Overview of recent theoretical work on two-photon exchange

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JPos17, September 12, 2017

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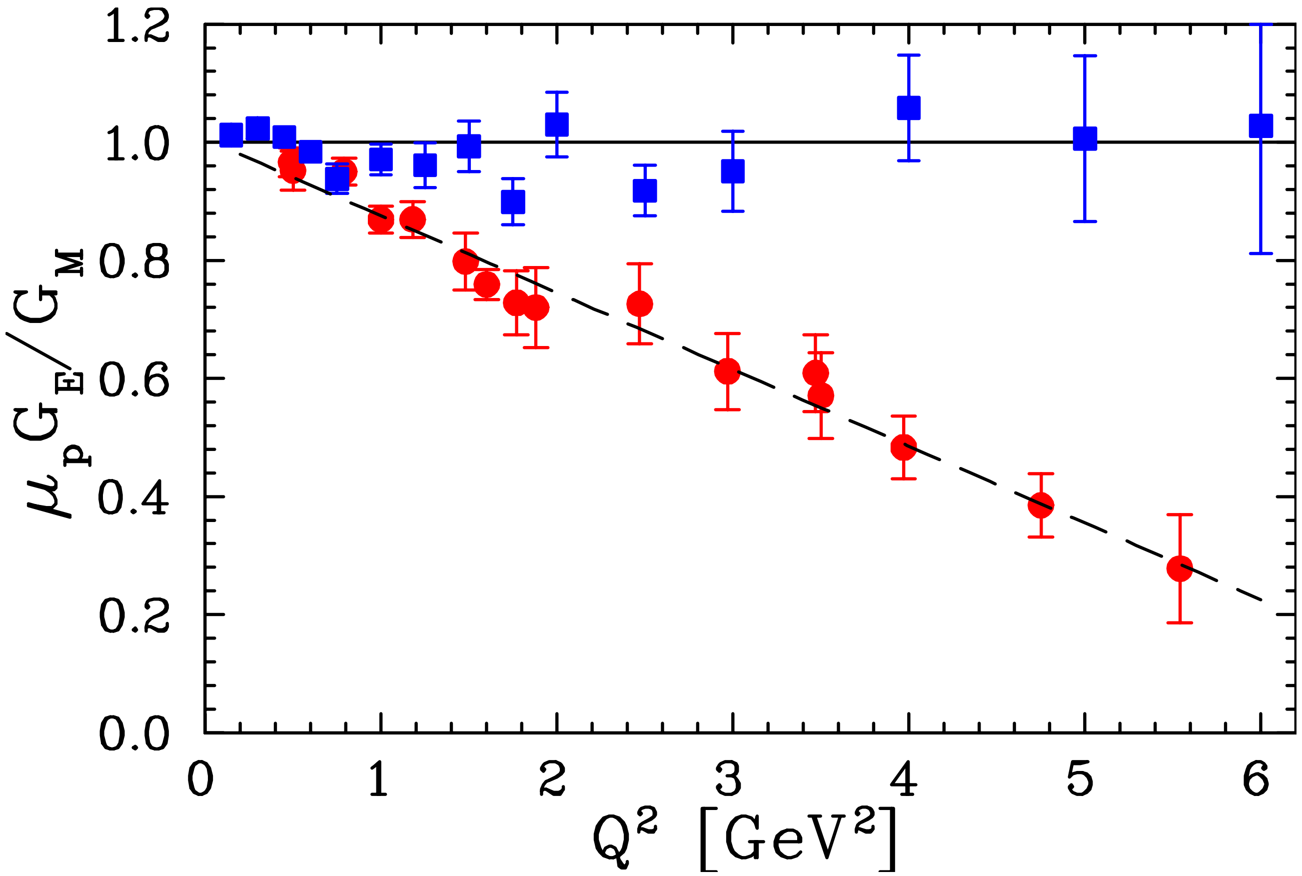
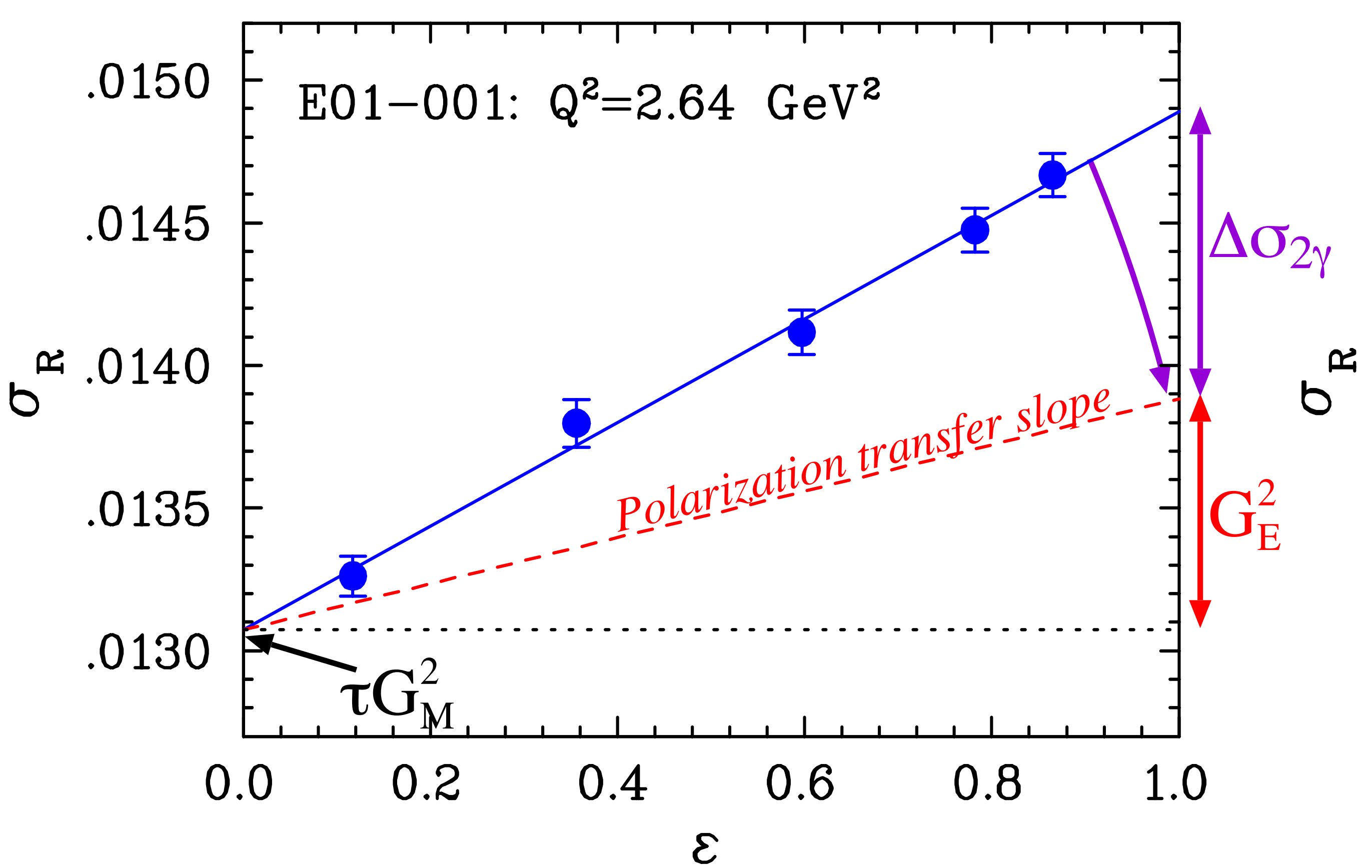
Assuming OPE



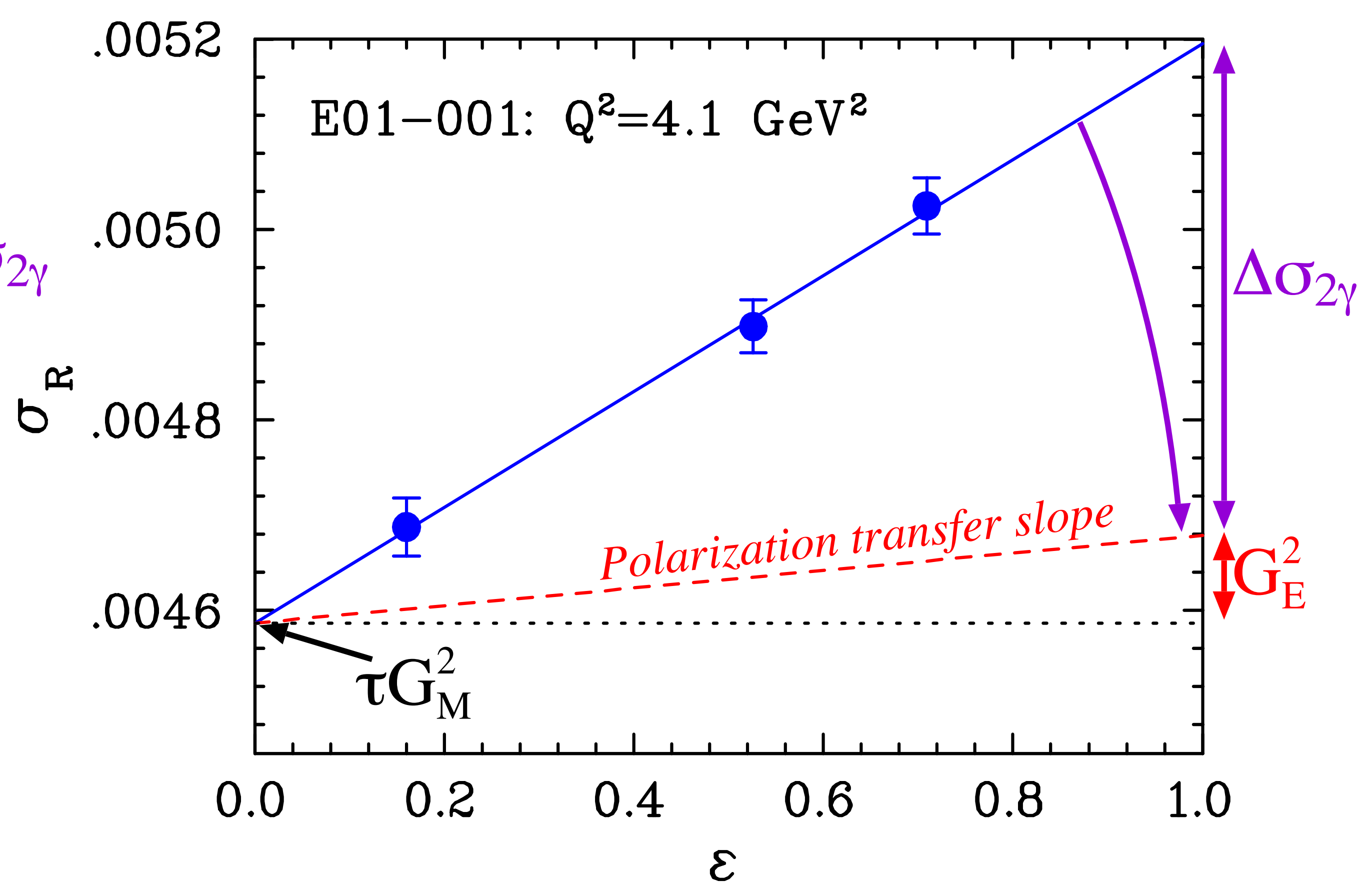
Rosenbluth

Pol. transfer

about 50% TPE + ??

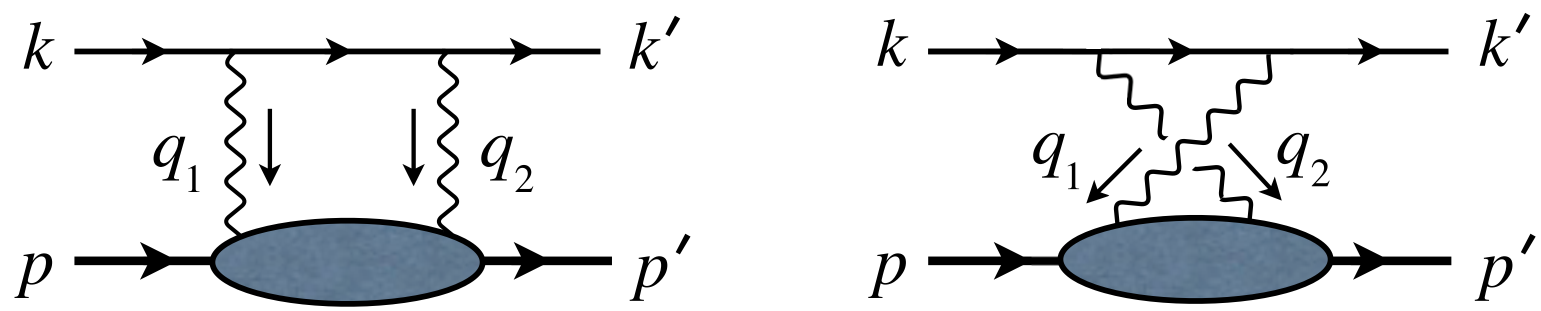


about 80% TPE + ??



Hadronic and Partonic Approaches

Low to moderate Q^2:



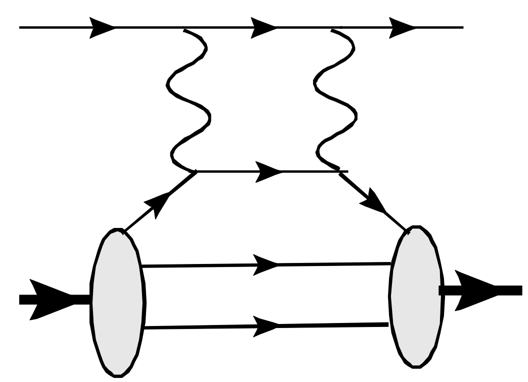
hadronic: $N + \Delta + N^*$ etc.

- as Q^2 increases more and more parameters
- Loop integration using sum of monopole transition form factors fit to spacelike Q^2

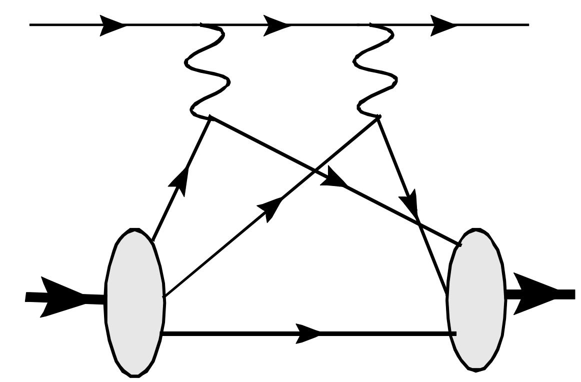
PGB, Melnitchouk, & Tjon, PRL **91**, 142304 (2003)

Moderate to high Q^2:

- GPD approach: assumption of hard photon interaction with 1 active quark
  - Embed in nucleon using Generalized Parton Distributions
  - Valid only in certain kinematic range ($|s,t,u| \gg M^2$)



"handbag"

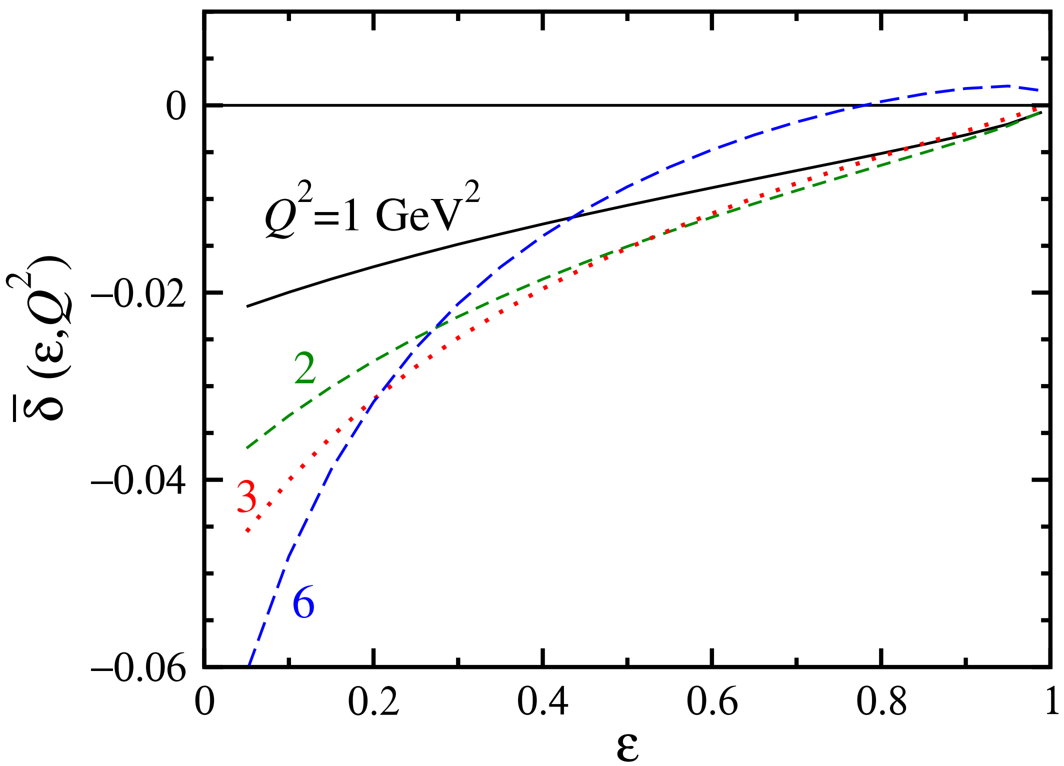


"cat's ears"

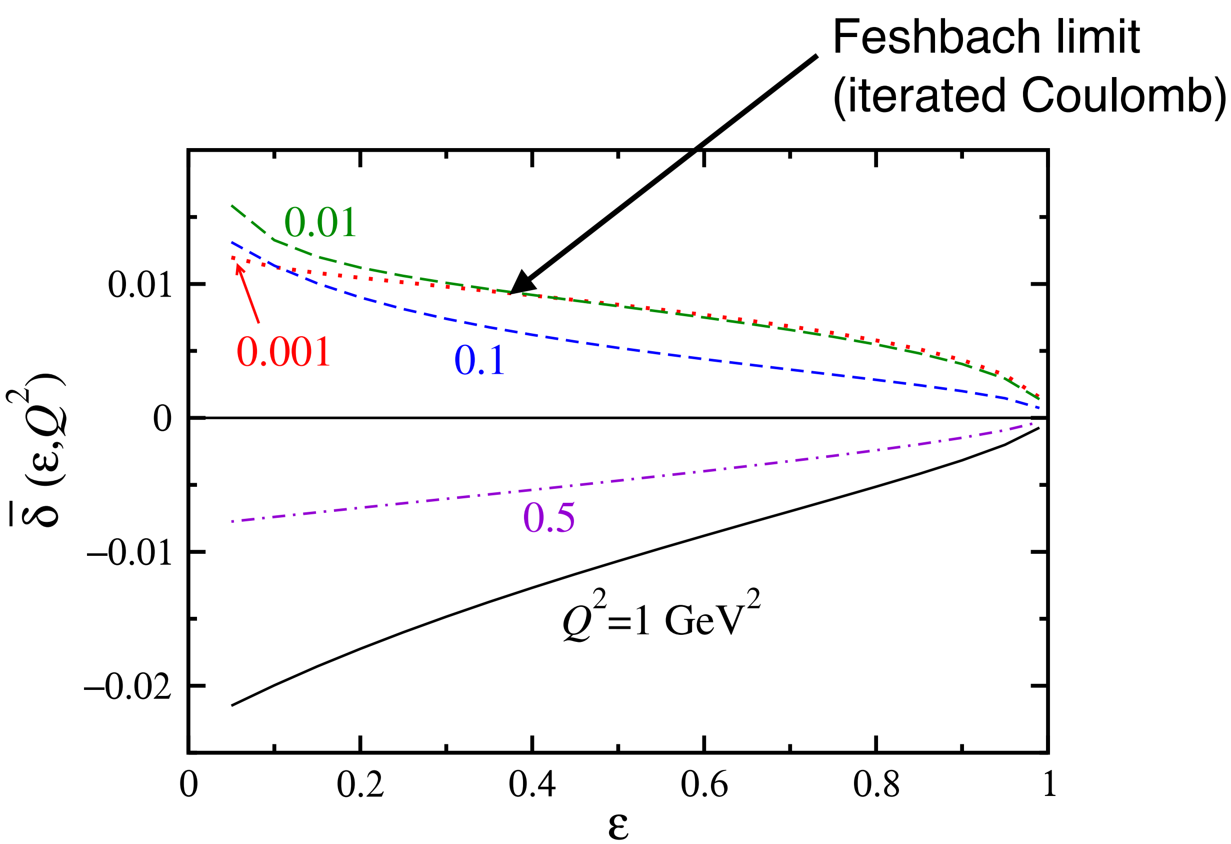
Afanasev *et al.*, PRD **72**, 013008 (2005)

- pQCD: recent work indicates two active quarks dominate

Nucleon (elastic) intermediate state

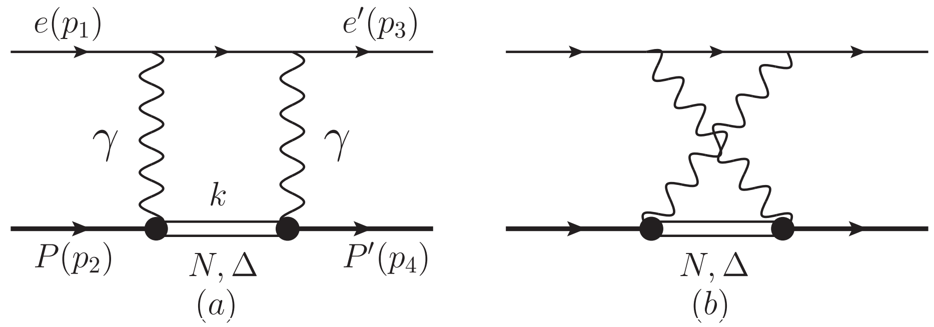


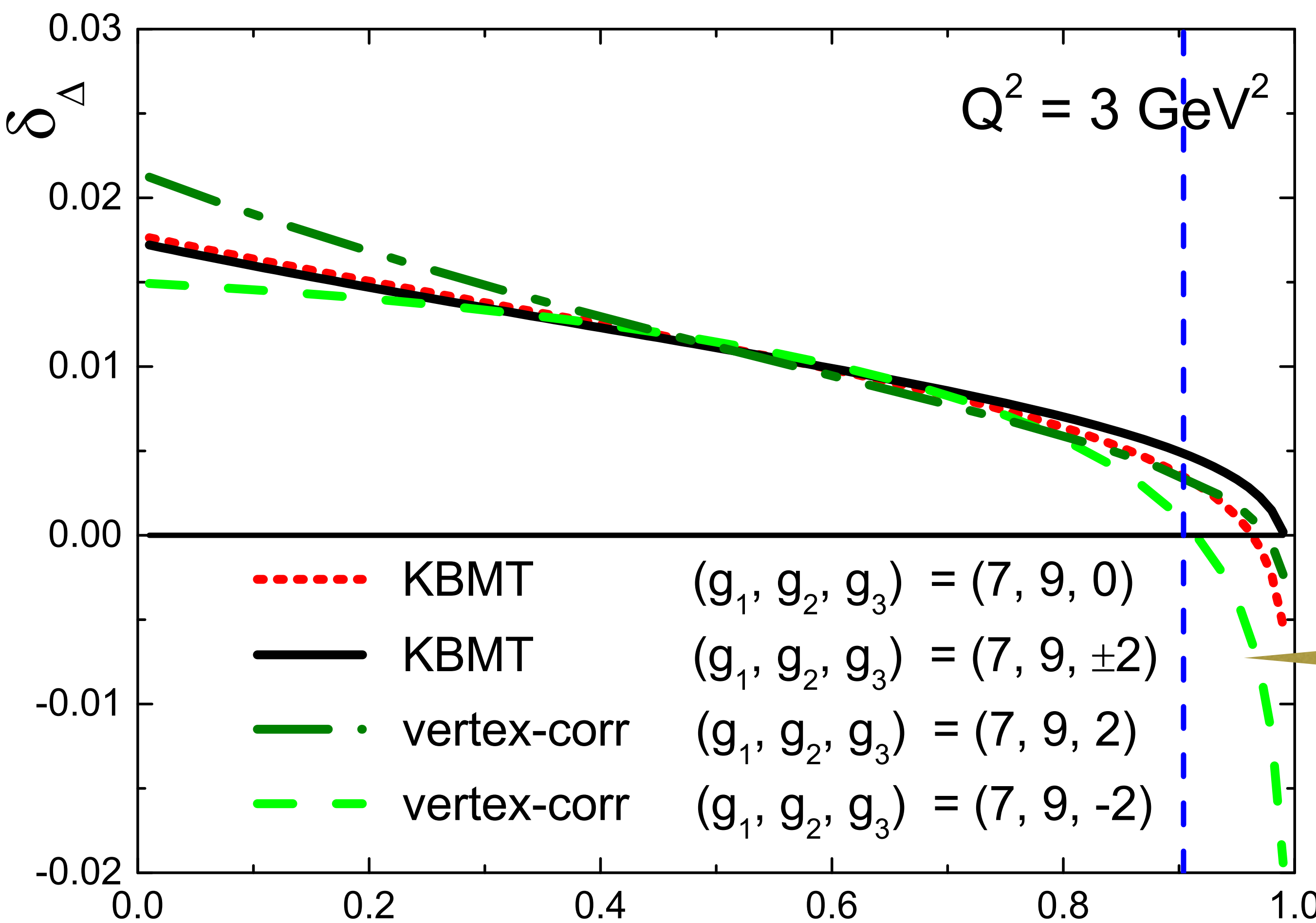
- positive slope
- vanishes as $\varepsilon \to 1$
- nonlinearity grows with increasing Q^2
- G_M dominates in loop integral
- Right order of magnitude and sign to explain G_E/G_M ratio



- changes sign at $Q^2 \approx 0.4$ GeV^2
- agrees with static (Feshbach) limit for point particle (no form factors in loop and $Q^2 \to 0$)
- G_E dominates in loop integral

Δ and N* intermediate states





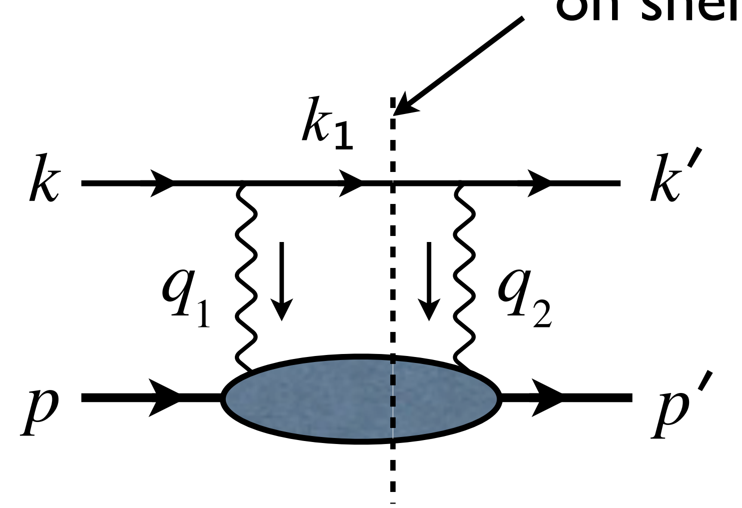
Direct loop integration method

Kondratyuk *et al.*, PRL **95**, 172503 (2005)
Zhou & Yang, Eur. Phys. J. A. **51**, 105 (2015)

Unphysical divergence

- Include all 3 $N \to \Delta$ multipoles, with form factors fit to CLAS data
- Opposite sign to nucleon contribution
- Qualitatively correct, BUT diverges as $\varepsilon \to 1$, implying a violation of unitarity (Froissart bound)

Solution: Dispersive method



$$S = 1 + i\mathcal{M}$$

$$S^\dagger = 1 - i\mathcal{M}^\dagger$$

$$SS^\dagger = 1$$

Unitarity → $-i\left(\mathcal{M} - \mathcal{M}^\dagger\right) = 2\Im m\,\mathcal{M} = \mathcal{M}^\dagger \mathcal{M}$

$$\Im m\,\langle f|\mathcal{M}|i\rangle = \frac{1}{2}\int d\rho\,\sum_n \langle f|\mathcal{M}^*|n\rangle\langle n|\mathcal{M}|i\rangle$$

$$d\rho = \frac{d^3k_1}{(2\pi)^3 2E_{k_1}} \sim dW_n\,dQ_1^2\,dQ_2^2$$

- Imaginary part determined by unitarity
- Uses only on-shell form factors
  - Use form factors directly fit to data, not reparametrized by sum of monopoles
- Real part determined from dispersion relations

TPE using dispersion relations

Generalized form factors

$$\mathcal{M}_{\gamma\gamma} \to (\gamma_\mu)^{(e)} \otimes \left(F_1'(Q^2,\nu)\gamma^\mu + F_2'(Q^2,\nu)\frac{i\sigma^{\mu\nu}q_\nu}{2M} \right)^{(p)} + (\gamma_\mu\gamma_5)^{(e)} \otimes \left(G_a'(Q^2,\nu)\gamma^\mu\gamma_5 \right)^{(p)}$$

$$\delta_{\gamma\gamma} = 2\mathrm{Re}\, \frac{\varepsilon G_E(F_1' - \tau F_2') + \tau G_M(F_1' + F_2') + \nu(1-\varepsilon)G_M G_a'}{\varepsilon G_E^2 + \tau G_M^2}$$

Dispersion relations

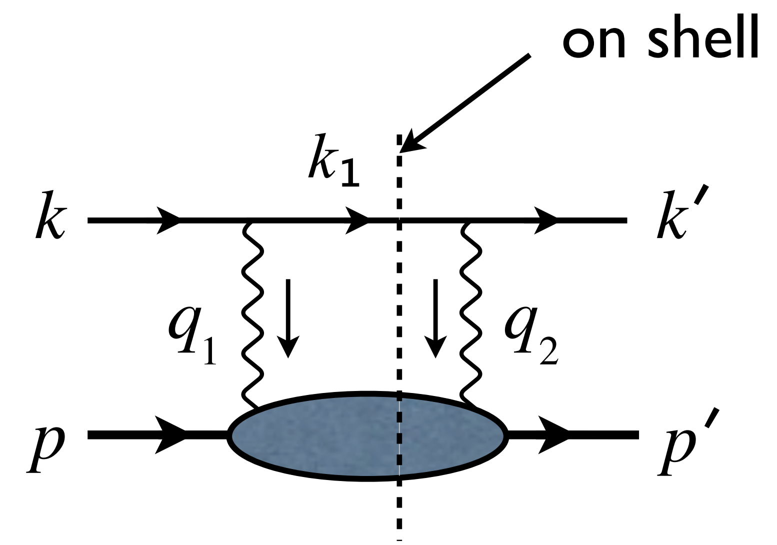
$$\mathrm{Re}\; F_1'(Q^2,\nu) = \frac{2}{\pi}\mathcal{P}\int_{-\tau}^{\infty} d\nu' \; \frac{\nu}{\nu'^2 - \nu^2}\, \mathrm{Im}\; F_1'(Q^2,\nu')\,,$$

$$\mathrm{Re}\; F_2'(Q^2,\nu) = \frac{2}{\pi}\mathcal{P}\int_{-\tau}^{\infty} d\nu' \; \frac{\nu}{\nu'^2 - \nu^2}\, \mathrm{Im}\; F_2'(Q^2,\nu')\,,$$

$$\mathrm{Re}\; G_a'(Q^2,\nu) = \frac{2}{\pi}\mathcal{P}\int_{-\tau}^{\infty} d\nu' \; \frac{\nu'}{\nu'^2 - \nu^2}\, \mathrm{Im}\; G_a'(Q^2,\nu')\,.$$

Integral extends into ``unphysical region'' down to zero energy ($\cos\theta < -1$)

A few technical details



$$\frac{\alpha}{4\pi} Q^2 \frac{1}{i\pi^2} \int d^4 q_1 \; \frac{\mathrm{Im}\left\{L_{\alpha\mu\nu} H^{\alpha\mu\nu}\right\}}{(q_1^2 - \lambda^2)(q_2^2 - \lambda^2)}$$

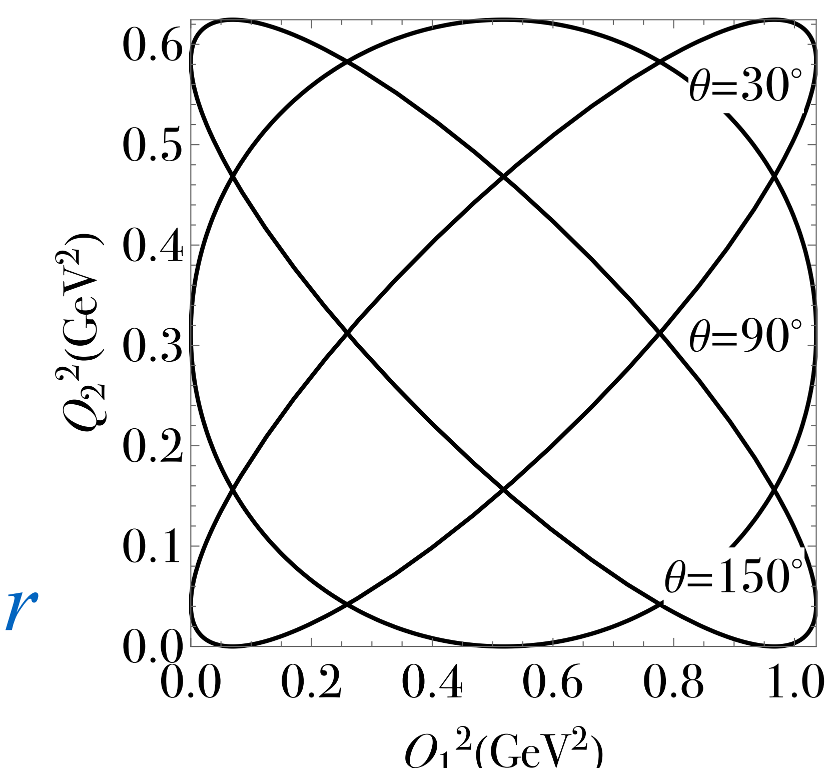
$$\rightarrow \quad \frac{s - W^2}{4s} \int d\Omega_{k_1} \; \frac{f\left(Q_1^2, Q_2^2\right) G_1(Q_1^2)\, G_2(Q_2^2)}{\left(Q_1^2 + \lambda^2\right)\left(Q_2^2 + \lambda^2\right)}$$

- L and H are leptonic and hadronic tensors
- f is a polynomial in photon virtualities Q_1^2 and Q_2^2
- $G_i(Q_i^2)$ is a transition form factor with poles in the complex Q_i^2 plane

Use numerical contour integration

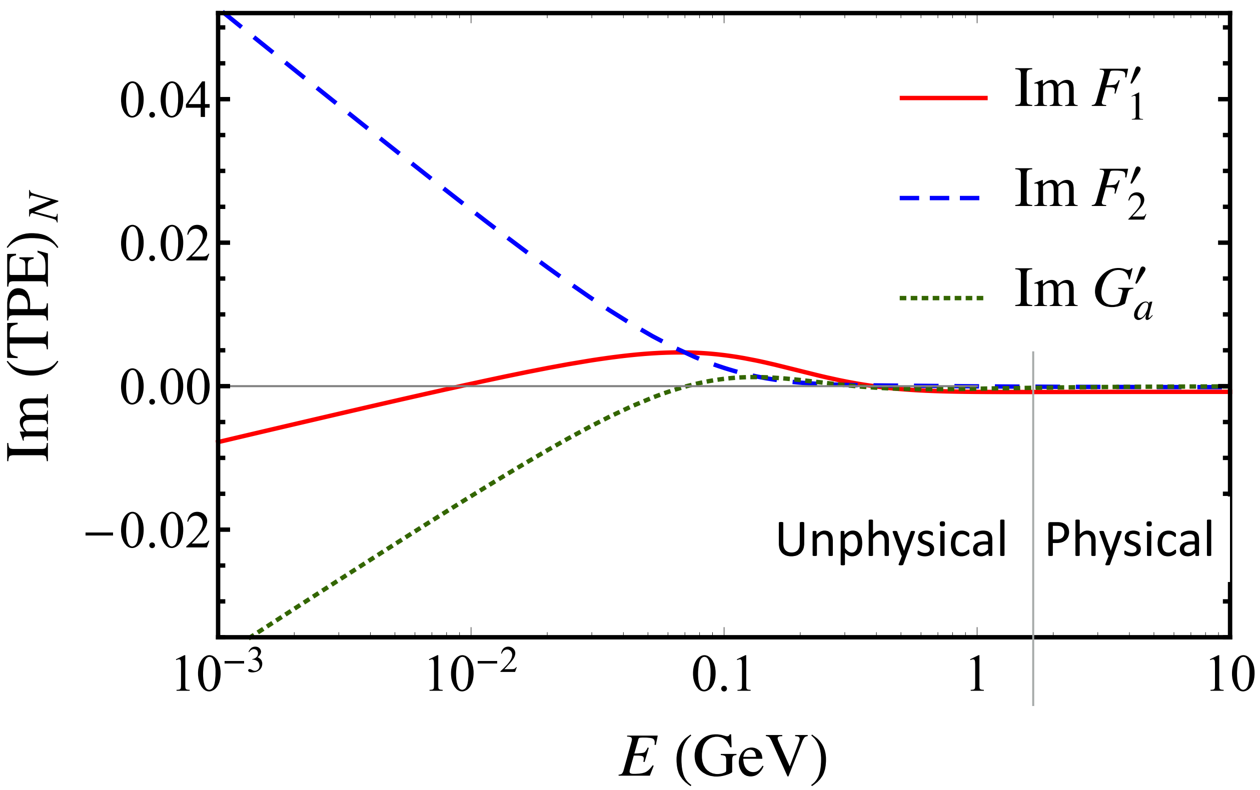
→ Allows for use of arbitrary functional forms for transition form factors $G_i(Q_i^2)$

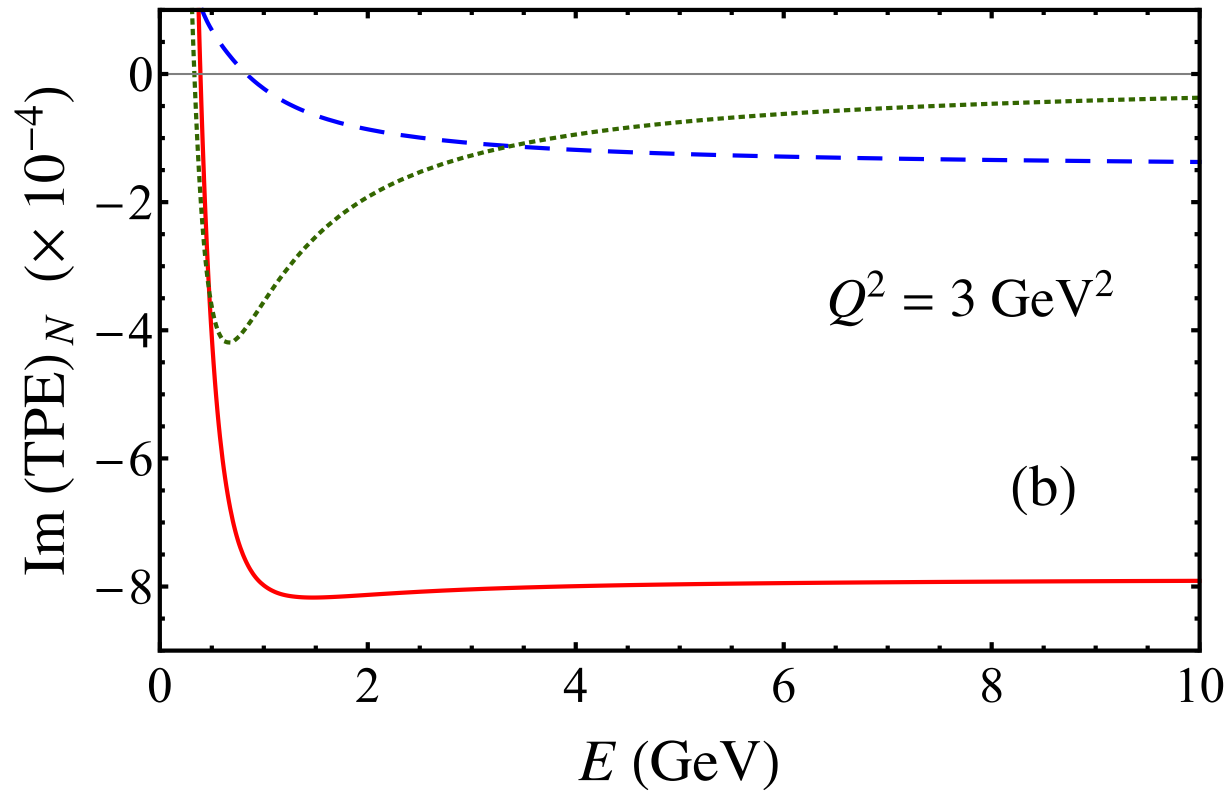
Contours are concentric ellipses of radial parameter r



Nucleon (elastic) intermediate state

$Q^2 = 3\ \mathrm{GeV}^2$

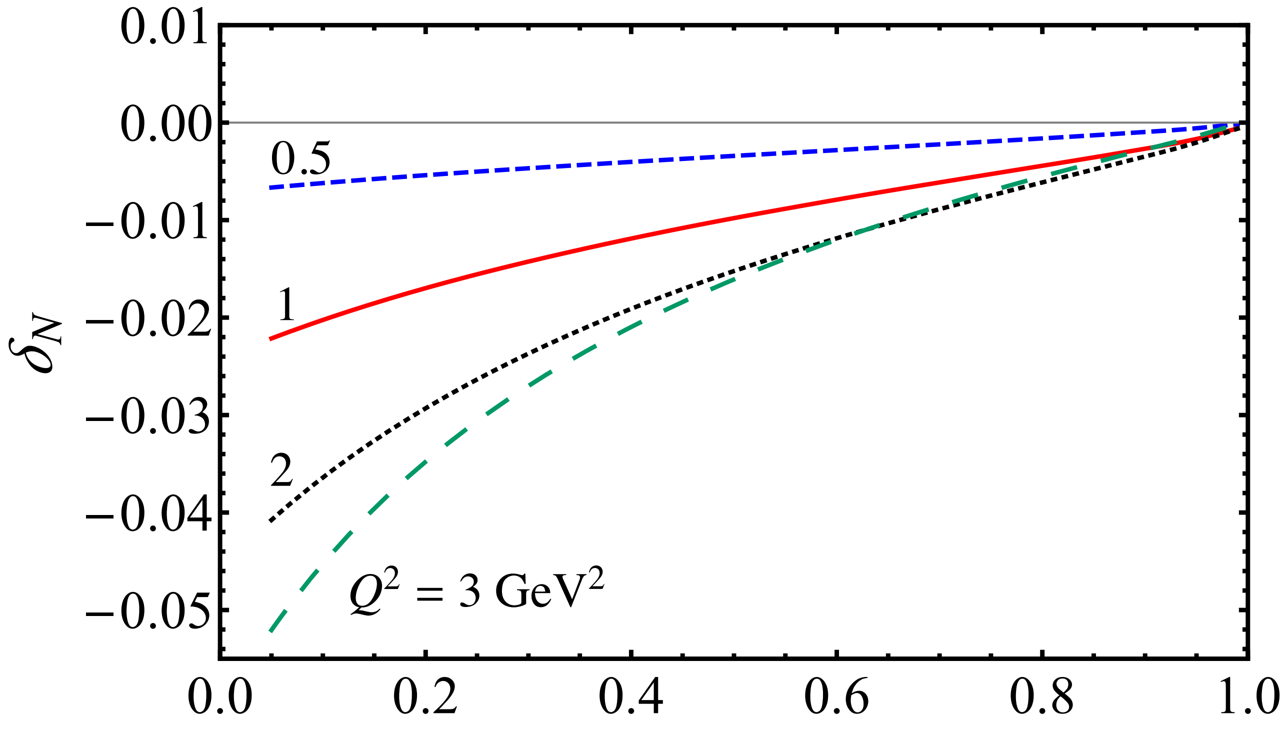




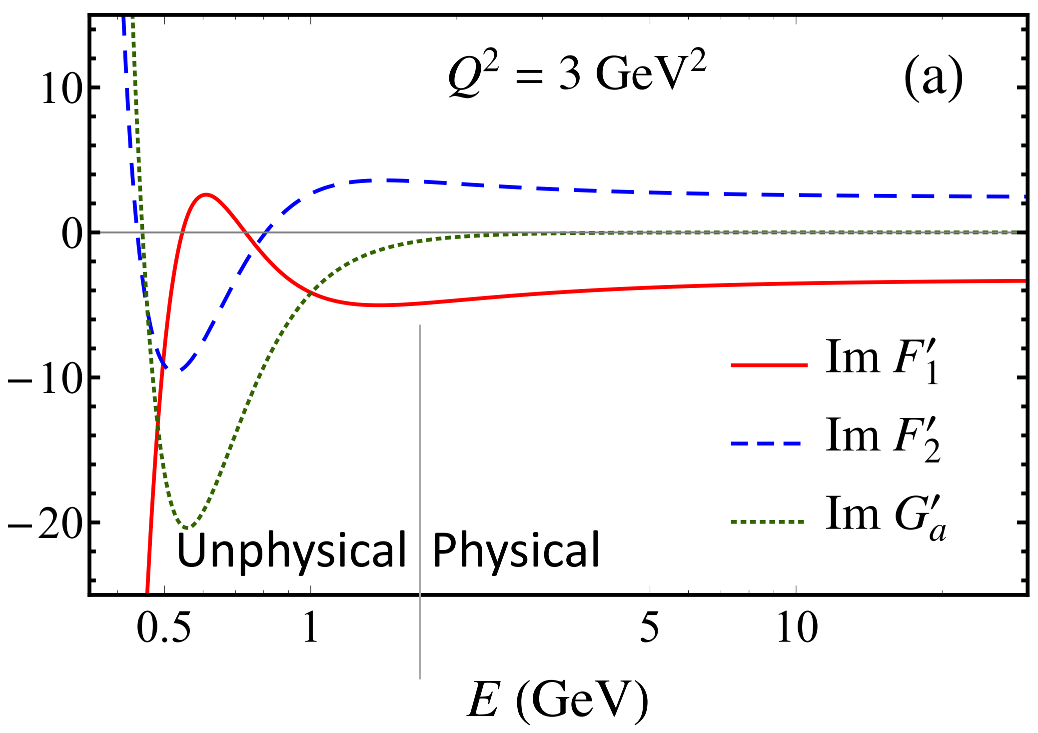
Logarithmic divergence at low energies

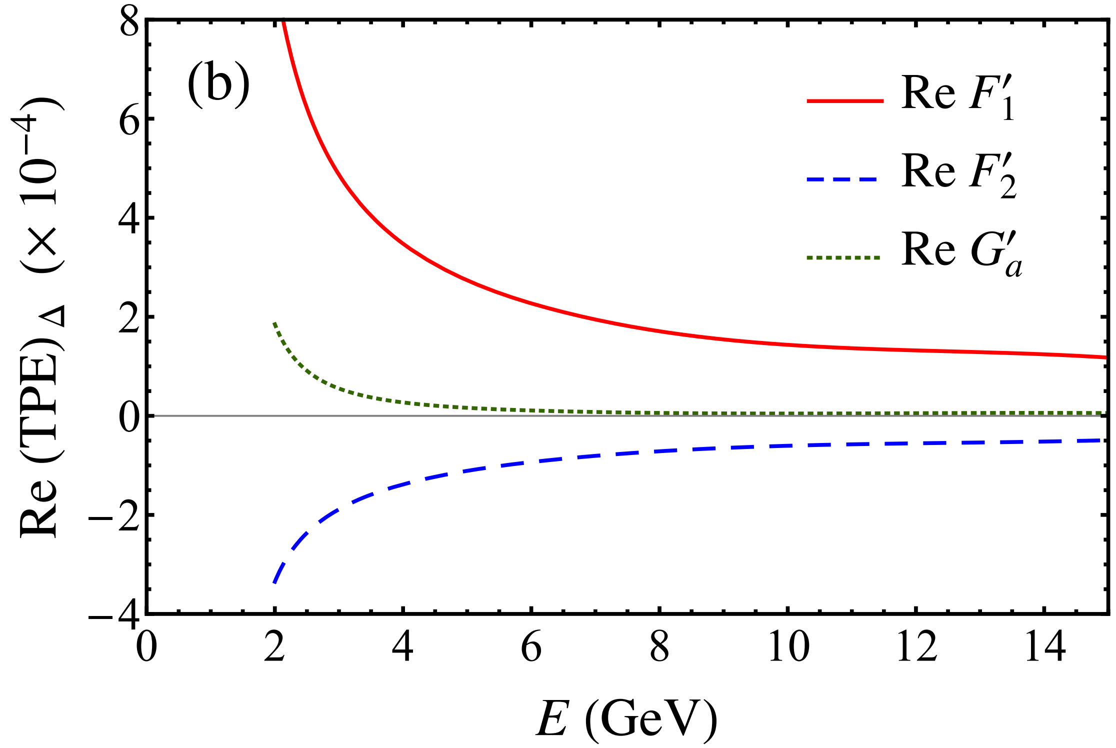
No subtractions needed

Agrees with old loop integration method

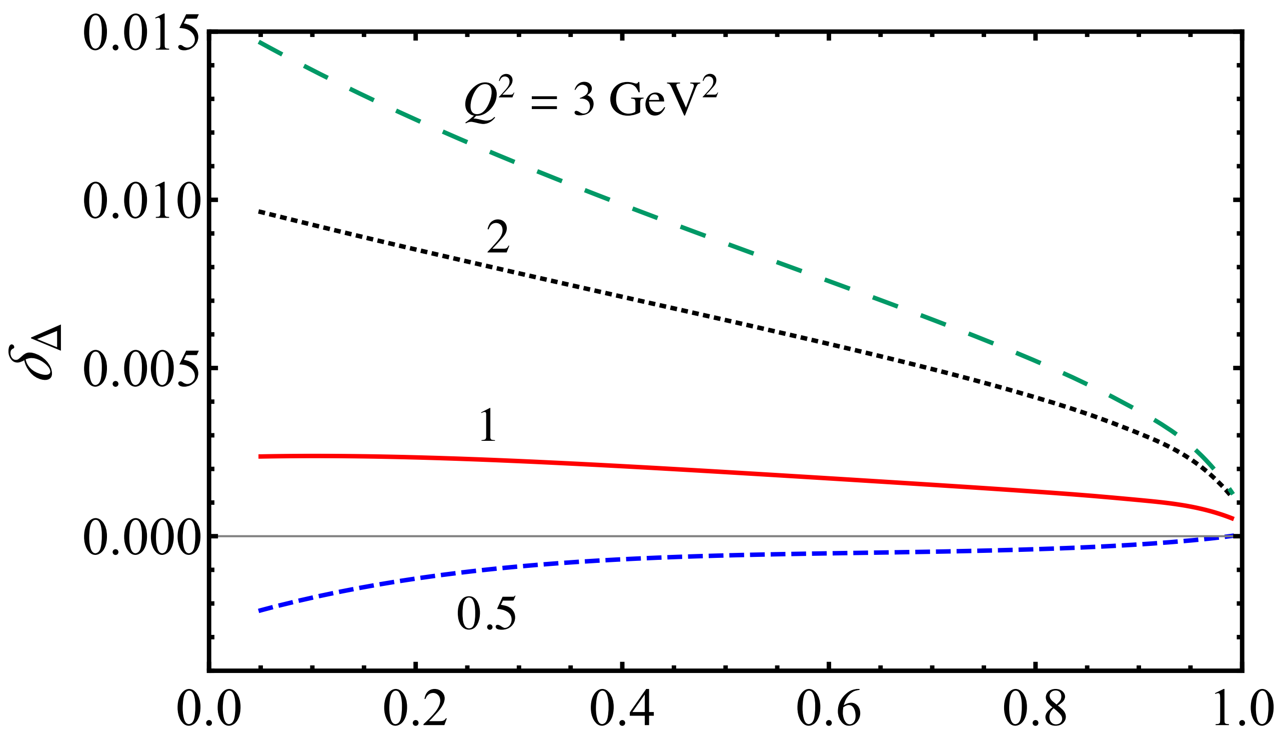


Δ intermediate state (zero width approximation)





- Include all 3 multipoles, with form factors fit to recent CLAS data
- G_M^* x G_M^* dominates, but G_M^* x G_E^* interference is significant



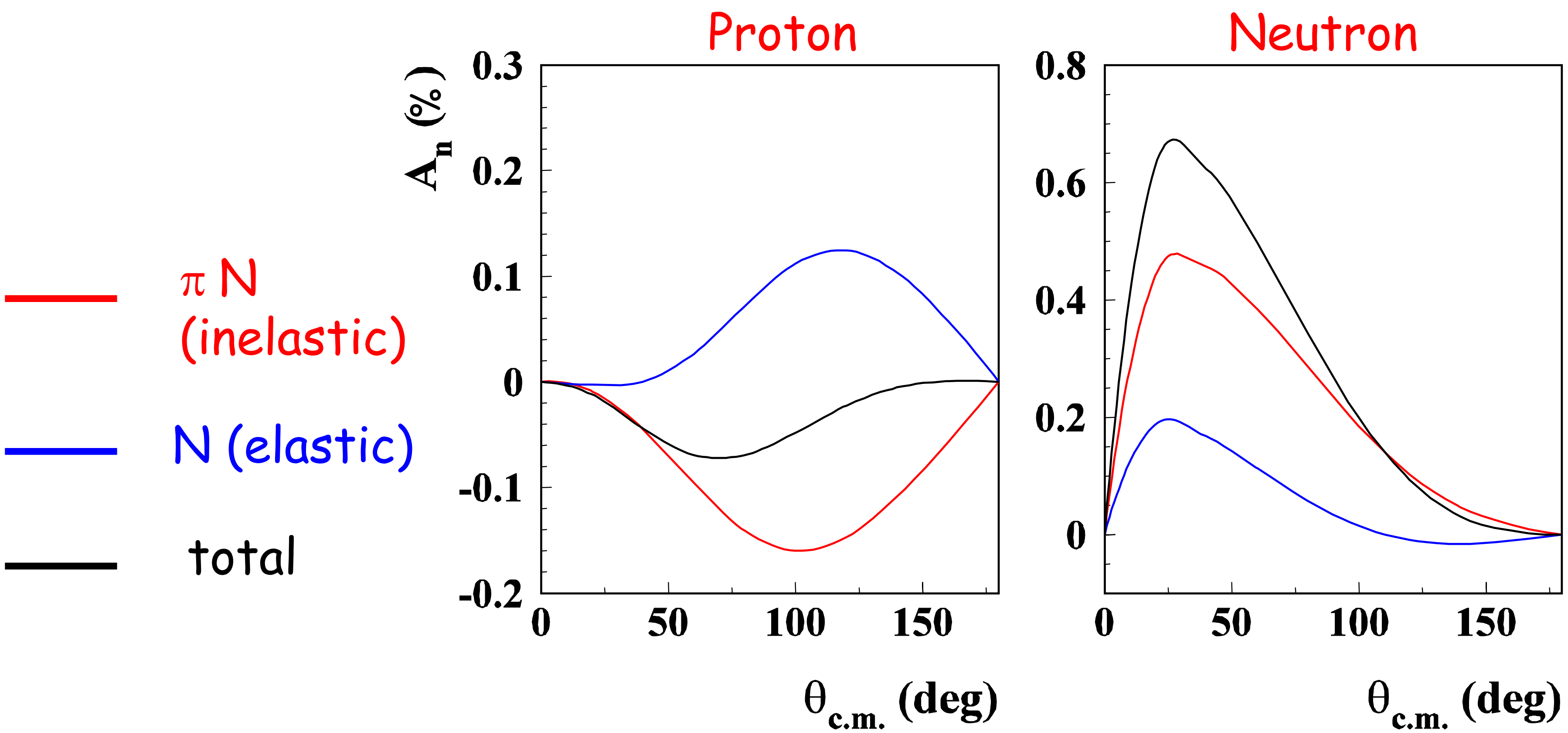
No unphysical divergence at $\varepsilon \to 1$

changes sign at $Q^2 \approx 0.6$ GeV^2

Direct measurements of Im part

Target normal spin asymmetry

Ee = 0.570 GeV



(taken from Pasquini & Vanderhaeghen)

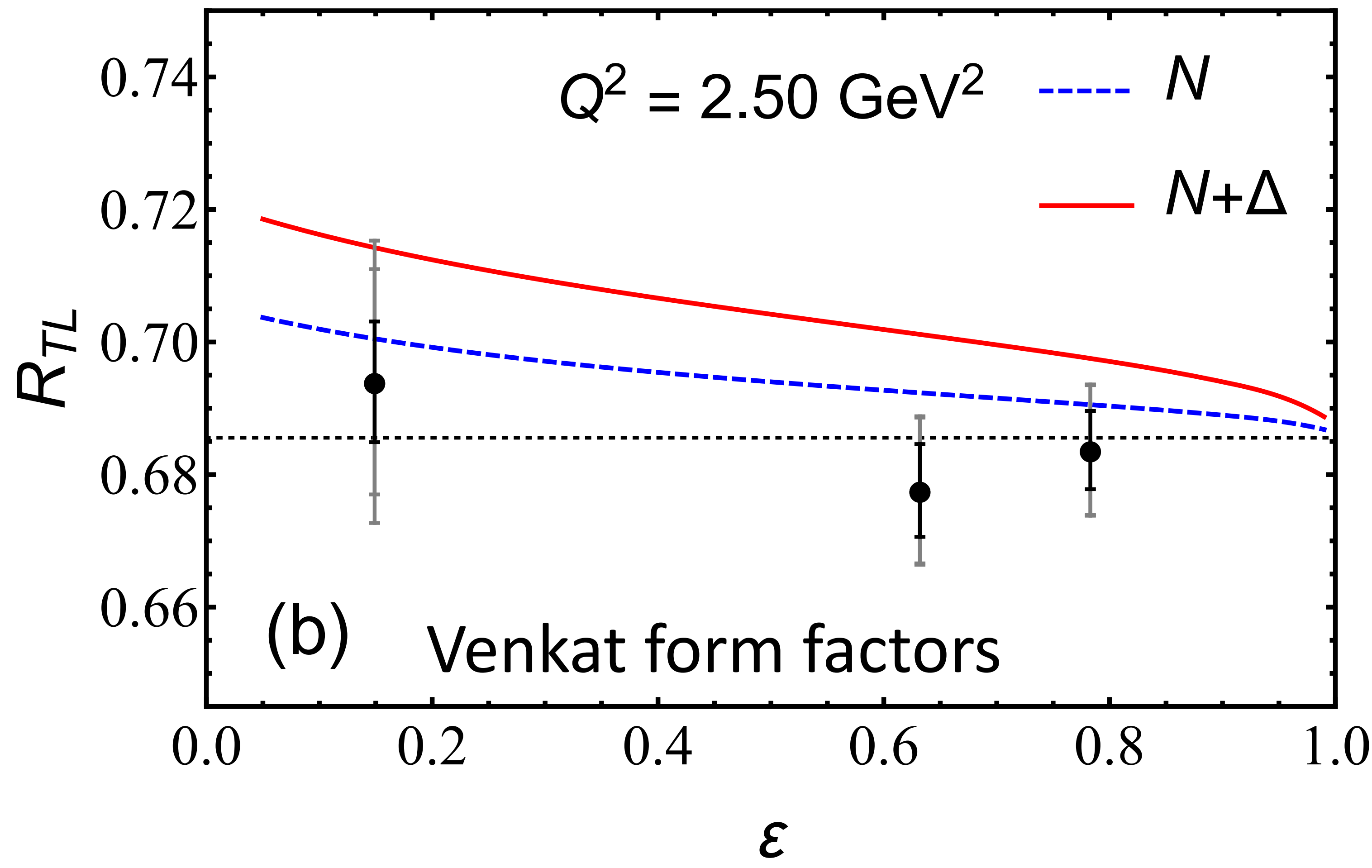
$$A_n = \sqrt{\frac{2\,\varepsilon\,(1+\varepsilon)}{\tau}}\;\left(G_M^2 + \frac{\varepsilon}{\tau}\,G_E^2\right)^{-1}$$

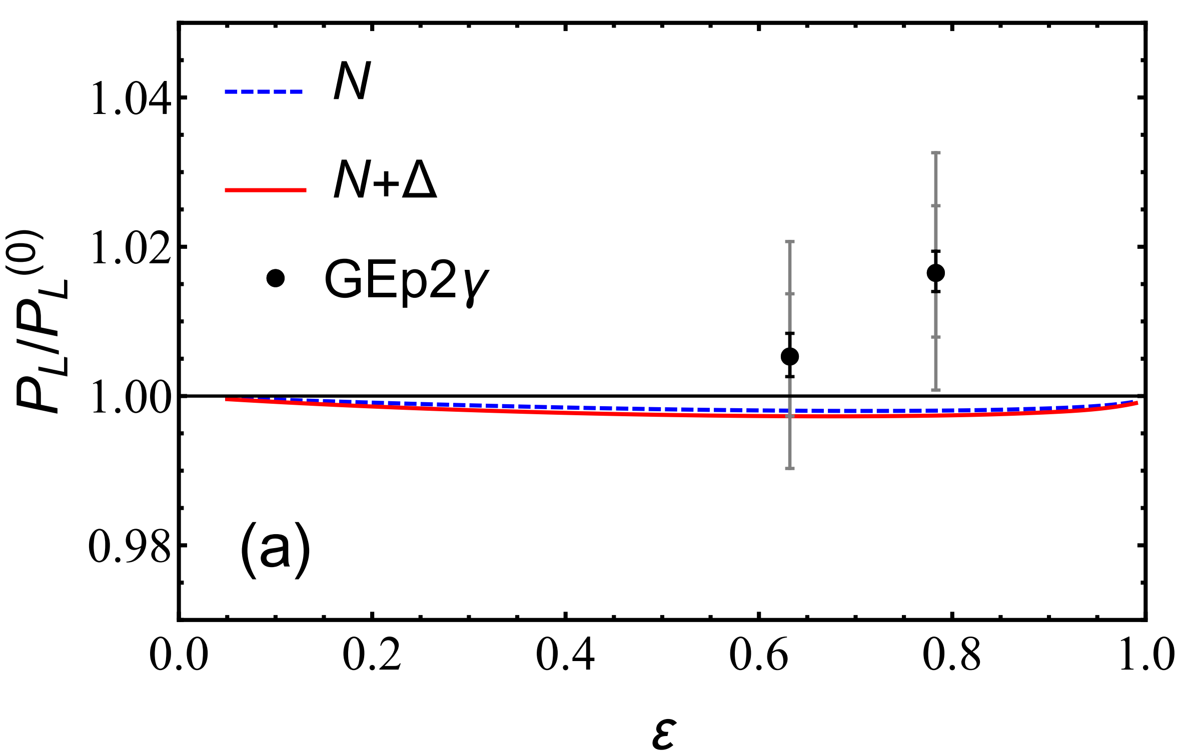
$$\times\;\left\{ -G_M\,\mathcal{I}m\left(\delta\tilde{G}_E + \frac{\nu}{M^2}\tilde{F}_3\right) + G_E\,\mathcal{I}m\left(\delta\tilde{G}_M + \left(\frac{2\varepsilon}{1+\varepsilon}\right)\frac{\nu}{M^2}\tilde{F}_3\right)\right\}$$

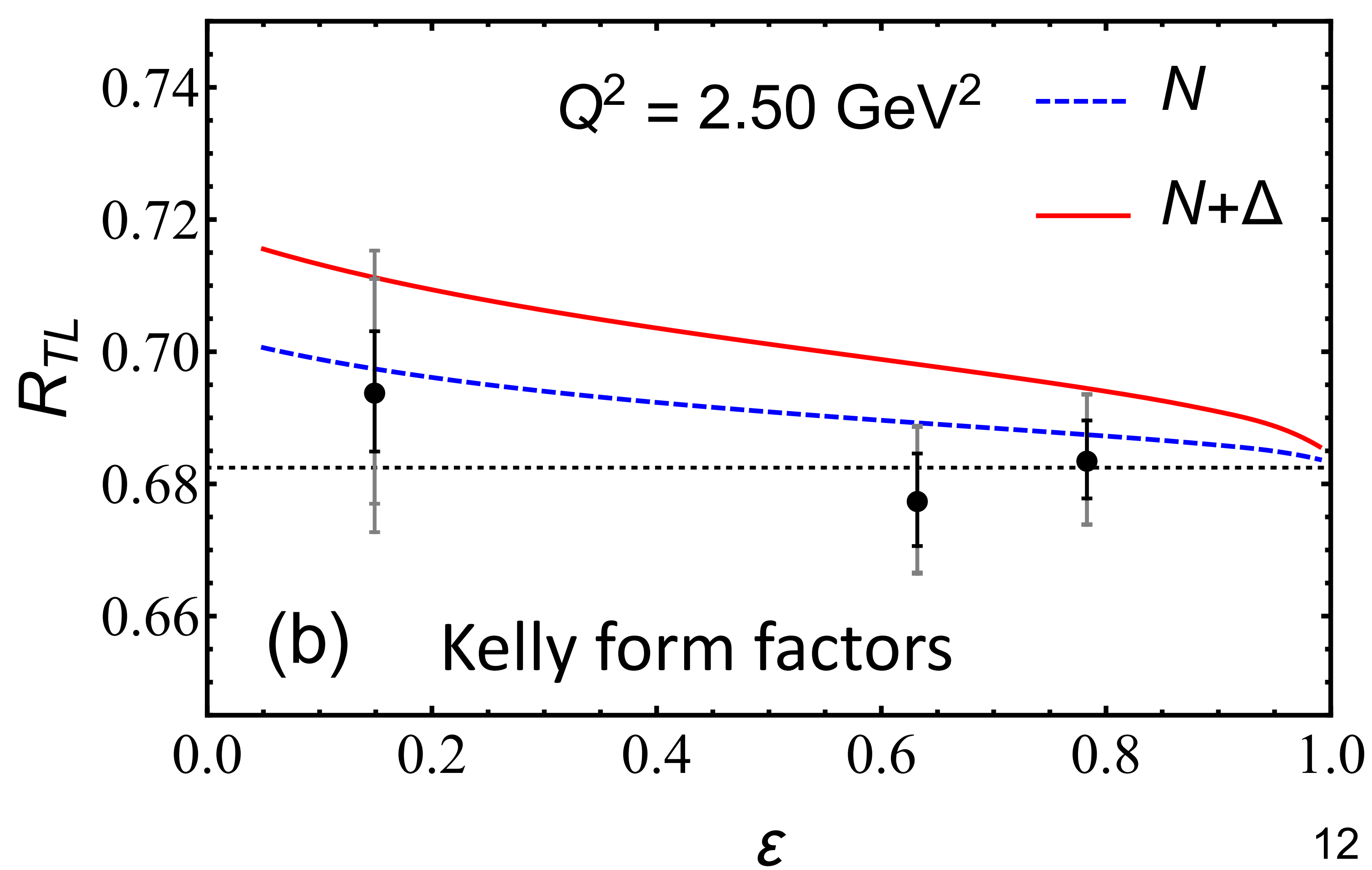
This is all in the physical region.

Polarization data

R_{TL} indicates mild sensitivity to G_E form factor at low ε





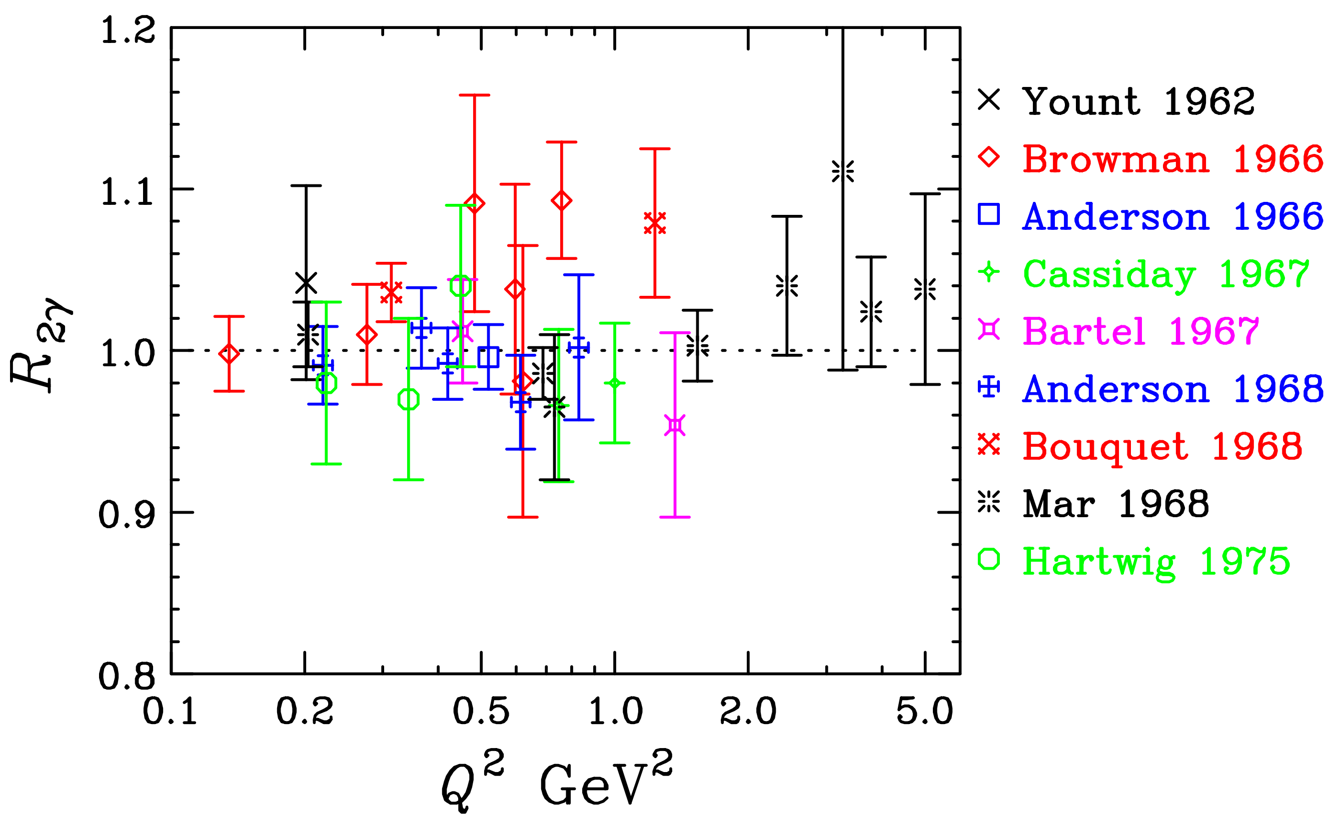


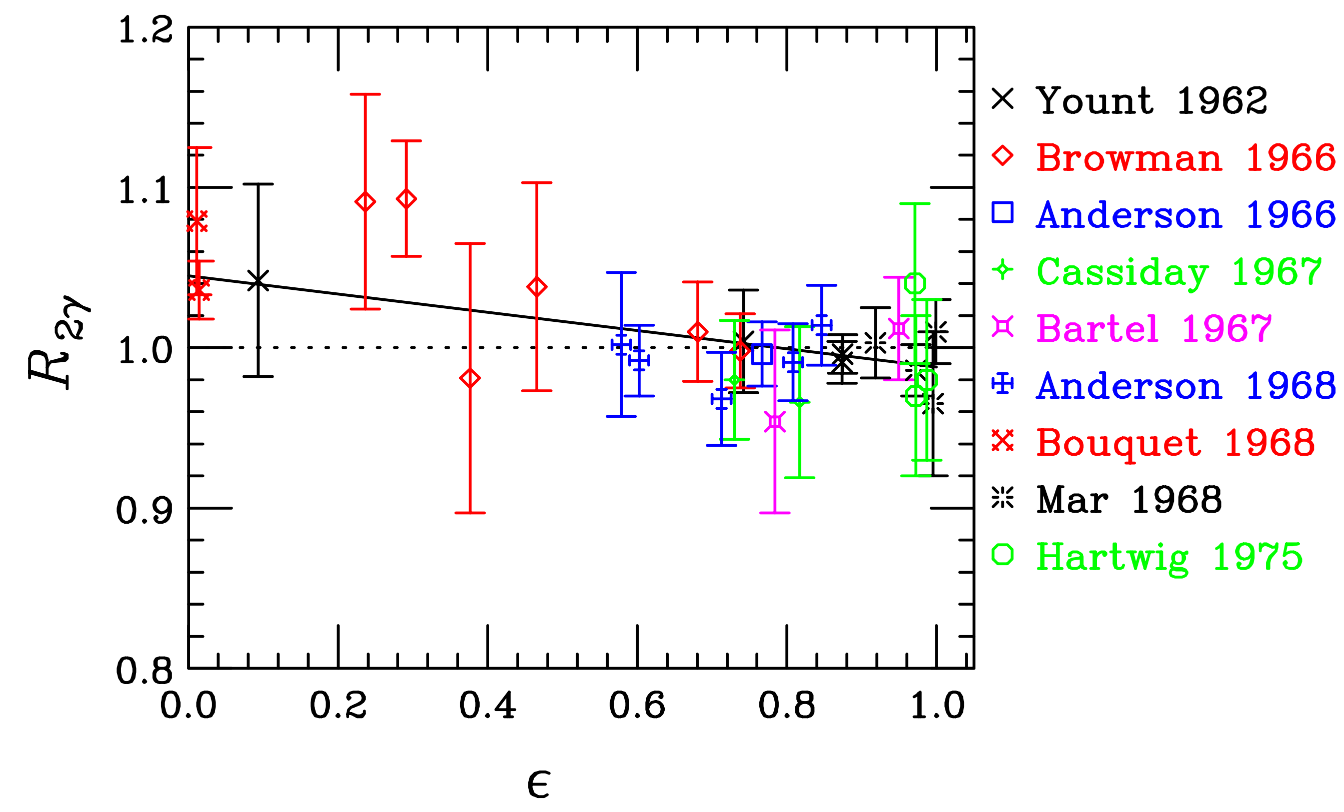
TPE effect on ratio of $e^{+}p$ to $e^{-}p$ cross sections

TPE interference changes sign for positrons vs electrons

$$R_{2\gamma} = \frac{\sigma^{e^+}}{\sigma^{e^-}} \approx 1 - 2\delta_{\gamma\gamma}$$

Old data from 1960-1970's



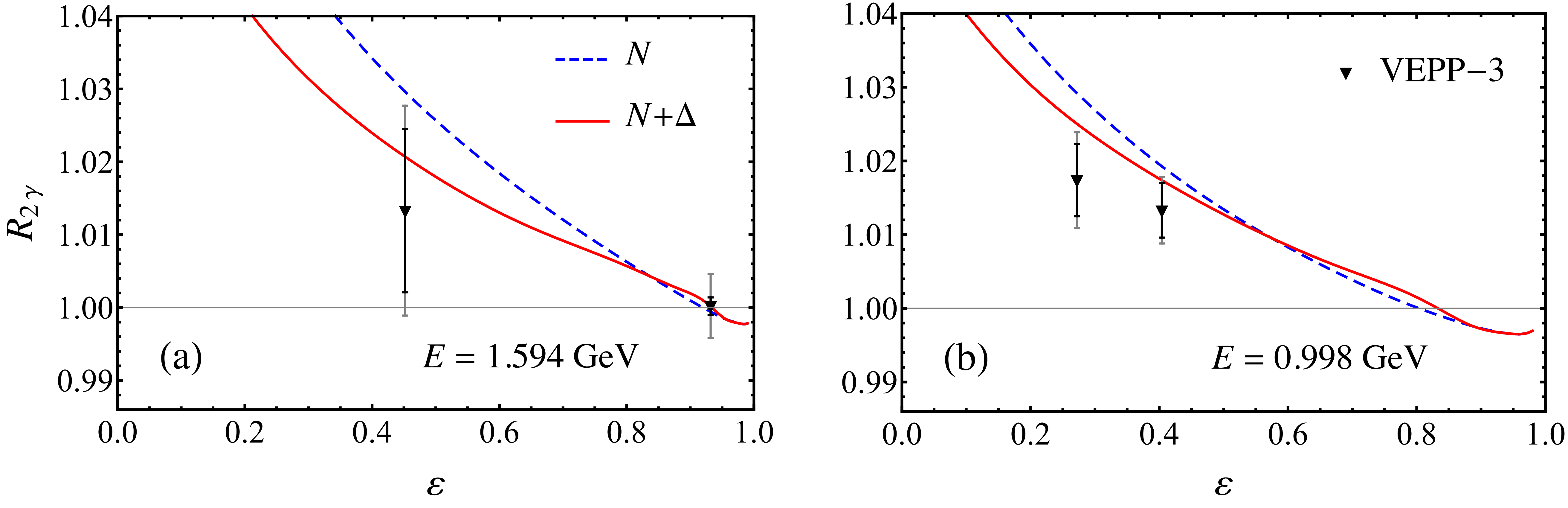


TPE effect on ratio of e^+p to e^-p cross sections

TPE interference changes sign for positrons vs electrons

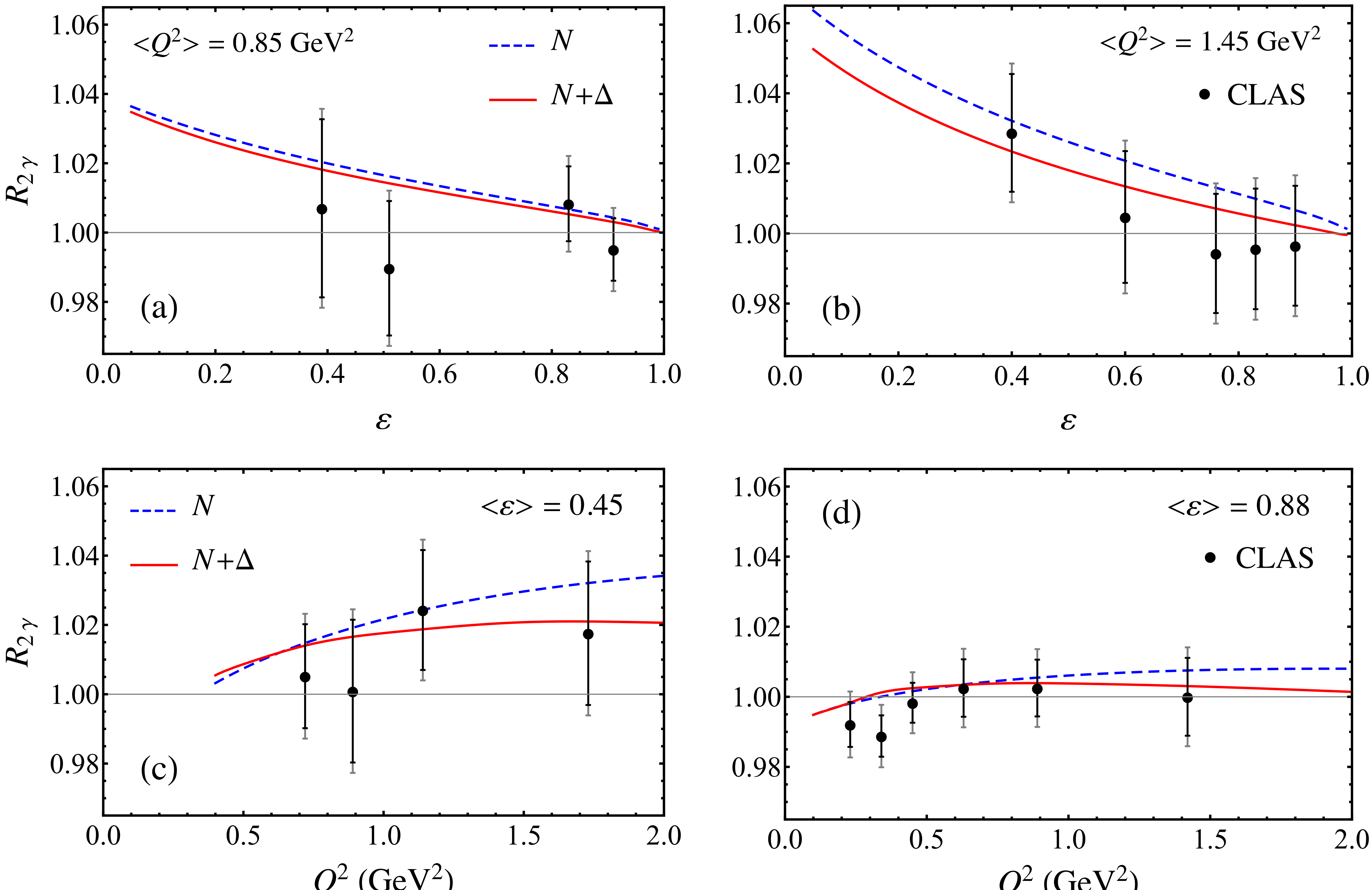
$$R_{2\gamma} = \frac{\sigma^{e^+}}{\sigma^{e^-}} \approx 1 - 2\delta_{\gamma\gamma}$$

VEPP-3 (Novosibirsk)

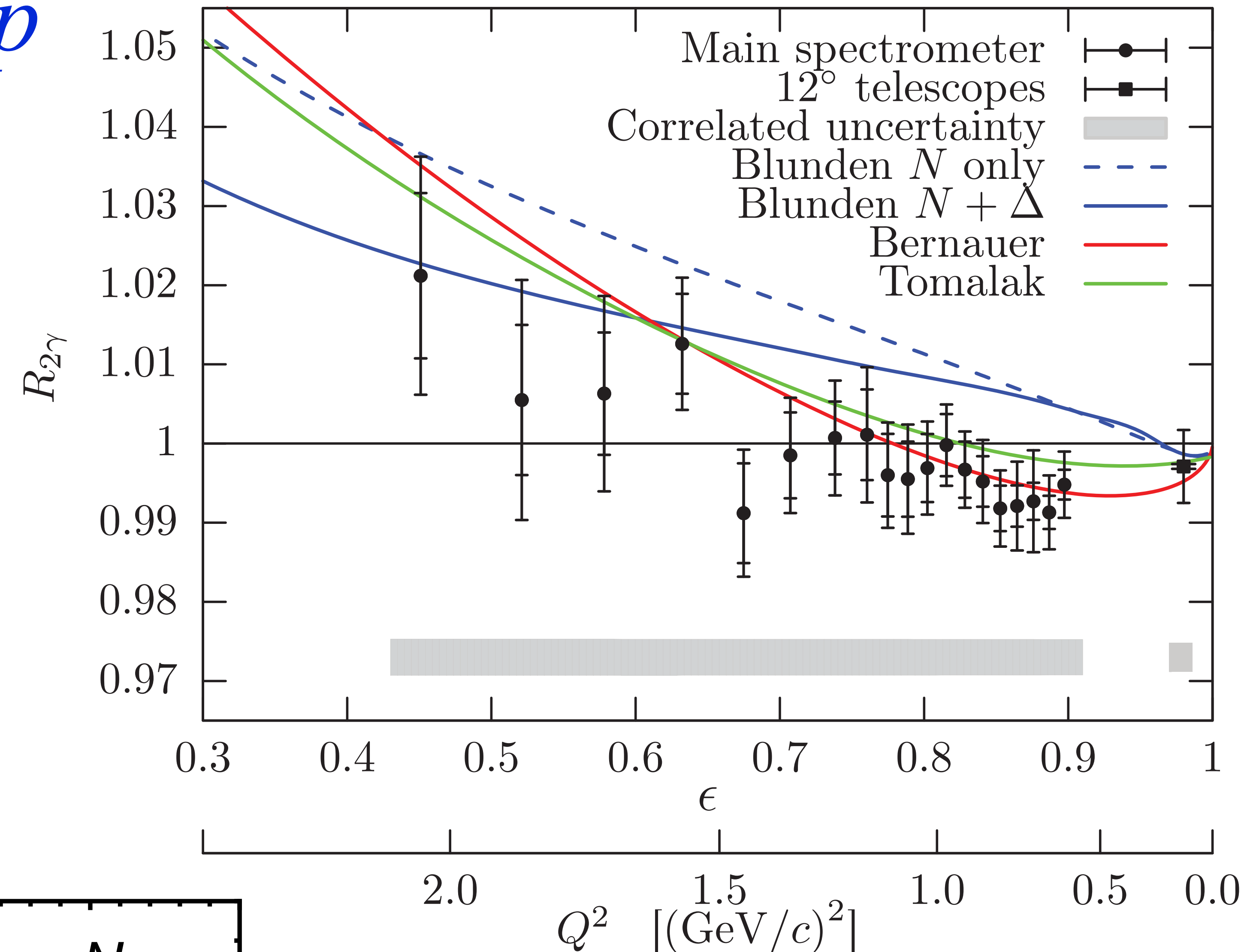


TPE effect on ratio of e^+p to e^-p cross sections

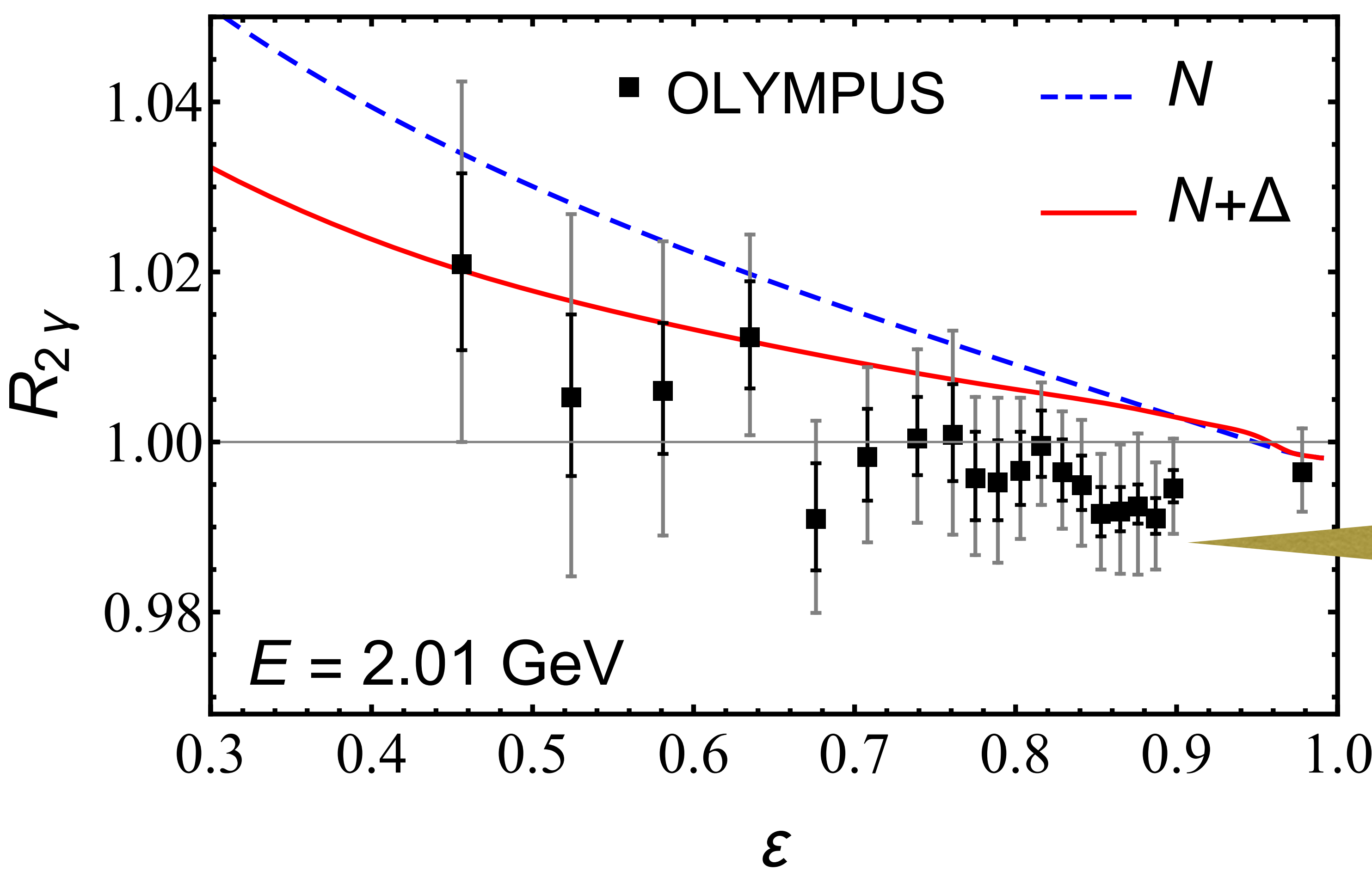
CLAS (Jefferson Lab)



TPE effect on ratio of e^+p to e^-p cross sections

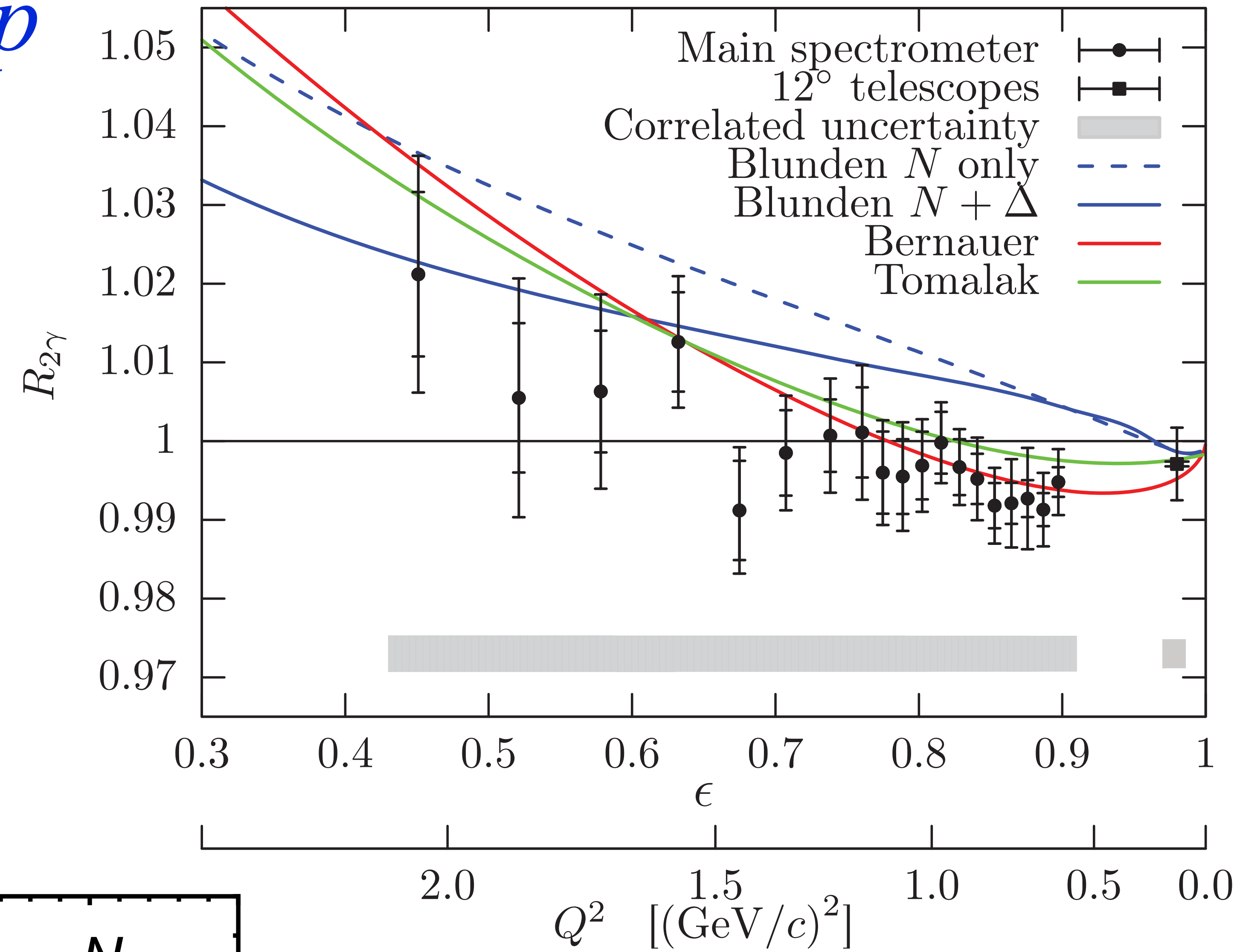


OLYMPUS
(Doris ring @ DESY)

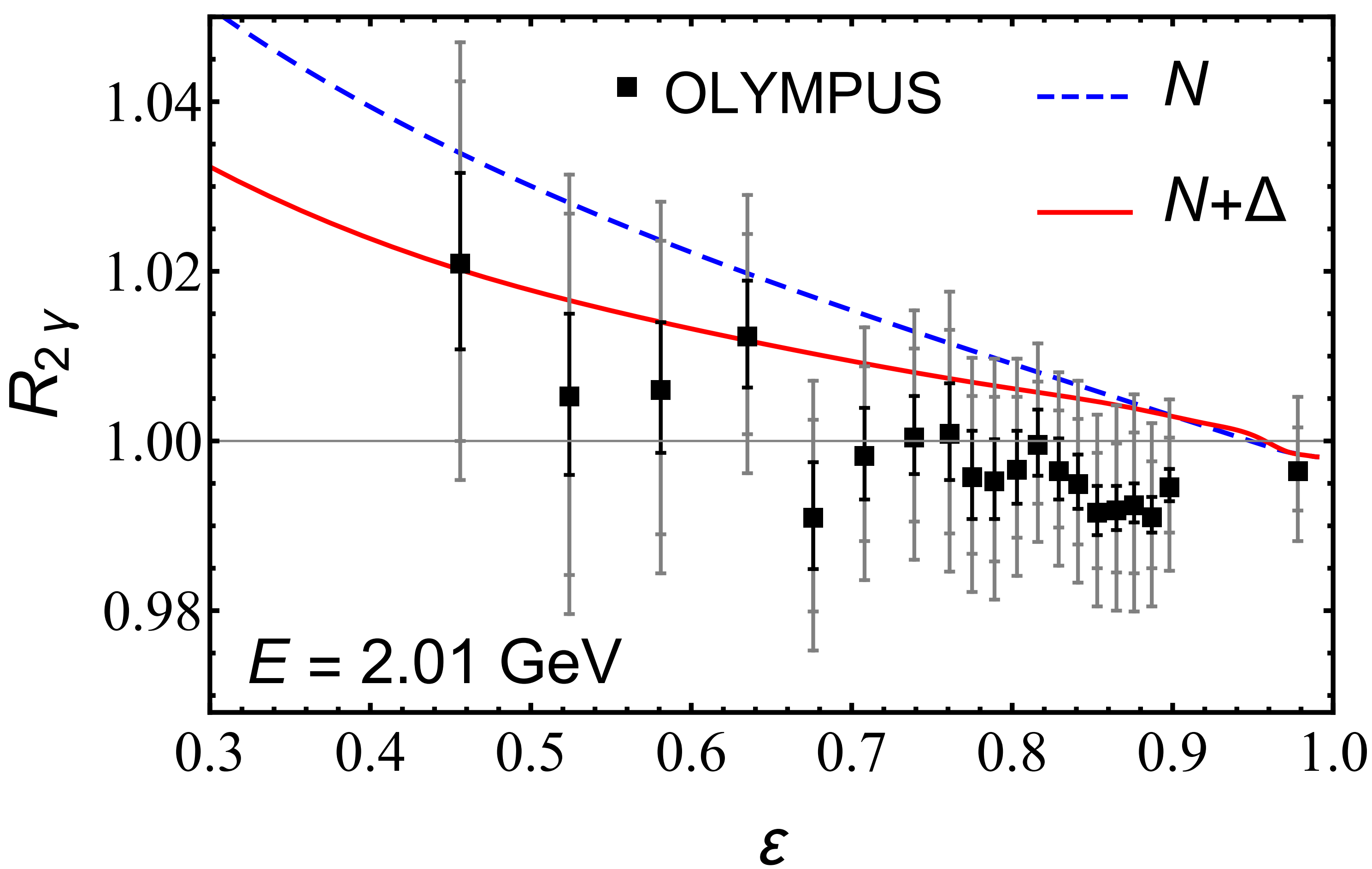


What is going on at low Q^2?

TPE effect on ratio of e^+p to e^-p cross sections

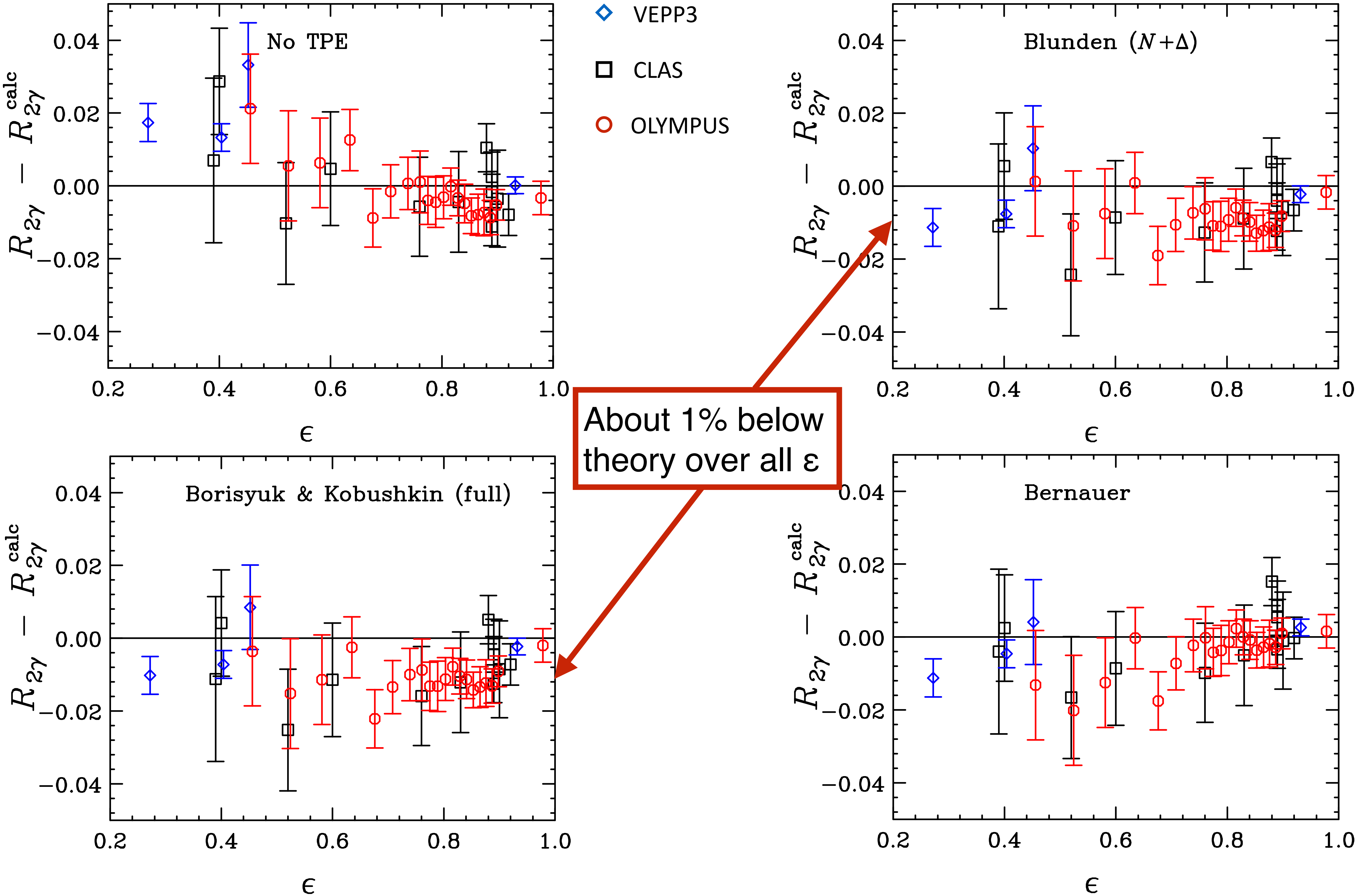


OLYMPUS
(Doris ring @ DESY)

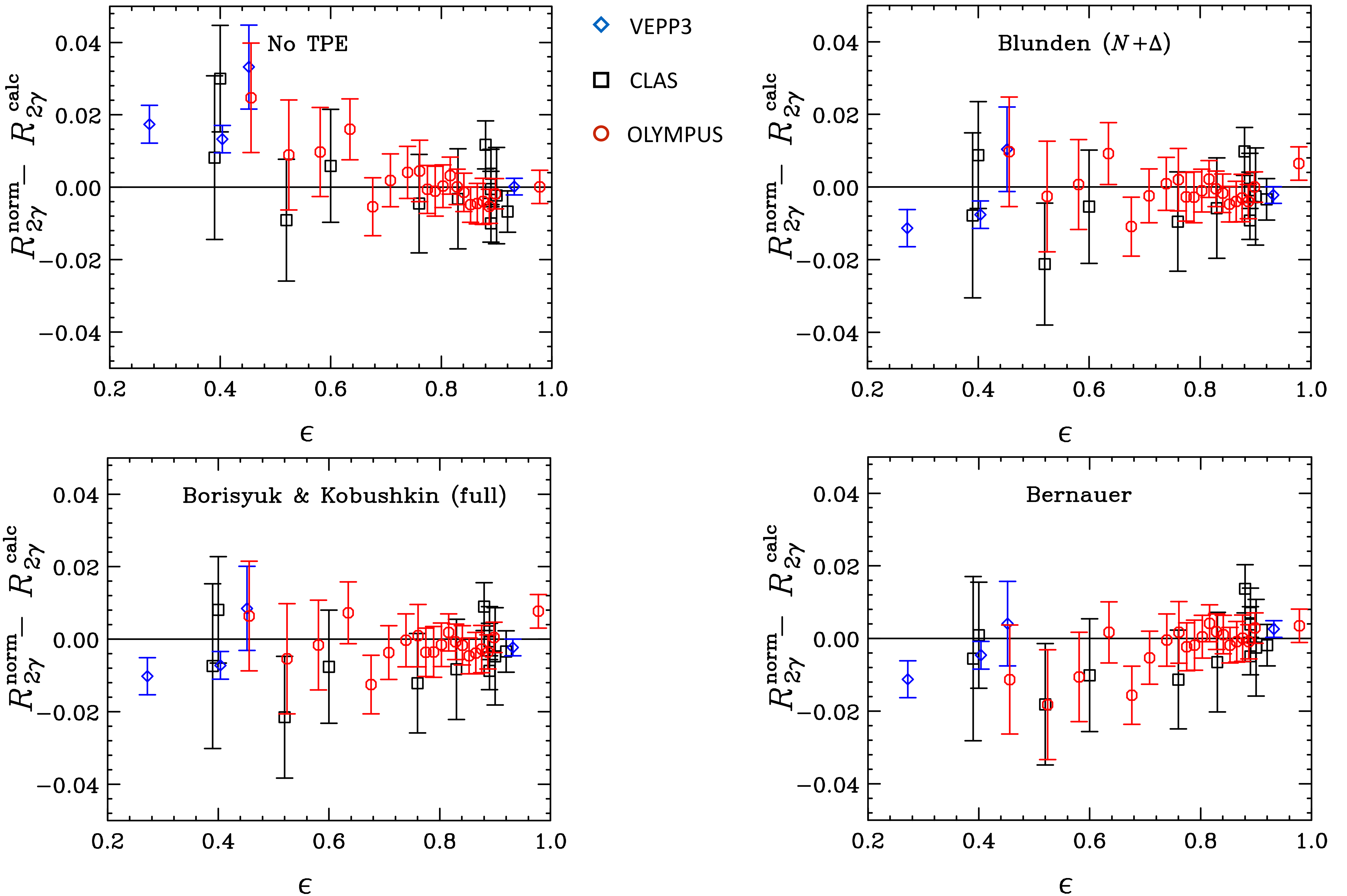


Includes systematic errors

Comparing theory and experiment



Allowing normalization to float

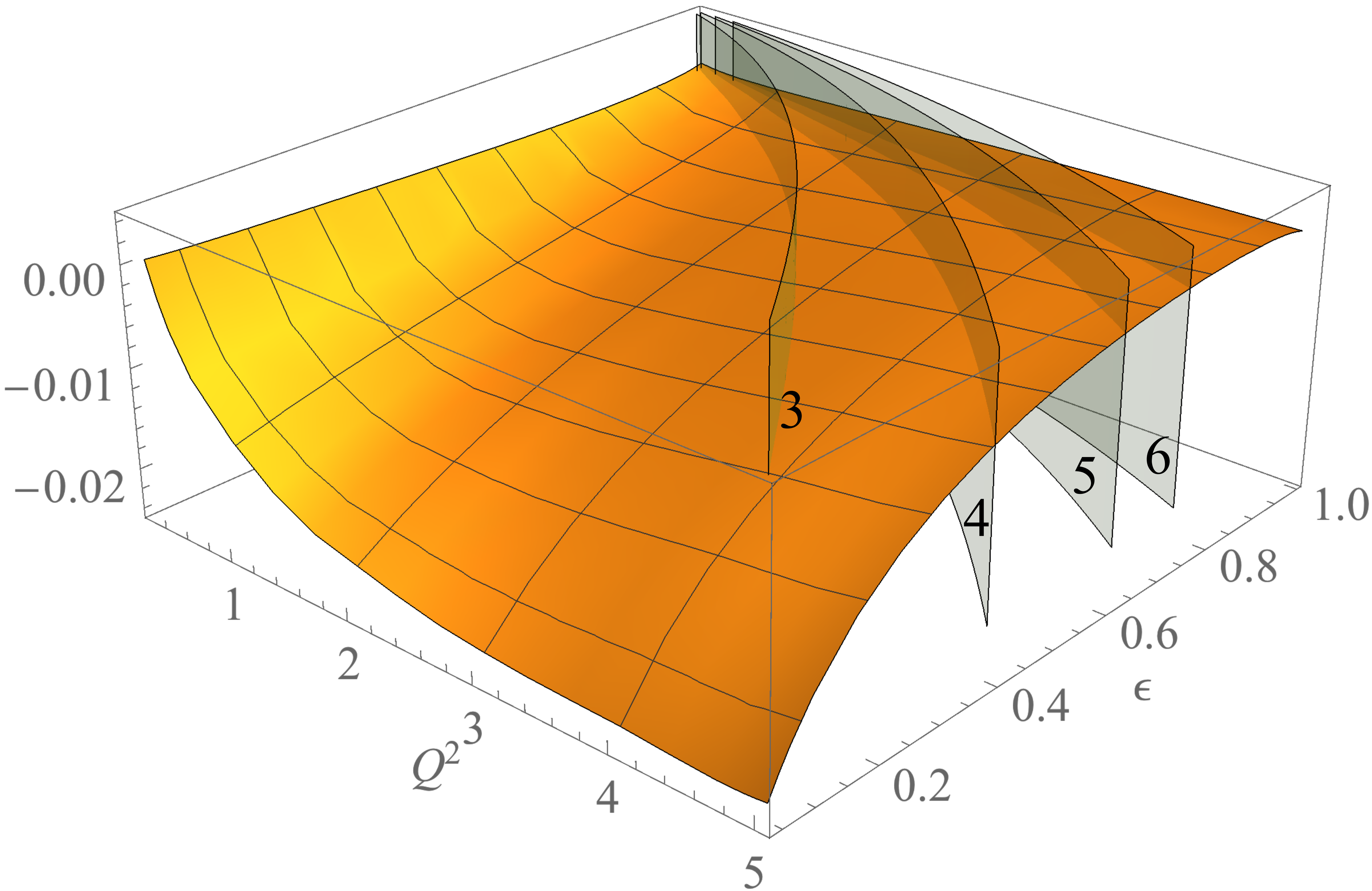


Allowing normalization to float

Data set	No normalization χ^2_ν	No normalization ν	With normalization χ^2_ν	With normalization ν	$\mathcal{N}$	$\left(\frac{\mathcal{N}-1}{\delta R^{\rm norm}_{2\gamma}}\right)$
Model: $\delta_{\gamma\gamma} = 0$						
VEPP-3	7.97	4	7.97	4	–	–
CLAS	0.99	12	1.25	11	1.0012	0.40
OLYMPUS	0.64	20	0.68	19	1.0034	0.76
All	1.57	36	1.73	34	–	–
Model: Blunden & Melnitchouk [54]						
VEPP-3	2.62	4	2.62	4	–	–
CLAS	0.90	12	0.91	11	1.0032	1.07
OLYMPUS	1.57	20	0.64	19	1.0082	1.82
All	1.46	36	0.96	34	–	–
Model: Borisyuk & Kobushkin [58]						
VEPP-3	2.28	4	2.28	4	–	–
CLAS	1.02	12	0.94	11	1.0038	1.27
OLYMPUS	2.15	20	0.75	19	1.0097	2.16
All	1.79	36	1.00	34	–	–
Model: Bernauer *et al.* [35]						
VEPP-3	1.90	4	1.90	4	–	–
CLAS	0.74	12	0.90	11	0.9985	−0.40
OLYMPUS	0.46	20	0.51	19	1.0019	0.42
All	0.71	36	0.80	34	–	–

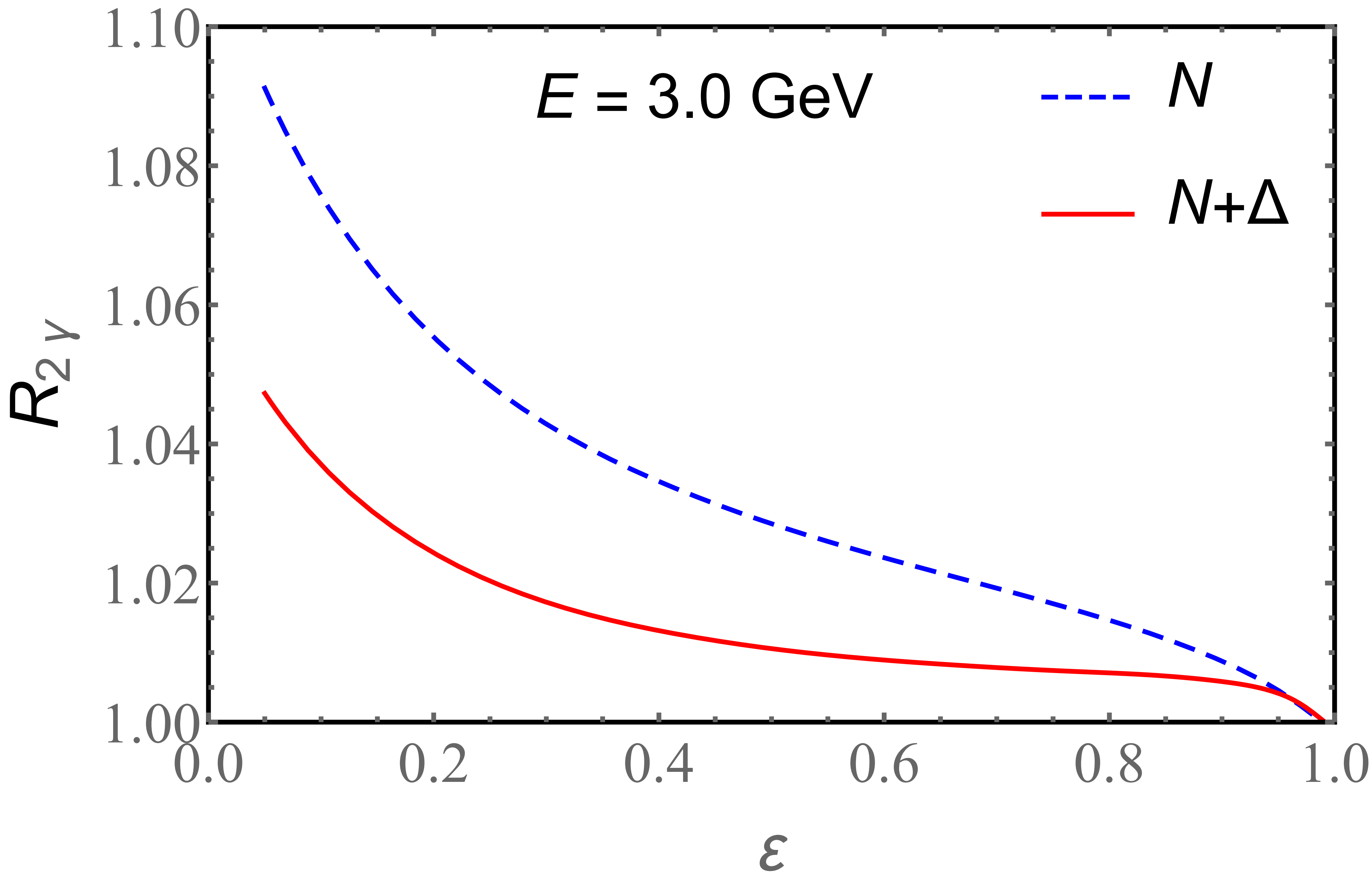
- For CLAS and OLYMPUS, allow normalization to float, with a penalty determined by normalization uncertainty of each data set
- Rules out no-TPE hypothesis at $> 90\%$ level

$\delta_{\gamma\gamma}$ plot vs. Q^2 and ε showing constant energy slices (in GeV)



E = 2-3 GeV is optimal for full coverage

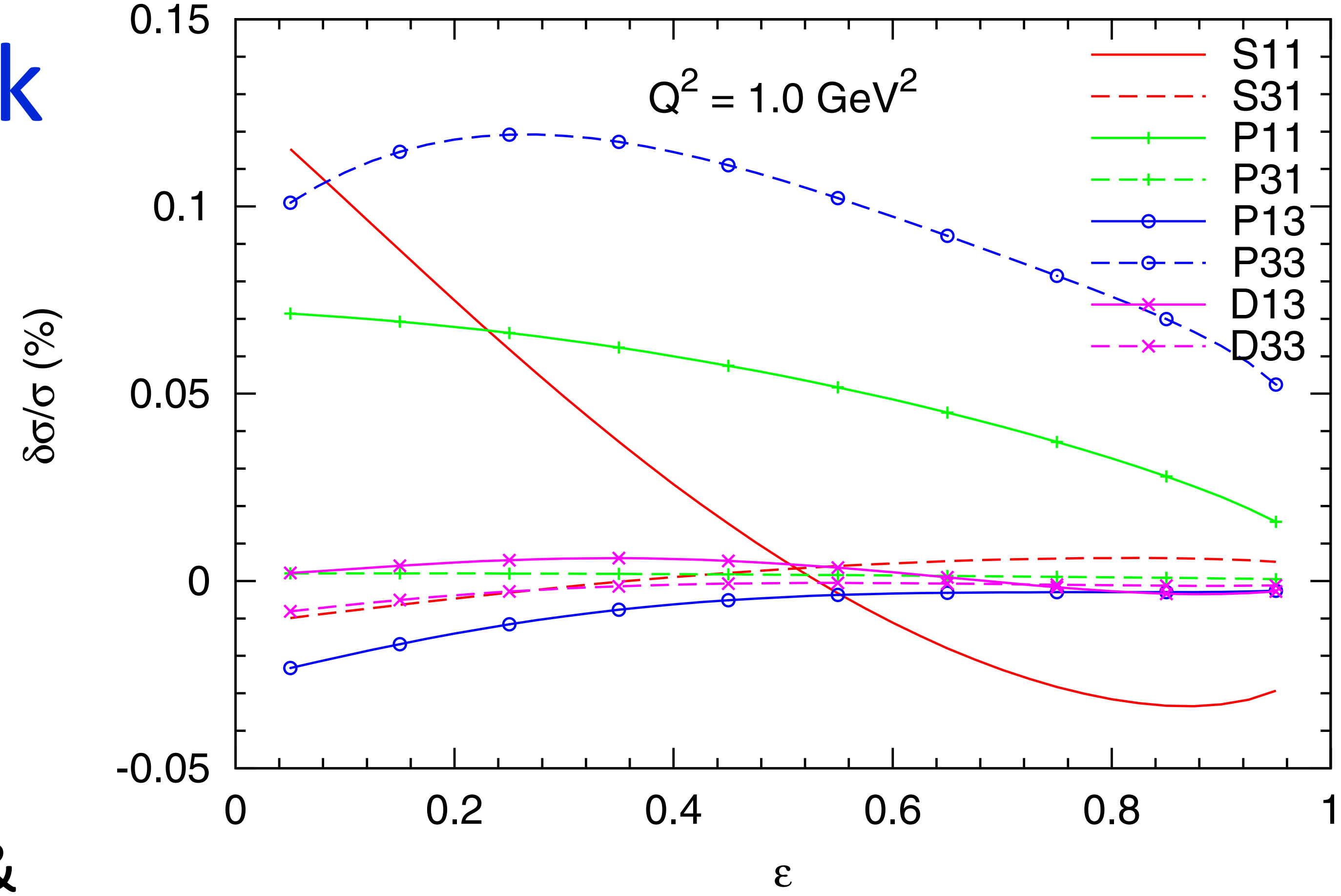
$R_{2\gamma}$ for fixed $E = 3.0$ GeV

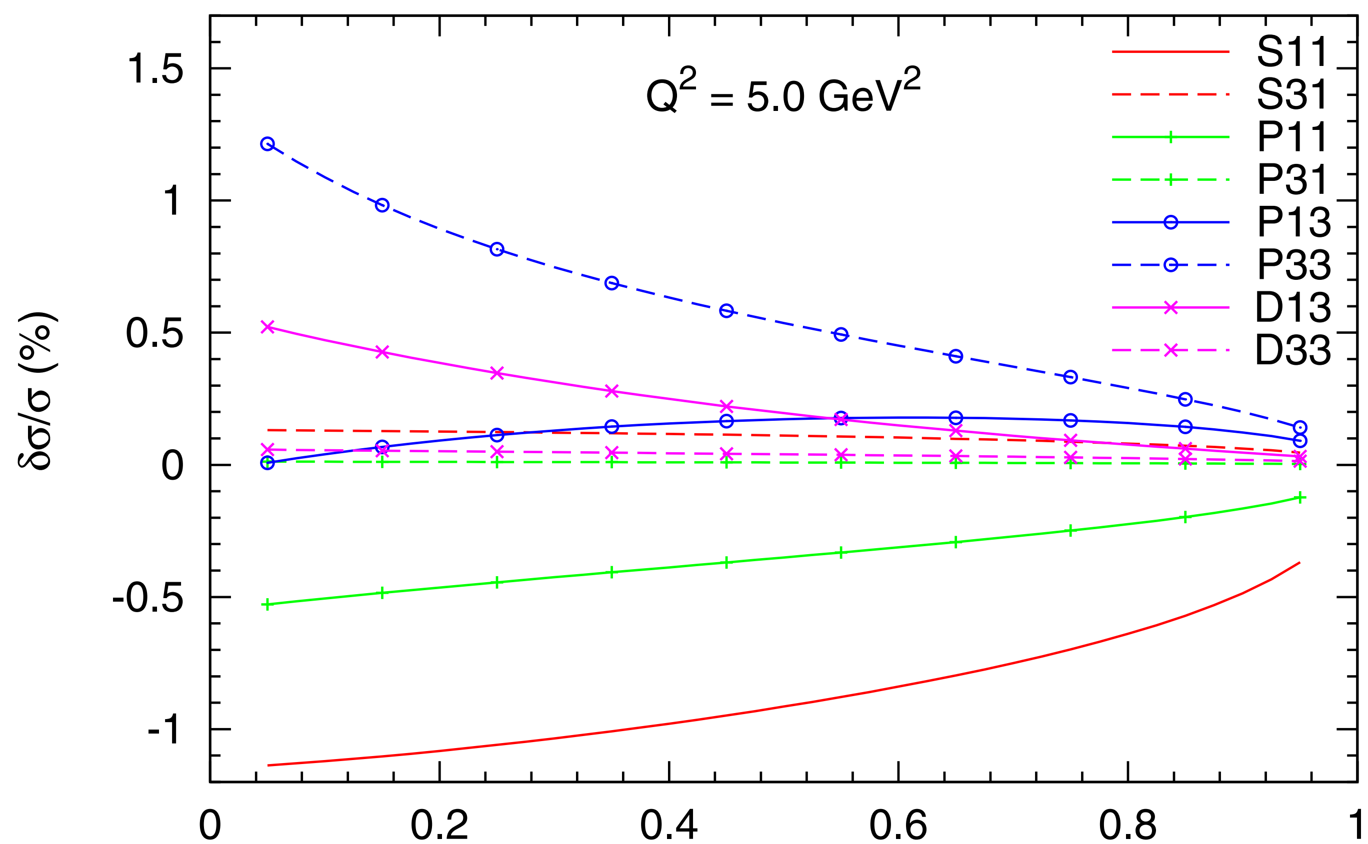


Additional theoretical work

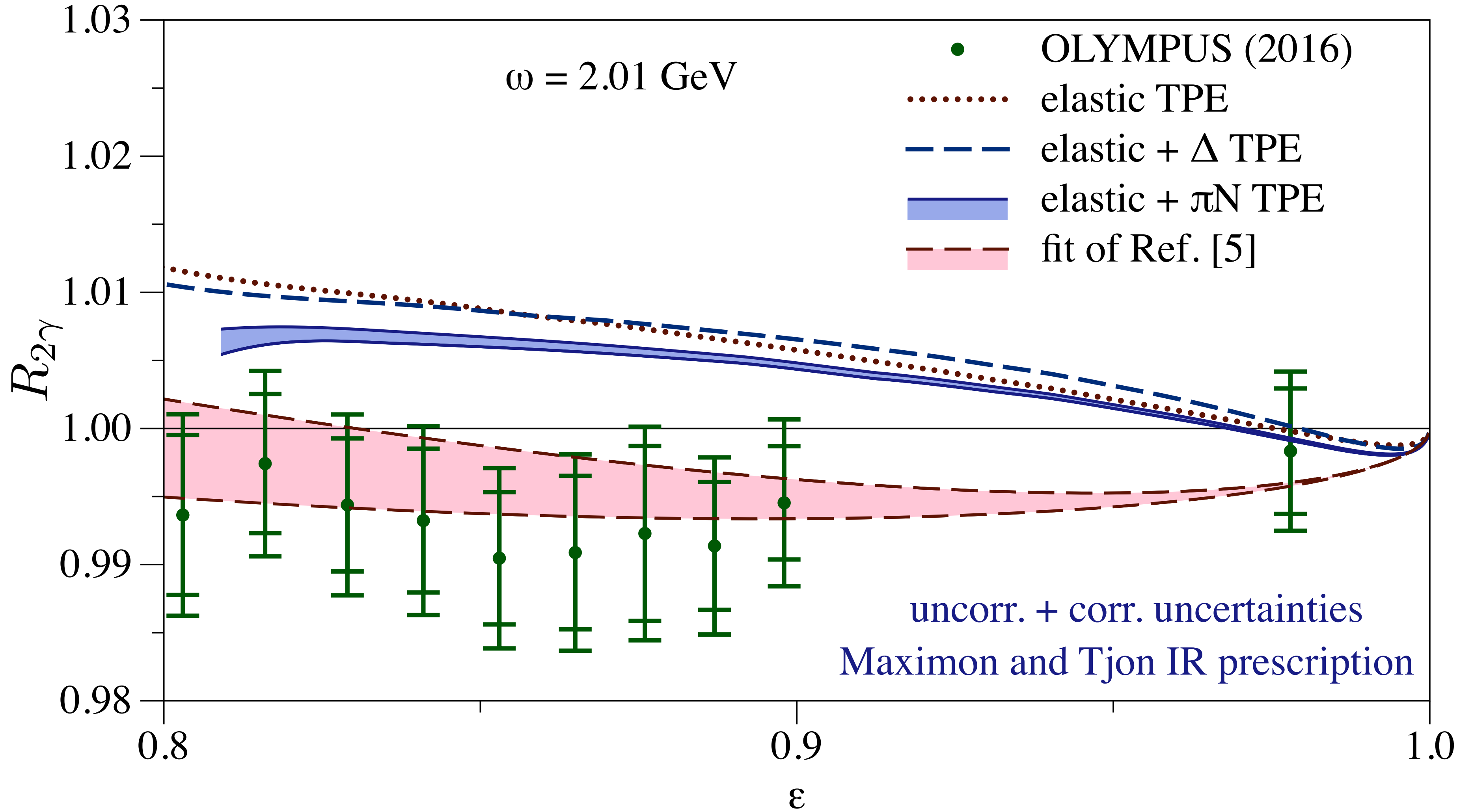
- Include πN spin $1/2$ and $3/2$ resonances + background using MAID helicity amplitudes
- Includes a finite width
- P_{33} and S_{11} dominate
- Contributions tend to cancel, in qualitative agreement with Kondratyuk & Blunden (2007) result

- Not a full dispersive calculation
- Sum of monopoles form factors is limiting





Borisyuk & Kobushkin, PRC**92**, 035204 (2015)



Tomalak, Pasquini & Vanderhaeghen, arXiv:1708.03303 (2017)

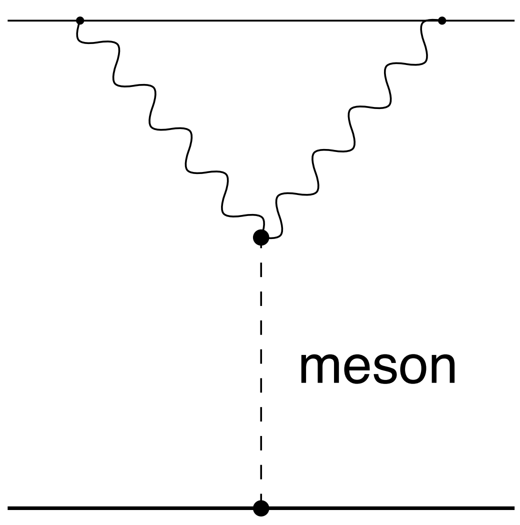
- Also include resonant and background πN states using MAID helicity amplitudes
- Full dispersive analysis

- πN continuum handled in unphysical region by analytic continuation from physical region
- Limited (for now) to relatively low Q^2 $(0.064 \lesssim Q^2 \lesssim 1\ \mathrm{GeV}^2)$

Other possible contributions

Meson pole terms (no imaginary part, so not included in dispersive analysis)

- Chen & Zhou, PRC **90**, 045205 (2014)
- Koshchii & Afanasev, PRD **94**, 116007 (2016)
- Borisyuk, arXiv: 1707.06513 (2017)



- Proportional to electron mass m_e
- Small for ep scattering, but may be important for μp (MUSE)
- May be important at very low Q^2, or for atomic physics (charge radius problem)

Summary

- Lots of interesting new theoretical work motivated by new experimental results
- Dispersive method only feasible approach, with connection to data in forward angle limit
  - A similar approach is essential for the γZ box in Qweak parity-violation kinematics
- Efforts underway to incorporate electroproduction data throughout the resonance region, including background
  - In forward angle limit the dispersive approach allows one to use total photonuclear cross section data (Gorchtein)
- Clear need for definitive e^+p measurements at high Q^2, low ε