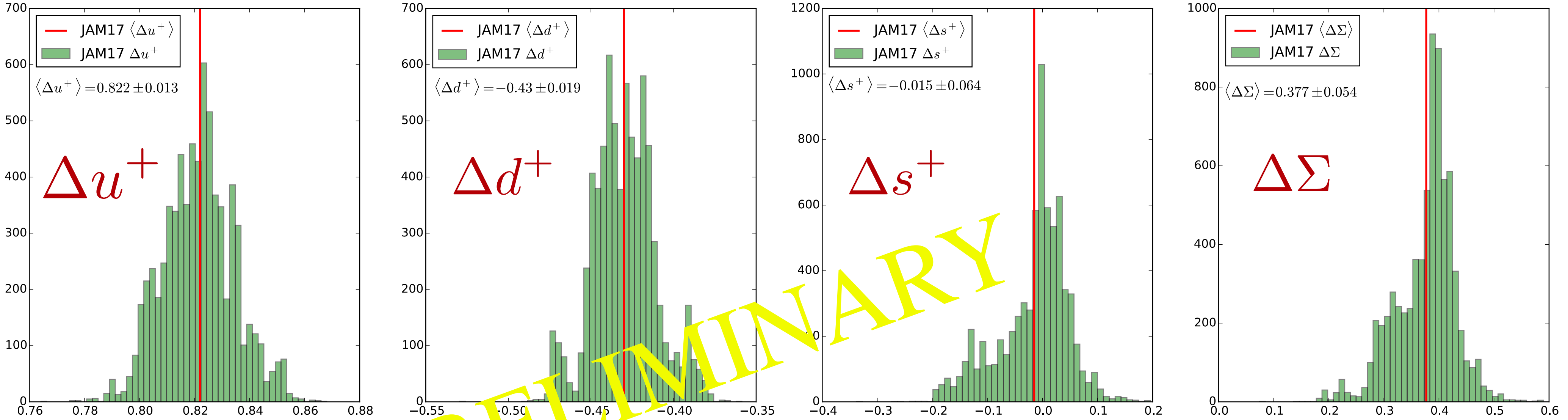
→ compare Lund string fragmentation with "CSS" (Collins-Soper-Sterman) type factorization

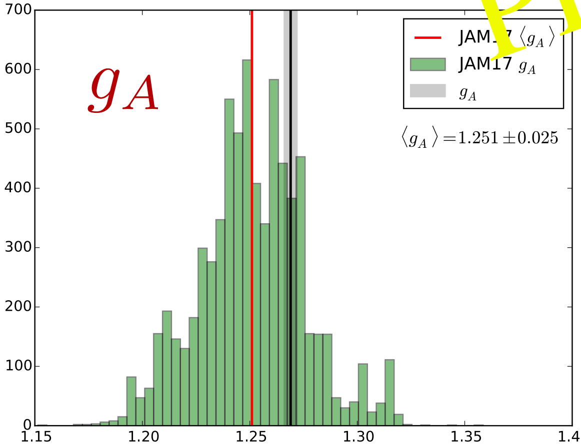
→ develop MC event generator for TMDs, including spin

→ *Markus Diefenthaler's talk*

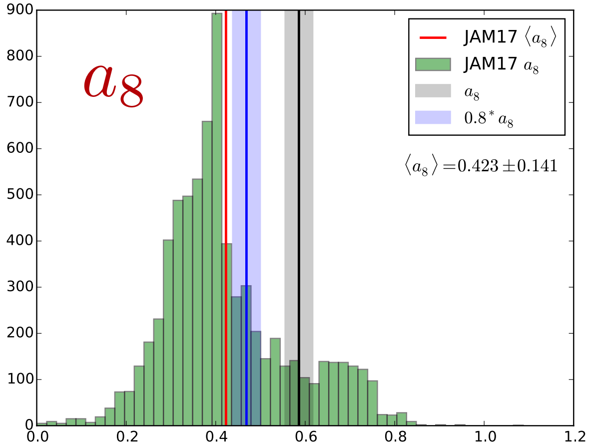
Simultaneous PDF + FF analysis

- First combined analysis of DIS + SIDIS + SIA data, with *simultaneous* extraction of PDFs and fragmentation functions





test of Bjorken sum rule



test of SU(3) symmetry

J. Ethier (2017)

Role of PDFs in 3-D structure

- Factorization in TMD observables

$$\begin{aligned}\Gamma &= \Gamma \\ &= \mathbf{T}_{\rm TMD}\Gamma + [\Gamma - \mathbf{T}_{\rm TMD}\Gamma] \\ &= \underbrace{\mathbf{T}_{\rm TMD}\Gamma}_{\mathbf{W}} + \underbrace{\mathbf{T}_{\rm coll}\,[\Gamma - \mathbf{T}_{\rm TMD}\Gamma]}_{\mathbf{Y}} + \mathcal{O}(m^2/Q^2)\Gamma\end{aligned}$$

Γ = any TMD observable

- Region of $q_T \ll Q$
  - TMD approx. dominates → $\Gamma \approx \mathbf{T}_{\rm TMD}\Gamma$
  - $\mathbf{Y}$ term small

"TMD jargon"

$$\mathbf{W} = \mathbf{T}_{\rm TMD}\Gamma$$
$$\mathbf{FO} = \mathbf{T}_{\rm coll}\Gamma$$
$$\mathbf{ASY} = \mathbf{T}_{\rm coll}\mathbf{T}_{\rm TMD}\Gamma$$
$$\mathbf{Y} = \mathbf{FO} - \mathbf{ASY}$$

- Region of $q_T \gtrsim Q$
  - Collinear approx. dominates → $\Gamma \approx \mathbf{T}_{\rm coll}\Gamma$
  - At large Q, $\mathbf{T}_{\rm TMD}\Gamma$ is mostly perturbative

Collins, Gamberg, Prokudin, Rogers, Sato, Wang
*PRD **94**, 034014 (2016)*

Role of PDFs in 3-D structure

- **Cross section and structure functions (SIDIS)**

$$\begin{aligned}\frac{d^5\sigma(S_\perp)}{dx_B\, dQ^2\, dz_h\, d^2P_{h\perp}} = \sigma_0\Big[& F_{UU} + \sin(\phi_h - \phi_s)\; F_{UT}^{\sin(\phi_h-\phi_s)} \\ & + \sin(\phi_h + \phi_s)\, \frac{2(1-y)}{1+(1-y)^2}\; F_{UT}^{\sin(\phi_h+\phi_s)} + \ldots\Big]\end{aligned}$$

- **CSS formalism**

$$\begin{aligned} F_{UU} &= H_{\text{SIDIS}}\frac{1}{z_h^2}\int_0^\infty \frac{db\, b}{(2\pi)} J_0(q_{h\perp} b)\, \widetilde{W}_{UU}(b_*) + Y_{UU} \\ F_{UT}^{\sin(\phi_h-\phi_s)} &= -H_{\text{SIDIS}}\frac{M_P}{z_h^2}\int_0^\infty \frac{db\, b^2}{(2\pi)} J_1\,(q_{h\perp} b)\; \widetilde{W}_{UT}^{\sin(\phi_h-\phi_s)}(b_*) + Y_{UT}^{\sin(\phi_h-\phi_s)} \\ F_{UT}^{\sin(\phi_h+\phi_s)} &= H_{\text{SIDIS}}\frac{M_h}{z_h^2}\int_0^\infty \frac{db\, b^2}{(2\pi)} J_1(q_{h\perp} b)\, \widetilde{W}_{UT}^{\sin(\phi_h+\phi_s)}(b_*) + Y_{UT}^{\sin(\phi_h+\phi_s)} \end{aligned}$$

hard scattering

$$\boxed{\begin{aligned} b_* &\to b, & b &\ll b_{\max} \\ &\to b_{\max}, & b &\gg b_{\max} \end{aligned}}$$

“Y” term

Role of PDFs in 3-D structure

- **W term formulation in b_T space**

$$\widetilde{W}_{UU}(b_*) \equiv e^{-S_{pert}(Q,b_*)-S_{\rm NP}^{\rm f_1}(Q,b)-S_{\rm NP}^{\rm D_1}(Q,b)}\ \widetilde{F}_{UU}(b_*)$$

$$\widetilde{W}_{UT}^{\sin(\phi_h-\phi_s)}(b_*) \equiv e^{-S_{pert}(Q,b_*)-S_{\rm NP}^{\rm f_{1T}^{\perp}}(Q,b)-S_{\rm NP}^{\rm D_1}(Q,b)}\ \widetilde{F}_{UT}^{\sin(\phi_h-\phi_s)}(b_*)$$

$$\widetilde{W}_{UT}^{\sin(\phi_h+\phi_s)}(b_*) \equiv e^{-S_{pert}(Q,b_*)-S_{\rm NP}^{\rm h_1}(Q,b)-S_{\rm NP}^{\rm H_1^{\perp}}(Q,b)}\ \widetilde{F}_{UT}^{\sin(\phi_h+\phi_s)}(b_*)$$

- **Small b_T contribution**

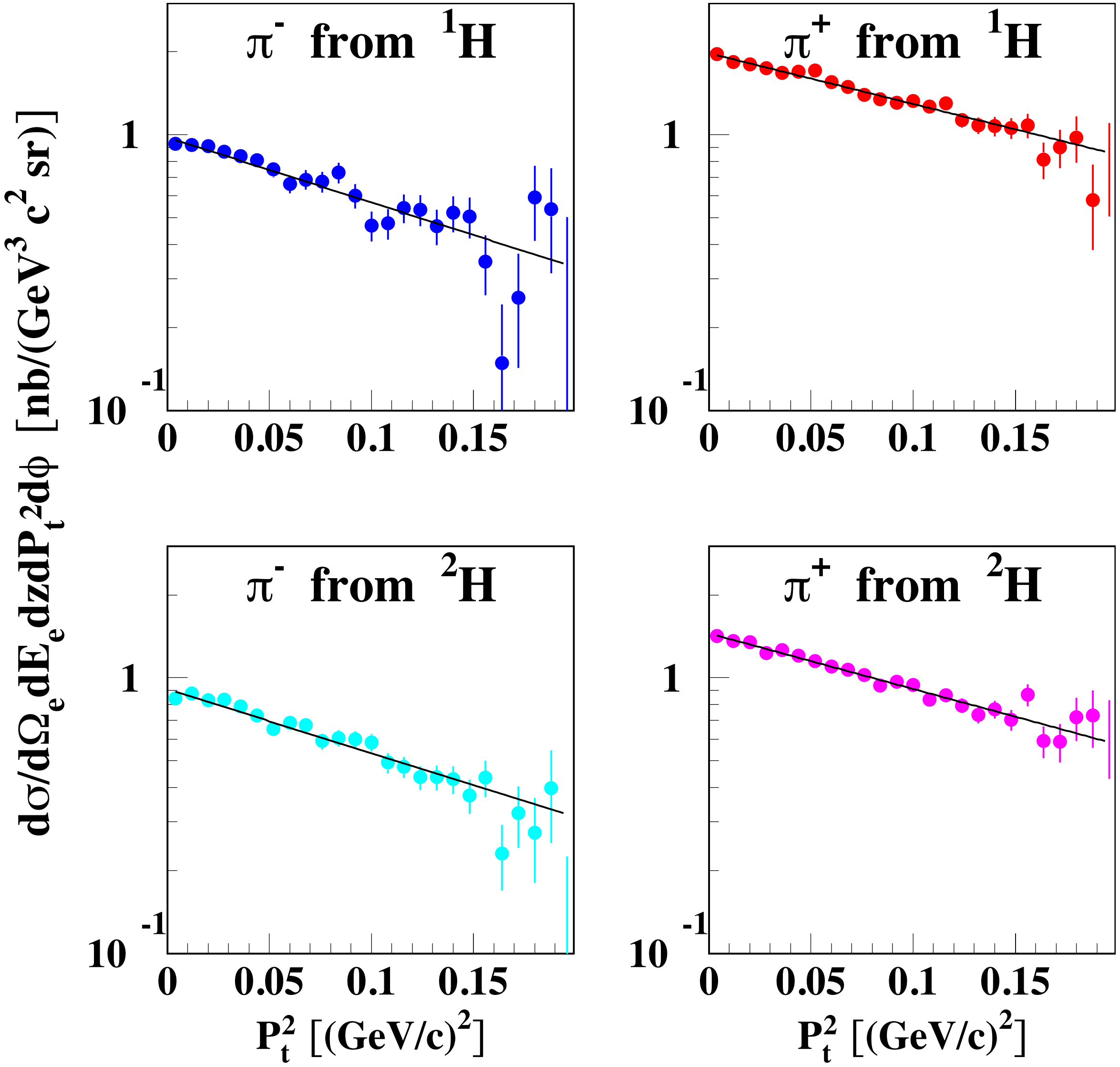
$$\widetilde{F}_{UU}(b_*) = \sum_q e_q^2 \left(C_{q\leftarrow i}^{\rm f_1} \otimes f_1^i(x_B,\mu_b)\right)\left(\hat{C}_{j\leftarrow q}^{\rm D_1} \otimes D_{h/j}(z_h,\mu_b)\right)$$

$$\widetilde{F}_{UT}^{\sin(\phi_h-\phi_s)}(b_*) = \sum_q e_q^2 \left(C_{q\leftarrow i}^{f_{1T}^{\perp}} \otimes f_{1T}^{\perp(1)i}(x_B,\mu_b)\right)\left(\hat{C}_{j\leftarrow q}^{\rm D_1} \otimes D_{h/j}(z_h,\mu_b)\right)$$

$$\widetilde{F}_{UT}^{\sin(\phi_h+\phi_s)}(b_*) = \sum_q e_q^2 \left(\delta C_{q\leftarrow i}^{h_1} \otimes h_1^i(x_B,\mu_b)\right)\left(\delta\hat{C}_{j\leftarrow q}^{\rm H_1^{\perp}} \otimes \hat{H}_1^{\perp(1)j}(z_h,\mu_b)\right)$$

collinear PDFs and FFs!

Can this describe TMD cross sections at low energies?

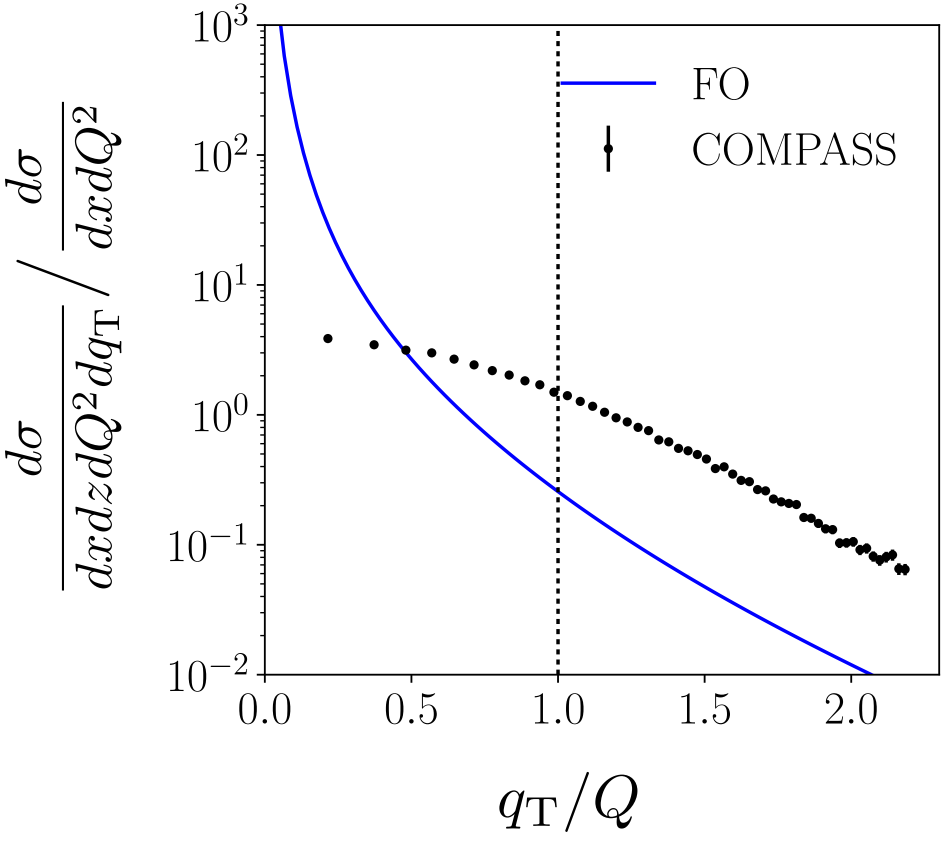


→ can fit low-p_T data with Gaussian distributions

→ *fit by Albright, Gamberg, Prokudin et al. (2016)*

Asaturyan et al. ***[JLab Hall C]****, PRC* ***85****, 015202 (2012)*

Can this describe TMD cross sections at low energies?



Sato, Wang, Rogers… (2016)

$Q^2 = 1.92\ \mathrm{GeV}^2$
$x = 0.0318$
$z = 0.375$

→ fixed-order (collinear) calculation *should* describe high-q_{T} region…

→ SIDIS cross section *must* be understood for any TMD analysis of JLab12 data!

Outlook

- Goal for collinear distributions
  — "universal" QCD analysis of all observables sensitive to collinear (unpolarized & polarized) PDFs & FFs in IMC framework

- Longer−term goal
  — apply IMC technology (where appropriate) to global QCD analysis of TMD PDFs and FFs

- Need to understand realm of applicability of TMD factorization at low energies
  — vital for analysis and interpretation of JLab12 data


THANK YOU!