

International Workshop on Physics  
with Positrons at Jefferson Lab (JPos17)

# Charge and Spin Asymmetries in Elastic Lepton-Nucleon Scattering

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The George Washington University

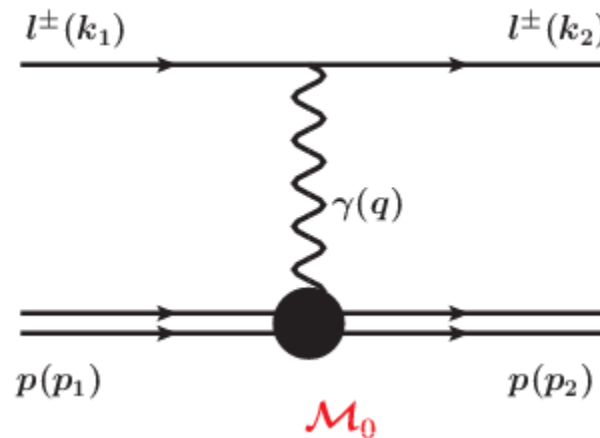
Jefferson Lab,  
Newport News, VA, September 15, 2017

# Outline

- Charge asymmetry in elastic  $lp$  scattering
- The proton radius puzzle and MUSE experiment
- Soft TPE in approach of Tsai and bremsstrahlung interference contributions for massive leptons
- Helicity-flip transitions:  $\sigma$ -meson exchange in the  $t$ -channel in kinematics of MUSE
- Target-normal single-spin asymmetries (SSA)
- Conclusion

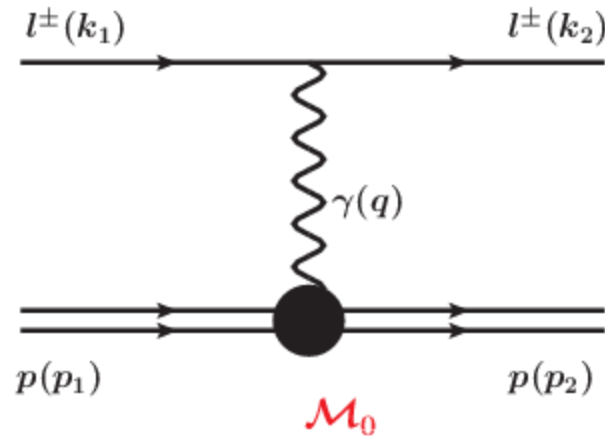
# Intro: Charge Asymmetry

- Charge asymmetry in unpolarized elastic lepton-proton scattering:



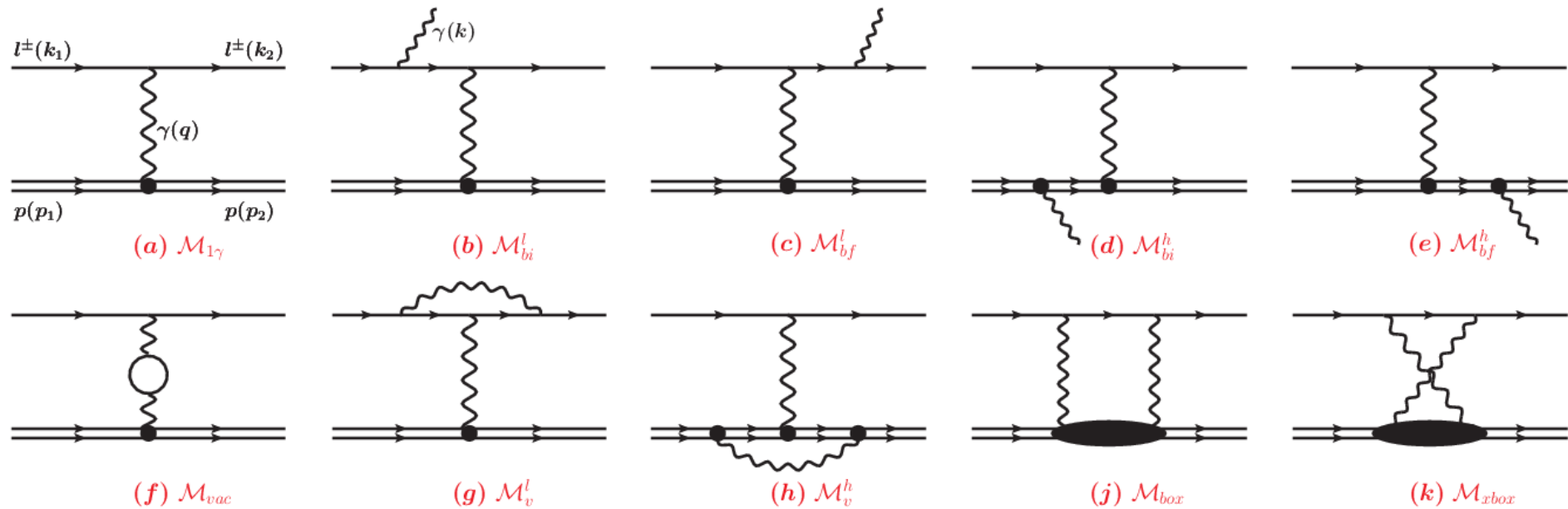
$$A \equiv \frac{d\sigma^+ - d\sigma^-}{d\sigma^+ + d\sigma^-}$$

# Lepton-Proton Scattering: Born Approximation



- Differential Cross Section:  $d\sigma \propto |M_0|^2$
- Lepton Coupling:  $\pm i e \gamma_\mu$       Coupling Constant:  $\alpha = \frac{e^2}{4\pi}$
- No asymmetry in Born ( $\propto \alpha^2$ ) Approximation

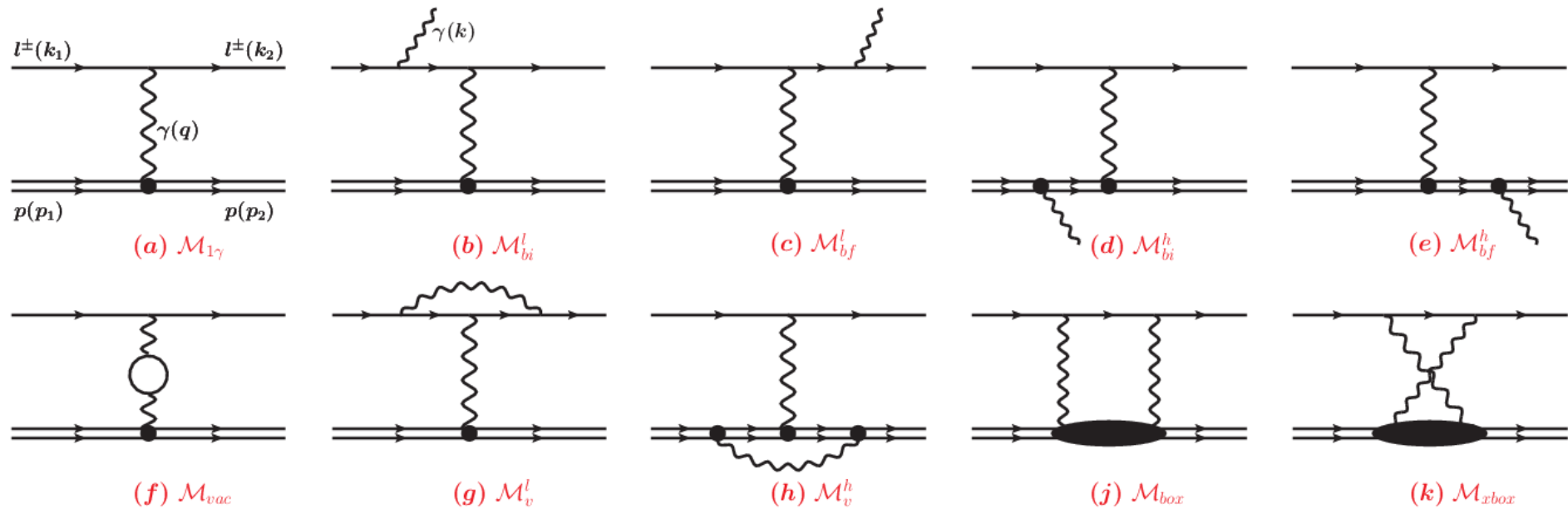
# Lepton-Proton Scattering: Higher Order Diagrams



➤ Leading and next-to-leading order contributions:

$$\begin{aligned}
 |M|^2 = & |M_0|^2 + |M_{1\gamma}^l|^2 + |M_{1\gamma}^h|^2 + 2\text{Re}[M_0^* M_{vac}] + 2\text{Re}[M_0^* M_{vert}^l] \\
 & + 2\text{Re}[M_0^* M_{vert}^h] + 2\text{Re}\left[(M_{1\gamma}^l)^* M_{1\gamma}^h\right] + 2\text{Re}[M_0^* M_{2\gamma}] + O(\alpha^4)
 \end{aligned}$$

# Lepton-Proton Scattering: Higher Order Diagrams



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 & + 2 \operatorname{Re} [M_0^* M_{vert}^h] + 2 \operatorname{Re} \left[ (M_{1\gamma}^l)^* M_{1\gamma}^h \right] + 2 \operatorname{Re} [M_0^* M_{2\gamma}] + O(\alpha^4)
 \end{aligned}$$

# Charge-Dependent Contributions

➤ Lepton Coupling:  $\pm i e \gamma_\mu$

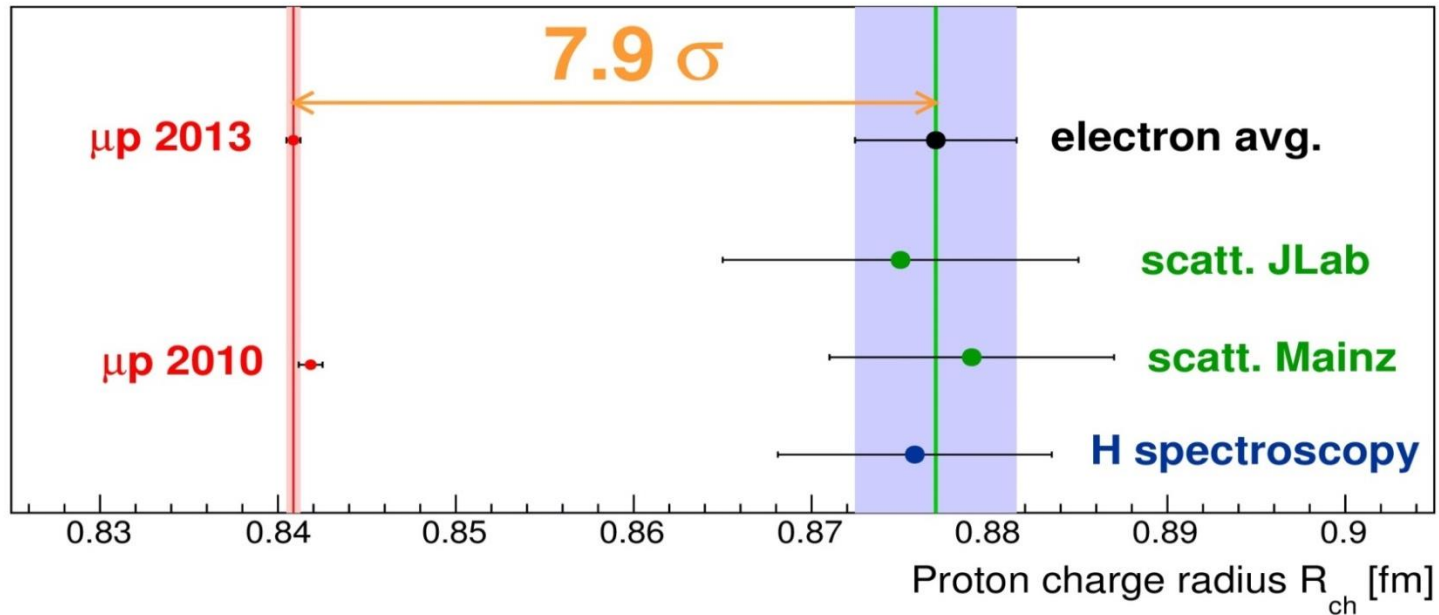
$$M_0^* M_{2\gamma} : \left( \text{Diagram } M_0 \right)^* \times \left( \text{Diagram } M_{box} + \text{Diagram } M_{xbox} \right)$$

The diagram  $M_0$  shows a fermion line with a photon loop. The diagram  $M_{box}$  shows a fermion line with a photon loop and a box diagram. The diagram  $M_{xbox}$  shows a fermion line with a photon loop and a crossed box diagram.

$$\left( M_{1\gamma}^l \right)^* M_{1\gamma}^h : \left( \text{Diagram } M_{1\gamma,i}^l + \text{Diagram } M_{1\gamma,f}^l \right)^* \times \left( \text{Diagram } M_{1\gamma,i}^h + \text{Diagram } M_{1\gamma,f}^h \right)$$

The diagram  $M_{1\gamma,i}^l$  shows a fermion line with a photon loop and an incoming photon. The diagram  $M_{1\gamma,f}^l$  shows a fermion line with a photon loop and an outgoing photon. The diagram  $M_{1\gamma,i}^h$  shows a fermion line with a photon loop and an incoming photon. The diagram  $M_{1\gamma,f}^h$  shows a fermion line with a photon loop and an outgoing photon.

# The Proton Radius Puzzle

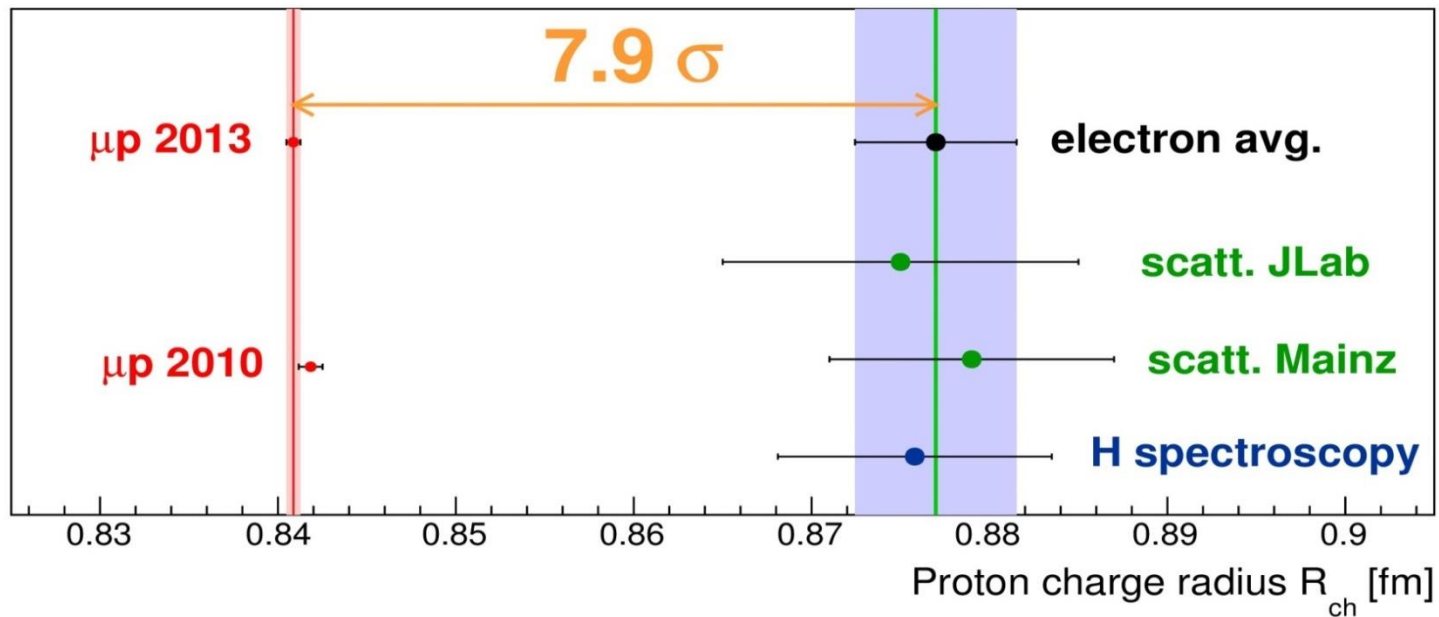


[<https://www.psi.ch/muonic-atoms/>]

	Muon	Electron
Spectroscopy	<b>0.8409(4)</b>	<b>0.8758(77)</b>
Scattering	???	<b>0.8770(60)</b>



# The Proton Radius Puzzle



[<https://www.psi.ch/muonic-atoms/>]

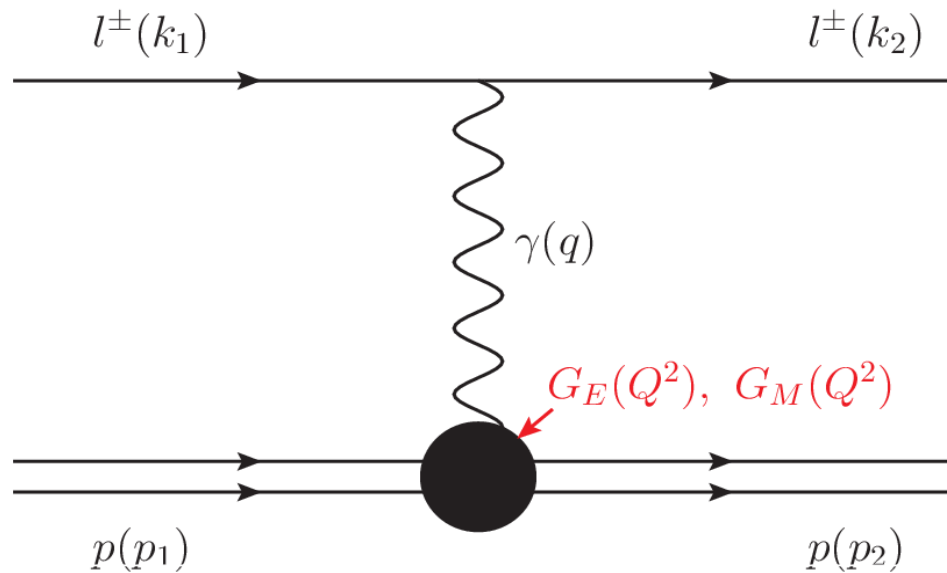
	Muon	Electron
Spectroscopy	0.8409(4)	0.8758(77)
Scattering	???	0.8770(60)

Muon Scattering  
Experiment (MUSE)  
at PSI, Switzerland

# MUSE at PSI

- Will measure simultaneously elastic  $e^\pm p$  and  $\mu^\pm p$  scattering:
  - Direct Access to TPE Corrections
- First significant  $\mu p$  scattering radius determination, at roughly the same level as done in previous scattering experiments:
  - Theoretical estimations beyond the Born approximation are required. Ultrarelativistic limit,  $\varepsilon \gg m$ , cannot be applied to scattering of muons in kinematics of MUSE.

# Theoretical Background: Born Approximation



Lab Frame :

$$k_1 = (\varepsilon_1, \vec{k}_1),$$

$$k_2 = (\varepsilon_2, \vec{k}_2),$$

$$p_1 = (M, 0),$$

$$p_2 = (E_2, \vec{p}_2),$$

$$Q^2 = -q^2 = -(k_1 - k_2)^2 > 0.$$

$$G_E(Q^2), G_M(Q^2)$$



**Electric and Magnetic  
form factors**

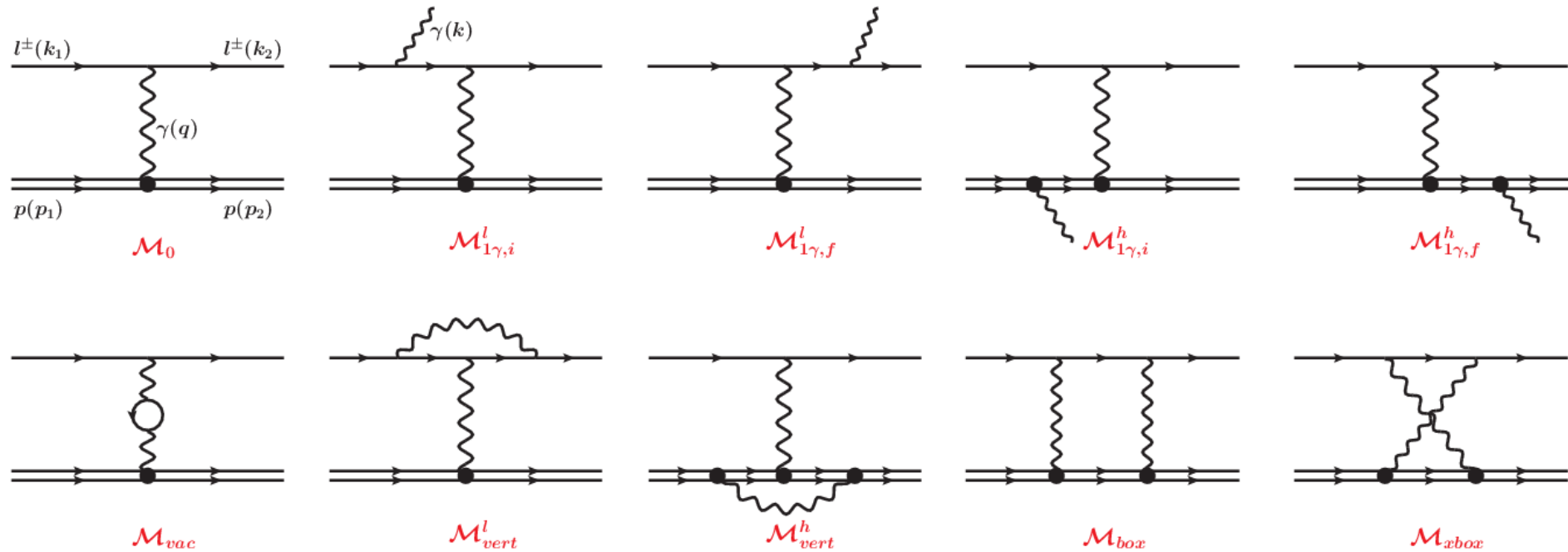
**Charge radius definition:**

$$\langle r^2 \rangle = -6 \left. \frac{dG_E(Q^2)}{dQ^2} \right|_{Q^2=0}$$

**Rosenbluth separation:**

$$\frac{d\sigma_0}{d\Omega} \propto G_E^2(Q^2) + \frac{\tau}{\epsilon} G_M^2(Q^2)$$

# Standard Higher Order Corrections



$$|M|^2 = \sum_i |M_i|^2 = |M_0|^2 (1 + \delta)$$

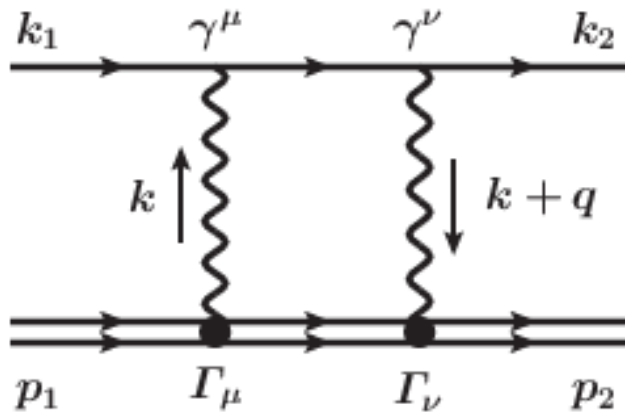
$$d\sigma^{\text{exp}} = d\sigma_0 (1 + \delta)$$

Need to know this value  
to extract the radius!

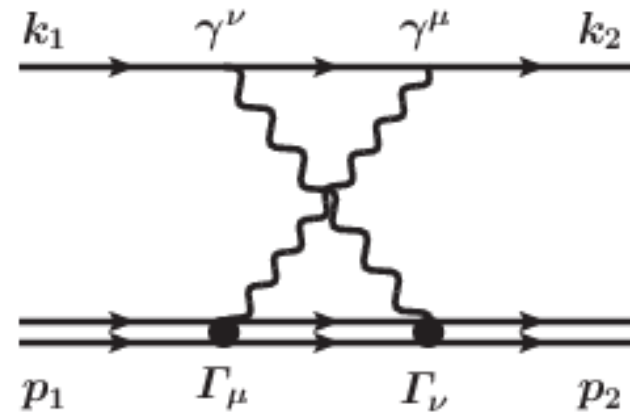


Need estimations of  $\delta$ !

# Model-independent TPE Calculation



(a)  $\mathcal{M}_{box}$



(b)  $\mathcal{M}_{xbox}$

Soft Photon Approximation:  $k \rightarrow 0$

Two prescriptions:

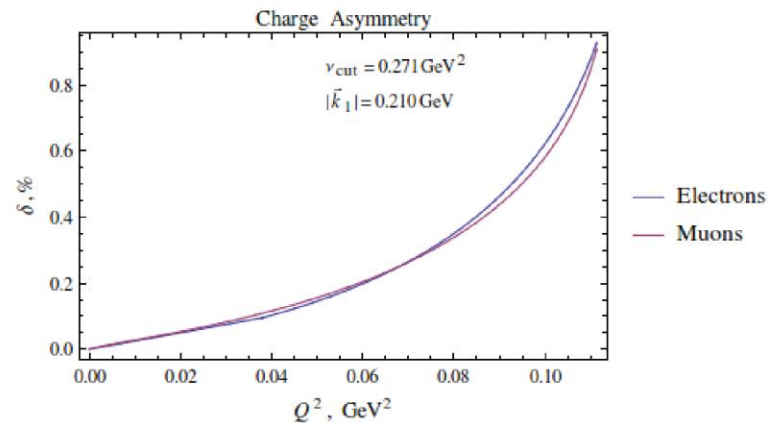
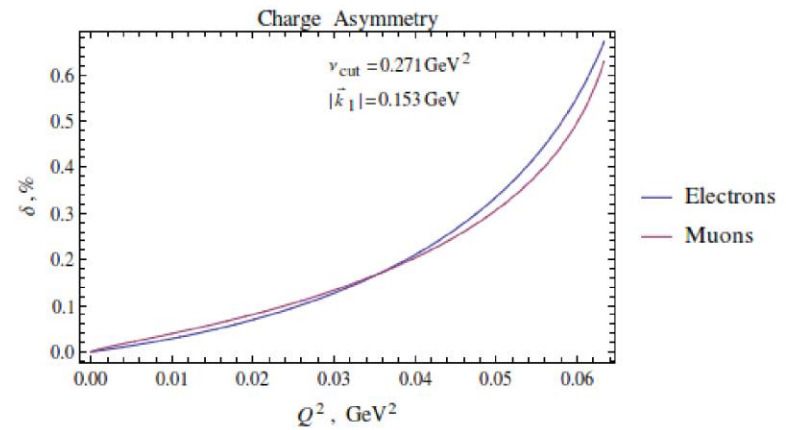
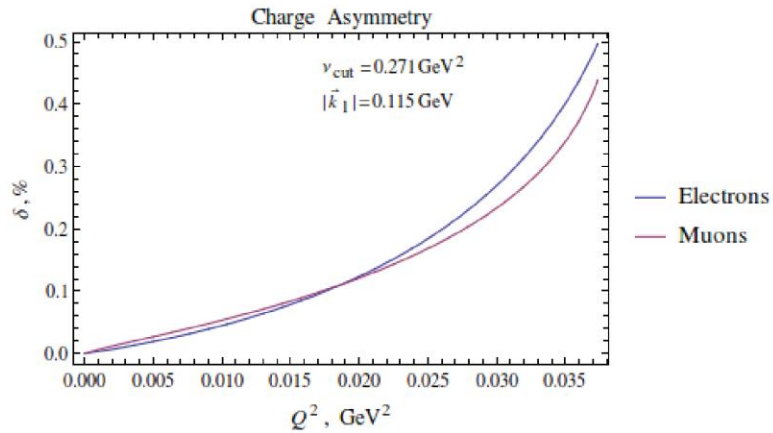
[Mo, Tsai, Rev Mod Phys 1969]

[Maximon, Tjon, Phys Rev C 2000]

Comparison:

[Gerasimov, Fadin, Phys At Nucl 2015]

# Asymmetry Comparison

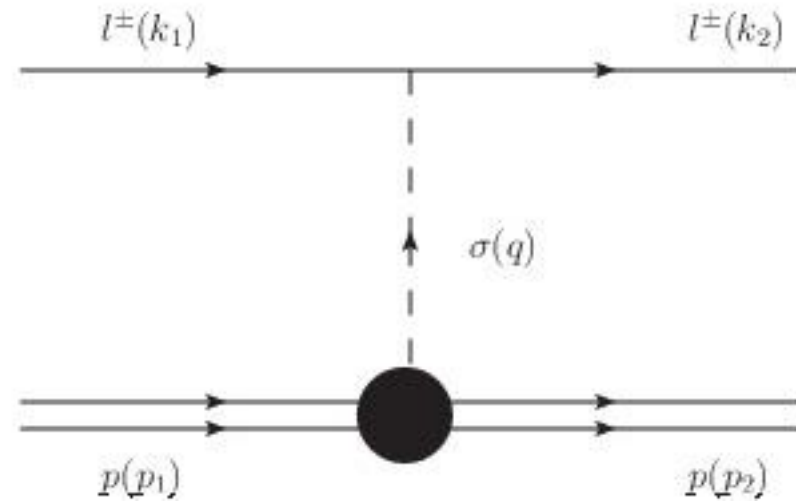
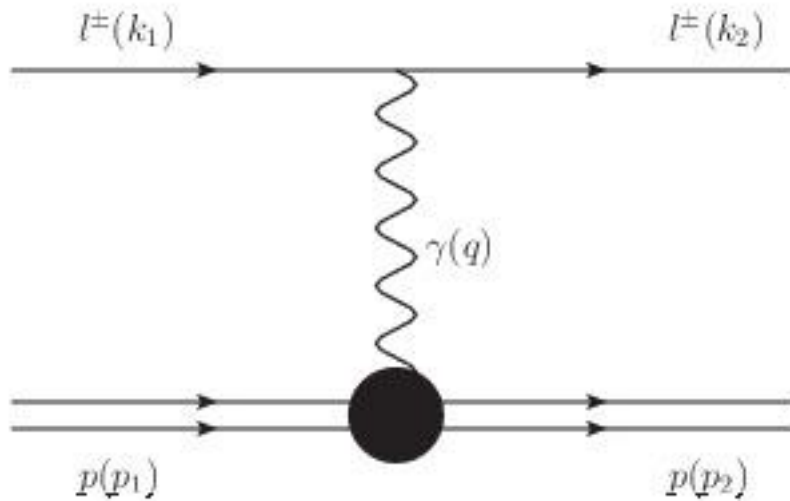


[Koshchii, Afanasev, Phys Rev D, 2017]

Extra contribution to be considered:  
helicity-flip transitions ( $\sim m_l$ )

# $\sigma$ -meson exchange in t-channel

Consider the interference between following diagrams:



$$j_\mu^\nu = \bar{u}(k_2)\gamma_\mu u(k_1)$$

$$J_\mu^\nu = \bar{U}(p_2) \left( \gamma_\mu F_1(Q^2) + \frac{i\sigma_{\mu\nu}q_\nu}{2M} F_2(Q^2) \right) U(p_1)$$

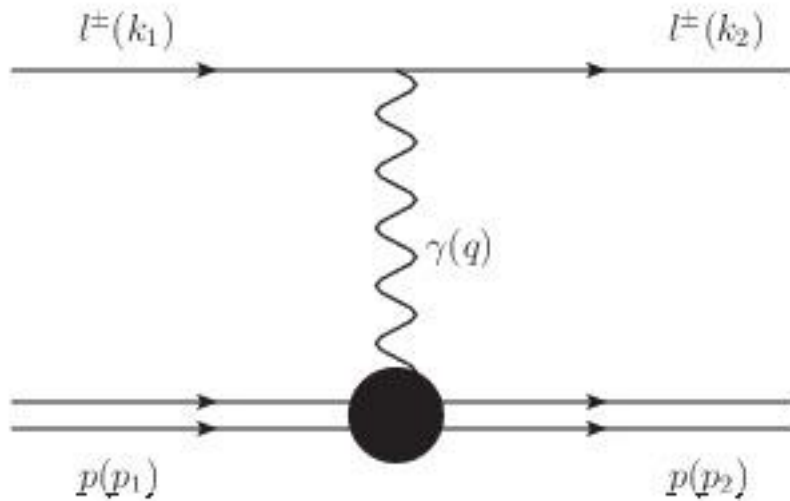
$$j_\mu^s = f_s \bar{u}(k_2)u(k_1)$$

$$J_\mu^s = \bar{U}(p_2)U(p_1)$$

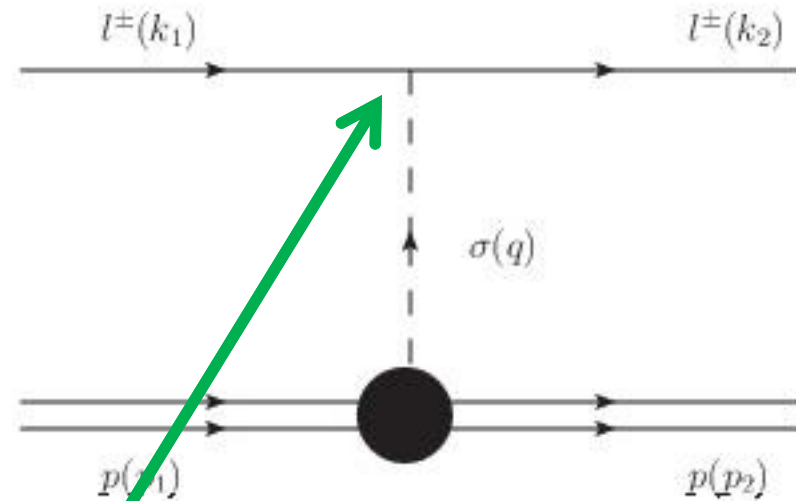


# $\sigma$ -meson exchange in t-channel

Consider the interference between following diagrams:



×



$$j_\mu^\nu = \bar{u}(k_2)\gamma_\mu u(k_1)$$

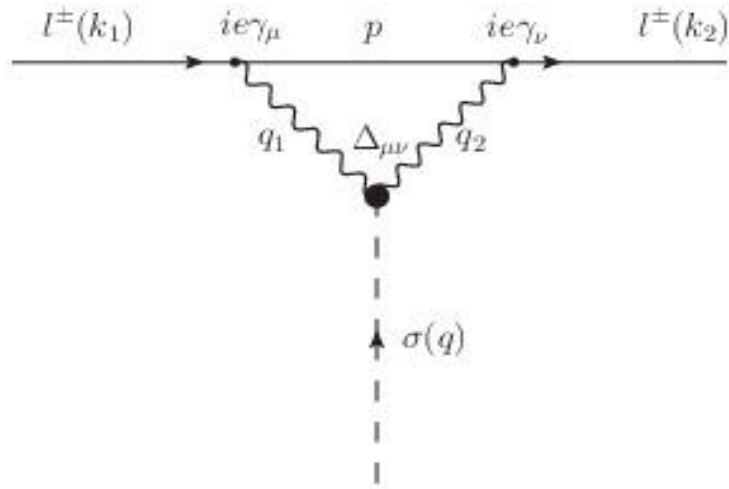
$$J_\mu^\nu = \bar{U}(p_2) \left( \gamma_\mu F_1(Q^2) + \frac{i\sigma_{\mu\nu}q_\nu}{2M} F_2(Q^2) \right) U(p_1)$$

$$j_\mu^s = f_s \bar{u}(k_2)u(k_1)$$

$$J_\mu^s = \bar{U}(p_2)U(p_1)$$

The coupling of  $\sigma$  to lepton is described via form factor  $f_s$

# Model to calculate $f_S$

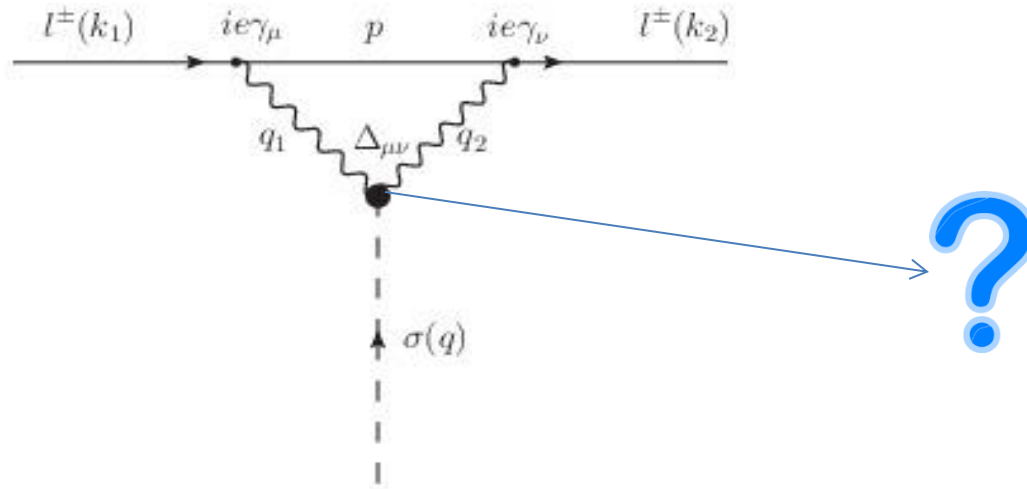


**Corresponding amplitude**

$$T = ie^4 \int \frac{d^4 p}{(2\pi)^4} \bar{u}(k_2) \frac{\gamma_\nu (\not{p} + m) \gamma_\nu}{p^2 - m^2} u(k_1) \frac{1}{q_1^2} \Delta_{\mu\nu} \frac{1}{q_2^2}$$

**Everything that is sandwiched between spinors is the form factor!**

# Model to calculate $f_S$



**Corresponding amplitude**

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**Everything that is sandwiched between spinors is the form factor!**

# Vertex description

**The most general form to describe the vertex:**

$$\Delta_{\mu\nu} = A_s(q^2; q_1^2, q_2^2) \left( g_{\mu\nu} (q_1 \cdot q_2) - q_1^\nu q_2^\mu \right) + B_s(q^2; q_1^2, q_2^2) \left( q_1^2 q_2^\mu - (q_1 \cdot q_2) q_1^\mu \right) \left( q_2^2 q_1^\nu - (q_1 \cdot q_2) q_2^\nu \right)$$

**[A.E. Dorokhov et. al. Eur. Phys. J. C (2012)]**

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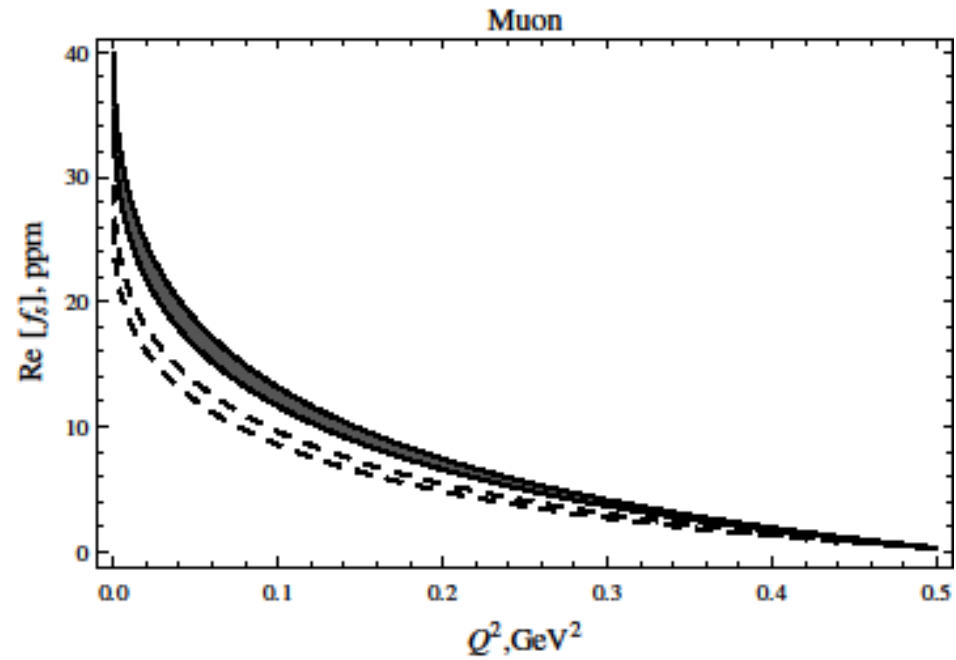
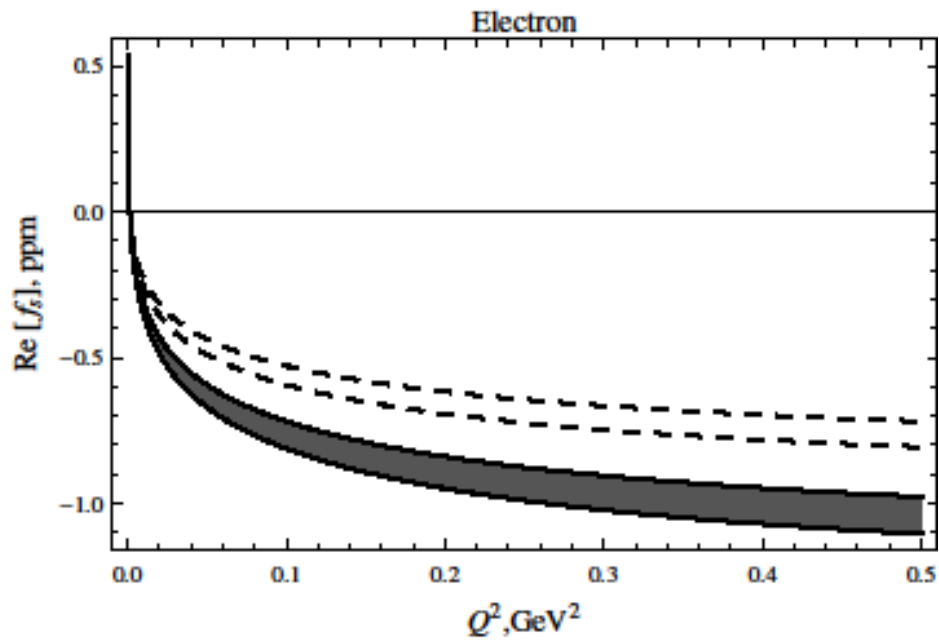
[A.E. Dorokhov et. al. Eur. Phys. J. C (2012)]

Vector meson dominance (VMD) model for transverse photons:

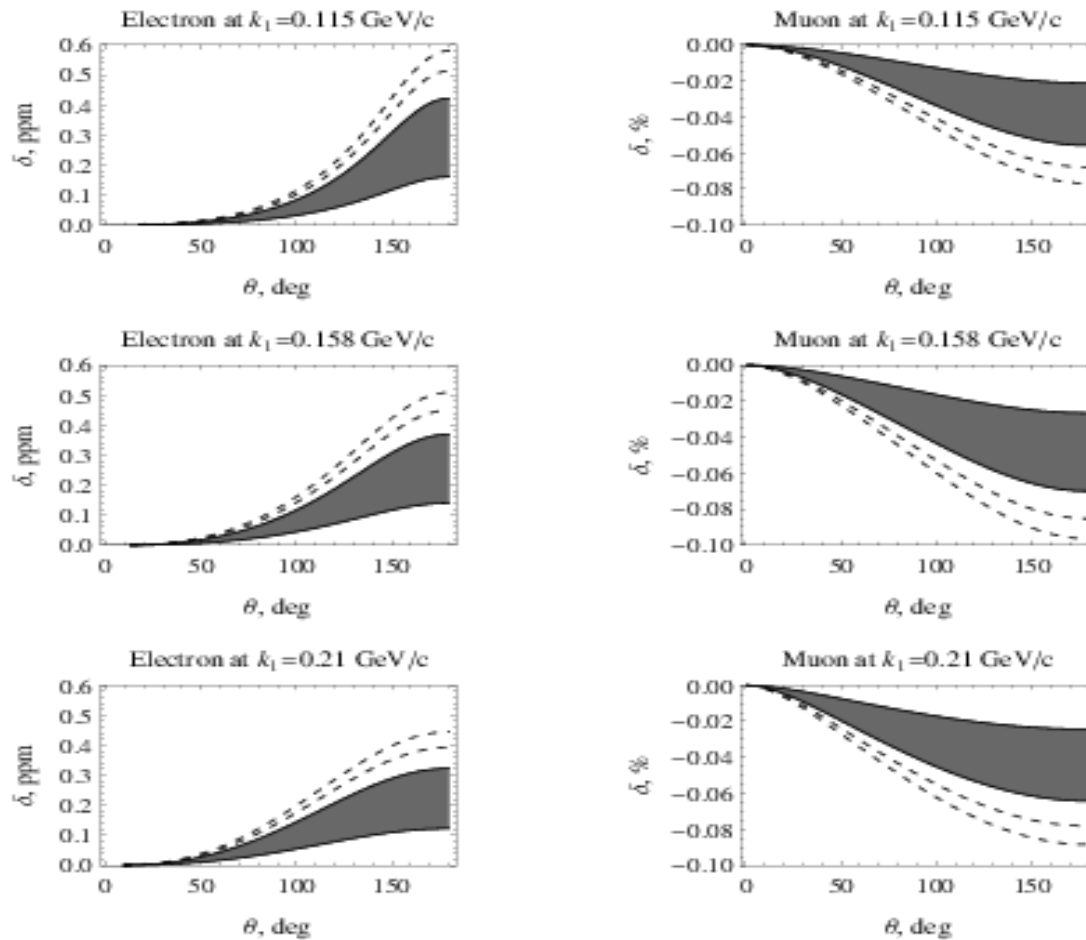
$$A_s(q^2; q_1^2, q_2^2) = \frac{g_{\sigma\gamma} m_\rho^4}{(m_\rho^2 - q_1^2)(m_\rho^2 - q_2^2)}$$

Obtained experimentally

# Results: Form Factor $f_S$



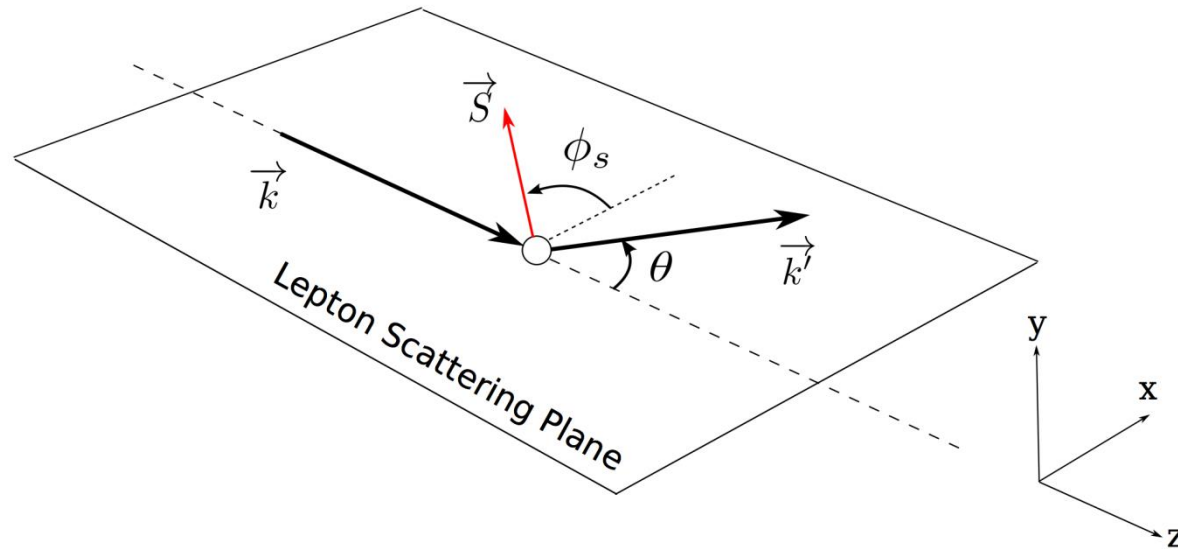
# Results: Correction $\delta$



[Koshchii, Afanasev, Phys Rev D, 2016]

# Target Normal Single Spin Asymmetry (SSA)

- Target normal SSA in elastic lepton-nucleon scattering:



$$A_n \equiv \frac{d\sigma_{\uparrow} - d\sigma_{\downarrow}}{d\sigma_{\uparrow} + d\sigma_{\downarrow}}$$



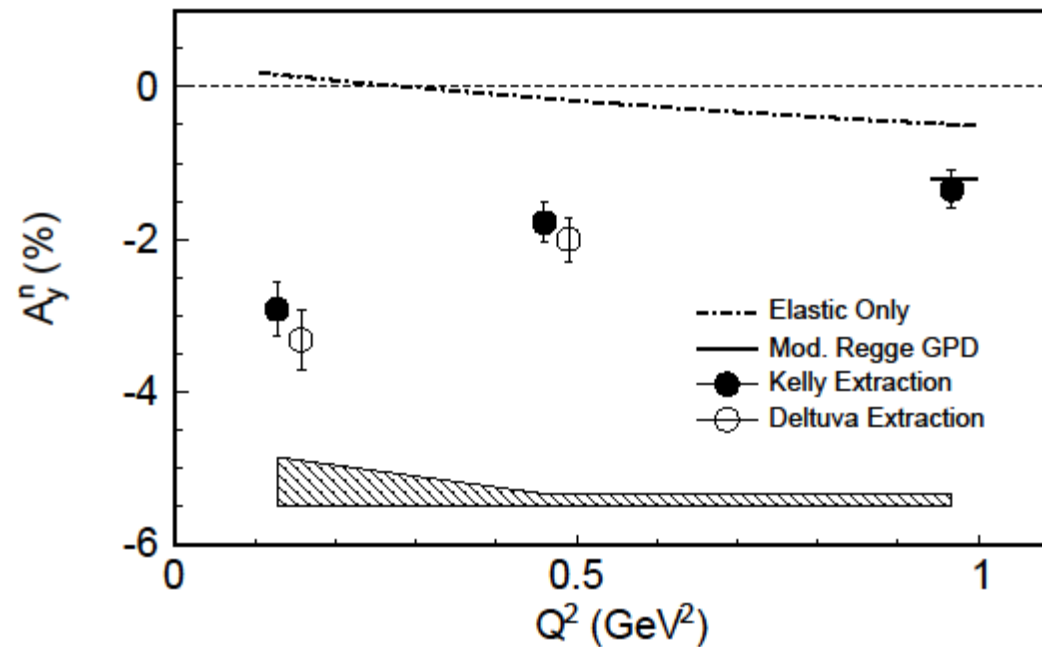
# Normal SSA: Theory

$$A_n \approx \frac{2 \operatorname{Im} \left( \sum_{spin} T_{1\gamma}^* \cdot \operatorname{Abs} [ T_{2\gamma} ] \right)}{\sum_{spin} |T_{1\gamma}|^2}$$

[de Rujula, Kaplan, Rafael, Nucl. Phys. B 1971]

- Target-Normal SSA in elastic  $l^\pm N$  scattering contribute with different signs. Corresponding sum will provide information on contributions beyond TPE
- Absorptive part of TPE amplitude can be used to obtain real part of the TPE amplitude through dispersive relations

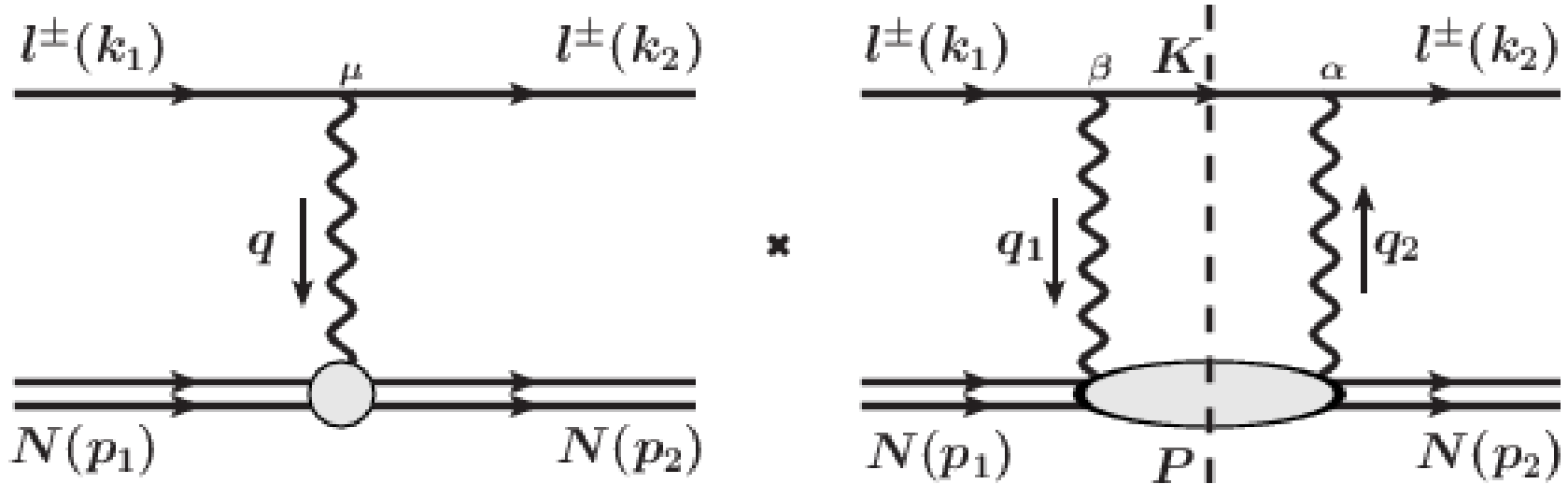
# Target-Normal SSA: Neutron



[Y.-W. Zhang, et al, Phys Rev Lett 2015]

- Polarized  $^3\text{He}$  target – effective neutron target
- Beam energies: 1.245, 2.425, 3.605 GeV

# Target-Normal SSA Calculation



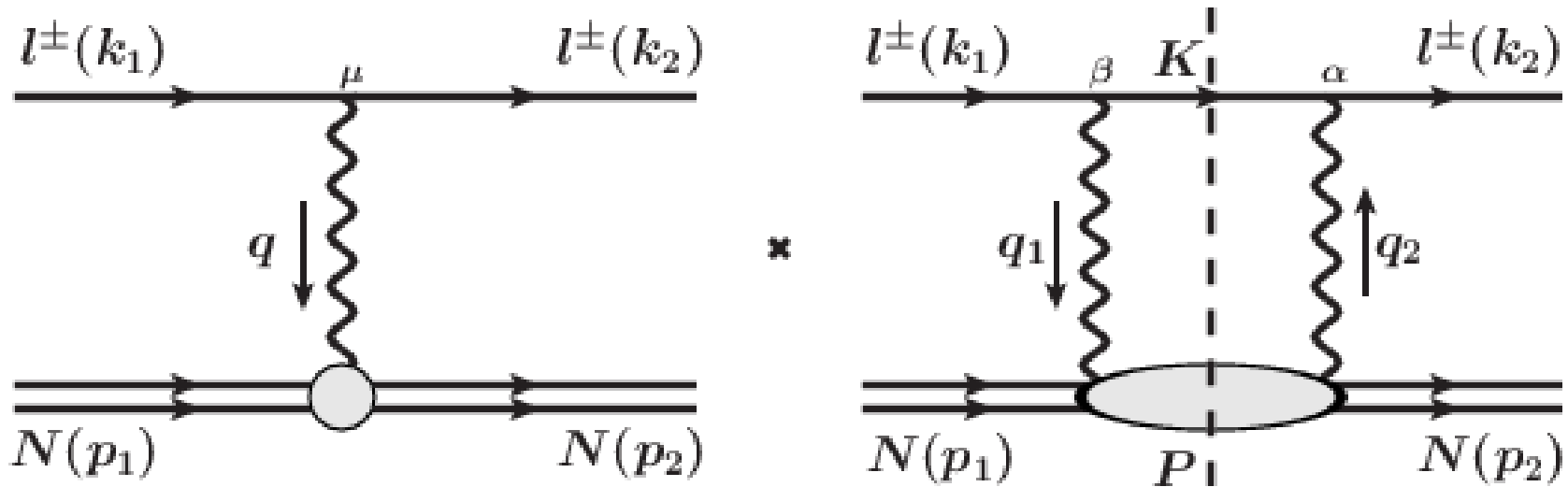
$$A_n = \frac{\alpha Q^2}{8\pi D(s, t, u)} \int \frac{d^3 K^*}{2E_K} \frac{1}{Q_1^2 Q_2^2} \text{Im} \left( \sum_{spin} L^{\mu\alpha\beta} H_{\mu\alpha\beta} \right)$$

$$\sum_{spin} H_{\mu\alpha\beta} = \frac{1}{2} \text{Tr} \left[ (p_1 + M)(1 - \gamma_5 \mathcal{S}) \Gamma_\mu (p_2 + M) W_{\alpha\beta} \right]$$

The most general form for  $W_{\alpha\beta}$ :

[R. Tarrach, IL Nuovo Cimento 1975] - 18 invariant amplitudes

# Target-Normal SSA Calculation



**Optical Theorem Approach ( $Q^2 \ll s$ ):**

[Afanasev, Merenkov, Phys Rev D 2004] Beam-normal SSA

**Electroabsorption Amplitudes ( $P \leq 2 \text{ GeV}$ ):**

[Pasquini, Vanderhaeghen, Phys Rev C 2004] Beam- and target-normal SSA

# $W_{\alpha\beta}$ parametrization

**Elastic contribution:**

$$W_{\alpha\beta} = 2\pi\delta\left((p_1 + q_1)^2 - M^2\right)\bar{U}(p_1)\Gamma_\beta(q_1)(p_1 + q_1 + M)\Gamma_\alpha(q_2)U(p_2)$$

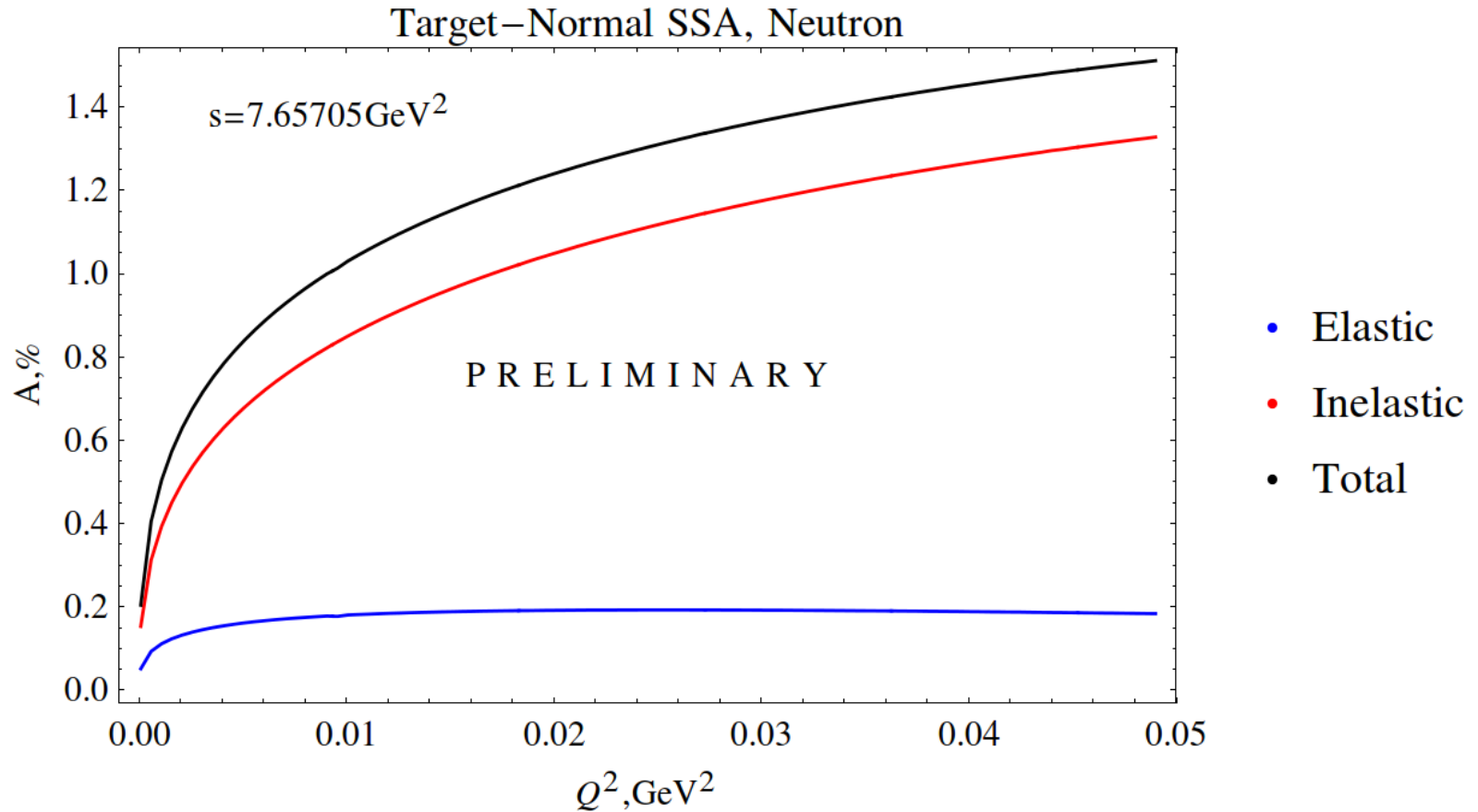
**Nearly forward inelastic parametrization:**

$$W_{\alpha\beta} = \left(\tau_1\right)_{\alpha\beta} W_1 + \left(\tau_2\right)_{\alpha\beta} W_2$$

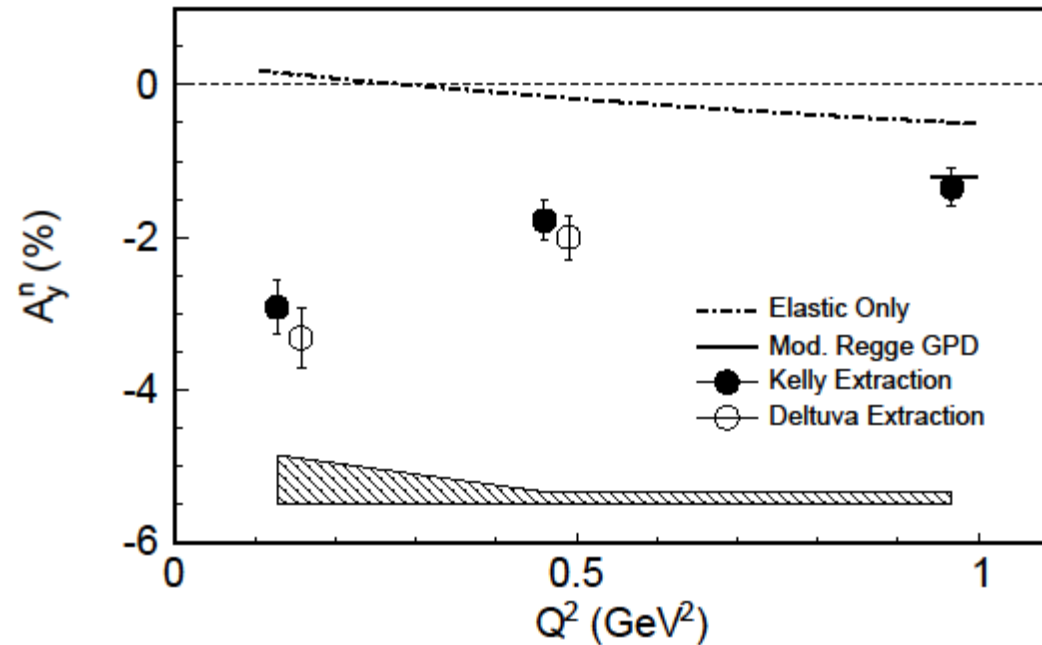
$W_1, W_2$  - unpolarized neutron structure functions

$\left(\tau_1\right)_{\alpha\beta}, \left(\tau_2\right)_{\alpha\beta}$  - [R. Tarrach, IL Nuovo Cimento 1975]

# Target-Normal SSA: Preliminary Results



# Target-Normal SSA: Neutron



[Y.-W. Zhang, et al, Phys Rev Lett 2015]

- Polarized  $^3\text{He}$  target – effective neutron target
- Beam energies: 1.245, 2.425, 3.605 GeV

# Conclusion

- Analytically generalized the existing soft-photon approximation approach of Tsai to include the lepton's mass in final expressions
- Estimated the major helicity-flip contribution in kinematics of MUSE coming from the scalar  $\sigma$ -meson exchange in the t-channel
- Calculated (preliminary) neutron-normal SSA using nearly forward Compton scattering tensor

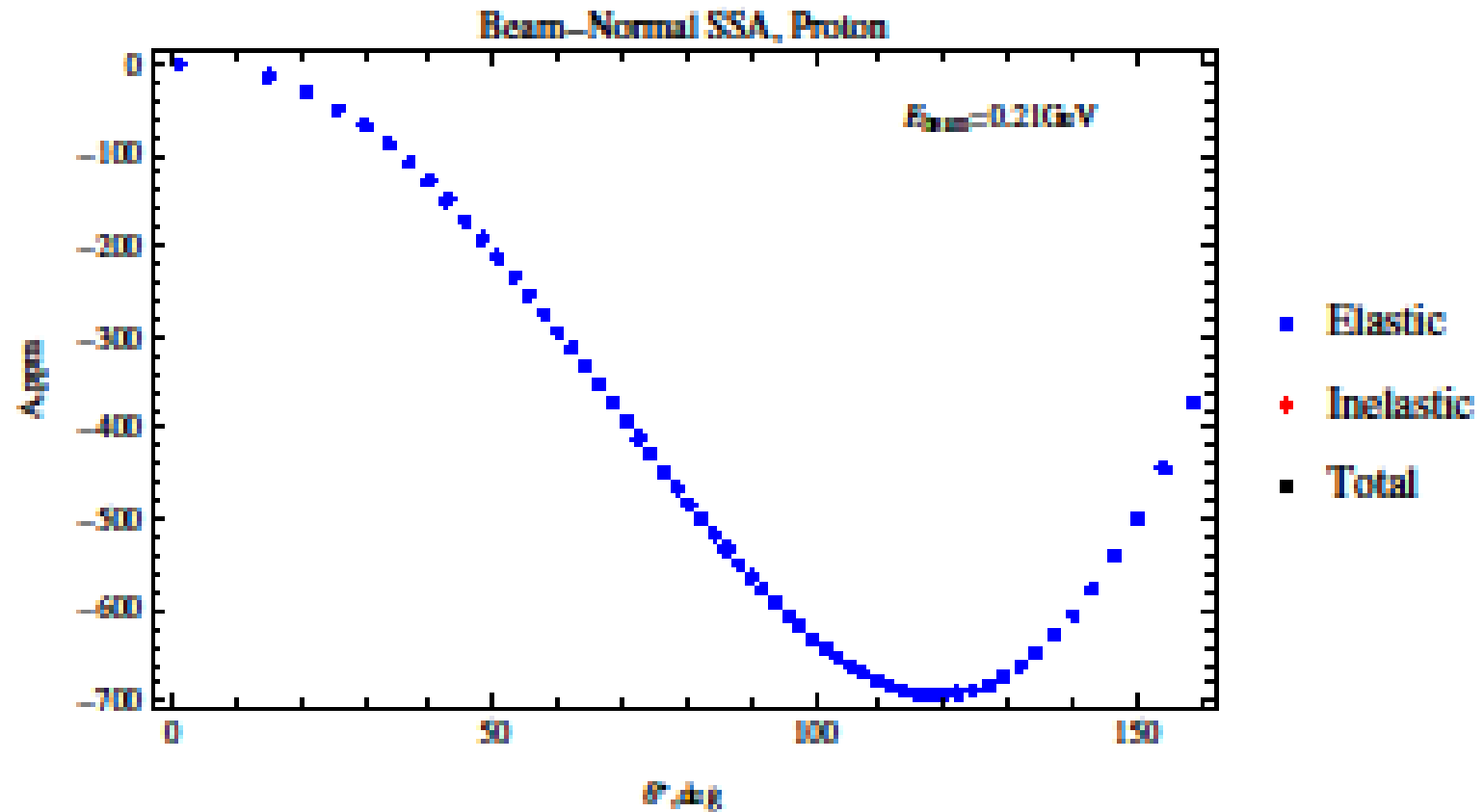


# Acknowledgement

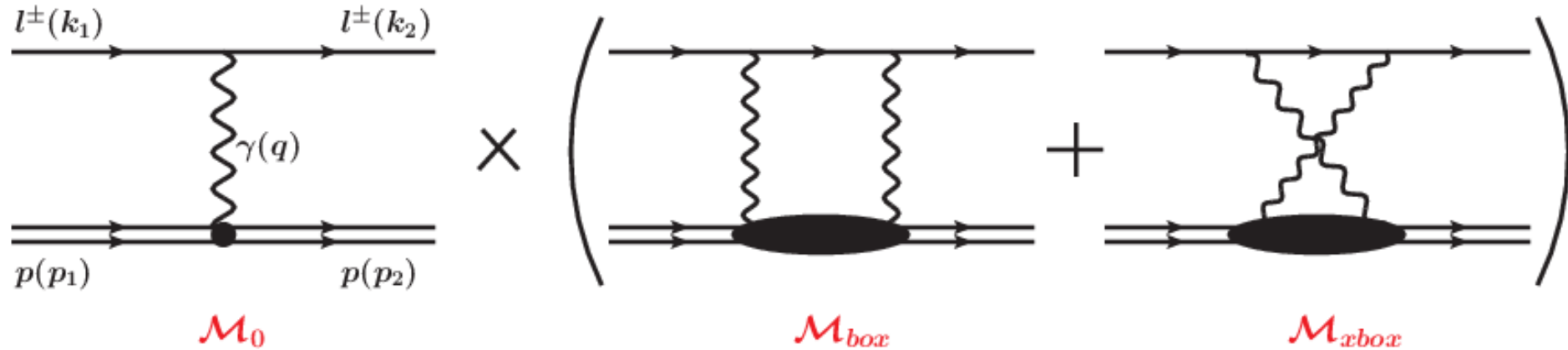
- This presentation was made possible with the support coming from a JLab/JSA fellowship award 2016/17



# Back-up



# Charge-Dependent Contributions: Calculation



**Direct loop calculation:**

[Blunden, Melnitchouk, Tjon, 2003 & 2005]

[Tomalak, Vanderhaeghen, Phys Rev D 2014]

**Dispersive treatment:**

[Gorstein, Phys Lett B 2007]

[Borisjuk, Kobushkin, Phys Rev C 2008]

[Tomalak, Vanderhaeghen, EPJ A 2015]