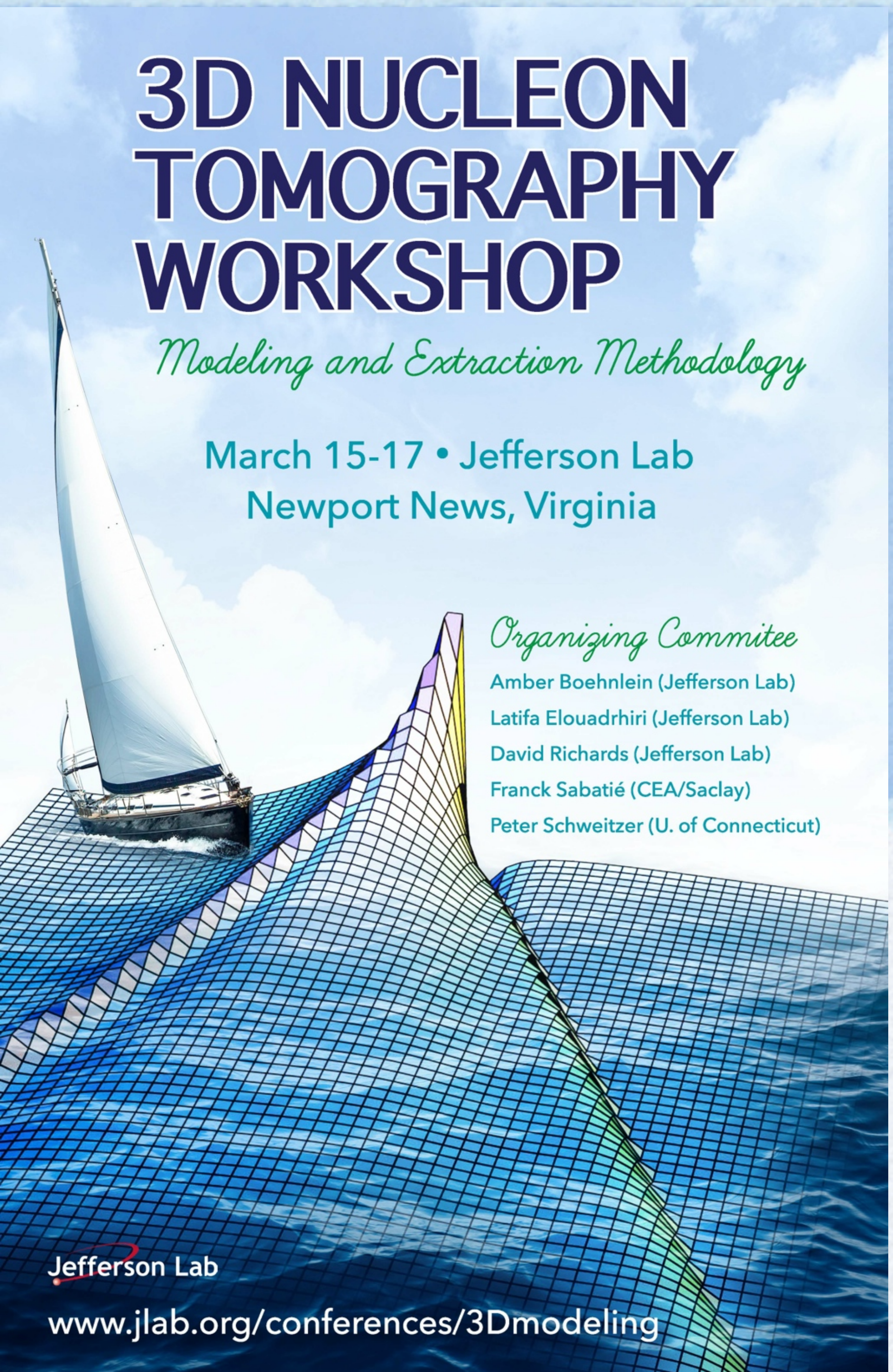
PRiSMA

SFB 1044
THE LOW-ENERGY FRONTIER
OF THE STANDARD MODEL

JOHANNES GUTENBERG UNIVERSITÄT MAINZ
JG|U


3D NUCLEON TOMOGRAPHY WORKSHOP
Modeling and Extraction Methodology

March 15-17 • Jefferson Lab
Newport News, Virginia

Organizing Commitee
Amber Boehnlein (Jefferson Lab)
Latifa Elouadrhiri (Jefferson Lab)
David Richards (Jefferson Lab)
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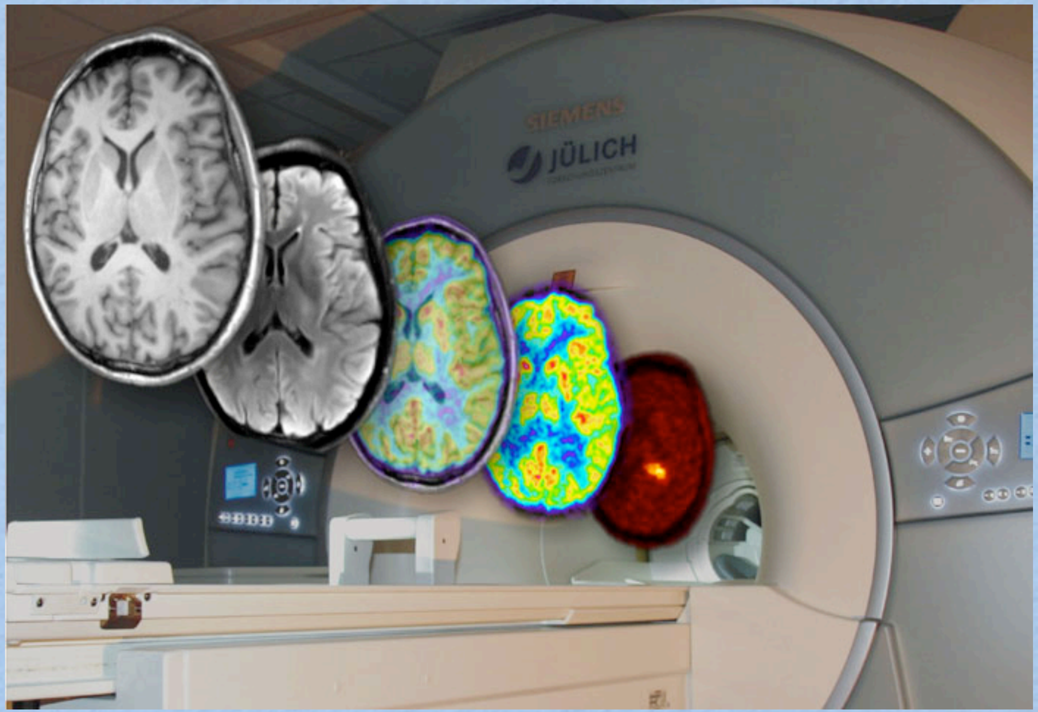
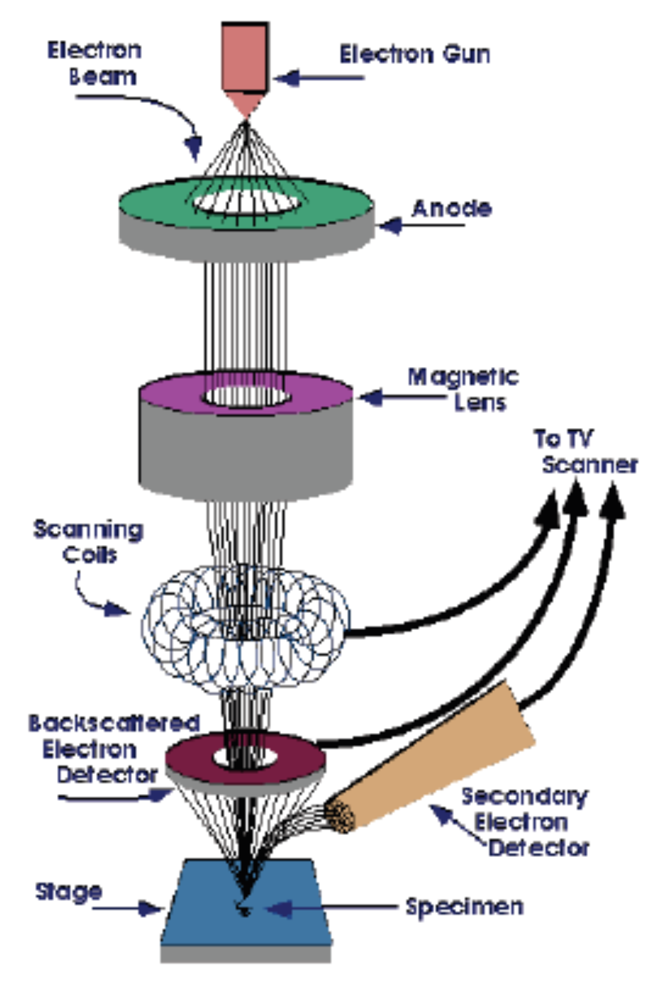
GPDs and DVCS



Marc Vanderhaeghen

Jefferson Lab, March 15-17, 2017

how to image a system

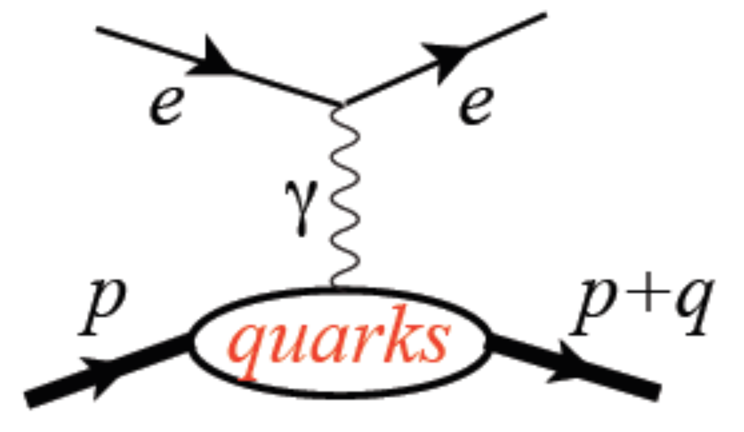


electron microscopy

when target is static ($m_{constituent}$, $m_{target} >> Q$)

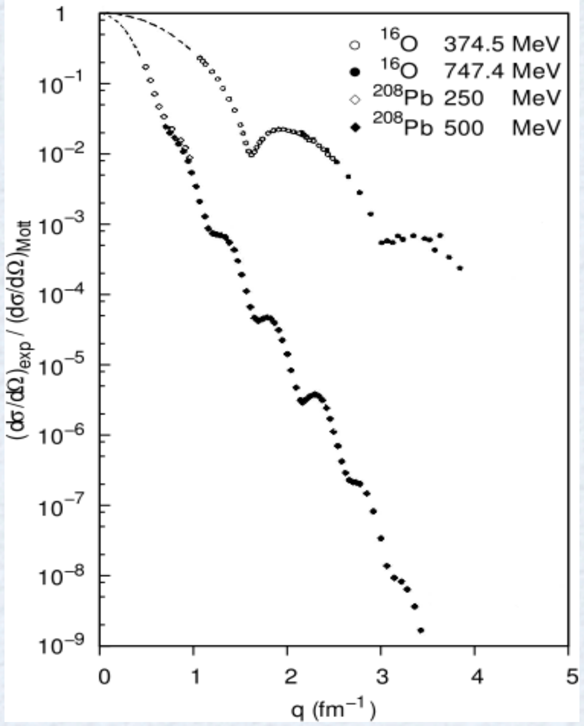
the 3-dim Fourier transform of **form factors** gives

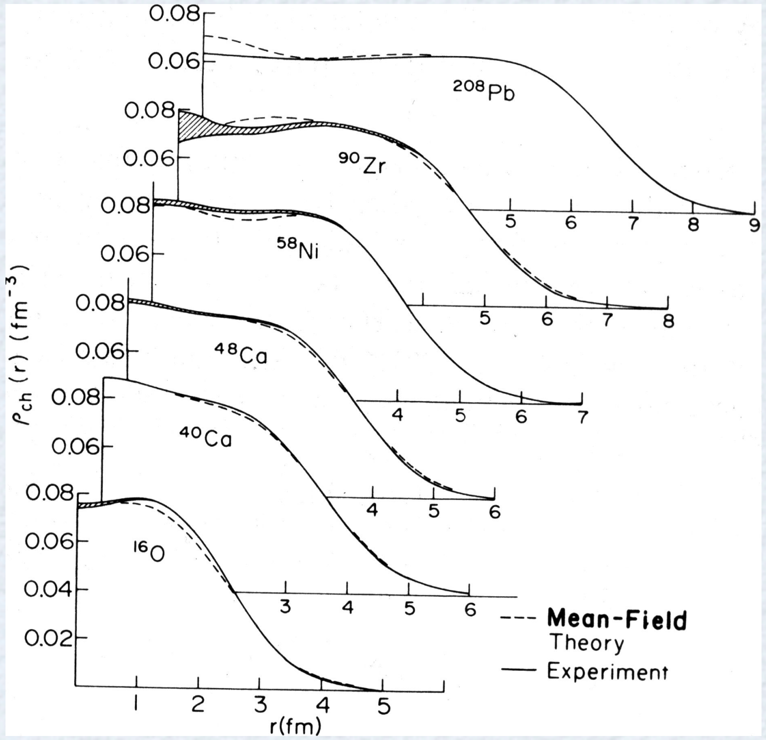
the distribution of electric charge and magnetization



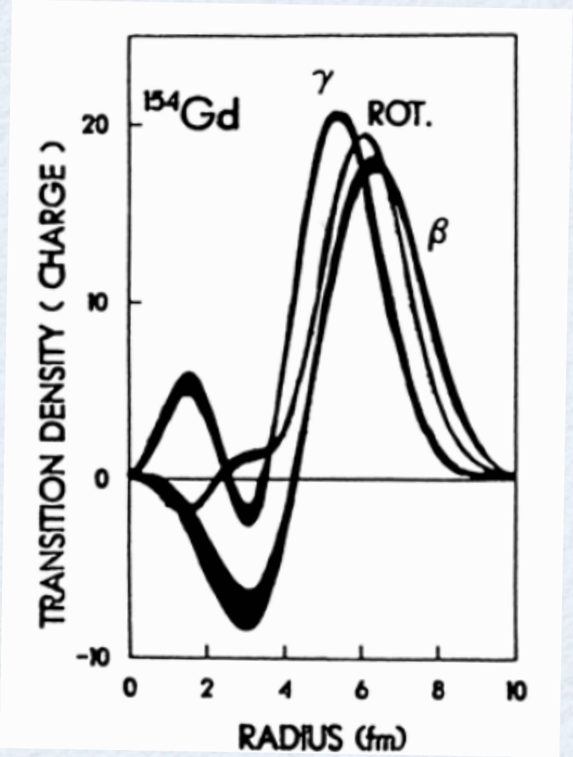
what do we know about spatial distributions of charges in nuclei?

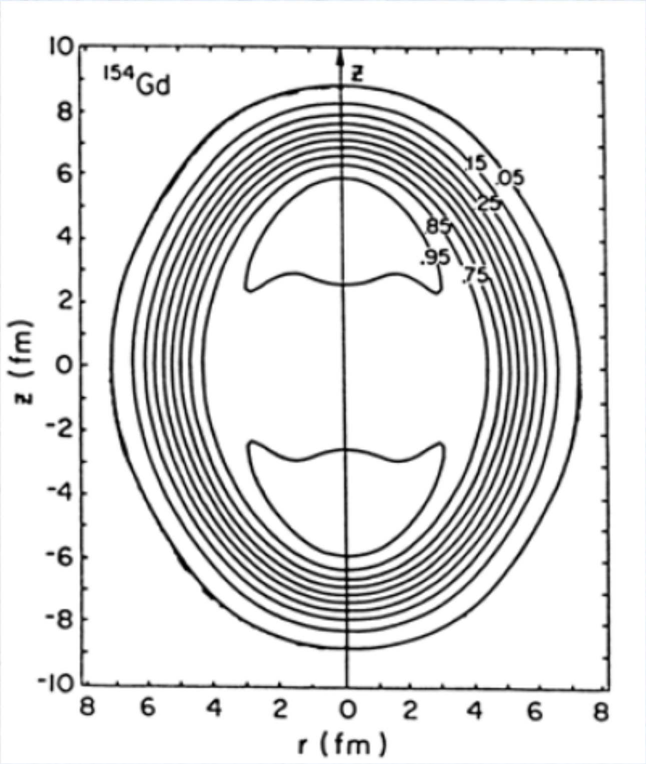
sizes of nuclei:
as revealed through
elastic electron scattering





shapes of nuclei:
as revealed through
inelastic electron scattering



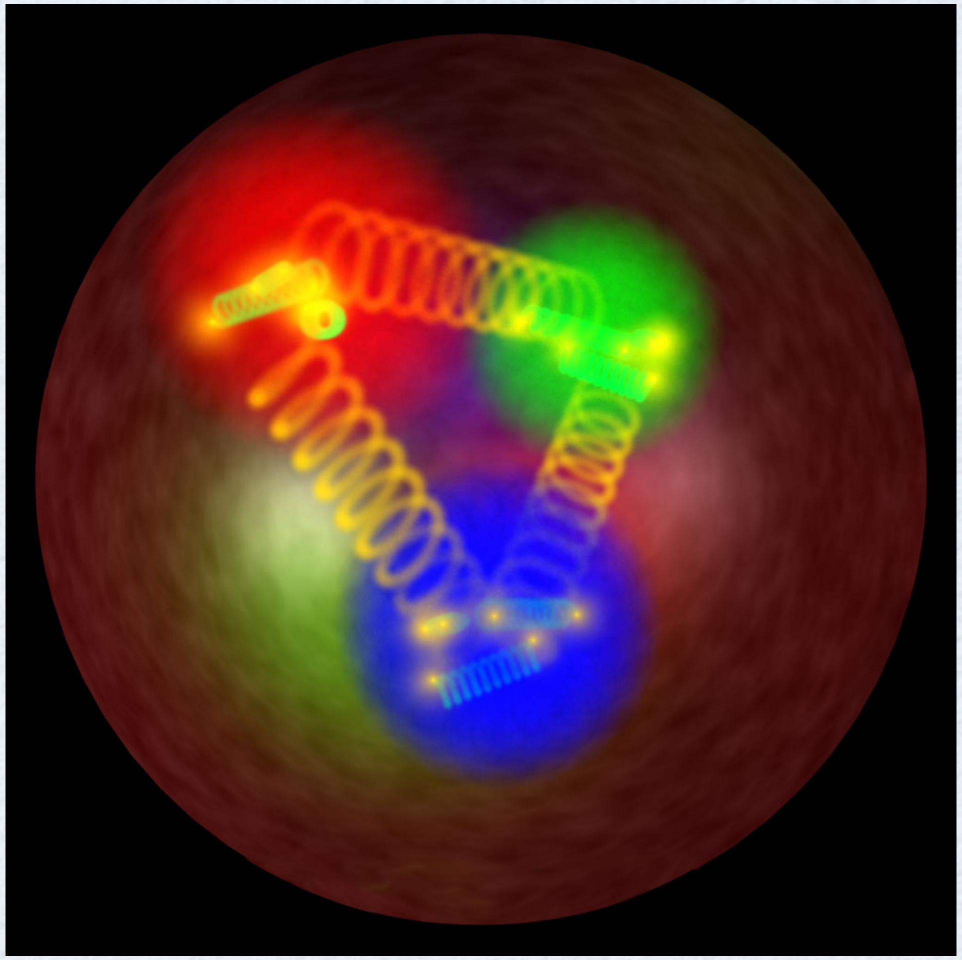


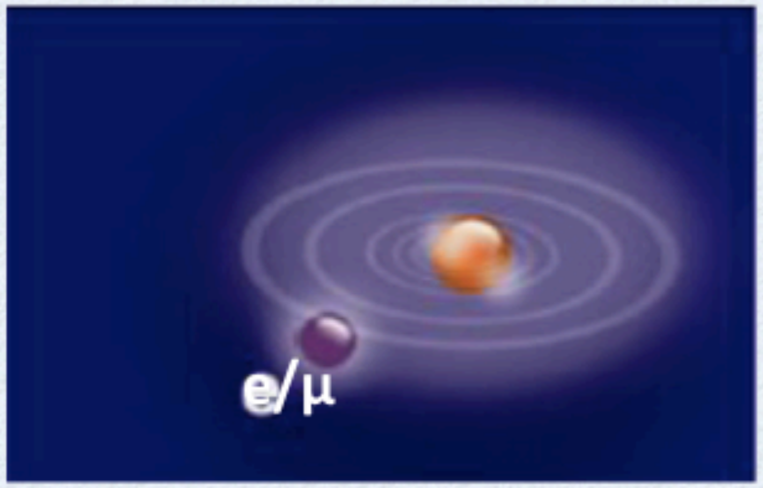
what do we know about the proton size and its charge distributions?

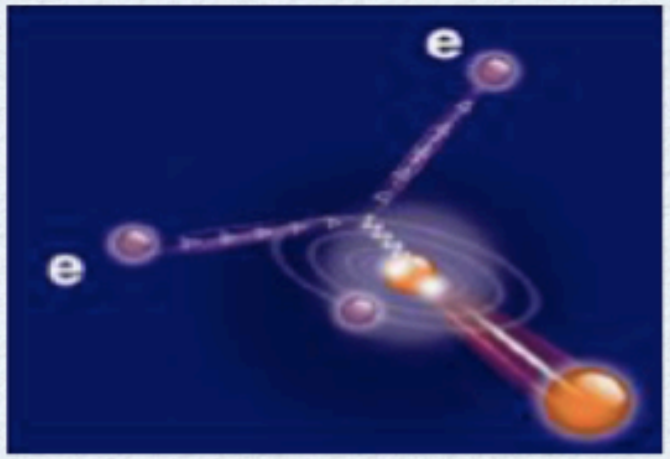
- proton **size**: charge radius R_E
  very low Q^2 **elastic** electron scattering,
  atomic spectroscopy (Lamb shift)

- proton **spatial (charge) distributions**
  elastic electron scattering
  e.m. FFs: $F_1(Q^2) \rightarrow \rho(\mathbf{b})$

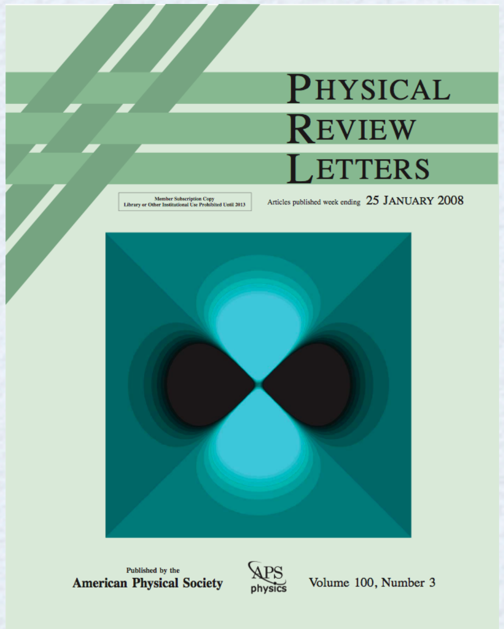
- proton **3D transverse spatial/ longitudinal momentum distributions**
  deeply virtual Compton scattering
  GPDs $H(x, \xi, t) \rightarrow \rho(x, \mathbf{b})$ for $\xi=0$







proton size, proton e.m. form factors, charge distributions



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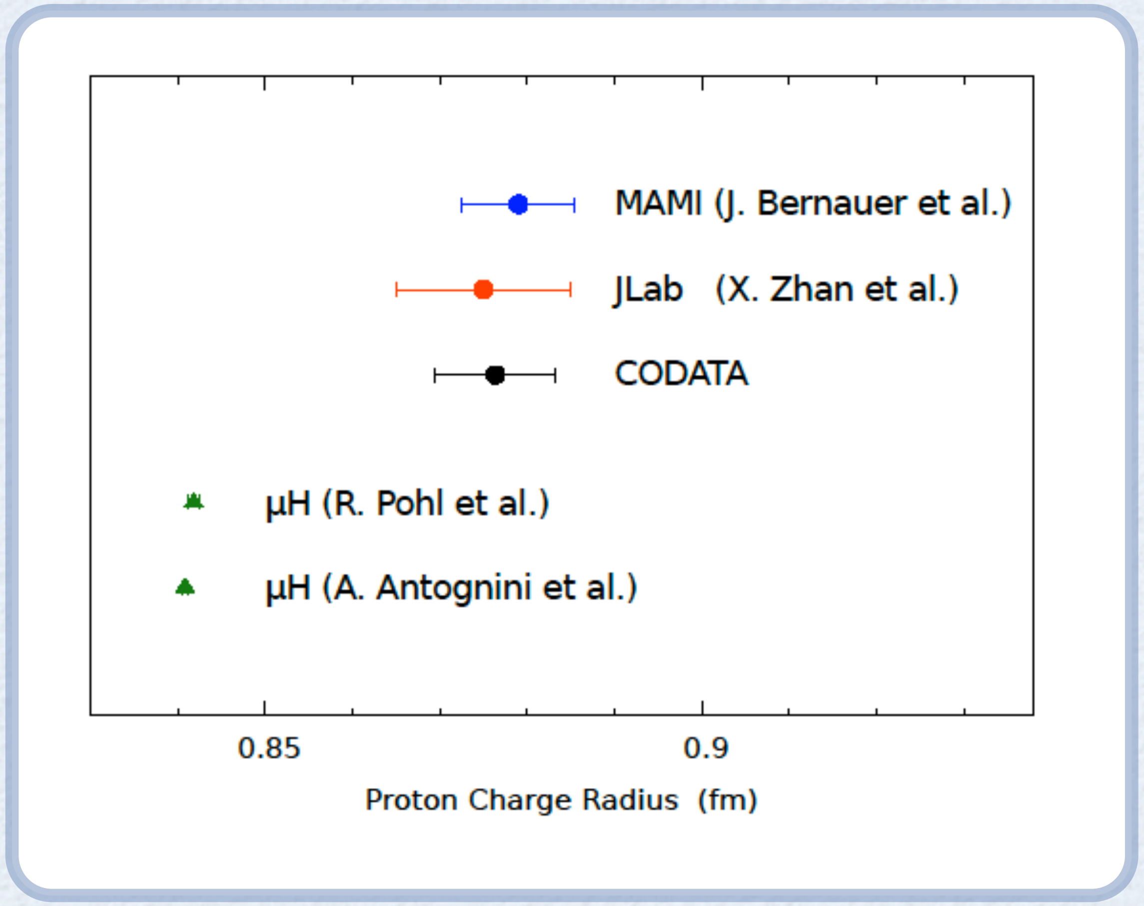
APS physics

Volume 100, Number 3

Proton radius puzzle



μH data:

Pohl et al.(2010)

Antognini et al.(2013)

$R_E = 0.8409 \pm 0.0004$ fm

7 σ difference !?

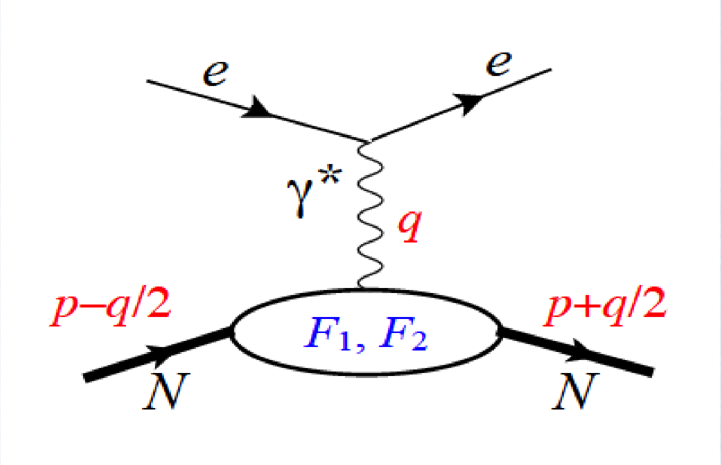
ep data:

$R_E = 0.8775 \pm 0.0051$ fm

CODATA(2012)

spin-1/2 electromagnetic form factors

- **(in)elastic electron scattering** is our microscope to investigate hadron structure



- in the **1-photon exchange approximation:**

nucleon (spin 1/2 target) structure is parameterized by 2 **form factors (FFs)**

$$\boxed{\langle p+\frac{q}{2}, \lambda' \,|\, J^\mu(0) \,|\, p-\frac{q}{2}, \lambda\rangle \;=\; \bar{u}(p+\frac{q}{2}, \lambda') \left[\underset{\uparrow}{F_1}(Q^2)\gamma^\mu + \underset{\uparrow}{F_2}(Q^2)\frac{i}{2M}\sigma^{\mu\nu}q_\nu \right] u(p-\frac{q}{2}, \lambda)}$$

Dirac FF **Pauli FF**

for proton: $F_1(Q^2=0) = 1$ $F_2(Q^2=0) = \kappa_p = 1.79$

- equivalently: in experiment one often uses **Sachs FFs** with $\tau \equiv \frac{Q^2}{4M^2}$

$$\boxed{\begin{aligned} G_M(Q^2) &= F_1(Q^2) + F_2(Q^2) \\ G_E(Q^2) &= F_1(Q^2) - \tau F_2(Q^2) \end{aligned}}$$

⟶ **magnetic FF**

⟶ **electric FF**

$$\boxed{G_E(Q^2) = 1 - \frac{1}{6}\underset{\uparrow}{\langle r_E^2\rangle}\, Q^2 + \mathcal{O}(Q^4)}$$

charge radius

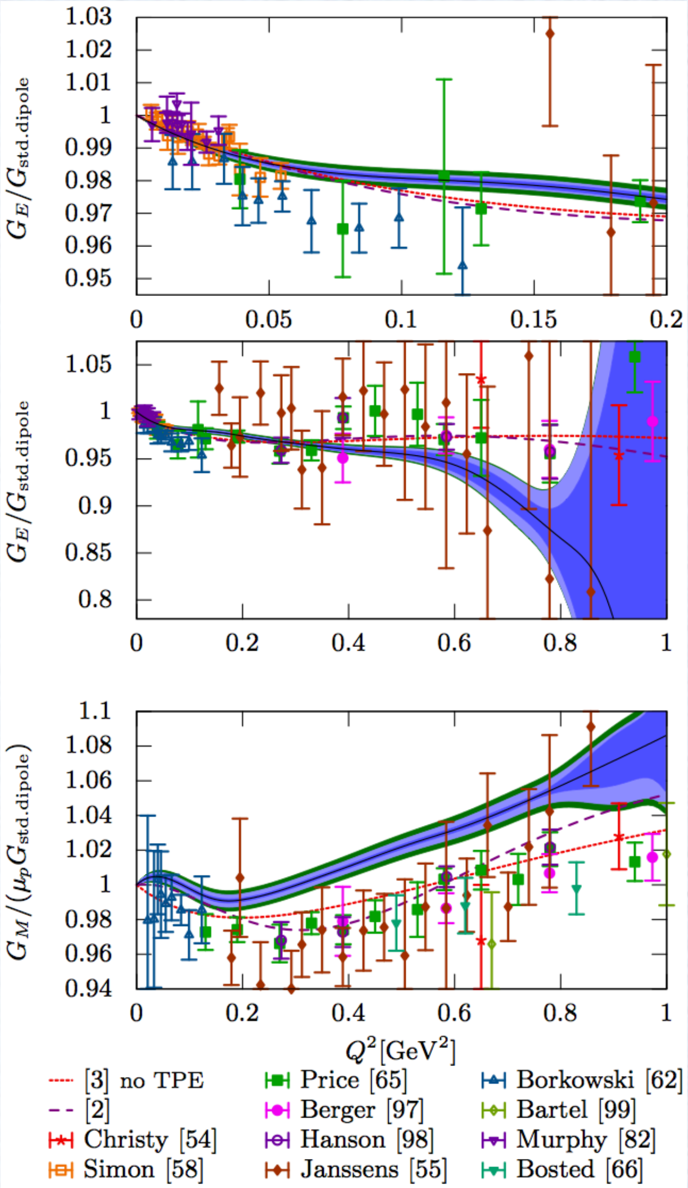
e^- scattering cross sections

Electron scattering facilities JLab (12 GeV), MAMI (1.6 GeV):

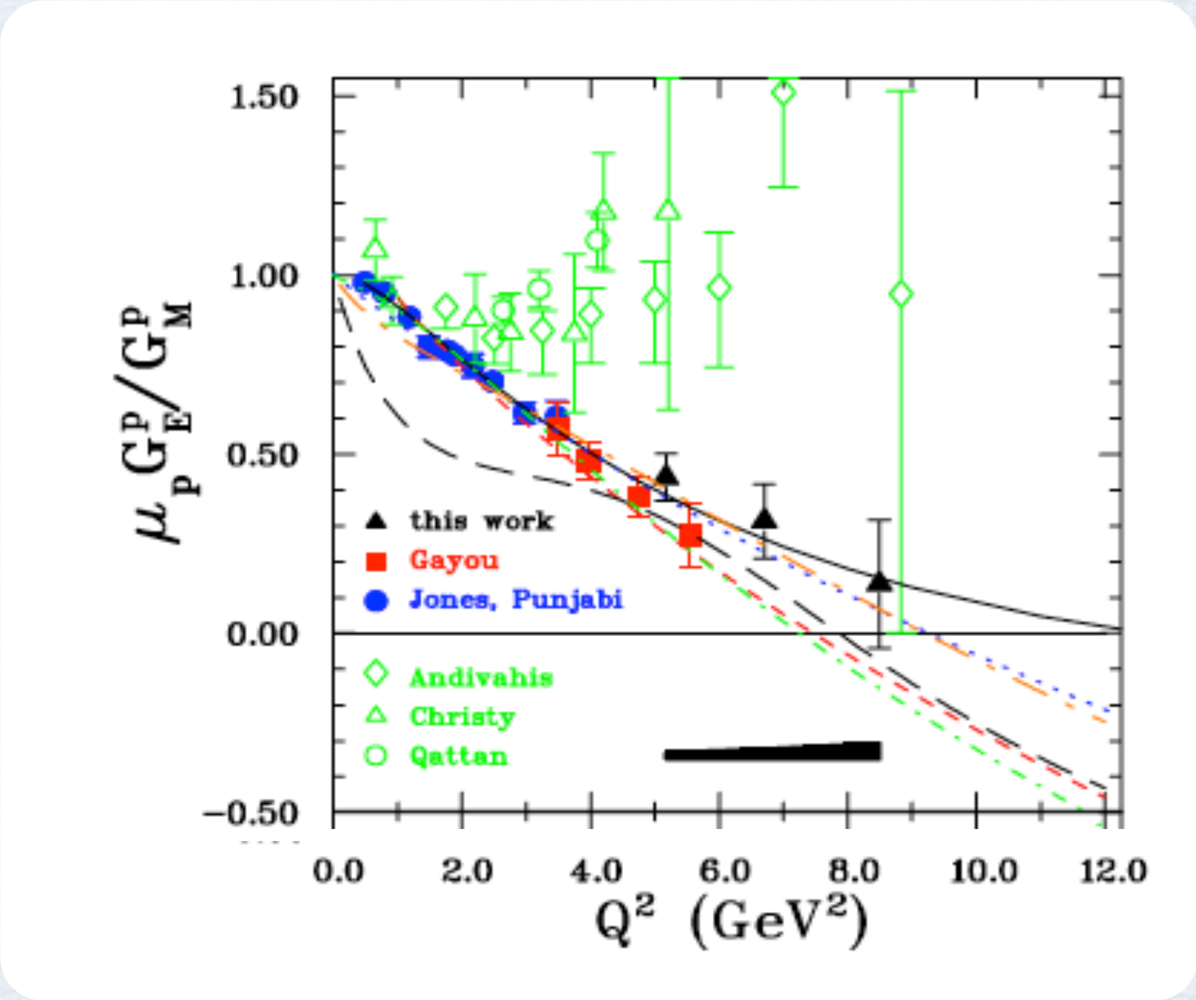
uniquely positioned to deliver high precision data

MAMI/A1 achieched < 1% measurement of proton charge radius R_E

JLab polarization transfer measurements: G_{Ep} / G_{Mp} difference with Rosenbluth



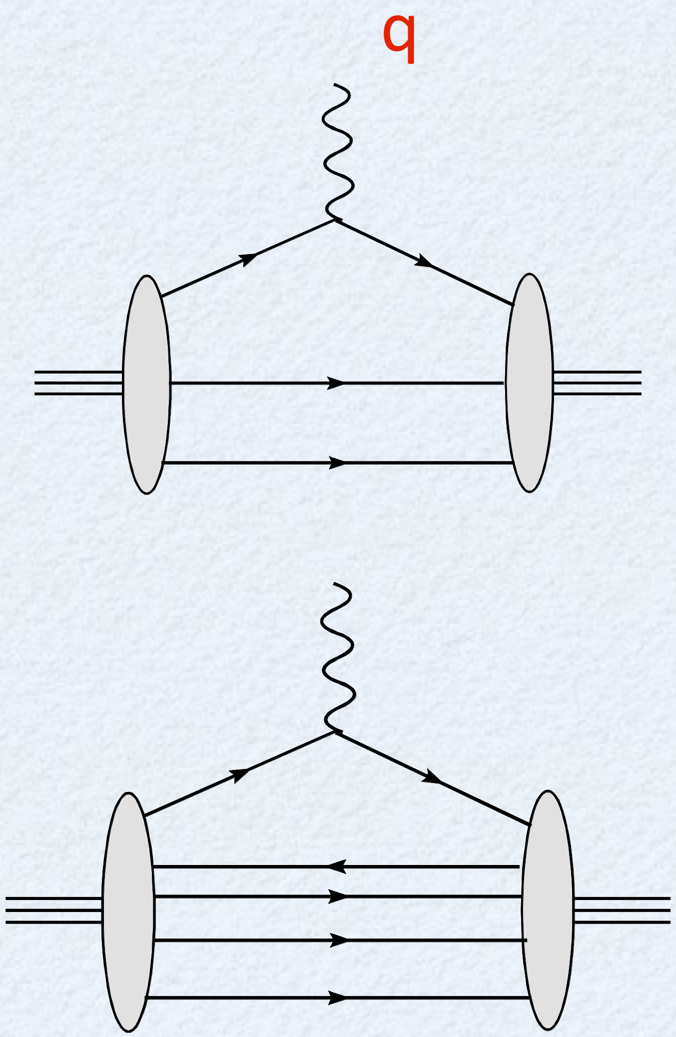
Bernauer et al.(2010, 2013)



Jones et al.(2000) Punjabi et al.(2005)

Gayou et al.(2002) Puckett et al.(2010)

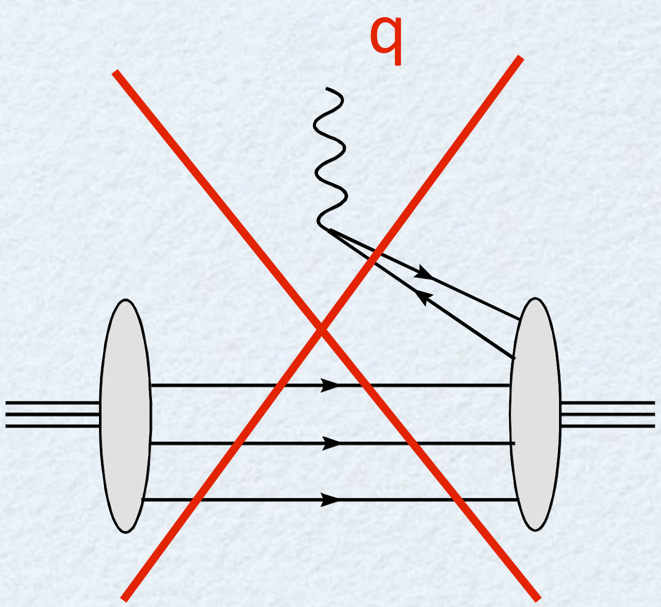
Interpretation of form factor as quark density



overlap of wave function

Fock components

with same number of quarks



overlap of wave function
Fock components

with different number of quarks

NO probability / charge density

interpretation

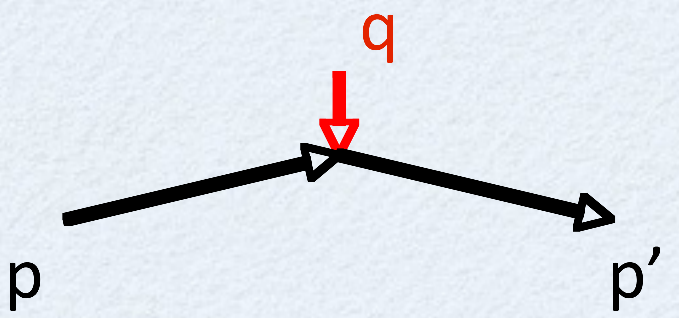
absent in a light-front frame!

$$q^{+} = q^{0} + q^{3} = 0$$

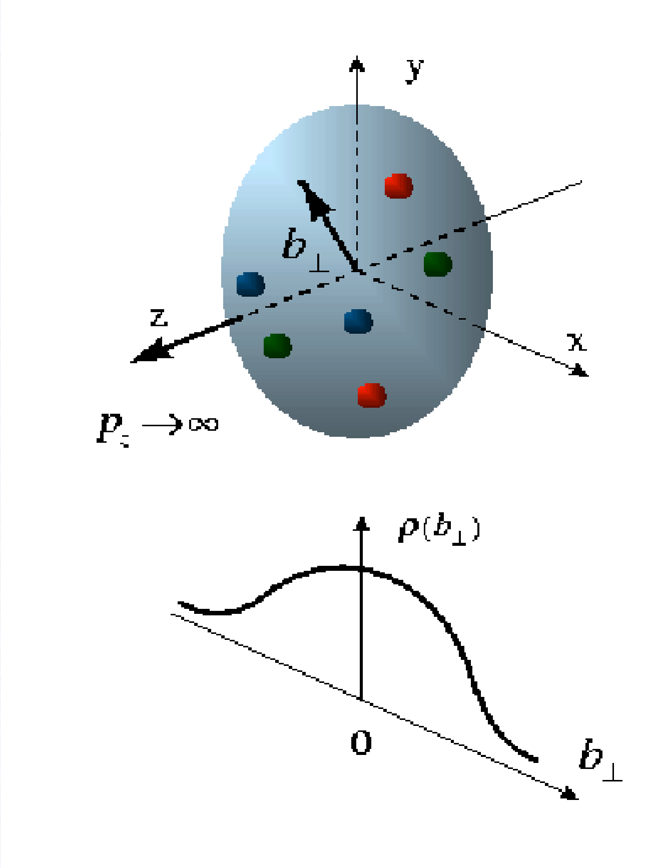
quark transverse charge densities in nucleon (1)

- light-front

$$q^+ = q^0 + q^3 = 0$$



photon only couples to forward moving quarks



- quark charge density operator

$$J^+ = J^0 + J^3 = \bar{q}\gamma^+ q = 2 q_+^\dagger q_+$$

with $q_+ \equiv \frac{1}{4}\gamma^-\gamma^+ q$

- longitudinally polarized nucleon

$$\rho_0^N(\vec{b}) \equiv \int \frac{d^2\vec{q}_\perp}{(2\pi)^2} e^{-i\vec{q}_\perp \cdot \vec{b}} \frac{1}{2P^+} \langle P^+, \frac{\vec{q}_\perp}{2}, \lambda | J^+(0) | P^+, -\frac{\vec{q}_\perp}{2}, \lambda \rangle$$
$$= \int_0^\infty \frac{dQ}{2\pi} Q J_0(bQ) F_1(Q^2)$$

Soper (1997)

Burkardt (2000)

Miller (2007)

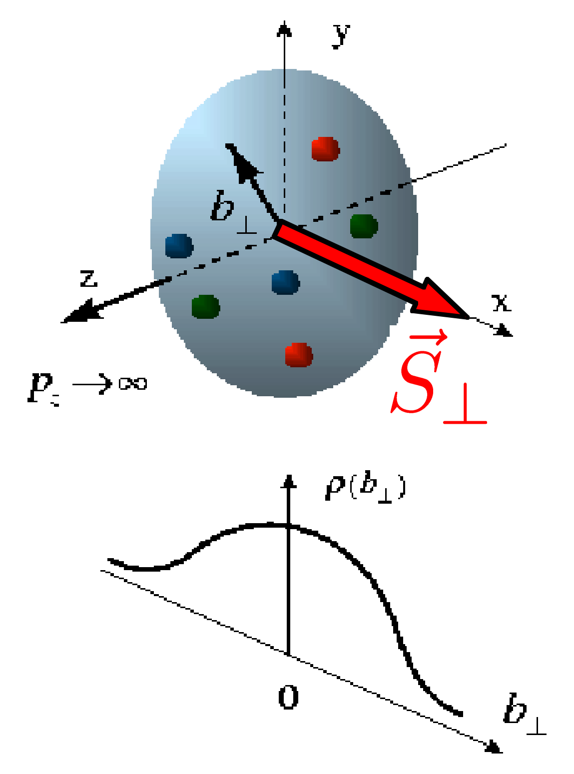
quark transverse charge densities in nucleon (2)

- transversely polarized nucleon

transverse spin $\vec{S}_\perp = \cos\phi_S\,\hat{e}_x + \sin\phi_S\,\hat{e}_y$

e.g. along x-axis $\phi_S = 0$

$$\vec{b} = b(\cos\phi_b\,\hat{e}_x + \sin\phi_b\,\hat{e}_y)$$

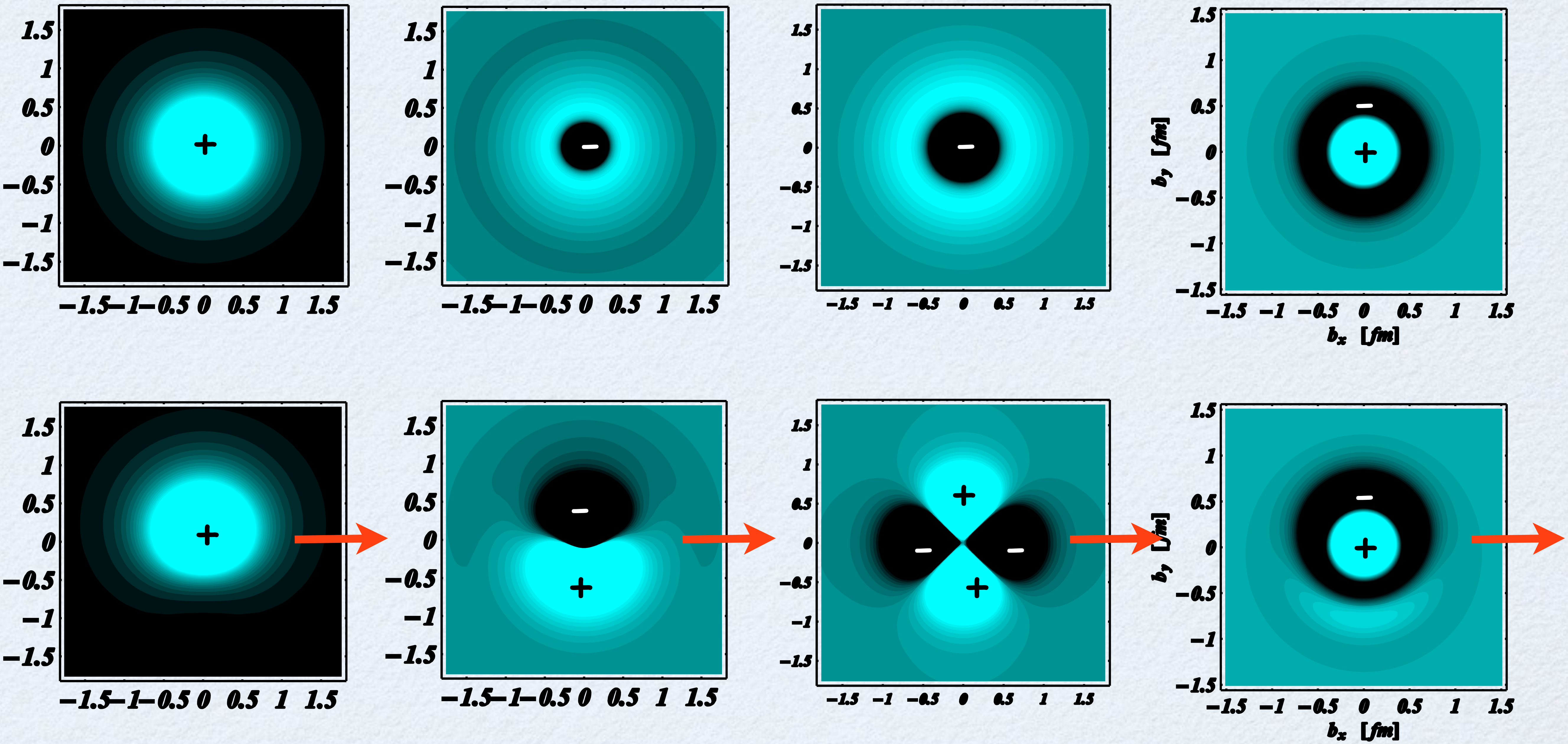


-
$$\rho_T^N(\vec{b}) \equiv \int \frac{d^2\vec{q}_\perp}{(2\pi)^2}\, e^{-i\vec{q}_\perp\cdot\vec{b}}\, \frac{1}{2P^+} \langle P^+, \frac{\vec{q}_\perp}{2}, s_\perp = +\frac{1}{2} | J^+(0) | P^+, -\frac{\vec{q}_\perp}{2}, s_\perp = +\frac{1}{2} \rangle$$
$$= \rho_0^N(b) + \sin(\phi_b - \phi_S) \int_0^\infty \frac{dQ}{2\pi}\, \frac{Q^2}{2M} J_1(bQ) F_2(Q^2)$$

dipole field pattern

Carlson, Vdh (2007)

spatial imaging of hadrons



proton

neutron

p -> Δ^+

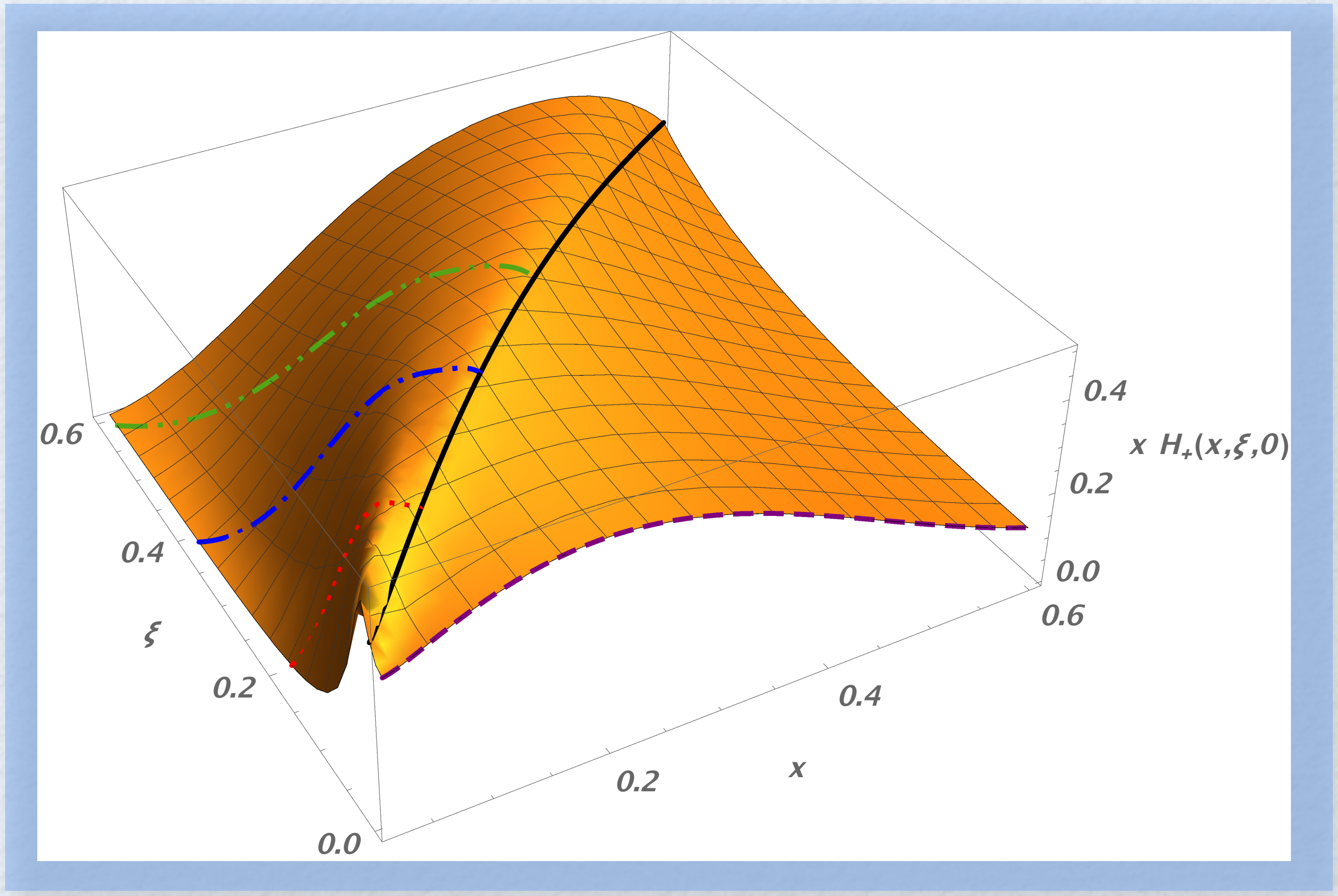
p -> N^* (1440)

Miller(2007)

Carlson, Vdh(2007)

Tiator, Vdh(2007)

Generalized Parton Distributions and DVCS



Correlations in transverse position/longitudinal momentum

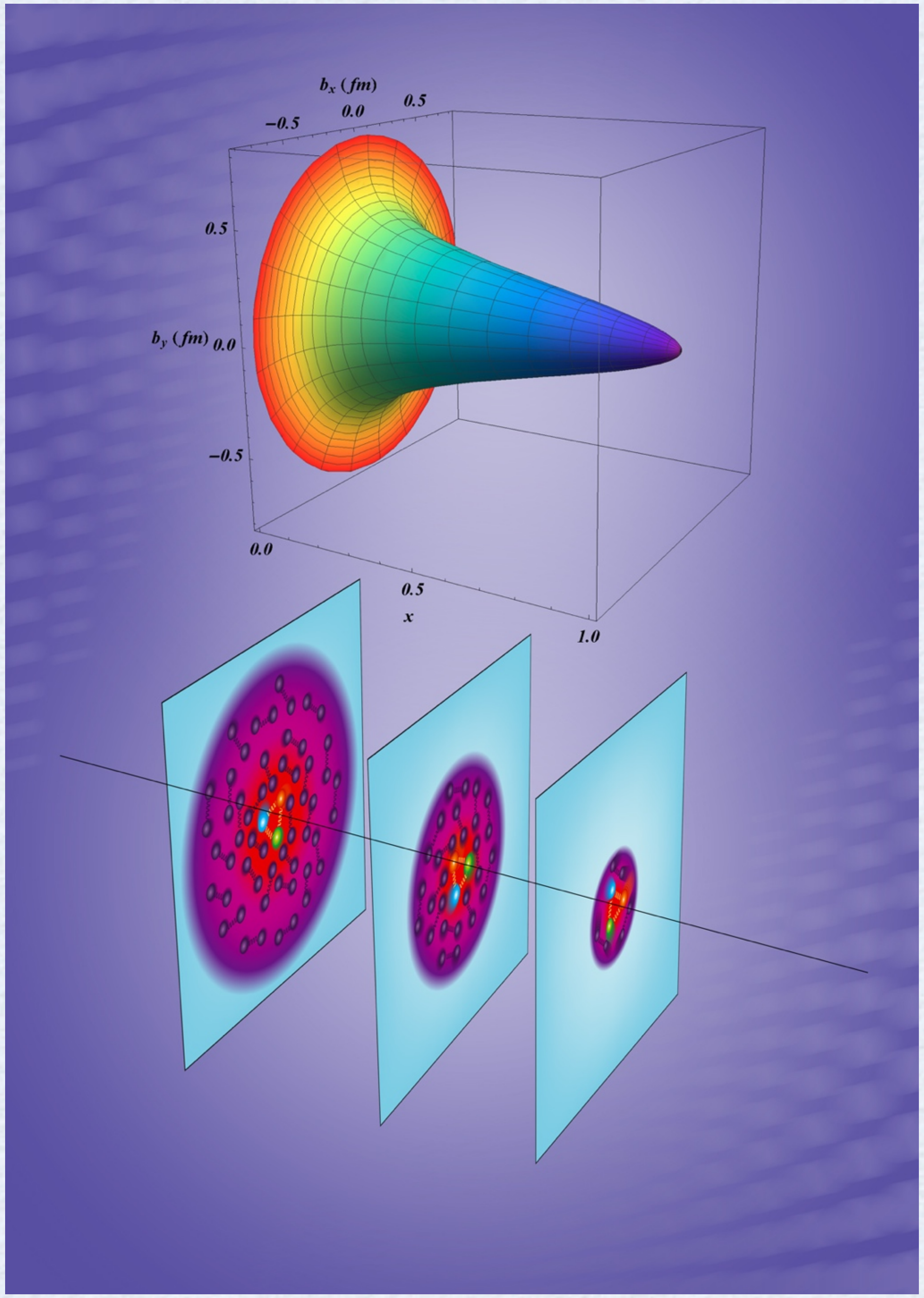
elastic scattering → **GPDs** ← **DIS**

quark distributions in **transverse position space**

proton
3D imaging

Burkardt(2000, 2003)

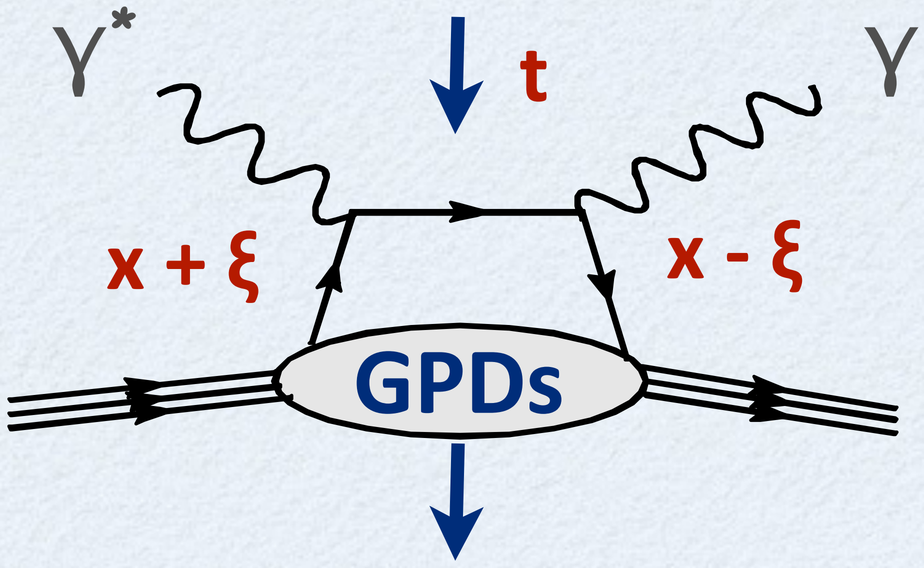
Belitsky, Ji, Yuan (2004)



quark distributions in **longitudinal momentum**

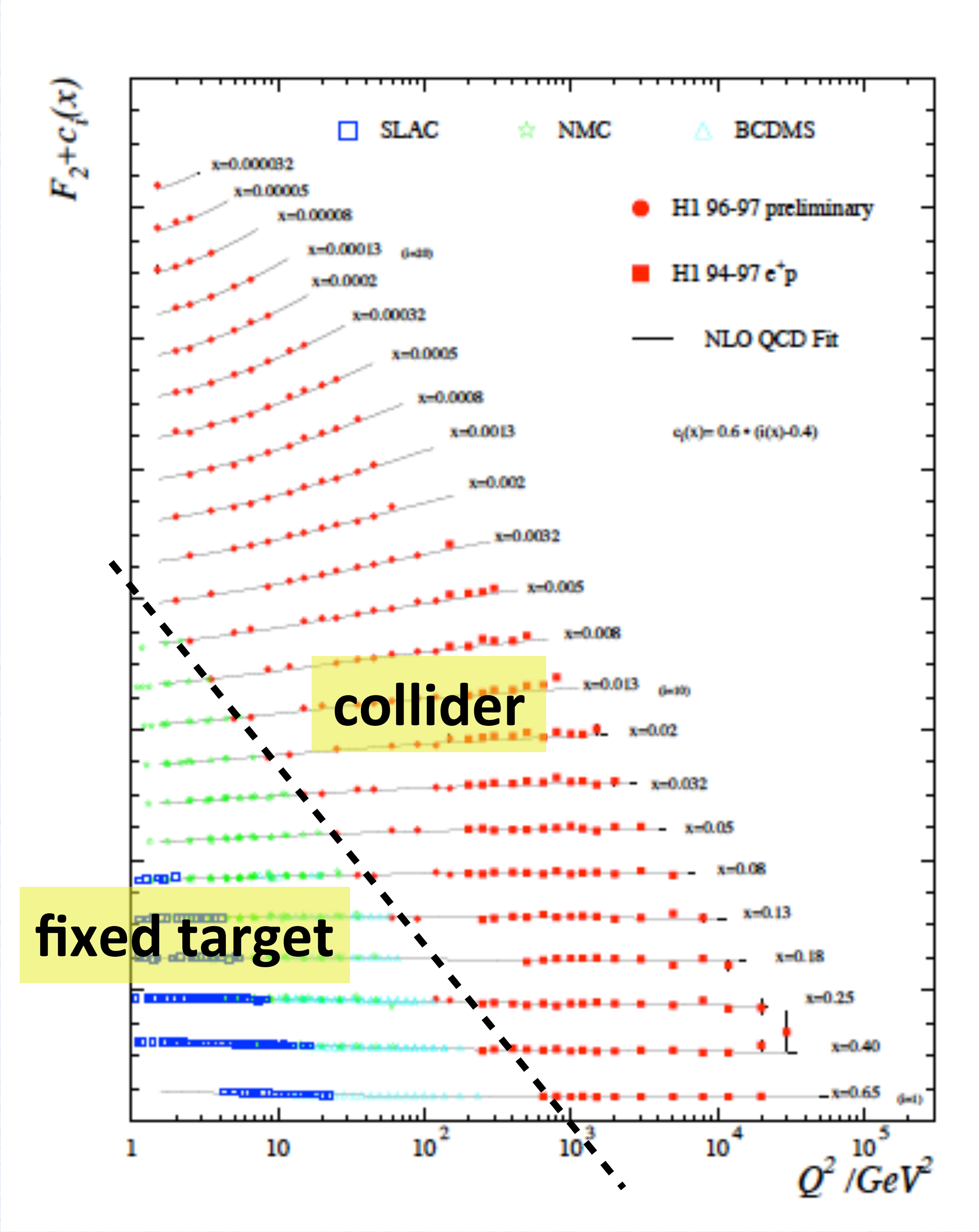
DVCS: tool to access GPDs

$Q^2 >> 1$ GeV^2



GPD(x, ξ, t)

world data on proton F_2



- at large Q^2: **QCD factorization theorem**

  Müller et al (1994)

  Ji (1995) Radyushkin (1995)

  Collins, Frankfurt, Strikman (1996)

  at twist-2: **4 quark helicity conserving GPDs**

- key: Q^2 leverage needed to test **QCD scaling**

GPDs: known limits

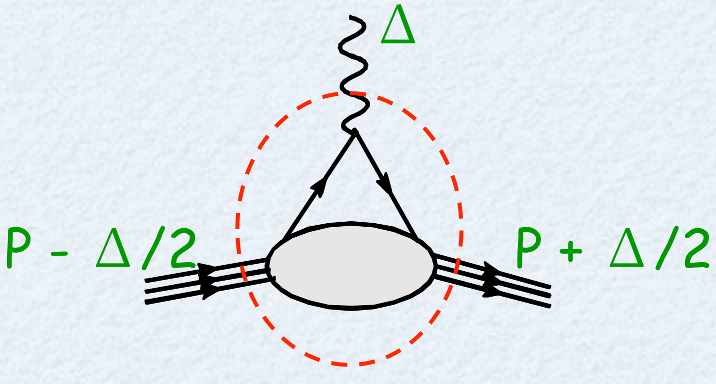
- in forward kinematics (ξ=0, t = 0) : **PDF limit**

$$H^q(x, \xi = 0, t = 0) = q(x)$$

$$\tilde{H}^q(x, \xi = 0, t = 0) = \Delta q(x)$$

$E, \tilde{E}^q$ do not appear in forward kinematics (DIS) ➡ **new information**

- first moments of GPDs : **elastic form factor limit**



t = Δ²

$$\int_{-1}^{+1} dx H^q(x, \xi, t) = F_1^q(t)$$ → Dirac FF

$$\int_{-1}^{+1} dx E^q(x, \xi, t) = F_2^q(t)$$ → Pauli FF

$$\int_{-1}^{+1} dx \tilde{H}^q(x, \xi, t) = G_A^q(t)$$ → axial FF

$$\int_{-1}^{+1} dx \tilde{E}^q(x, \xi, t) = G_P^q(t)$$ → pseudoscalar FF

GPDs: higher moments, total quark angular momentum

$$\int_{-1}^{+1} dx x H^q(x,\xi,t) = A(t)(t) + \xi^2 C(t)(t)$$

$$\int_{-1}^{+1} dx x E^q(x,\xi,t) = B(t)(t) - \xi^2 C(t)(t)$$

form factors of energy-momentum tensor

Polyakov, Weiss(1999)

Polyakov(2003)

Goeke, Schweitzer et al.(2007)

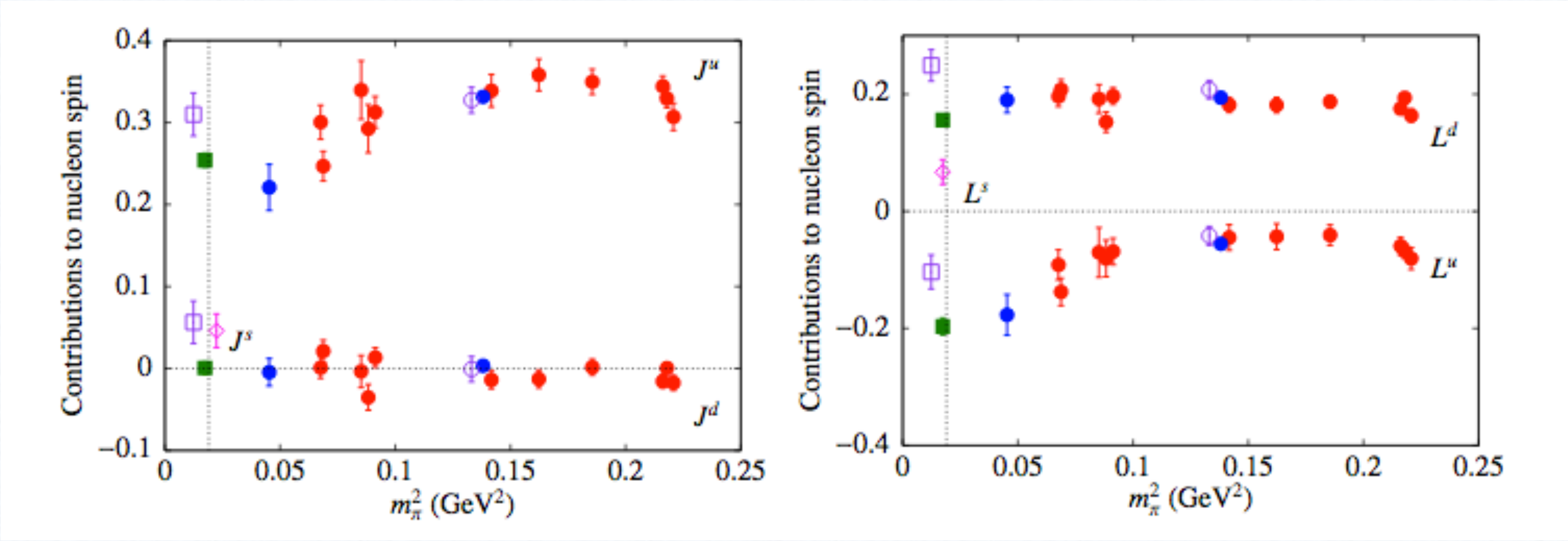
- Ji’s angular momentum sum rule

$$\int_{-1}^{+1} dx \, \{H^q(x,\xi,0) + E^q(x,\xi,0)\} = A(0) + B(0) = 2J^q$$

- lattice QCD calculations at the physical point

e.g. twisted mass fermions

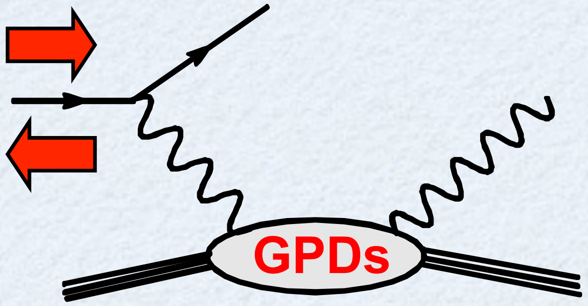
Alexandrou et al.(2016)



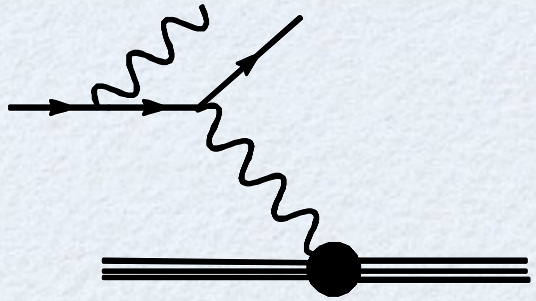
d, s-quarks carry very small total angular momentum, u-quark carries around 50%

DVCS beam spin asymmetries: first observations around 2000

$$A_{LU} = \frac{(BH) * Im(DVCS) * \sin \Phi}{(BH^2+DVCS^2)}$$

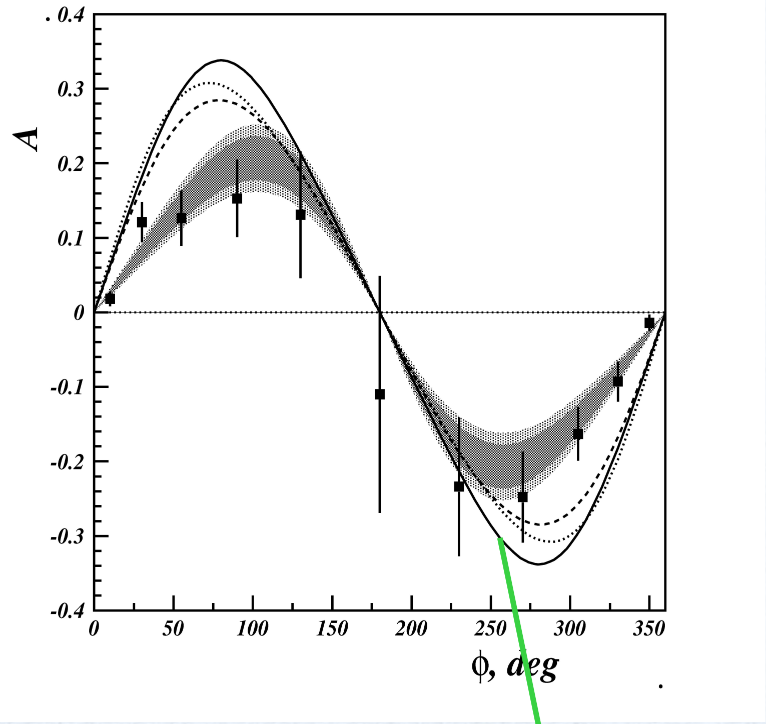


DVCS



Bethe-Heitler

CLAS



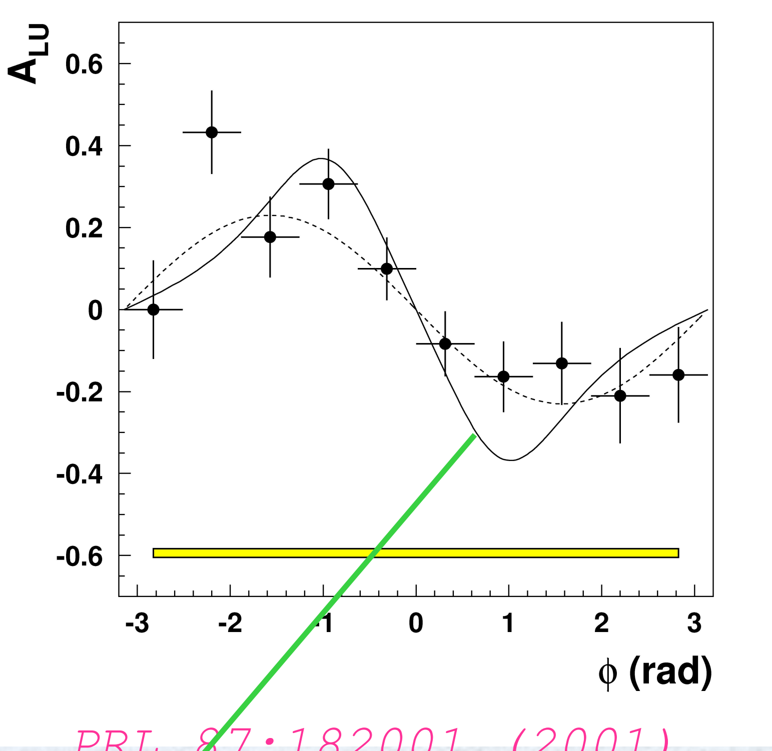
Q^2 = 1.25 GeV^2 ,

x_B = 0.19,

-t = 0.19 GeV^2

PRL 87:182002 (2001)

HERMES



Q^2 = 2.6 GeV^2 ,

x_B = 0.11,

-t = 0.27 GeV^2

PRL 87:182001 (2001)

twist-2 + twist-3

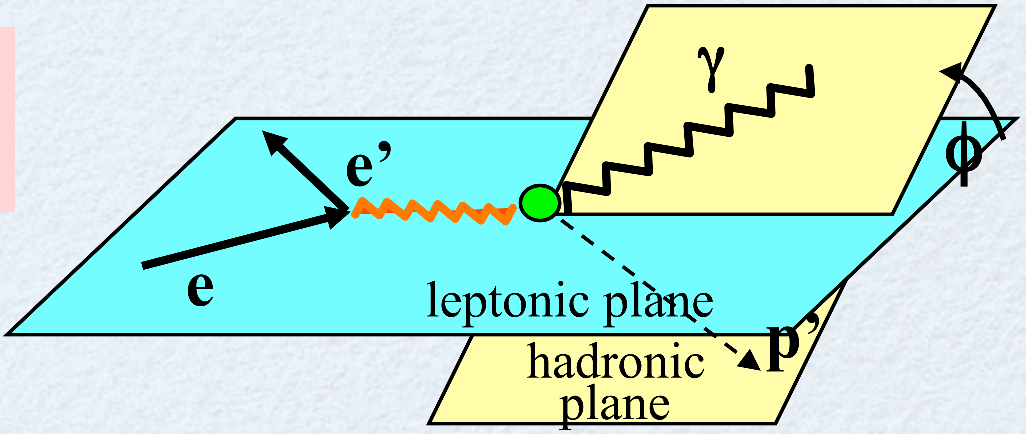
Vdh, Guichon, Guidal (1999)

Kivel, Polyakov, Vdh (2000)

DVCS observables

$$A = \frac{\sigma^{+} - \sigma^{-}}{\sigma^{+} + \sigma^{-}} = \frac{\Delta\sigma}{2\sigma}$$

$\xi = xB/(2-xB)$
$k = -t/4M^2$



Polarized beam, unpolarized proton target:

$$\Delta\sigma LU \sim \sin\phi\ \mathbf{Im}\{F1H + \xi(F1+F2)\tilde{H} + kF2E\}d\phi \Rightarrow \mathbf{Hp}, \tilde{H}p, Ep$$

Kinematically suppressed

Unpolarized beam, longitudinal proton target:

$$\Delta\sigma UL \sim \sin\phi\mathbf{Im}\{F1\tilde{H} + \xi(F1+F2)(H + \ldots\}d\phi \Rightarrow Hp, \mathbf{\tilde{H}p}$$

Unpolarized beam, transverse proton target:

$$\Delta\sigma UT \sim \sin\phi\mathbf{Im}\{k(F2H - F1E) + \ldots\ldots\}d\phi \Rightarrow \mathbf{Hp, Ep}$$

Polarized beam, unpolarized neutron target:

$$\Delta\sigma LU \sim \sin\phi\ \mathbf{Im}\{F1H + \xi(F1+F2)\tilde{H} - kF2E\}d\phi \Rightarrow Hn, \tilde{H}n, \mathbf{En}$$

Suppressed because F1(t) is small

Suppressed because of cancellation between PPD's of u and d quarks

$\mathbf{Hp}(x, \xi, t) = 4/9\ \mathbf{Hu}(x, \xi, t) + 1/9\ \mathbf{Hd}(x, \xi, t)$
$\mathbf{Hn}(x, \xi, t) = 1/9\ \mathbf{Hu}(x, \xi, t) + 4/9\ \mathbf{Hd}(x, \xi, t)$

DVCS accesses Compton Form Factors:
8 CFFs at twist-2

$$\mathcal{H}_{Re}(\xi,t) \equiv \mathcal{P}\int_0^1 dx \left\{ \frac{1}{x-\xi} + \frac{1}{x+\xi} \right\} H_+(x,\xi,t)$$

$$\mathcal{H}_{Im}(\xi,t) \equiv H_+(\xi,\xi,t)$$

$$\tilde{\mathcal{H}}_{Re}(\xi,t) \equiv \mathcal{P}\int_0^1 dx \left\{ \frac{1}{x-\xi} - \frac{1}{x+\xi} \right\} \tilde{H}_+(x,\xi,t)$$

$$\tilde{\mathcal{H}}_{Im}(\xi,t) \equiv \tilde{H}_+(\xi,\xi,t)$$

and analogous formulas for GPDs $E, \tilde{E}^q$ respectively

with singlet GPD combinations (quark + anti-quark):

$$H_+(x,\xi,t) \equiv H(x,\xi,t) - H(-x,\xi,t)$$

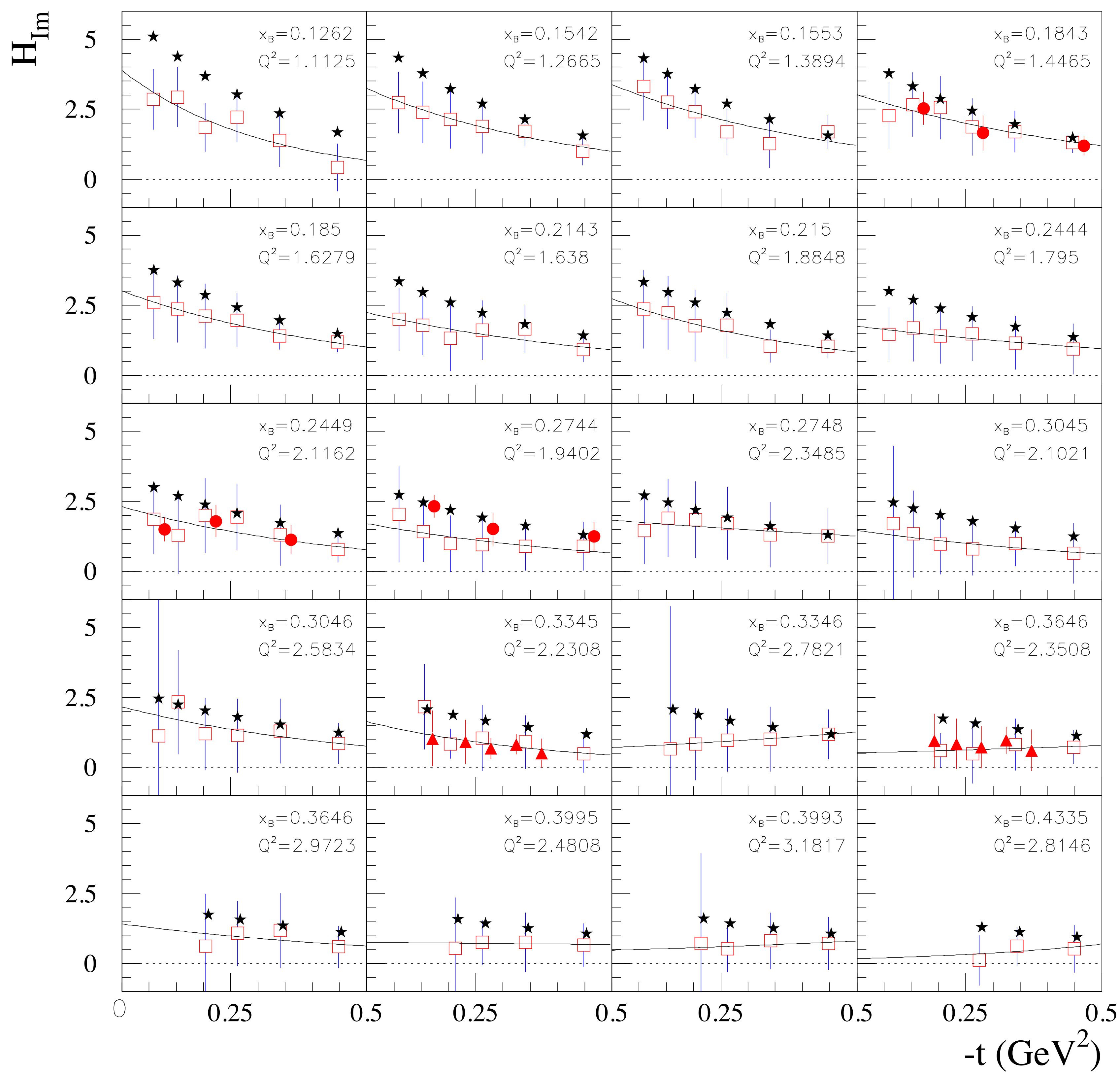
$$\tilde{H}_+(x,\xi,t) \equiv \tilde{H}(x,\xi,t) + \tilde{H}(-x,\xi,t)$$

CFF fit extractions from data:

`Guidal(2008,...)` `Guidal, Moutarde(2009,...)`

`Kumericki, Mueller, Passek-Kumericki(2008,...)`

global analysis of JLab 6 GeV data



$$\mathcal{H}_{Im}(\xi, t)$$

red solid circles:

CLAS: σ, A_{LU}, A_{UL}, A_{LL}

red open squares:

CLAS: σ, A_{LU}

red triangles:

Hall A: σ, A_{LU}

black stars

VGG model values

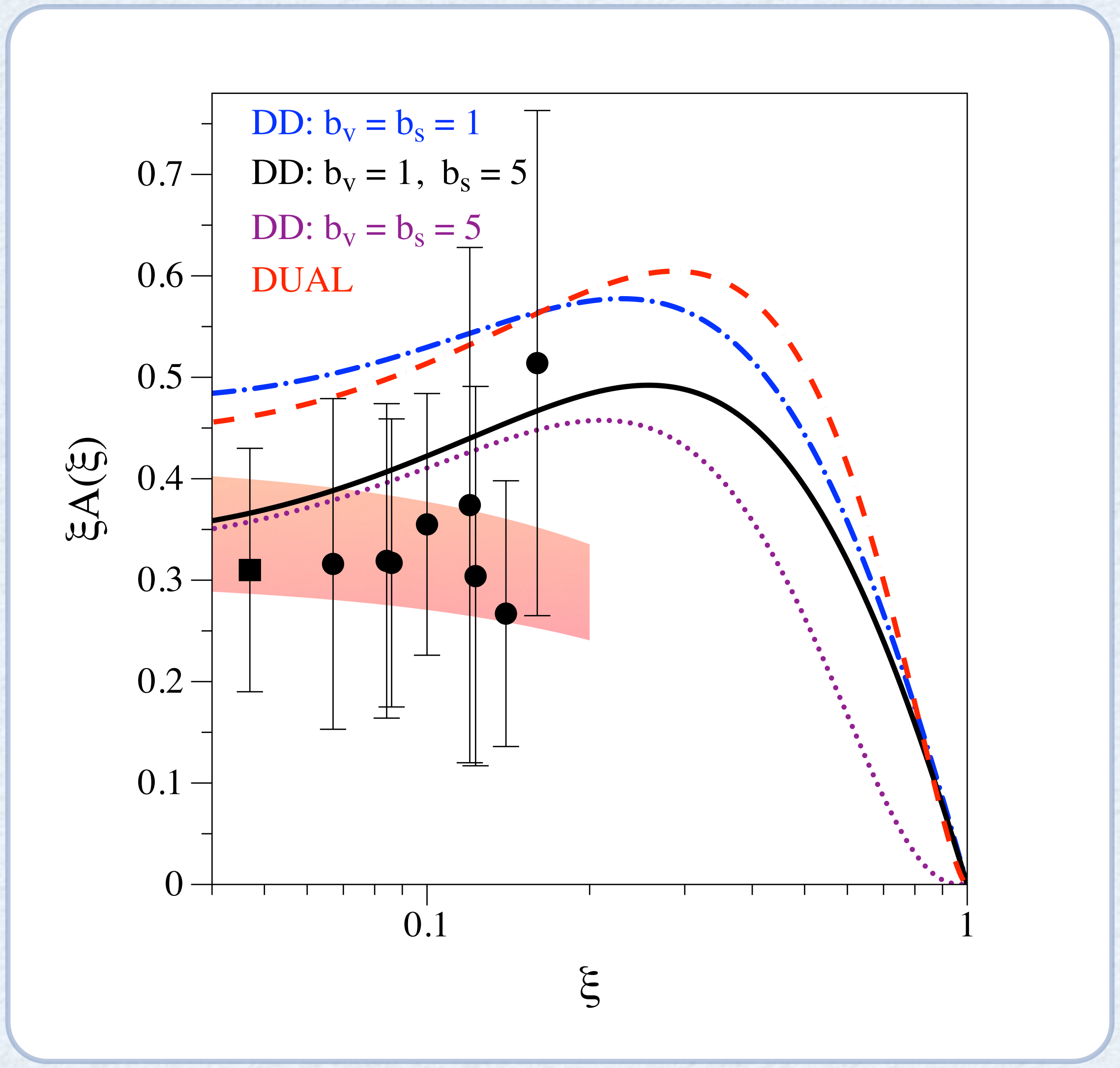
Dupré, Guidal, Vdh(2017)

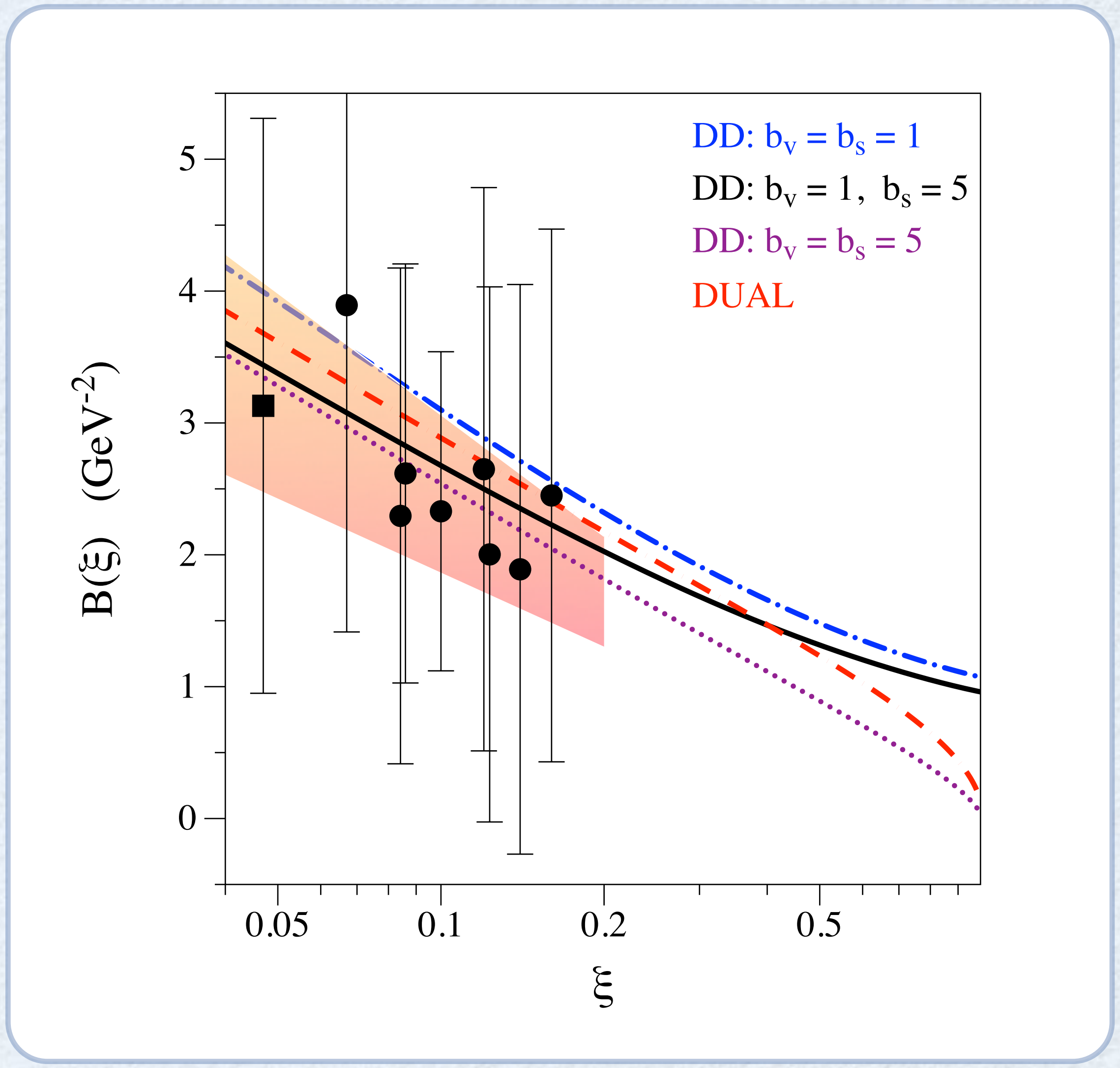
CFF $\mathcal{H}_{\mathrm{Im}}$:

$$\mathcal{H}_{Im}(\xi, t) = A(\xi)e^{B(\xi)t}$$

black circles: CFF fit of JLab data **Dupré, Guidal, Vdh(2017)**

black squares: CFF fit of HERMES data **Guidal, Moutarde(2009)**





$$A(\xi) = a_A(1 - \xi)/\xi$$

red bands:
1- parameter
fits of data

$$B(\xi) = a_B \ln(1/\xi)$$

3D imaging

➡ $$\boxed{\rho^q(x,\mathbf{b}_\perp) = \int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} e^{-i\mathbf{b}_\perp\cdot\mathbf{\Delta}_\perp} H^q_-(x,\xi=0,-\mathbf{\Delta}_\perp{}^2)}$$

Burkardt(2000)

number density of quarks (q) with longitudinal momentum x
at a transverse distance **b**$_\perp$ in proton

➡ non-singlet (valence quark) GPDs: $H^q_-(x,0,t) \equiv H^q(x,0,t) + H^q(-x,0,t)$

➡ x-dependent radius

$$\boxed{\langle b_\perp^2\rangle^q(x) \equiv \frac{\int d^2\mathbf{b}_\perp \mathbf{b}_\perp^2 \rho^q(x,\mathbf{b}_\perp)}{\int d^2\mathbf{b}_\perp \rho^q(x,\mathbf{b}_\perp)} = -4\frac{\partial}{\partial\mathbf{\Delta}_\perp^2} \ln H^q_-(x,0,-\mathbf{\Delta}_\perp{}^2)\Big|_{\mathbf{\Delta}_\perp=0}}$$

$$H^q_-(x,0,t) = q_v(x)e^{B_0(x)t} \longrightarrow \langle b_\perp^2\rangle^q(x) = 4B_0(x)$$

➡ x-independent radius

$$\boxed{\langle b_\perp^2\rangle^q = \frac{1}{N_q}\int_0^1 dx\, q_v(x)\,\langle b_\perp^2\rangle^q(x)}$$

N$_u$=2, N$_d$=1

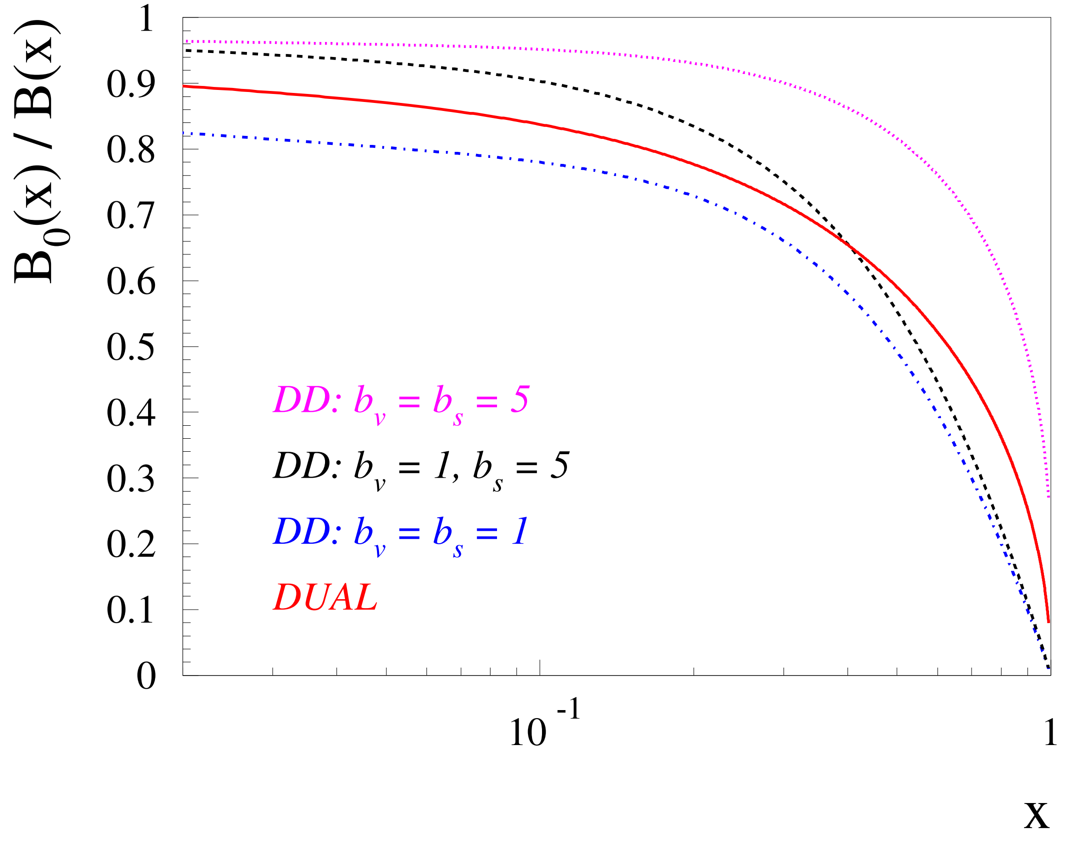
$$\langle b_\perp^2\rangle = 2e_u\langle b_\perp^2\rangle^u + e_d\langle b_\perp^2\rangle^d = 2/3\langle r_1^2\rangle = 0.43 \pm 0.01 \text{ fm}^2$$

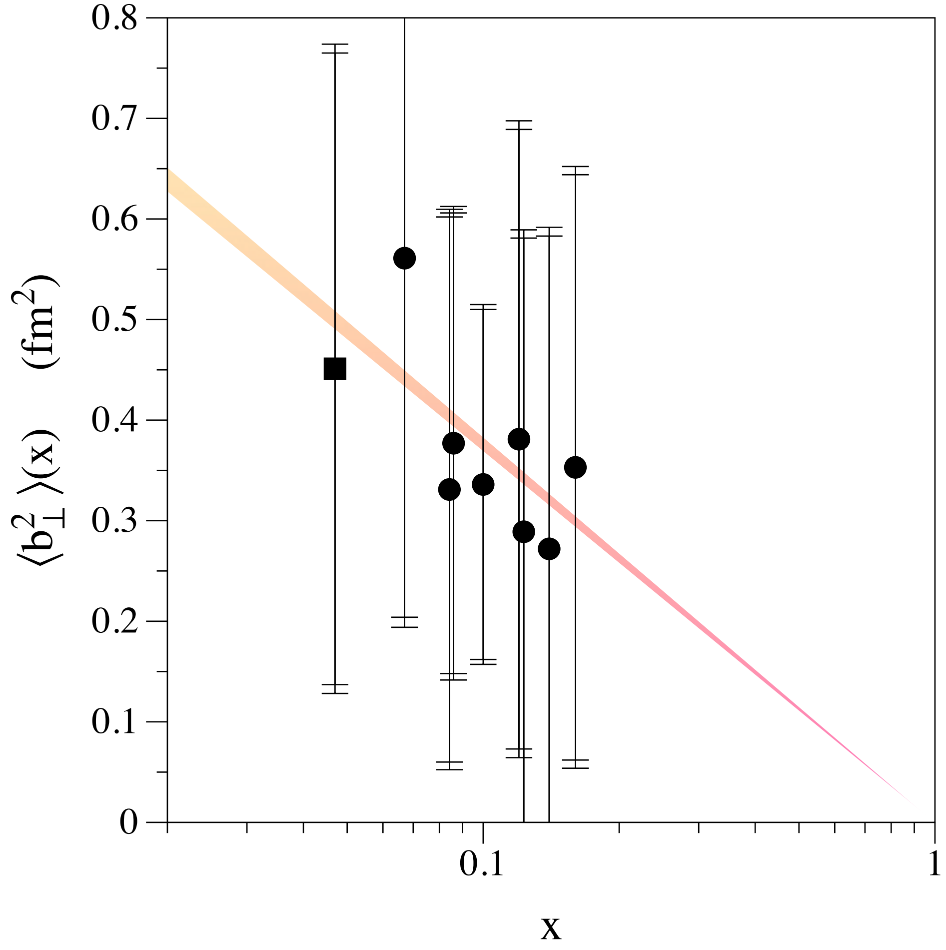
Bernauer(2014)

x-dependent radius in proton

black circles: CFF fit of JLab data

black squares: CFF fit of HERMES data



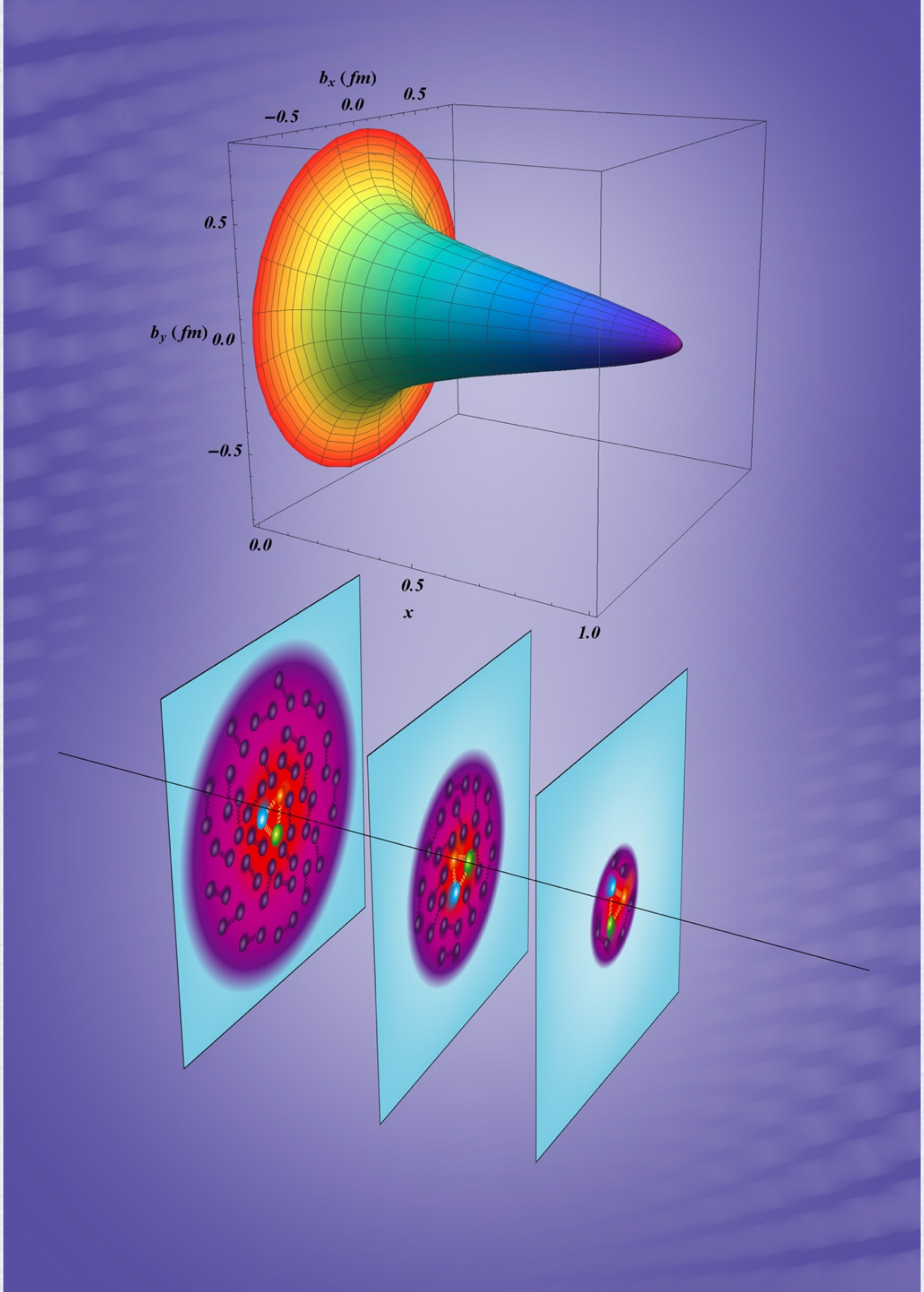


for x < 0.15: $B_0 / B > 0.9$

band: using $B_0(x) = a_{B_0} \ln(1/x)$

a_{B_0} fixed from elastic scattering

Dupré, Guidal, Vdh(2017)



3D imaging of proton

Dupré, Guidal, Vdh(2017)

CFF $\mathcal{H}_{\text{Re}}$: dispersion relation formalism

Anikin, Teryaev(2007) **Diehl, Ivanov(2007)** **Polyakov, Vdh(2008)**

Kumericki-Passek,Mueller,Passek (2008) **Goldstein,Liuti(2009)** **Guidal,Moutarde,Vdh (2013)**

➡ once-subtracted fixed-t dispersion relation

$$\mathcal{H}_{Re}(\xi,t) = -\Delta(t) + \mathcal{P}\int_0^1 dx\, H_+(x,x,t)\left[\frac{1}{x-\xi}+\frac{1}{x+\xi}\right]$$

ξ-independent subtraction function

known from CFF $\mathcal{H}_{Im}(x,t)$

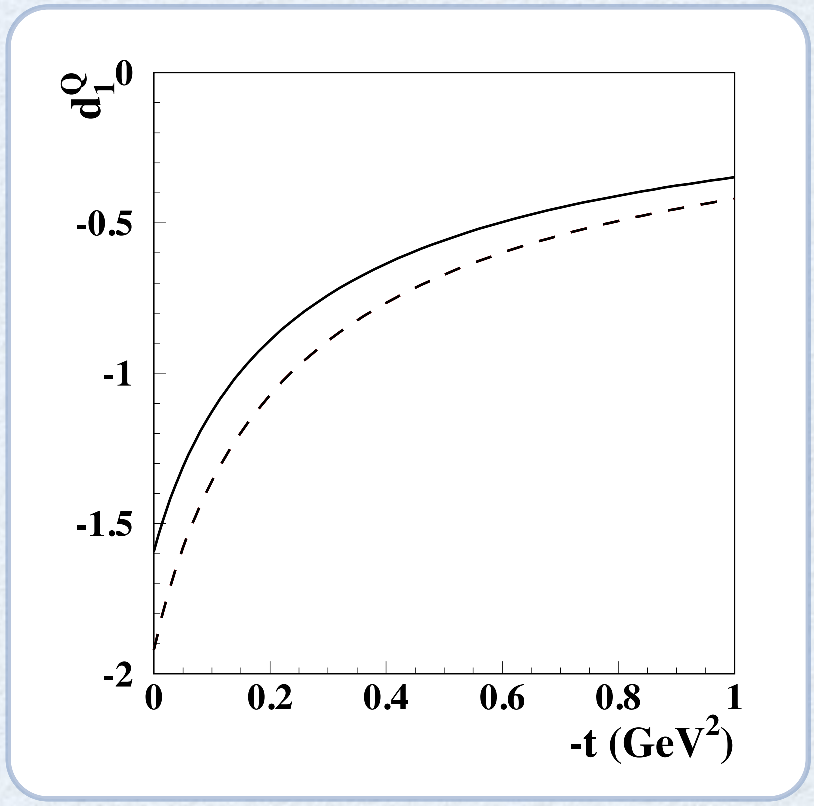
$$\Delta(t) \equiv \frac{2}{N_f}\int_{-1}^{1} dz\, \frac{D(z,t)}{1-z}$$

D-term

Polyakov, Weiss (1999)

$$D(z,t) = (1-z^2)\sum_{\substack{n=1\\ n\ odd}}^{\infty} d_n(t)\, C_n^{3/2}(z)$$

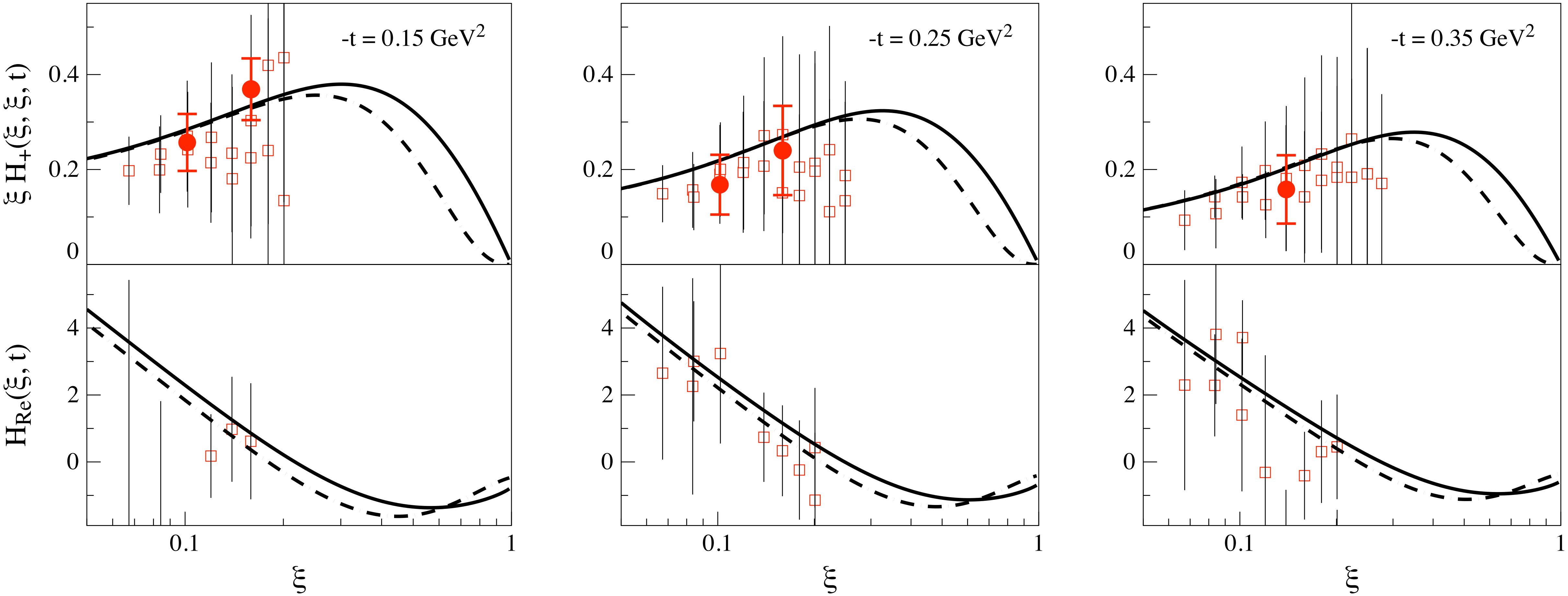
Pasquini,Polyakov,Vdh(2014)



experimental strategy for CFF $\mathcal{H}_{\text{Re}}$: direct extraction vs dispersion formalism

red solid circles: CLAS: σ, A_{LU}, A_{UL}, A_{LL}

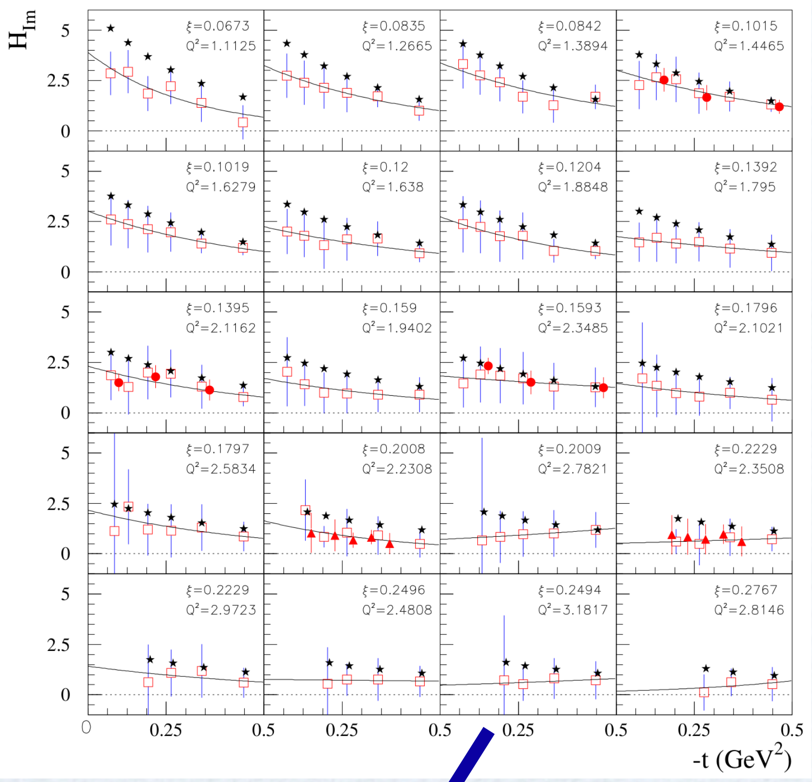
red open squares: CLAS: σ, A_{LU}



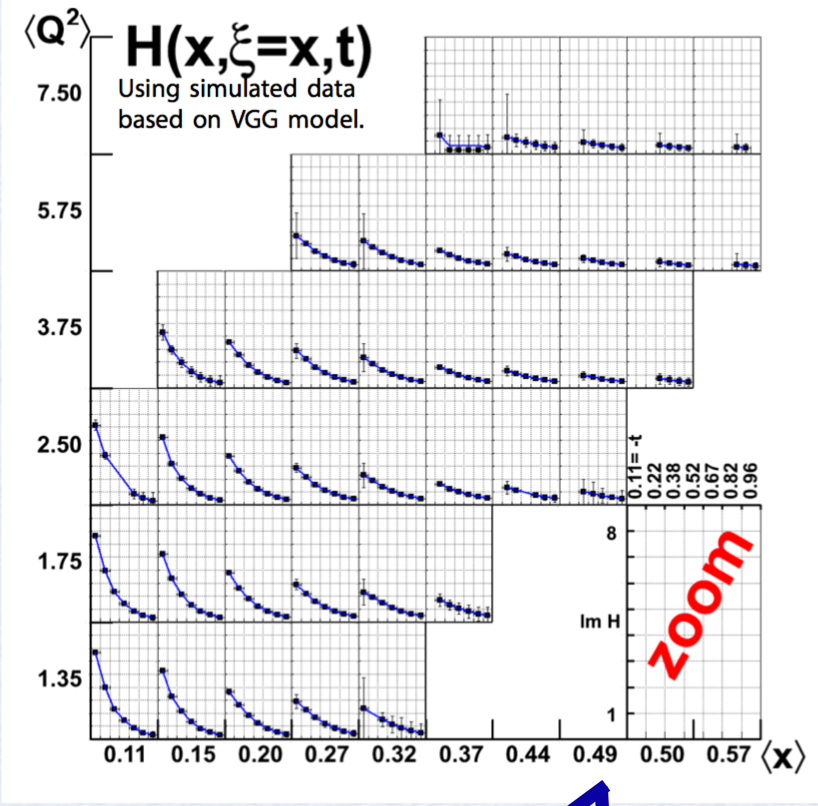
Dupré, Guidal, Niccolai, Vdh (in progress)

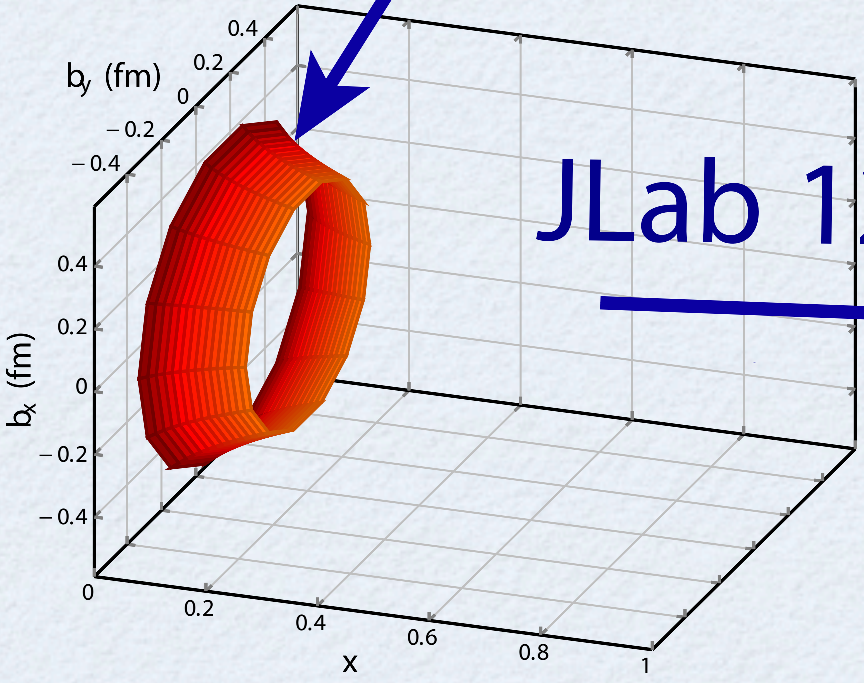
Projections for CFFs at JLab 12 GeV

Düpré-Guidal-Vanderhaeghen-PRD **95** 011501 (R) (2017)

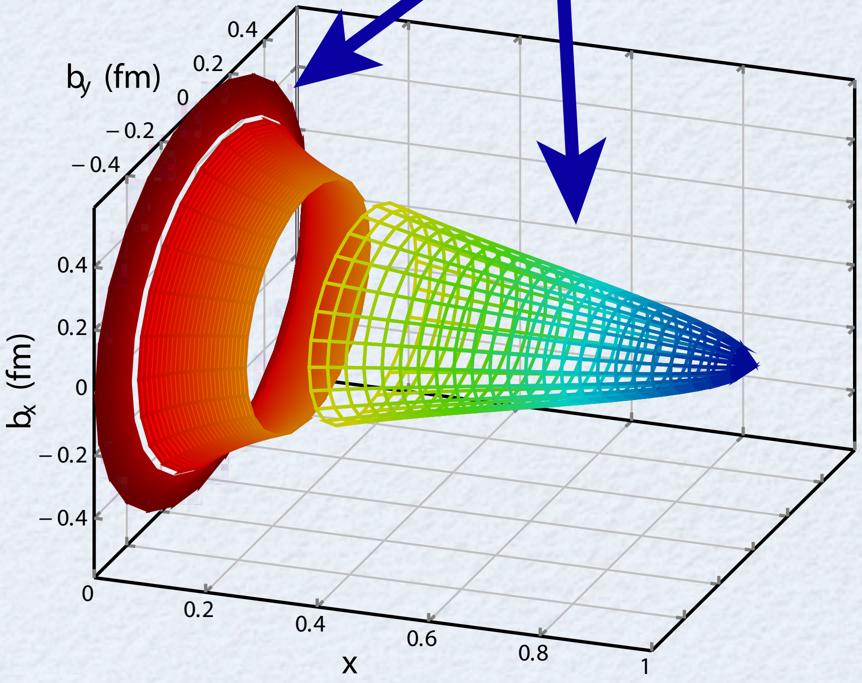


CLAS12 projections E12-06-119 with DVCS A_{UL} and A_{LU}





JLab 12 GeV



courtesy of Z.E. Meziani

Outlook

- elastic / transition FFs have allowed to get a first glimpse at the spatial distributions of quarks in nucleons
- GPDs allow for a proton imaging in longitudinal momentum and transverse position
- global analysis of JLab 6 GeV data have shown a proof of principle of such 3D imaging (tools available: fitters, dispersive analyses)
- systematic 3D imaging is possible now: COMPASS, JLab 12 GeV,...EIC