Finite-Energy Sum Rules

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GHENT UNIVERSITY

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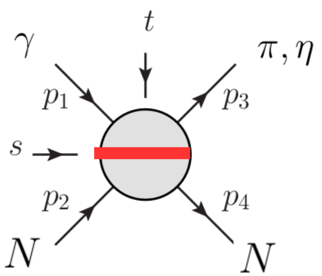
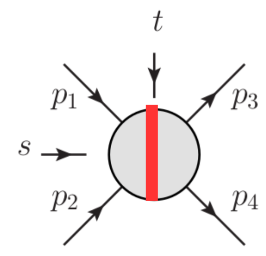
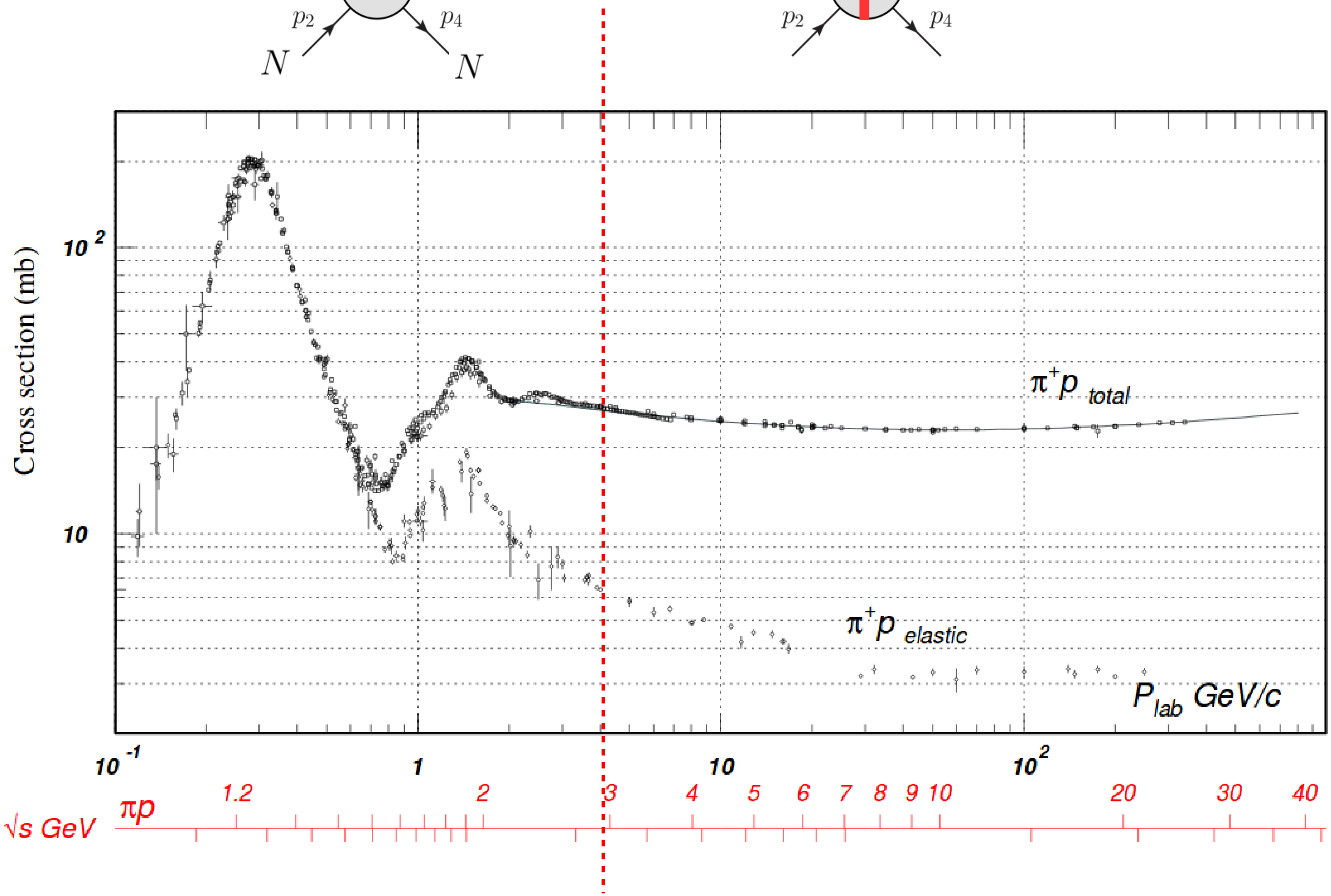
Finite-Energy Sum Rules

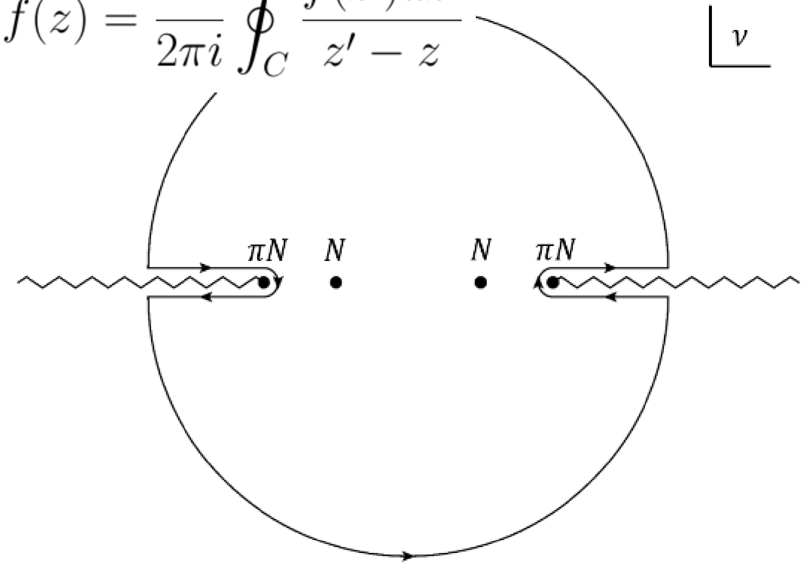


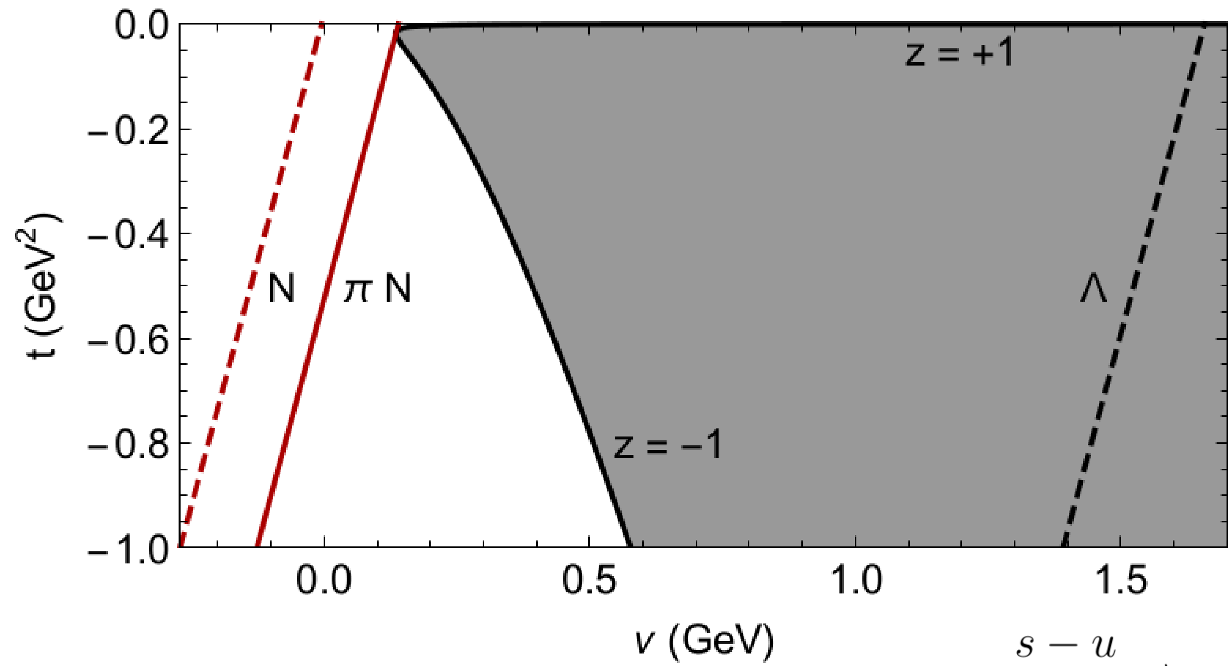




Finite-Energy Sum Rules

$$f(z) = \frac{1}{2\pi i}\oint_C \frac{f(z')dz'}{z'-z}$$

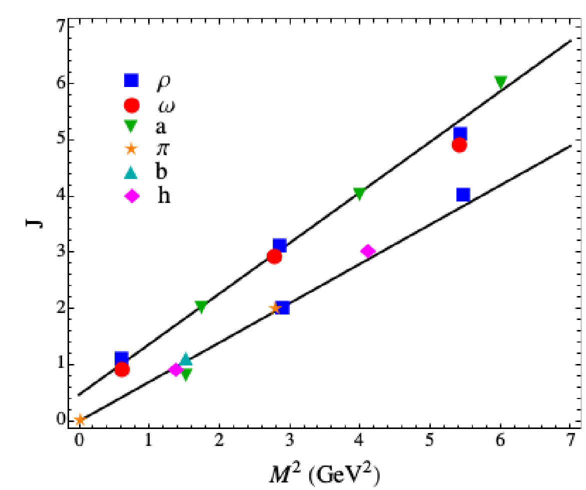




$$\nu = \frac{s-u}{2} \to s$$

$$A^e_{\mu_f,\mu_i\,\mu_\gamma}(s,t) = -\beta^e_{\mu_\gamma}(t)\beta^e_{\mu_i\mu_f}(t)\frac{\tau_e + e^{-i\pi\alpha_e(t)}}{2\sin\pi\alpha_e(t)}(r_e\nu)^{\alpha_e(t)}$$

$$\frac{\pi B_i^\sigma(t)}{\Lambda}\left(\frac{\nu_N}{\Lambda}\right)^k + \int_{\nu_\pi}^{\Lambda} \text{Im}\ A_i^\sigma(\nu',t)\left(\frac{\nu'}{\Lambda}\right)^k\frac{d\nu'}{\Lambda} = \beta_i^\sigma(t)\frac{(r_i\Lambda)^{\alpha(t)-1}}{\alpha(t)+k},$$



Finite-Energy Sum Rules

Given the s-dependence at high energies,
one can **predict the t-dependence at *high energies***,
using only ***low-energy* models**

Amplitudes

$$M_1 = \frac{1}{2}\gamma_5\gamma_\mu\gamma_\nu F^{\mu\nu} ,$$
$$M_2 = 2\gamma_5 q_\mu P_\nu F^{\mu\nu} ,$$
$$M_3 = \gamma_5\gamma_\mu q_\nu F^{\mu\nu} ,$$
$$M_4 = \frac{i}{2}\epsilon_{\alpha\beta\mu\nu}\gamma^\alpha q^\beta F^{\mu\nu} .$$

$$\gamma p \to \pi^+ n : \sqrt{2}\left(A^{(0)} + A^{(-)}\right)$$
$$\gamma n \to \pi^- p : \sqrt{2}\left(A^{(0)} - A^{(-)}\right)$$
$$\gamma p \to \pi^0 p : \quad A^{(+)} + A^{(0)}$$
$$\gamma n \to \pi^0 n : \quad A^{(+)} - A^{(0)}$$

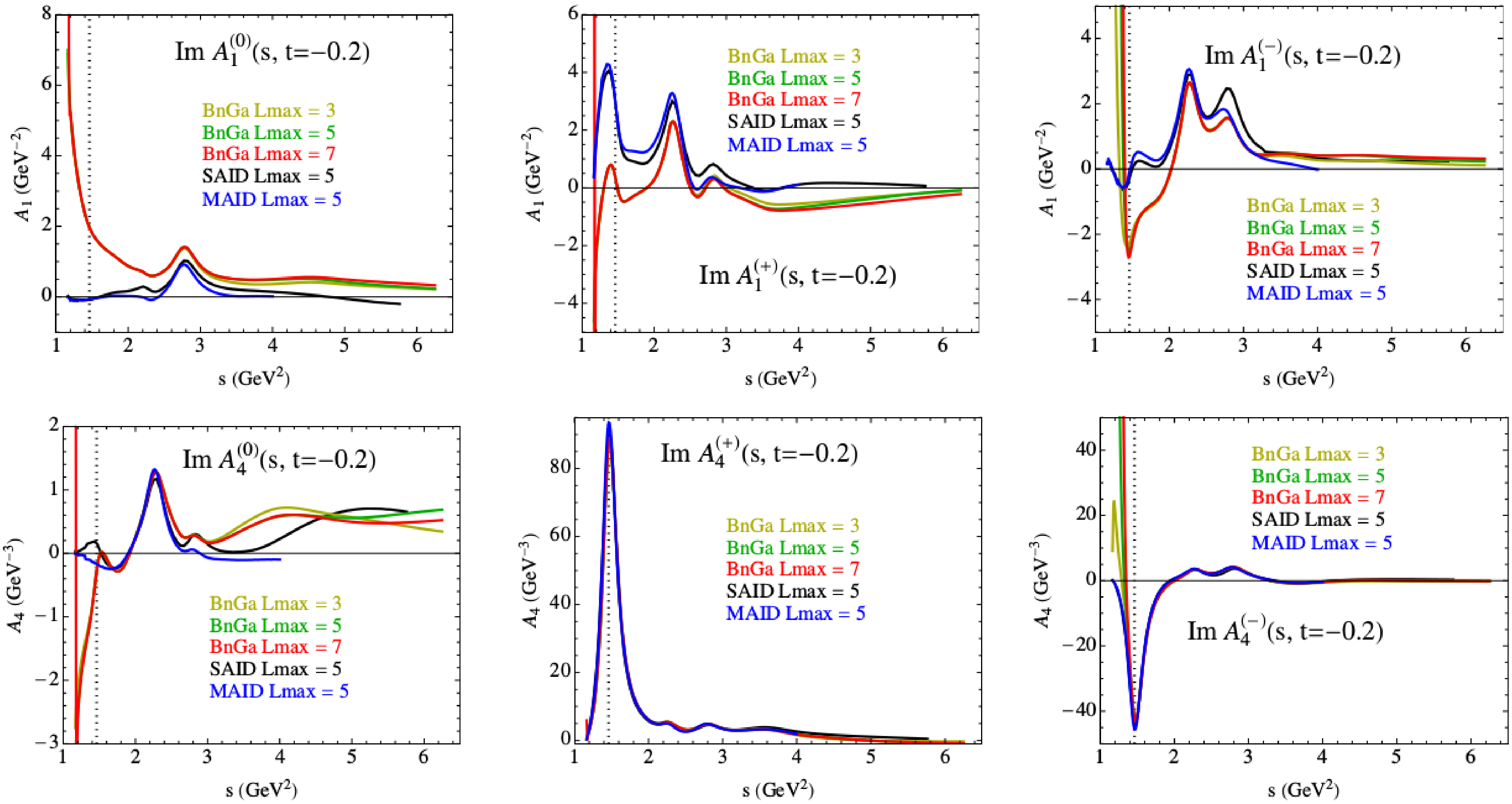
$A_i^{(\sigma)}$	I^G	$P(-1)^J$	τ	J^{PC}	Lowest spin
$A_{1,4}^{(0)}$	1^+	$+1$	-1	$(1,3,5,...)^{--}$	$\rho(770)$
$A_{1,4}^{(+)}$	0^-	$+1$	-1	$(1,3,5,...)^{--}$	$\omega(782)$
$A_{1,4}^{(-)}$	1^-	$+1$	$+1$	$(2,4,6,...)^{++}$	$a_2(1320)$
$A_2'^{(0)}$	1^+	-1	-1	$(1,3,5,...)^{+-}$	$b_1(1235)$
$A_2'^{(+)}$	0^-	-1	-1	$(1,3,5,...)^{+-}$	$h_1(1170)$
$A_2'^{(-)}$	1^-	-1	$+1$	$(0,2,6,...)^{-+}$	$\pi(140)$
$A_3^{(0)}$	1^+	-1	$+1$	$(2,4,6,...)^{--}$	$\rho_2(?)$
$A_3^{(+)}$	0^-	-1	$+1$	$(2,4,6,...)^{--}$	$\omega_2(?)$
$A_3^{(-)}$	1^-	-1	-1	$(1,2,5,...)^{++}$	$a_1(1260)$

$$\gamma p \to \eta p , \qquad A = (\omega + h + \omega_2) + (\rho + b + \rho_2) ,$$
$$\gamma n \to \eta n , \qquad A = (\omega + h + \omega_2) - (\rho + b + \rho_2) .$$

$\boldsymbol{A_i}$	$\boldsymbol{I^G}$	$\boldsymbol{J^{PC}}$	$\boldsymbol{\eta}$	**Leading exchanges**
A_1	$0^-, 1^+$	$(1,3,5,...)^{--}$	$+1$	$\rho(770), \omega(782)$
A_2'	$0^-, 1^+$	$(1,3,5,...)^{+-}$	-1	$h_1(1170), b_1(1235)$
A_3	$0^-, 1^+$	$(2,4,...)^{--}$	-1	$\rho_2(??), \omega_2(??)$
A_4	$0^-, 1^+$	$(1,3,5,...)^{--}$	$+1$	$\rho(770), \omega(782)$

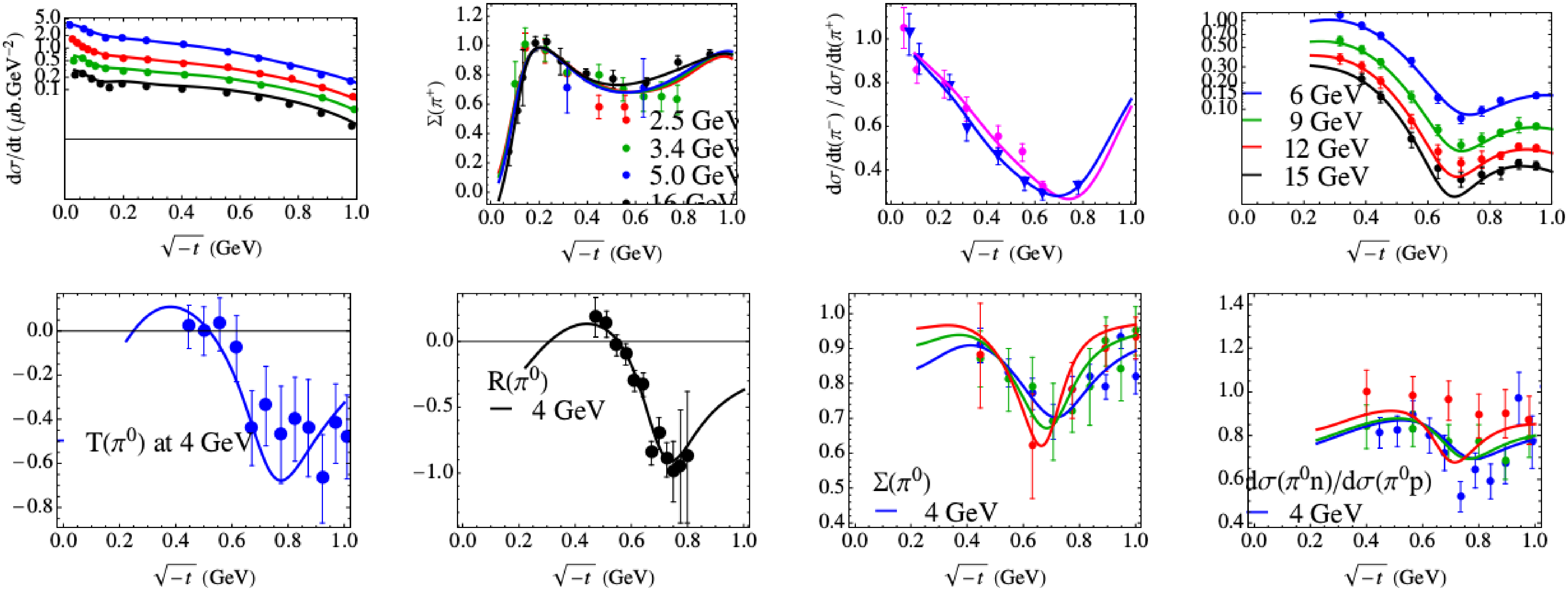
$$\Sigma = \frac{\left(|A_1|^2 - t\,|A_4|^2\right) - \left(|A_2'|^2 - t\,|A_3|^2\right)}{\left(|A_1|^2 - t\,|A_4|^2\right) + \left(|A_2'|^2 - t\,|A_3|^2\right)}$$

Pion photoproduction



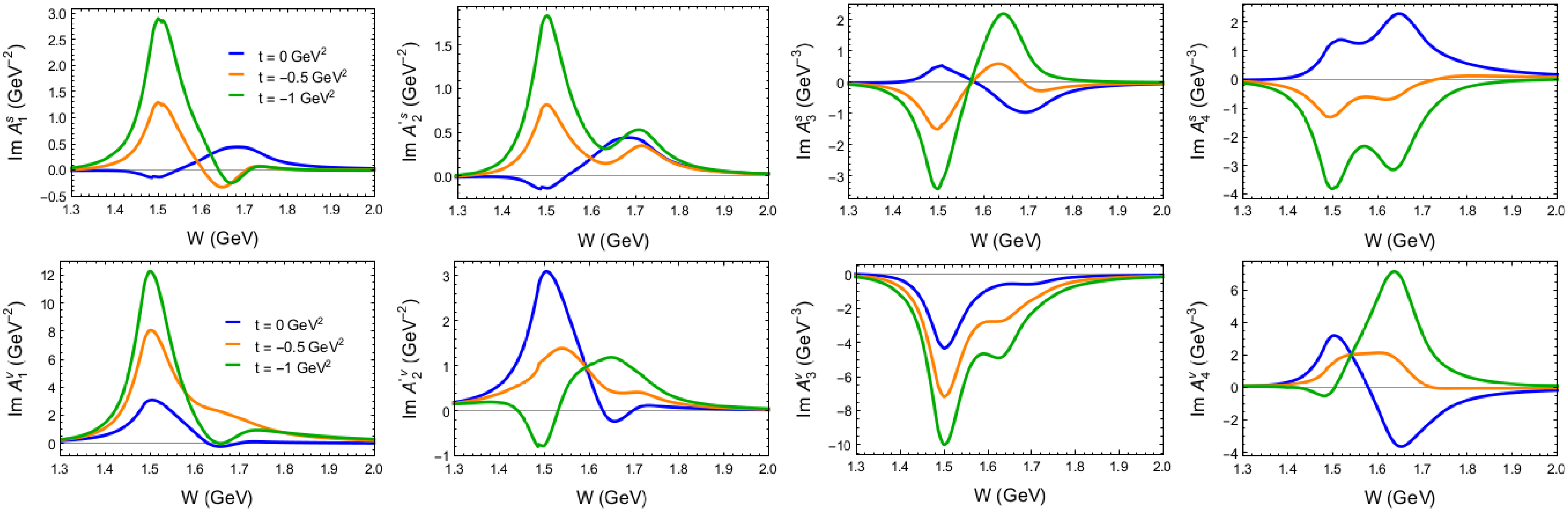
V. Mathieu (in prep.)

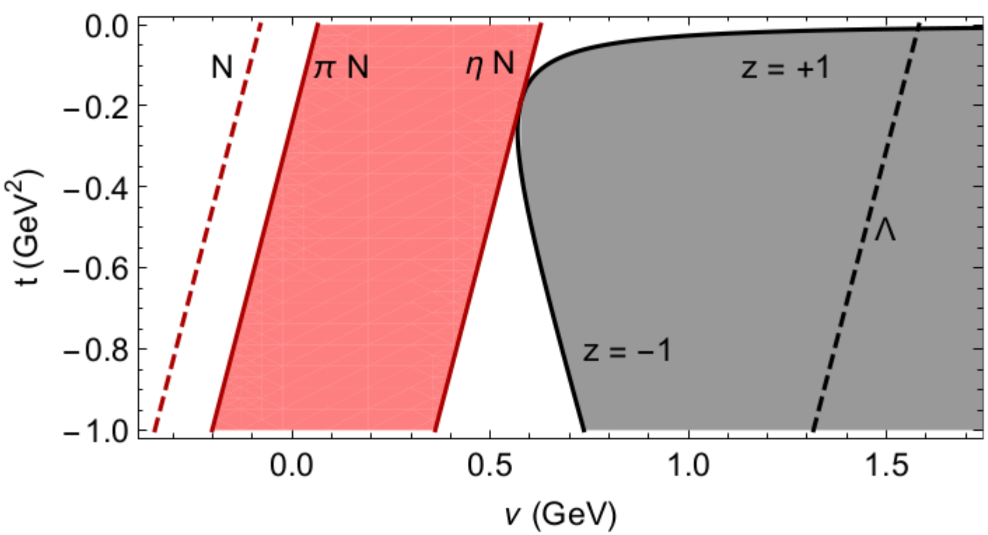
Pion photoproduction



$$A^e_{\mu_f,\mu_i\,\mu_\gamma}(s,t) = -\beta^e_{\mu_\gamma}(t)\beta^e_{\mu_i\mu_f}(t)\frac{\tau_e + e^{-i\pi\alpha_e(t)}}{2\sin\pi\alpha_e(t)}(r_e\nu)^{\alpha_e(t)}$$

Eta photoproduction





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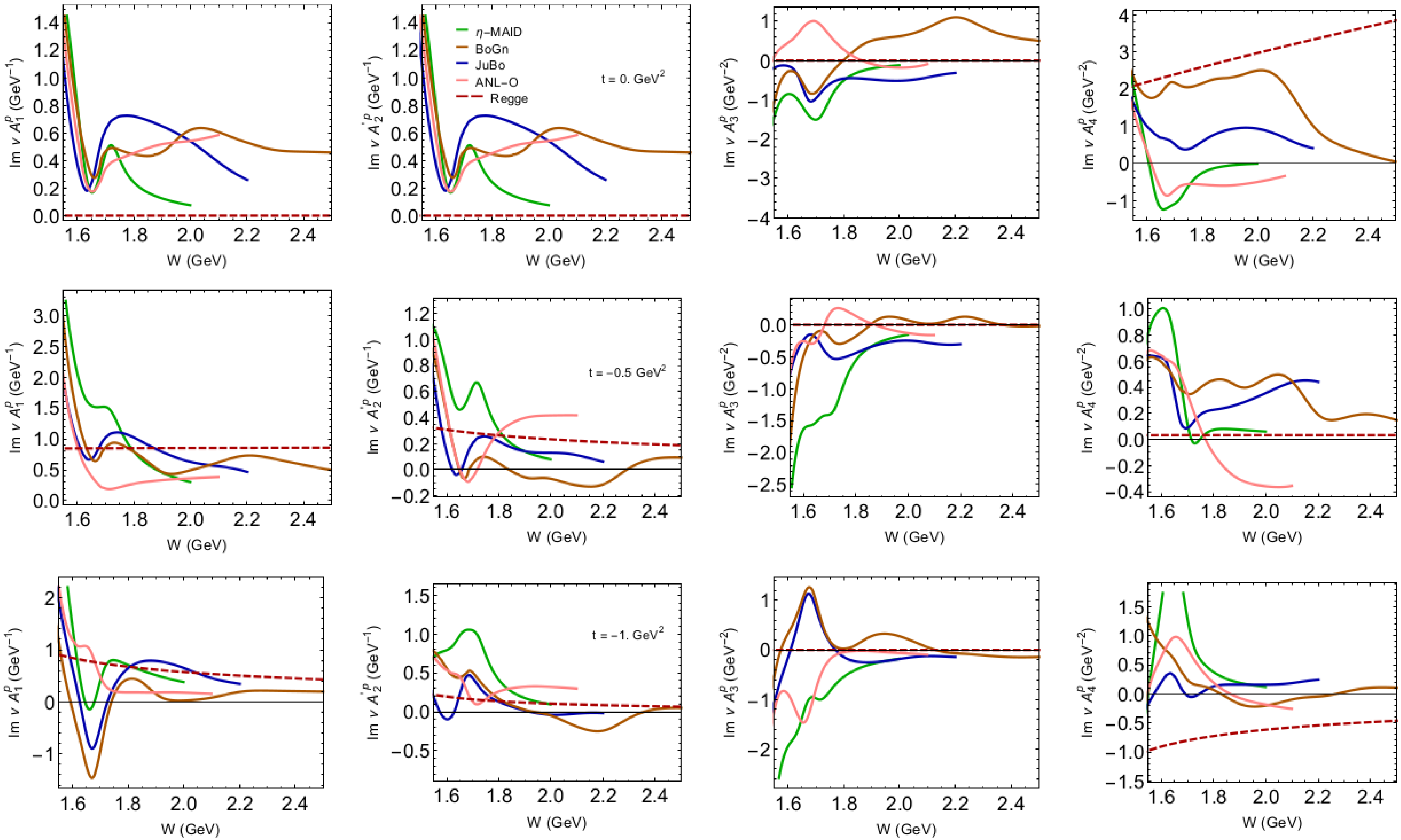
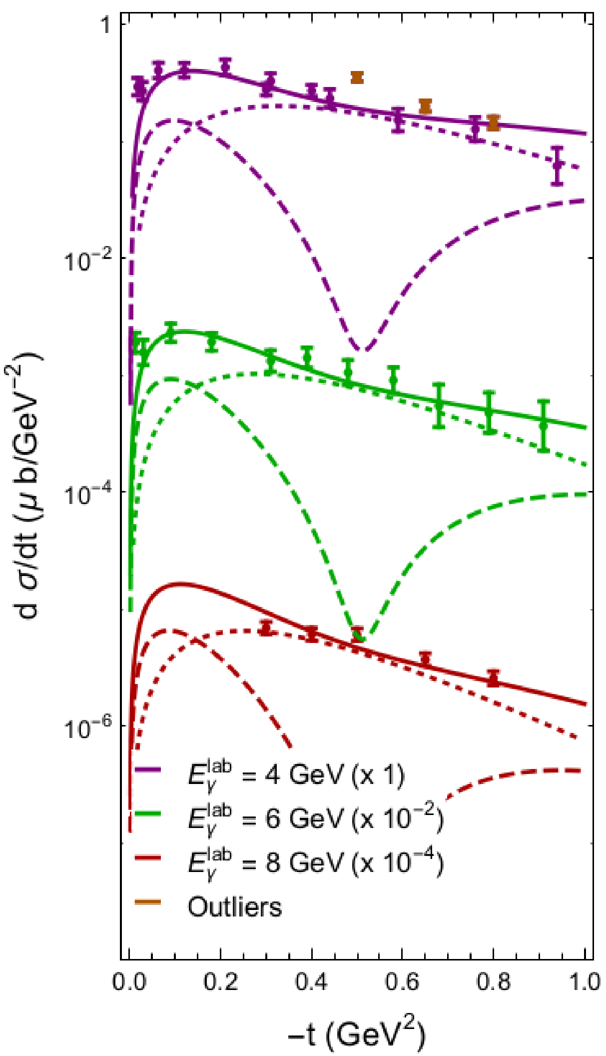
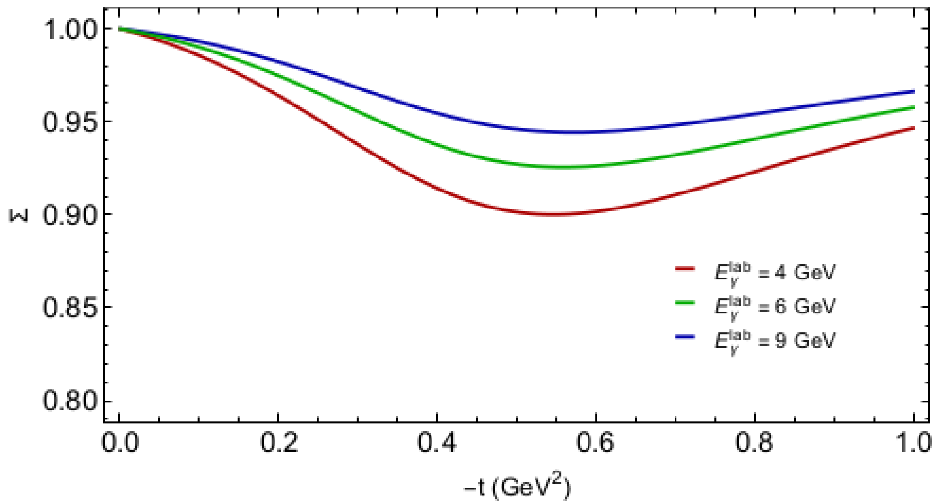
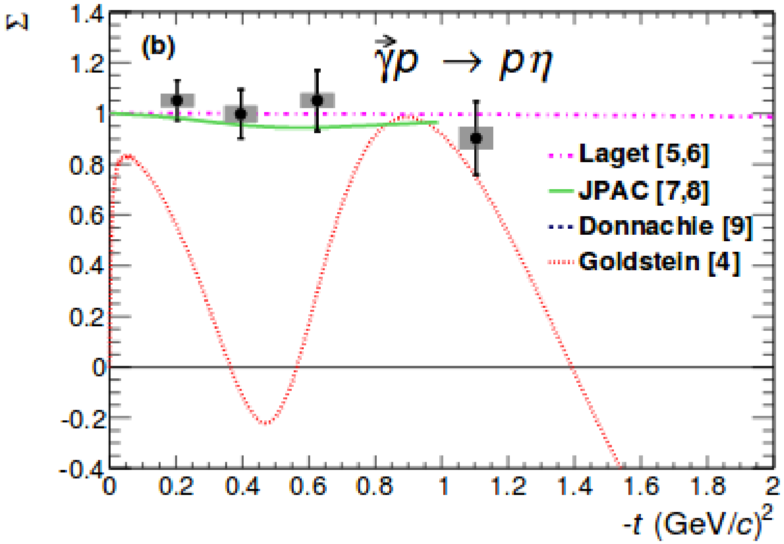


Eta photoproduction

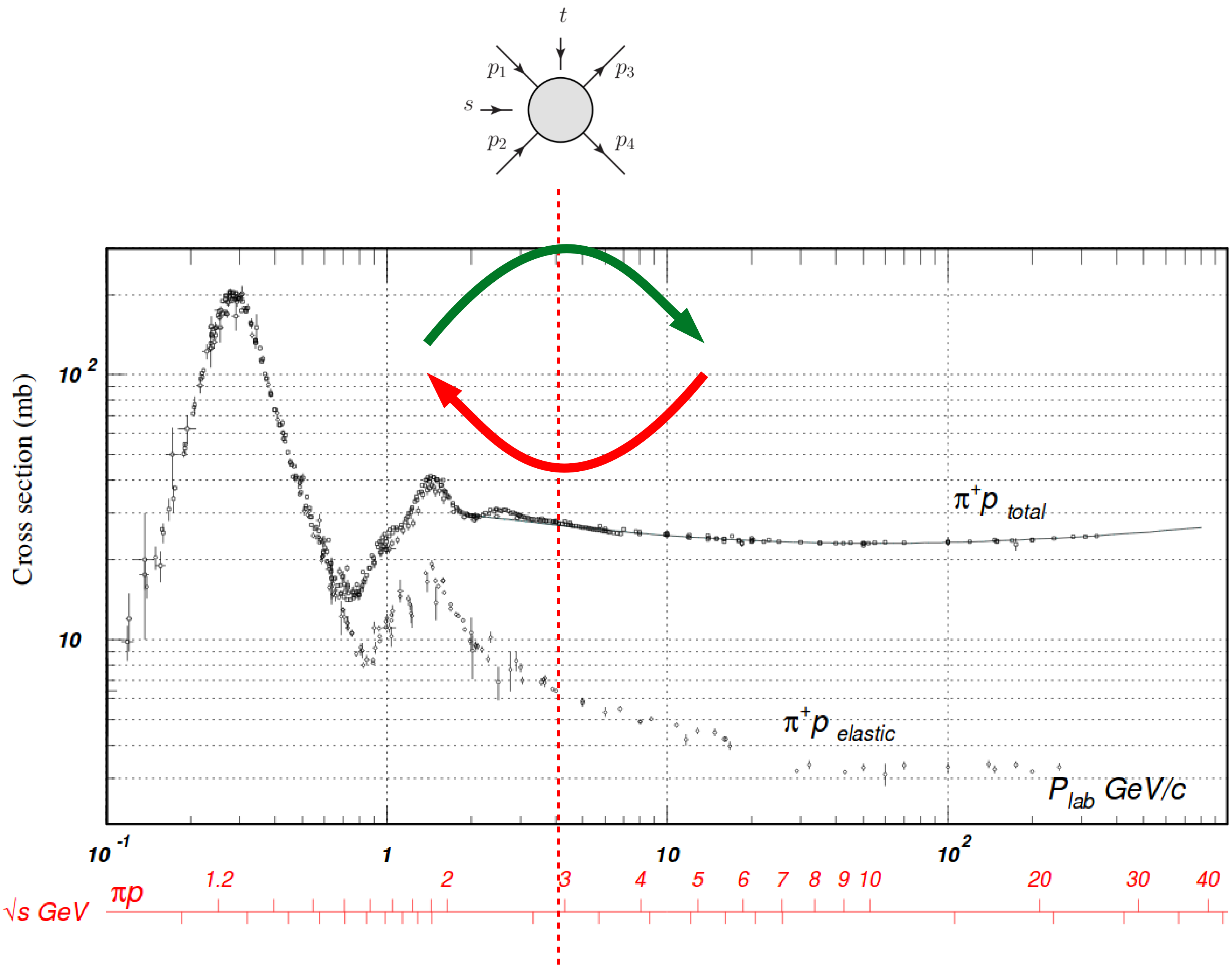






Al Ghoul *et al.* (GlueX) 1701.08123

Finite-Energy Sum Rules



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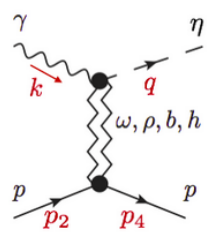
$\gamma p \to \eta p$



We present the model published in [Nys16] .
The differential cross section for $\gamma p \to \eta p$ is computed with Regge amplitudes in the domain $E_\gamma \geq 4$ GeV and $0 \leq -t \leq 1$ (in GeV^2).
We use the CGLN invariant amplitudes A_i defined in [Chew57a].
See the section Formalism for the definition of the variables.
The model and its context is detailed in [Nys16] . We report here only the main features of the model.

Formalism

The differential cross section is a function of 2 kinematic variables. The first is the beam energy in the laboratory frame E_γ (in GeV) or the total energy squared s (in (... scattering angle in the rest frame $\cos\theta$ or the momentum transfer ... The momenta of the particles are k (photon), q (eta), p_2 (target) ... μ and the proton mass is M_N. The Mandelstam variables, $s = (k$ ...

- **Publication:** [Nys16]
- **C/C++ observables:** C-code main, Input file, C-code source, C-code header, Eta-MAID 2001 multipoles
- **C/C++ minimal script to calculate the amplitudes:** C-code zip
- **Data:** Dewire , Braunschweig
- **Contact person:** Jannes Nys
- **Last update:** November 2016

Step-by-step introduction to calculating the model amplitudes of the high-energy model.

[hide] [show]

Run the code

The simulation is temporarily not working properly

Choose the beam energy in the lab frame E_γ, the other variable (t or $\cos\theta$) and its minimal, maximal, and increment values.
If you choose t (cos) only the min, max and step values of t ($\cos\theta$) are read.
Only physical t-values are calculated. Hence, for example $t = 0$ will be set to $t(\cos\theta = +1)$. Below $W = 2$ GeV, we use the Eta-MAID 2001 model using the lowest $l \leq 5$ multipoles. Above $W = 2$ GeV, the Regge model is evaluated. There is no smooth transition.

E_γ in GeV [9]

(•) t ( ) cos

t in GeV^2 (min max step) [-1] [0] [0.01]

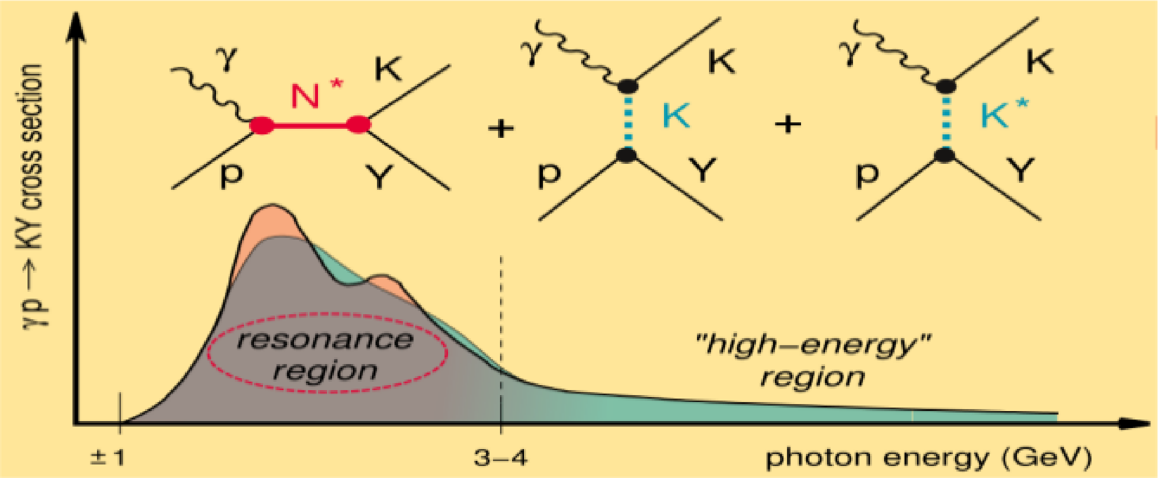
$\cos\theta$ (min max step) [0.85] [1] [0.01]

[Start] [reset]

http://www.indiana.edu/~jpac/

Something else: $\gamma^{(*)}p \to K^+\Lambda$

Regge-plus-resonance (RPR) approach [PRC86 (2012) 015212]



- Regge background: exchange of $K(494)$ and $K^*(892)$ Regge trajectories in t channel
- Enrich Reggeized background with $N^\star$: $J = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}$ with $M_{N^\star} \leq 2$ GeV

RPR-2011 (PDG-2010)

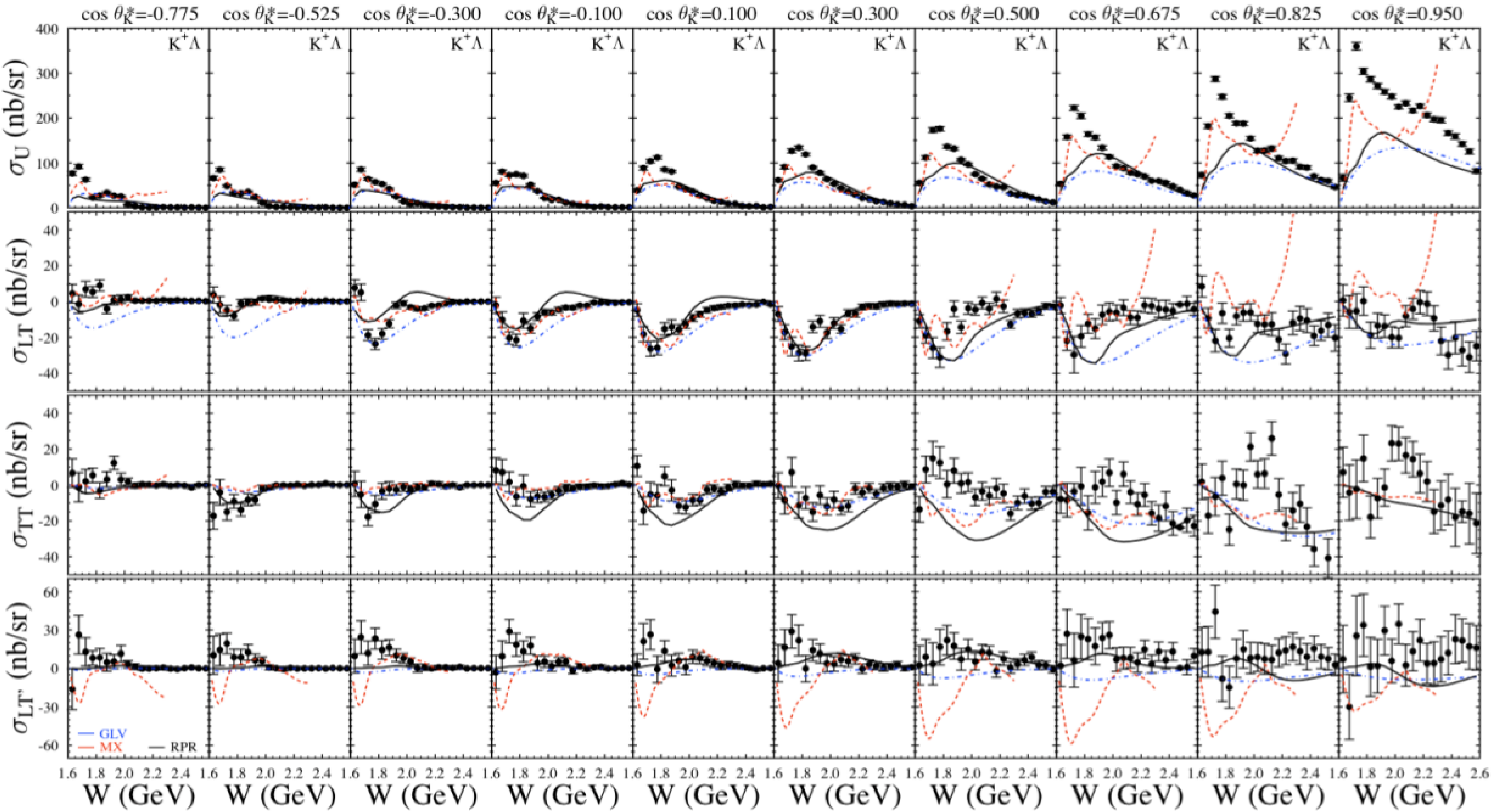
- $S_{11}(1535)$ ★★★★
- $S_{11}(1650)$ ★★★★
- $D_{15}(1675)$ ★★★★
- $F_{15}(1680)$ ★★★★
- $D_{13}(1700)$ ★★★
- $P_{11}(1710)$ ★★★
- $P_{13}(1720)$ ★★★★
- $D_{13}(1875)$ m
- $P_{13}(1900)$ ★★
- $P_{11}(1900)$ m
- $F_{15}(2000)$ ★★★

Bayesian inference of the resonance content of $p(\gamma, K^+)\Lambda$ [PRL108 (2012) 182002]

$S_{11}(1535)$, $S_{11}(1650)$, $F_{15}(1680)$, $P_{13}(1720)$,
$D_{13}(1875)$, $P_{13}(1900)$, $P_{11}(1900)$, and $F_{15}(2000)$

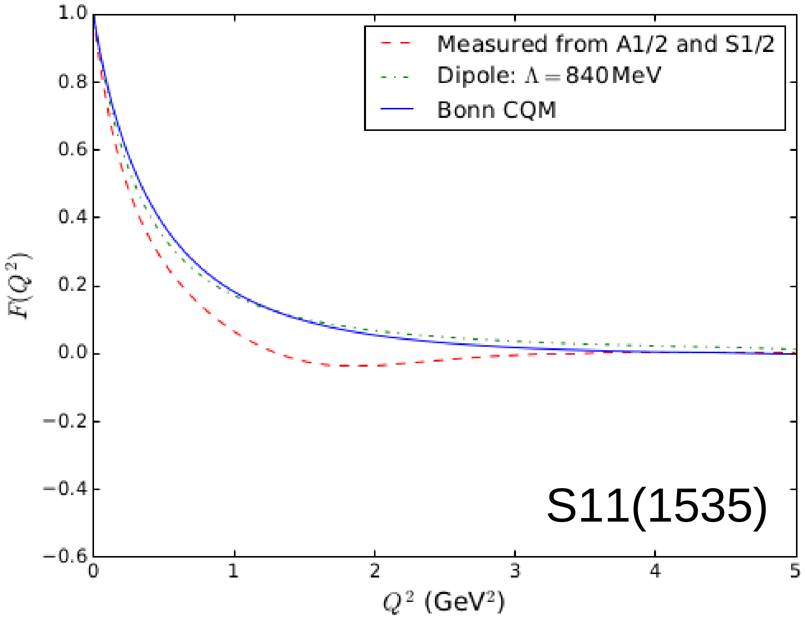
- **17 parameters**

Something else: $\gamma^{(*)}p \to K^+\Lambda$

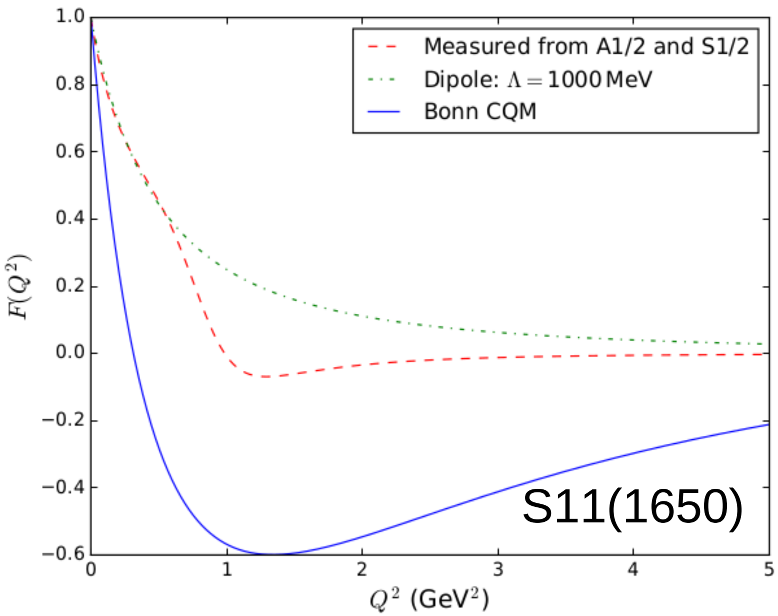


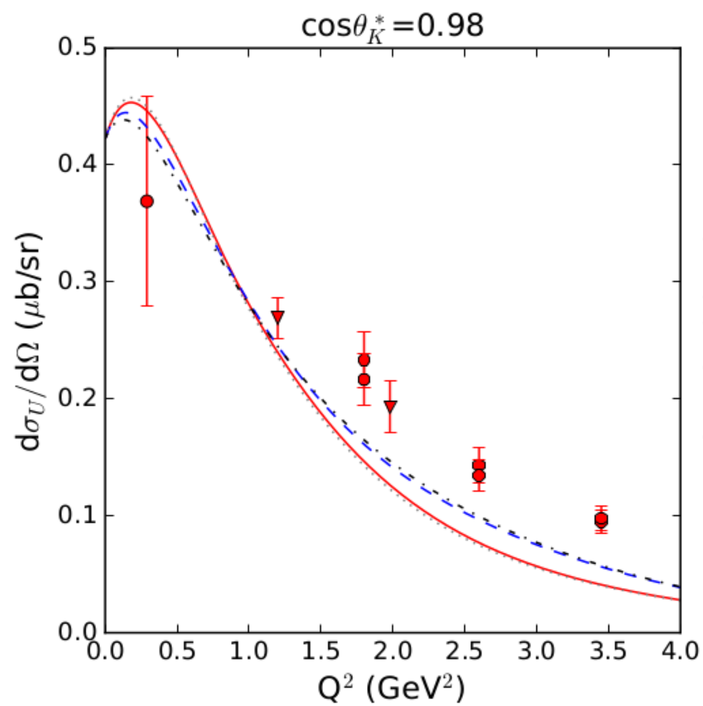
E = 5.499 GeV, W: thr – 2.6 GeV, Q^2 = 1.80, 2.60, 3.45 GeV² *[Carman et al., PRC 87, 025204 (2013)]*

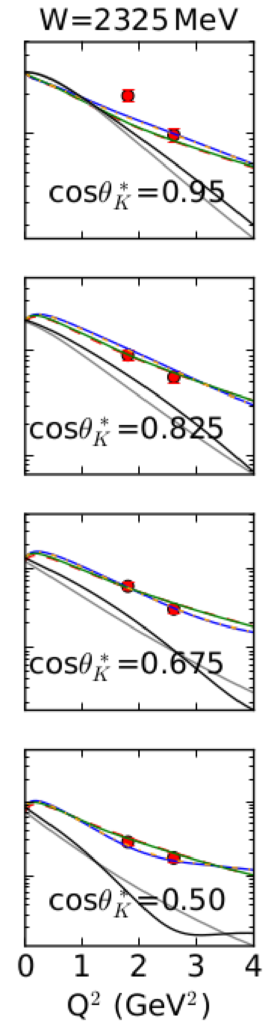
Something else: $\gamma^{(*)}p \rightarrow K^+\Lambda$



$$f_2^{\pm}(Q^2) = \frac{\mp 1}{Q_\pm^3}\left[\frac{Q^2}{|\boldsymbol{p}|}\mathcal{M}^0_{\frac{1}{2},\frac{1}{2}} \mp \frac{m_R \pm m_p}{2}\mathcal{M}^+_{\frac{1}{2},-\frac{1}{2}}\right]$$

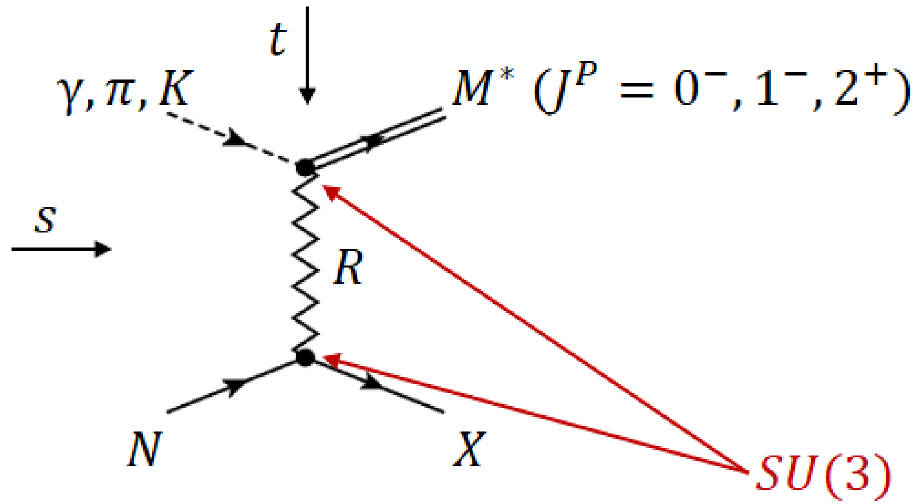


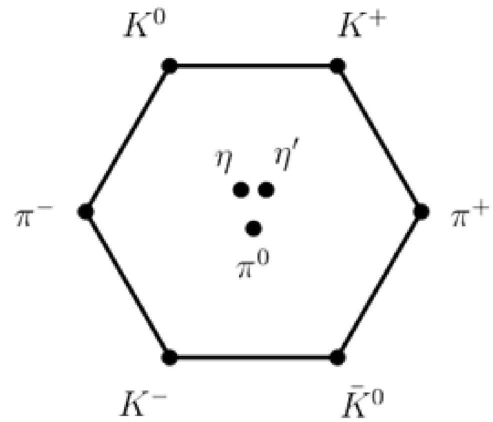




Coupled Regge analysis of meson resonance production

[Fox and Hey, Nucl. Phys. B 56 (1973) 386]





Ingredients

- Regge pole exchange
- Residue factorization
- $SU(3)$ symmetry
- Vector-meson dominance

$$H^{(s)}_{\mu_3\mu_4,\mu_2\mu_1} = \frac{V(t)}{\sin \pi\alpha(t)}\underbrace{g_{\mu_3\mu_1}(t)\,g_{\mu_4\mu_2}(t)}_{\text{factorization}}\left(\frac{\nu}{-t}\,\frac{1-\cos\vartheta_s}{2}\right)^{\frac{|\mu_i-\mu_f|}{2}}\left(\frac{1+\cos\vartheta_s}{2}\right)^{\frac{|\mu_i+\mu_f|}{2}}$$

Data collection

- γ, π, K beam
- Nucleon target
- $0^-, 1^-, 2^+$ meson production
- Low $-t$, high s
- Data format:

E_{lab}	t	t_{max}	t_{min}	O	$+\Delta O$	$-\Delta O$	$Reaction$	Obs	Ref
8.9	-0.0062301713197332464	-0.001230171319733353	-0.011230171319733362	13.15	1.68	-1.68	GAMMA P --> OMEGA P	DSIGDT	Abr76o
8.9	-0.016230171319733255	-0.011230171319733362	-0.02123017131973337	10.63	1.52	-1.52	GAMMA P --> OMEGA P	DSIGDT	Abr76o
8.9	-0.026230171319733264	-0.02123017131973337	-0.03123017131973338	9.45	1.46	-1.46	GAMMA P --> OMEGA P	DSIGDT	Abr76o
8.9	-0.03623017131973327	-0.03123017131973338	-0.04123017131973339	8.05	1.28	-1.28	GAMMA P --> OMEGA P	DSIGDT	Abr76o
8.9	-0.04623017131973328	-0.04123017131973339	-0.0512301713197334	7.27	1.23	-1.23	GAMMA P --> OMEGA P	DSIGDT	Abr76o

(Observable: O, $+\Delta O$, $-\Delta O$)

Current status

- 7176 data points
- 55 reactions

Summary

- Finite-energy sum rules connect low and high energy regime
- Successful predictions at high energies based on low-energy models
- Provides information at the amplitude level

- Extending RPR (Ghent) from photoproduction to electroproduction