



# 3D Nucleon Tomography Workshop

*Modeling and Extracting Methodology*

*March 15-17, 2017*

*Jefferson Lab, Newport News, VA*

## **TMD Theory and TMD Topical Collaboration**

**Jianwei Qiu**

*Theory Center, Jefferson Lab*





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## TMD Theory and TMD Topical Collaboration

*Why TMDs?*

*What is the TMD Topical Collaboration?*

*What does the collaboration want to achieve?*

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*Theory Center, Jefferson Lab*

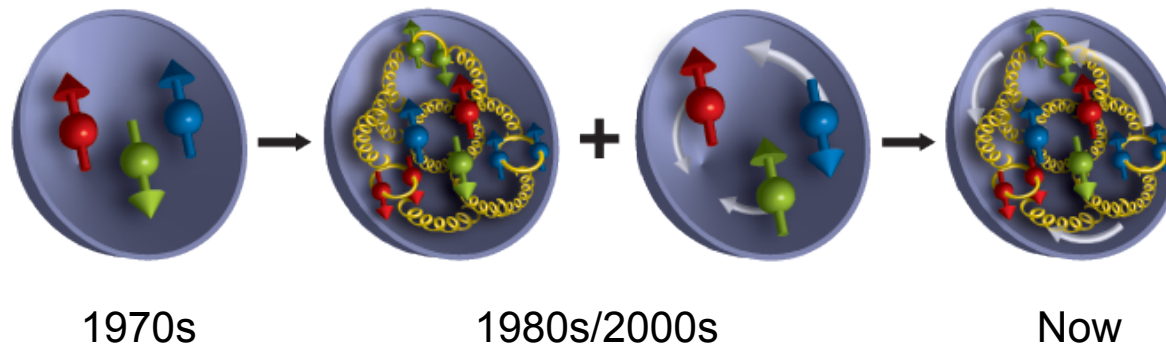


# Hadron and Hadron Structure

- ❑ Nucleons are the fundamental building blocks of all atomic nuclei, and make up essentially almost all the mass of the visible universe
- ❑ Understanding the structure of hadrons in terms of QCD's quarks and gluons is one of the central goals of modern nuclear physics – The 2015 *LONG RANGE PLAN* for *NUCLEAR SCIENCE*

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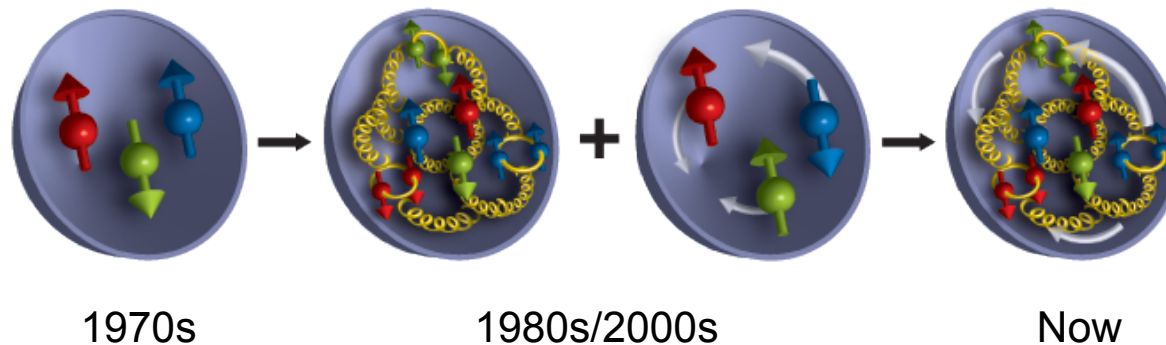
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In QCD, nucleon emerges as a strongly interacting, relativistic bound state of quarks and gluons

- ❑ This has been identified as a major theme and the great intellectual challenge of the DOE Nuclear Theory subprogram

# Hadron structure in QCD

## □ What do we need to know for the structure?

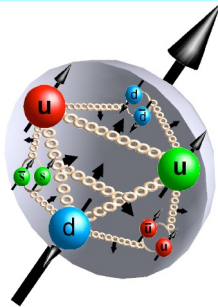
✧ In theory:  $\langle P, S | \mathcal{O}(\bar{\psi}, \psi, A^\mu) | P, S \rangle$  – Hadronic matrix elements

with all possible operators:  $\mathcal{O}(\bar{\psi}, \psi, A^\mu)$

✧ In fact: *None of these matrix elements is a direct physical observable in QCD – color confinement!*

✧ In practice: Accessible hadron structure  
= hadron matrix elements of quarks and gluons, which

- 1) can be related to physical cross sections of hadrons and leptons with controllable approximation; and/or
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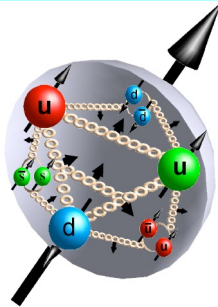
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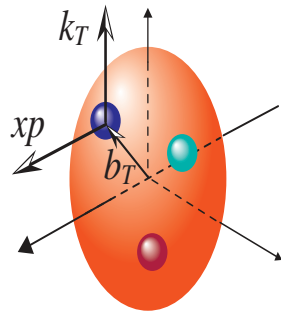


## □ Single-parton structure “seen” by a short-distance probe:

✧ 5D structure: 1)  $\int d^2 b_T \longrightarrow f(x, k_T, \mu)$  – TMDs: 2D confined motion!

2)  $\int d^2 k_T \longrightarrow F(x, b_T, \mu)$  – GPDs: 2D spatial imaging!

3)  $\int d^2 k_T d^2 b_T \longrightarrow f(x, \mu)$  – PDFs: Number density!



# Hadron structure in QCD

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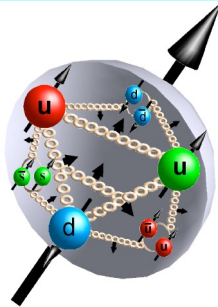
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## □ Multi-parton correlations:

$$\sigma(Q, \vec{s}) \propto \left| \begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \\ \text{Diagram 3} \\ \vdots \end{array} \right|^2 \left( \frac{\langle k_\perp \rangle}{Q} \right)^n \text{ – Expansion}$$

The diagrams show a series of Feynman-like diagrams for a scattering process. The first diagram shows a vertex with incoming lines labeled  $p, \vec{s}$  and  $k$ , and outgoing lines labeled  $t \sim 1/Q$ . The second diagram shows a similar vertex with a wavy line (gluon) connecting to another vertex. The third diagram shows a similar vertex with a wavy line connecting to another vertex. The diagrams are summed and then squared to give the cross section.

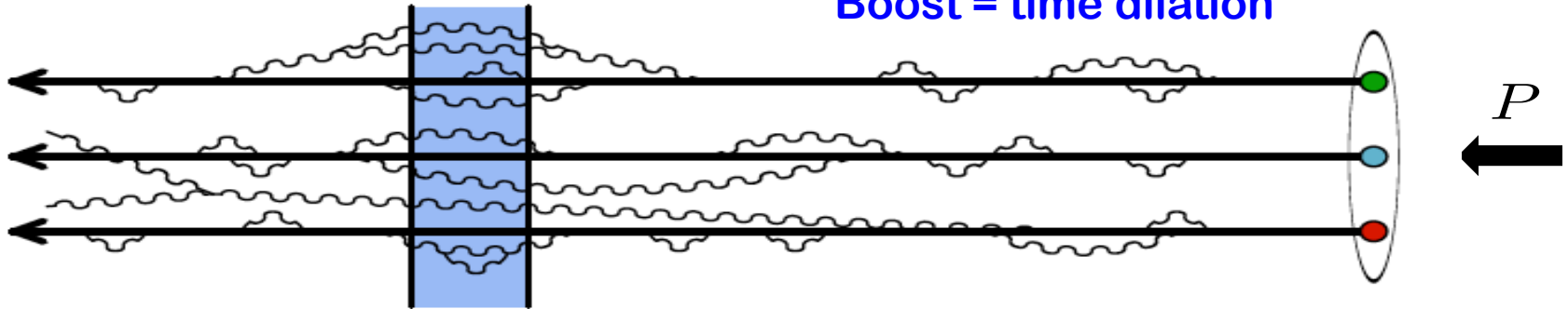
Quantum interference  $\longrightarrow$  3-parton matrix element – not a probability!



# How to “see” the hadron structure?

□ Need high energy probes to “see” the **boosted** structure:

Boost = time dilation



*Hard probe ( $t \sim 1/Q < fm$ ): Catches the quantum fluctuation!*

✧ Longitudinal momentum fraction –  $x$ :

$$xP \sim Q$$

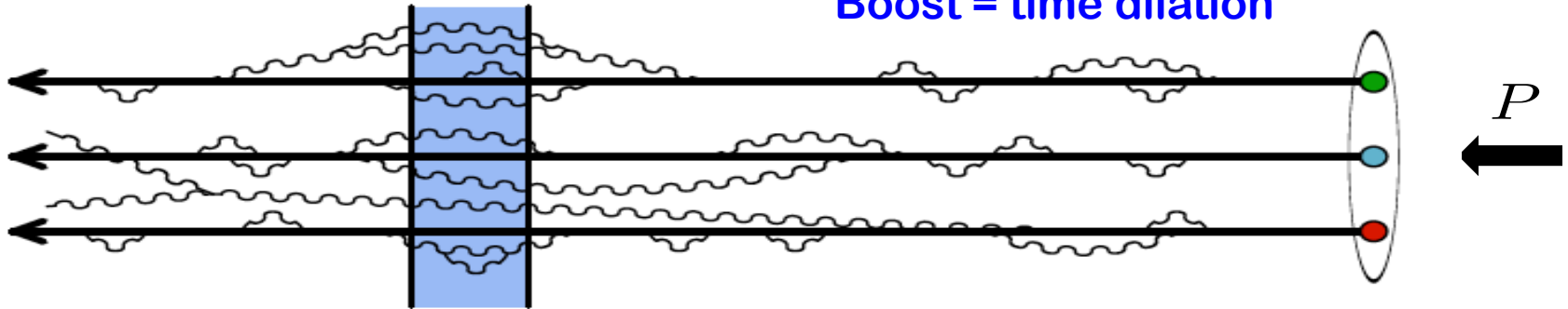
✧ Transverse momentum – **confined motion**:

$$1/R \sim \Lambda_{\text{QCD}} \ll Q$$

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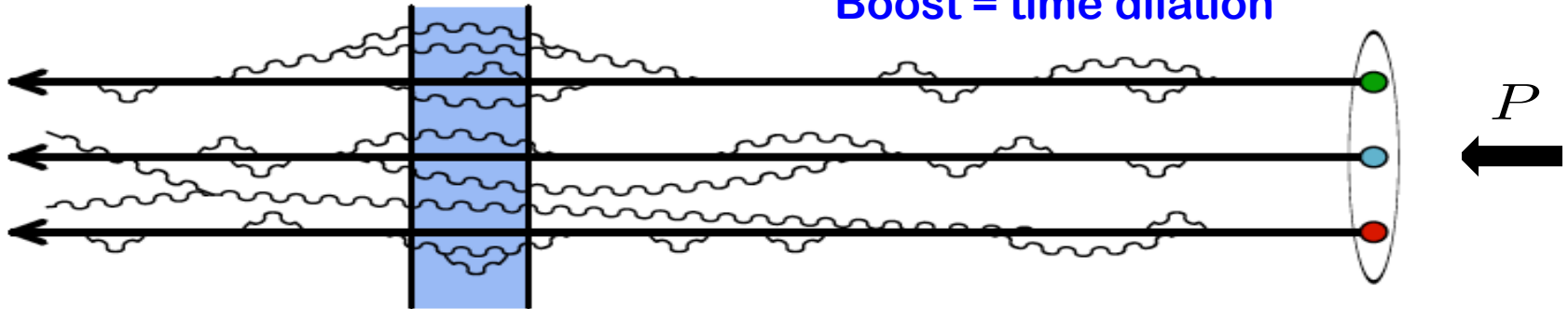
- ❑ Challenge:

**No modern detector can see quarks and gluons in isolation!**

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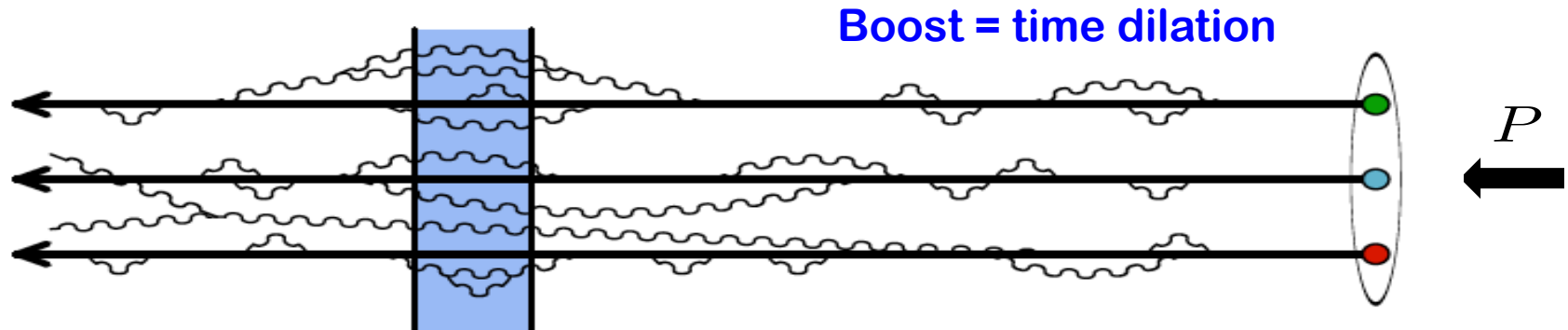
**No modern detector can see quarks and gluons in isolation!**

- ❑ Question:

How to quantify the hadron structure if we cannot see quarks and gluons?

# How to “see” the hadron structure?

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How to quantify the hadron structure if we cannot see quarks and gluons?

- ❑ Answer:

**QCD factorization!** *Not exact, but, controllable approximation!*

# QCD factorization - approximation

- ❑ Cross section with identified hadron(s) is **NON-Perturbative!**

# QCD factorization - approximation

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$$\sigma_{\text{DIS}}(x, Q^2) = \left| \text{Diagram 1} \right|^2 = \left| \text{Diagram 2} + \text{Diagram 3} + \text{Diagram 4} + \dots \right|^2$$

The equation shows the deep inelastic scattering (DIS) cross-section  $\sigma_{\text{DIS}}(x, Q^2)$  as the squared magnitude of a sum of Feynman diagrams. The first term is a single diagram with a blue oval representing a non-perturbative hadronic structure, a blue wavy line for the incoming photon, and several black lines for the outgoing hadrons. The subsequent terms in the sum show the same structure but with additional green wavy lines (representing gluons) and black dots (representing quarks) attached to the hadronic structure, illustrating the perturbative expansion of the non-perturbative object. The diagrams are separated by plus signs, and the entire sum is enclosed in large vertical bars with a superscript 2, indicating the squared magnitude of the sum.

# QCD factorization - approximation

□ Cross section with identified hadron(s) is **NON-Perturbative!**

$$\begin{aligned}
 \sigma_{\text{DIS}}(x, Q^2) &= \left| \text{Diagram 1} \right|^2 = \left| \text{Diagram 2} + \text{Diagram 3} + \text{Diagram 4} + \dots \right|^2 \\
 &= \text{Diagram 5} + \text{Diagram 6} + \text{Diagram 7} + \text{Diagram 8} + \dots \\
 &= c_q \otimes q(x, Q^2) + c_g \otimes g(x, Q^2) + c_{qg} \otimes T_{qg}(\{x\}, Q^2) + c_{gg} \otimes T_{gg}(\{x\}, Q^2) \\
 &\quad + \mathcal{O}(\langle k_T^n \rangle / Q^n, \langle F^{2n} \rangle / Q^n) + \dots
 \end{aligned}$$

The diagrams illustrate the expansion of the DIS cross-section. The first row shows the cross-section as the square of a sum of diagrams. The second row shows the expansion of the square into a sum of diagrams. The third row shows the factorization of these diagrams into coefficient functions (c\_q, c\_g, c\_{qg}, c\_{gg}) and parton distribution functions (q, g, T\_{qg}, T\_{gg}).

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Leading power  
Linear contribution  
DGLAP regime

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**Leading power**
**Power corrections**

**Linear contribution**
**Non-Linear contribution**

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**Leading power**  
**Linear contribution**  
**DGLAP regime**

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**Power corrections**  
**Non-Linear contribution**  
**Multi-parton correlations**

$$\approx c_q \otimes q(x, Q^2) + c_g \otimes g(x, Q^2) + \mathcal{O}\left(\frac{\langle k_T^2 \rangle}{Q^2}, \frac{\langle F^2 \rangle}{Q^2}, \dots\right)$$

**Approximation – Leading power/twist factorization!**

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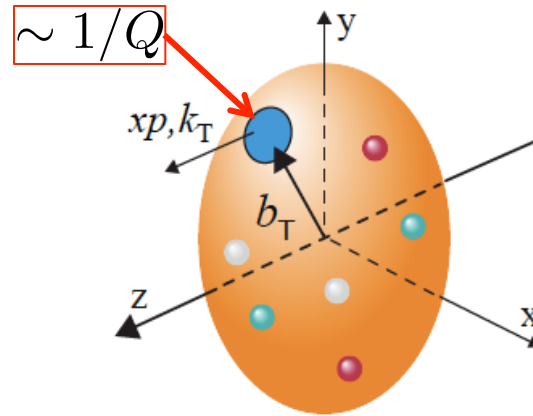
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**Non-perturbative**  
**physics neglected**  
**or in input PDFs!**

**Approximation – Leading power/twist factorization!**

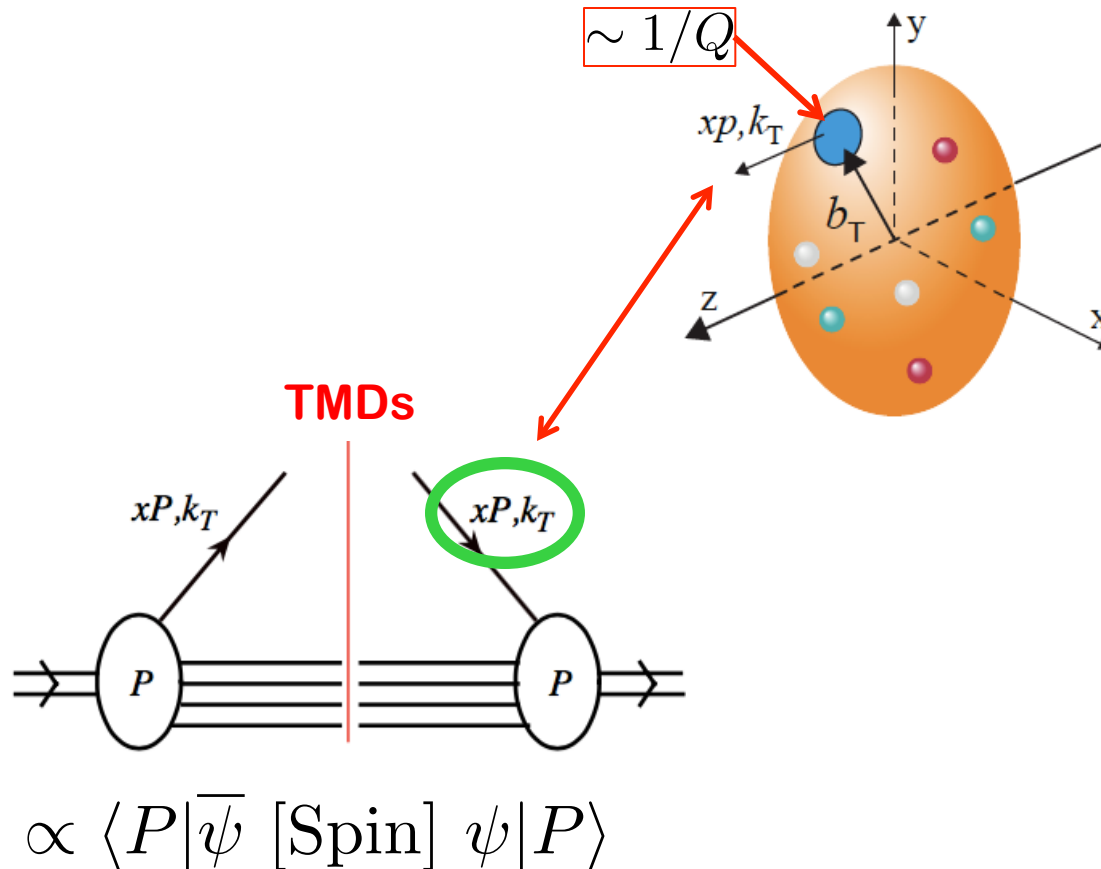
# How to quantify the hadron structure?

## □ Quantifying the “structure”:



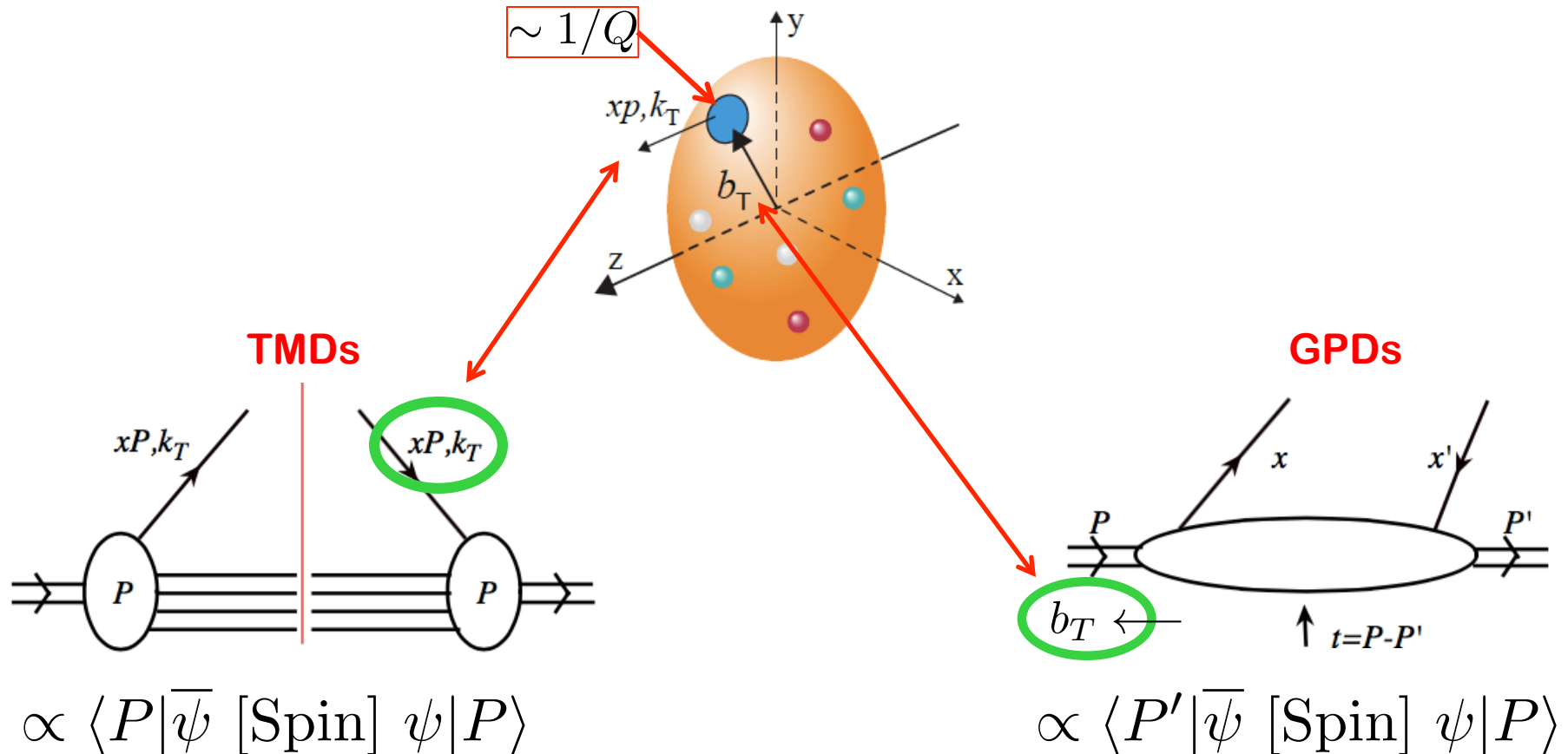
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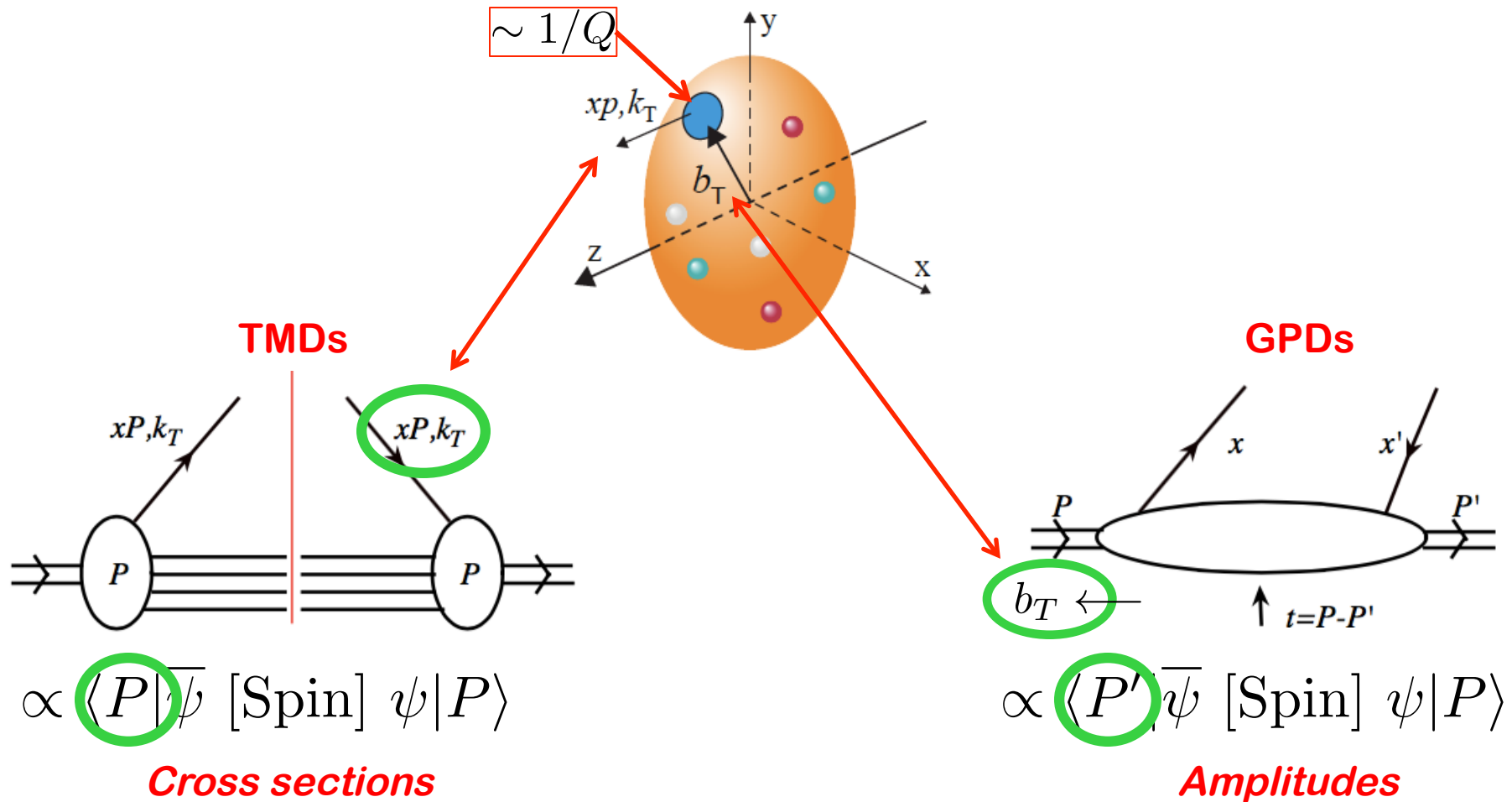
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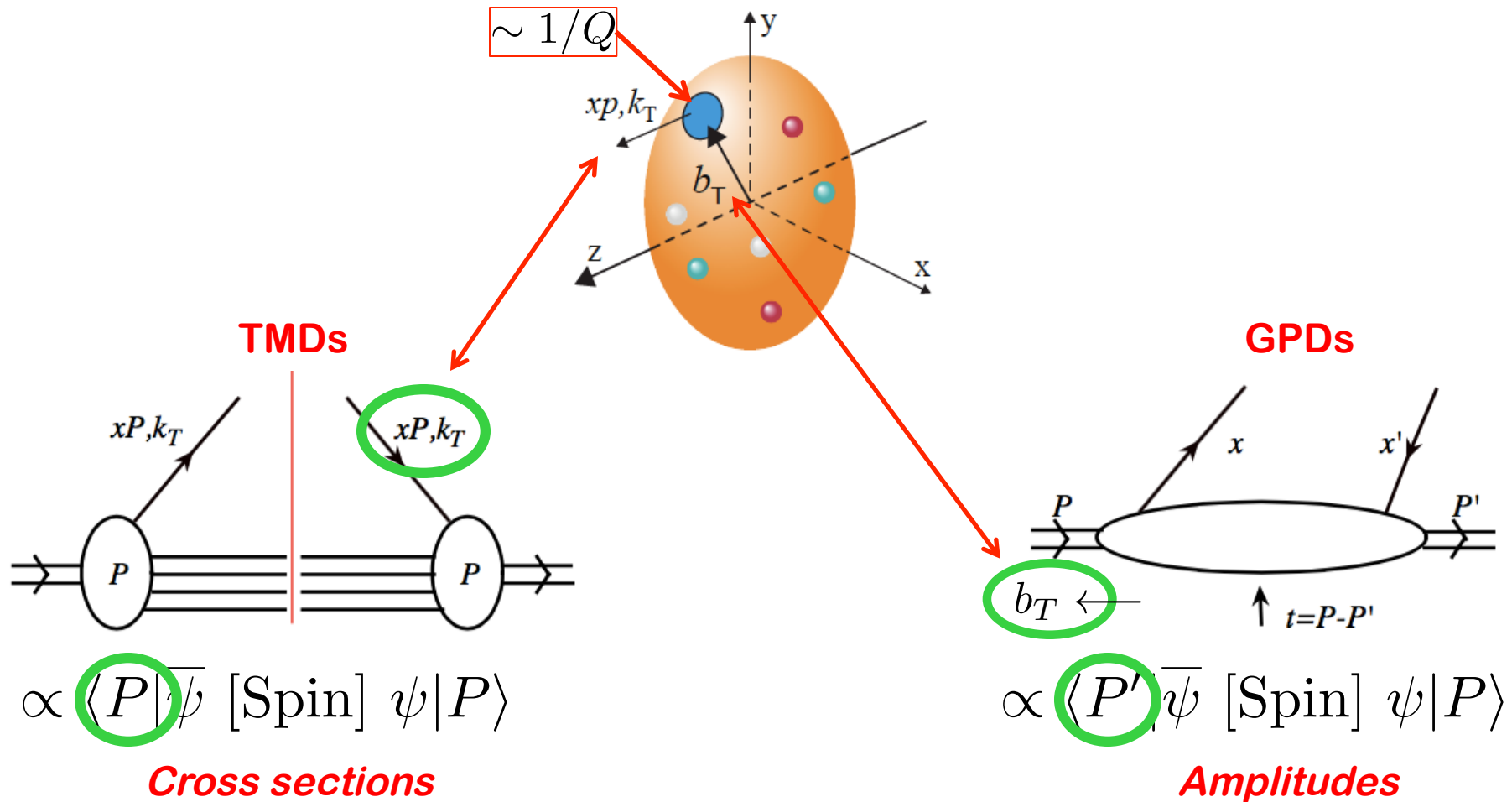
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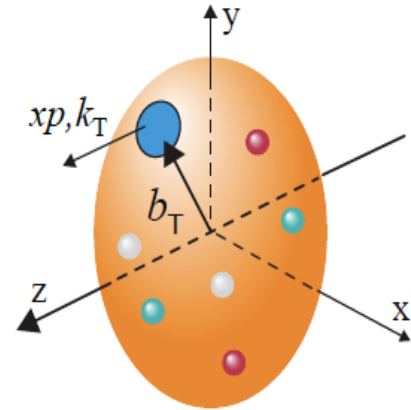


# Two-momentum-scale observables

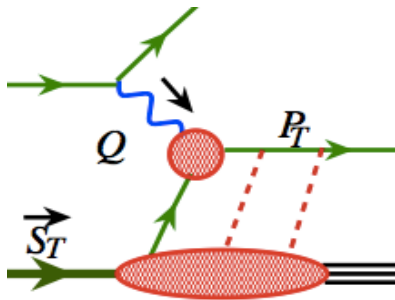
## □ Cross sections with two-momentum scales observed:

$$Q_1 \gg Q_2 \sim 1/R \sim \Lambda_{\text{QCD}}$$

- ✧ **Hard scale:**  $Q_1$  localizes the probe to see the quark or gluon d.o.f.
- ✧ **“Soft” scale:**  $Q_2$  could be more sensitive to hadron structure, e.g., confined motion

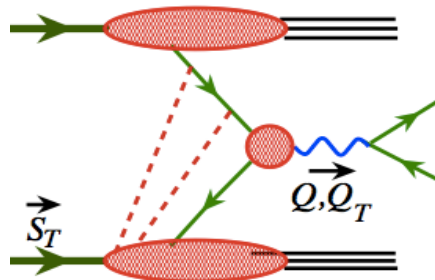


## □ Two-scale observables with the hadron **broken**:



**SIDIS:  $Q \gg P_T$**

+



**DY:  $Q \gg P_T$**

+

**Two-jet momentum imbalance in SIDIS, ...**



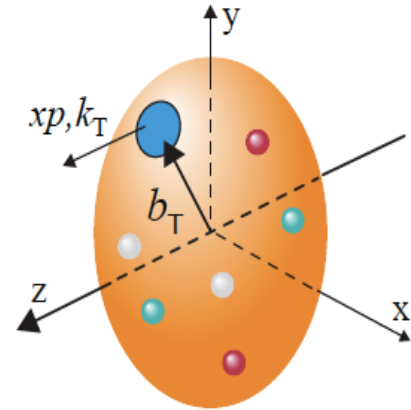
- ✧ **Natural observables with TWO very different scales**
- ✧ **TMD factorization:** partons' confined motion is encoded into TMDs

# Two-momentum-scale observables

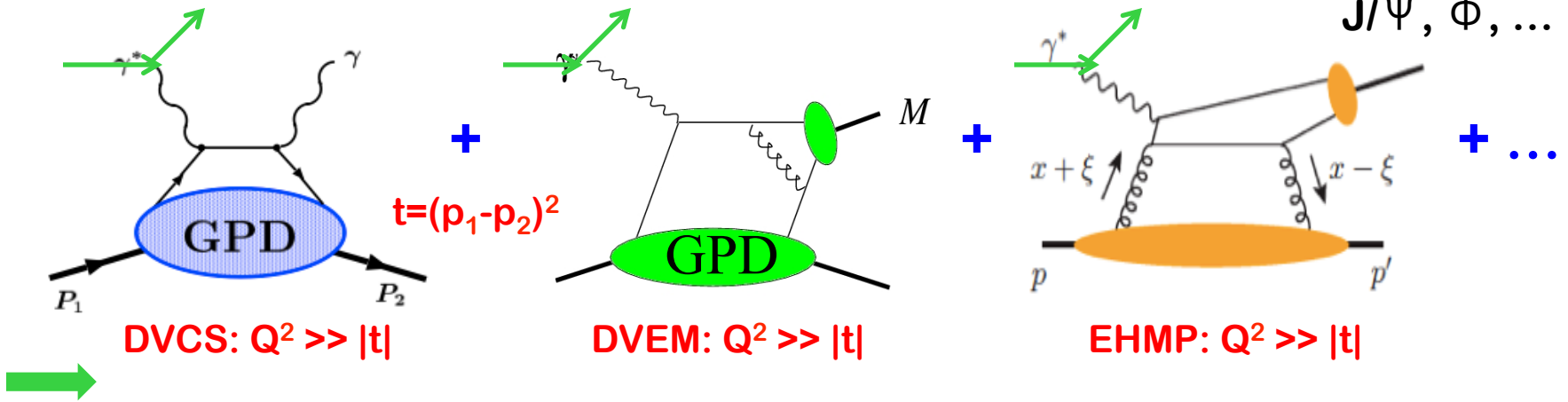
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## □ Two-scale observables with the hadron **unbroken**:



- ✧ Natural observables with TWO very different scales
- ✧ GPDs: Fourier Transform of  $t$ -dependence gives spatial  $b_T$ -dependence

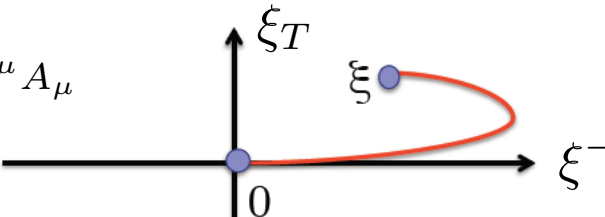
# Definition of TMDs

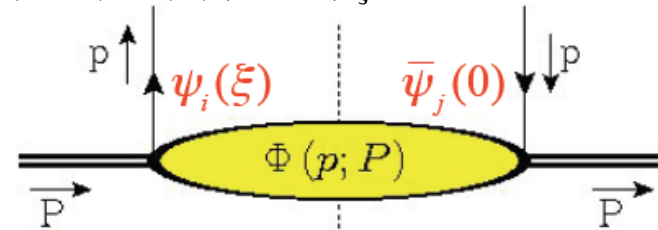
## □ Non-perturbative definition:

✧ In terms of matrix elements of parton correlators:

$$\Phi^{[U]}(x, p_T; n) = \int \frac{d\xi^- d^2\xi_T}{(2\pi)^3} e^{i p \cdot \xi} \langle P, S | \bar{\psi}(0) U(0, \xi) \psi(\xi) | P, S \rangle_{\xi^+ = 0}$$

✧ Depends on the choice of the gauge link:

$$U(0, \xi) = e^{-ig \int_0^\xi ds^\mu A_\mu}$$




✧ Decomposes into a list of TMDs:

$$\Phi^{[U]}(x, p_T; n) = \left\{ f_1^{[U]}(x, p_T^2) - f_{1T}^{\perp [U]}(x, p_T^2) \frac{\epsilon_T^{p_T S_T}}{M} + g_{1s}^{[U]}(x, p_T) \gamma_5 \right. \\ \left. + h_{1T}^{[U]}(x, p_T^2) \gamma_5 \not{s}_T + h_{1s}^{\perp [U]}(x, p_T) \frac{\gamma_5 \not{p}_T}{M} + i h_1^{\perp [U]}(x, p_T^2) \frac{\not{p}_T}{M} \right\} \frac{\not{P}}{2},$$

□ Gives “unique” TMDs, IF we knew proton wave function!



But, we do NOT know proton wave function (calculate it on lattice?)

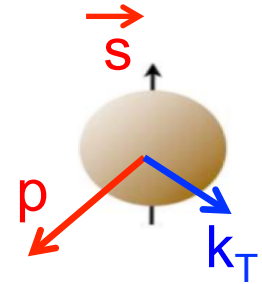
***TMDs are NOT direct physical observables!***

# TMDs: confined motion, its spin correlation

## □ Power of spin – many more correlations:

		Quark Polarization		
		Un-Polarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	$f_1 = \odot$		$h_1^\perp = \odot - \odot$ Boer-Mulders
	L		$g_{1L} = \odot \rightarrow - \odot \rightarrow$ Helicity	$h_{1L}^\perp = \odot \rightarrow - \odot \rightarrow$
	T	$f_{1T}^\perp = \odot \uparrow - \odot \downarrow$ Sivers	$g_{1T}^\perp = \odot \uparrow - \odot \uparrow$	$h_1 = \odot \uparrow - \odot \uparrow$ Transversity $h_{1T}^\perp = \odot \uparrow - \odot \uparrow$

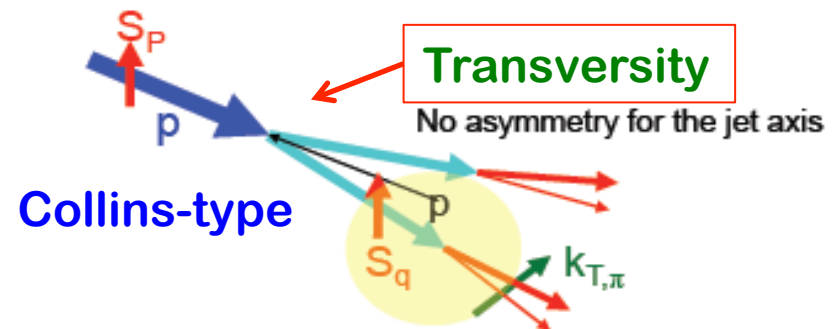
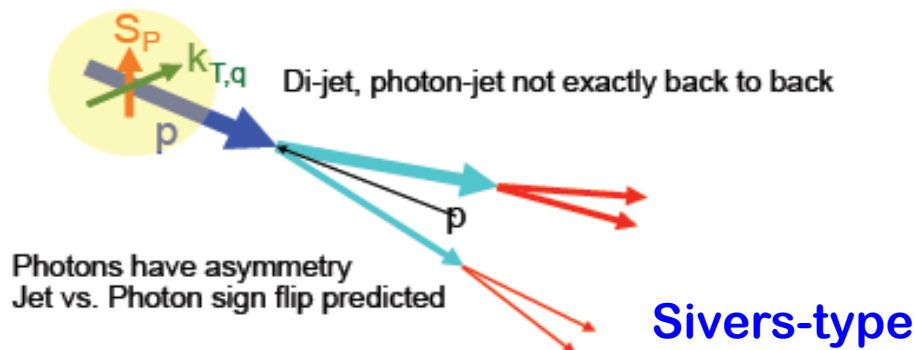
 Nucleon Spin     
  Quark Spin     
 *Similar for gluons*



Require **two** Physical scales

More than one TMD contribute to the same observable!

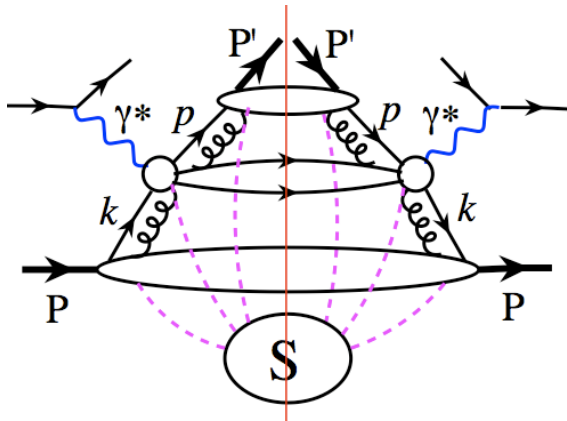
## □ $A_N$ – single hadron production:



# TMDs extracted from data

□ Perturbative definition – in terms of TMD factorization:

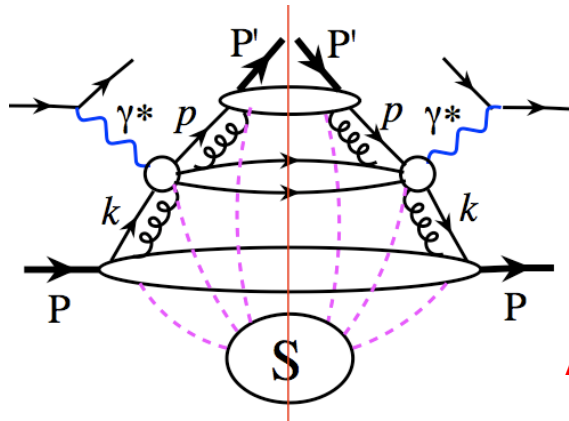
SIDIS as an example:



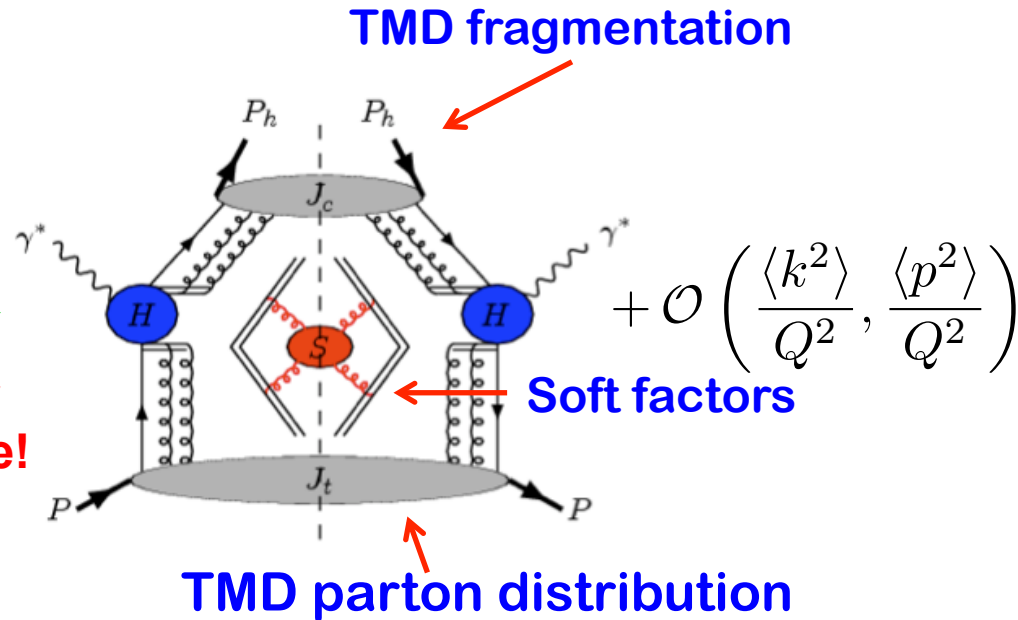
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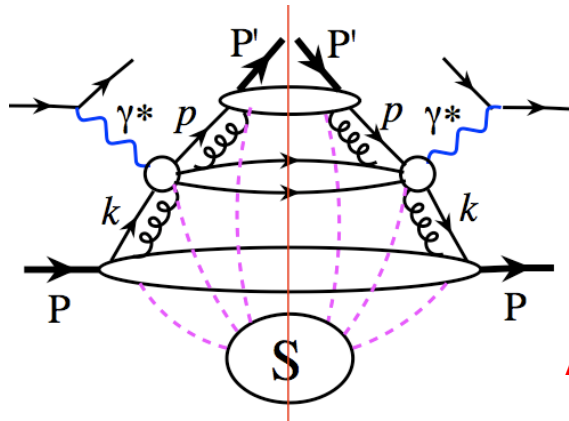
**Theory  
Advance!**



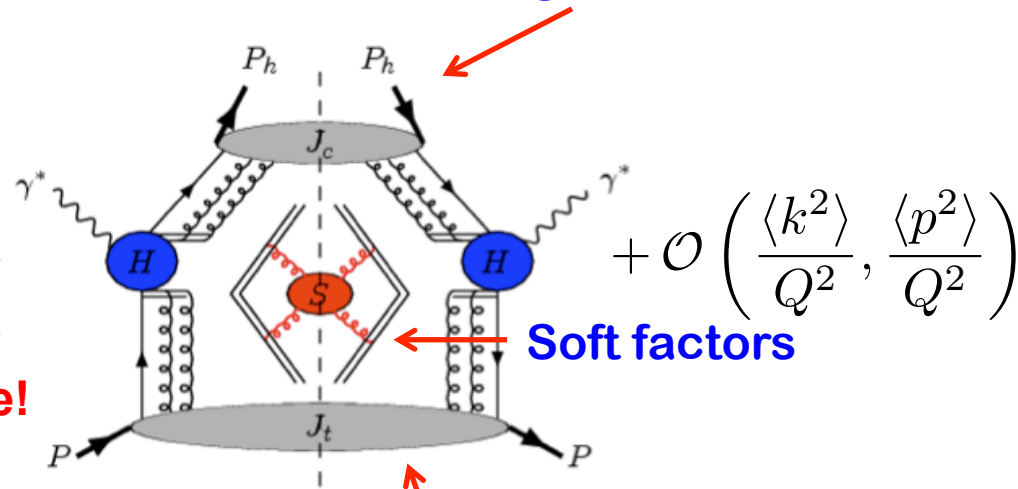
# TMDs extracted from data

## □ Perturbative definition – in terms of TMD factorization:

SIDIS as an example:



**Theory Advance!**



**TMD parton distribution**

## □ Extraction of TMDs:

$$\sigma_{\text{SIDIS}}(Q, P_{h\perp}, x_B, z_h) = \hat{H}(Q) \otimes \Phi_f(x, k_\perp) \otimes \mathcal{D}_{f \rightarrow h}(z, p_\perp) \otimes \mathcal{S}(k_{s\perp}) + \mathcal{O}\left[\frac{P_{h\perp}}{Q}\right]$$

**TMDs are extracted by fitting DATA using the factorization formula**

- ✧ Depending on the perturbatively calculated  $\hat{H}(Q; \mu)$  perturbative orders, renormalization, factorization schemes, ...
- ✧ Depending on the approximation of neglecting the power corrections, ..

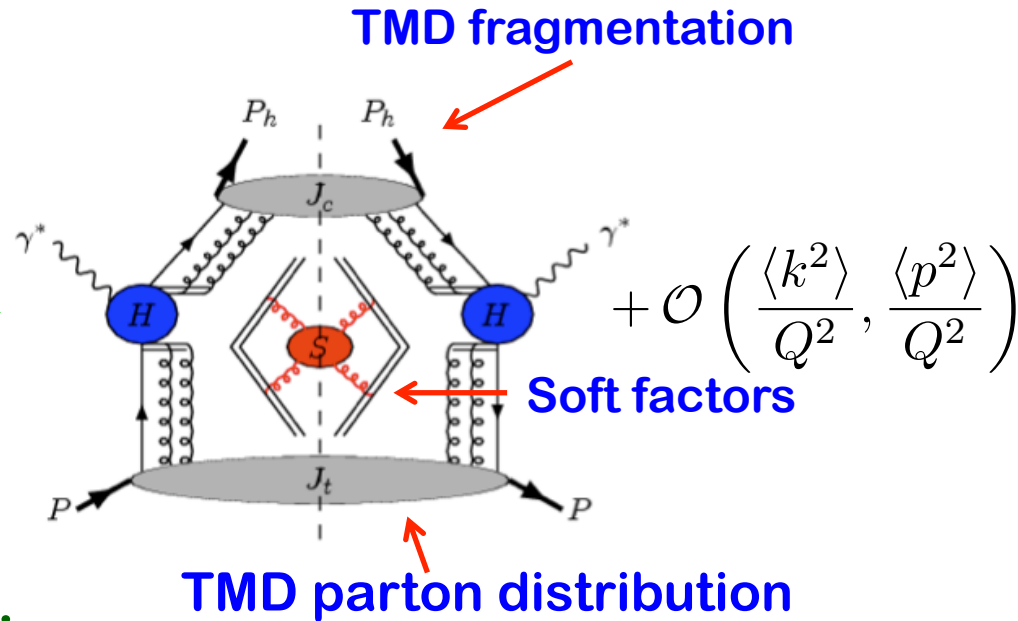
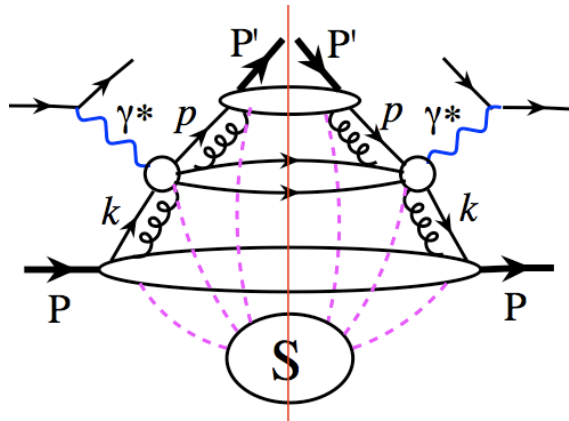


**Importance of lattice QCD calculations, ...**

# TMDs extracted from data

## □ Perturbative definition – in terms of TMD factorization:

SIDIS as an example:



## □ Low $P_{hT}$ – TMD factorization:

$$\sigma_{\text{SIDIS}}(Q, P_{h\perp}, x_B, z_h) = \hat{H}(Q) \otimes \Phi_f(x, k_\perp) \otimes \mathcal{D}_{f \rightarrow h}(z, p_\perp) \otimes \mathcal{S}(k_{s\perp}) + \mathcal{O}\left[\frac{P_{h\perp}}{Q}\right]$$

## □ High $P_{hT}$ – Collinear factorization:

$$\sigma_{\text{SIDIS}}(Q, P_{h\perp}, x_B, z_h) = \hat{H}(Q, P_{h\perp}, \alpha_s) \otimes \phi_f \otimes D_{f \rightarrow h} + \mathcal{O}\left(\frac{1}{P_{h\perp}}, \frac{1}{Q}\right)$$

## □ $P_{hT}$ Integrated - Collinear factorization:

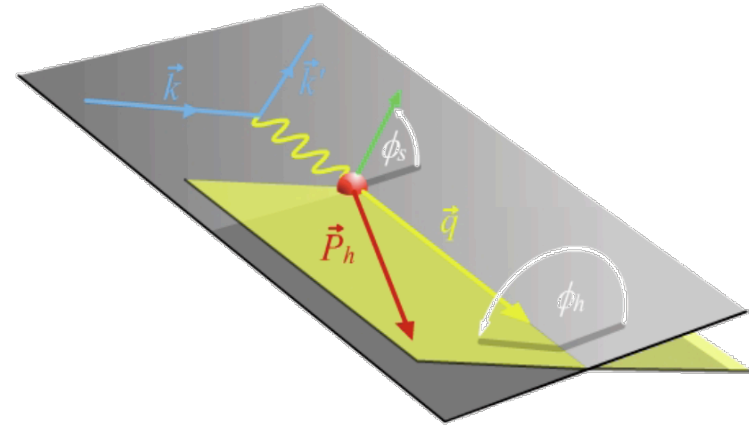
$$\sigma_{\text{SIDIS}}(Q, x_B, z_h) = \tilde{H}(Q, \alpha_s) \otimes \phi_f \otimes D_{f \rightarrow h} + \mathcal{O}\left(\frac{1}{Q}\right)$$



# SIDIS is the best for probing TMDs

## □ Naturally, two scales & two planes:

$$\begin{aligned}
 A_{UT}(\varphi_h^l, \varphi_S^l) &= \frac{1}{P} \frac{N^\uparrow - N^\downarrow}{N^\uparrow + N^\downarrow} \\
 &= A_{UT}^{\text{Collins}} \sin(\phi_h + \phi_S) + A_{UT}^{\text{Sivers}} \sin(\phi_h - \phi_S) \\
 &\quad + A_{UT}^{\text{Pretzelosity}} \sin(3\phi_h - \phi_S)
 \end{aligned}$$

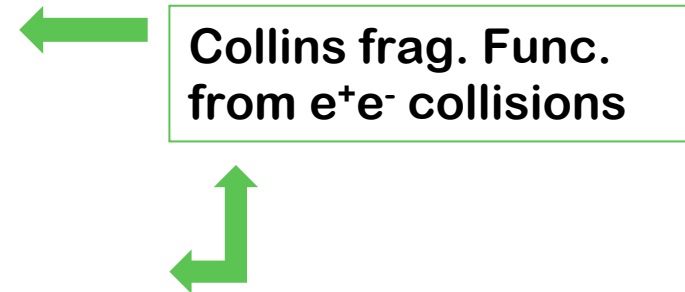


## □ Separation of TMDs:

$$A_{UT}^{\text{Collins}} \propto \langle \sin(\phi_h + \phi_S) \rangle_{UT} \propto h_1 \otimes H_1^\perp$$

$$A_{UT}^{\text{Sivers}} \propto \langle \sin(\phi_h - \phi_S) \rangle_{UT} \propto f_{1T}^\perp \otimes D_1$$

$$A_{UT}^{\text{Pretzelosity}} \propto \langle \sin(3\phi_h - \phi_S) \rangle_{UT} \propto h_{1T}^\perp \otimes H_1^\perp$$



***Hard, if not impossible, to separate TMDs in hadronic collisions***

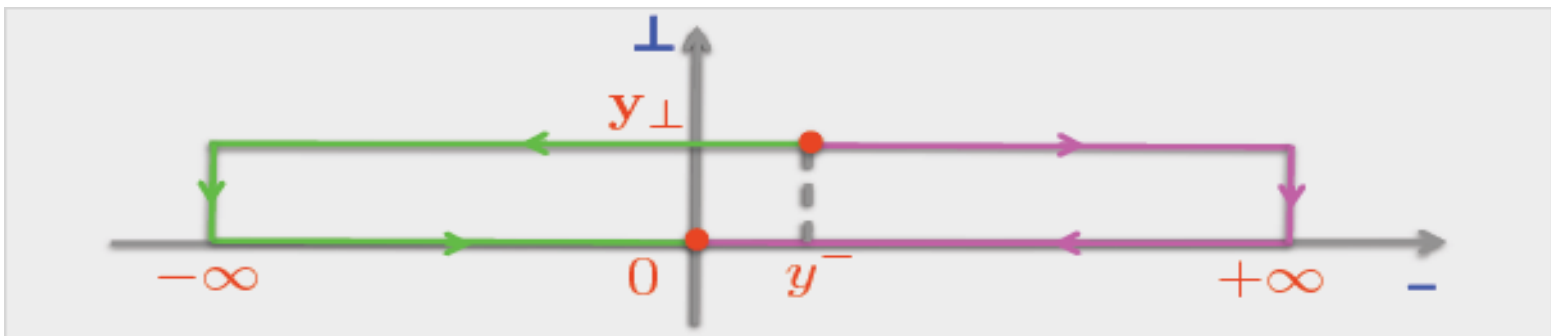
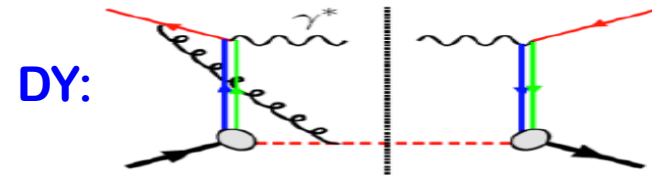
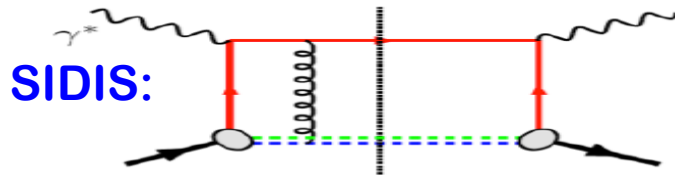
Using a combination of different observables (not the same observable):  
jet, identified hadron, photon, ...

# Modified universality for TMDs

## □ Definition:

$$f_{q/h\uparrow}(x, \mathbf{k}_\perp, \vec{S}) = \int \frac{dy^- d^2 y_\perp}{(2\pi)^3} e^{ixp^+ y^- - i\mathbf{k}_\perp \cdot \mathbf{y}_\perp} \langle p, \vec{S} | \bar{\psi}(0^-, \mathbf{0}_\perp) \boxed{\text{Gauge link}} \frac{\gamma^+}{2} \psi(y^-, \mathbf{y}_\perp) | p, \vec{S} \rangle$$

## □ Gauge links:



## □ Process dependence:

$$f_{q/h\uparrow}^{\text{SIDIS}}(x, \mathbf{k}_\perp, \vec{S}) \neq f_{q/h\uparrow}^{\text{DY}}(x, \mathbf{k}_\perp, \vec{S})$$

Collinear factorized PDFs are process independent

# Critical test of TMD factorization

## □ Parity – Time reversal invariance:

$$f_{q/h\uparrow}^{\text{SIDIS}}(x, \mathbf{k}_\perp, \vec{S}) = f_{q/h\uparrow}^{\text{DY}}(x, \mathbf{k}_\perp, -\vec{S})$$

## □ Definition of Sivers function:

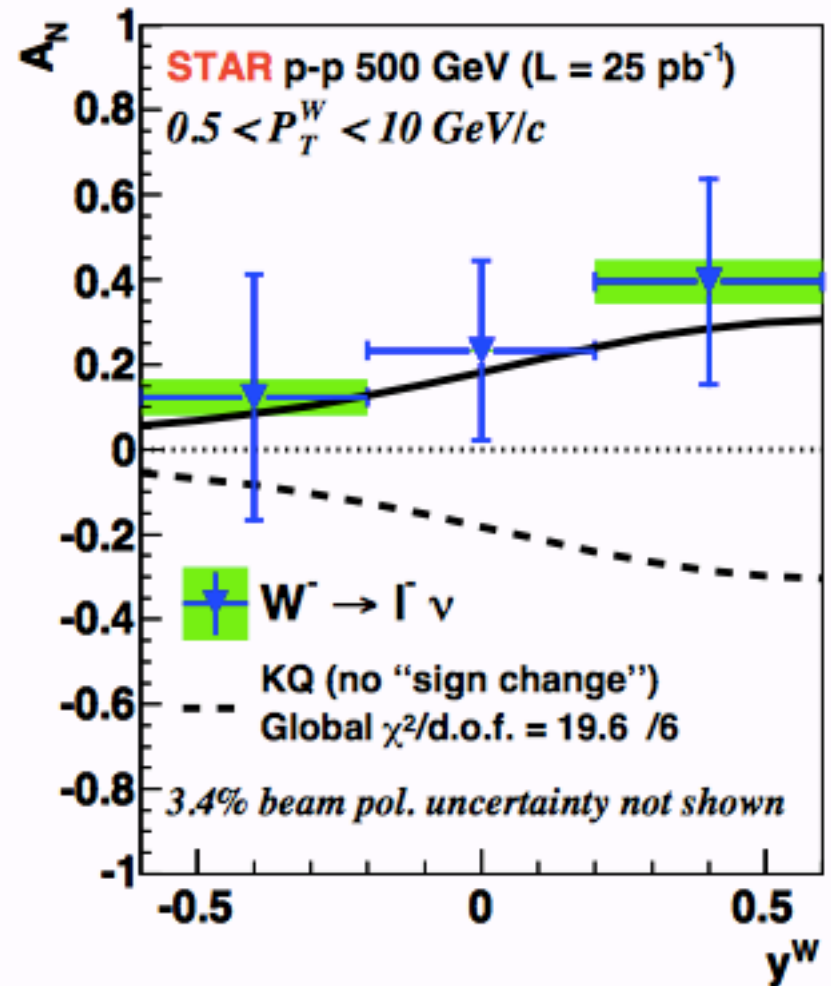
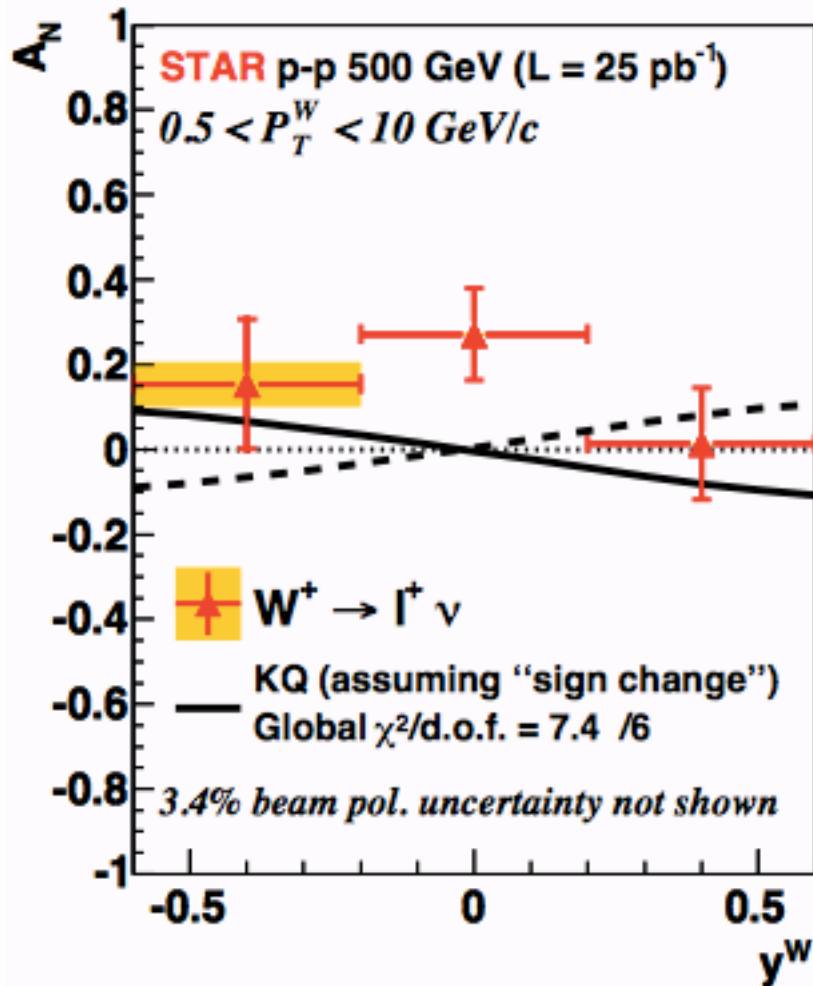
$$f_{q/h\uparrow}(x, \mathbf{k}_\perp, \vec{S}) \equiv f_{q/h}(x, k_\perp) + \frac{1}{2} \Delta^N f_{q/h\uparrow}(x, k_\perp) \vec{S} \cdot \hat{p} \times \hat{\mathbf{k}}_\perp$$

## □ Modified universality:

$$\Delta^N f_{q/h\uparrow}^{\text{SIDIS}}(x, k_\perp) = -\Delta^N f_{q/h\uparrow}^{\text{DY}}(x, k_\perp)$$

The spin-averaged part of this TMD is process independent,  
but, spin-averaged Boer-Mulder's TMD requires the sign change!  
Same PT symmetry examination needs for TMD gluon distributions!

# Hint of the sign change: $A_N$ of W production



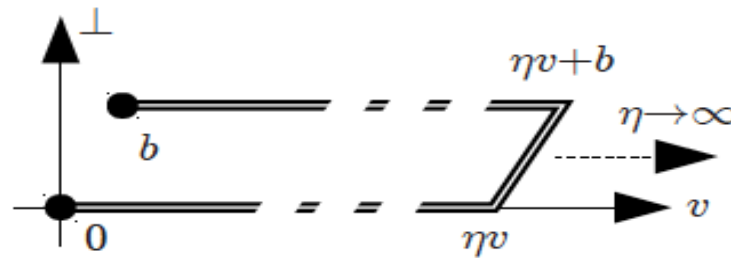
*Data from STAR collaboration on  $A_N$  for W-production are consistent with a sign change between SIDIS and DY*

# Hint of the sign change from lattice QCD

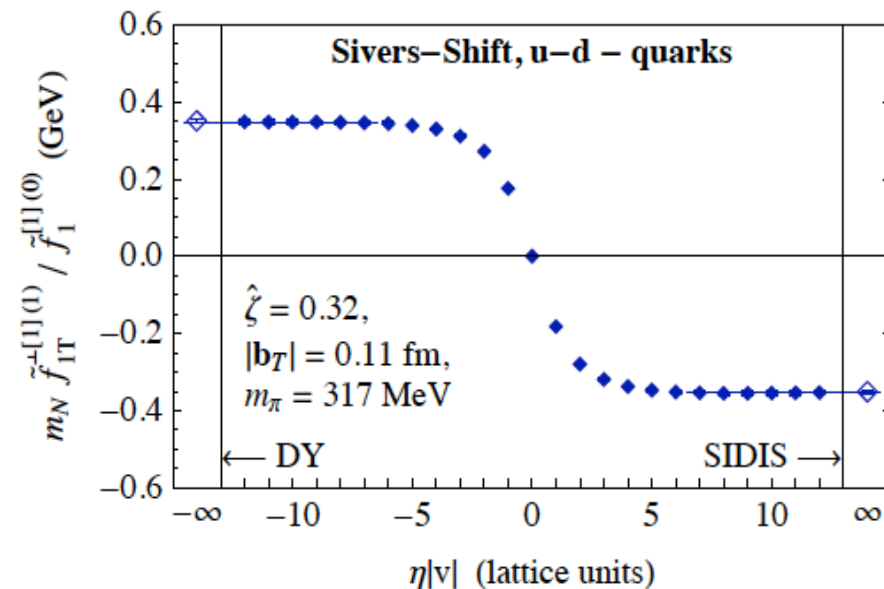
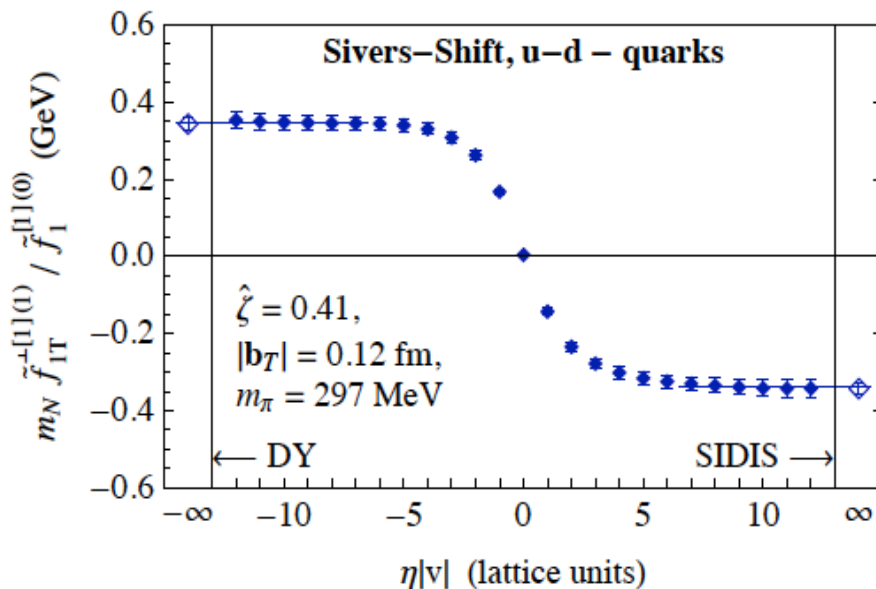
Engelhardt@TMD  
Collaboration meeting

## □ Gauge link for lattice calculation:

Staple-shaped gauge link  $\mathcal{U}[0, \eta v, \eta v + b, b]$



## □ Normalized moment of Sivers function – at given $b_T$ :

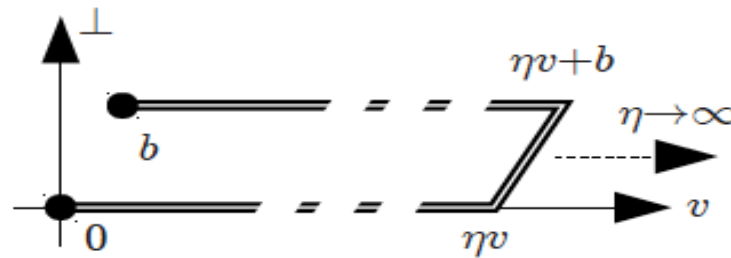


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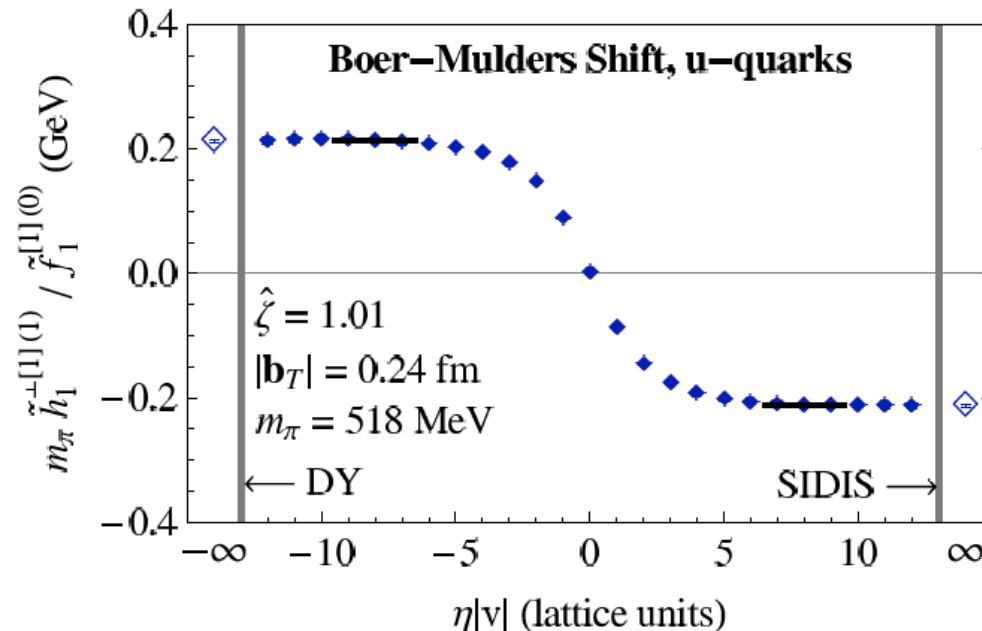
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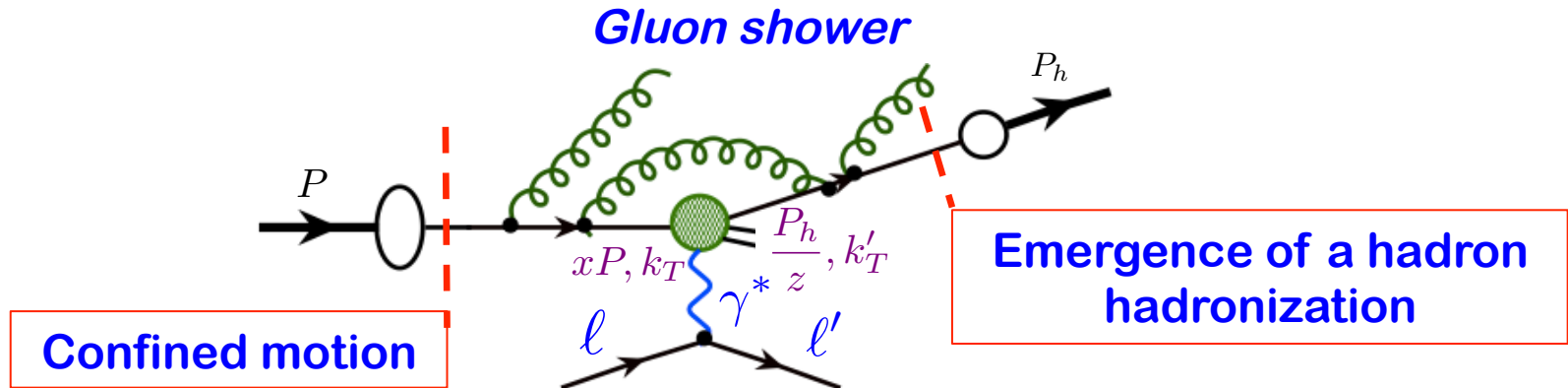


## □ Normalized moment of Boer-Mulders function – at given $b_T$ :



# Parton $k_T$ at the hard collision

## □ Sources of parton $k_T$ at the hard collision:



## □ Large $k_T$ generated by the shower (caused by the collision):

- ✧  $Q^2$ -dependence – **linear** evolution equation of TMDs in **b-space**
  - ✧ The evolution kernels are perturbative at small  $b$ , but, not large  $b$
- ➡ **The nonperturbative inputs at large  $b$  could impact TMDs at all  $Q^2$**

## □ Challenge: to extract the “true” parton’s confined motion:

- ✧ Separation of perturbative shower contribution from nonperturbative hadron structure – **QCD evolution** - not as simple as PDFs

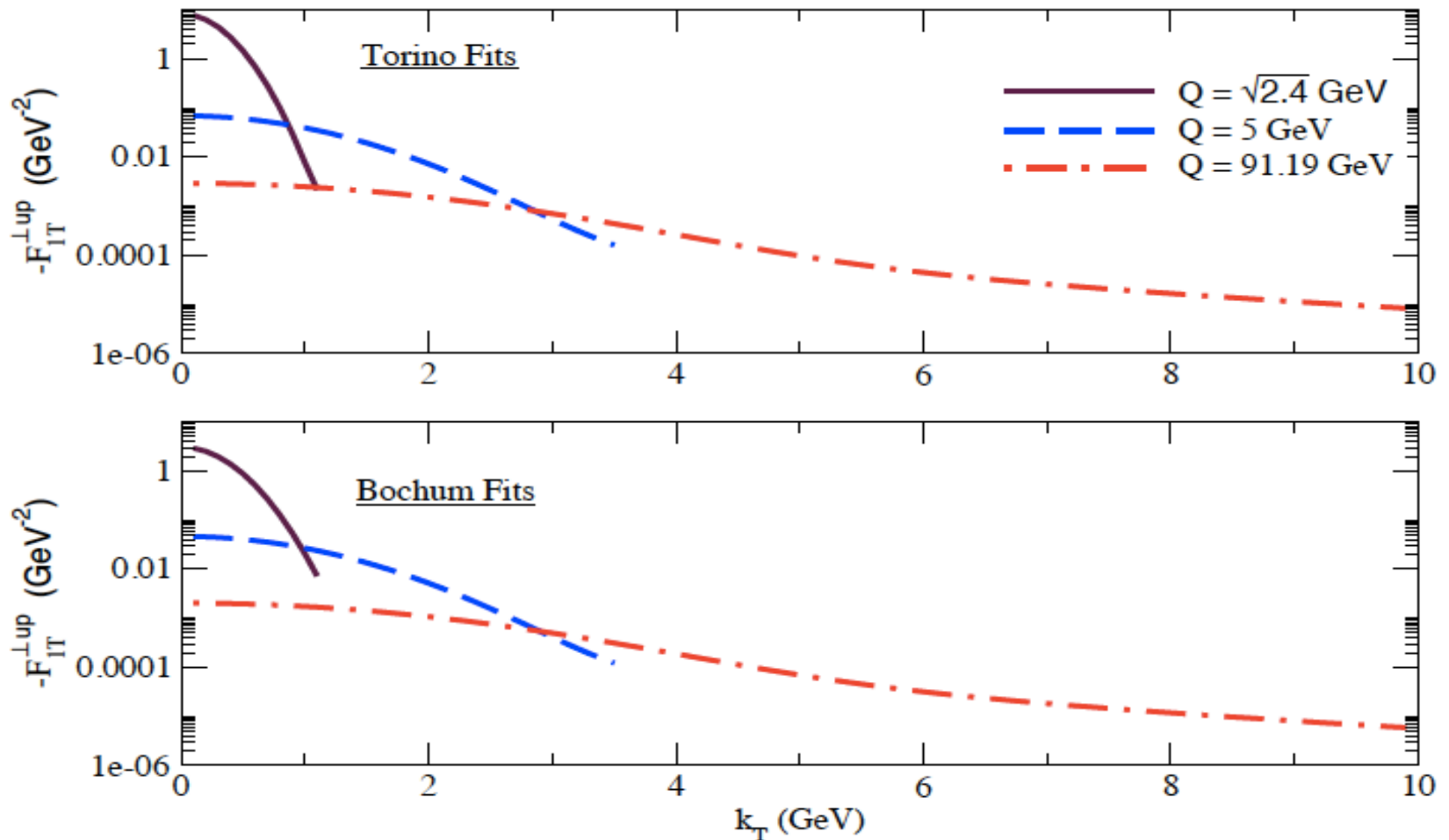
- ✧ Role of lattice QCD?

*Task of the DOE supported TMD collaboration*

# Evolution of Sivers function

Aybat, Collins, Qiu, Rogers, 2011

## □ Up quark Sivers function:

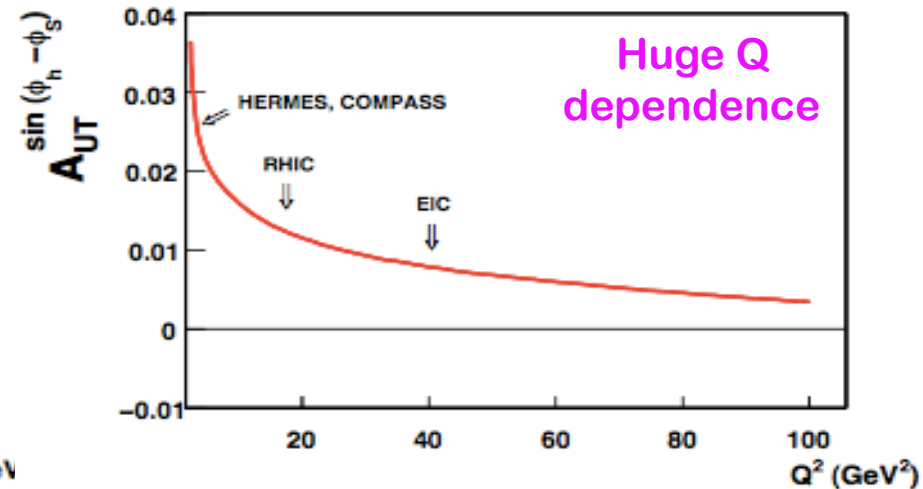
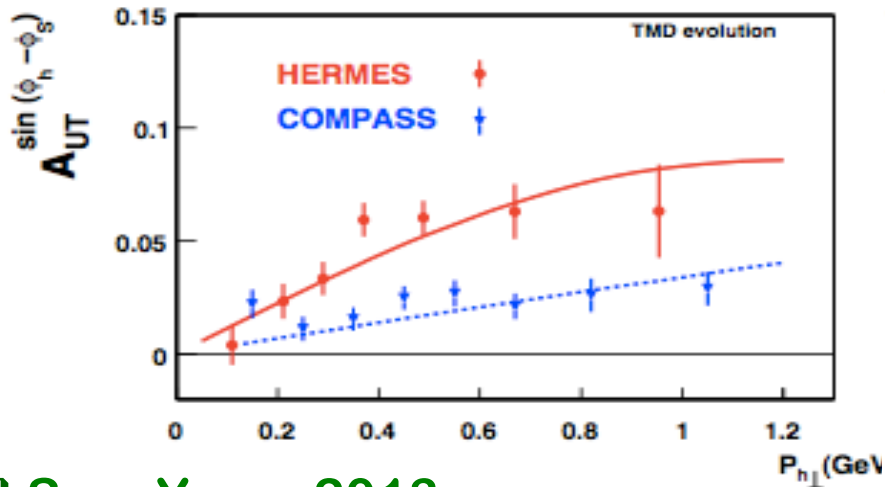


*Very significant growth in the width of transverse momentum*

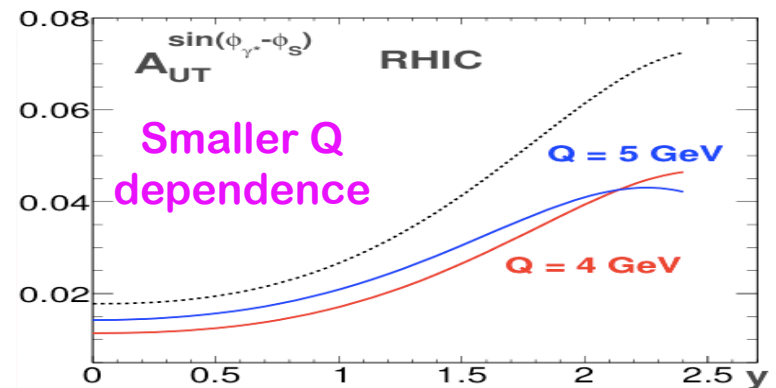
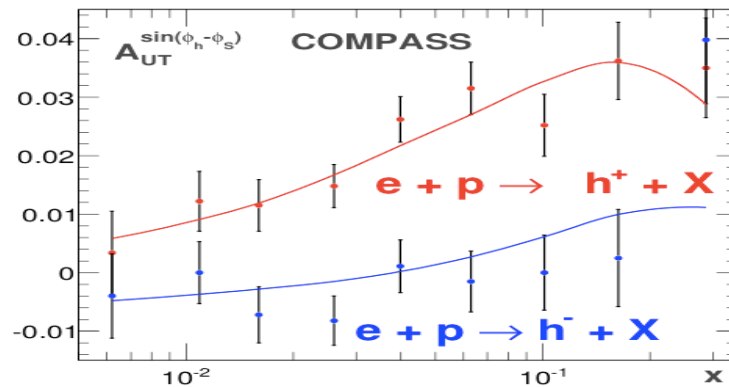


# Different fits – different Q-dependence

□ Aybat, Prokudin, Rogers, 2012:



□ Sun, Yuan, 2013:



*No disagreement on evolution equations!*

Issues: Extrapolation to non-perturbative large b-region

Choice of the Q-dependent "form factor"

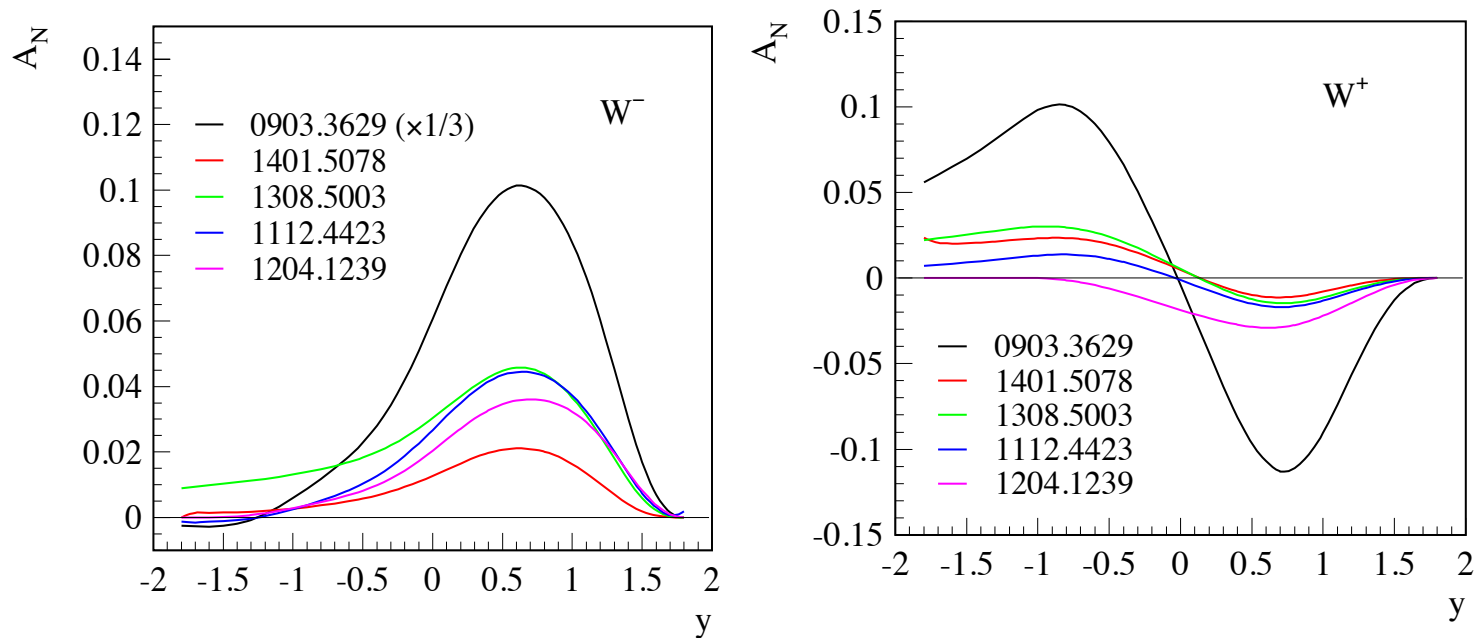
# “Predictions” for $A_N$ of W-production at RHIC?

See also talk by Marcia Quaresma on Wed. for COMPASS

## □ Siverson Effect:

- ✧ Quantum correlation between the **spin direction** of colliding hadron and the preference of **motion direction** of its confined partons
- ✧ QCD Prediction: **Sign change** of Siverson function from SIDIS and DY

## □ Current “prediction” and uncertainty of QCD evolution:



**TMD collaboration proposal: Lattice, theory & Phenomenology**  
**RHIC is the excellent and unique facility to test this (W/Z – DY)!**

# What happened?

## □ **Sivers function:**

Differ from PDFs!

$$F_{1T}^{\perp f}(x, k_T; \mu, \zeta_F) = \frac{-1}{2\pi k_T} \int_0^\infty db_T b_T J_1(k_T b_T) \tilde{F}_{1T}^{\prime \perp f}(x, b_T; \mu, \zeta_F)$$

*Need non-perturbative large  $b_T$  information for any value of  $Q$ !*       $Q = \mu$

# What happened?

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*Need non-perturbative large  $b_T$  information for any value of  $Q$ !*  $Q = \mu$

## ❑ What is the “correct” $Q$ -dependence of the large $b_T$ tail?

$$\begin{aligned} \tilde{F}_{f/P}(x, \mathbf{b}_T; Q, Q^2) &= \overbrace{\sum_j \int_x^1 \frac{d\hat{x}}{\hat{x}} \tilde{C}_{f/j}(x/\hat{x}, b_*; \mu_b^2, \mu_b, g(\mu_b)) f_{j/P}(\hat{x}, \mu_b)}^{\text{AA}} \\ &\times \overbrace{\exp \left\{ \ln \frac{Q}{\mu_b} \tilde{K}(b_*; \mu_b) + \int_{\mu_b}^Q \frac{d\mu'}{\mu'} \left[ \gamma_F(g(\mu'); 1) - \ln \frac{Q}{\mu'} \gamma_K(g(\mu')) \right] \right\}}^{\text{BB}} \\ &\times \underbrace{\exp \left\{ g_{f/P}(x, b_T) + g_K(b_T) \ln \frac{Q}{Q_0} \right\}}_{\text{CC}} \end{aligned}$$

Nonperturbative  
“form factor”

$$g_{f/P}(x, b_T) + g_K(b_T) \ln \frac{Q}{Q_0} \equiv - \left[ g_1 + g_2 \ln \frac{Q}{2Q_0} + g_1 g_3 \ln(10x) \right] b_T^2$$

# What happened?

## ❑ Siverson function:

Differ from PDFs!

$$F_{1T}^{\perp f}(x, k_T; \mu, \zeta_F) = \frac{-1}{2\pi k_T} \int_0^\infty db_T b_T J_1(k_T b_T) \tilde{F}_{1T}'^{\perp f}(x, b_T; \mu, \zeta_F)$$

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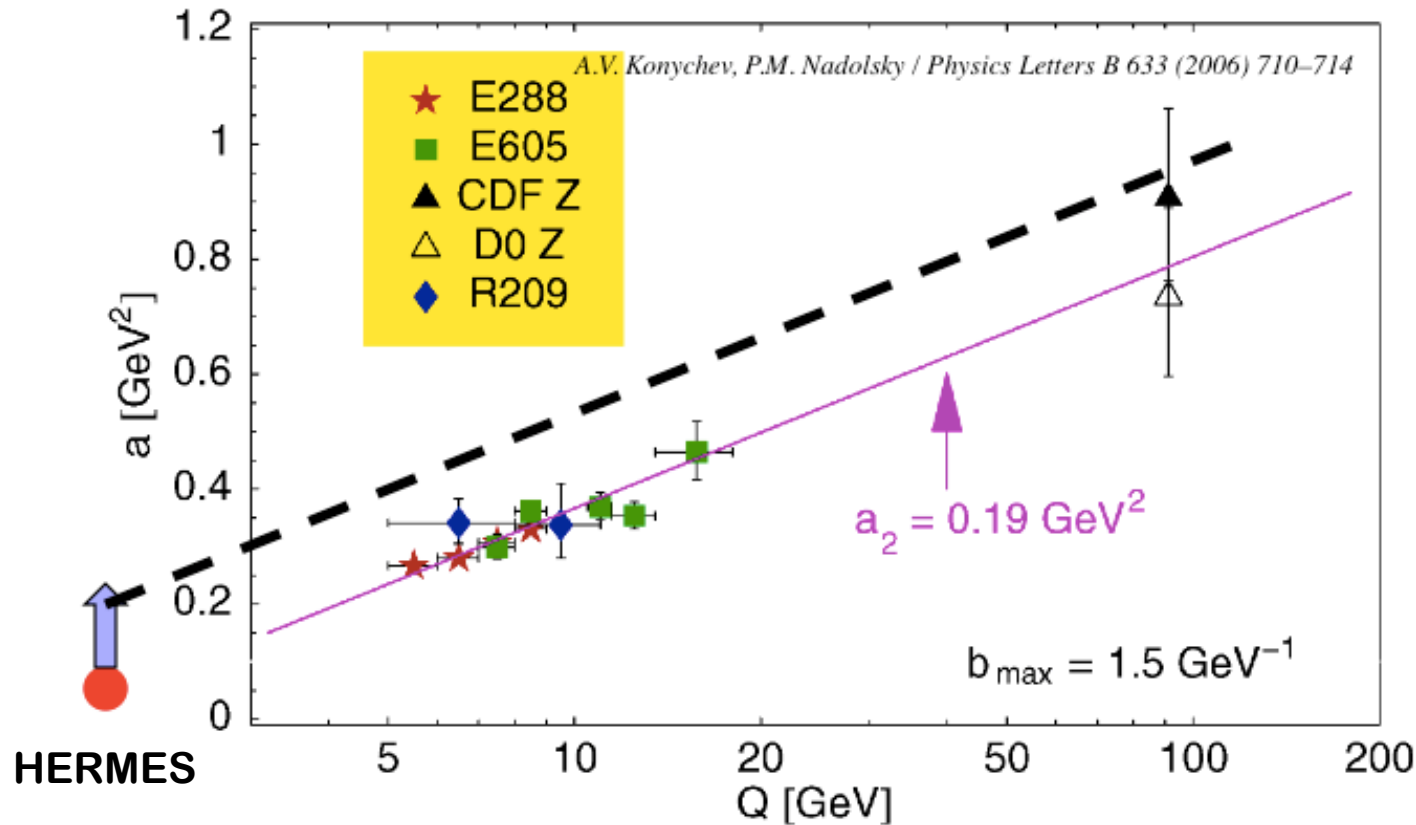
*Is the  $\log(Q)$  dependence sufficient? Choice of  $g_2$  &  $b_*$  affects  $Q$ -dep.*

*The “form factor” and  $b_*$  change perturbative results at small  $b_T$ !*

# Q-dependence of the “form” factor

## □ Q-dependence of the “form factor” :

Konychev, Nadolsky, 2006



$$\mathcal{F}^{\text{NP}}(b, Q) = a(Q^2) b^2$$

At  $Q \sim 1 \text{ GeV}$ ,  $\ln(Q/Q_0)$  term may not be the dominant one!

$$\mathcal{F}^{\text{NP}} \approx b^2(a_1 + a_2 \ln(Q/Q_0) + a_3 \ln(x_A x_B) + \dots) + \dots$$

Power correction?  $(Q_0/Q)^n$ -term? Better fits for HERMES data?

# Global QCD analysis: extraction of TMDs

## □ QCD TMD factorization:

- Connect cross sections, asymmetries to TMDs
- ✧ Factorization is known or expected to be valid for SIDIS, Drell-Yan ( $\Upsilon^*$ ,  $W/Z$ ,  $H^0$ , ...), 2-Jet imbalance in DIS, ...
- ✧ *Same level of reliability as collinear factorization for PDFs, up to the sign change*

## □ QCD evolution of TMDs:

- TMDs evolve when probed at different momentum scales
- ✧ Evolution equations are for TMDs in  $b_T$ -space (Fourier Conjugate of  $k_T$ )

FACT: QCD evolution does NOT fully fix TMDs in momentum space, even with TMDs fixed at low  $Q$  – large  $b_T$ -input!!!

- ✧ *Very different from DGLAP evolution of PDFs – in momentum space*

FACT: QCD evolution uniquely fix PDFs at large  $Q$ , once the PDFs is determined at lower  $Q$  – linear evolution in momentum space

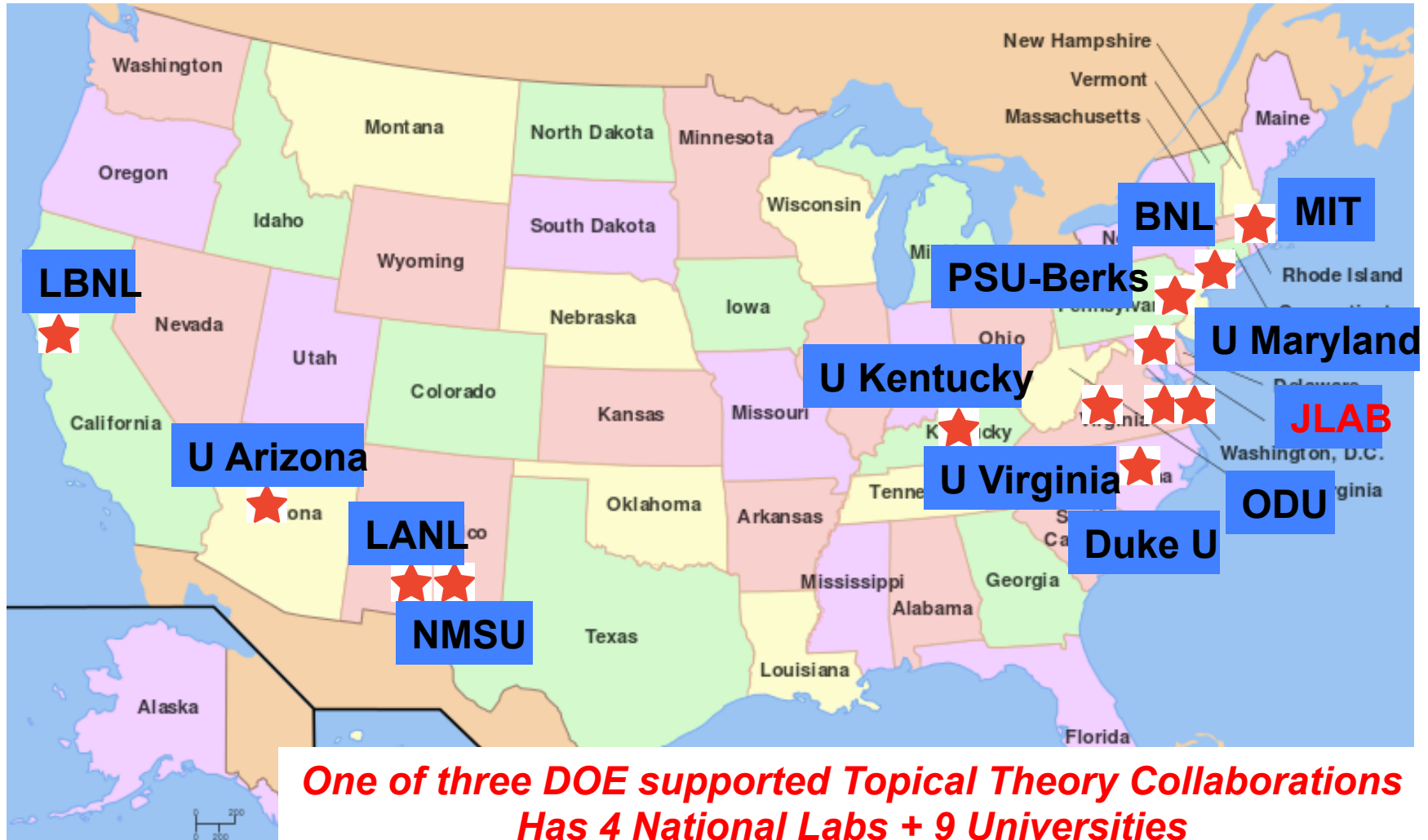
## □ Challenges:

Predictive power, extraction of hadron structure, ...

# TMD Topical Theory Collaboration

Coordinated Theoretical Approach to Transverse Momentum  
Dependent Hadron Structure in QCD (TMD Collaboration)

Co-spokespersons: W. Detmold, J.W. Qiu



***One of three DOE supported Topical Theory Collaborations  
Has 4 National Labs + 9 Universities***

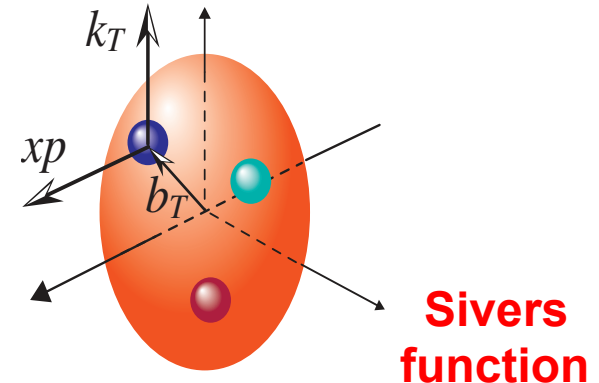


# TMD Topical Theory Collaboration

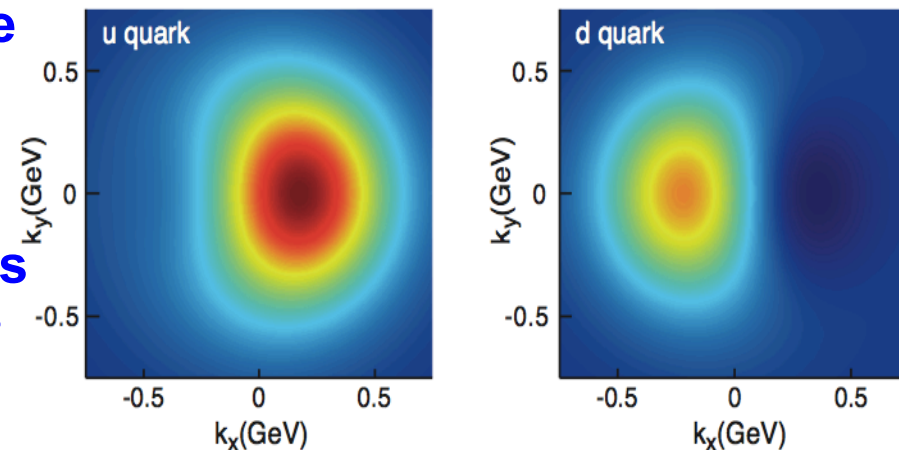
## □ Objectives/Deliverables – 3D Confined Motion:



*Unique three pronged scientific effort:  
(1) theory, (2) phenomenology and  
(3) lattice QCD, to explore 3D hadron  
structure – 3D confined motion!*



- ✧ Matching x-section to parton motion
  - QCD factorization
- ✧ Parton motion vs. probing scale
  - QCD quantum evolution
  - RHIC Run17 – W program
- ✧ Lattice QCD calculation of TMDs
  - QCD 1<sup>st</sup> principle prediction?
- ✧ Fast software to extract TMDs
  - Service to community
- ✧ JLab12 data, ...



*Density distribution of an  
unpolarized  
quark in a proton moving in z*

# Proposed research projects – examples

## ❑ Factorization and definition of TMDs

Stewart (coordinator), Ji, Lee, Prokudin, Rogers, Vitev, Yuan, ...

## ❑ Evolution of TMDs

Prokudin (coordinator), Gamberg, Lee, Metz, Rogers, Yuan, ...

## ❑ Nonperturbative Input to TMD Evolution

Rogers (coordinator), Engelhardt, Fleming, Gamberg, Lee, Mehen, Qiu, Stewart, Vitev, Yuan, ...

## ❑ QCD Global Analysis of the TMDs

Yuan (coordinator), Gamberg, Lee, Metz, Prokudin, Qiu, Rogers, Vitev, Yuan, ...

## ❑ Relation between TMDs and collinear PDFs

Metz (coordinator), Qiu, Rogers, Sterman, Yuan, ...

# Proposed research projects – examples

- ❑ New physical observables sensitive to TMDs

Gamberg (coordinator), Engelhardt, Mehen, Metz, Prokudin, Rogers, Stewart , ...

- ❑ Lattice calculations of PDFs and TMDs – factorization

Qiu (coordinator), Detmold, Fleming, Ji, Mehen, Stewart, ...

- ❑ Lattice calculations of hadron structure – nonperturbative

Detmold (coordinator), Engelhardt, Liu, Negele, ...

- ❑ TMDs and parton orbital angular momentum

Negele (coordinator), Burkardt, Engelhardt, Liu, Liuti, Metz, ...

- ❑ TMDs at small-x and in nuclei

Venugopalan (coordinator), Detmold, Fleming, Qiu, Stewart, Yuan, ...

# TMD WIKI page



Search this site

<https://sites.google.com/a/lbl.gov/tmdwiki/>

## Navigation

### Main

- Overview
- Collaboration calendar
- Meetings
- Publications
- Conference Talks
- Code Packages
- Jobs
- Working groups
- Documents
- Collaboration news
- Contacts
- Links
- TMD Summer School 2017:  
June 22-28,2017
- Members Only
- Sitemap

**173**  
days since

**First Collaboration  
Meeting at BNL**

## Recent site activity

TMD Summer School 2017:  
June 22-28,2017  
edited by Sean Fleming

## Topical Collaboration for the Coordinated Theoretical Approach to Transverse Momentum Dependent Hadron Structure in QCD

### Collaboration News

**TMD Collaboration Summer School** We are pleased to announce the first TMD Collaboration Summer School. It will take place at Temple University in Philadelphia, from Thursday, June 22 to Wednesday, June 28, 2017. The ...

Posted Dec 21, 2016, 7:18 AM by Sean Fleming






**Kentucky** The University of Kentucky has hired **Yibo Yang** as a postdoc starting in Fall 2016. Welcome to the TMD collaboration!

Posted Apr 21, 2016, 12:26 PM by Will Detmold


**PSU/ODU** Penn State and Old Dominion Universities have hired **Daniel Pitonyak** as a joint postdoctoral associate. Welcome to the TMD collaboration!

Posted Apr 21, 2016, 12:33 PM by Will Detmold

### TMD Collaboration Calendar

Today				March 2017			 Print	 Week	Mon
Sun	Mon	Tue	Wed	Thu	Fri	Sat			
26	27	28	Mar 1	2	3	4			
5	6	7	8	9	10	11			
12	13	14	15	16	17	18			
19	20	21	22	23	24	25			
26	27	28	29	30	31	Apr 1			

Events shown in time zone: Pacific Time

 GoogleCalendar

+ Google Calendar

# TMD Summer School @ Temple University

Travel

Program

Email Us

## TMD Collaboration Summer School 2017

June 22 - 28, 2017  
Temple University  
Philadelphia, USA

### Organizing Committee:

Matthias Burkardt (New Mexico State University)  
Martha Constantinou (Temple University)  
Sean Fleming, Co-Chair (University of Arizona)  
Leonard Gamberg (Penn State University-Berks)  
Keh-Fei Liu (University of Kentucky)  
Andreas Metz, Co-Chair (Temple University)  
Alexei Prokudin (Penn State University-Berks)

<http://www.physics.arizona.edu/~fleming/Main.html>

### Topics and Speakers:

- QCD and Parton Model: Pavel Nadolsky (SMU)
- TMD Phenomenology: Alessandro Bacchetta (U. di Pavia) & Andrea Signori (ULAB)
- TMD Factorization and Evolution: Ted Rogers (ODU)
- TMDs in Experiment: Matthias Grosse-Perdekamp (UIUC)
- Lattice QCD: Will Detmold (MIT)
- SCET: Iain Stewart (MIT)
- Quasi-PDFs: Martha Constantinou (Temple U.)
- GPDs and Generalized TMDs: Cedric Lorce (Ecole Polytechnique)
- TMDs in Lattice QCD: Michael Engelhardt (NMSU)
- TMDs at small x: Feng Yuan (LBNL)

# PDFs, TMDs, GPDs, and hadron structure

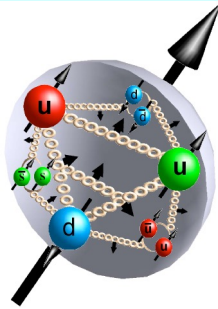
## □ What do we need to know for full hadron structure?

✧ In theory:  $\langle P, S | \mathcal{O}(\bar{\psi}, \psi, A^\mu) | P, S \rangle$  – Hadronic matrix elements  
with ALL possible operators  $\mathcal{O}(\bar{\psi}, \psi, A^\mu)$

✧ In fact: *None of these matrix elements is a direct physical observable in QCD – color confinement! – need probes!!!*

✧ In practice: Accessible hadron structure  
= hadron matrix elements of quarks and gluons, which

- 1) can be related to physical cross sections of hadrons and leptons with controllable approximation – factorization;
- 2) can be calculated in lattice QCD



## □ Multi-parton correlations – beyond single parton distributions:

$$\sigma(Q, \vec{s}) \propto \left| \underbrace{\text{[Feynman diagrams: a series of diagrams showing a hadron (represented by a circle with three lines) interacting with a probe (represented by a circle with two lines) via a gluon (represented by a wavy line) and a quark (represented by a straight line). The diagrams show different ways the probe can interact with the hadron's internal structure, including single and double parton interactions.]} + \dots \right|^2 \left( \frac{\langle k_\perp \rangle}{Q} \right)^n \text{ – Expansion}$$

Quantum interference/correlation ➡ Multi-parton matrix elements

# Summary

- ❑ TMDs and GPDs are NOT direct physical observables
  - could be defined differently
- ❑ Knowledge of nonperturbative inputs at large  $b_T$  is crucial in determining the TMDs from fitting the data
- ❑ QCD factorization is necessary for any controllable “probe” for hadron’s quark-gluon structure!
- ❑ Jlab12, COMPASS, ... will provide rich information on hadron structure via TMDs and/or GPDs in years to come!
- ❑ EIC is a ultimate QCD machine, and will provide answers to many of our questions on hadron structure, in particular, the confined transverse motions (TMDs), spatial distributions (GPDs), and multi-parton correlations, ...

**Thank you!**

**Backup slides**



# Evolution equations for TMDs

## □ TMDs in the b-space:

J.C. Collins, in his book on QCD

$$\tilde{F}_{f/P\uparrow}(x, \mathbf{b}_T, S; \mu; \zeta_F) = \tilde{F}_{f/P\uparrow}^{\text{unsub}}(x, \mathbf{b}_T, S; \mu; y_P - (-\infty)) \sqrt{\frac{\tilde{S}_{(0)}(\mathbf{b}_T; +\infty, y_s)}{\tilde{S}_{(0)}(\mathbf{b}_T; +\infty, -\infty)\tilde{S}_{(0)}(\mathbf{b}_T; y_s, -\infty)}} Z_F Z_2$$

## □ Collins-Soper equation:

Renormalization of the soft-factor

$$\frac{\partial \tilde{F}_{f/P\uparrow}(x, \mathbf{b}_T, S; \mu; \zeta_F)}{\partial \ln \sqrt{\zeta_F}} = \tilde{K}(b_T; \mu) \tilde{F}_{f/P\uparrow}(x, \mathbf{b}_T, S; \mu; \zeta_F) \quad \zeta_F = M_P^2 x^2 e^{2(y_P - y_s)}$$

$$\tilde{K}(b_T; \mu) = \frac{1}{2} \frac{\partial}{\partial y_s} \ln \left( \frac{\tilde{S}(b_T; y_s, -\infty)}{\tilde{S}(b_T; +\infty, y_s)} \right)$$

Introduced to regulate the rapidity divergence of TMDs

## □ RG equations:

Wave function Renormalization

$$\frac{d\tilde{K}(b_T; \mu)}{d \ln \mu} = -\gamma_K(g(\mu))$$

Evolution equations are only valid when  $b_T \ll 1/\Lambda_{\text{QCD}}$  !

$$\frac{d\tilde{F}_{f/P\uparrow}(x, \mathbf{b}_T, S; \mu; \zeta_F)}{d \ln \mu} = \gamma_F(g(\mu); \zeta_F/\mu^2) \tilde{F}_{f/P\uparrow}(x, \mathbf{b}_T, S; \mu; \zeta_F).$$

## □ Momentum space TMDs:

Need information at large  $b_T$  for all scale  $\mu$  !

$$F_{f/P\uparrow}(x, \mathbf{k}_T, S; \mu, \zeta_F) = \frac{1}{(2\pi)^2} \int d^2 \mathbf{b}_T e^{i\mathbf{k}_T \cdot \mathbf{b}_T} \tilde{F}_{f/P\uparrow}(x, \mathbf{b}_T, S; \mu, \zeta_F)$$

# Evolution equations for Sivers function

## □ Sivers function:

Aybat, Collins, Qiu, Rogers, 2011

$$F_{f/P\uparrow}(x, k_T, S; \mu, \zeta_F) = F_{f/P}(x, k_T; \mu, \zeta_F) - F_{1T}^{\perp f}(x, k_T; \mu, \zeta_F) \frac{\epsilon_{ij} k_T^i S^j}{M_p}$$

## □ Collins-Soper equation:

Its derivative obeys the CS equation

$$\frac{\partial \ln \tilde{F}_{1T}^{\perp f}(x, b_T; \mu, \zeta_F)}{\partial \ln \sqrt{\zeta_F}} = \tilde{K}(b_T; \mu)$$

$$\tilde{F}_{1T}^{\perp f}(x, b_T; \mu, \zeta_F) \equiv \frac{\partial \tilde{F}_{1T}^{\perp f}(x, b_T; \mu, \zeta_F)}{\partial b_T}$$

## □ RG equations:

$$\frac{d \tilde{F}_{1T}^{\perp f}(x, b_T; \mu, \zeta_F)}{d \ln \mu} = \gamma_F(g(\mu); \zeta_F / \mu^2) \tilde{F}_{1T}^{\perp f}(x, b_T; \mu, \zeta_F)$$

$$\frac{d \tilde{K}(b_T; \mu)}{d \ln \mu} = -\gamma_K(g(\mu)) \quad \longrightarrow \quad \frac{\partial \gamma_F(g(\mu); \zeta_F / \mu^2)}{\partial \ln \sqrt{\zeta_F}} = -\gamma_K(g(\mu)),$$

## □ Sivers function in momentum space:

$$F_{1T}^{\perp f}(x, k_T; \mu, \zeta_F) = \frac{-1}{2\pi k_T} \int_0^\infty db_T b_T J_1(k_T b_T) \tilde{F}_{1T}^{\perp f}(x, b_T; \mu, \zeta_F)$$

Ji, Ma, Yuan, 2004  
 Idilbi, et al, 2004,  
 Boer, 2001, 2009,  
 Kang, Xiao, Yuan, 2011  
 Aybat, Prokudin, Rogers, 2012  
 Idilbi, et al, 2012,  
 Sun, Yuan 2013, ...

# Extrapolation to large $b_T$

## □ CSS $b^*$ -prescription:

Aybat and Rogers, arXiv:1101.5057  
Collins and Rogers, arXiv:1412.3820

$$\begin{aligned}
 \tilde{F}_{f/P}(x, b_T; Q, Q^2) &= \overbrace{\sum_j \int_x^1 \frac{d\hat{x}}{\hat{x}} \tilde{C}_{f/j}(x/\hat{x}, b_*; \mu_b^2, \mu_b, g(\mu_b)) f_{j/P}(\hat{x}, \mu_b)}^{\text{AA}} \\
 &\times \overbrace{\exp \left\{ \ln \frac{Q}{\mu_b} \tilde{K}(b_*; \mu_b) + \int_{\mu_b}^Q \frac{d\mu'}{\mu'} \left[ \gamma_F(g(\mu'); 1) - \ln \frac{Q}{\mu'} \gamma_K(g(\mu')) \right] \right\}}^{\text{BB}} \\
 &\times \underbrace{\exp \left\{ g_{f/P}(x, b_T) + g_K(b_T) \ln \frac{Q}{Q_0} \right\}}_{\text{CC}} \leftarrow \text{Nonperturbative "form factor"}
 \end{aligned}$$

$$b_* = \frac{b_T}{\sqrt{1 + b_T^2/b_{\text{max}}^2}} \quad \text{with } b_{\text{max}} \sim 1/2 \text{ GeV}^{-1}$$

## □ Nonperturbative fitting functions

Various fits correspond to different choices for  $g_{f/P}(x, b_T)$  and  $g_K(b_T)$   
e.g.

$$g_{f/P}(x, b_T) + g_K(b_T) \ln \frac{Q}{Q_0} \equiv - \left[ g_1 + g_2 \ln \frac{Q}{2Q_0} + g_1 g_3 \ln(10x) \right] b_T^2$$

*Different choice of  $g_2$  &  $b_*$  could lead to different over all  $Q$ -dependence!*