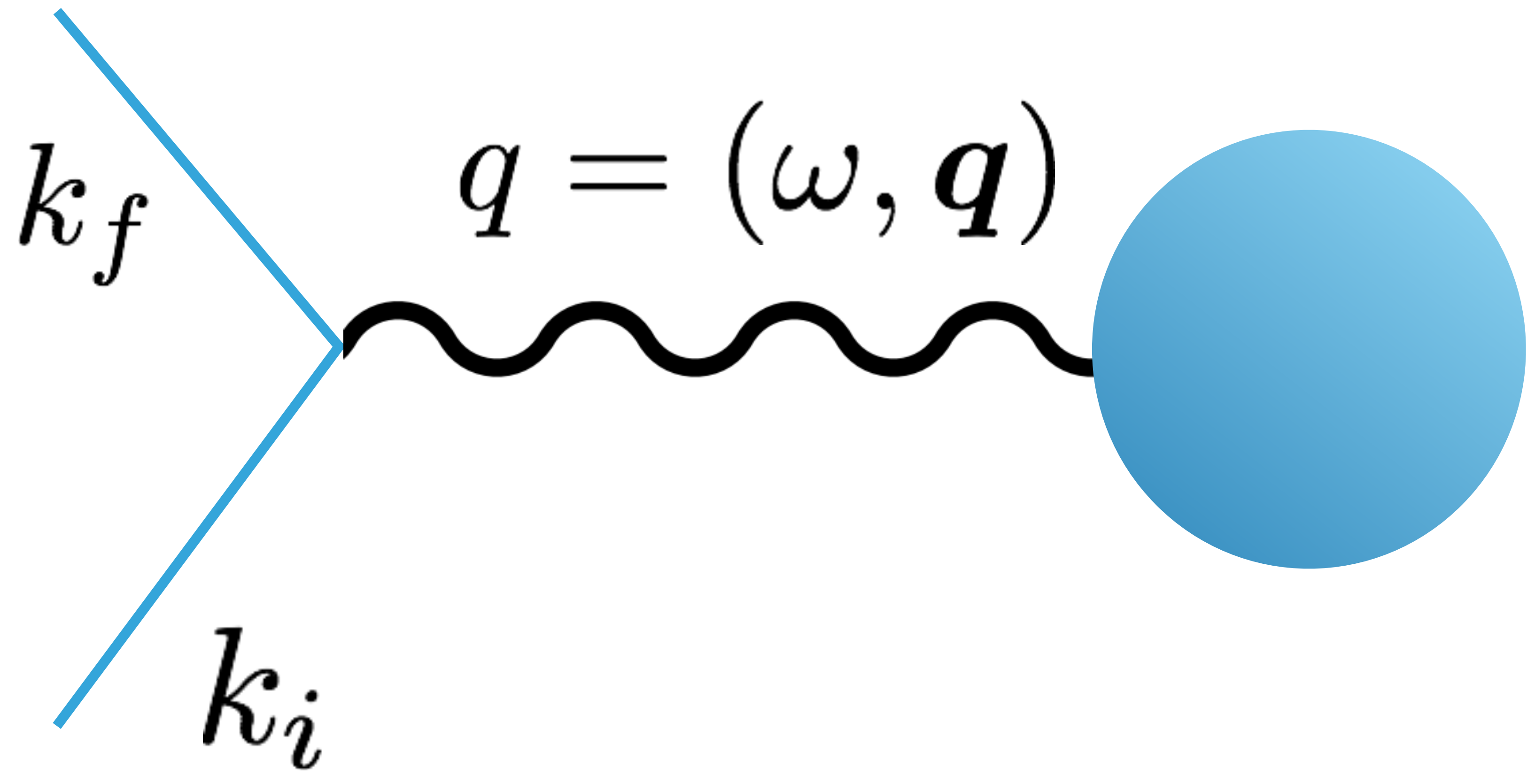
MICHAEL PAOLONE

TEMPLE UNIVERSITY

FOR THE E05-110 COLLABORATION.

MEASURING THE COULOMB SUM RULE AT JLAB

COULOMB SUM RULE



Inclusive electron scattering cross-section:

$$\frac{d^2\sigma}{d\Omega d\omega} = \sigma_{\text{Mott}} \left[\frac{q^4}{|\boldsymbol{q}|^4} R_L(\omega, |\boldsymbol{q}|) + \left(\frac{q^2}{2|\boldsymbol{q}|^2} + \tan^2 \frac{\theta}{2} \right) R_T(\omega, |\boldsymbol{q}|) \right]$$

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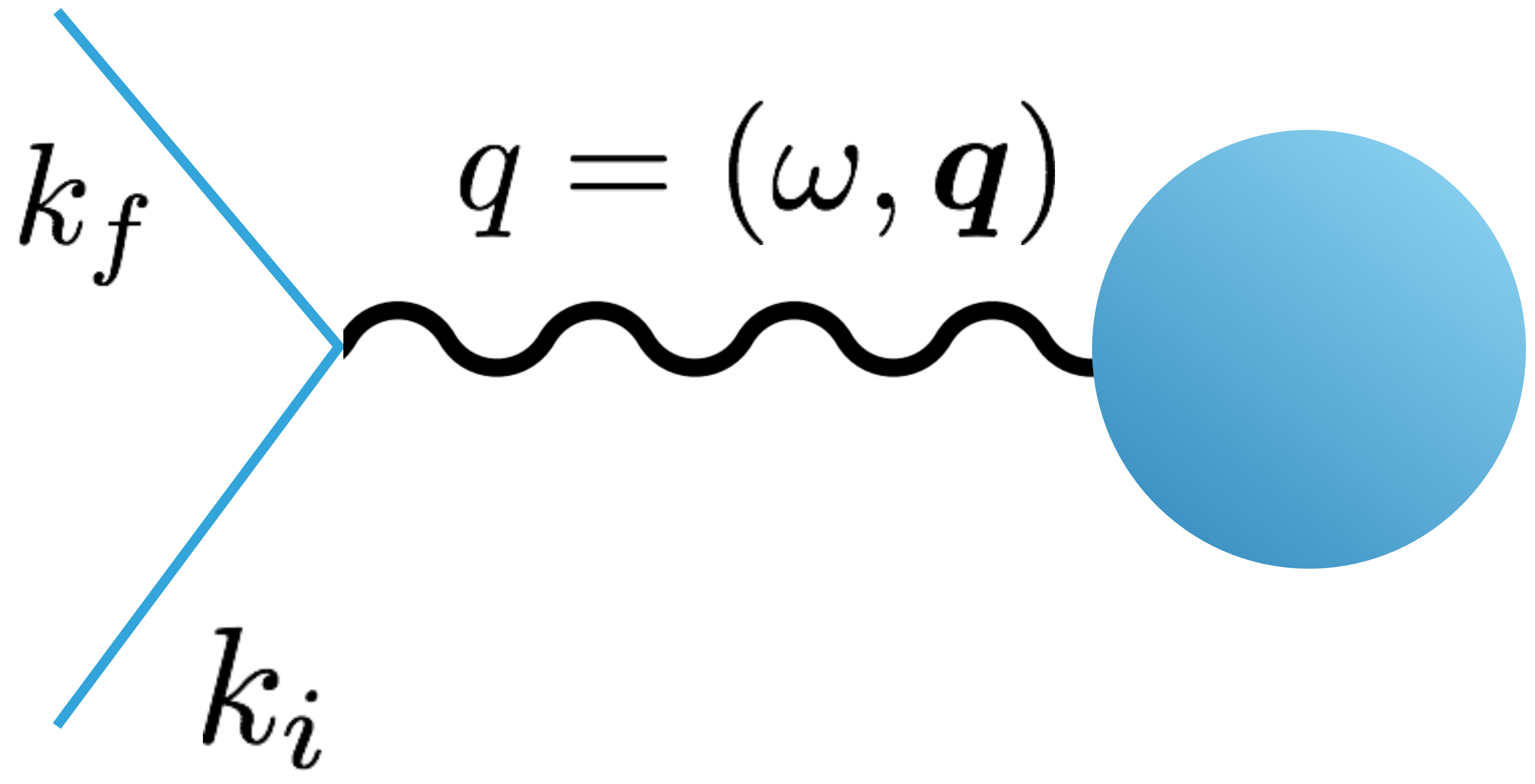

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Scattering response due to **charge** properties

Scattering response due to **magnetic** properties

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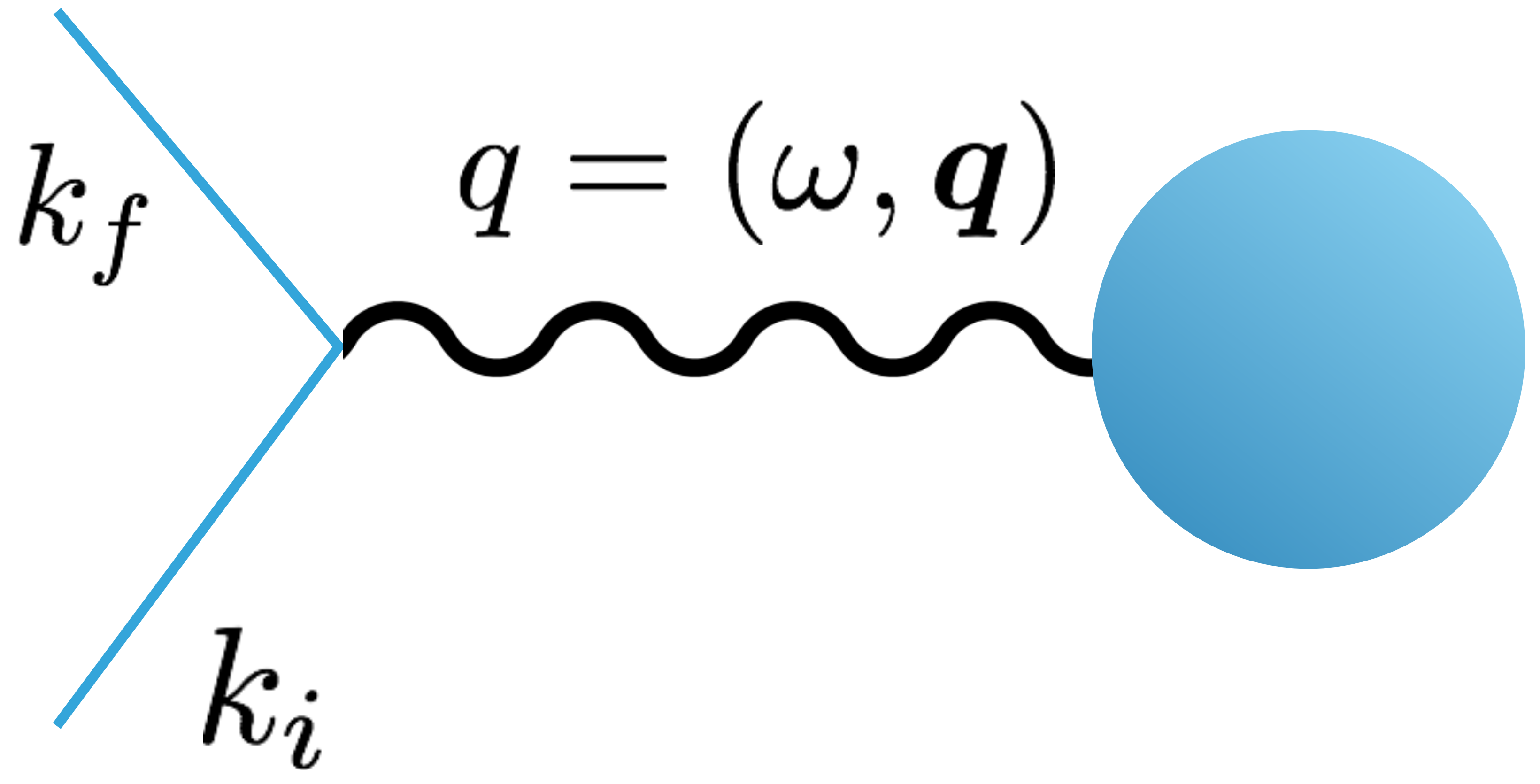

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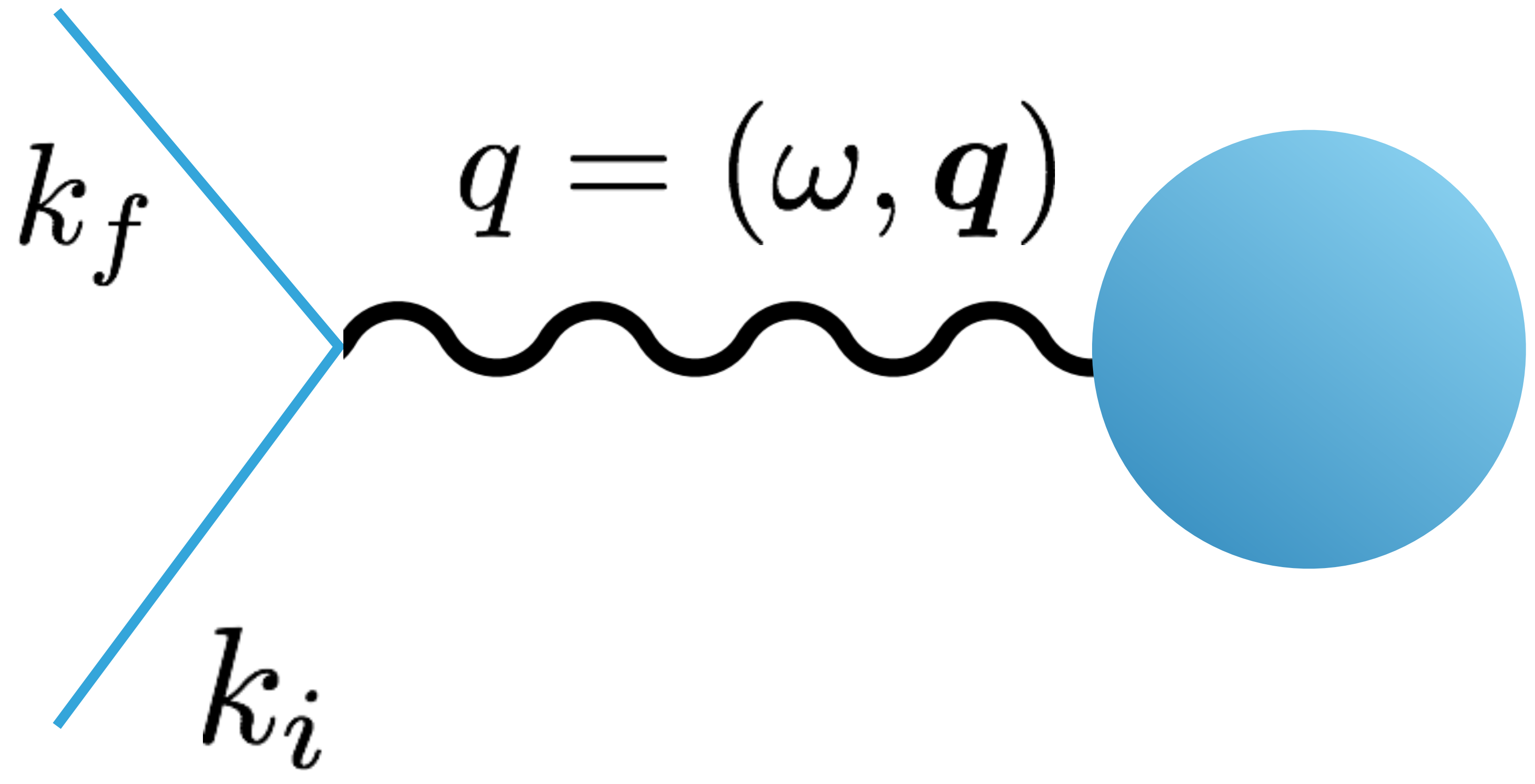
Coulomb Sum Rule definition:

$$S_L(|\boldsymbol{q}|) = \int_{\omega^+}^{|\boldsymbol{q}|} d\omega \frac{R_L(\omega, |\boldsymbol{q}|)}{Z\tilde{G}_{Ep}^2(Q^2) + N\tilde{G}_{En}^2(Q^2)}$$

If one integrates the charge response divided by the total charge form factor over all available virtual photon energies, naively one might expect the integral to go to unity.

COULOMB SUM RULE



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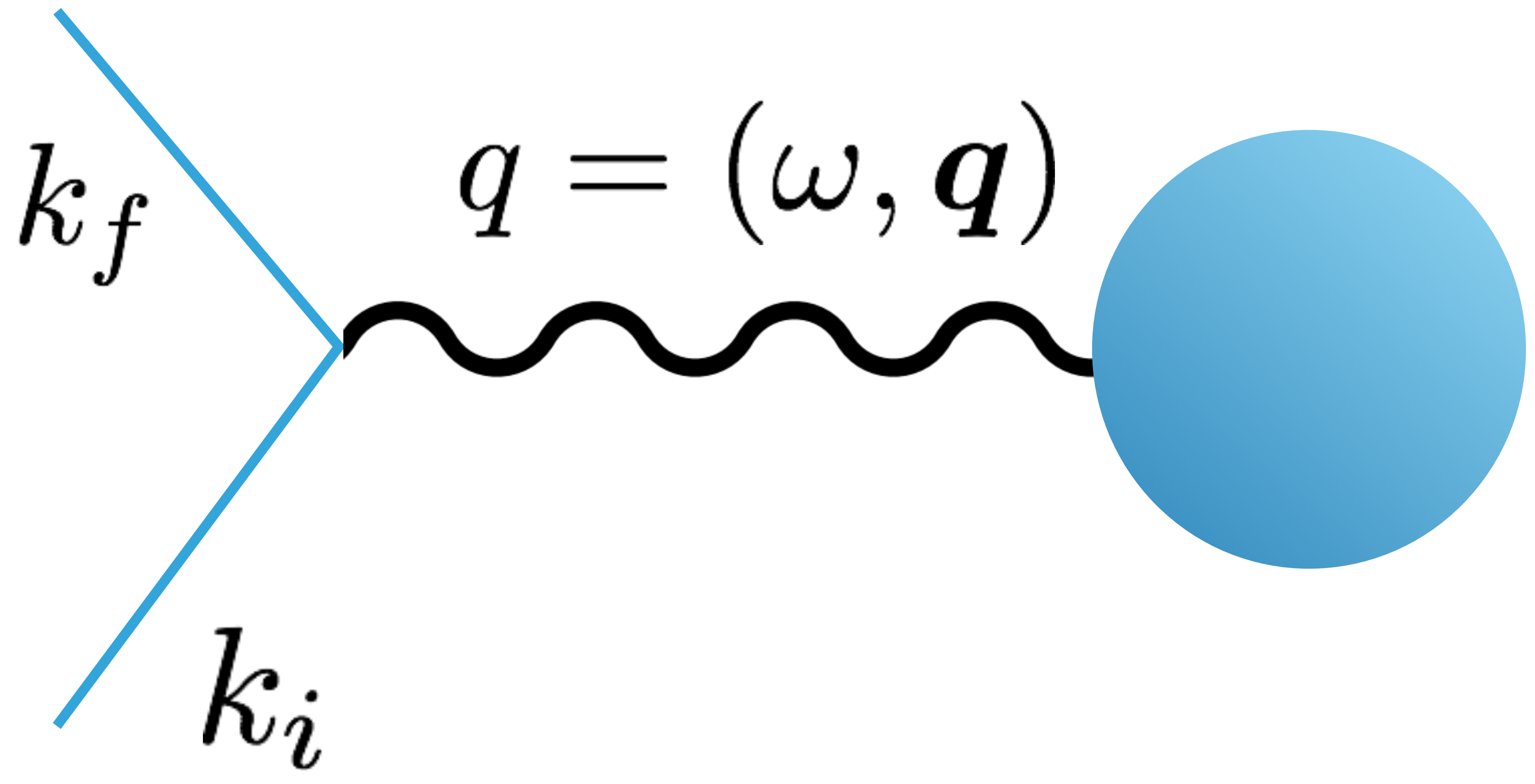

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At large |q| >> 2k_f , S_L should go to 1. Any significant* deviation from this would be an indication of relativistic or medium effects distorting the nucleon form factor!

*Short range correlations will also quench S_L, but only by < 10%

COULOMB SUM RULE

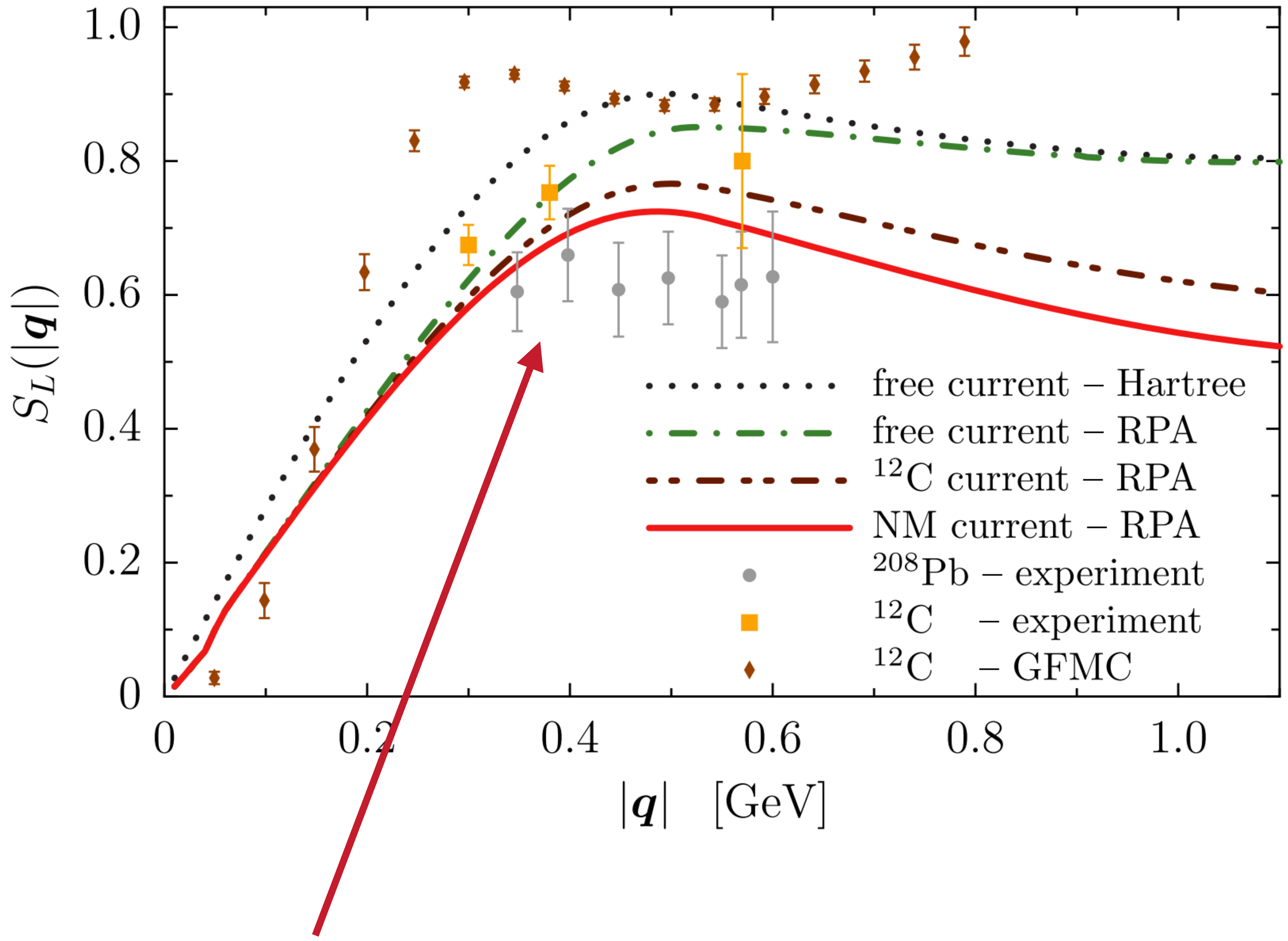
- Long standing issue with many years of theoretical interest.
- Even most state-of the-art models cannot predict existing data.
- New precise data at larger |q| would provide crucial insight and constraints to modern calculations.

$$S_L(|\boldsymbol{q}|) = \int_{\omega^+}^{|\boldsymbol{q}|} d\omega \frac{R_L(\omega, |\boldsymbol{q}|)}{Z\tilde{G}_{Ep}^2(Q^2) + N\tilde{G}_{En}^2(Q^2)}$$

PRL 116, 032701 (2016) PHYSICAL REVIEW LETTERS week ending 22 JANUARY 2016

Relativistic and Nuclear Medium Effects on the Coulomb Sum Rule

Ian C. Cloët,[1] Wolfgang Bentz,[2] and Anthony W. Thomas[3]
[1]Physics Division, Argonne National Laboratory, Argonne, Illinois 60439, USA
[2]Department of Physics, School of Science, Tokai University, Hiratsuka-shi, Kanagawa 259-1292, Japan
[3]CSSM and ARC Centre of Excellence for Particle Physics at the Terascale, Department of Physics, University of Adelaide, Adelaide South Australia 5005, Australia
(Received 23 June 2015; published 19 January 2016)



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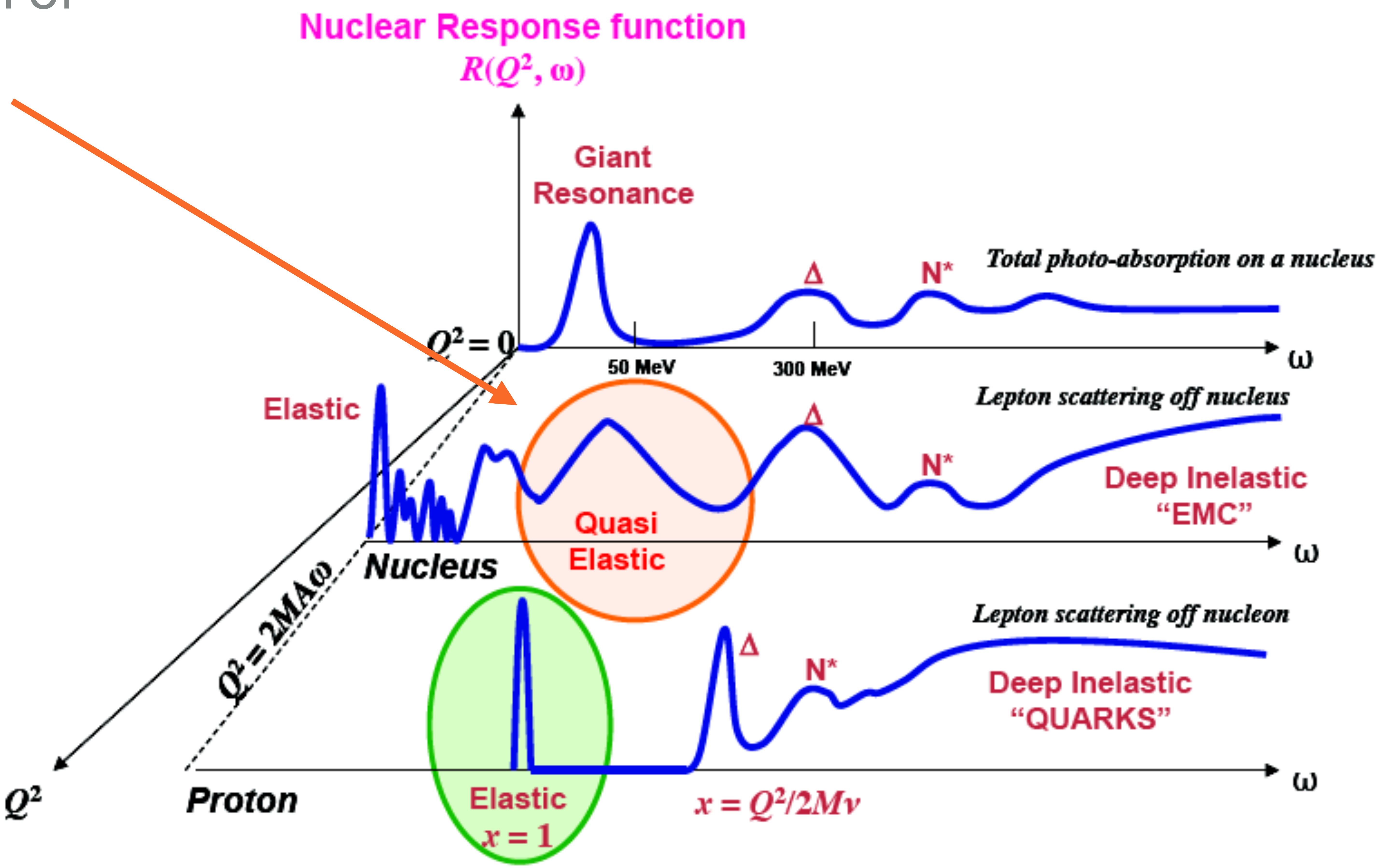
QUASI-ELASTIC SCATTERING

$$S_L(|\boldsymbol{q}|) = \int_{\omega^+}^{|\boldsymbol{q}|} d\omega \frac{R_L(\omega, |\boldsymbol{q}|)}{Z\tilde{G}^2_{Ep}(Q^2) + N\tilde{G}^2_{En}(Q^2)}$$

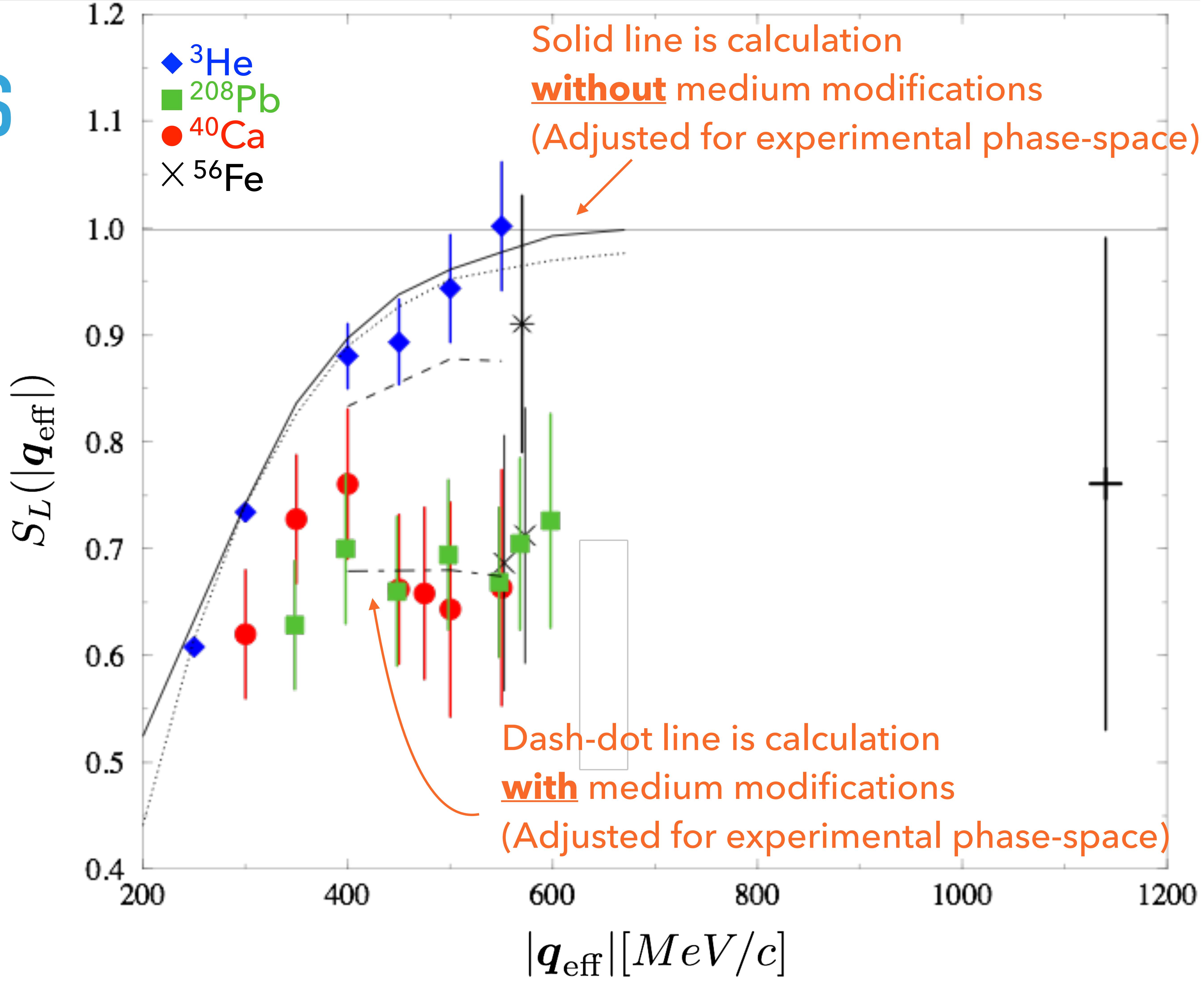
We want to integrate above the coherent elastic peak:
Quasi-elastic is "elastic" scattering on constituent nucleons inside nucleus.

- Quasi-elastic scattering at intermediate Q^2 is the region of interest for our experiment:
  - Nuclei investigated:
    - ^{4}He
    - ^{12}C
    - ^{56}Fe
    - ^{208}Pb



PUBLISHED EXPERIMENTAL RESULTS

- First group of experiments from Saclay, Bates, and SLAC show a quenching of S_L consistent with medium modified form-factors.



$|\mathbf{q}_{\rm eff}|$ is $|\mathbf{q}|$ corrected for a nuclei dependent mean coulomb potential. Methodology agreed on by Andreas Aste, Steve Wallace and John Tjon.

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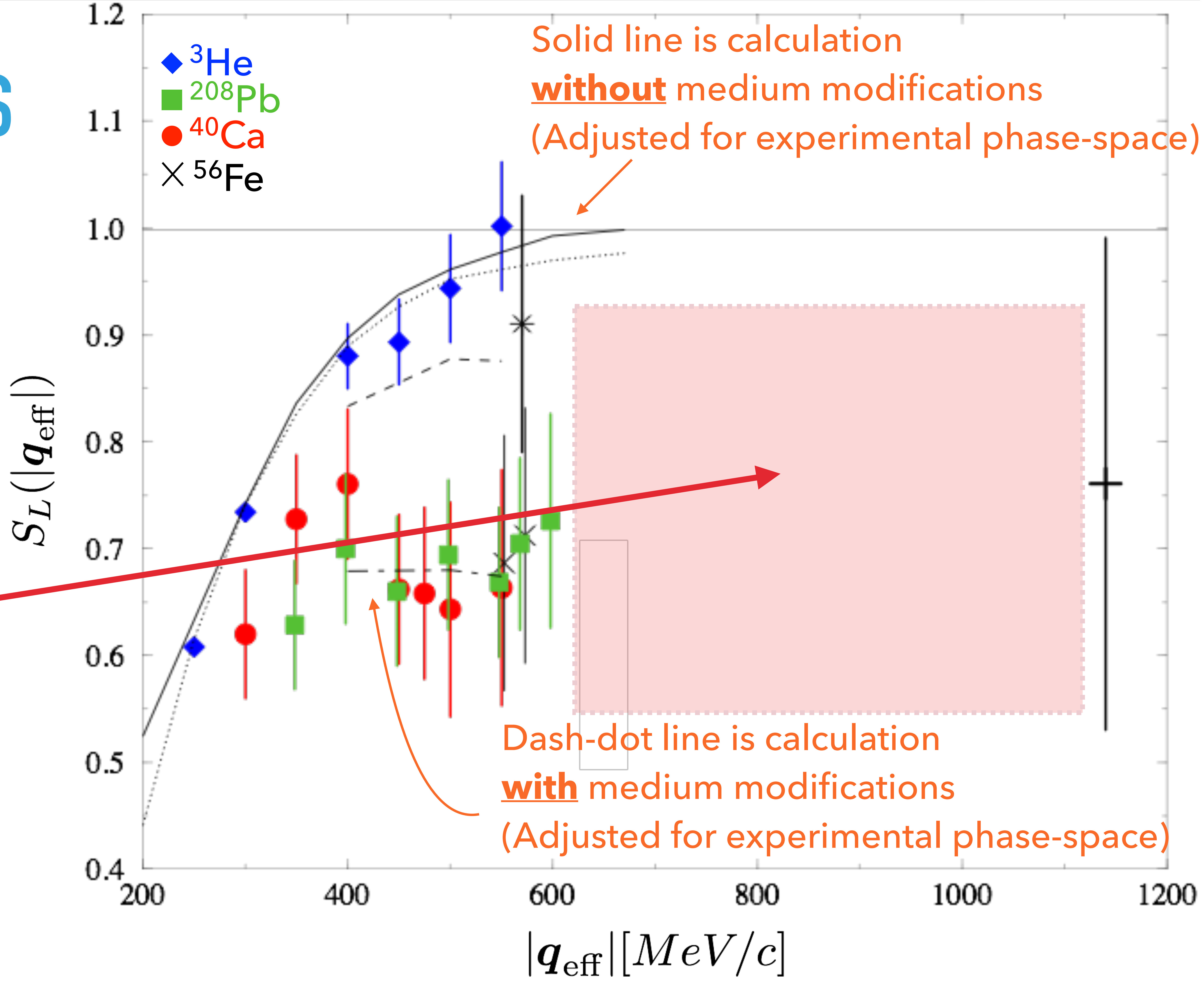
- Very little data above |**q**| of 600 MeV/c, where the cleanest signal of medium effects should exist!
    - Sarclay, Bates limited in beam energy reach up to 800 MeV.
    - SLAC limited in kinematic coverage of scattered electron at |q| below 1150 MeV/c.



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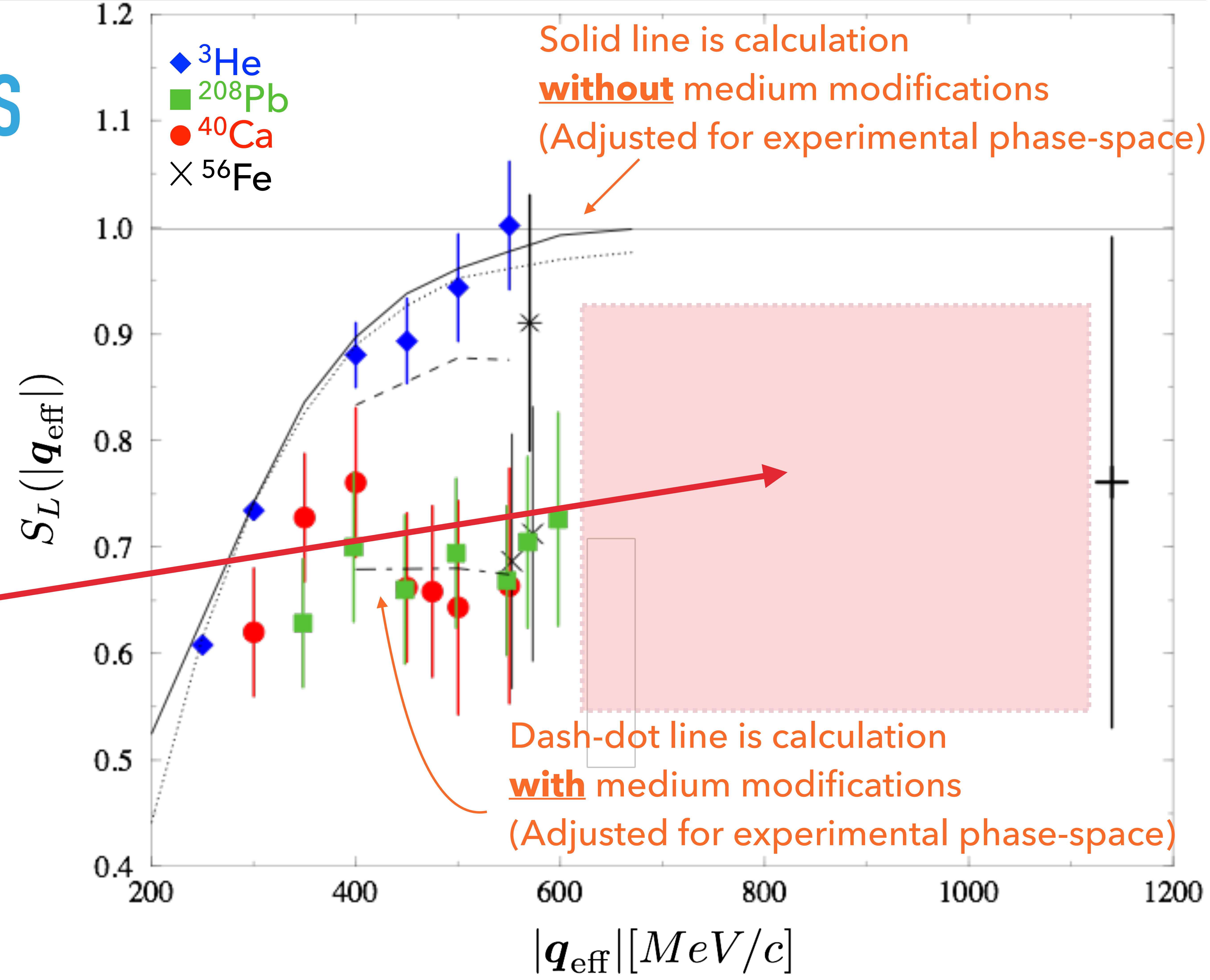

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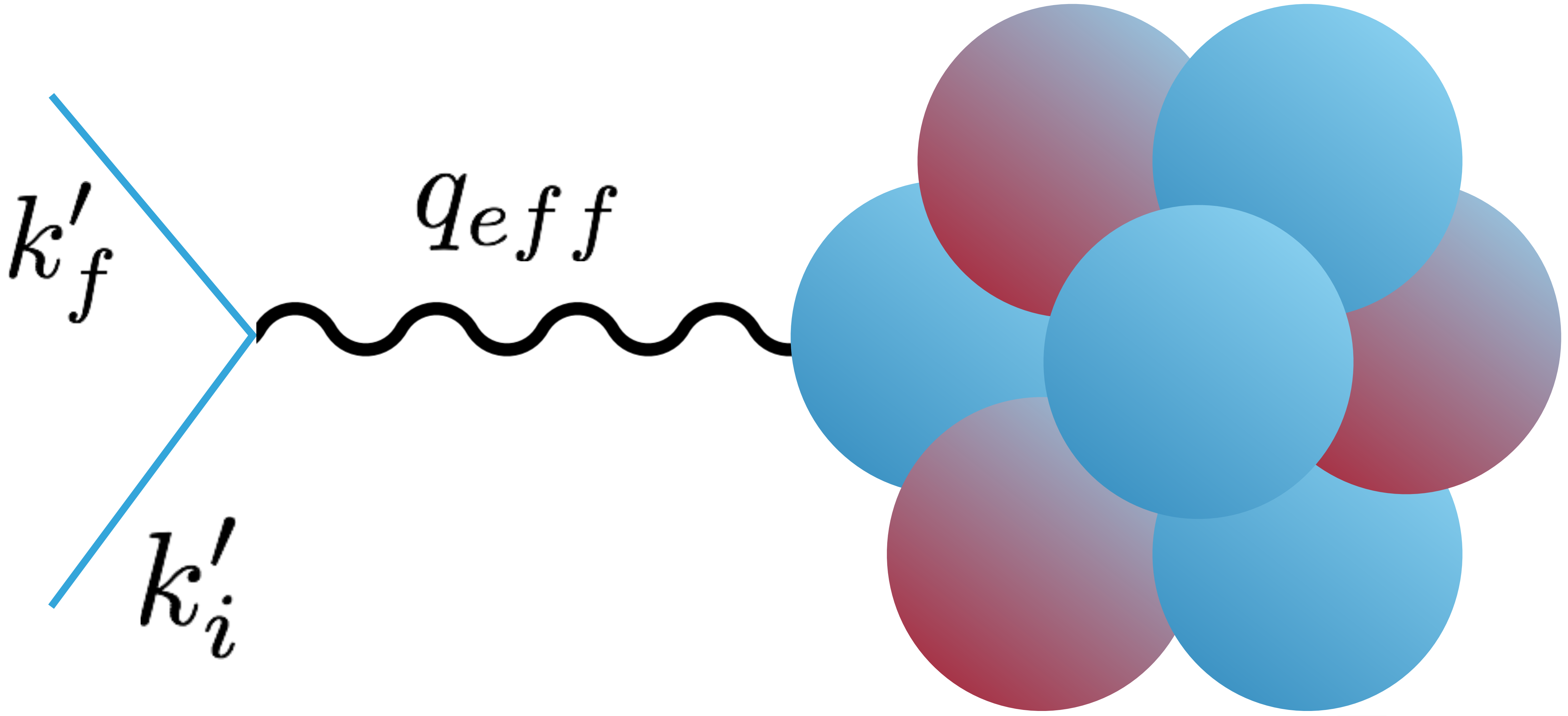
MEAN COULOMB POTENTIAL, EMA, AND POSITRON SCATTERING

- An effective momentum approximation (EMA) takes into account the mean field potential of the target nucleus during quasi-elastic scattering.



$$V_0 = \frac{3\alpha Z}{2r_c}$$

$$k'_i = k_i - \kappa_A \frac{V_0}{c} \qquad k'_f = k_f - \kappa_A \frac{V_0}{c}$$

$$\omega' = (k'_i - k'_f) = (k_i - k_f) = \omega$$

$$Q'^2 = 4(k'_i)(k'_f)\sin^2\theta/2$$

$$q_{eff} = \sqrt{\omega^2 + Q'^2}$$

Nucleus	V_0 (MeV)
^{12}C	3.46 +/- 0.11
^{56}Fe	9.80 +/- 0.32
^{208}Pb	20.57 +/- 0.66

A. Aste, C. von Arx, and D. Trautmann, EPJ A26 167 (2005)

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~ 0.75 to 0.8

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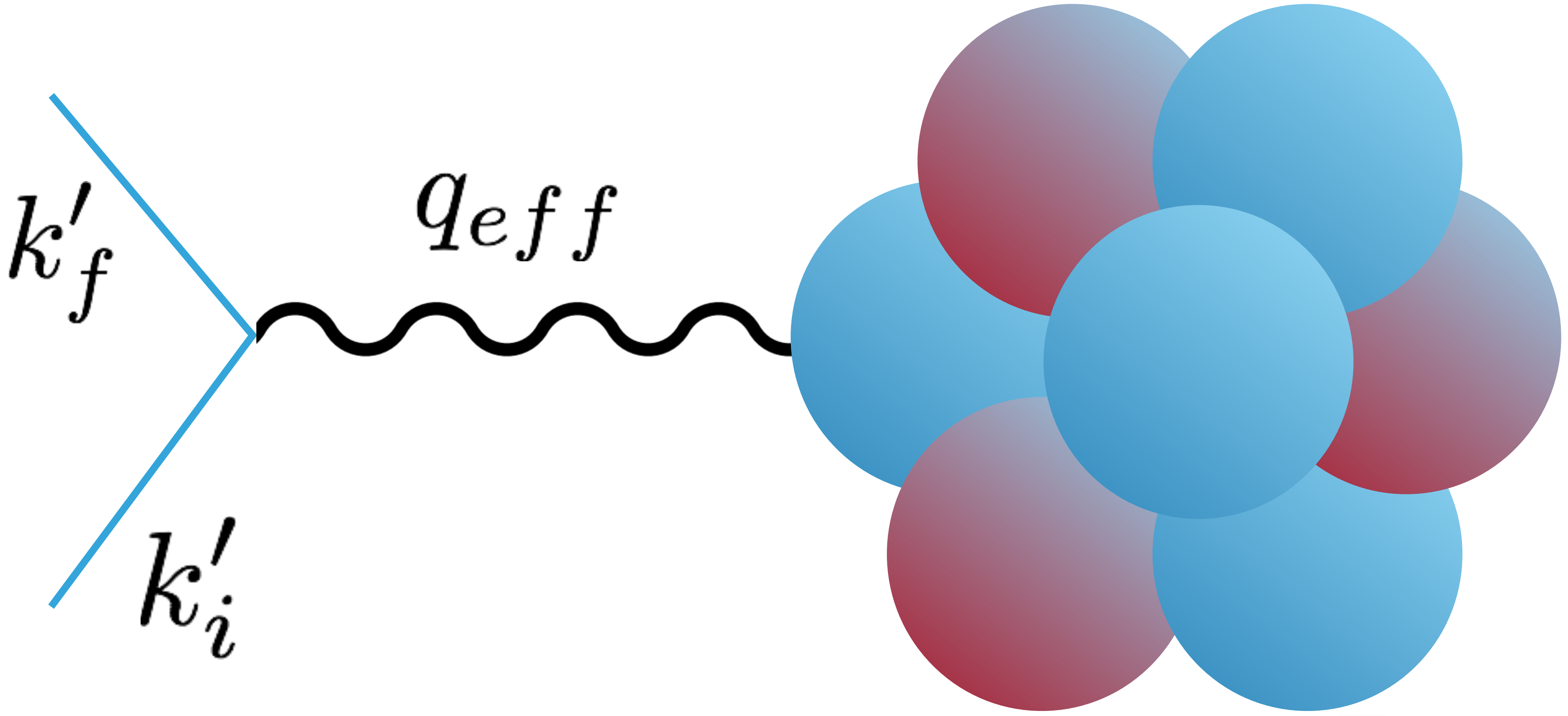

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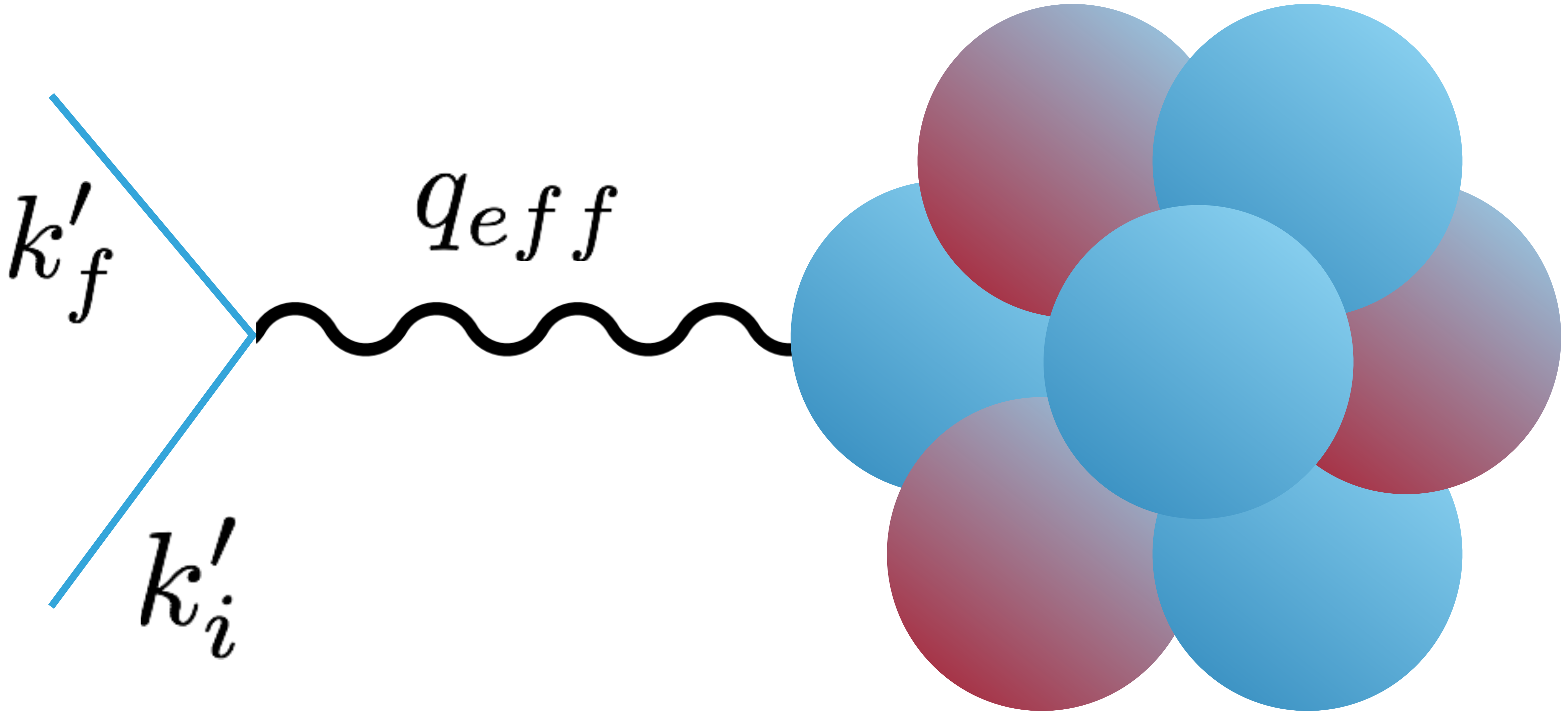

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When scattering with positrons, we effectively change the sign of the mean potential

A. Aste, C. von Arx, and D. Trautmann, EPJ A26 167 (2005)

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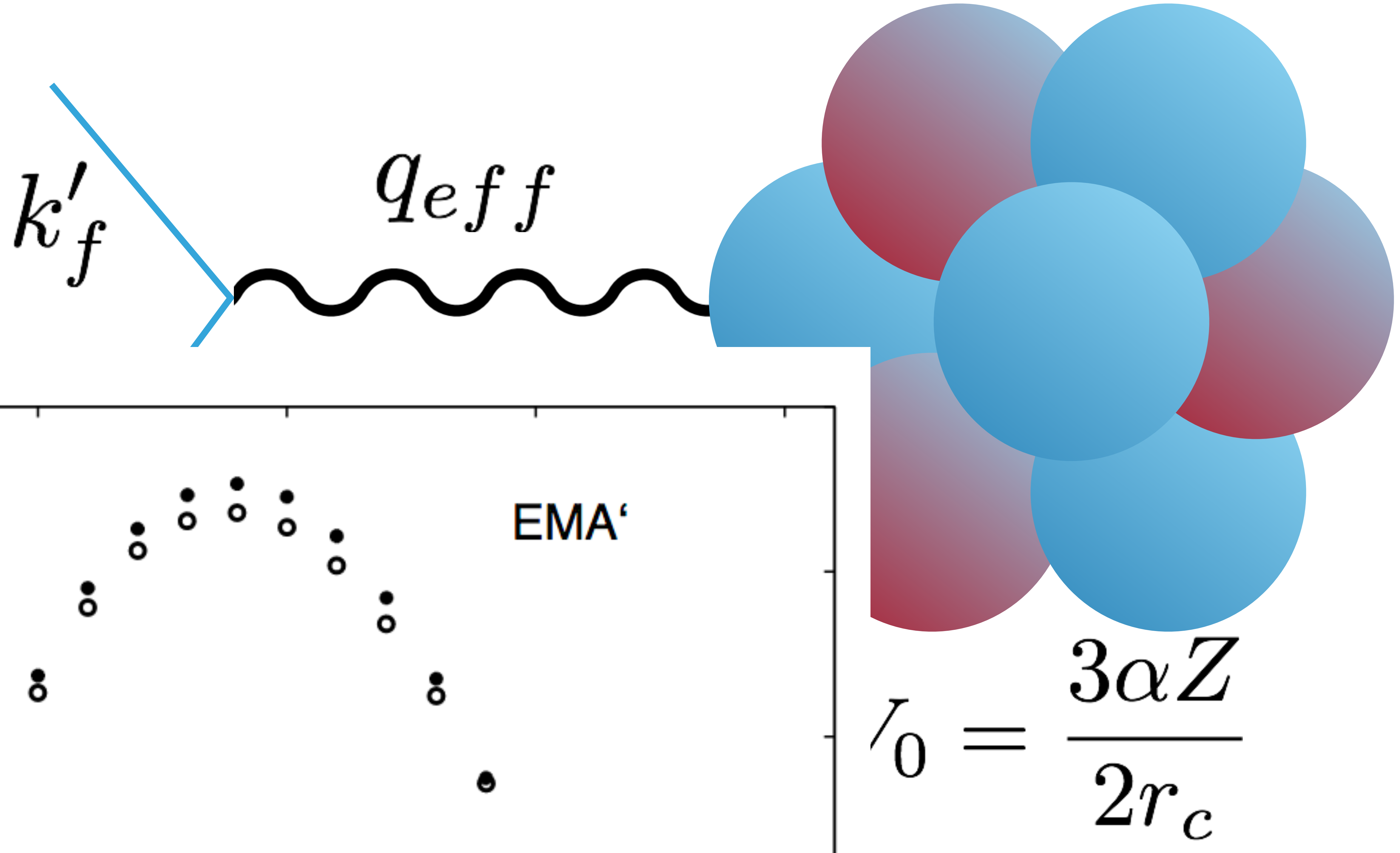
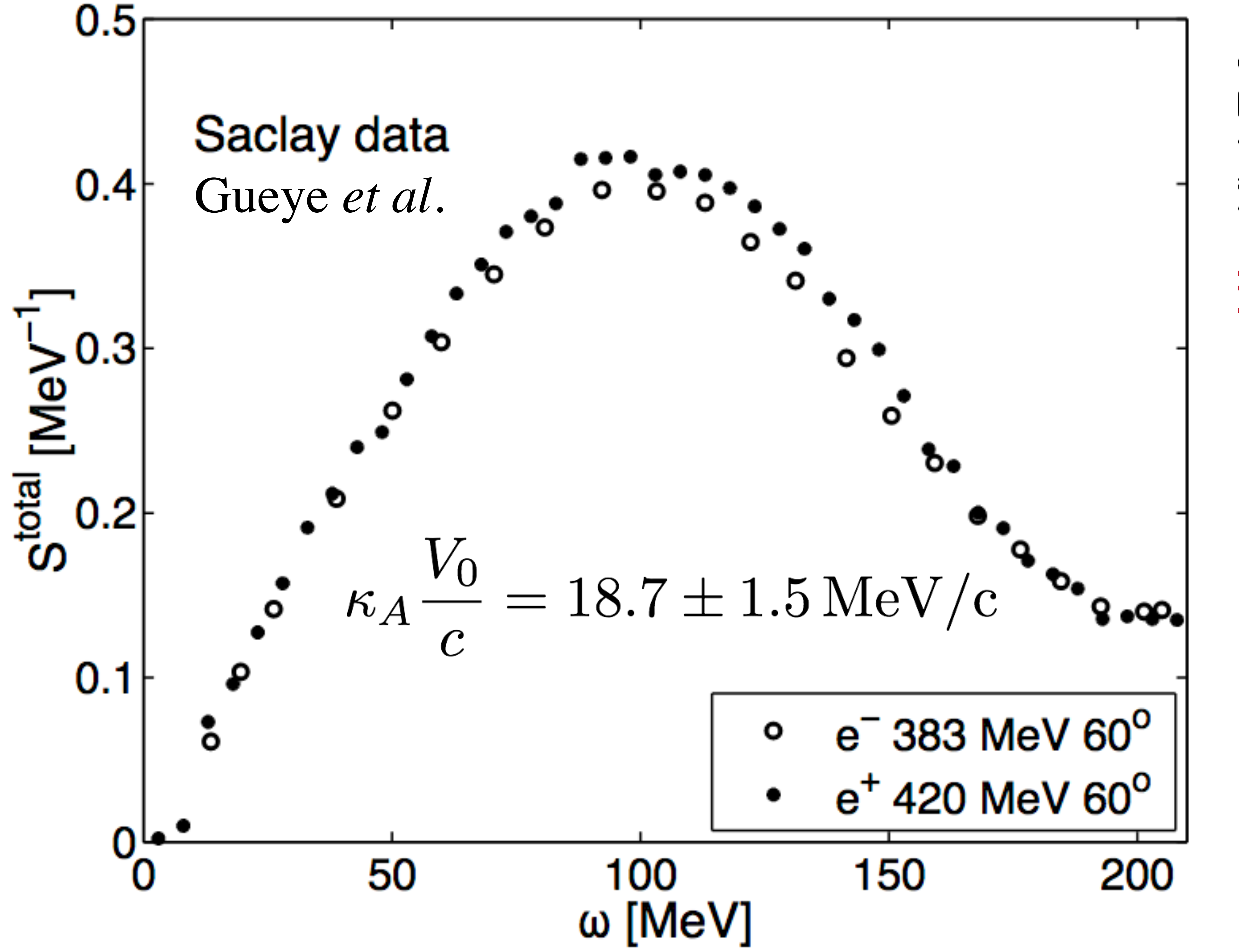
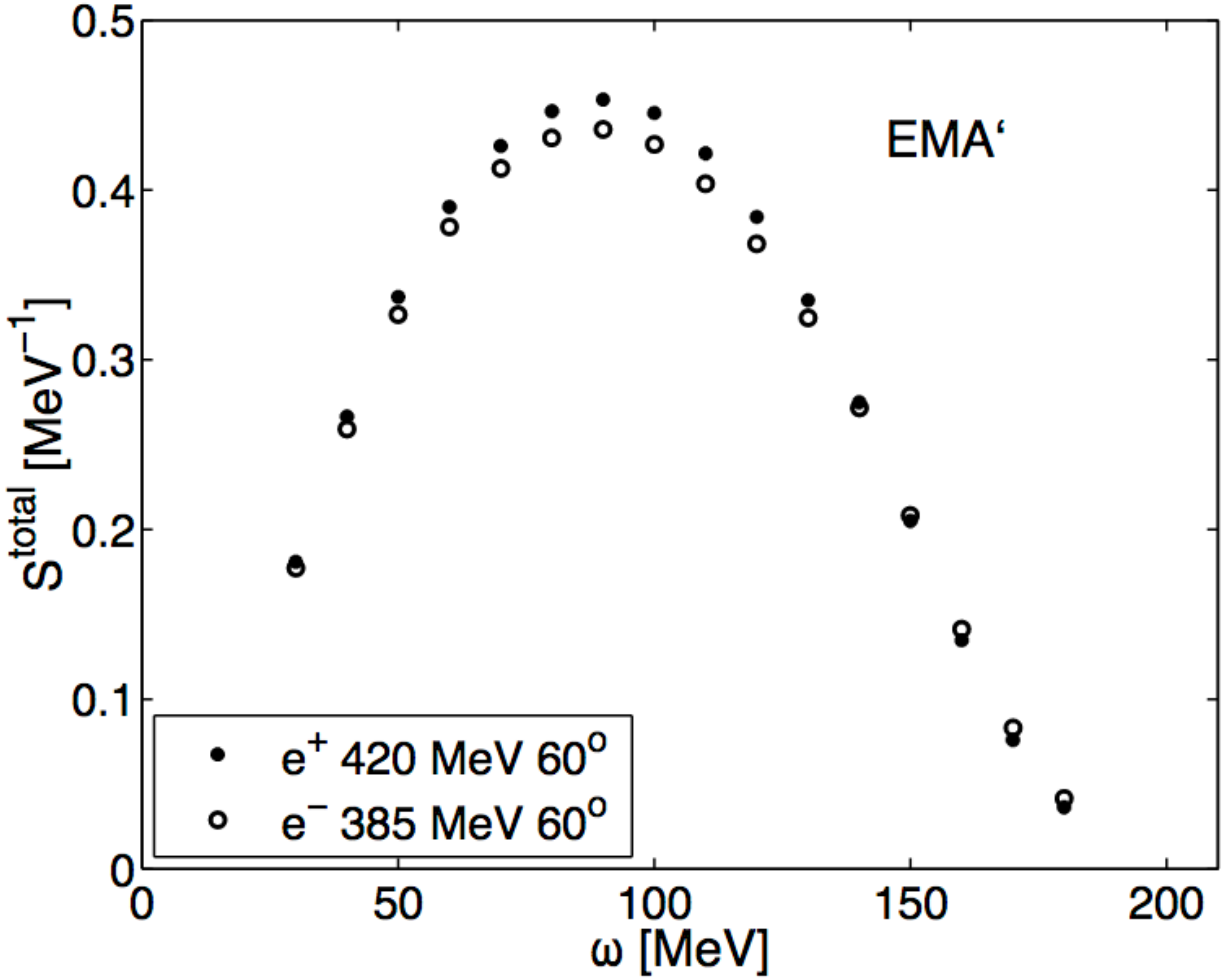
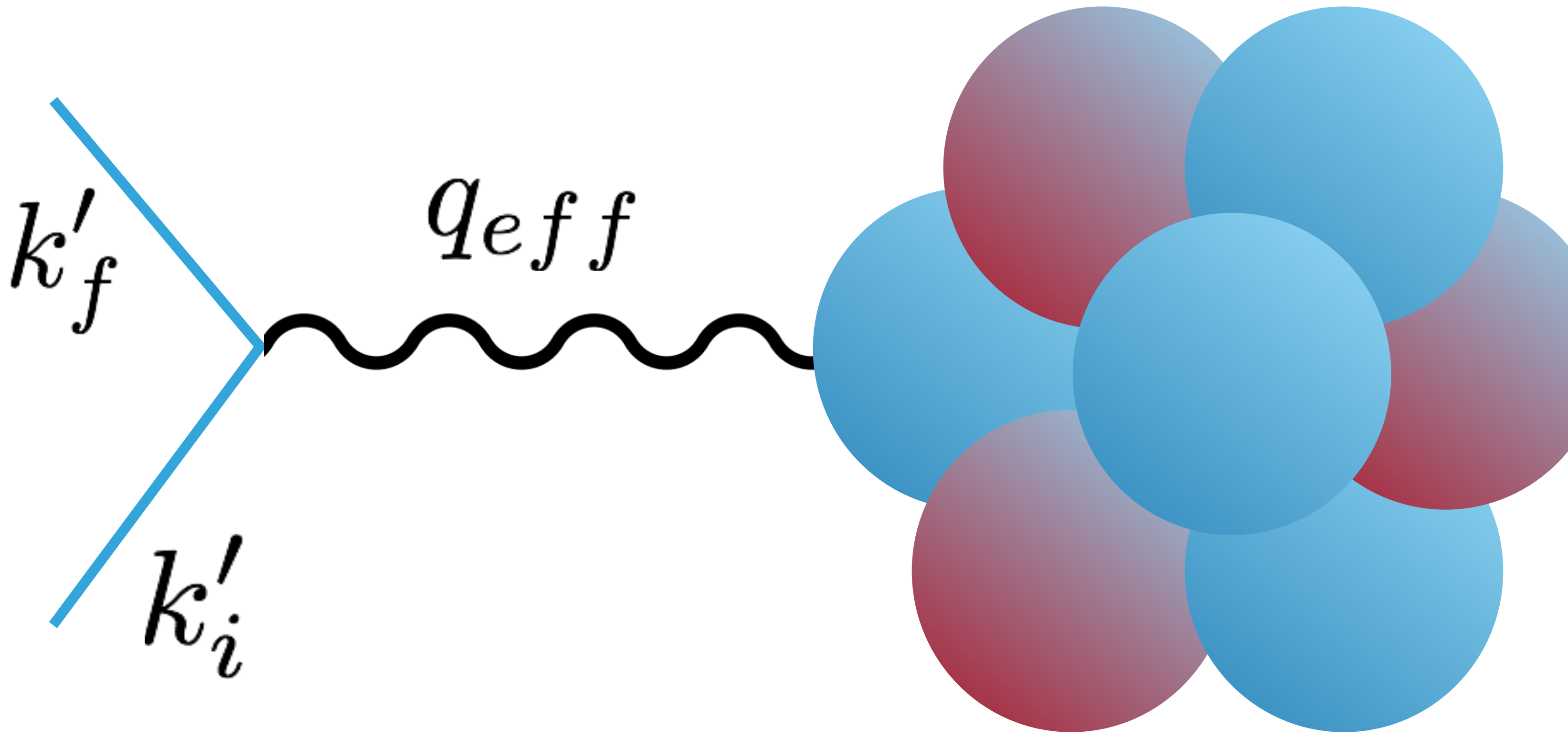






MEAN COULOMB POTENTIAL, EMA, AND POSITRON SCATTERING

- An effective momentum approximation (EMA) takes into account the mean field potential of the target nucleus during quasi-elastic scattering.
  - **Full treatment includes:**
    - Enhancement of electron (initial and final) momentum in vicinity of nucleus due to electrostatic potential.
    - Focusing of electron wave-function in nuclear region.
  - **The simplest EMA attempts to use the electrostatic potential in the lowest order of αZ**
    - PW scattering calculation occurs at center of nucleus.
    - Nucleus is perfectly spherical (charge is evenly distributed)
      - Electron wave-function remains constant inside nuclear volume.
    - Scattering length is zero.



Solutions to the Dirac equation for electron scattering in the presence of many-body nuclear fields are (laboriously) calculable with partial wave expansion and numerical calculation.

A modified EMA attempts to parameterize these effects into a term that modifies the potential:

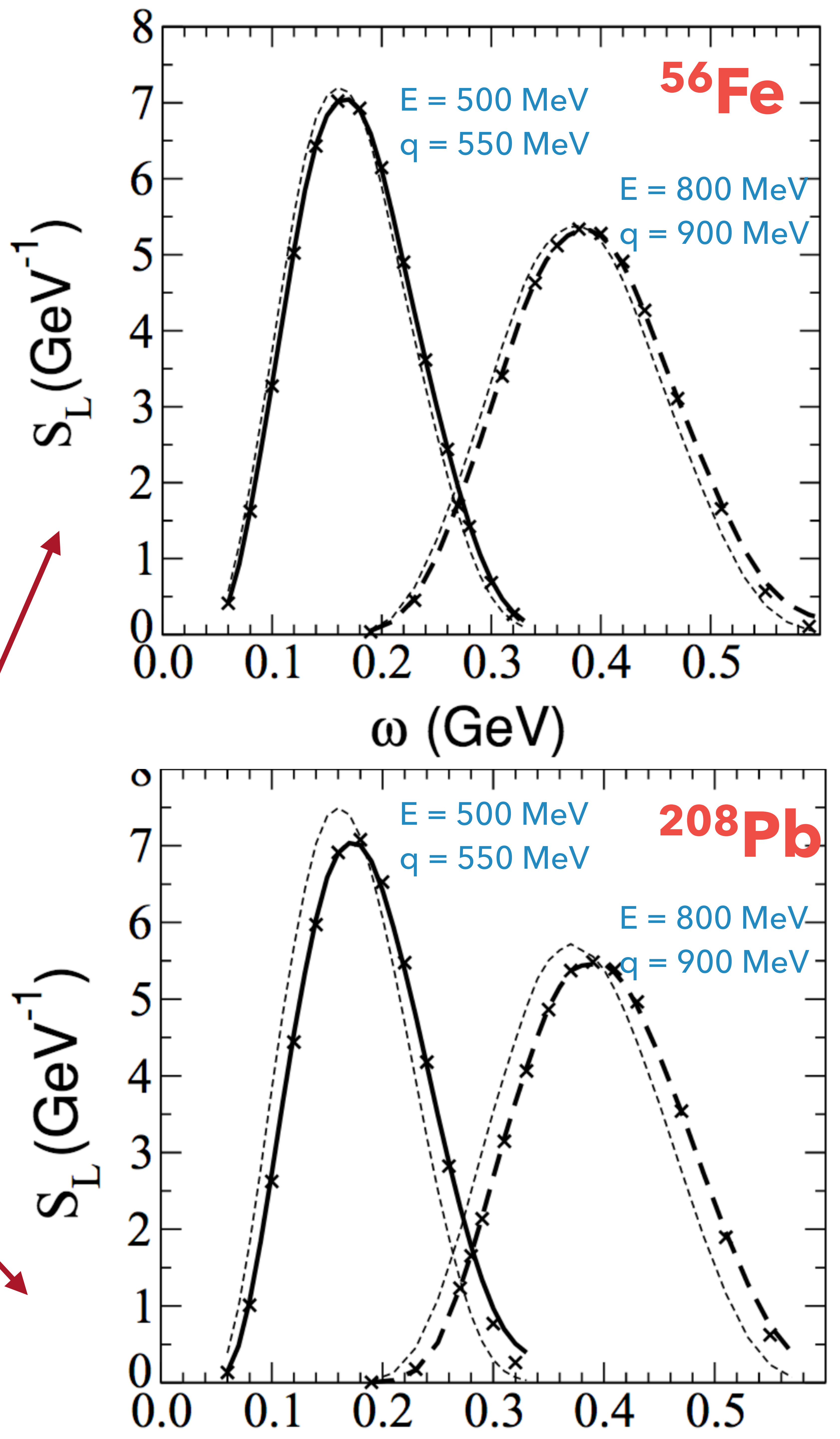
$$k'_i = k_i - \boxed{\kappa_A}\frac{V_0}{c}$$

MEAN COULOMB POTENTIAL, EMA

- An effective momentum approximation (EMA) takes into account the mean field potential of the target nucleus during quasi-elastic scattering.
    - Above slides follow the prescription from A. Aste, but a very similar treatment of the electrostatic potential is preformed by S. Wallace and J. Tjon.
        - An r-dependent integration provides a more accurate approximation (called EMAr) and calculations show the total expected effect on iron and lead targets.

Longitudinal Response (S_L) vs. Energy Transfer (ω)

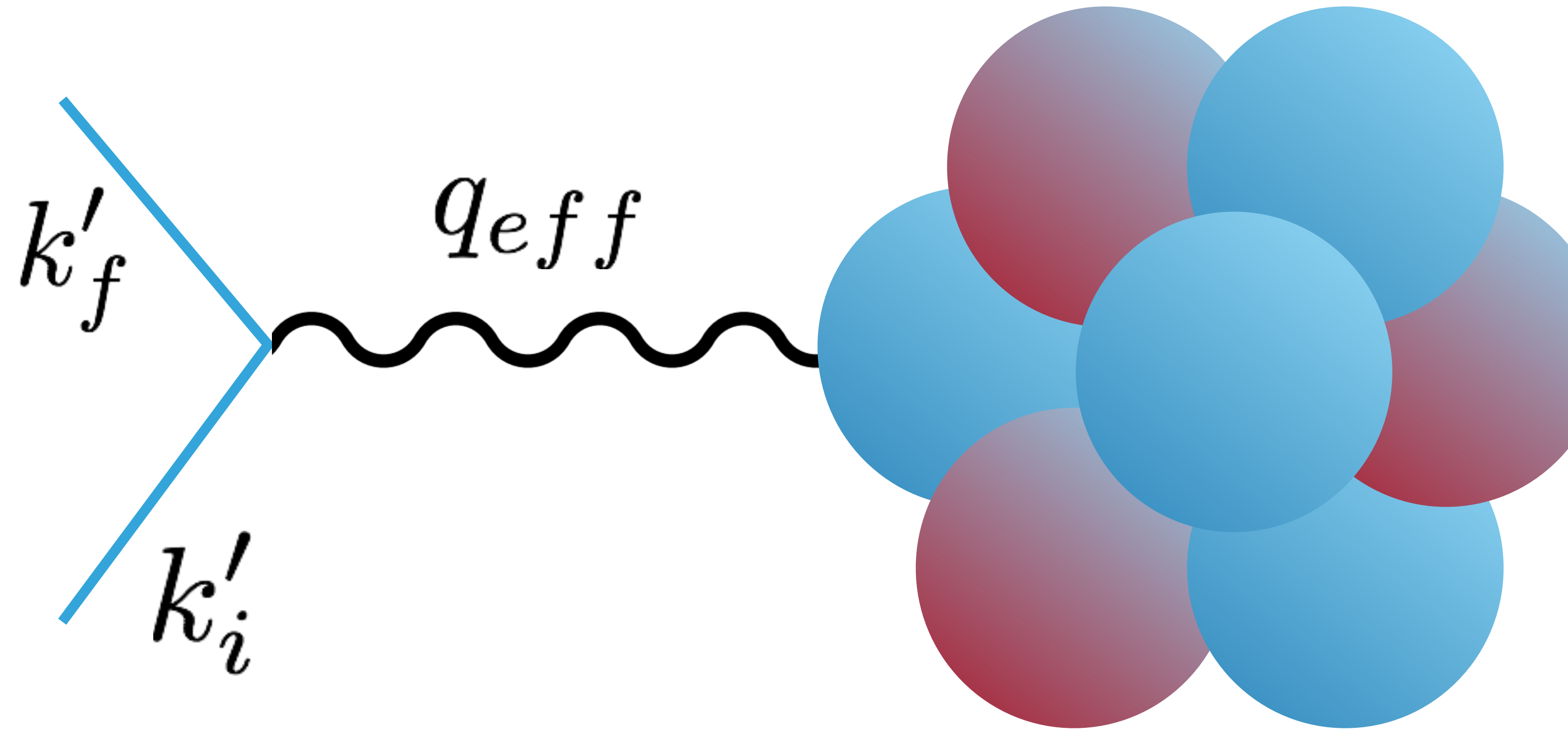
Solid/dashed is EMAr
Fine-dashed is PWIA (no coulomb distortion)



S. Wallace, J. Tjon, PRC 78 (2008)

MEAN COULOMB POTENTIAL, EMA, AND POSITRON SCATTERING

- An effective momentum approximation (EMA) takes into account the mean field potential of the target nucleus during quasi-elastic scattering.
  - **Important to note:**
    - For the least complicated nuclei (low A), the coulomb corrections to the momentum are small (close to negligible).
    - For large nuclei, where we need the corrections are needed the most, the simple EMA approximations are most likely to break down!
  - High precision data with positron scattering would be extremely useful, especially for large A nuclei.



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THOMAS JEFFERSON NATIONAL ACCELERATOR FACILITY

- Located in Newport News, Virginia
- Four main experimental halls
- Recently completed upgrade allows electron beam energies up to 12 GeV

Jefferson Lab Accelerator Site

THOMAS JEFFERSON NATIONAL ACCELERATOR FACILITY

EXPERIMENTAL DESIGN

- Need R_L ⟶ Use Rosenbluth separation!

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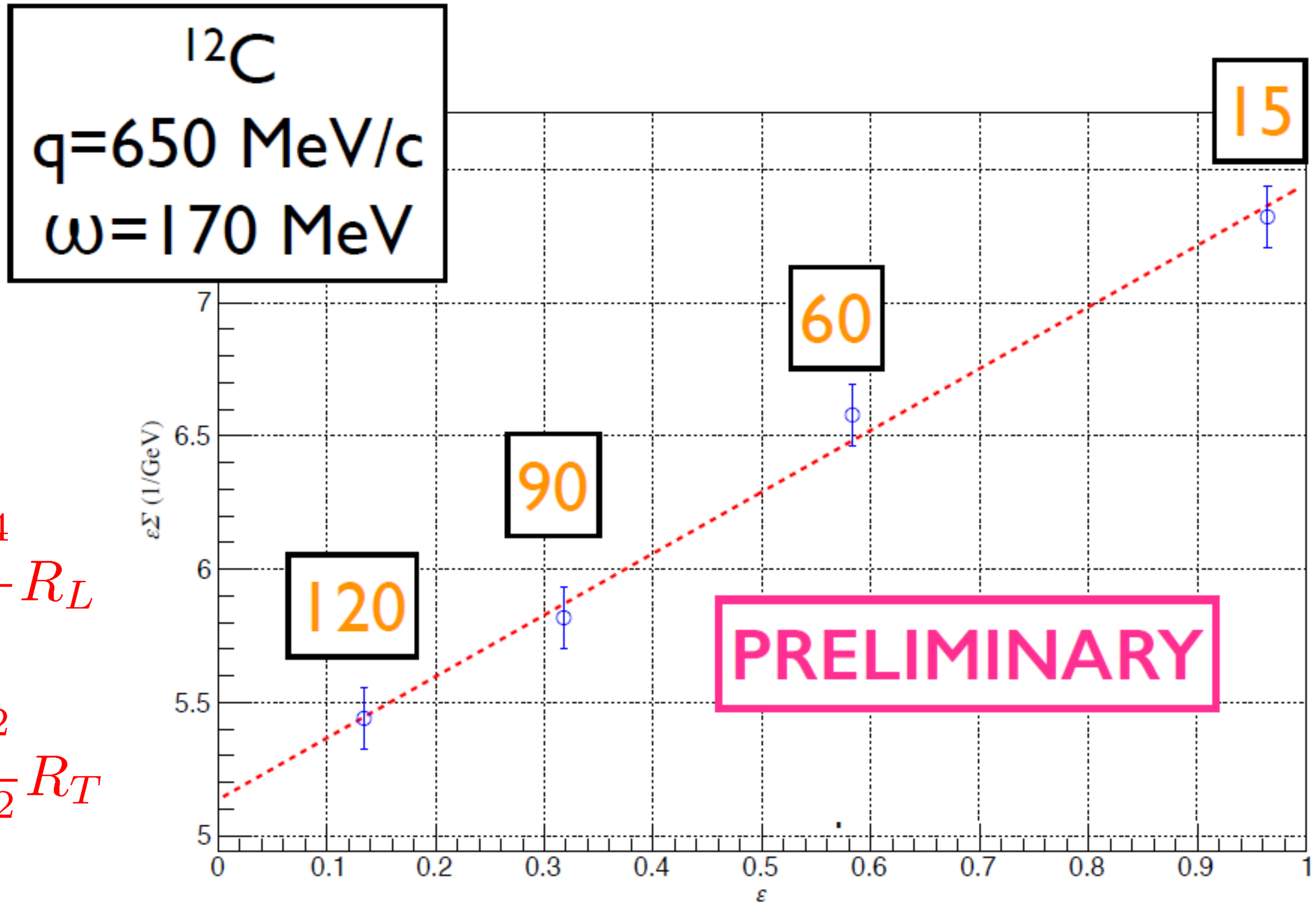
Slope = $\frac{Q^4}{\vec{q}^4} R_L$

Intercept = $\frac{Q^2}{2\vec{q}^2} R_T$



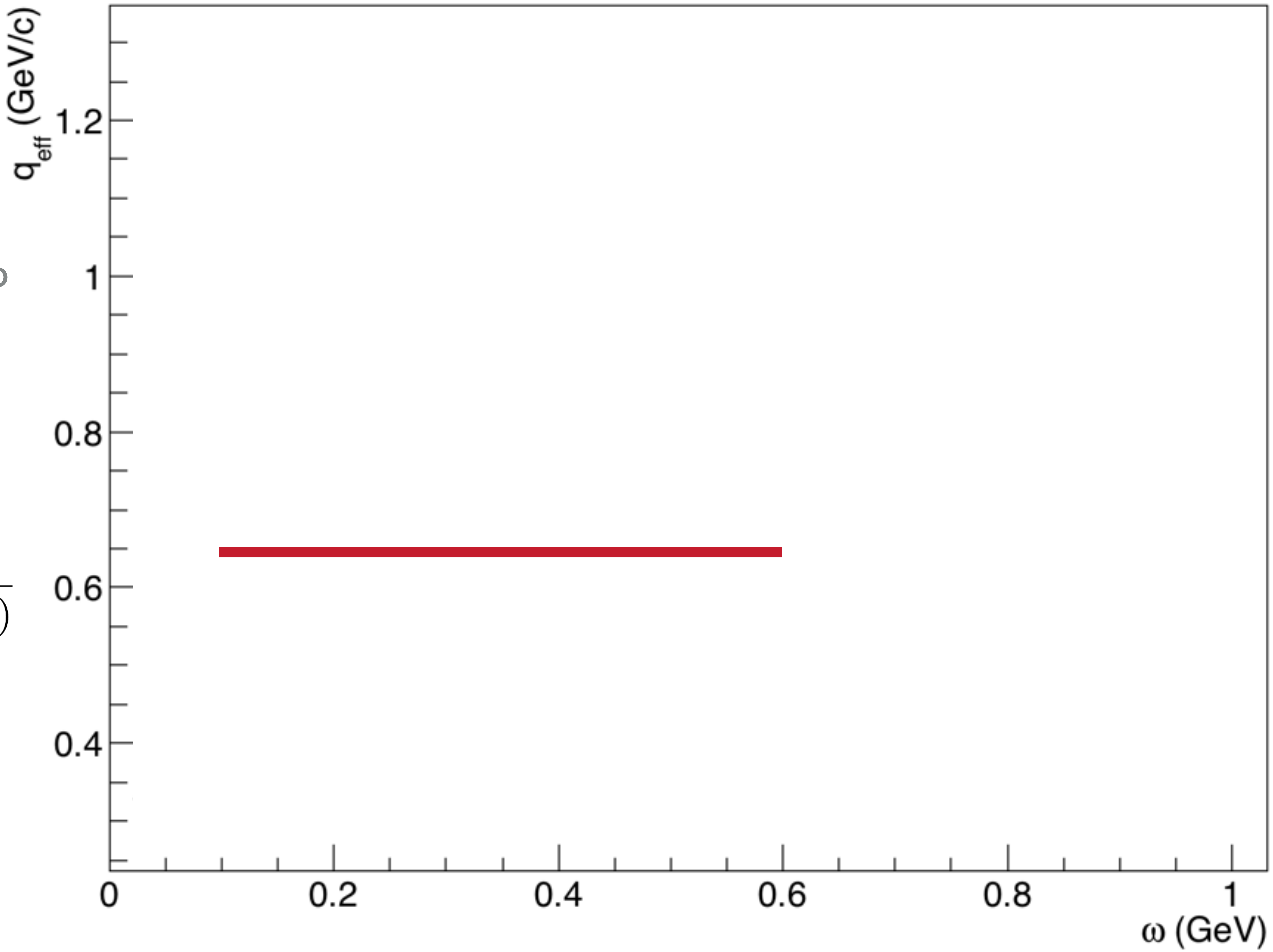
- Experiment run at 4 angles per target: 15, 60, 90, 120 degs. Very large lever arm for precise calculation of R_L!
- Need data for each angle at a constant |q| over an ω range starting above the elastic peak up to |q|.
  - When running a single arm experiment with fixed beam energy and scattering angle, |q| is NOT constant over your momentum acceptance.
    - Need to take data at varying beam energies, and “map-out” |q| and ω space.

EXPERIMENTAL DESIGN

- If one wants to measure from 100 to 600 MeV ω at constant |q| = 650 MeV/c

CSR calculated at constant |**q**| !!

$$S_L(|\boldsymbol{q}|) = \int_{\omega^+}^{|\boldsymbol{q}|} d\omega \frac{R_L(\omega, |\boldsymbol{q}|)}{Z\tilde{G}^2_{Ep}(Q^2) + N\tilde{G}^2_{En}(Q^2)}$$



EXPERIMENTAL DESIGN

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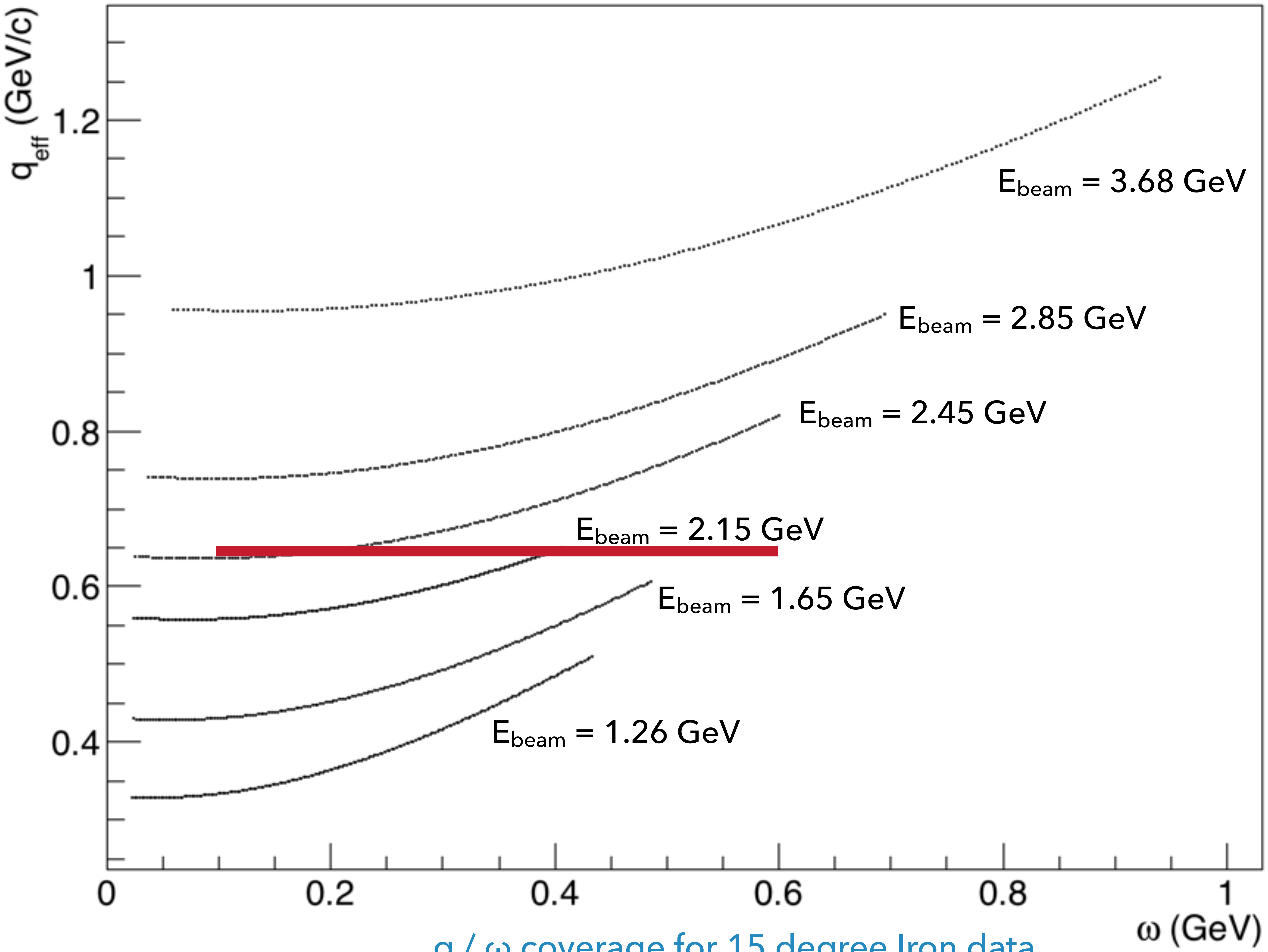
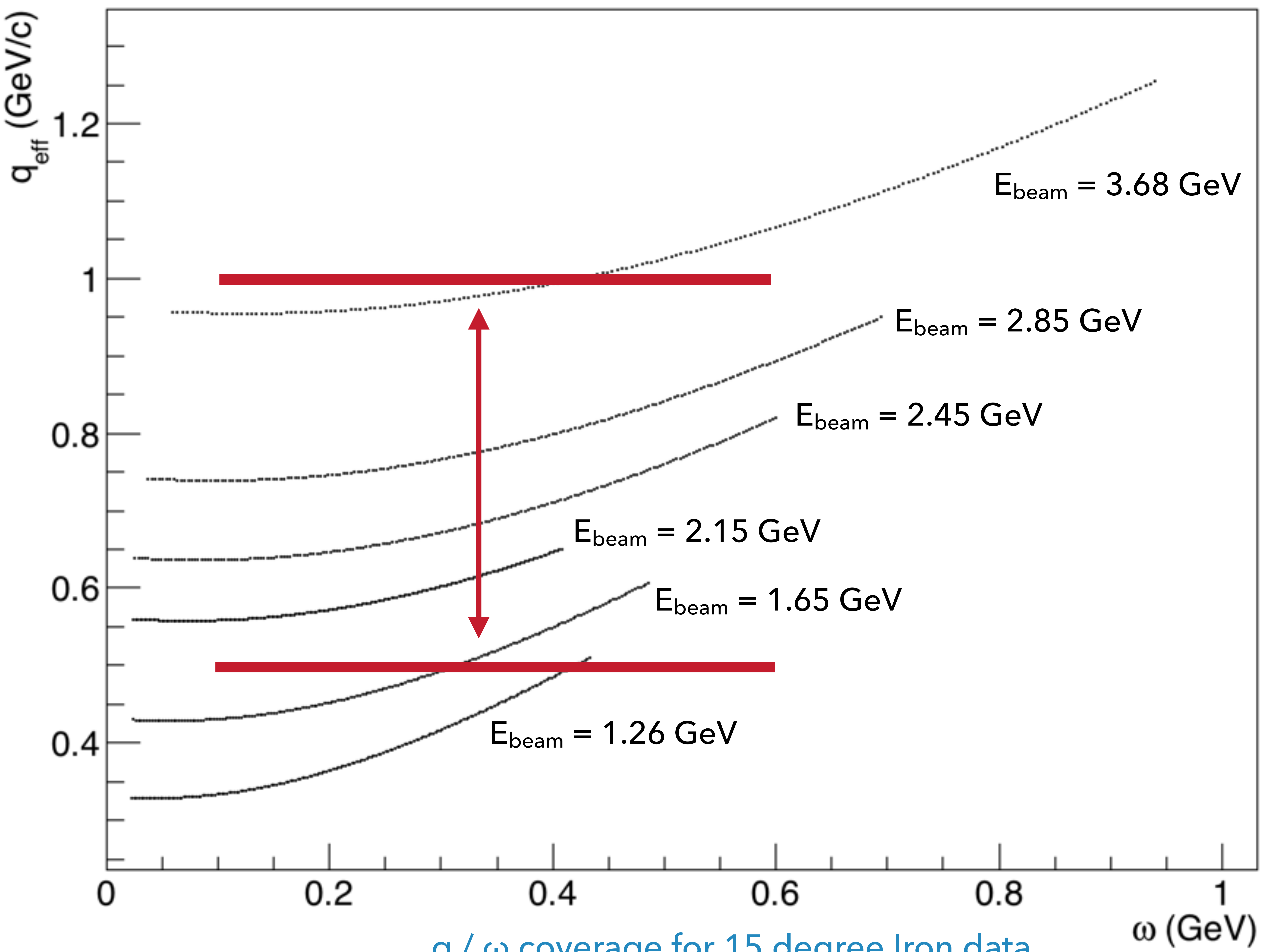
  - Take data at different beam energies, and interpolate to determine cross-section at constant |q|.



q / ω coverage for 15 degree Iron data

EXPERIMENTAL DESIGN

- If one wants to measure from 100 to 600 MeV ω at constant |q| = 650 MeV/c
  - Take data at different beam energies, and interpolate to determine cross-section at constant |q|.
  - |q| can be selected between 550 and 1000 MeV/c



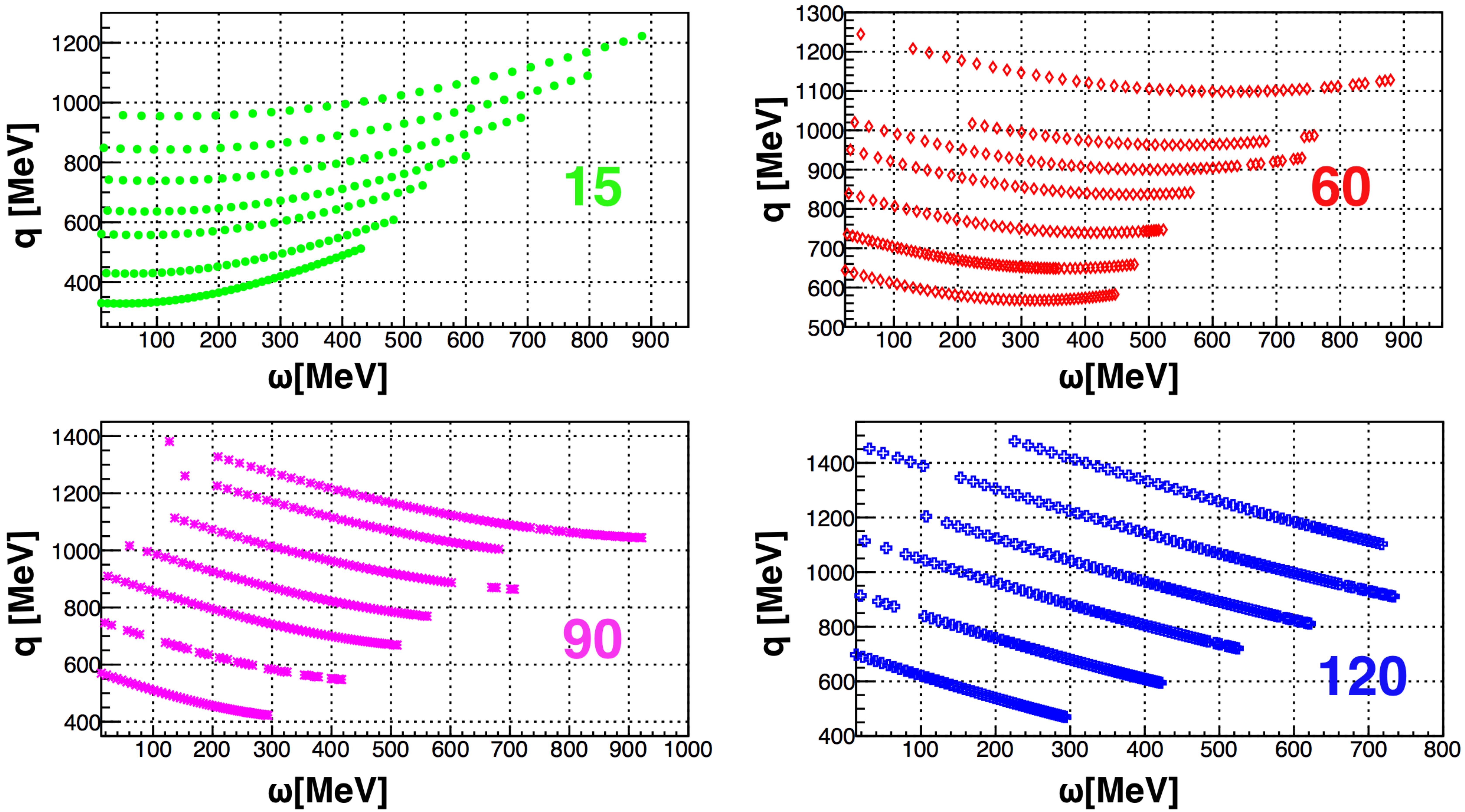
q / ω coverage for 15 degree Iron data

Repeat this “mapping” for 60, 90, and 120 degree spectrometer central angles.

EXPERIMENTAL SPECIFICS

- E05-110:
  - Data taken from October 23rd 2007 to January 16th 2008
  - 4 central angle settings: 15, 60, 90, 120 degs.
  - Many beam energy settings: 0.4 to 4.0 GeV
  - Many central momentum settings: 0.1 to 4.0 GeV
  - LHRS and RHRS independent (redundant) measurements for most settings
  - 4 targets: ^{4}He, ^{12}C, ^{56}Fe, ^{208}Pb.

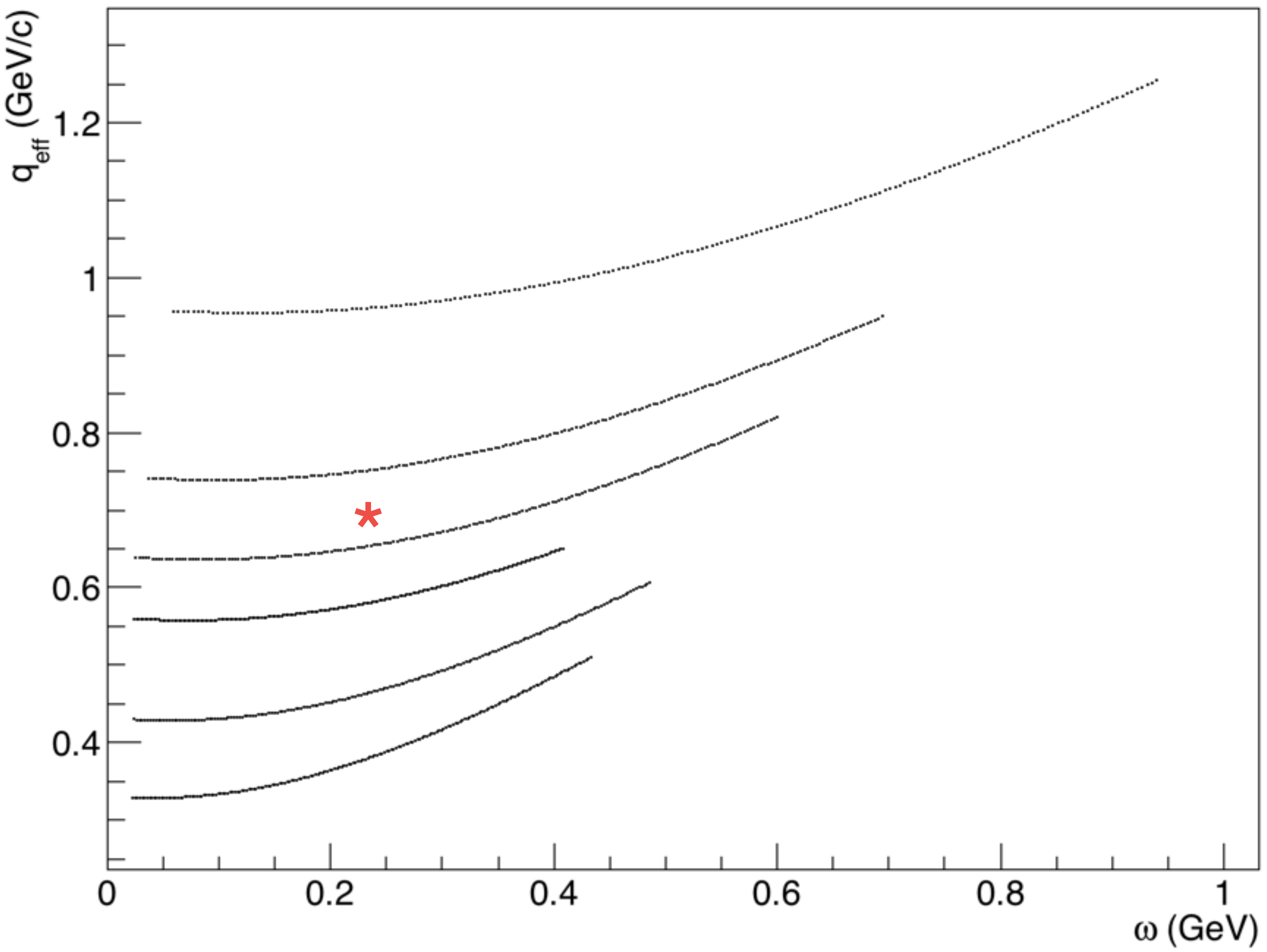
Each data line represents a constant beam-energy



INTERPOLATION TECHNIQUES

- Interpolation of |q|

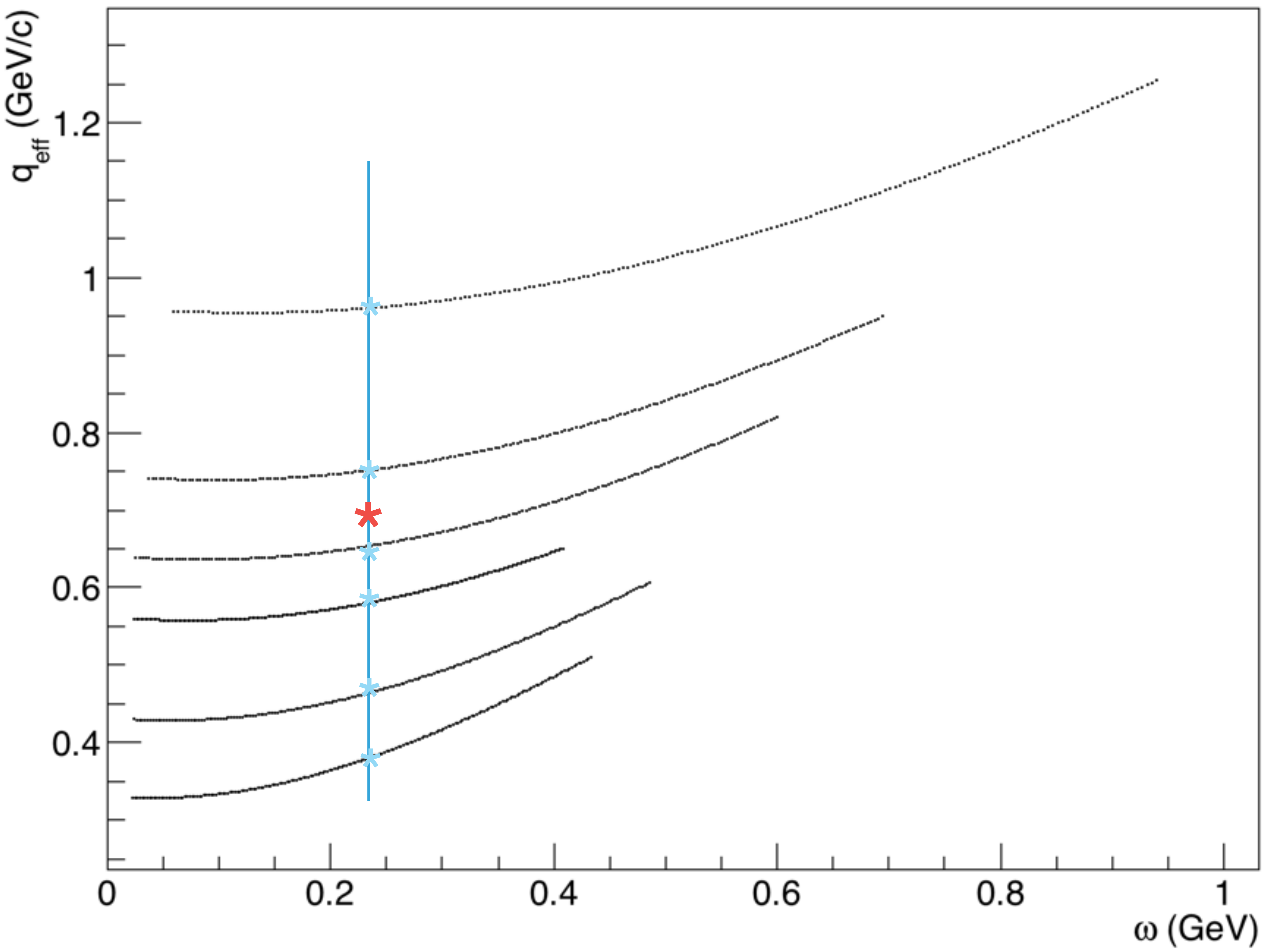
q / ω coverage for 15 degree Iron data



INTERPOLATION TECHNIQUES

- Interpolation of |q|
  - Could go along a constant ω line. Not the best option.

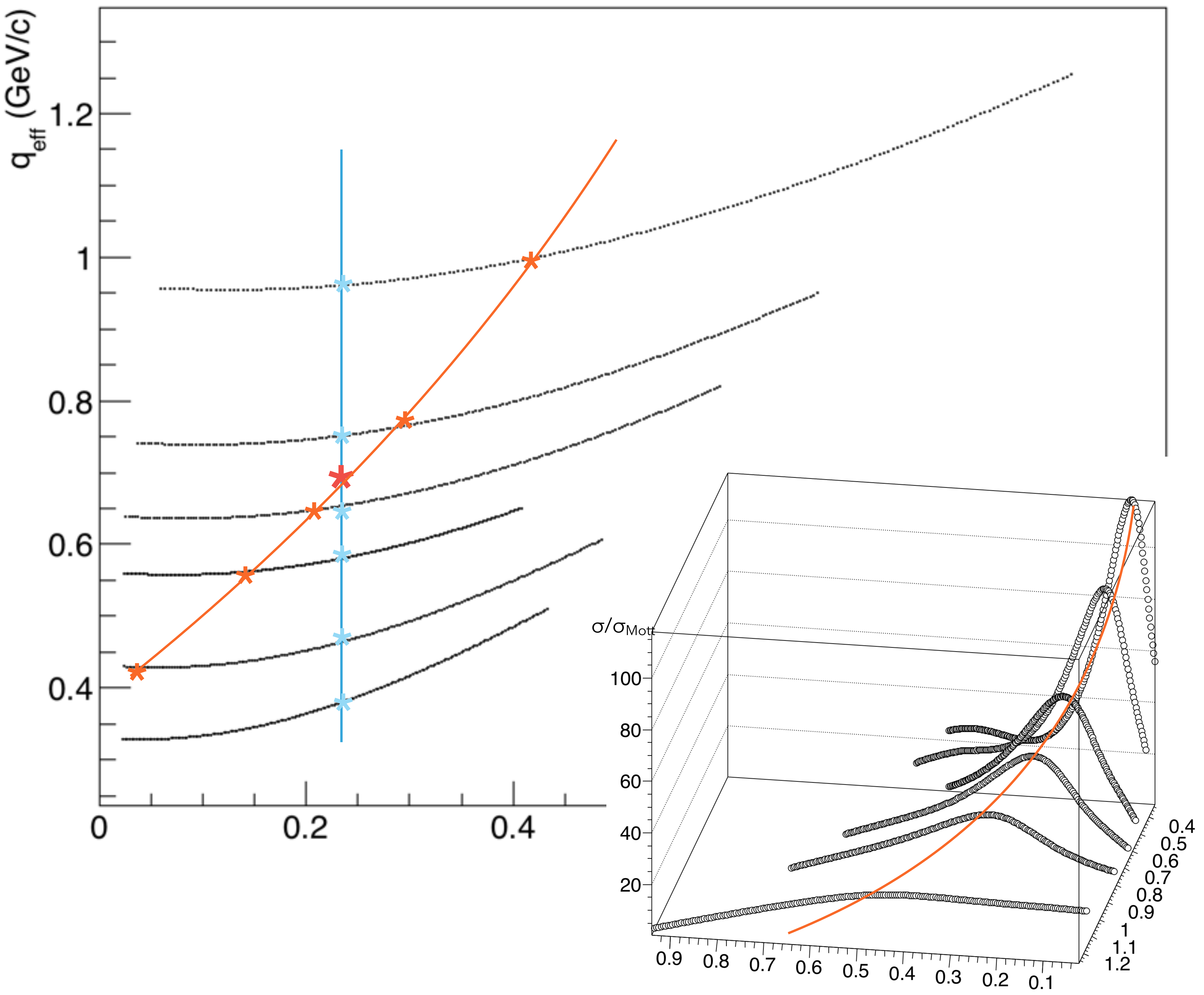
q / ω coverage for 15 degree Iron data



INTERPOLATION TECHNIQUES

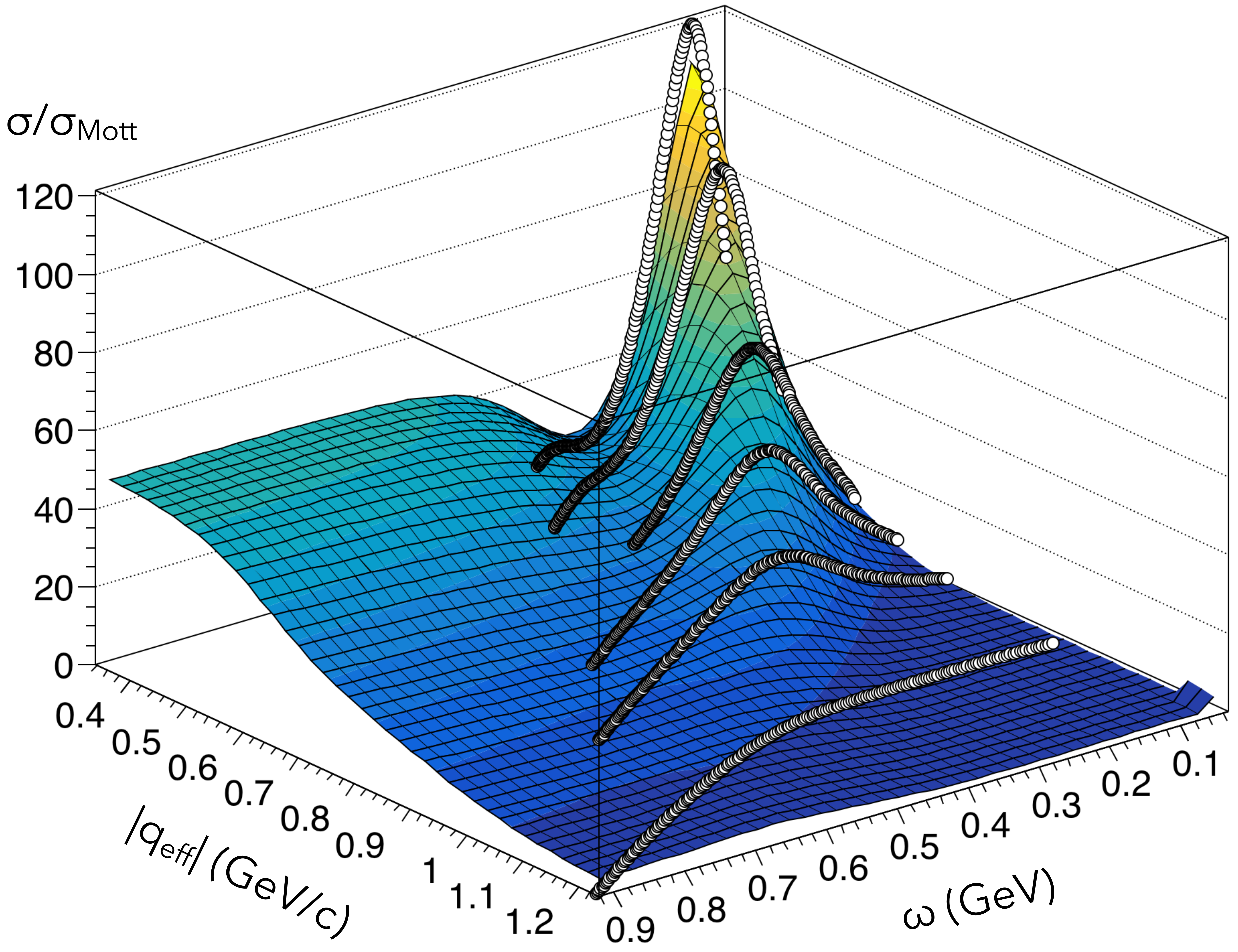
- Interpolation of |q|
  - Could go along a constant ω line. Not the best option.
  - Better: use a constant y line, which will follow the trend of quasi-elastic peak.
  - Alternative: use a constant W line, which should follow the Δ peak.
    - or even a combination of y and W.

q / ω coverage for 15 degree Iron data



INTERPOLATION TECHNIQUES

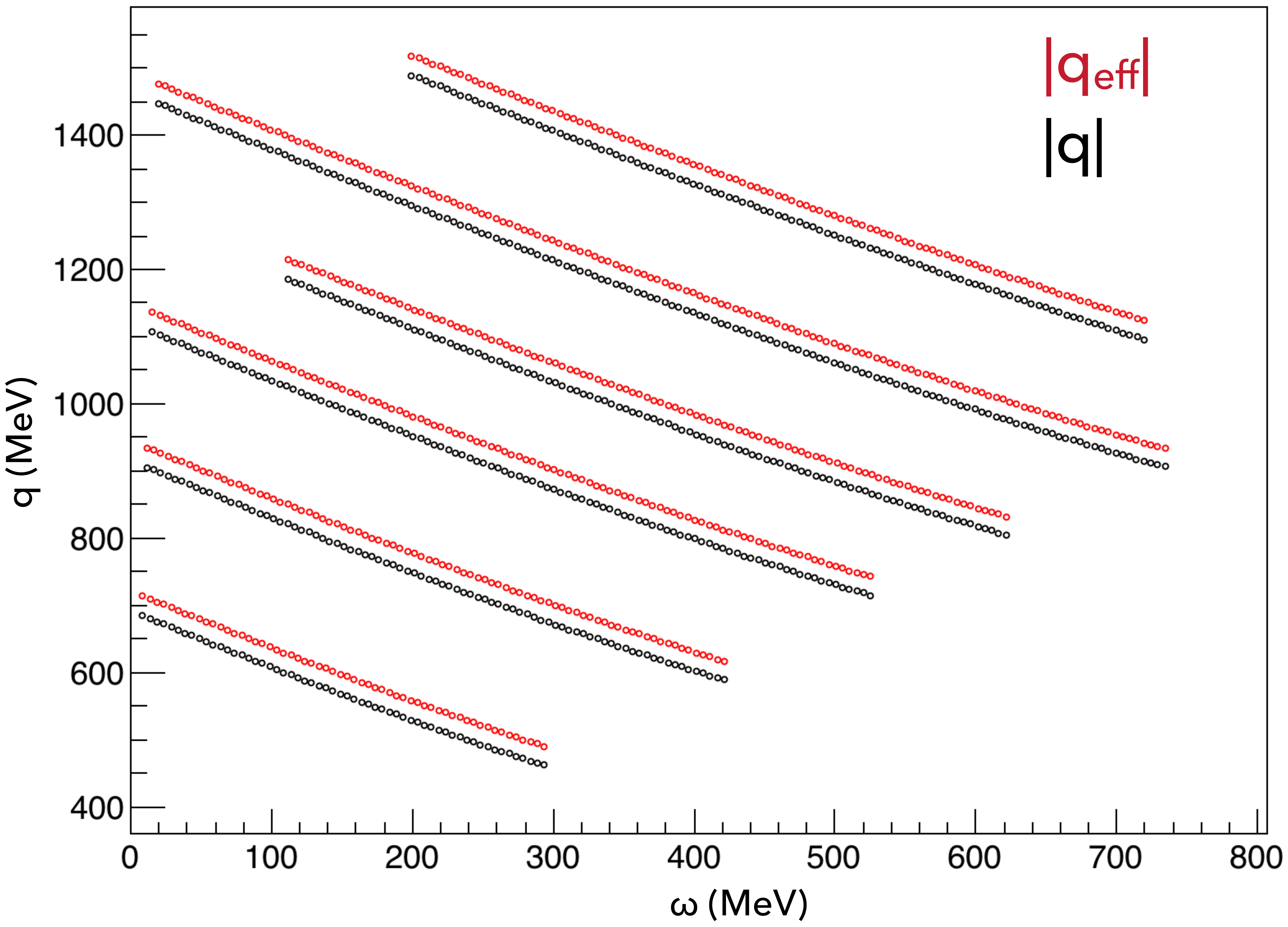
- 3D Machine learning techniques are also available:
  - Unsupervised Neural Network
    - Method uncertainty is hard to pin down.
  - Supervised Gaussian Process Regression.
    - Implemented from scratch.
    - Uncertainties are well constrained.



INTERPOLATION AND q_{eff}

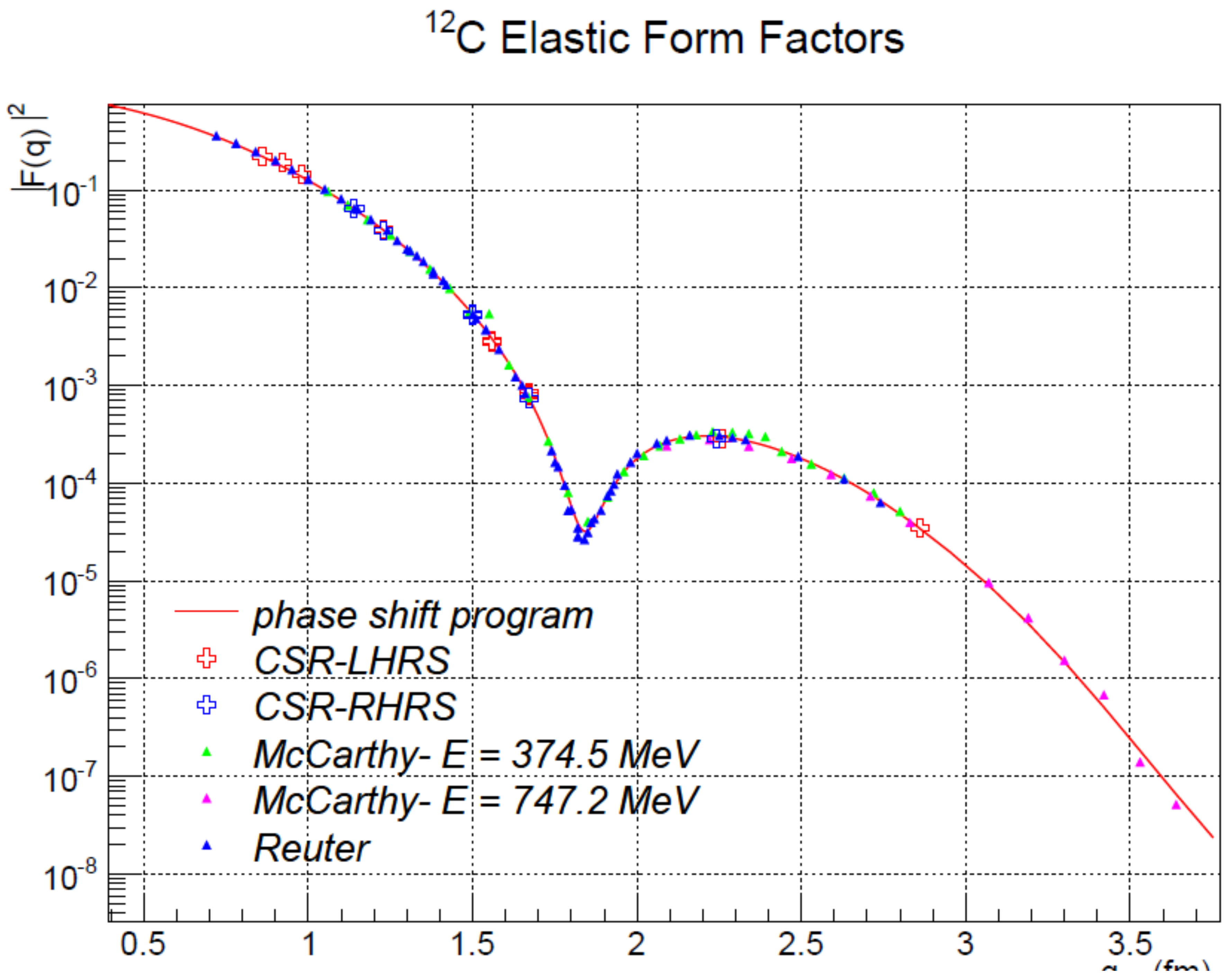
- The offset in the spectra when using the EMA corrected momentum transfer significantly affects the interpolation landscape.
  - Effect is largest at low momenta and in heavier targets

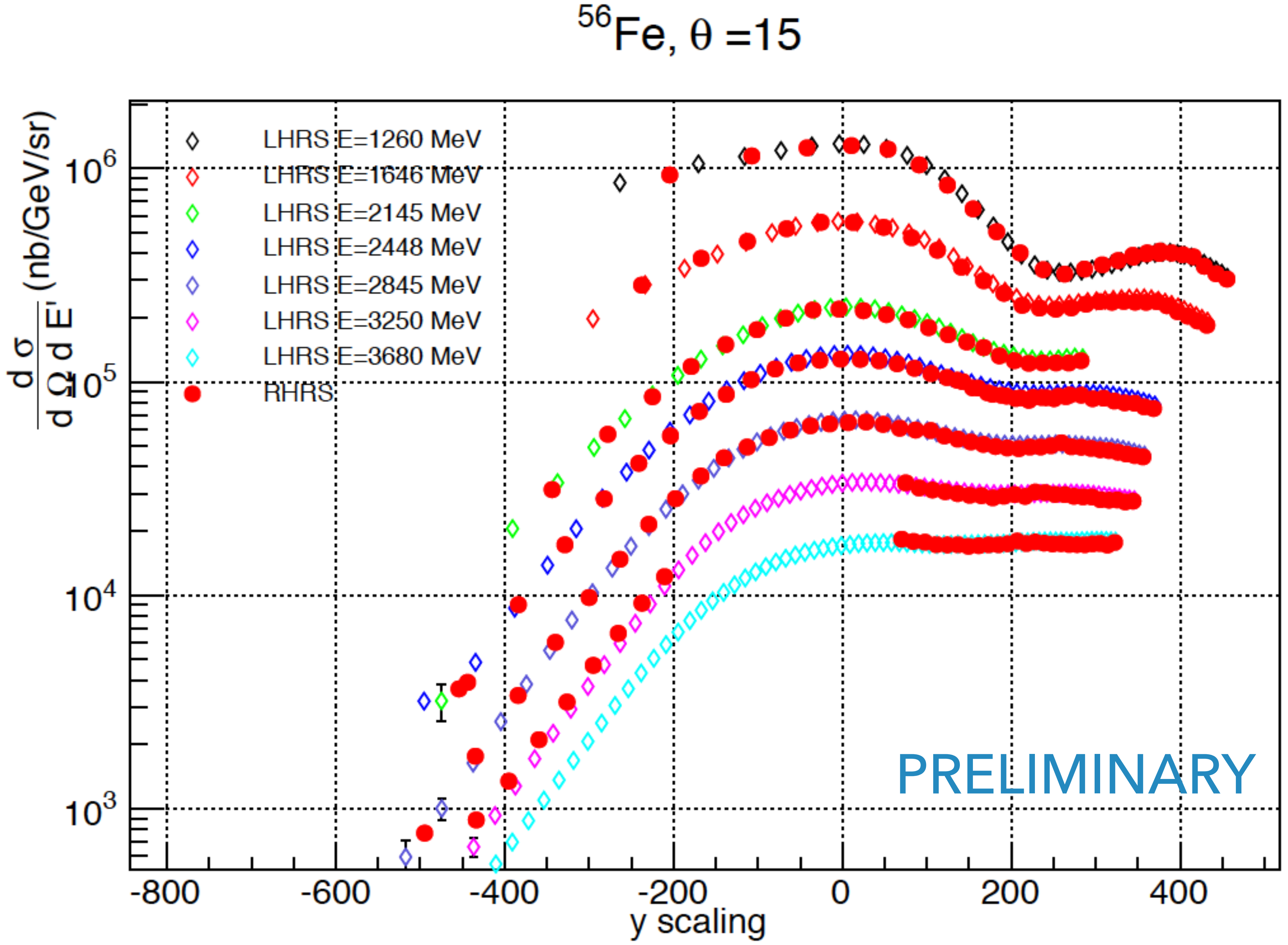
q / ω coverage for 120 degree Pb data



PRELIMINARY RESULTS: LHRS AND RHRS AGREEMENT

- Both LHRS and RHRS agree well :
  - with world data on ^{12}C elastic form factors
  - with each other for ^{56}Fe quasi-elastic cross-section.
- Each spectrometer arm is an independent measurement.
  - Agreement shows a good handle on acceptance and radiative corrections.

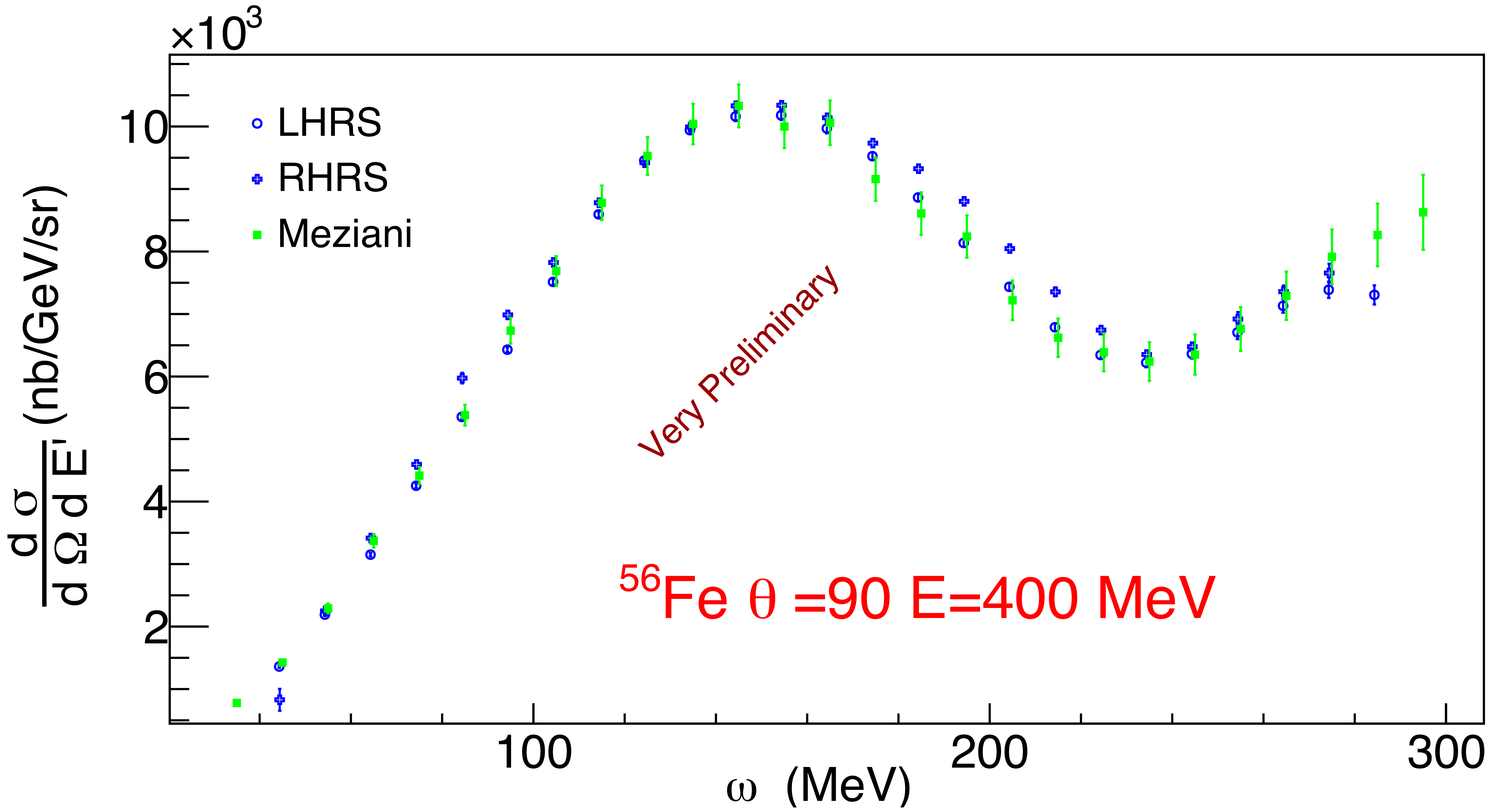




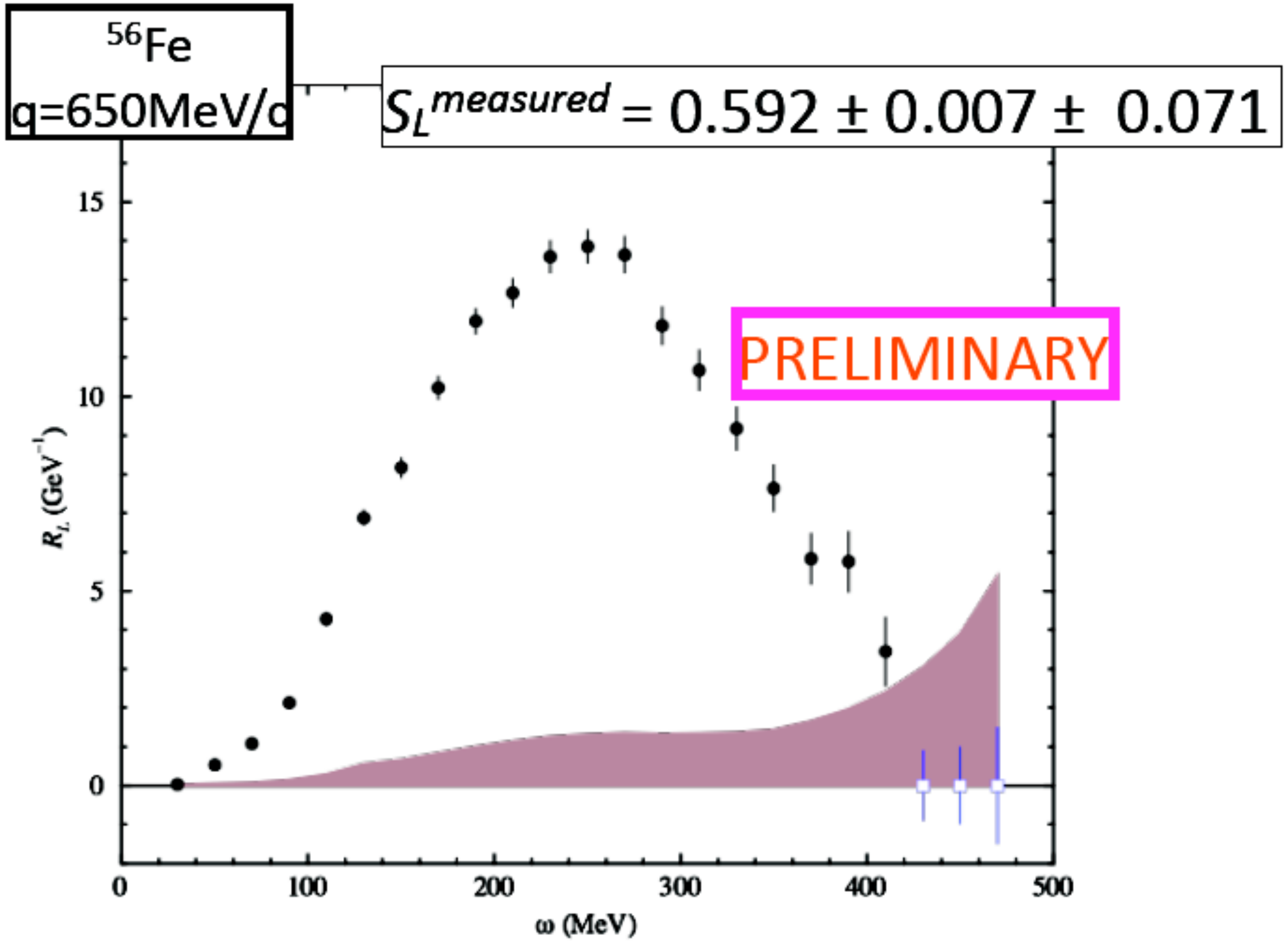
Analysis by Hamza Atac,
Temple Graduate Student

PRELIMINARY RESULTS: AGREEMENT WITH PREVIOUS MEASUREMENTS

- Not much world-data for Iron-targets at kinematics overlapping with E05-110.
- We do have one set of data at 90 degrees and 400 MeV from Saclay that we can directly compare to.
    - Good agreement between both arms and prior data.



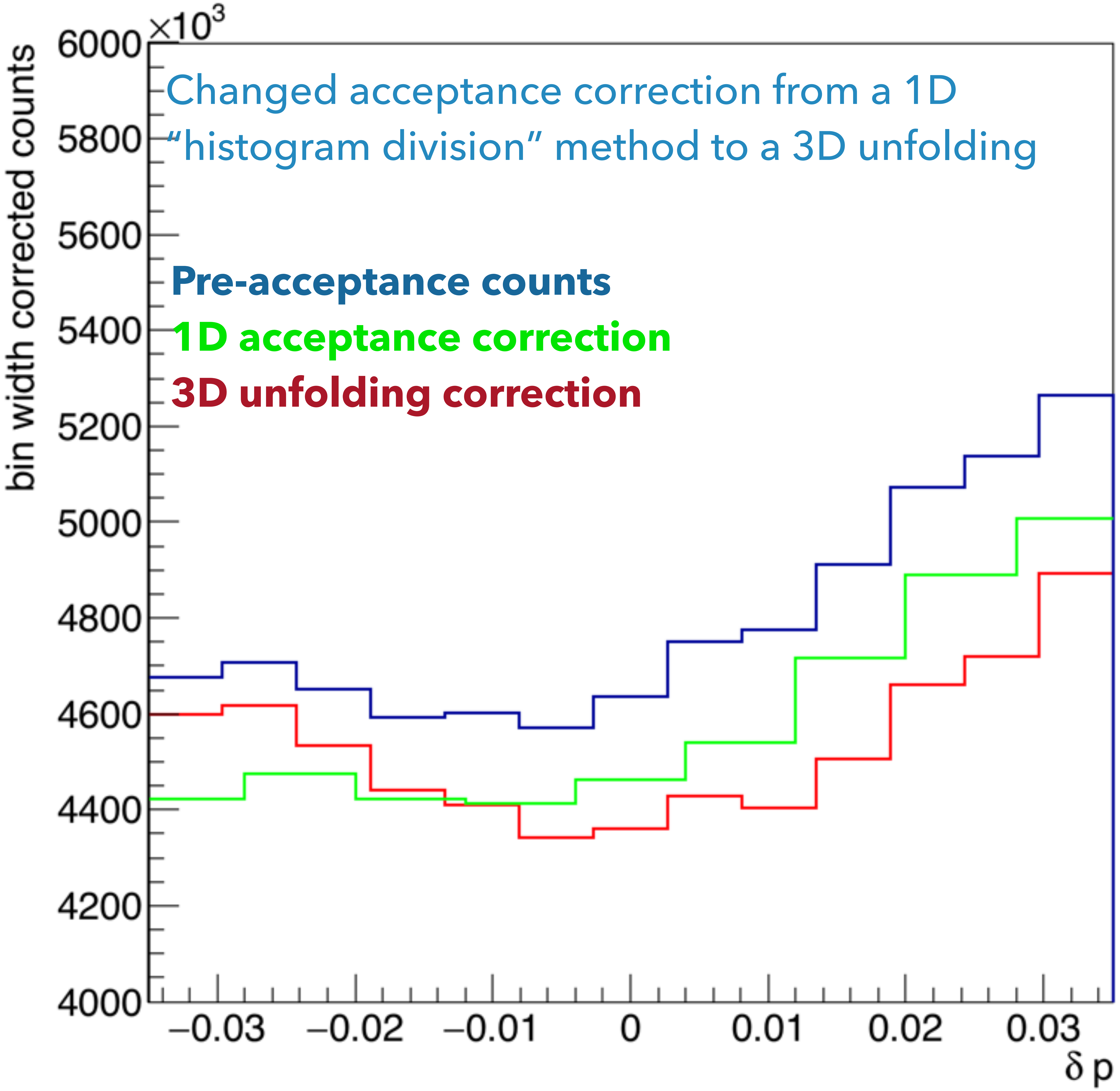
(OLD) PRELIMINARY RESULTS: ^{56}FE LONGITUDINAL RESPONSE FUNCTION



- Much has been done since this CSR was calculated:
  - New acceptance procedure
  - Updated optics
  - Newer sophisticated interpolation methods
  - Many studies and cross-checks of the radiative corrections.

Analysis by Dr. Yoomin Oh, PhD Graduate of Seoul National University

RECENT EFFORTS



- Much has been done since this CSR was calculated:
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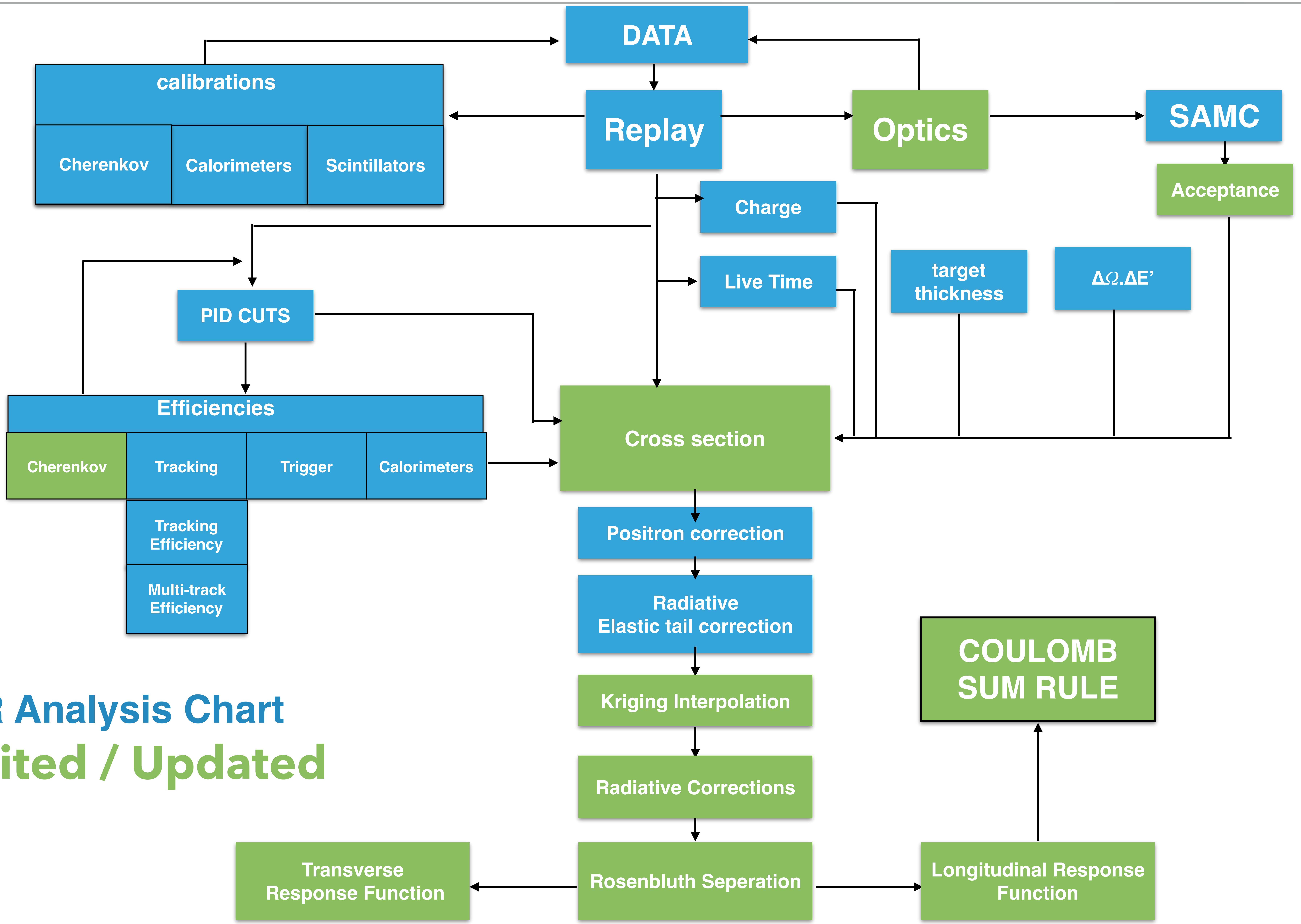

CSR Analysis Chart
Revisited / Updated

CONCLUSIONS

- Measuring the Coulomb Sum Rule on nuclei at large |q| is a straight forward method for testing medium modifications of the nucleon form factor.
    - There have been decades of theoretical and experimental interest in testing the CSR on nuclei.
- Jefferson-Lab experiment (E04-110) was run in 2007 and 2008, with many angles, energies, and nuclear targets, in order to measure S_L for the first time ever in the range of $550 < |q| < 1000$ MeV/c
    - Most regions of the S_L calculation are well understood.
        - Working systematically through all contributions to R_L with careful investigation of the large ω region where Rosenbluth separation is most sensitive.
    - Final results are very VERY close!!!
- A positron beam would allow one to confirm the recent measurements of E05-110, and would help pin down the coulomb potential for heavy nuclei at lower energies.

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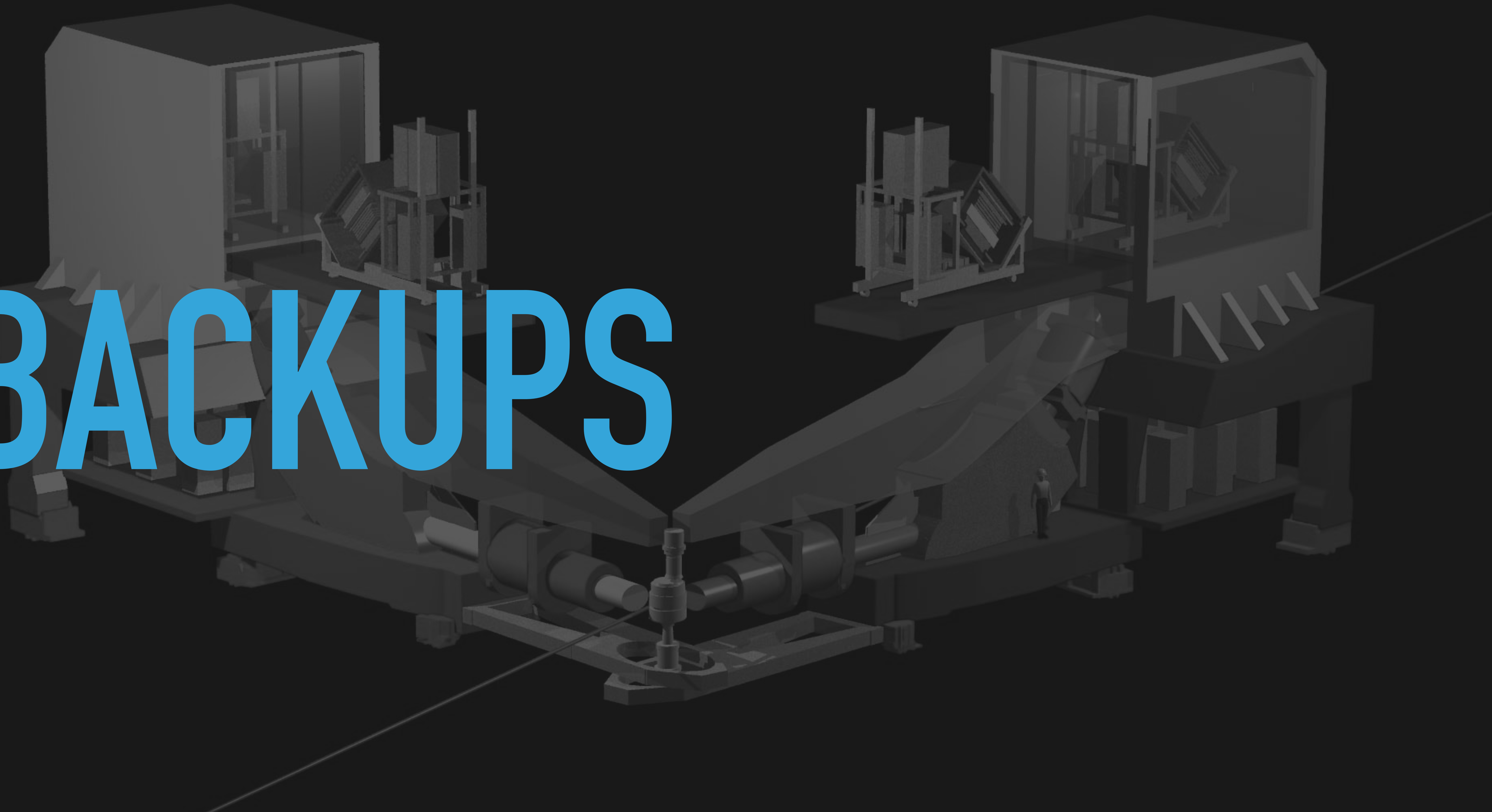
PEOPLE

Kalyan Allada, Korand Aniol, Jon Arrington, Hamza Atac, Todd Averett, Herat Bandara, Werner Boeglin, Alexandre Camsonne, Mustafa Canan, **Jian-Ping Chen**, Wei Chen, Khem Chirapatpimol, **Seonho Choi**, Eugene Chudakov, Evaristo Cisbani, Francesco Cusanno, Rafelle De Leo, Chiranjib Dutta, Cesar Fernandez-Ramirez, David Flay, Salvatore Frullani, Haiyan Gao, Franco Garibaldi, Ronald Gilman, Oleksandr Glamazdin, Brian Hahn, Ole Hansen, Douglas Higinbotham, Tim Holmstrom, Bitao Hu, Jin Huang, Yan Huang, Florian Itard, Liyang Jiang, Xiaodong Jiang, Kai Jin, Hoyoung Kang, Joe Katich, Mina Katramatou, Aidan Kelleher, Elena Khrosinkova, Gerfried Kumbartzki, John LeRose, Xiaomei Li, Richard Lindgren, Nilanga Liyanage, Joaquin Lopez Herraiz, Lagamba Luigi, Alexandre Lukhanin, Michael Paolone, Maria Martinez Perez, Dustin McNulty, **Zein-Eddine Meziani**, Robert Michaels, Miha Mihovilovic, Joseph Morgenstern, Blaine Norum, Yoomin Oh, Michael Olson, Makis Petratos, Milan Potokar, Xin Qian, **Yi Qiang, Arun Saha, Brad Sawatzky, Elaine Schulte**, Mitra Shabestari, Simon Sirca, Patricia Solvignon, Jeongseog Song, **Nikolaos Sparveris, Ramesh Subedi, Vincent Sulkosky**, Jose Udias, Javier Vignote, Eric Voutier, Youcai Wang, John Watson, Yunxiu Ye , Xinhu Yan, Huan Yao, Zhihong Ye, Xiaohui Zhan, Yi Zhang, Xiaochao Zheng, Lingyan Zhu

and
Hall-A collaboration

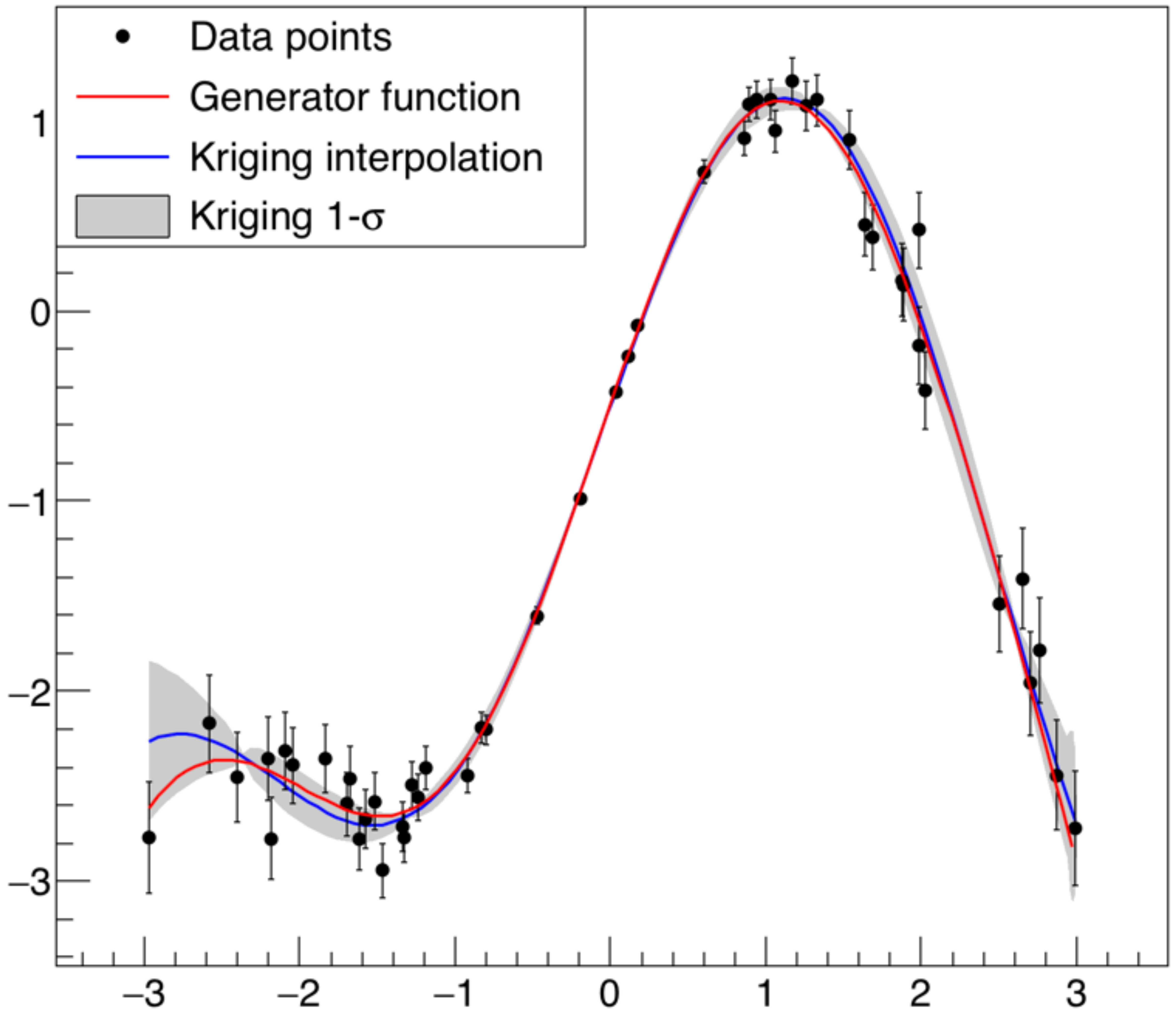
Spokespersons **Run Coordinators**

BACKUPS



SUPERVISED GAUSSIAN PROCESS REGRESSION (KRIGING)

- Advantages:
  - Can provide "smoothing" of distribution.
  - Does not need an input function (like least squared fitting).
  - Well constrained uncertainties.
- Disadvantages:
  - Interpolation options are still needed:
    - The exact covariant function (gaussian, matern)
    - The "scale" and "width" parameter of the covariant function must be set:
      - A small width parameter will pick out more "bumps".
        - As sigma goes to zero, the interpolation will directly go through every point.
      - A larger width will smooth the distribution.



VARIOUS DEFINITIONS AND CORRECTIONS

Basic kinematic definitions:

$$\epsilon(|\boldsymbol{q}|,\omega,\theta) = \left[1 + \frac{2\boldsymbol{q}^2}{\boldsymbol{q}^2 - \omega^2}\tan^2\frac{\theta}{2}\right]^{-1}$$

$$Q^2 = \boldsymbol{q}^2 - \omega^2$$

$$\frac{d\sigma}{d\Omega}_{\text{Mott}} = \frac{\alpha^2\cos^2(\theta/2)}{4E^2\sin^4(\theta/2)}$$

$$W^2 = M_N^2 + 2M_N\omega - Q^2$$

Relativistic correction to nucleon form-factor:

$$\tilde{G}_E^2 = G_E^2\frac{1 + Q^2/4M^2}{1 + Q^2/2M^2}$$

VERIFICATION OF RADIATIVE CORRECTIONS