Testing Lorentz symmetry in deep inelastic scattering with an electron-ion collider

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Abstract. We investigate the prospects for detecting tiny violations of Lorentz symmetry in deep inelastic scattering in the context of the future electron-ion collider. Preliminary results suggest estimated bounds on quark-sector coefficients for Lorentz violation can be extracted at sensitivities of parts in $10^3 - 10^6$.

INTRODUCTION

Lorentz invariance is a global symmetry of the Standard Model (SM) of particle physics and a local symmetry of General Relativity. While both theories have been fantastically successful in describing physics at currently attainable energies, it is widely expected that a fully quantum-theoretical description of all known physics including gravity will emerge at the Planck scale. One interesting possible consequence of this unification is the violation of Lorentz invariance. It has been shown that Lorentz violation can occur via spontaneous symmetry breaking in string field theory [1]. In this setting, a low-energy theory can gain terms in its Lagrange density that violate Lorentz symmetry, e.g.,

$$\mathcal{L}_{LV} \sim \frac{\lambda}{m_P^k} \langle T \rangle \cdot \bar{\psi} \Gamma (i\partial)^k \psi + h.c., \tag{1}$$

where λ is a dimensionless coupling constant, k is an integer exponent, and m_P is the Planck mass. The object $\langle T \rangle$ is a nonzero vacuum expectation value (vev) of a tensor field with suppressed spacetime indices, and Γ is a generic gamma-matrix structure. In Eq. (1), Lorentz symmetry is violated through the vev, which has orientation dependence. Even though the underlying theory is Poincaré invariant, interactions can destabilize the vacuum and generate effective background fields that induce Lorentz violation.

Given that the gap between currently accessible energies and the presumed scale of quantum gravity spans roughly 15 orders of magnitude, probing Planck-scale physics directly is infeasible. An alternative approach is to search for suppressed signals at attainable energies. Probing Nature in this way suggests the use of a low-energy, effective quantum field theory which completely accounts for all possible residual Lorentz-violating effects that presumably originate from mechanisms in a more fundamental theory. This framework is known as the Standard-Model Extension (SME) [2, 3, 4]. By construction, the SME contains the field content from all known fundamental physics with the addition of all possible terms built from fundamental fields that break Lorentz and CPT symmetry. These additional terms take the form of coefficients contracted with products of SM and gravitational fields. For example, one term in the quantum electrodynamics (QED) sector of the SME is given by

$$\mathcal{L}_{QED}^{SME} \supset -a_\mu \bar{\psi} \gamma^\mu \psi. \tag{2}$$

Here, the coefficient for Lorentz (and CPT) violation a_μ is a real quantity that can be thought of as a combination of a coupling constant, mass suppression scale, and tensor vev. An important property of the coefficients for Lorentz violation is that they transform as four-vectors under general coordinate transformations, called Lorentz observer transformations, but as scalars under transformations of the physical system itself, called Lorentz particle transformations. Because these coefficients represent preferred directions in spacetime, their presence implies a violation of

Lorentz symmetry. Since CPT symmetry is related to Lorentz symmetry through the CPT theorem [5], CPT-violating effects are also completely parametrized by the SME. Thus, the SME is the general phenomenological framework used to search for Lorentz- and CPT-violating suppressed signals arising from a more fundamental theory.

Constraints on many coefficients across all sectors of the SME have been placed to date [6]. Despite the large amount of work that has been carried out thus far, comparatively little attention has been paid to the quantum chromodynamics (QCD) sector of the SME. This is primarily due to the difficulties in bypassing the observed spectrum of states to access the fundamental degrees of freedom of QCD. Very recently, there has been a push towards exploring Lorentz violation in this sector [7, 8, 9, 10]. Much of this work may ultimately be relevant to the proposed electron-ion collider (EIC) [12], which is expected to usher in a new era of precision QCD studies of the hadrons. Since the EIC will have a unique ability to study QCD, it is interesting to consider the prospects for detecting Lorentz-violating QCD effects. This is the basis for the current document, which reports on forthcoming research that examines the prospects for detecting Lorentz violation at the EIC through the process of unpolarized electron-proton deep inelastic scattering (e-p DIS).

LORENTZ-VIOLATING EFFECTS IN UNPOLARIZED DIS

We now focus our attention on tree level Lorentz-violating effects in e-p DIS. This process was recently studied in the context of HERA collider data [11], and the discussion that follows is largely a summary of those results [10]. The general process is depicted in Fig. 1. For sufficiently large momentum transfer $-q^2 \gg M^2$, with M being the proton mass, interactions among the partons within the proton can be neglected due to asymptotic freedom. At zeroth order in the strong-interaction coupling constant g_s, the partons predominantly interact electromagnetically with the incident electron. Note that, at large enough $-q^2$, effects of Z^0 exchange have also been considered [10]. The dominant Lorentz-violating contributions here appearing in the quark-sector of the SME QCD lagrangian are

$$\mathcal{L}_{QCD}^{SME} \supset \sum_{f=u,d} \tfrac{1}{2}\left(\eta^{\mu\nu} + c_f^{\mu\nu}\right)\bar{\psi}_f \gamma_\mu i \overleftrightarrow{D}_\nu \psi_f, \tag{3}$$

where $i\overleftrightarrow{D}_\nu = i\overleftrightarrow{\partial}_\nu - 2q_f A_\nu$ is the usual QED covariant derivative. The coefficients $c_f^{\mu\nu}$ control the magnitude of Lorentz violation for each quark flavor f and are fixed in a given observer frame. In considering the Lorentz-violating contributions as written in Eq. (3), a number of additional factors have been taken into account. First, for simplicity, only the up- and down-quark flavor content is considered, as this is the dominant proton flavor content. Second, photon- and electron-sector bounds are constrained at much higher sensitivities than can be probed in this process [6], so these effects are neglected. Considering further only flavor-conserving couplings for the quarks and the fact that spin-dependent coefficients average to zero for unpolarized scattering leaves the written class of CPT-even coefficients in Eq. (3). Taking the right-hand side of Eq. (3) as the effective model, we treat the coefficients $c_f^{\mu\nu}$ as small perturbations and calculate the differential cross section at tree level for the unpolarized scattering process. The inclusion of

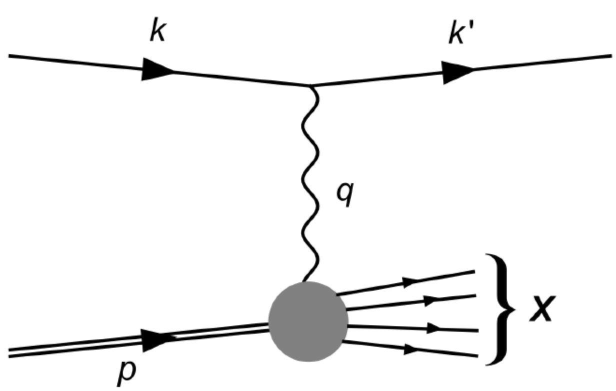

FIGURE 1. DIS: an electron with momentum k scatters off a proton of momentum p producing a generic final hadronic state $|X\rangle$. Figure taken from Ref. [10].

these Lorentz-violating effects leads to a modified quark current

$$J^{\mu}(x) = q_f \bar{\psi}_f(x)\Gamma^{\mu}_f \psi_f(x), \tag{4}$$

with $\Gamma^{\mu}_f \equiv \gamma^{\mu} + c_f^{\mu\nu}\gamma_{\nu}$. The scattering amplitude $\mathcal{M}$ for the DIS process illustrated in Fig. 1 is given by

$$i\mathcal{M} = (-ie)\bar{u}(k')\gamma_{\mu}u(k)\frac{-i}{q^2}(ie)\int d^4x\, e^{iq\cdot x}\langle X|\, J^{\mu}(x)\,|p\rangle\,. \tag{5}$$

To compute the cross section, the squared modulus of Eq. (5) must be calculated along with the flux factor $F = N_1 N_2|\vec{v}_1 - \vec{v}_2|$, where N_i and $\vec{v}_i$ are the colliding beam densities and group velocities, respectively. Effects of Lorentz violation on the flux factor are expected to be negligible for the DIS process [13]. The main difficulty here lies in the computation of the hadronic matrix element $\langle X|\, J^{\mu}(x)\,|p\rangle$, as the current contains quark creation and annihilation operators operating on hadronic states. This can be circumvented by first defining the hadronic tensor as the spin-average of the following quantity:

$$W^{\mu\nu}(p,q) = i\int d^4x\, e^{iq\cdot x}\langle p|\, T\{J^{\mu}(x)J^{\nu}(0)\}\,|p\rangle\,. \tag{6}$$

The optical theorem can now be applied to the hadronic tensor to reveal

$$2\mathrm{Im}\left[W^{\mu\nu}(p,q)\right] = \sum_X \int d\Pi_X \langle p|\, J^{\mu}(-q)\,|X\rangle\langle X|\, J^{\nu}(q)\,|p\rangle\,, \tag{7}$$

with $d\Pi_X$ being the final hadronic state phase space. Using Eq. (7), and recognizing that the squared modulus of the electon-photon vertex is essentially the leptonic tensor $L^{\mu\nu} = 2\left(k^{\mu}k'^{\nu} + k^{\nu}k'^{\mu} - k\cdot k'\eta^{\mu\nu}\right)$, leads to a compact form for the triply differential cross section

$$\frac{d^3\sigma}{dxdyd\phi} = \frac{\alpha^2 y}{2\pi q^4}L^{\mu\nu}\mathrm{Im}[W_{\mu\nu}], \tag{8}$$

where $y = p\cdot q/p\cdot k$ and α is the fine-structure constant. In the usual SM calculation of the cross section, there is no complicated dependence on the final-state electron azimuthal angle. However, in the presence of Lorentz violation, the angular dependence is nontrivial. What remains is to choose how to evaluate $\mathrm{Im}[W_{\mu\nu}]$. Here, the parton model can be used to approximate photon-proton scattering as a photon interacting with a parton of flavor f carrying a momentum fraction ξp of the proton's four-momentum p. The hadronic tensor can thus be approximated as

$$W_{\mu\nu} \approx i\int d^4x\, e^{iq\cdot x}\int_0^1 d\xi \sum_f \frac{f_f(\xi)}{\xi}\langle \xi p|\, T\{J_{\mu}(x)J_{\nu}(0)\}\,|\xi p\rangle\,, \tag{9}$$

where $f_f(\xi)$ is the parton distribution function (PDF) for a given flavor f. Evaluating the time-ordered product and subsequent Wick contractions and traces leads directly to the expression of the modified γ-exchange differential cross section in terms of SM and Lorentz-violating contributions,

$$\begin{aligned}\frac{d^3\sigma}{dxdyd\phi} &= \frac{\alpha^2 s}{q^4}\sum_f q_f^2 x f_f(x)\left[1 + (1-y)^2\right] \\ &+ \frac{\alpha^2}{q^4}\sum_f q_f^2 x f_f(x)\left[[C''] - \frac{2(1+(1-y)^2)}{y}\left([C'] + \left(\frac{1}{x} + \frac{d\ln f_f(x)}{dx}\right)[C]\right)\right],\end{aligned} \tag{10}$$

where

$$\begin{aligned}[C] &= c_f^{\mu\nu}\left[q_{\mu}q_{\nu} + x\left(q_{\mu}p_{\nu} + q_{\nu}p_{\mu}\right) + x^2 p_{\mu}p_{\nu}\right], \\ [C'] &= c_f^{\mu\nu}\left(p_{\mu}q_{\nu} + p_{\nu}q_{\mu} + 2xp_{\mu}q_{\nu}\right), \\ [C''] &= \frac{2y}{x}[C] \\ &\quad + c_f^{\mu\nu}\left[4\left(k'_{\mu}p_{\nu} + p_{\mu}k'_{\nu}\right) - \frac{4M^2}{s}\left(k_{\mu}k'_{\nu} + k'_{\mu}k_{\nu}\right) + \frac{4}{x}(1-y)k_{\mu}k_{\nu} + 4(1-y)\left(k_{\mu}p_{\nu} + p_{\mu}k_{\nu}\right) - 4xyp_{\mu}p_{\nu} - \frac{4}{x}k'_{\mu}k'_{\nu}\right].\end{aligned} \tag{11}$$

The first line of Eq. (10) is the usual leading-order SM contribution. The second line, which is proportional to $c_f^{\mu\nu}$, is the contribution from Lorentz-violating effects. In choosing a frame to analyze Eq. (10), special care must be taken because the coefficients for Lorentz violation depend on choice of observer frame. This implies that an Earth-based experiment will exhibit a sidereal time dependence in the cross section. It is therefore important to work initially in a suitable (approximately) inertial frame.

THE SUN-CENTERED CELESTIAL-EQUATORIAL FRAME AND HERA ANALYSIS

The standard choice of frame used for reporting bounds on coefficients for Lorentz violation is known as the Sun-centered celestial-equatorial frame (SCF) [16]. This frame is effectively inertial over the time scale of most Earth-based experiments. For simplicity, it is convenient to take the coefficients for Lorentz violation as constants in this frame, which preserves energy and momentum conservation. A series of rotations relates the SCF to the laboratory

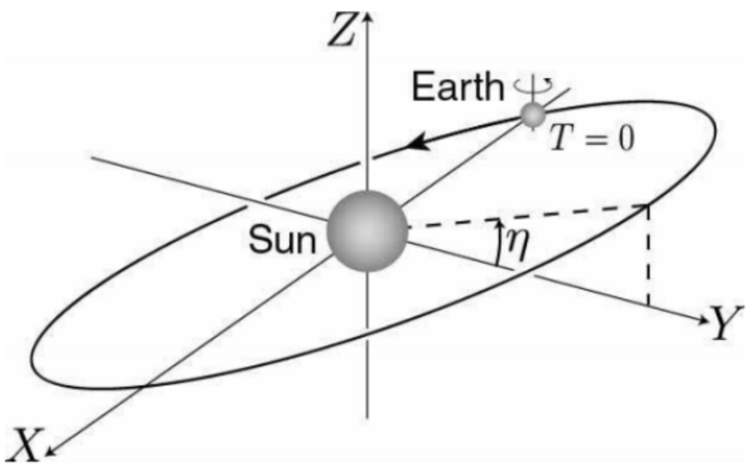

FIGURE 2. The SCF: in this frame the Z axis is taken to be parallel with the rotation axis of the Earth, and the X axis points towards the 2000 vernal equinox. Figure taken from Ref. [17].

frame on Earth that depend on three angles: the colatitude of the laboratory, the orientation of the colliding beamline relative to the cardinal points, and the product of Earth's sidereal frequency with the local sidereal time. Once the coefficients have been transformed to the laboratory frame, they exhibit a time dependence by virtue of the local sidereal time. More precisely, for a given flavor f, only six combinations of the nine independent components of $c_f^{\mu\nu}$ obtain time dependence after the transformation from the SCF to the laboratory frame. The first estimates of bounds on the time-dependent coefficients were placed in the study of HERA collider data [10]. The best estimated bounds obtained in this work are at at low $(x, -q^2)$ values and on the order of $10^{-4} - 10^{-6}$. A complete description of these results is contained in Table 1 of Ref. [10]. Sensitivity to the kinematical region of low $(x, -q^2)$ is convincing when examining the Lorentz-violating contribution to the cross section in Eq. (10). We also mention that these bounds are estimates obtained from binning the time-dependent contribution of the cross section in bins of sidereal time. For real bounds to be placed on the coefficients, time stamps of the events culminating in each measurement are needed, which are unavailable. We now turn our attention to a preliminary analysis of simulated EIC data.

PRELIMINARY RESULTS FOR THE EIC

The EIC is scheduled to be constructed at either Jefferson Lab (JLab) or Brookhaven National Lab (BNL). Initial design parameters for the JLab EIC (JLEIC) and BNL EIC (eRHIC) suggest a similar reach in terms of kinematic phase space [12, 15]. The main distinction between the two proposed designs is that JLEIC has lower center-of-mass system (CMS) energy range than eRHIC, but a higher luminosity. Whether the EIC is eventually built at JLab or BNL, each machine is capable of being upgraded to a similar maximum CMS energies and luminosity.

Simulated inclusive *e*-*p* DIS data can be used to estimate the sensitivities to Lorentz violation attainable at the future EIC [14]. Thus far, only a full analysis of JLEIC simulated data has been performed. Our preliminary results suggest estimated bounds on the time-dependent *and* time-independent coefficients on the order of $10^3 - 10^6$. We remark that bounds on the time-independent coefficients will be the first estimates placed to date. These coefficients can be examined for the future EIC because the simulated cross section data does not enter into the construction of the PDFs used to build the cross section. Because of this, there is no concern of the PDFs being contaminated with potential Lorentz-violating effects as in the case of real data. The sensitivities obtained for the time-dependent coefficients are at the a similar level to what was obtained in the analysis of HERA data. Nevertheless, as HERA and both potential EIC colliders have a unique geographic location, the linear combination of coefficients within the cross section for each orientation are unique. This implies that the level of sensitivity to a particular coefficient obtainable in one experiment is not necessarily indicative of the sensitivity in another. This fact further supports the case for exploring the sensitivities of an EIC to Lorentz violation with two distinct locations.

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