Hadron charges and role of excited states

David Richards
Jefferson Lab

The Proton Mass: at the Heart of most Visible Matter
ECT, 3rd -7th April, 2017*

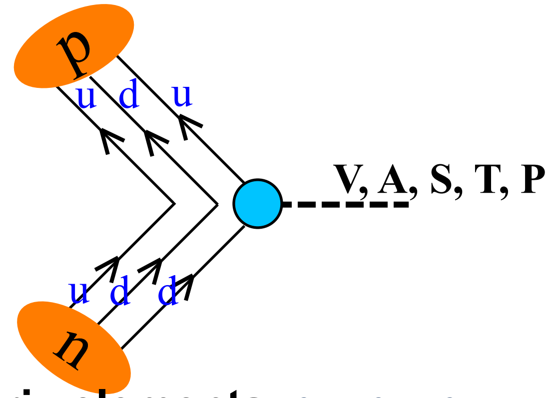
Outline

- Nucleon Charges and Matrix elements - *removing excited states.*
- The excited-state spectrum - *examining excited states and their mass origin*

Precise and Accurate Hadron Matrix Elements

Fundamental Symmetries: Nucleon Charges

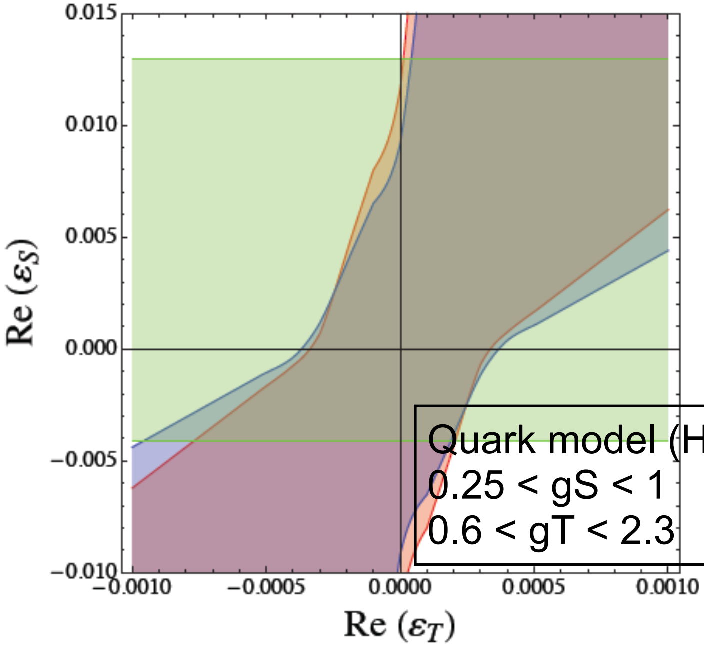
e.g. novel interactions probed in ultra-cold neutron decay



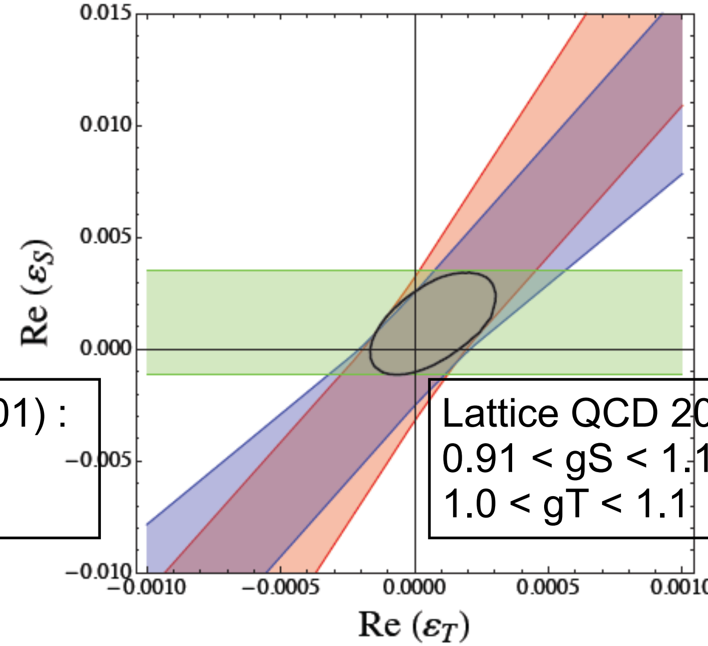
$$H_{eff} \supset G_F\left[\varepsilon_S\, \bar{u}d \times \bar{e}(1-\gamma_5)\nu_e + \varepsilon_T\, \bar{u}\sigma_{\mu\nu}d \times \bar{e}\sigma^{\mu\nu}(1-\gamma_5)\nu_e\right]$$

$$g_S = Z_S \langle p|\bar{u}d|n\rangle \quad g_T = Z_T \langle p|\bar{u}\sigma_{\mu\nu}\, d|n\rangle$$

Require precise knowledge of QCD nucleon matrix elements, g_A, g_T, g_S

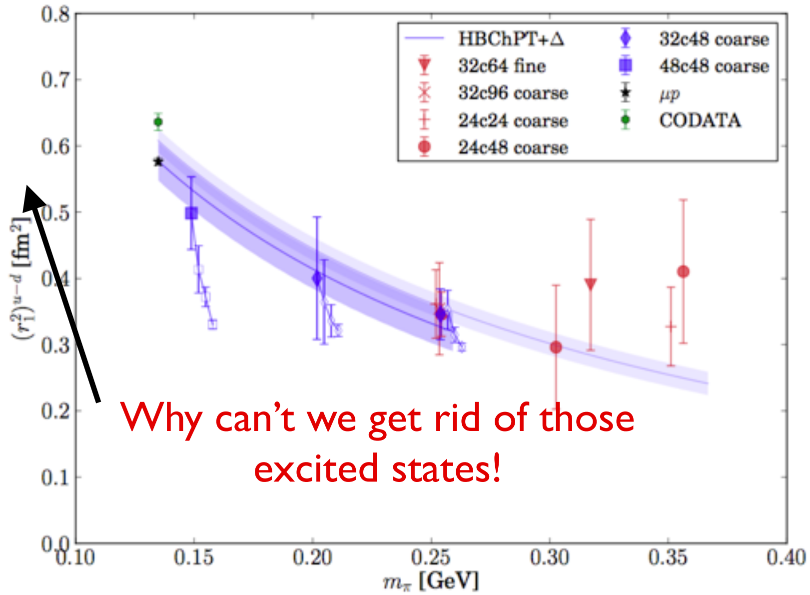


Quark model (Herczeg 2001) :
0.25 < gS < 1
0.6 < gT < 2.3



Lattice QCD 2014 (R. Gupta)
0.91 < gS < 1.13
1.0 < gT < 1.1

Hadron Structure

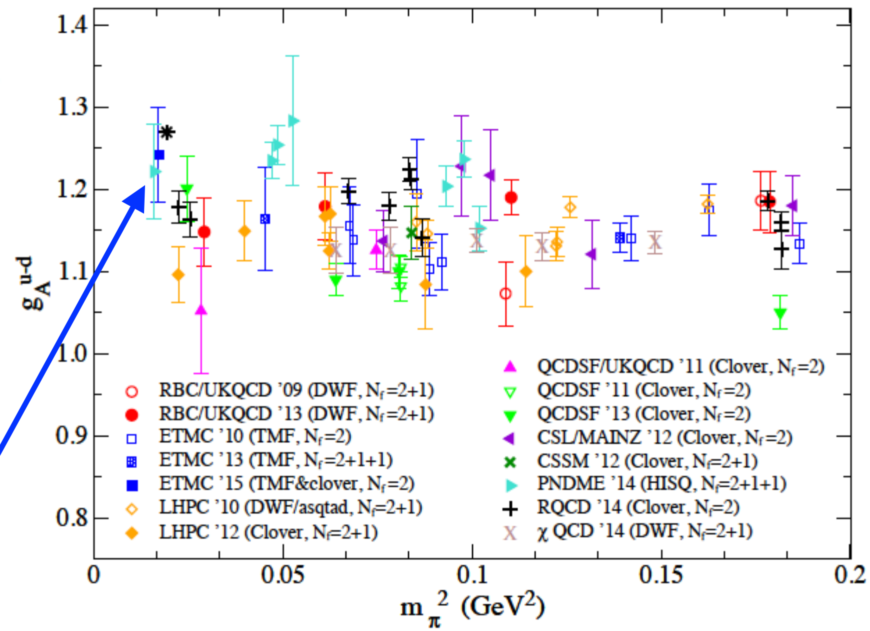


Why can't we get rid of those excited states!

Green et al, arXiv:1404.40

M Constantinou, arXiv:1511.00214

- Governs beta-decay rate
- Important for proton-proton fusion rate in solar models
- Benchmark for lattice QCD calculations of hadron structure



Luxury of large statistical errors! $m_\pi L < 4$

Calculation of Physics Observables

Our paradigm: nucleon mass

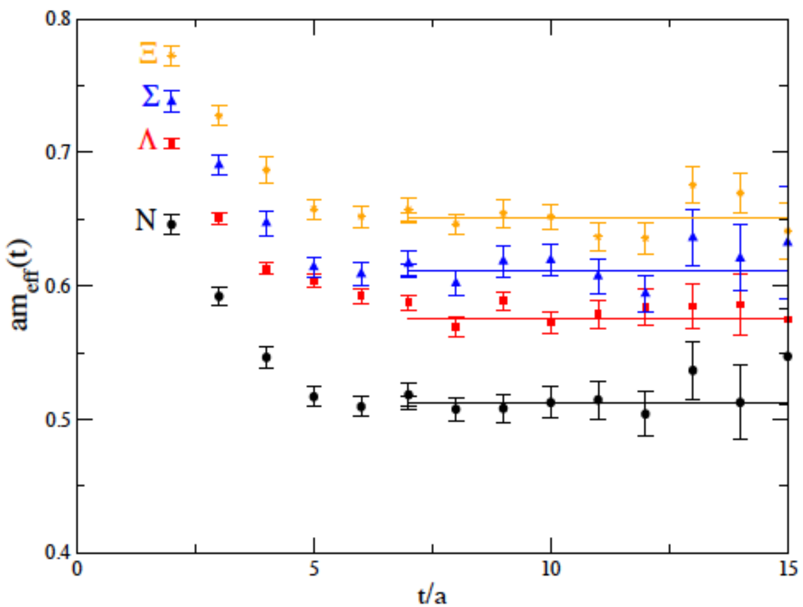
$$C(t) = \sum_{\vec{x}} \langle N(\vec{x},t)\bar{N}(0)\rangle = \sum_n A_n e^{-E_n t}$$

Noise:

$$C_{\sigma^2}(t) = \sum_{\vec{x}} \langle \bar{N}N(\vec{x},t)\bar{N}N(0)\rangle \propto \sum_n A_n e^{-3m_\pi t}$$

whence

$$\boxed{C(t)/\sqrt{C_{\sigma^2}(t)} \simeq e^{-(m_N - 3m_\pi/2)t}}$$



Alexandrou et al, arXiv:0910.2149

$$m_{\rm eff} = \ln C(t)/C(t+1) \to E_0$$

Variational Method

Subleading terms → *Excited states*

Construct matrix of correlators with *judicious choice of operators*

$$C_{ij}(t,0) = \frac{1}{V_3}\sum_{\vec{x},\vec{y}} \langle \mathcal{O}_i(\vec{x},t)\mathcal{O}_j^\dagger(\vec{y},0)\rangle = \sum_N \frac{Z_i^{N*} Z_j^N}{2E_N} e^{-E_N t}$$

$$Z_i^N = \langle N \mid \bar{N}_i(0) \mid 0\rangle$$

Delineate contributions using *variational method*: solve

$$C(t)v^{(N)}(t,t_0) = \lambda_N(t,t_0)C(t_0)v^{(N)}(t,t_0).$$

$$\lambda_N(t,t_0) \to e^{-E_N(t-t_0)}(1+\mathcal{O}(e^{-\Delta E(t-t_0)}))$$

Eigenvectors, with metric $C(t_0)$, are orthonormal and project onto the respective states

$$v^{(N')\dagger}C(t_0)v^{(N)} = \delta_{N,N'}$$

$$Z_i^N = \sqrt{2m_N}e^{m_N t_0/2}v_j^{(N)*}C_{ji}(t_0).$$

Excited States: Variational Method

Replace quark field by spatially extended (smeared) quark field

$$\psi \longrightarrow (1 - \sigma^2 \nabla^2 / 4N)^N \psi$$

Use local nucleon interpolating operators

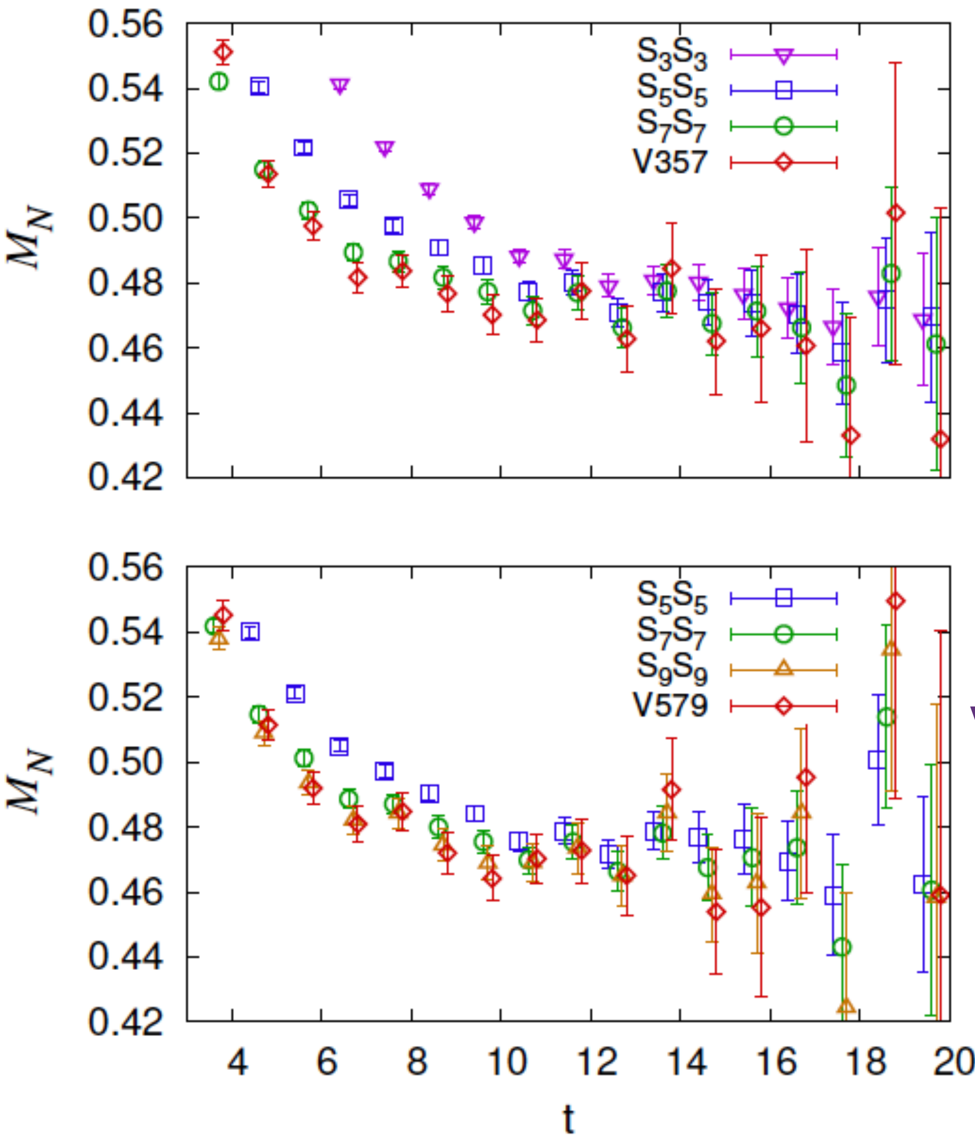
$$[u C \gamma_5 (1 \pm \gamma_4) d] u$$

ID	Method	Analysis	Smearing Parameters	t_{sep}	LP	HP
R1	AMA	2-state	$\{5, 60\}$	10,12,14,16,18	96	3
R2	LP	VAR	$\{3, 22\}$, $\{5, 60\}$, $\{7, 118\}$	12	96	
R3	AMA	VAR	$\{5, 46\}$, $\{7, 91\}$, $\{9, 150\}$	12	96	3
R4	AMA	2-state	$\{9, 150\}$	10,12,14,16,18	96	3

Only admits calculation of quasi-local operators - not orbital structure

Yoon et al., Phys. Rev. D 93, 114506 (2016)

Nucleon Mass - II



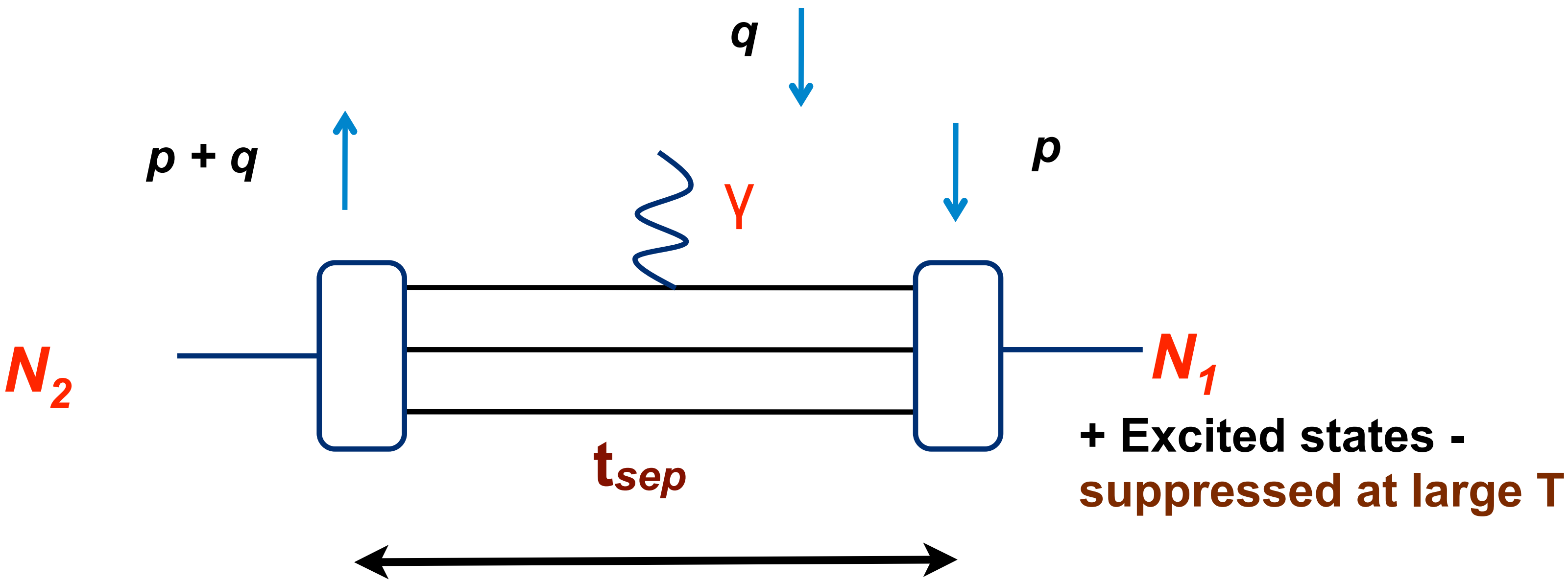
$$m_{\text{eff}} = \ln C(t)/C(t+1) \to E_0$$

Variational method: single-state domination nearer source
increase signal-to-noise

} **Fixed Smearing**

Variational

Matrix Elements



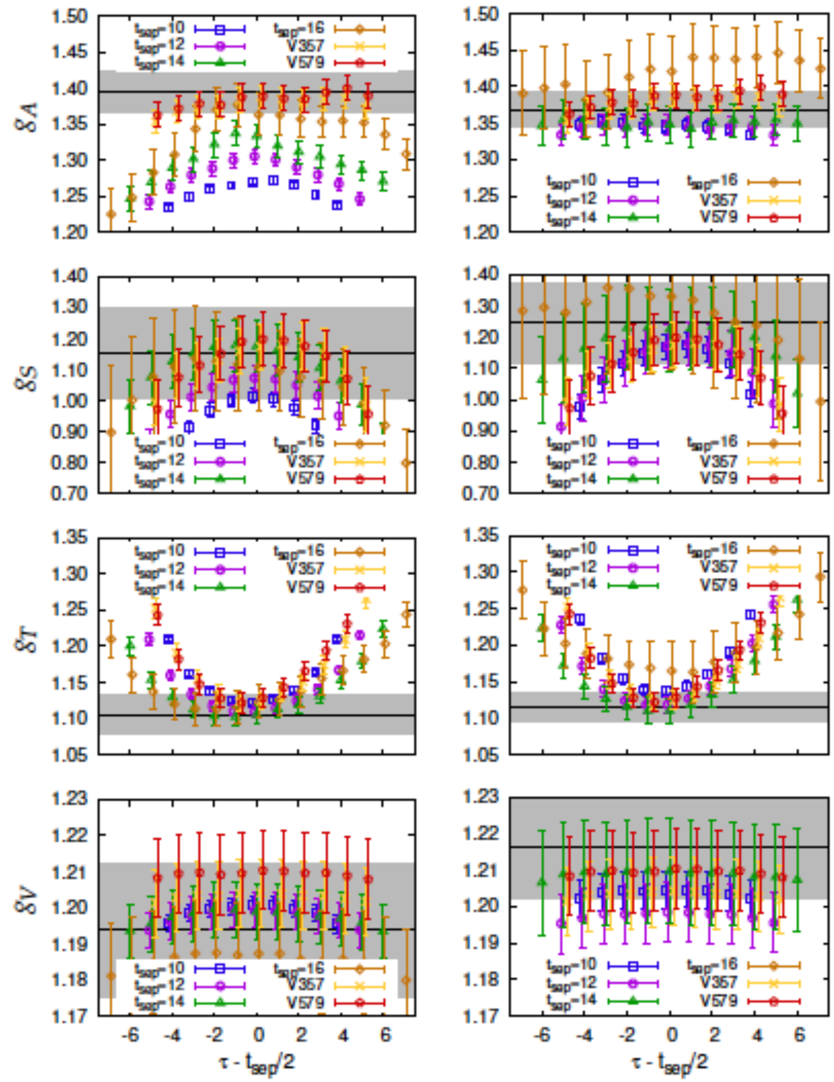
$$C_{3\mathrm{pt}}(t_{sep}, t; \vec{p}, \vec{q}) = \sum_{\vec{x},\vec{y}} \langle 0 \mid N(\vec{x}, t_{\mathrm{sep}}) V_\mu(\vec{y}, t) \bar{N}(\vec{0}, 0) \mid 0 \rangle e^{-i\vec{p}\cdot\vec{x}} e^{-i\vec{q}\cdot\vec{y}}$$

Optimal Interpolating Operator from Variational Method

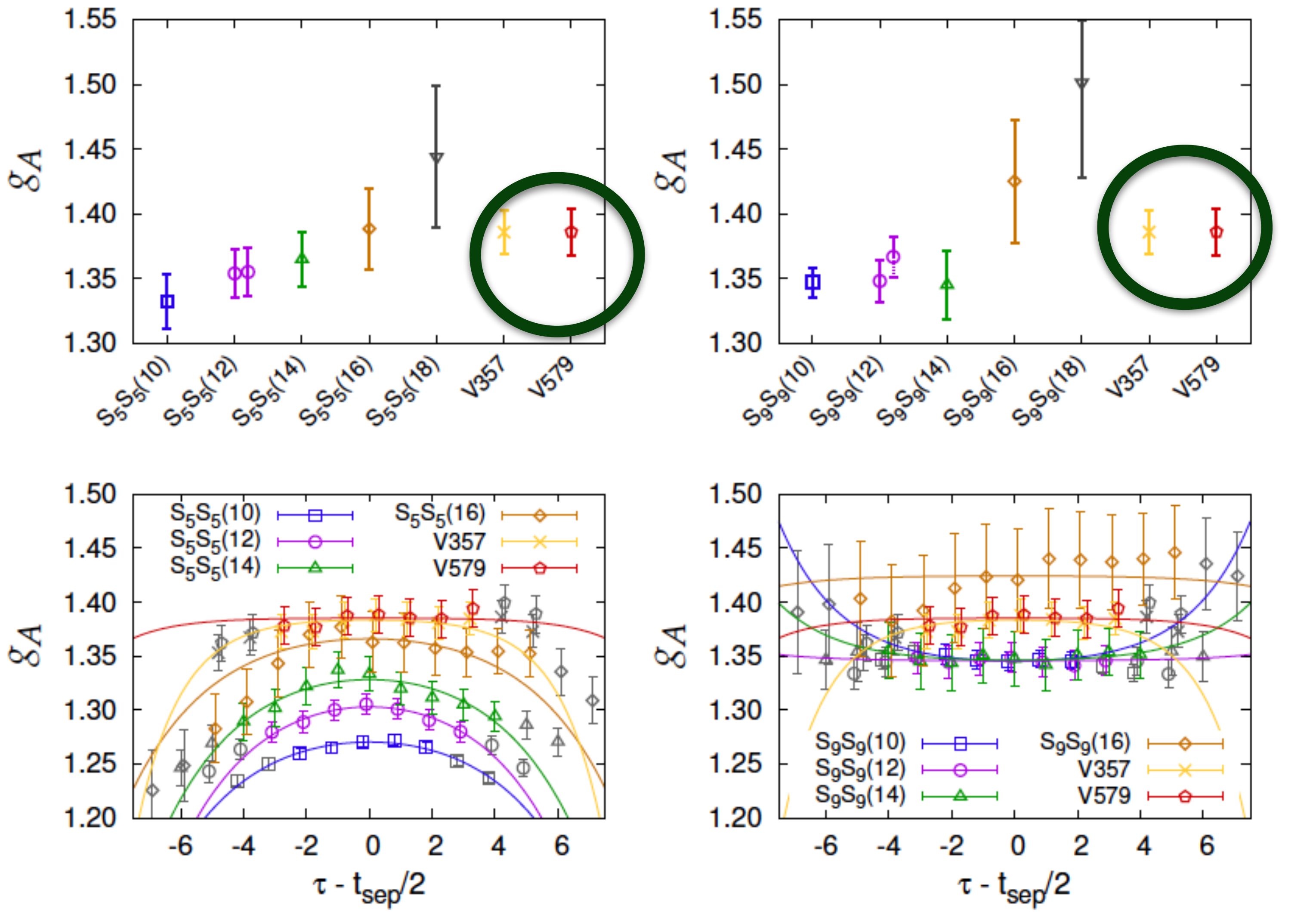
$$\longrightarrow \langle 0 \mid N \mid N, \vec{p}+\vec{q}\rangle \langle N, \vec{p}+\vec{q} \mid V_\mu \mid N \vec{p}\rangle \langle N, \vec{p} \mid \bar{N} \mid 0 \rangle e^{-E(\vec{p}+\vec{q})(t_{\mathrm{sep}}-t)} e^{-E(\vec{p})t}$$

Variational Method

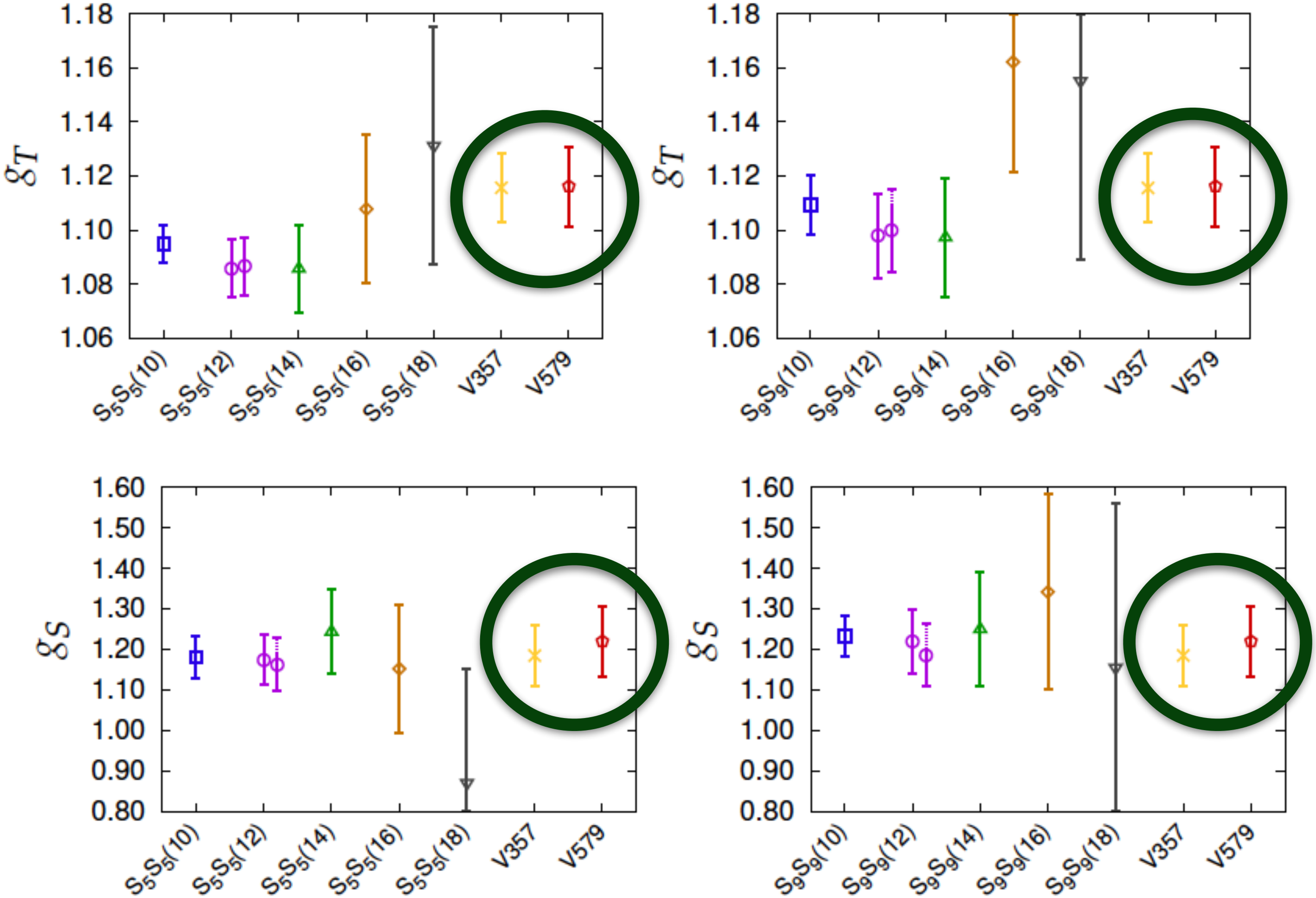
Grey Band : $t_{\text{sep}} \to \infty$



Variational Comparison - II



Variational - III



Controlling excited states essential for precision calculations!

Renormalized Charges

ID	Lattice Theory	a fm	M_π(MeV)	g_A^{u-d}	g_S^{u-d}	g_T^{u-d}	g_V^{u-d}
$a127m285$	2+1 clover-on-clover	0.127(2)	285(6)	1.249(28)	0.89(5)	1.023(21)	1.014(2
$a12m310$	2+1+1 clover-on-HISQ	0.121(1)	310(3)	1.229(14)	0.84(4)	1.055(36)	0.969(2
$a094m280$	2+1 clover-on-clover	0.094(1)	278(3)	1.208(33)	0.99(9)	0.973(36)	0.998(2
$a09m310$	2+1+1 clover-on-HISQ	0.089(1)	313(3)	1.231(33)	0.84(10)	1.024(42)	0.975(3
$a091m170$	2+1 clover-on-clover	0.091(1)	166(2)	1.210(19)	0.86(9)	0.996(23)	1.012(2
$a09m220$	2+1+1 clover-on-HISQ	0.087(1)	226(2)	1.249(35)	0.80(12)	1.039(36)	0.969(3
$a09m130$	2+1+1 clover-on-HISQ	0.087(1)	138(1)	1.230(29)	0.90(11)	0.975(38)	0.971(3

Consistency between different actions

ID	Type	$\langle 0\|\mathcal{O}_A\|1\rangle$	$\langle 0\|\mathcal{O}_S\|1\rangle$	$\langle 0\|\mathcal{O}_T\|1\rangle$	$\langle 0\|\mathcal{O}_V\|1\rangle$	$\langle 1\|\mathcal{O}_A\|1\rangle$	$\langle 1\|\mathcal{O}_S\|1\rangle$	$\langle 1\|\mathcal{O}_T\|1\rangle$	$\langle 1\|\mathcal{O}_V\|1\rangle$
		−0.179(21)		0.182(16)		−0.9(2.4)		−0.2(1.2)	
			−0.35(4)		−0.014(2)		0.6(1.1)		0.80(34)
a127m285	S_5S_5	−0.172(18)	−0.37(4)	0.210(15)	−0.015(2)	0.75(48)	0.8(9)	0.42(27)	0.87(28)
		−0.295(58)	−0.45(15)	0.167(40)	−0.014(6)	1.5(3.0)	1.8(1.4)	0.54(86)	0.86(55)
		−0.295(57)	−0.45(15)	0.166(47)	−0.014(6)	1.46(54)	1.8(1.4)	0.54(41)	0.86(28)

Matrix Elements of 1st excited state?

Probing (Gluon) Contributions to Mass in Excited-State Spectrum

Variational Method Revisited

Subleading terms → *Excited* states

Construct matrix of correlators with *judicious choice of operators*

$$\begin{aligned} C_{\alpha\beta}(t,t_0) &= \langle 0 \mid \mathcal{O}_\alpha(t)\mathcal{O}^\dagger_\beta(t_0) \mid 0\rangle \\ &\longrightarrow \sum_n Z^n_\alpha Z^{n\dagger}_\beta e^{-M_n(t-t_0)} \end{aligned}$$

Delineate contributions using variational method: solve

$$C(t)u(t,t_0) = \lambda(t,t_0)C(t_0)u(t,t_0)$$

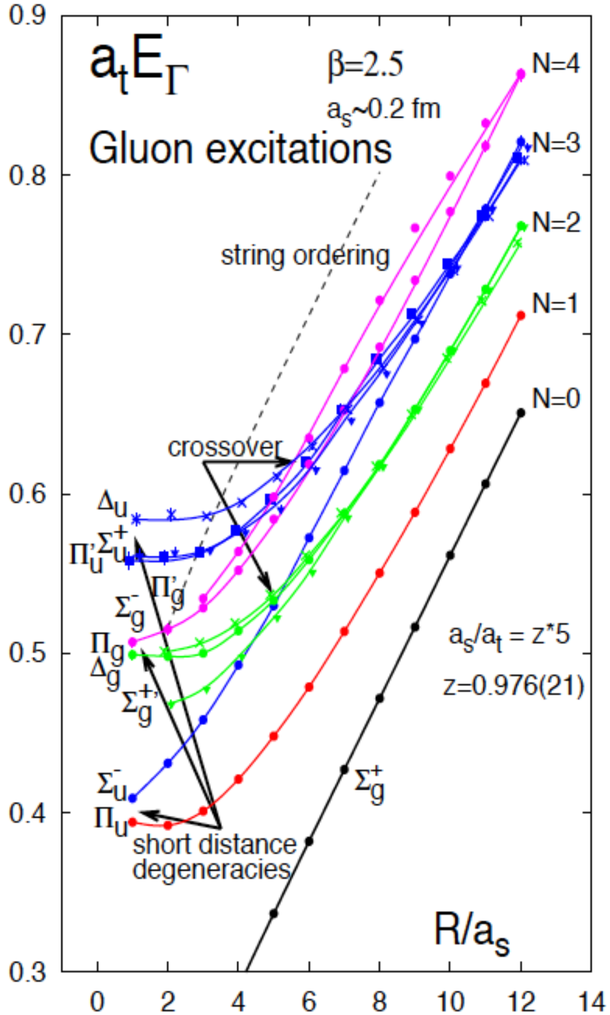
$$\lambda_i(t,t_0) \rightarrow e^{-E_i(t-t_0)}\left(1 + O(e^{-\Delta E(t-t_0)})\right)$$

Eigenvectors, with metric $C(t_0)$, are orthonormal and project onto the respective states

Signal-to-noise ratio degrades with increasing E - Solution: anisotropic lattice with $a_t < a_s$

Essential to almost all calculations of excited-state spectroscopy

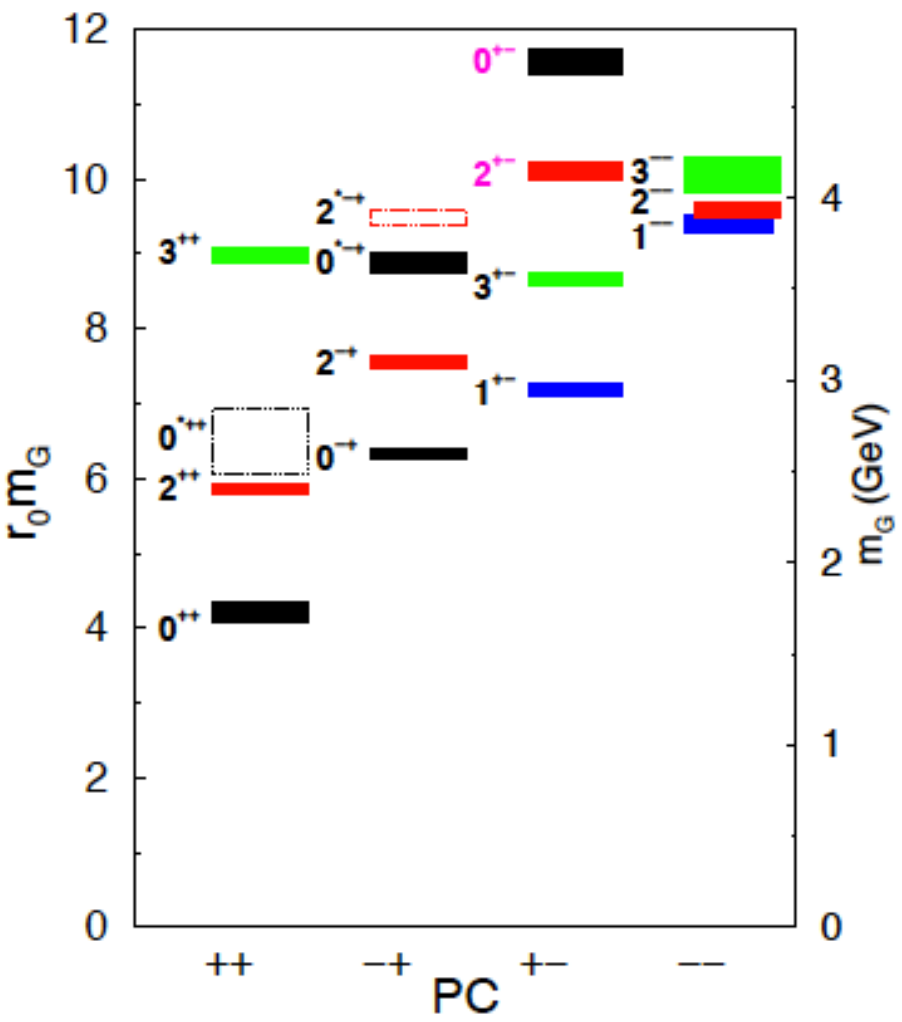
Excited-State Spectrum



Excitations of the string between infinitely heavy quarks

Juge, Kuti, Morningstar, 2002

Glueball Spectrum



Morningstar, Peardon 97,99

This is the pure Yang-Mills spectrum. Predicts existence of bound states with all mass coming from gluons!

Variational Method: Meson Operators

Aim: interpolating operators of *definite* (continuum) JM: O^{JM}

$$\langle 0 \mid O^{JM} \mid J', M' \rangle = Z^J \delta_{J,J'} \delta_{M,M'}$$

Starting point $\bar{\psi}(\vec{x}, t) \Gamma D_i D_j \ldots \psi(\vec{x}, t)$

Introduce circular basis:

$$\overleftrightarrow{D}_{m=-1} = \frac{i}{\sqrt{2}} \left(\overleftrightarrow{D}_x - i \overleftrightarrow{D}_y \right)$$
$$\overleftrightarrow{D}_{m=0} = i \overleftrightarrow{D}_z$$
$$\overleftrightarrow{D}_{m=+1} = -\frac{i}{\sqrt{2}} \left(\overleftrightarrow{D}_x + i \overleftrightarrow{D}_y \right).$$

Straighforward to project to definite spin - *for example J = 0, 1, 2*

$$(\Gamma \times D^{[1]}_{J=1})^{J,M} = \sum_{m_1, m_2} \left\langle 1, m_1; 1, m_2 \middle| J, M \right\rangle \bar{\psi} \Gamma_{m_1} \overleftrightarrow{D}_{m_2} \psi.$$

$D^{[2]}_{J=1}$ is the *signature* of hybrid

Caveat: rotational symmetry not a good symmetry of the lattice *but realized in practice for operators of "hadronic size"*

Baryon Operators

$$\langle 0 \mid O^{JM} \mid J', M' \rangle = Z^J \delta_{J,J'} \delta_{M,M'}$$

Starting point $B = \left(\mathcal{F}_{\Sigma_F} \otimes \mathcal{S}_{\Sigma_S} \otimes \mathcal{D}_{\Sigma_D}\right)\{\psi_1\psi_2\psi_3\}$

Introduce circular basis:

$$\overleftrightarrow{D}_{m=-1} = \frac{i}{\sqrt{2}}\left(\overleftrightarrow{D}_x - i\overleftrightarrow{D}_y\right)$$

$$\overleftrightarrow{D}_{m=0} = i\overleftrightarrow{D}_z$$

$$\overleftrightarrow{D}_{m=+1} = -\frac{i}{\sqrt{2}}\left(\overleftrightarrow{D}_x + i\overleftrightarrow{D}_y\right).$$

Straighforward to project to definite spin: *J = 1/2, 3/2, 5/2*

$$\left|[J,M]\right\rangle = \sum_{m_1,m_2} \left|[J_1,m_1]\right\rangle \otimes \left|[J_2,m_2]\right\rangle \langle J_1 m_1; J_2 m_2 | JM\rangle$$

$D^{[2]}_{J=1}$ is the *signature* of hybrid baryon

Distillation

Measure matrix of correlation functions:

$$C_{ij}(t) \equiv \sum_{\vec{x},\vec{y}} \langle N_i(\vec{x},t)\bar{N}_j(\vec{y},0)\rangle$$

M. Peardon *et al.*, PRD80,054506 (2009)

Can we evaluate such a matrix efficiently, for reasonable basis of operators?

Introduce $\tilde{\psi}(\vec{x},t) = L(\vec{x},\vec{y})\psi(\vec{y},t)$ **where L is 3D Laplacian**

Write $L \equiv (1-\kappa\nabla/n)^n = \sum_i f(\lambda_i)\xi^i \times \xi^{*i}$ **where λ_i and ξ_i are eigenvalues and eigenvectors of the Laplacian.**

We now truncate the expansion at i = N_{eigen} where N_{eigen} is sufficient to capture the low-energy physics.

Insert between each quark field in our correlation function.

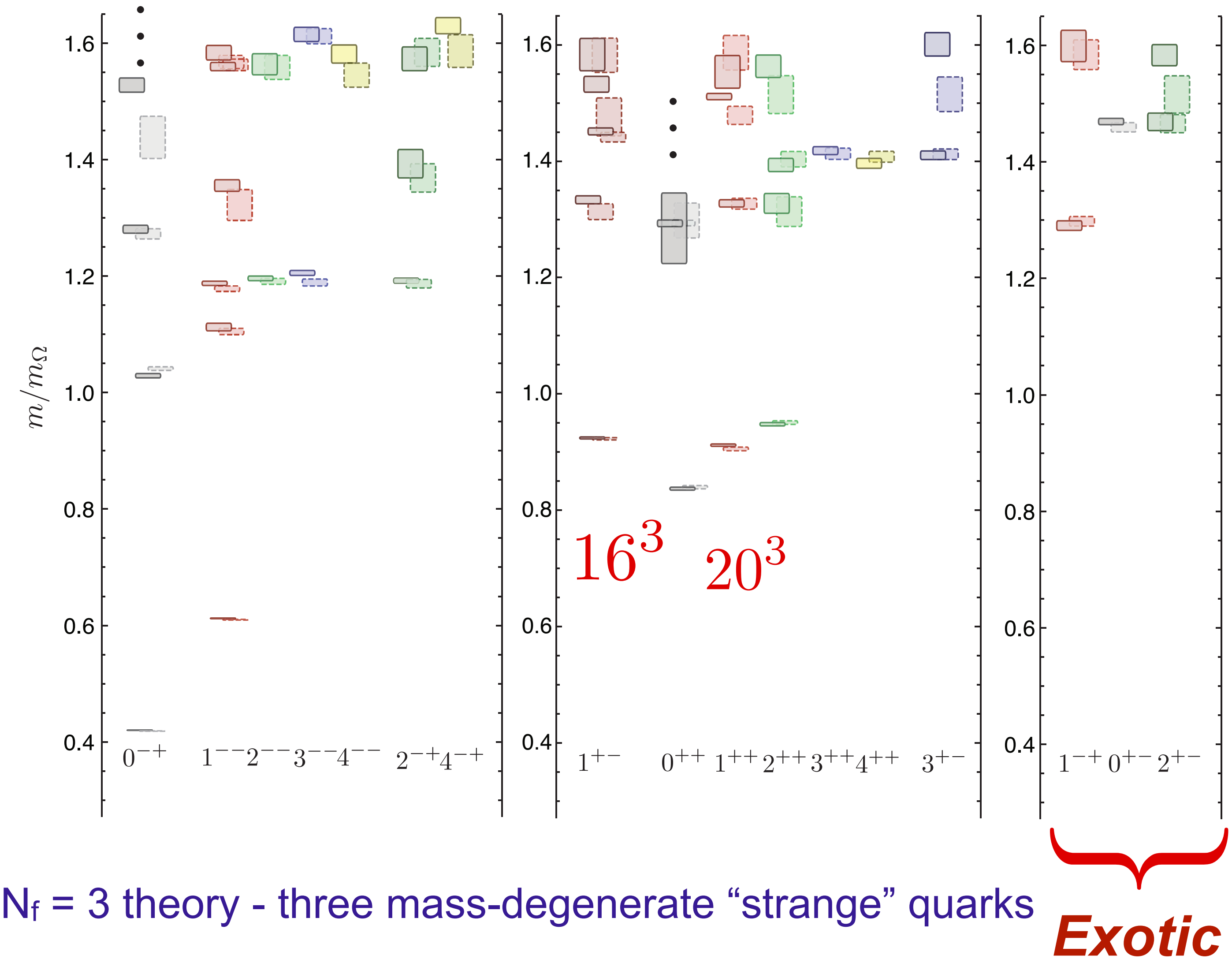
Perambulators $\tau^{ij}_{\alpha\beta}(t,0) = \xi^{*i}(t)M^{-1}(t,0)_{\alpha\beta}\xi^j$

$$C_{ij}(t) = \phi^{i,(pqr)}_{\alpha\beta\gamma)}(t)\phi^{j,(\bar{p}\bar{q}\bar{r})}_{\bar{\alpha}\bar{\beta}\bar{\gamma}}(0) \times \left[\tau^{p\bar{p}}_{\alpha\bar{\alpha}}(t,0)\tau^{q\bar{q}}_{\beta\bar{\beta}}(t,0)\tau^{r\bar{r}}_{\gamma\bar{\gamma}}(t,0) + \dots\right]$$

- Meson correlation functions N^3
- Baryon correlation functions N^4
- Stochastic sampling of eigenvectors - *stochastic LaPH*

Severely constrains baryon lattice sizes

Isovector Meson Spectrum - I



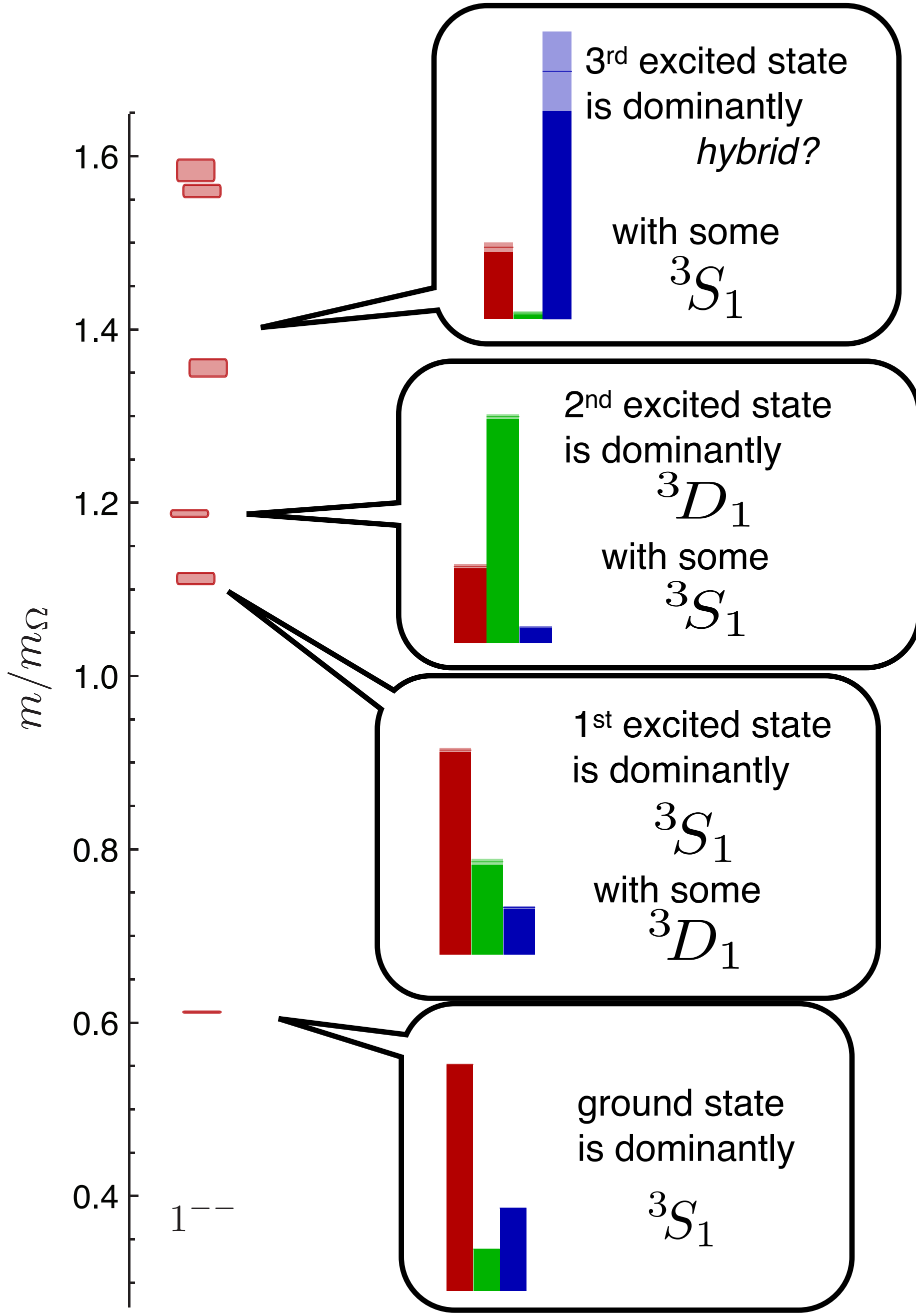
Dudek *et al*, PRL 103:262001 (2009)

Isovector spectrum with quantum numbers reliably identified

N_f = 3 theory - three mass-degenerate "strange" quarks

1^{--}

$$Z_i^N = \sqrt{2m_N} e^{m_N t_0/2} v_j^{(N)*} C_{ji}(t_0).$$



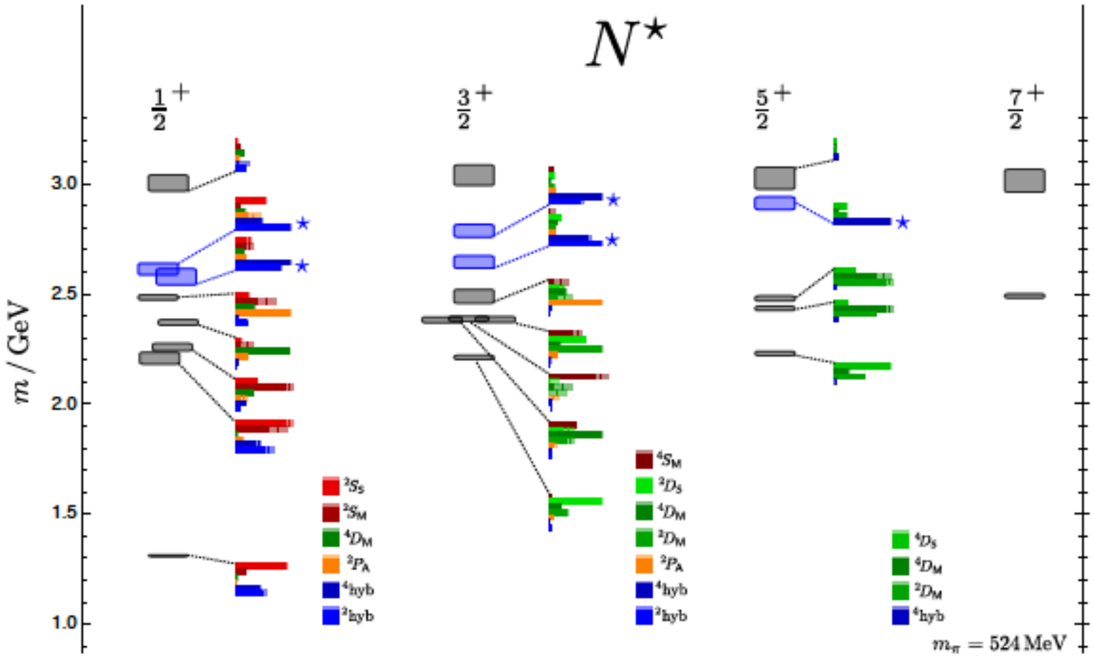
ρ	$^3S_1$
$\left(\rho \times D^{[2]}_{J=2}\right)^{J=1}$	$^3D_1$
$\left(\pi \times D^{[2]}_{J=1}\right)^{J=1}$	hybrid?

Anti-commutator of covariant derivative: vanishes for unit gauge!

Use lattice QCD to build phenomenology of bound states

Dudek, arXiv:1106.5515

N*: Interpolating Operators

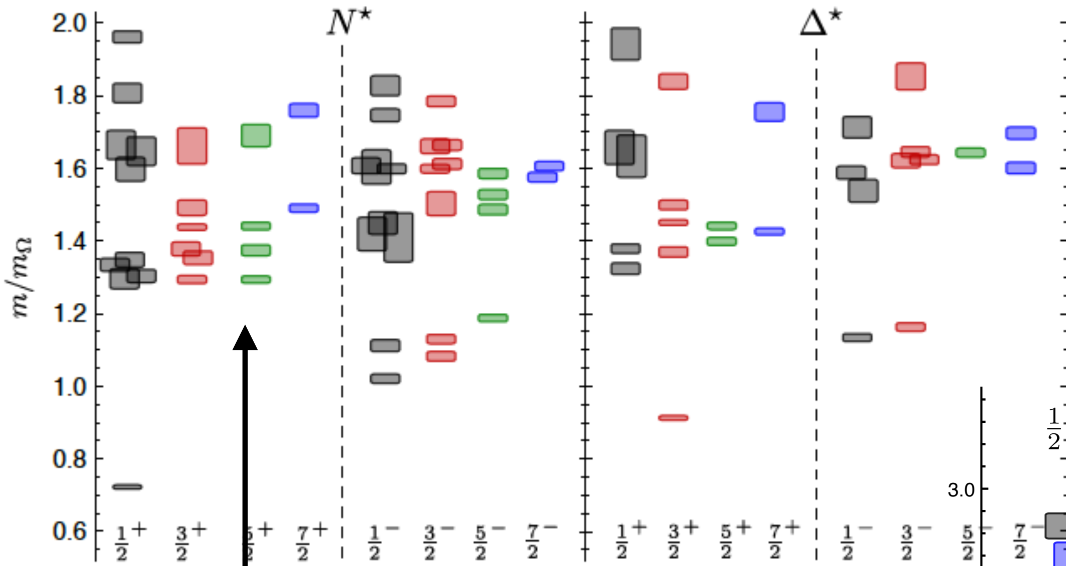


Examine overlaps onto different NR operators, i.e. containing upper components of spinors: *ground state has substantial hybrid component*

Variational method gives important *indications* of structure of hadrons

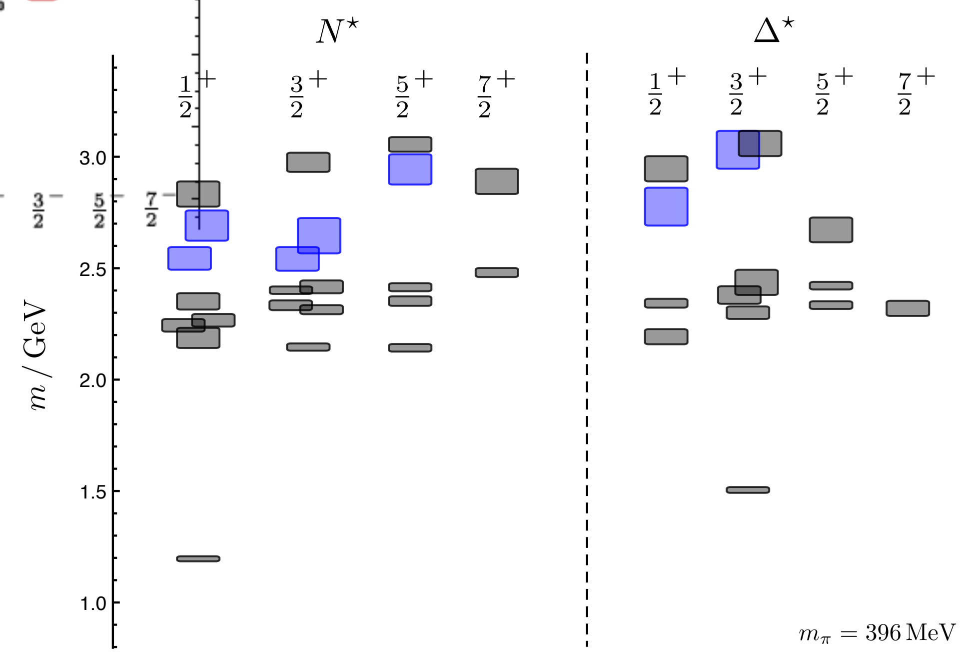
Excited Baryon Spectrum

R.G.Edwards et al., arXiv:1104.5152



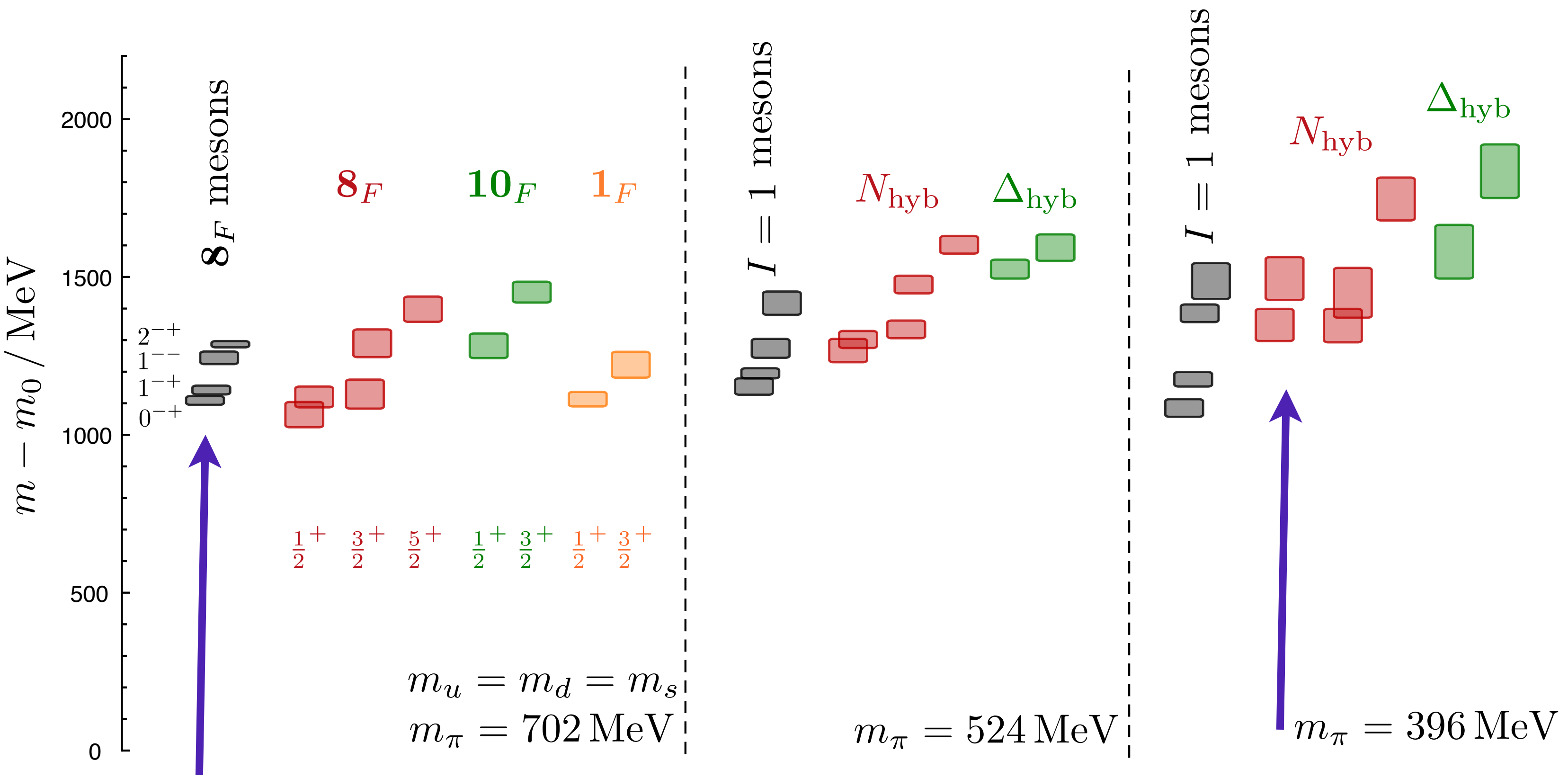
Quark-model Counting

Include hybrid operators of form $D^{[2]}_{l=1,M}$



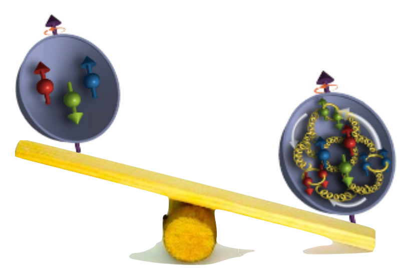
Dudek, Edwards, arXiv:1201.2349

A Picture Emerges



Subtract ρ

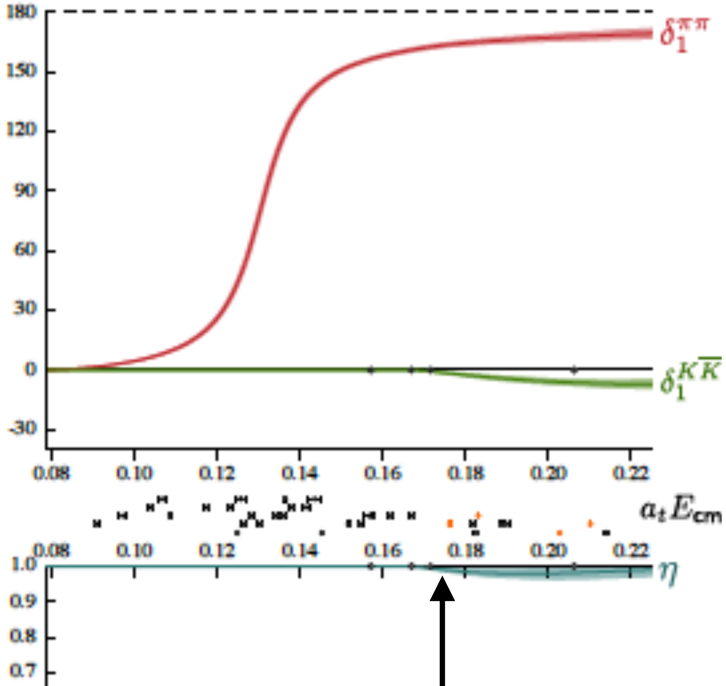
Subtract N

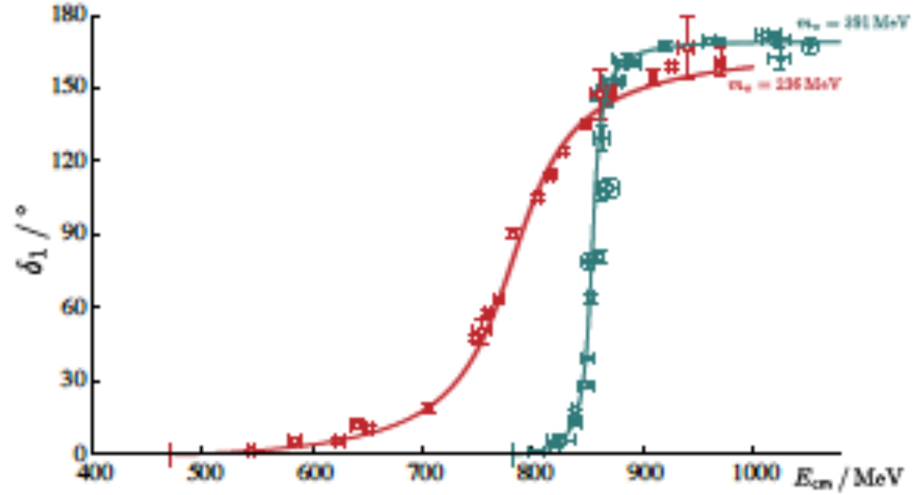


Common mechanism in meson and baryon hybrids: chromomagnetic field with *E_g ~ 1.2 - 1.3 GeV*

Resonant Phase Shift

We have treated excitations as stable states - *resonances under strong interaction*
Luscher: finite-volume energy levels to infinite-volume scattering phase shift





Decreasing Pion Mass

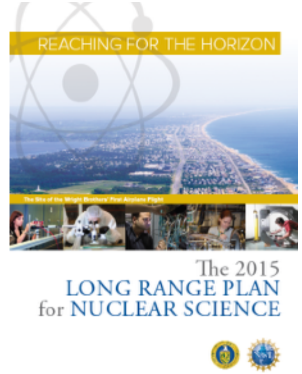
Inelastic Threshold

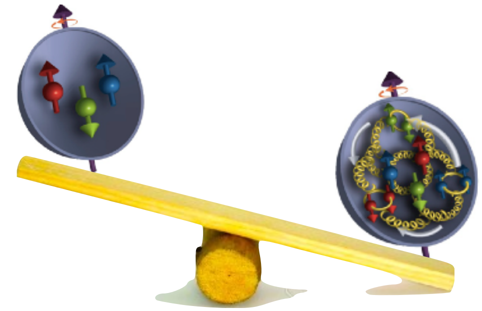
$$\mathcal{O}^{\Gamma,\gamma}_{\pi\pi}(|\vec{p}|) = \sum_m \mathcal{S}^{\ell,m}_{\Gamma,\gamma} \sum_{\hat{p}} Y^m_\ell(\hat{p})\, \mathcal{O}_\pi(\vec{p})\mathcal{O}_\pi(-\vec{p})$$

Wilson, Briceno, Dudek, Edwards, Thomas, arXiv:1507.02599

Energy-Momentum Tensor?

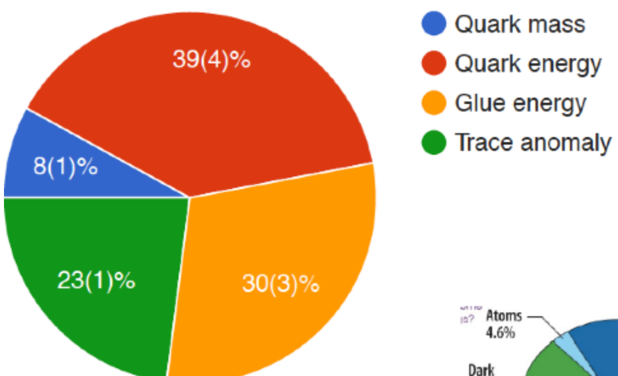
“Understanding the Glue That Binds Us All: The Next QCD Frontier in Nuclear Physics”



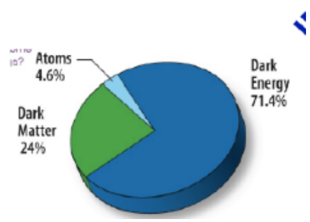


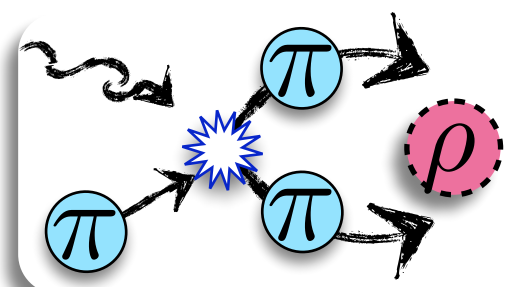
- Quark masses contribute only 1% to mass of proton: binding through gluon confinement
- Gluon spin and orbital angular momentum to spin of proton largely unknown

$$T_{\mu\nu} = \frac{1}{4}\bar{\psi}\gamma_{(\mu}D_{\nu)}\psi + G_{\mu\alpha}G_{\nu\alpha} - \frac{1}{4}\delta_{\mu\nu}G^2; \langle P \mid T_{\mu\nu} \mid P\rangle = P_\mu P_\nu / M$$



$$\text{Trace Anomaly: } T_{\mu\mu} = -(1+\gamma_m)\bar{\psi}\psi + \frac{\beta(g)}{2g}G^2$$

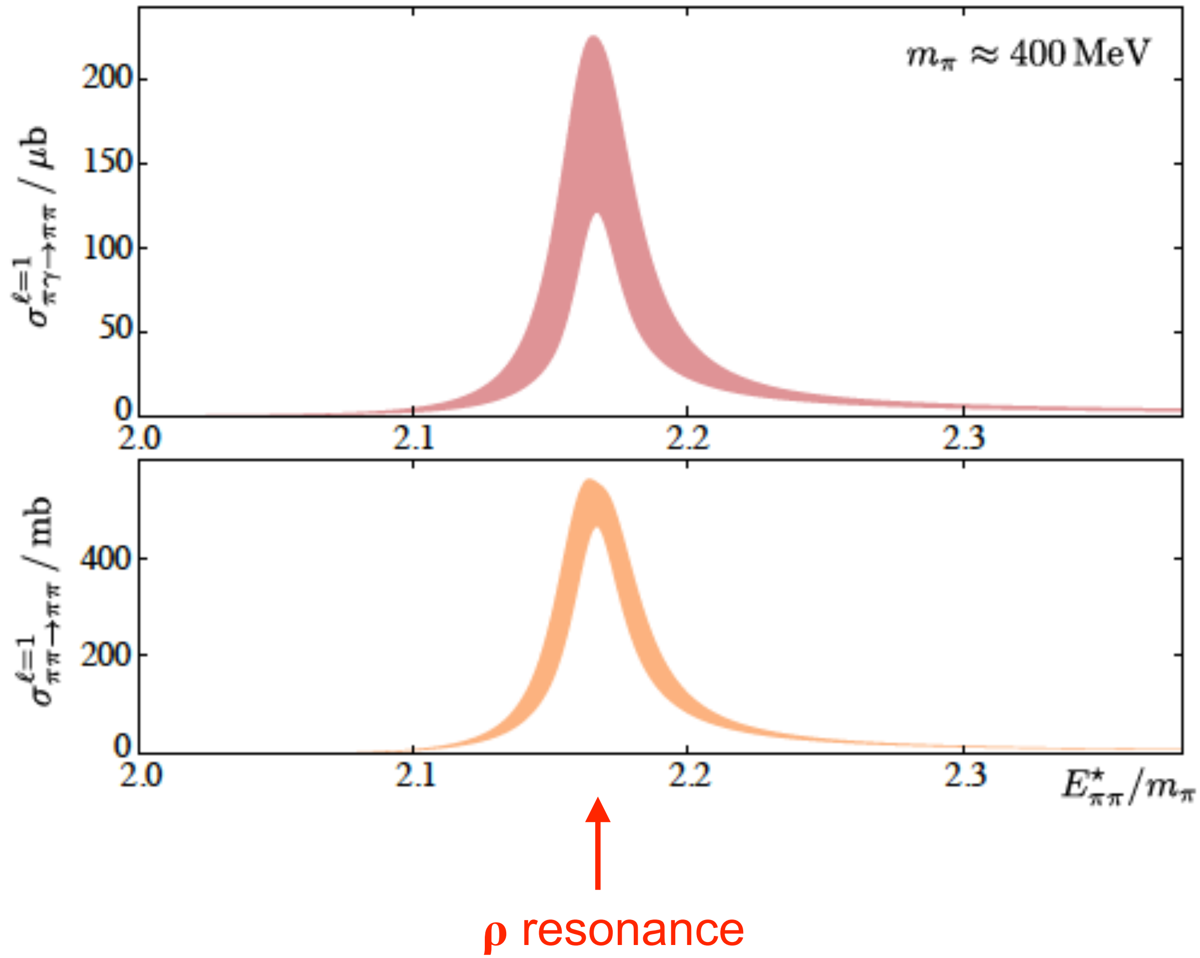




Yang, this meeting

Briceno, Hansen and Walker-Loud, PRD 91, 034501 (2015)

Transition form factor of ρ



Briceno et al., Phys. Rev. D 93, 114508 (2016)

Summary

- Controlling the contribution from excited states in study of hadron structure is a crucial for precise and accurate calculations
- The approach of the variational method is a powerful way of addressing systematic uncertainties due to excited states
- Structure of Excited-State Spectrum provides important insights into origins of mass
    - Chromomagnetic excitations of string responsible for hybrids in both mesons and baryonsEfficient implementation for nucleons a challenge - but making progress…
- Theoretical underpinning of computing the energy-momentum tensor for resonances is now in place.