



European Centre for Theoretical Studies
in Nuclear Physics and Related Areas



Trento Institute for
Fundamental Physics
and Applications

The gap equation interaction kernel and the QCD effective charge

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**The Proton Mass:
At the Hearth of Most Visible Matter
ECT*, April 3-7, 2017**

Gap equation's interaction kernel



infer
interaction
by fitting
data

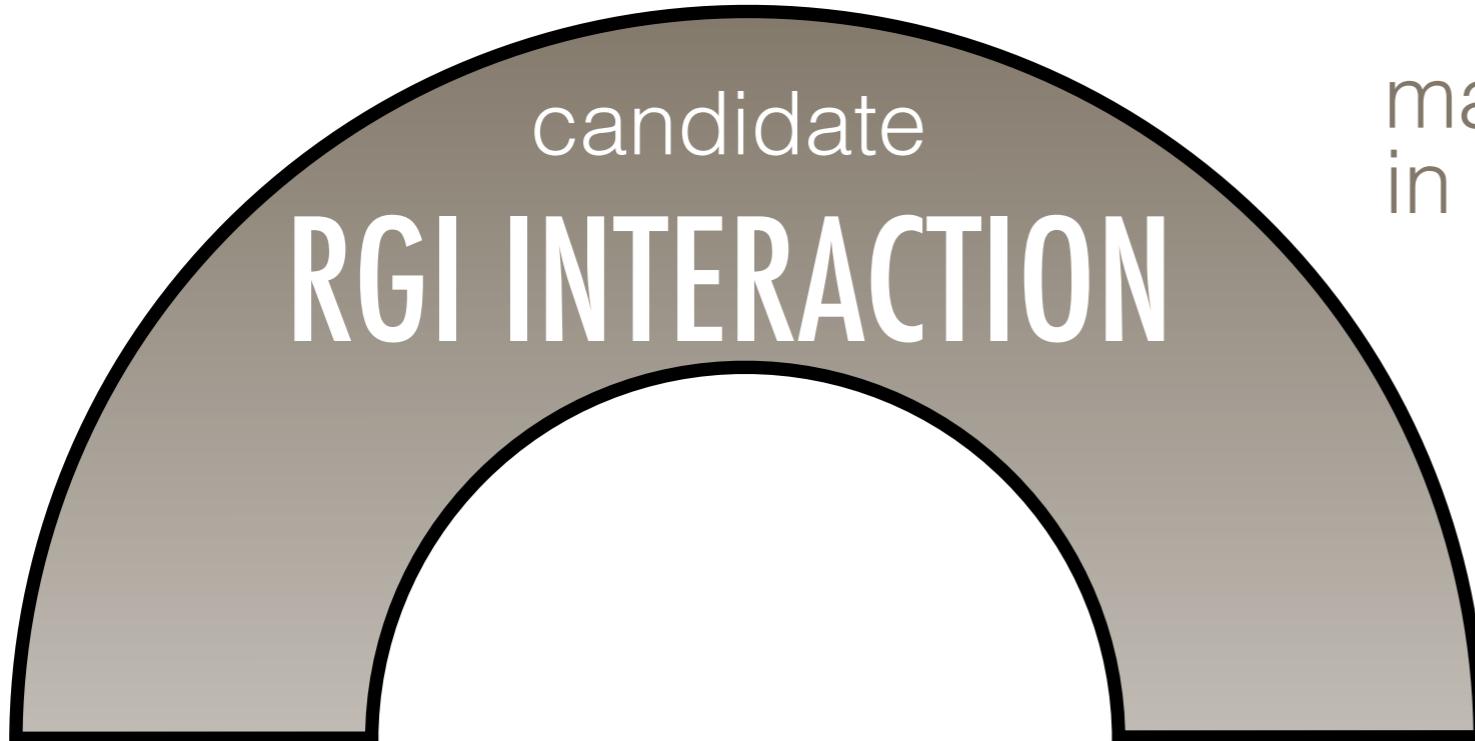
BOTTOM-
UP

TOP-
DOWN

ab-initio
computation
of interaction



Gap equation's interaction kernel



main ingredient
in D/B SE

infer
interaction
by fitting
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**BOTTOM-
UP**

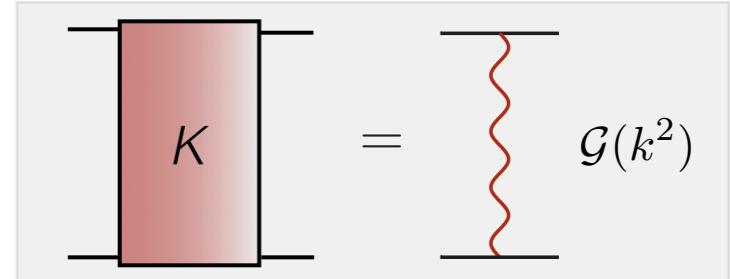
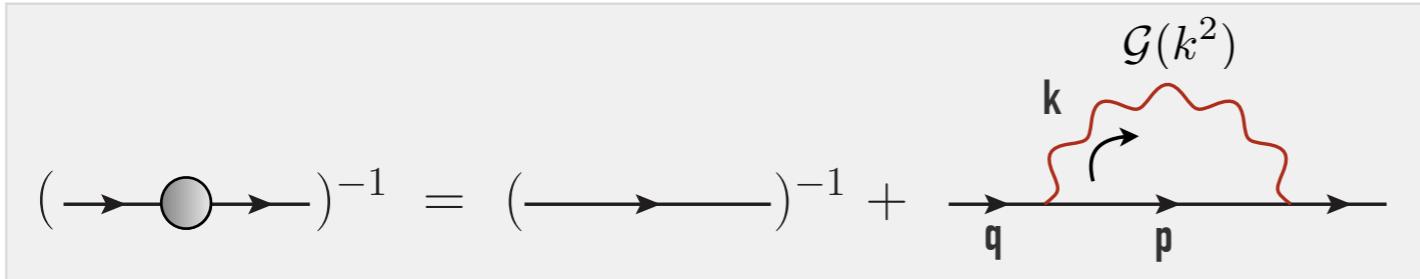
**TOP-
DOWN**

ab-initio
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RGI interaction: bottom-up approach - I



- Use rainbow-ladder truncation:
tree level vertex + one gluon exchange effective kernel



- Interaction strength Ansatz

IR: Gaussian; UV: perturbative tail

Maris, Roberts, Tandy, PRC 56 (1997); PRC 60 (1999)

$$\mathcal{I}(k^2) = k^2 \frac{\mathcal{G}_{\text{IR}}(k^2) + \mathcal{G}_{\text{UV}}(k^2)}{4\pi}$$

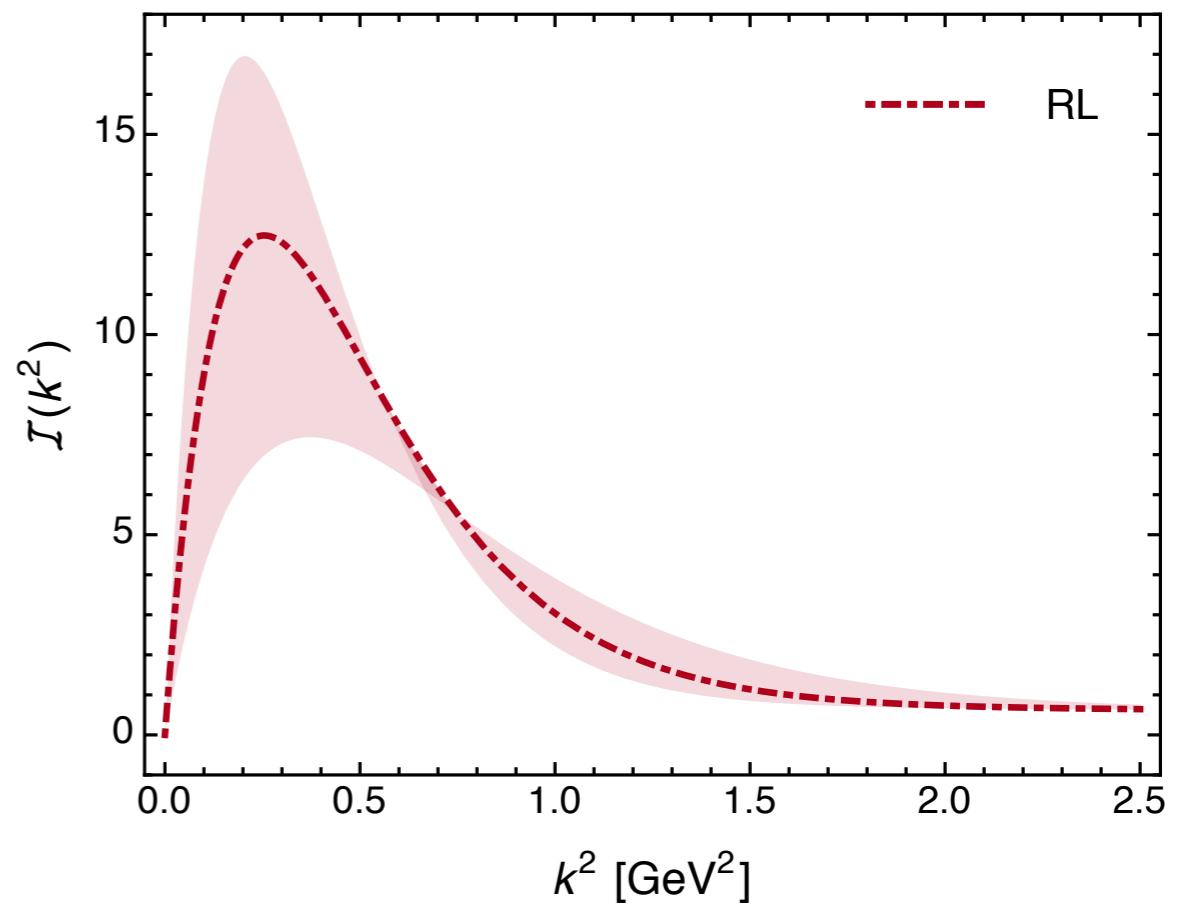
$$\mathcal{G}_{\text{IR}}(k^2) = \frac{8\pi^2}{\omega^5} \zeta^3 e^{-k^2/\omega^2}$$

$$\mathcal{G}_{\text{UV}}(k^2) = \frac{96\pi^2}{25} \frac{1 - e^{-k^2/1[\text{GeV}^2]}}{k^2 \log[e^2 - 1 + (1 + k^2/\Lambda^2)^2]}$$

- One parameter interaction ($\Lambda_{\text{MS}}=234$ [MeV])
 ζ fitted to obtain pion decay constant

$$\zeta_{\text{RL}} = 0.87 \text{ [GeV]}$$

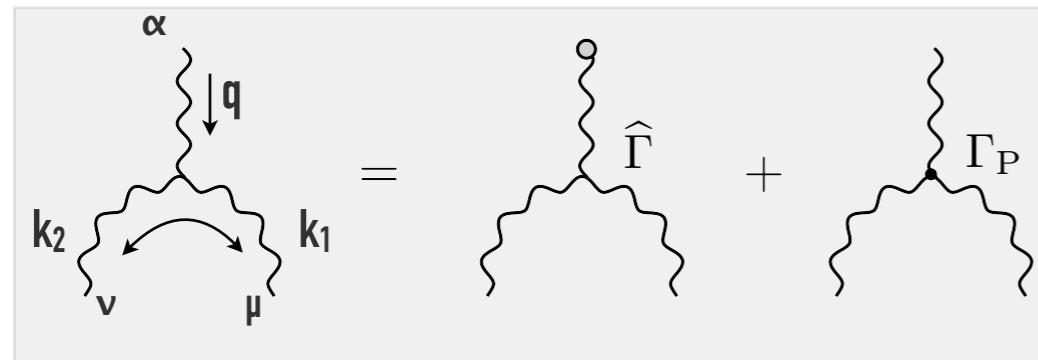
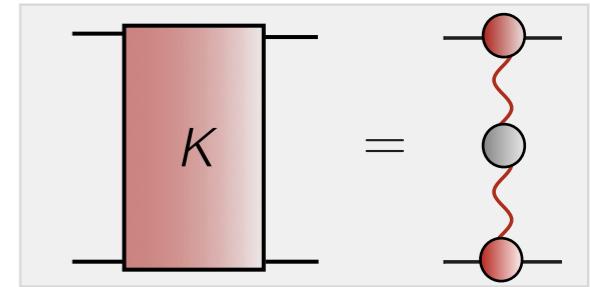
- Depends on truncation used: $\Gamma_\nu = \gamma_\nu$
- Many observables invariant while $\omega \in [0.4, 0.6]$



RGI interaction: top-down approach



- **Universal (process-independent) contribution:**
originates entirely from the gauge sector
- **Fundamental quantities: PT-BFM propagators/vertices**
satisfy Abelian-like Slavnov-Taylor (ST) identities
- **How to get them?**
use the PT algorithm
[Cornwall, Papavassiliou, PRD 40 \(1989\)](#)

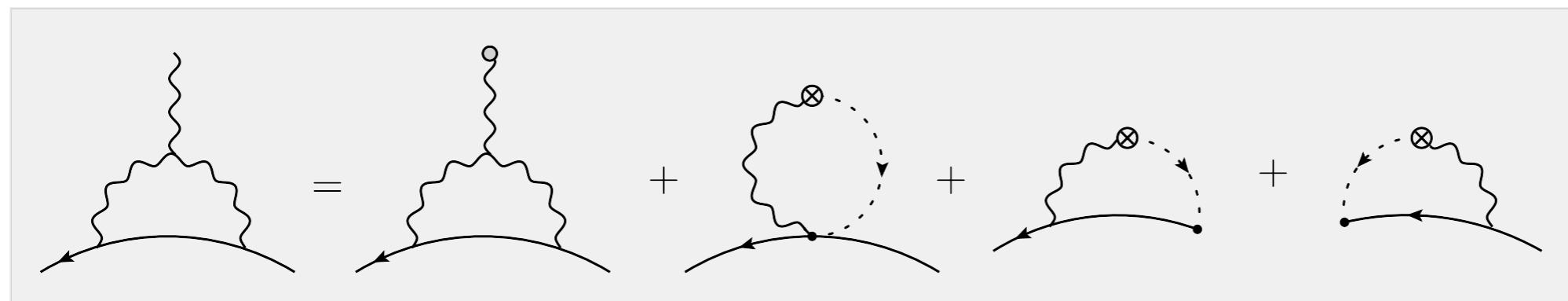


$$\hat{\Gamma}^{\alpha\mu\nu} = (k_2 - k_1)^\alpha g^{\mu\nu} + 2q^\nu g^{\alpha\mu} - 2q^\mu g^{\alpha\nu}$$

$$\Gamma_P^{\alpha\mu\nu} = k_1^\mu g^{\alpha\nu} - k_2^\nu g^{\alpha\mu}$$

- **longitudinal momenta**
trigger elementary Ward identities

- **Apply the PT to the quark-gluon vertex**
one loop result:



RGI interaction: top-down approach



- Allot pieces to different Green's functions

construct $\widehat{\Delta}$ and $\widehat{\Gamma}_\mu$

$$\widehat{\Delta}^{-1} = \text{---} + 2 \text{---} \otimes \text{---}$$

$$q^\mu \widehat{\Gamma}_\mu = S^{-1}(p_1) - S^{-1}(p_2)$$

$$\widehat{\Gamma} = \text{---} + \text{---} + \text{---} + \text{---}$$

vanish on-shell

- Crucial all-order equivalence: **PT=BFM**

yields Feynman rules for systematic calculation

$$\widehat{\Delta} \sim \frac{1}{q^2[1 + bg^2 \log q^2/\mu^2]}; \quad b = 11C_A/48\pi^2$$

- Absorbs all the RG logs as the photon in QED
- Renormalizes as Z_g^{-2}

- An additional equivalence holds: **antiBRST+BRST=BFM**

plethora of symmetry identities, in particular BQ identities

DB, Quadri, PRD 88 (2013)

$$\Delta(q^2) = [1 + G(q^2)]^2 \widehat{\Delta}(q^2)$$

$$\Lambda_{\mu\nu}(q) = \text{---} + \text{---}$$

$$= G(q^2)g_{\mu\nu} + L(q^2)\frac{q_\mu q_\nu}{q^2}$$

- **G special PT-BFM function:** determined by ghost-gluon dynamics
- **Combination 1+G appears in all BQIs** fundamental non-Abelian quantity
- **G is related (Landau gauge) to the ghost dressing:** use ghost gap equation to constrain 1+G, L

$$F^{-1}(q^2) = 1 + G(q^2) + L(q^2)$$

RGI interaction: top-down approach



- Convert vertices/propagators into PT-BFM ones
new RG invariant combination appears

$$\hat{d}(k^2) = \alpha(\mu^2)\hat{\Delta}(k^2; \mu^2)$$

- Use symmetry identity
to identify the interaction strength

Aguilar, DB, Papavassiliou, Rodriguez-Quintero, PRD 90 (2009)
DB, Chang, Papavassiliou, Roberts, PLB 742 (2015)

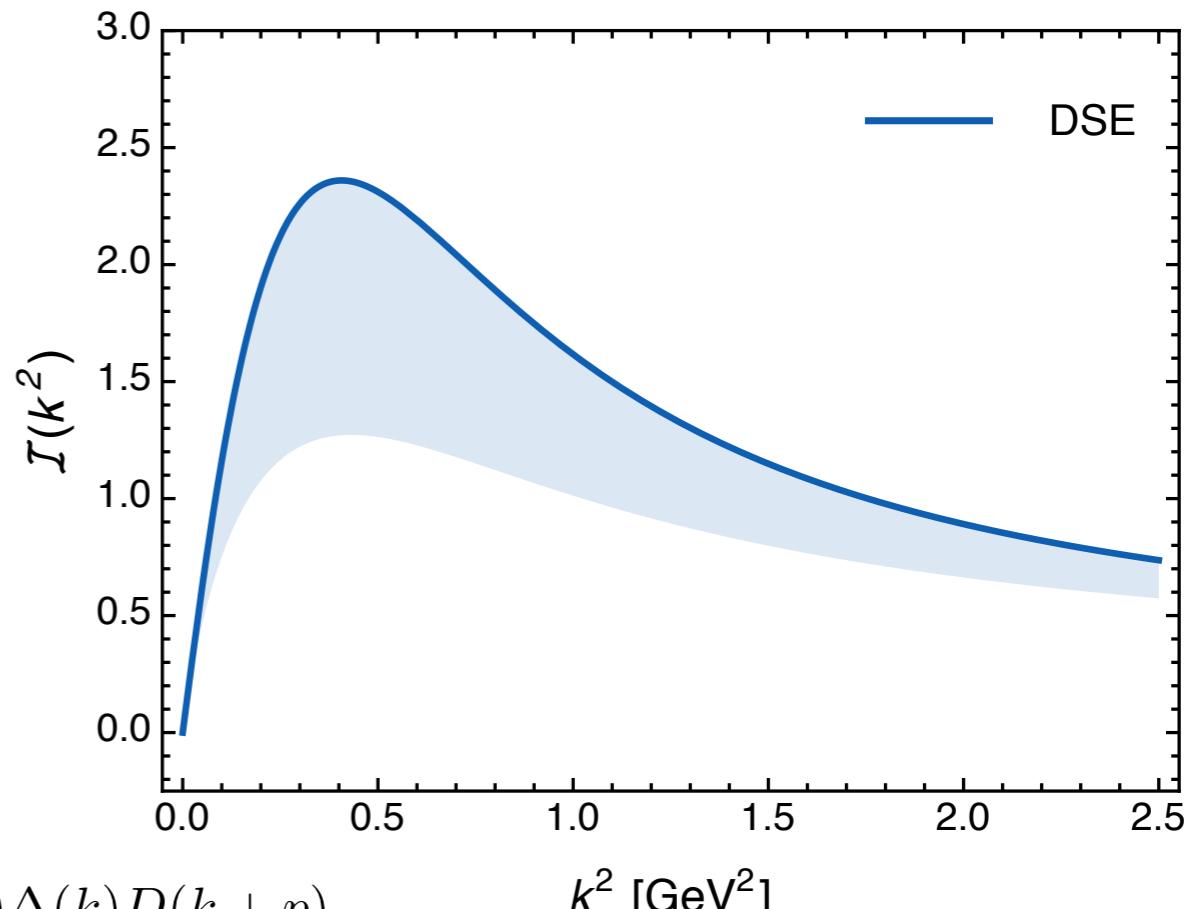
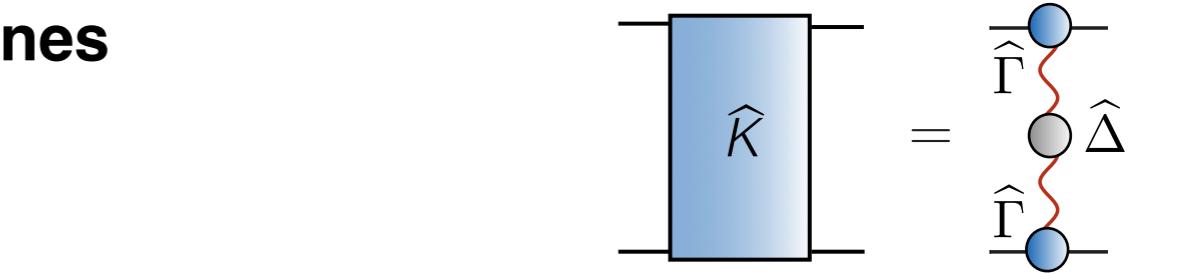
$$\mathcal{I}(k^2) = k^2 \hat{d}(k^2)$$

$$\hat{d}(k^2) = \frac{\alpha(\mu^2)\Delta(k^2; \mu^2)}{[1 + G(k^2; \mu^2)]^2}$$

- $1+G$ and L determined by their own SDEs
under simplifying assumptions:

$$1 + G(p^2) = Z_c + \frac{g^2 C_A}{d-1} \int_k \left[(d-2) + \frac{(k \cdot p)^2}{k^2 p^2} \right] B_1(-k, 0, k) \Delta(k) D(k+p)$$

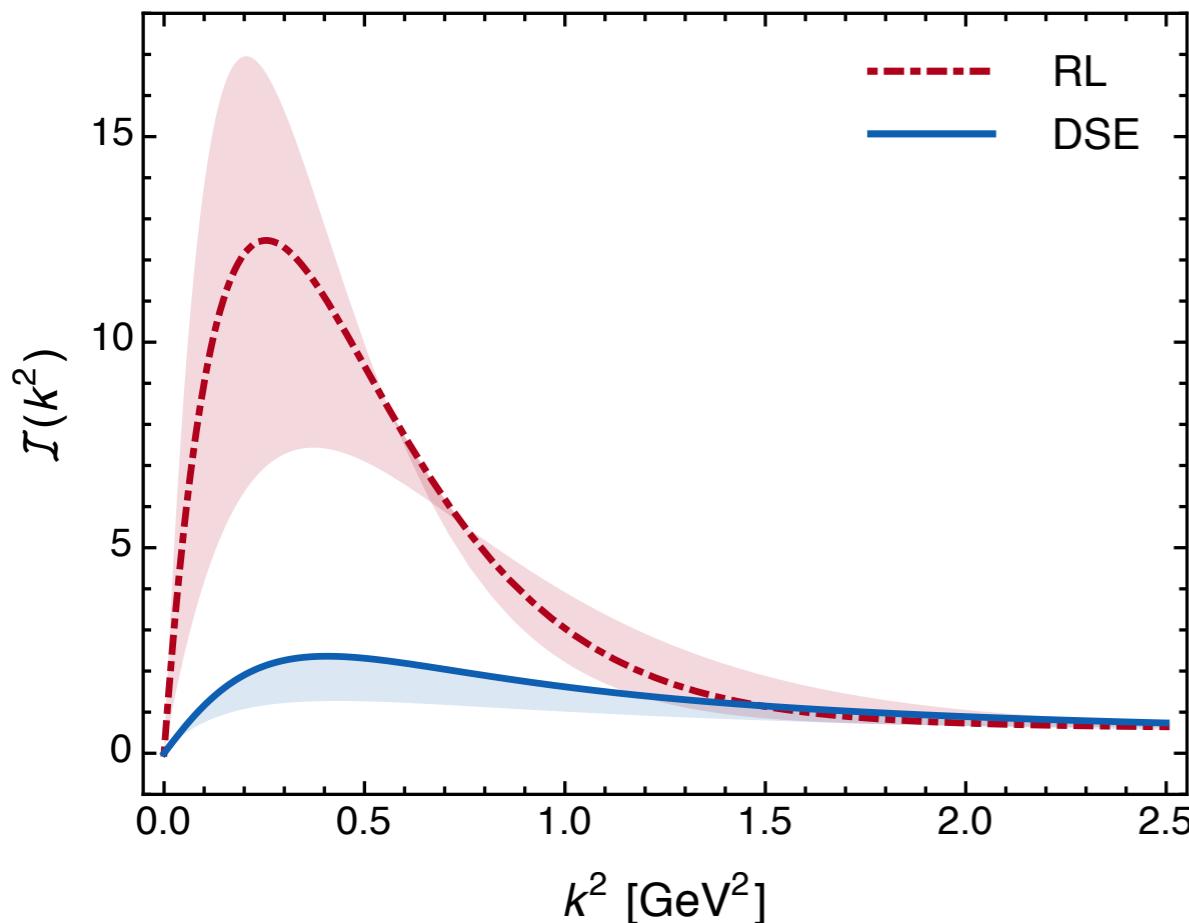
$$L(p^2) = \frac{g^2 C_A}{d-1} \int_k \left[1 - \frac{(k \cdot p)^2}{k^2 p^2} \right] B_1(-k, 0, k) \Delta(k) D(k+p)$$



- Main source of uncertainties:
needs assumptions on ghost vertex behavior
- Parametrized by $\delta \in [0, 1]$
lower bound ($\delta=0$): $1/F = 1+G$



Top-down vs bottom-up comparison - I



infer
interaction
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RGI INTERACTION

candidate



main ingredient
in D/B SE

%!#\$!#@!!!



ab-initio
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RGI interaction: bottom-up approach - II



- **Go beyond RL truncation:**
implement dynamical symmetry breaking into bound-state equations

- **Same RL interaction strength Ansatz**

IR: Gaussian; UV: perturbative tail

Qin, Chang, Liu, Roberts, Wilson, PRC 84 (2011)

$$\mathcal{I}(k^2) = k^2 \frac{\mathcal{G}_{\text{IR}}(k^2) + \mathcal{G}_{\text{UV}}(k^2)}{4\pi}$$

$$\mathcal{G}_{\text{IR}}(k^2) = \frac{8\pi^2}{\omega^5} \zeta^3 e^{-k^2/\omega^2}$$

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- **Quark-gluon vertex:**

$$\Gamma_\nu = \Gamma_\nu^{\text{BC}} + \Gamma_\nu^{\text{ACM}}$$

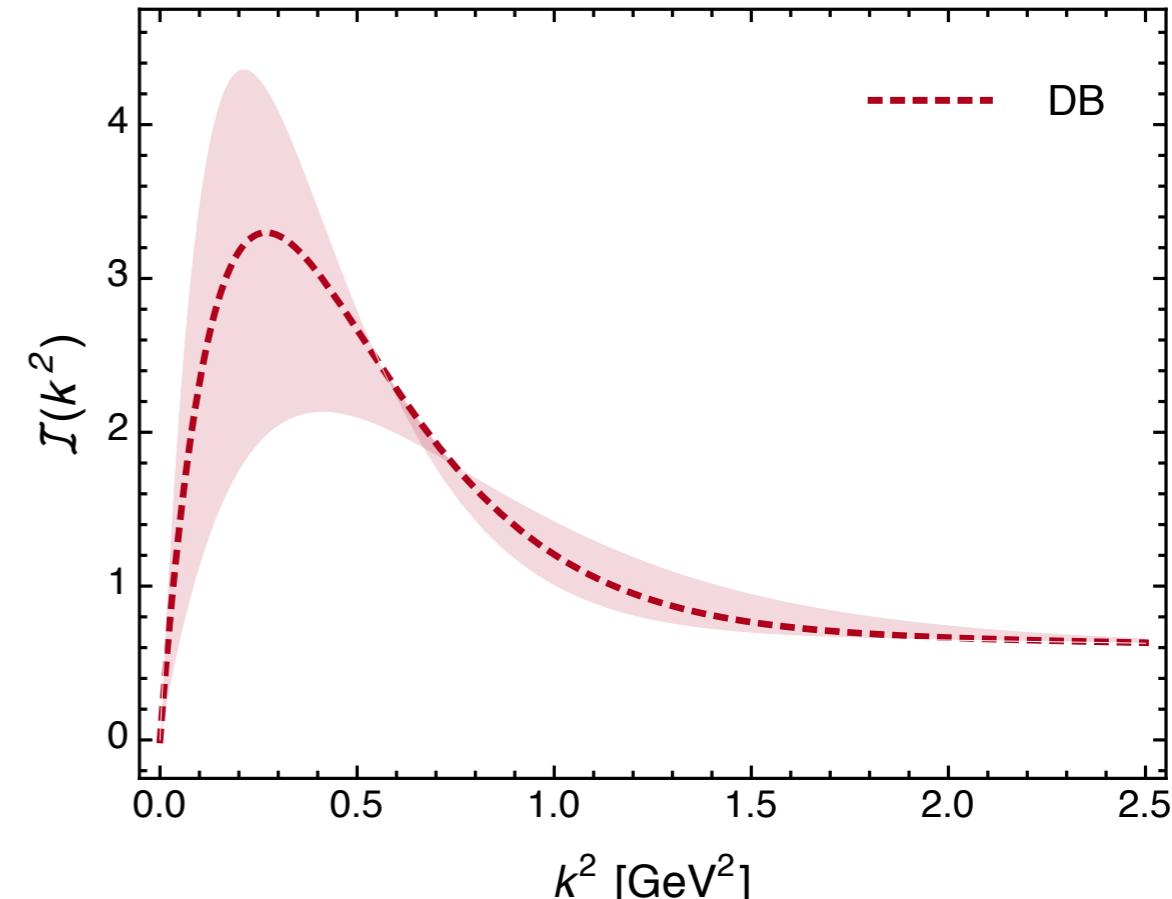
- **Ball-Chiu vertex**

completely determined by fermion propagator
Ball, Chiu PRD 22 (1980)

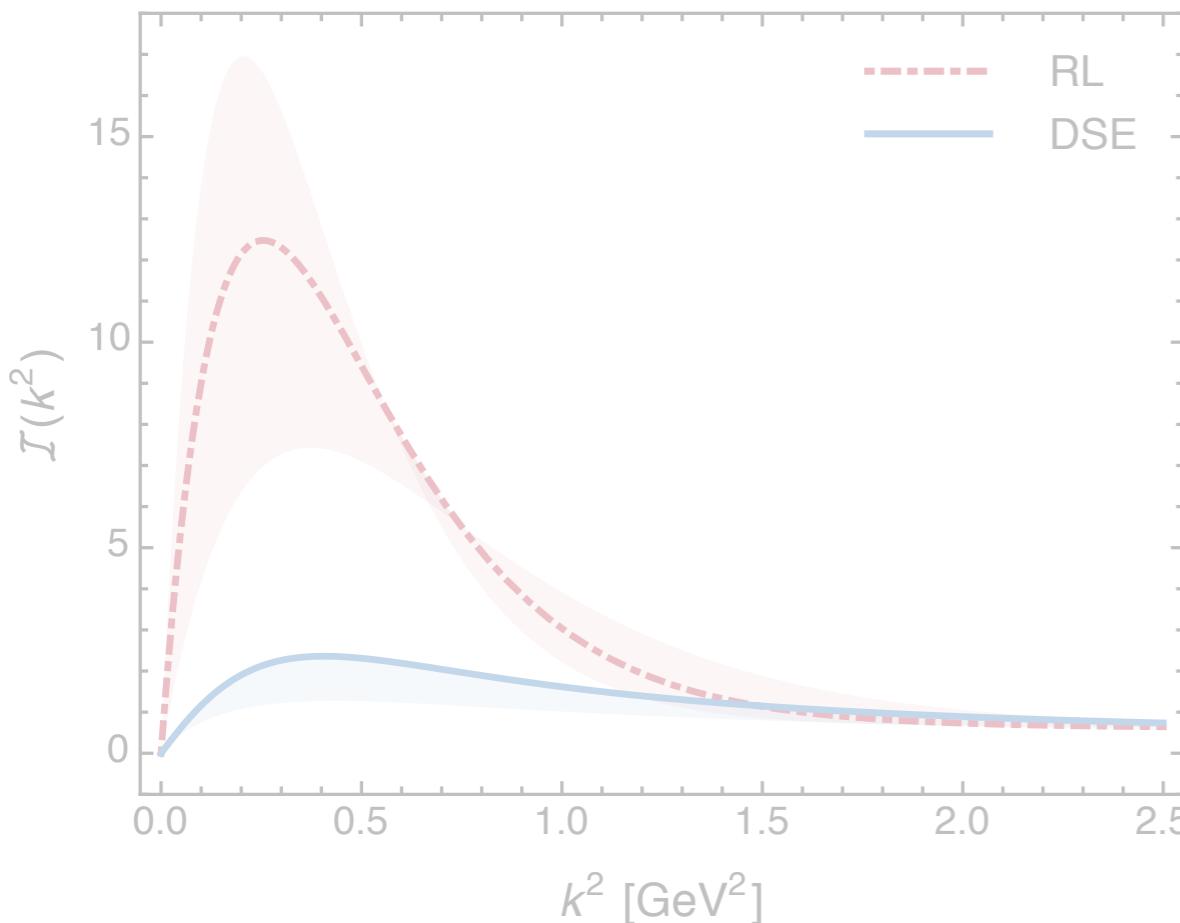
- **Anomalous chromo-magnetic vertex**
transverse part (undetermined by STI)

- ζ fitted to

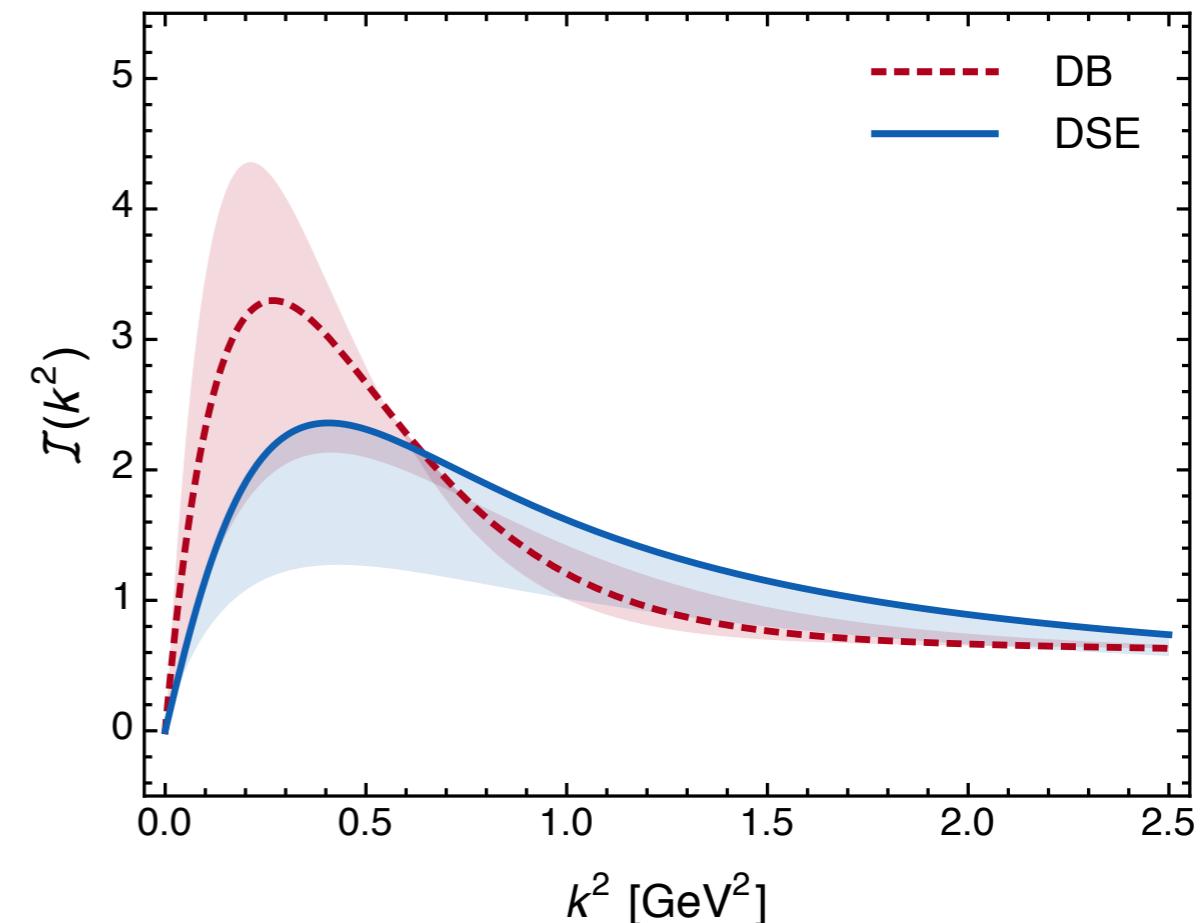
$$\zeta_{\text{DB}} = 0.55 \text{ [GeV]}$$

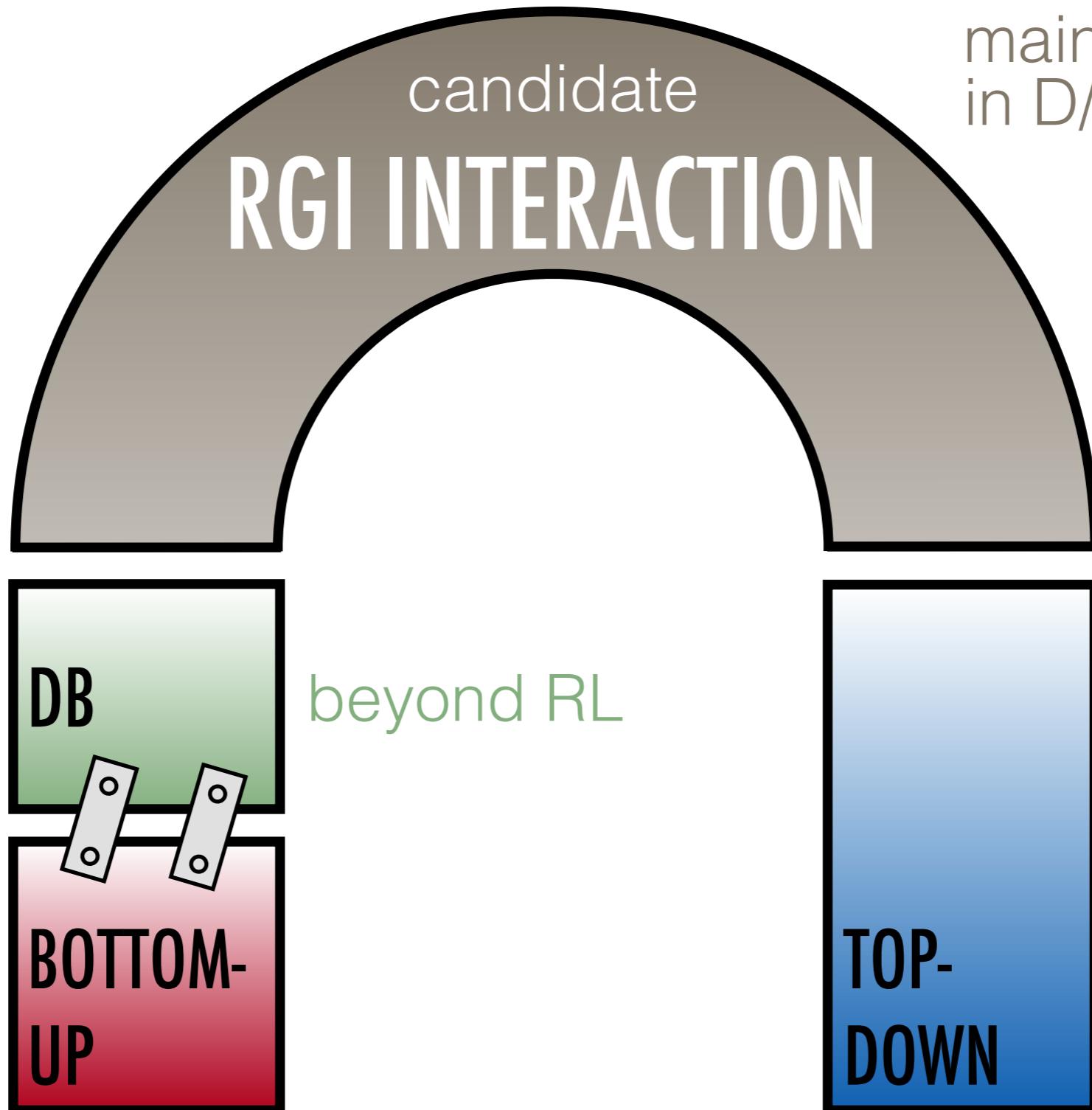


Top-down vs bottom-up comparison - II



DB, Chang, Papavassiliou, Roberts, PLB 742 (2015)





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QCD Effective charge



- **Remarkable feature of QCD:**

$\hat{d}(k^2)$ saturates in the IR

DB, Chang, Papavassiliou, Roberts, PLB 742 (2015)

$$\hat{d}(0) = \frac{\alpha_0}{m_0^2} \approx \frac{0.9\pi}{(m_P/2)^2}$$

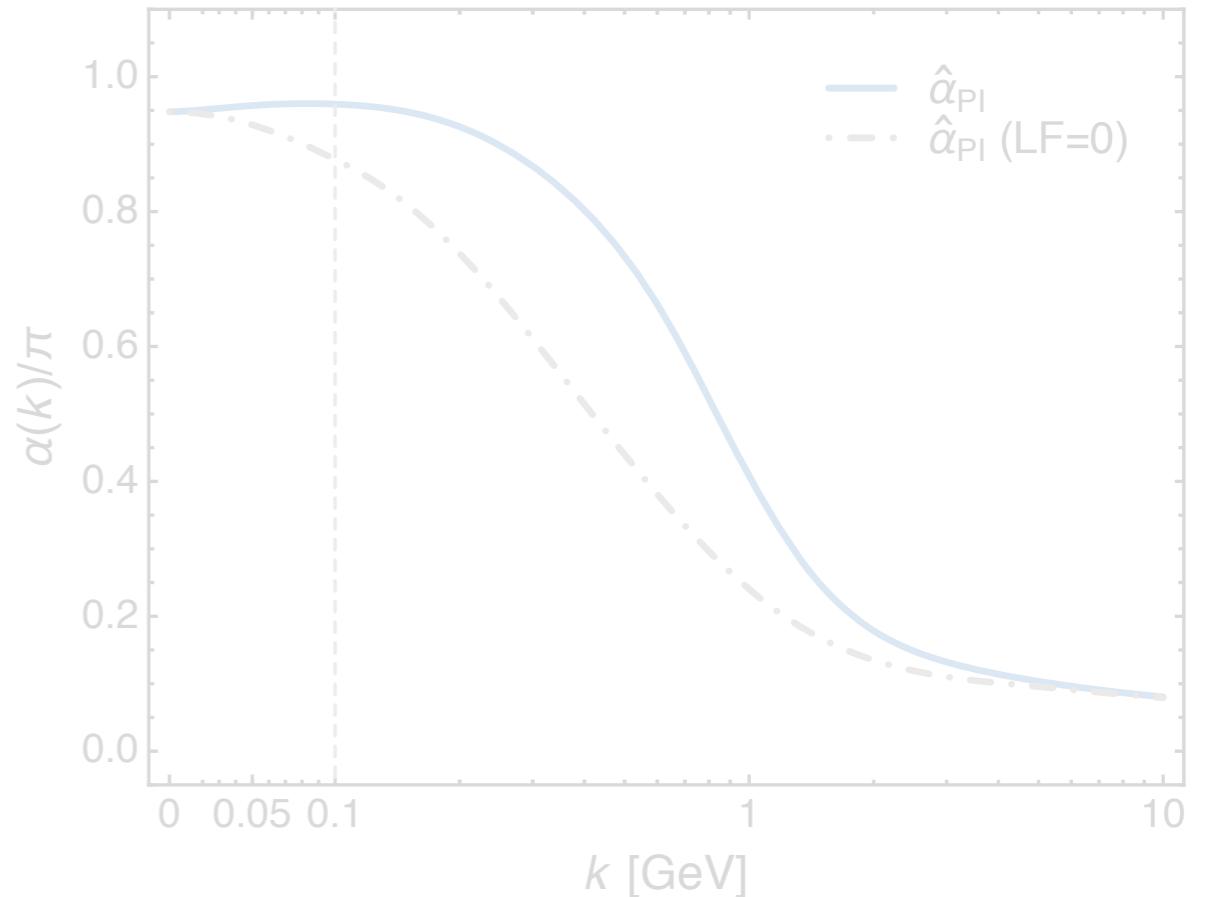
- **Define the RG invariant function**

$$\mathcal{D}(k^2) = \frac{\Delta(k^2; \mu^2)}{\Delta(0; \mu^2)m_0^2}$$

- **Extract (process independent) coupling**
using the quark gap equation

DB, Mezrag, Papavassiliou, Roberts, Rodriguez-Quintero, 1612.04835

$$\hat{\alpha}(k^2) = \frac{\hat{d}(k^2)}{\mathcal{D}(k^2)} \xrightarrow{k^2 \gg m_0^2} \mathcal{I}(k^2)$$



- Parameter free
completely determined from 2-point sector
- No Landau pole
physical coupling showing an IR fixed point
- Smoothly connects IR and UV domains
no need for matching procedures
- Essentially non-perturbative result
continuum/lattice results plus setting of single mass scale
- Ghost gluon dynamics critical
produces enhancement at intermediate momenta

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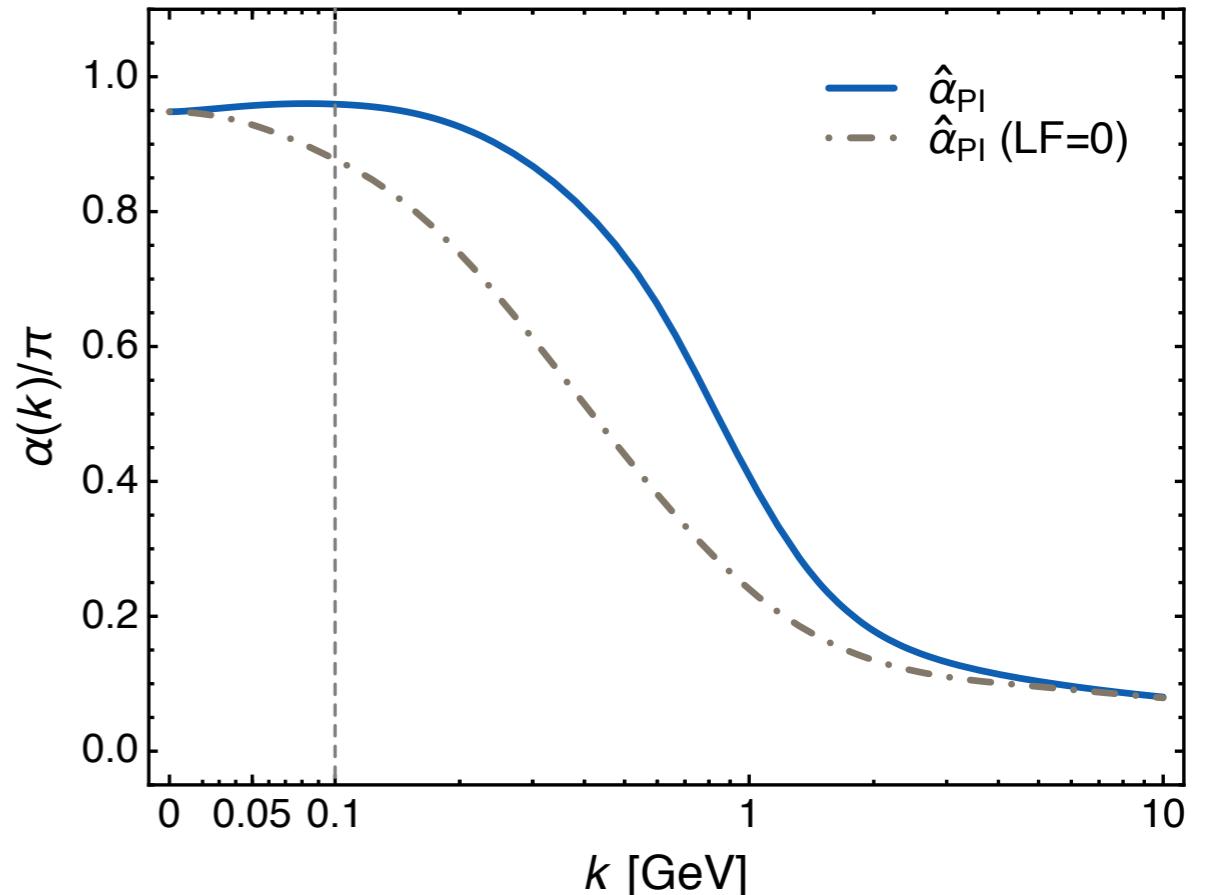
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QCD Effective charge

- **Process dependent effective charges**

fixed by the leading-order term in the expansion of a given observable

Grunberg, PRD 29 (1984)

- **Bjorken sum rule**

defines such a charge

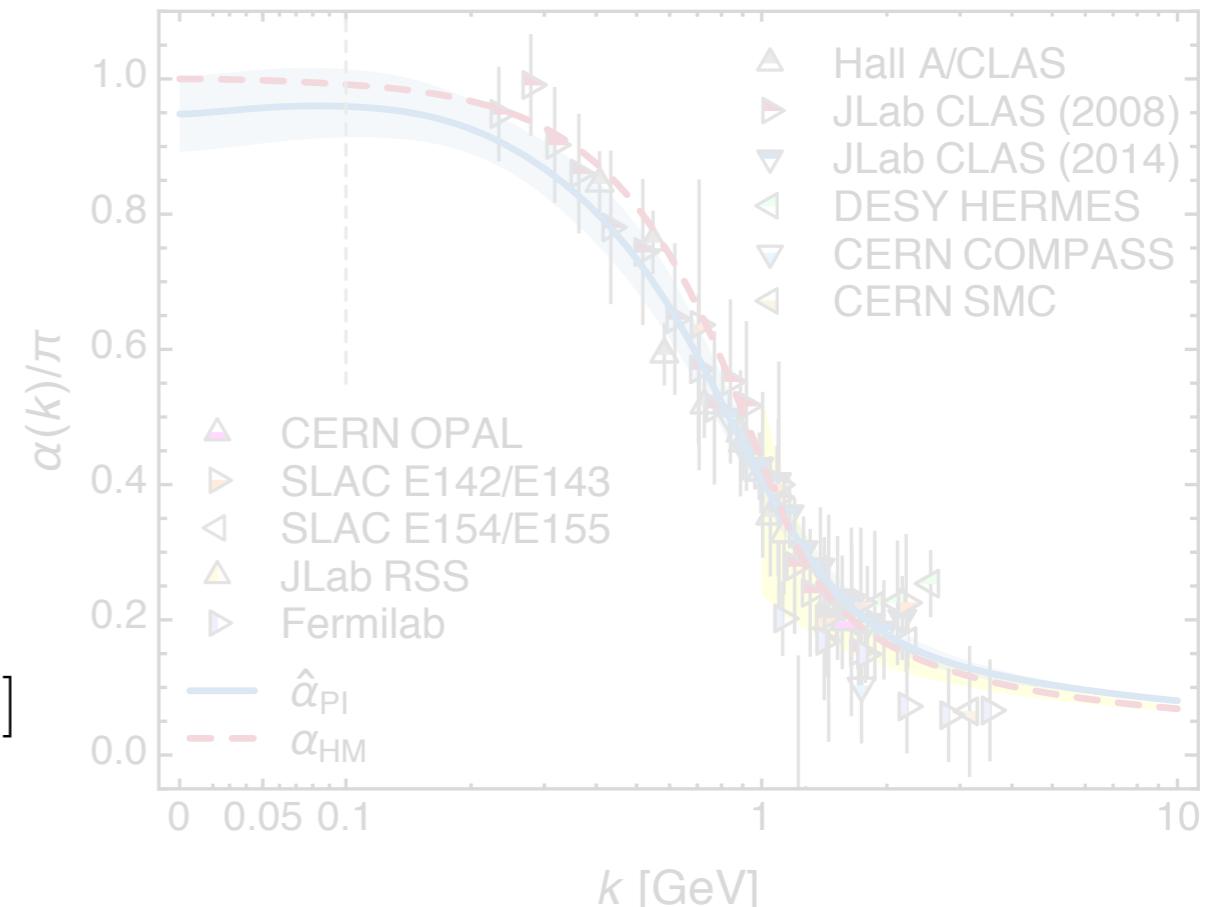
Bjorken, PR 148 (1966); PRD 1 (1970)

$$\int_0^1 dx [g_1^p(x, k^2) - g_1^n(x, k^2)] = \frac{g_A}{6} [1 - \alpha_{g_1}(k^2)/\pi]$$

- $g_1^{p,n}$ **spin dependent p/n structure functions**
extracted from measurements using unpolarized targets
- g^A **nucleon flavour-singlet axial charge**

- **Many merits**

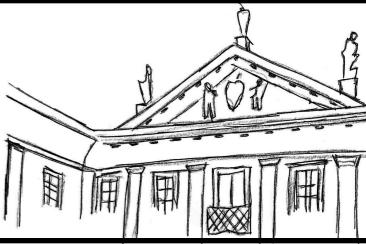
- **Existence of data**
for a wide momentum range
- **Tight sum rules constraints on the integral**
at IR and UV extremes
- **Isospin non-singlet**
suppress contributions from hard-to-compute
processes



- **Equivalence in the perturbative domain**
reasonable definitions of the charge

$$\alpha_{g_1}(k^2) = \alpha_{\overline{\text{MS}}}(k^2)[1 + 1.14\alpha_{\overline{\text{MS}}}(k^2) + \dots]$$

$$\hat{\alpha}_{PI}(k^2) = \alpha_{\overline{\text{MS}}}(k^2)[1 + 1.09\alpha_{\overline{\text{MS}}}(k^2) + \dots]$$
- **Equivalence in the non-perturbative domain**
highly non-trivial (ghost-gluon interactions)
- **Agreement with light-front holography**
model for α_{g_1}
Deur, Brodsky, de Teramond, PPNP 90 (2016)



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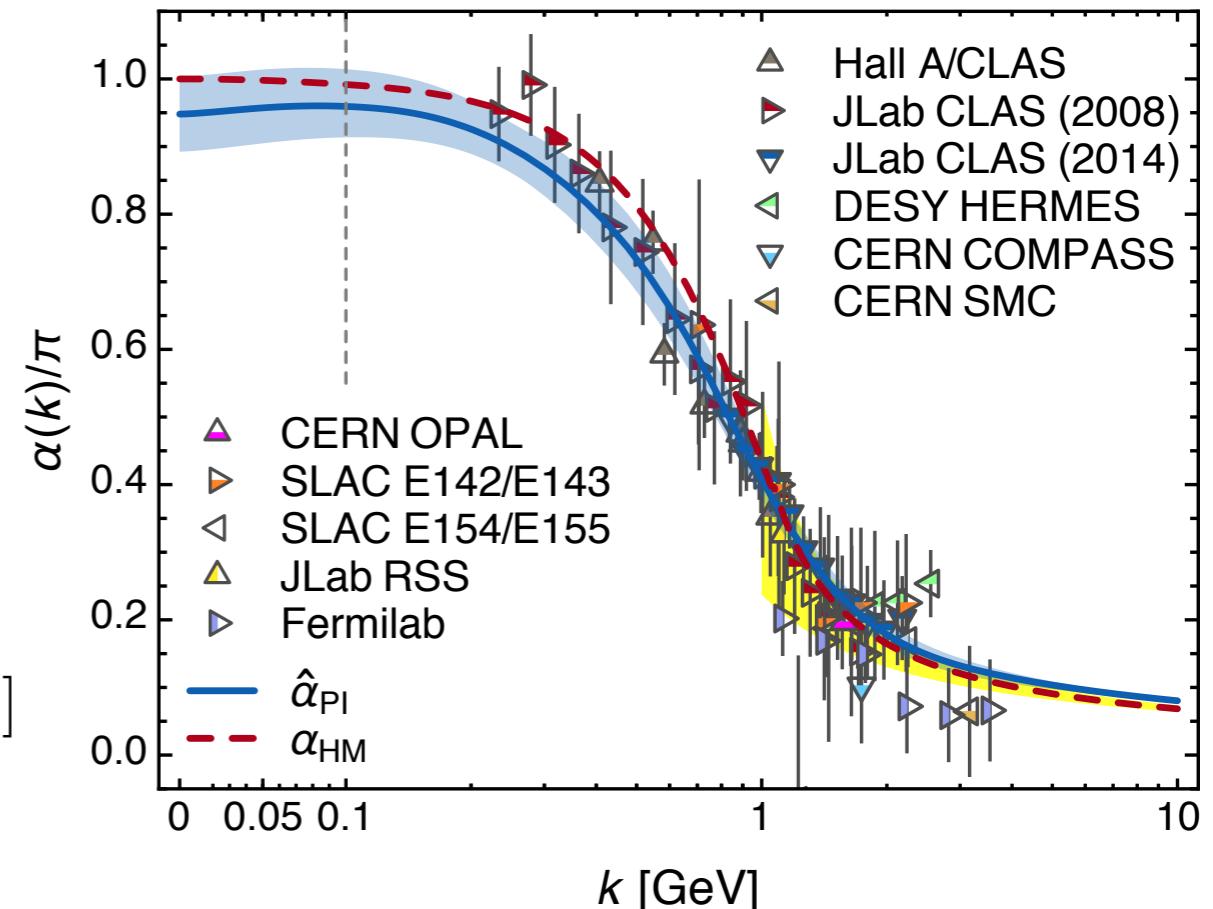
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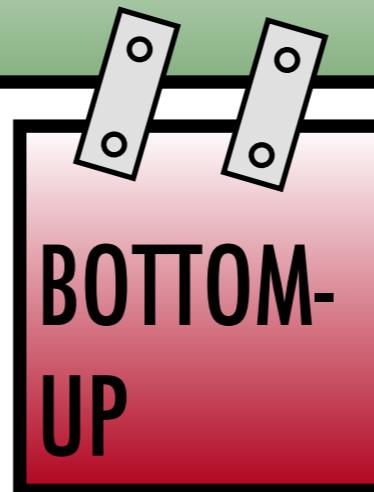
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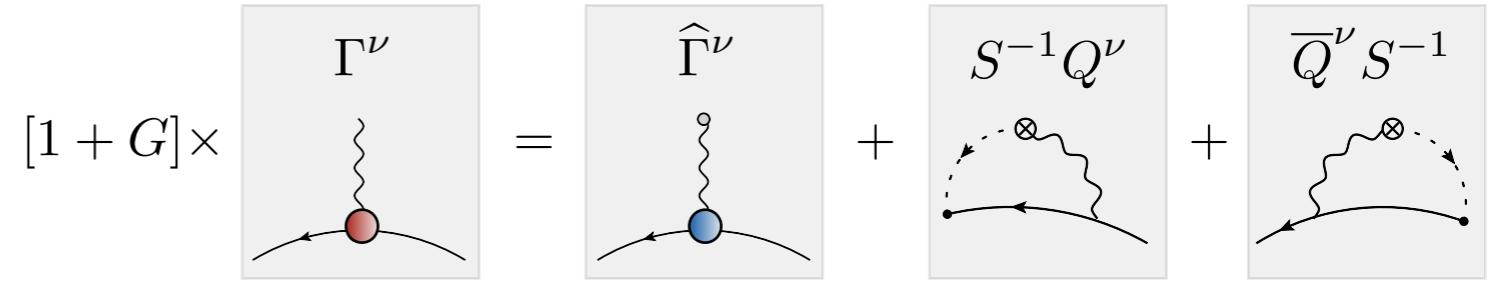
TOP-
DOWN

Quark-gluon vertex



- **Complete quark-gluon vertex**
solves all symmetry identities

Aguilar, DB, Ibañez, Papavassiliou, PRD 90 (2014)



$$\mathcal{G}_q \Gamma_1^L = [1 - L(q^2)F(q^2)] \left\{ \widehat{\Gamma}_1^L + A_1 \left[\frac{1}{2}(q \cdot t)K_3^L - (p_1 \cdot t)K_4^L \right] - B_1 K_1^L \right. \\ \left. + A_2 \left[-\frac{1}{2}(q \cdot t)\overline{K}_3^L + (p_2 \cdot t)\overline{K}_4^L \right] - B_2 \overline{K}_1^L \right\},$$

$$\mathcal{G}_q \Gamma_2^L = [1 - L(q^2)F(q^2)] \left\{ \widehat{\Gamma}_2^L + A_1 \left[\frac{1}{2}K_3^L + \frac{p_1 \cdot q}{q \cdot t}K_4^L \right] - B_1 K_2^L \right. \\ \left. + A_2 \left[\frac{1}{2}\overline{K}_3^L - \frac{p_2 \cdot q}{q \cdot t}\overline{K}_4^L \right] - B_2 \overline{K}_2^L \right\},$$

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$$\mathcal{G}_q \Gamma_4^L = [1 - L(q^2)F(q^2)] \left\{ \frac{A_1}{2} [-K_1^L + (q \cdot t)K_2^L] - B_1 K_4^L \right. \\ \left. + \frac{A_2}{2} [\overline{K}_1^L + (q \cdot t)\overline{K}_2^L] - B_2 \overline{K}_4^L \right\},$$

$$\mathcal{G}_q \Gamma_1^T = \widehat{\Gamma}_1^T + A_1 \left[-\frac{1}{q \cdot t}K_1^T + (p_1 \cdot t)K_2^T + K_3^T - K_6^T \right] - B_1 K_1^T \\ + A_2 \left[\frac{1}{q \cdot t}\overline{K}_1^T + (p_2 \cdot t)\overline{K}_2^T + \overline{K}_3^T + \overline{K}_6^T \right] - B_2 \overline{K}_1^T \\ + \frac{2}{q^2}L(q^2)F(q^2) \left\{ A_1 \left[\frac{p_1 \cdot q}{q \cdot t}K_1^L + (p_1 \cdot t)K_2^L \right] - B_1 K_3^L + \right. \\ \left. + A_2 \left[\frac{p_2 \cdot q}{q \cdot t}\overline{K}_1^L + (p_2 \cdot t)\overline{K}_2^L \right] - B_2 \overline{K}_3^L - \frac{B_1 - B_2}{q \cdot t} \right\}$$

$$\mathcal{G}_q \Gamma_2^T = \widehat{\Gamma}_2^T + A_1 \left[-\frac{1}{q \cdot t}K_4^L + \frac{1}{2}K_1^T + \frac{1}{2}(p_1 \cdot q)K_4^T - \frac{1}{2}K_7^T \right] - B_1 K_2^T \\ + A_2 \left[-\frac{1}{q \cdot t}\overline{K}_4^L + \frac{1}{2}\overline{K}_1^T - \frac{1}{2}(p_2 \cdot q)\overline{K}_4^T - \frac{1}{2}\overline{K}_7^T \right] - B_2 \overline{K}_2^T \\ + \frac{2}{q^2}L(q^2)F(q^2) \left\{ A_1 \left[\frac{1}{2}K_3^L + \frac{p_1 \cdot q}{q \cdot t}K_4^L \right] - B_1 K_2^L + \right. \\ \left. + A_2 \left[\frac{1}{2}\overline{K}_3^L - \frac{p_2 \cdot q}{q \cdot t}\overline{K}_4^L \right] - B_2 \overline{K}_2^L + \frac{A_1 - A_2}{2(q \cdot t)} \right\},$$

$$\mathcal{G}_q \Gamma_3^T = \widehat{\Gamma}_3^T + \frac{A_1}{2} \left[\frac{1}{2}(q \cdot t)K_1^T - \frac{1}{2}(p_1 \cdot t)(q \cdot t)K_4^T - K_5^T \right] - B_1 K_3^T \\ + \frac{A_2}{2} \left[-\frac{1}{2}(q \cdot t)\overline{K}_1^T + \frac{1}{2}(p_2 \cdot t)(q \cdot t)\overline{K}_4^T - \overline{K}_5^T \right] - B_2 \overline{K}_3^T \\ + \frac{1}{q^2}L(q^2)F(q^2) \left\{ A_1 \left[\frac{1}{2}(q \cdot t)K_3^L - (p_1 \cdot t)K_4^L \right] - B_1 K_1^L \right. \\ \left. + A_2 \left[-\frac{1}{2}(q \cdot t)\overline{K}_3^L + (p_2 \cdot t)\overline{K}_4^L \right] - B_2 \overline{K}_1^L + \frac{1}{2}(A_1 + A_2) \right\},$$

$$\mathcal{G}_q \Gamma_4^T = \widehat{\Gamma}_4^T + A_1 \left[K_2^T - \frac{2}{q \cdot t}K_3^T + \frac{1}{q \cdot t}K_8^T \right] - B_1 K_4^T + A_2 \left[\overline{K}_2^T + \frac{2}{q \cdot t}\overline{K}_3^T - \frac{1}{q \cdot t}\overline{K}_8^T \right] - B_2 \overline{K}_4^T \\ + \frac{4}{q^2(q \cdot t)}L(q^2)F(q^2) \left\{ \frac{A_1}{2} [-K_1^L + (q \cdot t)K_2^L] - B_1 K_4^L + \frac{A_2}{2} [\overline{K}_1^L + (q \cdot t)\overline{K}_2^L] - B_2 \overline{K}_4^L \right\},$$

$$\mathcal{G}_q \Gamma_5^T = \widehat{\Gamma}_5^T + \frac{A_1}{2} [-K_1^L - q^2 K_3^T - (q \cdot t)K_6^T - (p_1 \cdot t)K_8^T] - B_1 K_5^T \\ + \frac{A_2}{2} [-\overline{K}_1^L - q^2 \overline{K}_3^T - (q \cdot t)\overline{K}_6^T - (p_2 \cdot t)\overline{K}_8^T] - B_2 \overline{K}_5^T,$$

$$\mathcal{G}_q \Gamma_6^T = \widehat{\Gamma}_6^T + \frac{A_1}{2} \left[-K_3^L - \frac{q^2}{2}K_1^T + \frac{q^2}{2}(p_1 \cdot t)K_4^T - K_5^T - (p_1 \cdot t)K_7^T \right] - B_1 K_6^T \\ + \frac{A_2}{2} \left[\overline{K}_3^L + \frac{q^2}{2}\overline{K}_1^T - \frac{q^2}{2}(p_2 \cdot t)\overline{K}_4^T + \overline{K}_5^T + (p_2 \cdot t)\overline{K}_7^T \right] - B_2 \overline{K}_6^T,$$

$$\mathcal{G}_q \Gamma_7^T = \widehat{\Gamma}_7^T + A_1 \left[-K_2^L - \frac{q^2}{q \cdot t}K_3^T - K_6^T + \frac{p_1 \cdot q}{q \cdot t}K_8^T \right] - B_1 K_7^T \\ + A_2 \left[-\overline{K}_2^L + \frac{q^2}{q \cdot t}\overline{K}_3^T + \overline{K}_6^T + \frac{p_2 \cdot q}{q \cdot t}\overline{K}_8^T \right] - B_2 \overline{K}_7^T \\ + \frac{2}{q \cdot t}L(q^2)F(q^2) \left\{ \frac{A_1}{2} [-K_1^L + (q \cdot t)K_2^L] - B_1 K_4^L + \frac{A_2}{2} [\overline{K}_1^L + (q \cdot t)\overline{K}_2^L] - B_2 \overline{K}_4^L \right\},$$

$$\mathcal{G}_q \Gamma_8^T = \widehat{\Gamma}_8^T + A_1 \left[K_4^L - K_5^T + \frac{1}{2}(q \cdot t)K_7^T \right] - B_1 K_8^T + A_2 \left[-\overline{K}_4^L - \overline{K}_5^T - \frac{1}{2}(q \cdot t)\overline{K}_7^T \right] - B_2 \overline{K}_8^T.$$

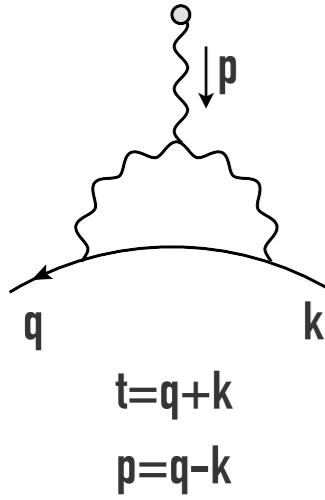
Quark-gluon vertex model



- **Complete quark-gluon vertex**
solves all symmetry identities

Aguilar, DB, Ibañez, Papavassiliou, PRD 90 (2014)

$$[1 + G] \times \Gamma^\nu = \widehat{\Gamma}^\nu + S^{-1} Q^\nu + \bar{Q}^\nu S^{-1}$$



- **Abelian-like ST identity**
determines vertex up to transverse parts

$$\widehat{\Gamma}^\nu = \Gamma_{BC}^\nu + \widehat{\Gamma}_T^\nu$$

- **Ball-Chiu vertex:** completely specified by the fully dressed quark propagator

Ball, Chiu PRD 22 (1980)

$$\Gamma_{BC}^\nu = \gamma^\nu \Sigma_A + \frac{1}{2} t_\nu [(\gamma \cdot t) \Delta_A - i \Delta_B]$$

$$\Sigma_\Phi(q^2, k^2) = \frac{1}{2} [\Phi(q^2) + \Phi(k^2)]$$

$$\Delta_\Phi(q^2, k^2) = \frac{\Phi(q^2) - \Phi(k^2)}{q^2 - k^2}$$

- **Transverse part:** minimum required for meson spectrum description, including scalar meson and radial excitations

$$a_T^\nu = a^\nu + (a \cdot p)p^\nu/p^2$$

$$\tau_1(q, k) = a_1 \Delta_A$$

$$T_1^\nu(q, k) = \frac{i}{2} t_T^\nu;$$

$$T_2^\nu(q, k) = \frac{1}{2} (\gamma \cdot t) t_T^\nu;$$

$$\tau_3(q, k) = -2(k \cdot q) a_3 \Delta_A$$

$$T_3^\nu(q, k) = \gamma_T^\nu;$$

$$\tau_4(q, k) = a_4 \frac{4\Delta_B}{t_T \cdot t_T}$$

$$T_4^\nu(q, k) = -\frac{1}{2} t_T^\nu k^\mu q^\rho \sigma_{\mu\rho};$$

$$T_5^\nu(q, k) = \sigma^{\nu\mu} p_\mu$$

$$\tau_5(q, k) = a_5 \Delta_B$$

$$T_6^\nu(q, k) = -\gamma^\nu (q^2 - k^2) + t^\nu \gamma \cdot p$$

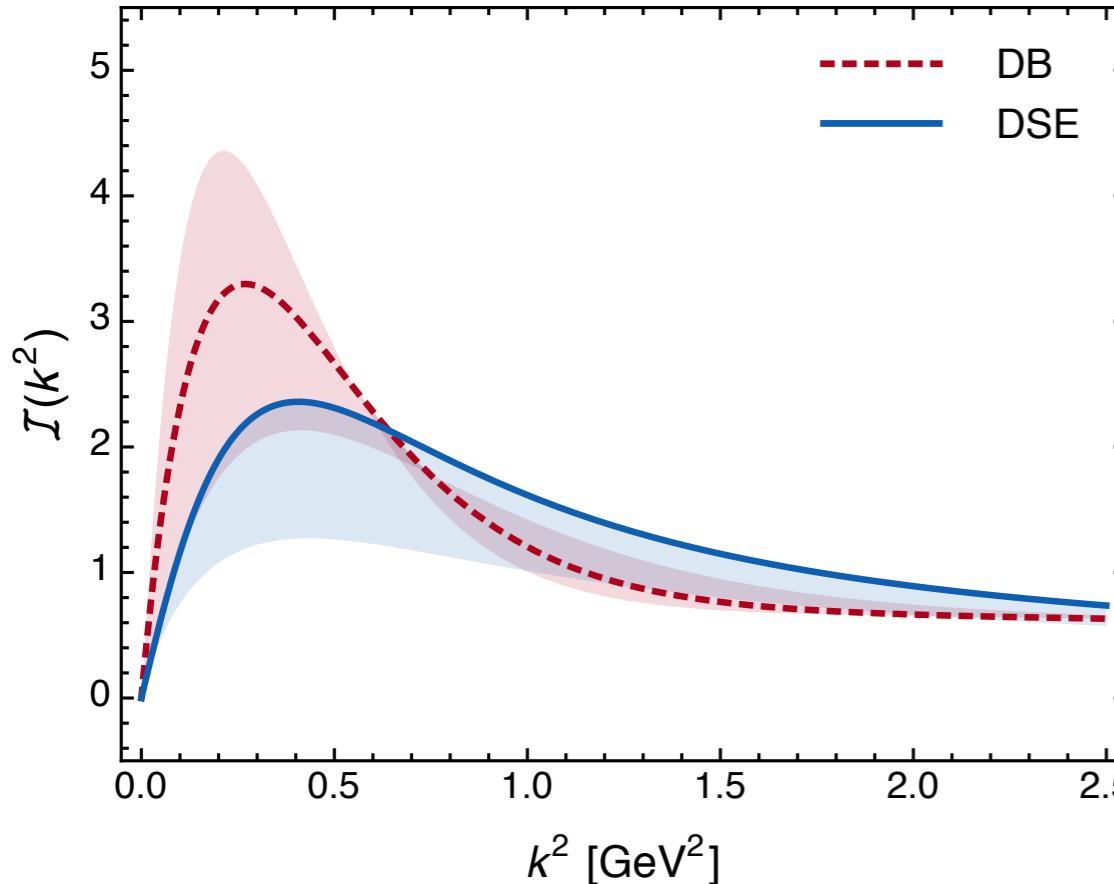
$$T_7^\nu(q, k) = \frac{i}{2} (q^2 - k^2) [\gamma^\nu (\gamma \cdot t) - t^\nu] + t^\nu k^\mu q^\rho \sigma_{\mu\rho}$$

$$T_8^\nu(q, k) = q^\nu \gamma \cdot k - k^\nu (\gamma \cdot q) - i \gamma^\nu k^\mu q^\rho \sigma_{\mu\rho} \quad \tau_8(q, k) = a_8 \Delta_A$$

Natural constraints on the vertex



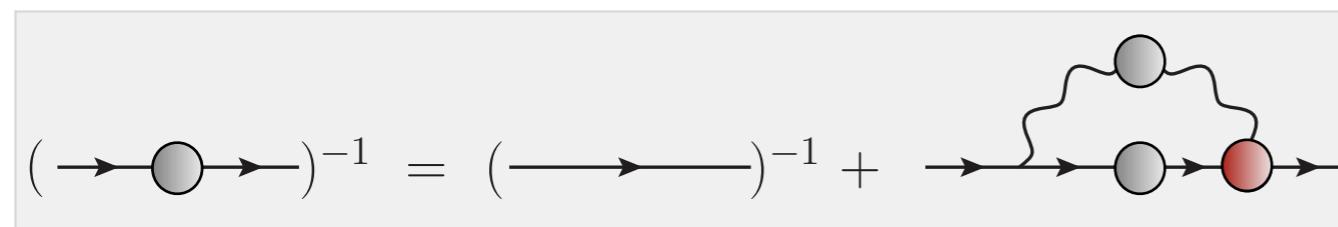
- Interaction strength



- Solve gap equation

in this new scheme

DB, Chang, Papavassiliou, Qin, Roberts PRD(R) 95 (2017)



- Interaction vertex

$$[1 + G] \times \begin{array}{c} \Gamma^\nu \\ \swarrow \searrow \end{array} = \begin{array}{c} \widehat{\Gamma}^\nu \\ \swarrow \searrow \end{array} + \begin{array}{c} S^{-1} Q^\nu \\ \circlearrowleft \circlearrowright \end{array} + \begin{array}{c} \overline{Q}^\nu S^{-1} \\ \circlearrowleft \circlearrowright \end{array}$$

$$\widehat{\Gamma}^\nu = \Gamma_{BC}^\nu + \widehat{\Gamma}_T^\nu$$

$$\Gamma_{BC}^\nu = \gamma^\nu \Sigma_A + \frac{1}{2} t_\nu [(\gamma \cdot t) \Delta_A - i \Delta_B]$$

$$\begin{aligned} \widehat{\Gamma}_T^\nu = & a_1 \Delta_A T_1^\nu - 2(k \cdot q) a_3 \Delta_B T_3^\nu \\ & + 4a_4 \frac{\Delta_B}{t^T \cdot t^T} + a_5 T_5^\nu + a_8 \Delta_A T_8^\nu \end{aligned}$$

- Many degeneracies

form factors enter in the following combinations

$$\tau_1 - \Delta_B \quad \Sigma_A + \tau_3 \quad \tau_4 - 3\tau_5$$

- Scan parameter space

look for solutions varying $a_{1,3,45,8}$ ($a_{45} = a_4 - 3a_5$)

$$\mathbb{V}_4 = \{(a_1, a_3, a_{45}, a_8)\}$$

$$|a_1, a_3 \in [-1, 1], a_{45} \in [-7, 5], a_8 \in [-5, 1]\}$$



Natural constraints on the vertex

- **Scanning method**

1. Generate a quadruplet $\mathbf{q} = (a_1, a_3, a_{45}, a_8)$
2. Construct the quark-gluon vertex ${}^q\Gamma_\nu$
3. Solve the gap equation with the t-d/b-u RGI interaction
4. Repeat 1.66 million times

- **Filtering method**

1. Check if solution has converged
2. Categorise the solutions as acceptable iff:
 - (i) express DCSB of sufficient strength:
 $M_{0.25}^{0.45}$ [GeV]
 - (ii) have non-negative ACM distribution

$$\kappa(\mathfrak{m}) = 2\mathfrak{m} \frac{(a_5 - 1 + a_1/2)\delta_B + \mathfrak{m}(1 - a_8)\delta_A}{\sigma_A + 2\mathfrak{m}^2(a_3 - 1)\delta_A + 2\mathfrak{m}(1 - a_1/2)\delta_B}$$

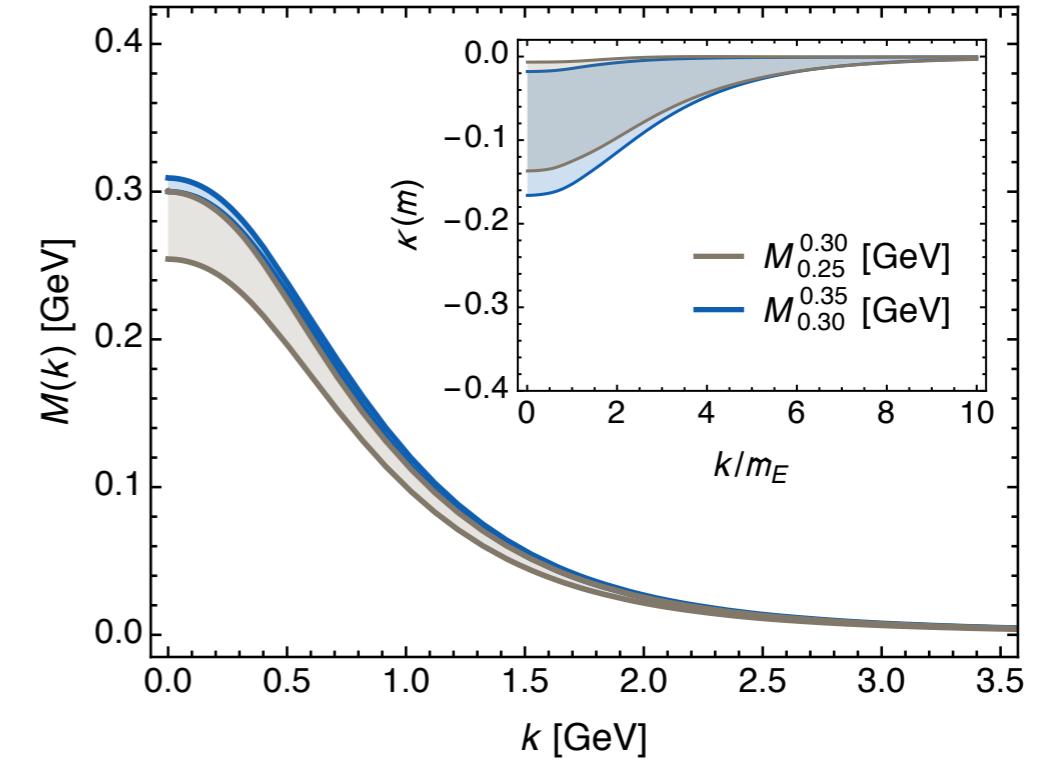
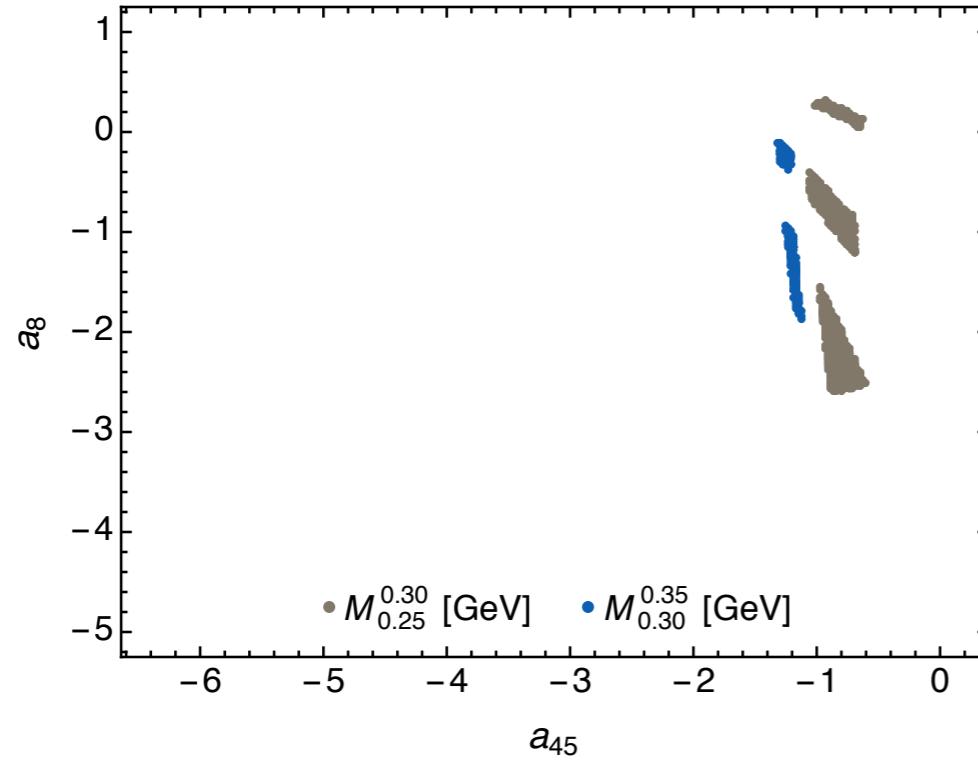
- (iii) f_π within 5% of chiral value (0.088 [GeV])

Even a small selection of observables places extremely tight bounds on the domain of acceptable, realistic vertex Ansätze

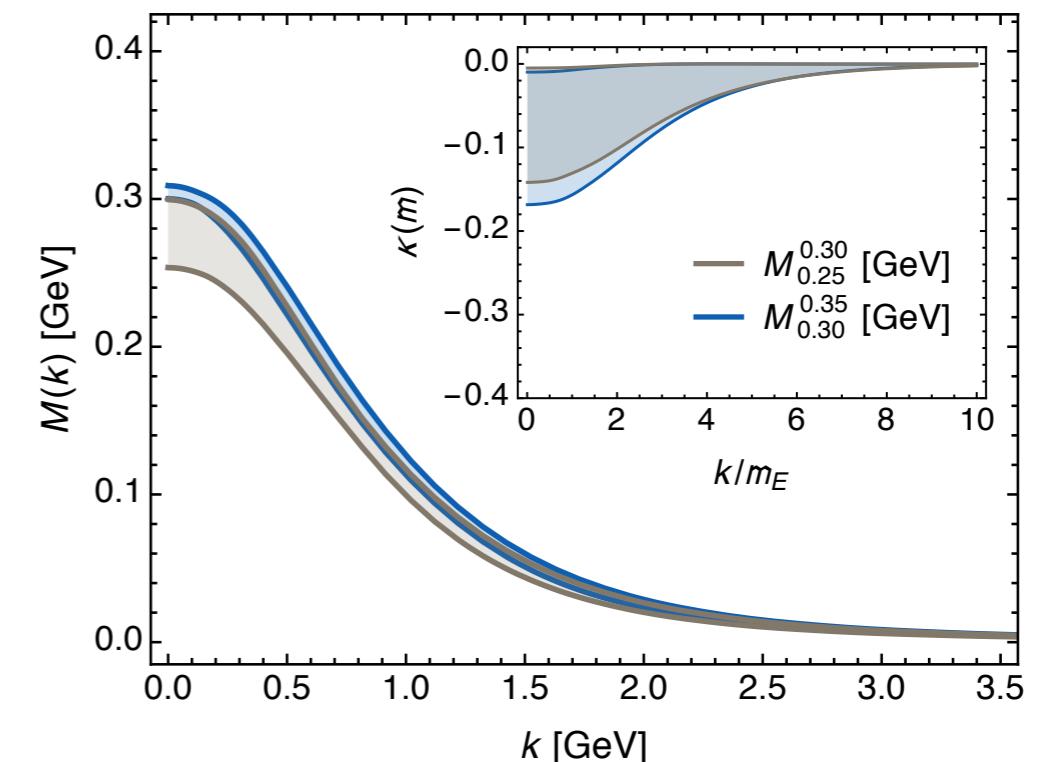
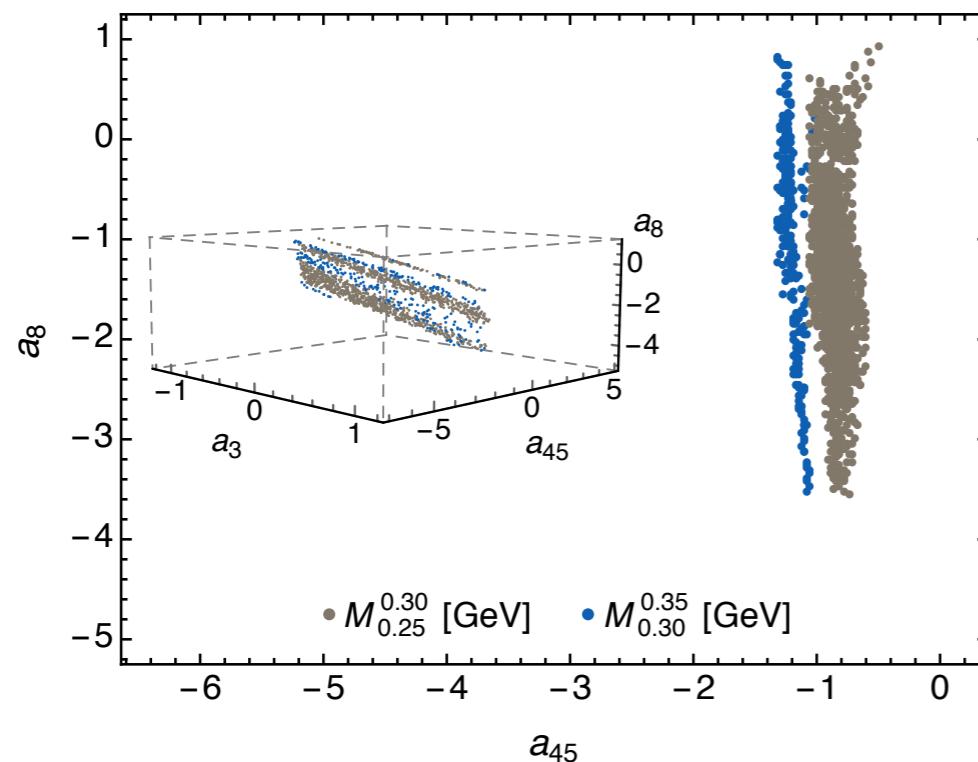
Natural constraints on the vertex



- Case A ($a_1=a_3=0$, $360k \text{ q}$)



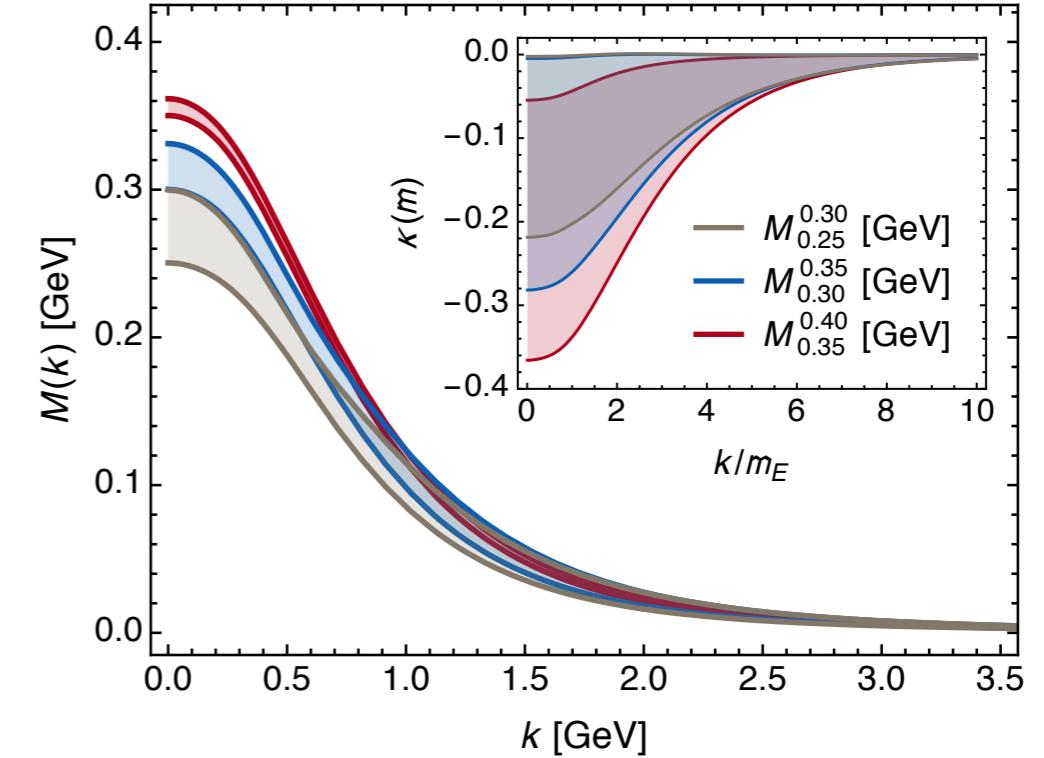
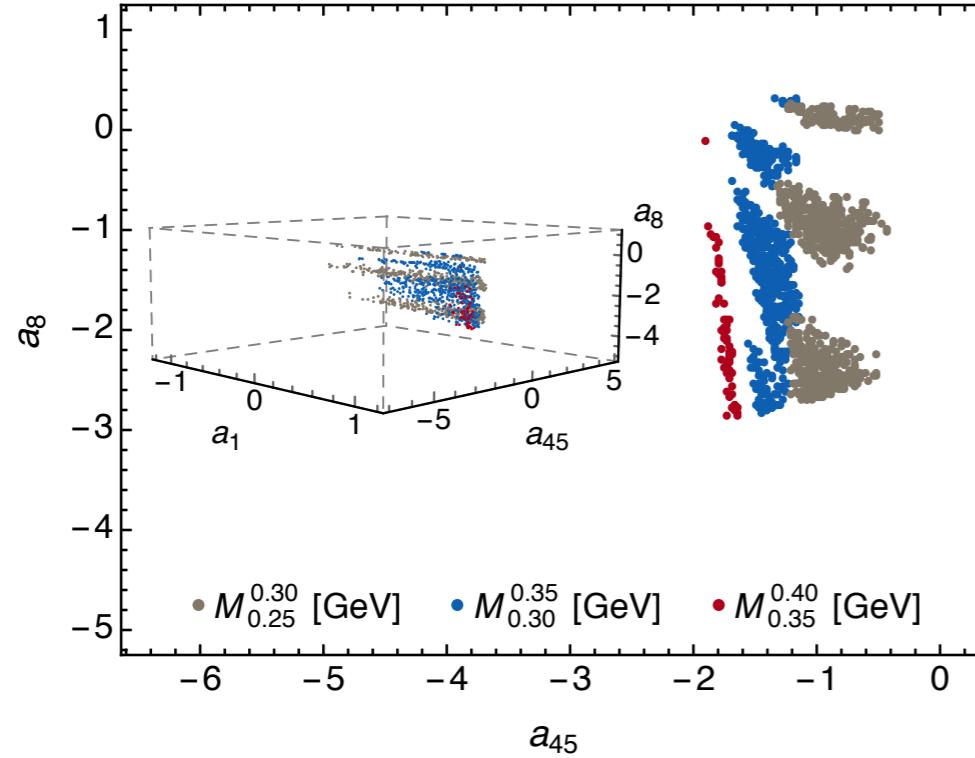
- Case B ($a_1=0$, $400k \text{ q}$)



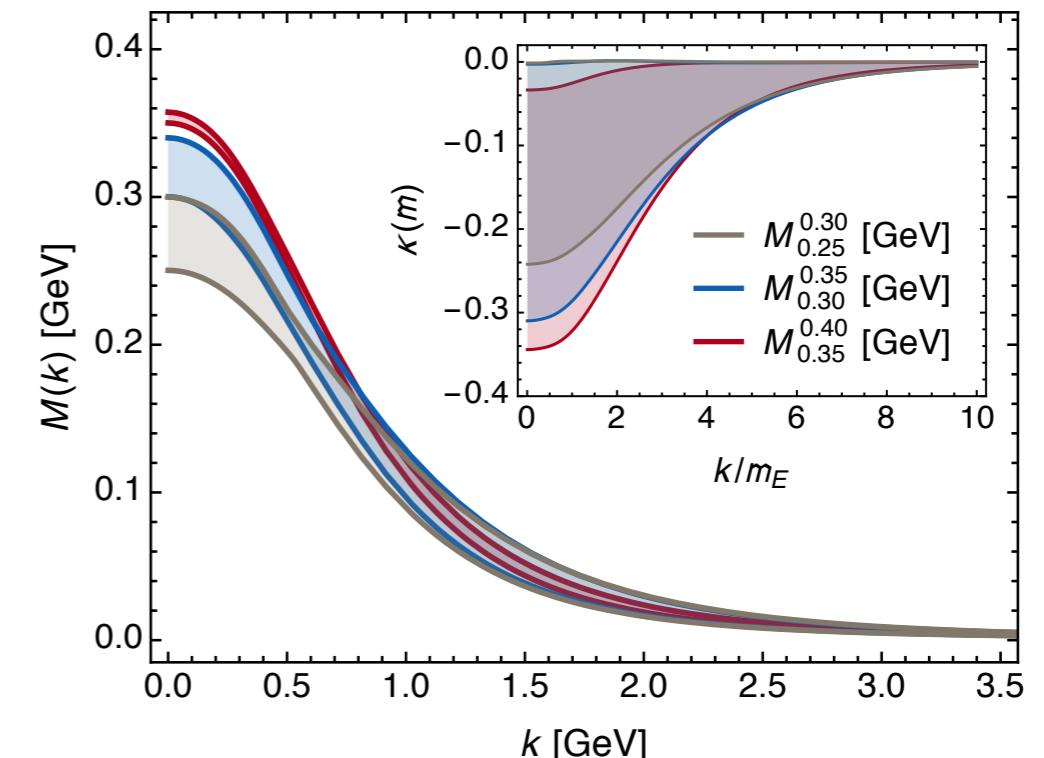
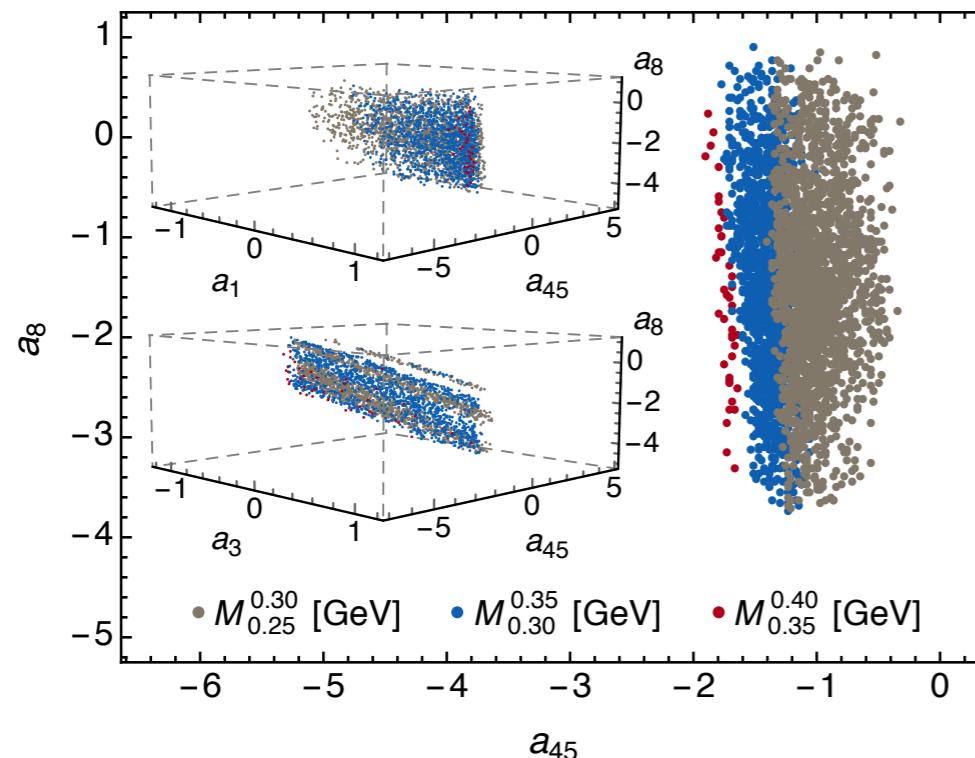
Natural constraints on the vertex



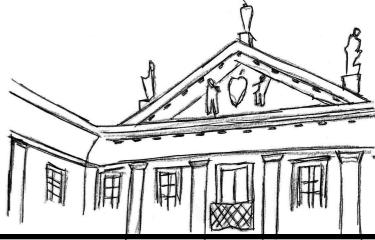
- Case C ($a_3=0$, $360k q$)



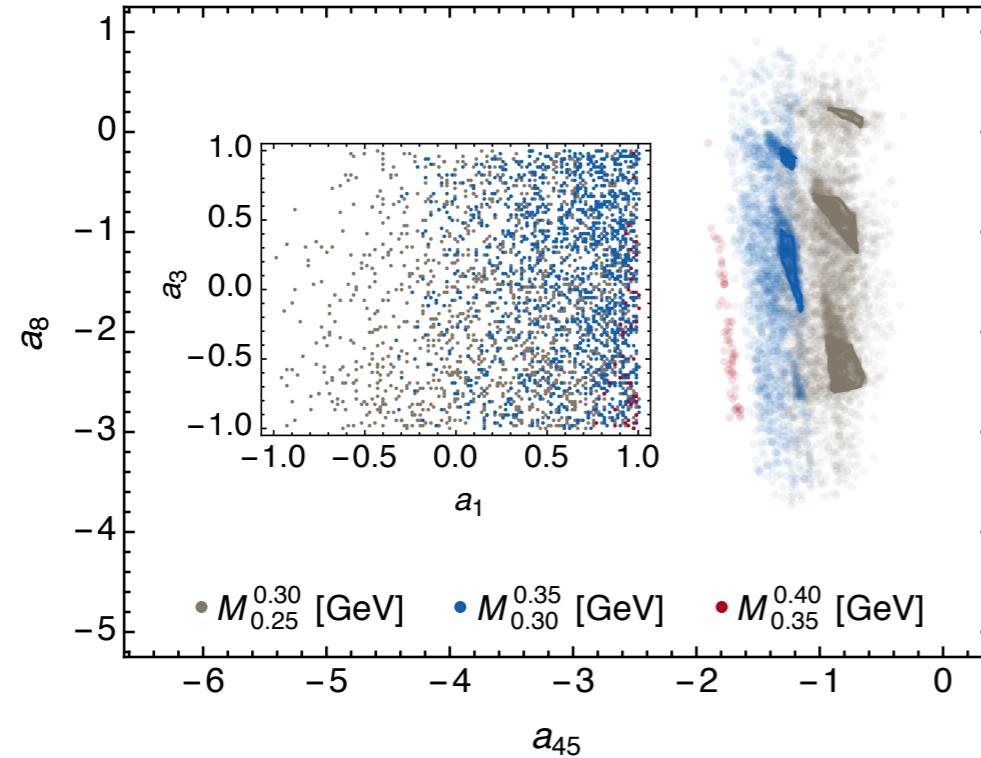
- Case D ($a_{1,3}\neq 0$, $560kq$)



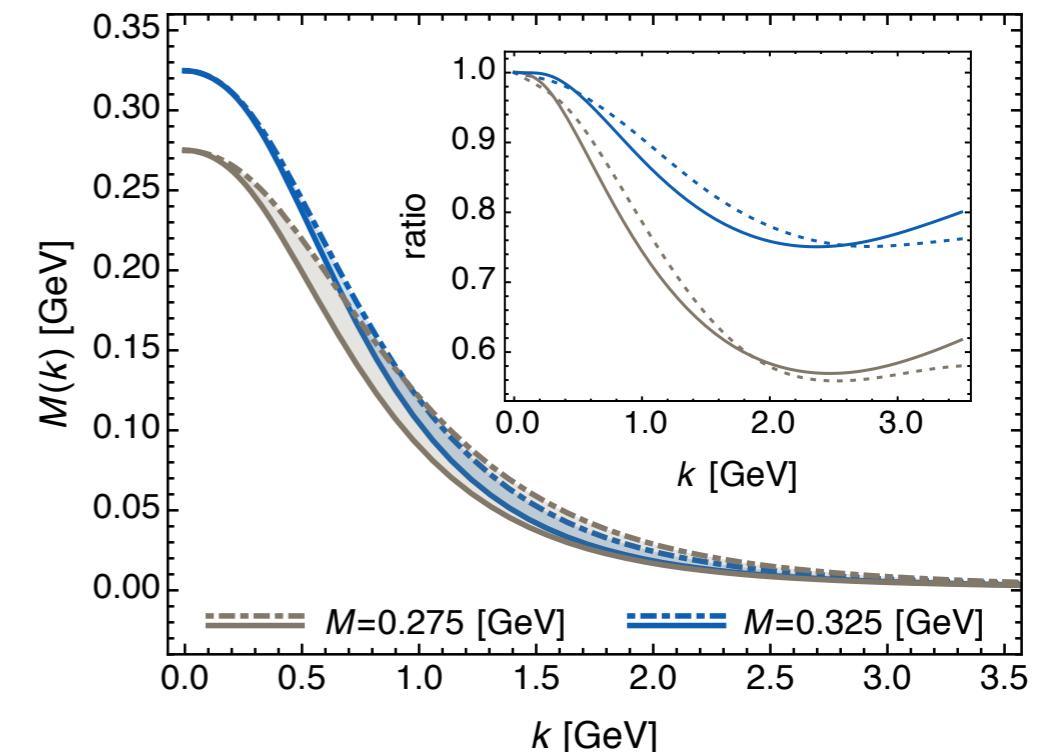
Natural constraints on the vertex



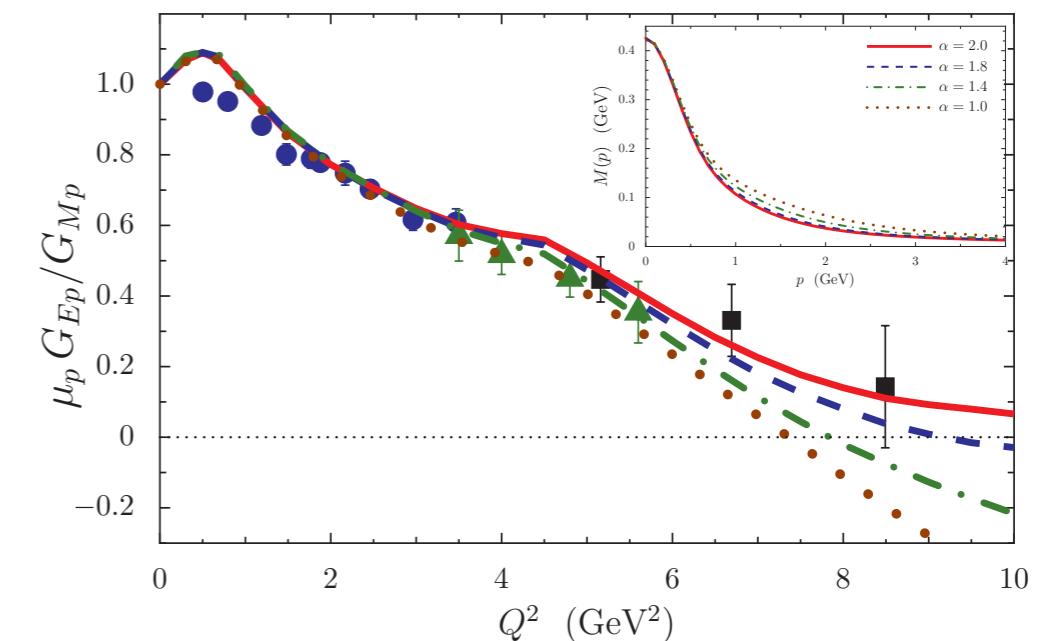
- All domains



- Running



- Impacts on observable:
possible existence of a zero in the ratio of p elastic form factors
- Probe the mass running rate
and ultimately the structure of the quark-gluon vertex





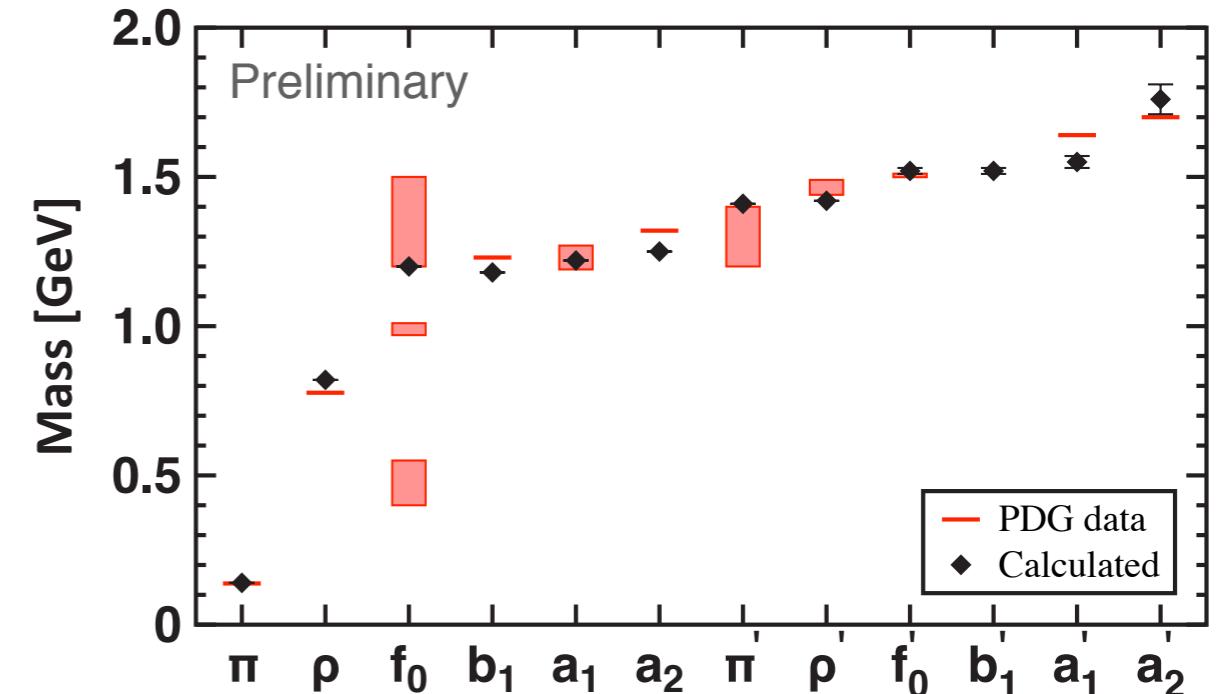
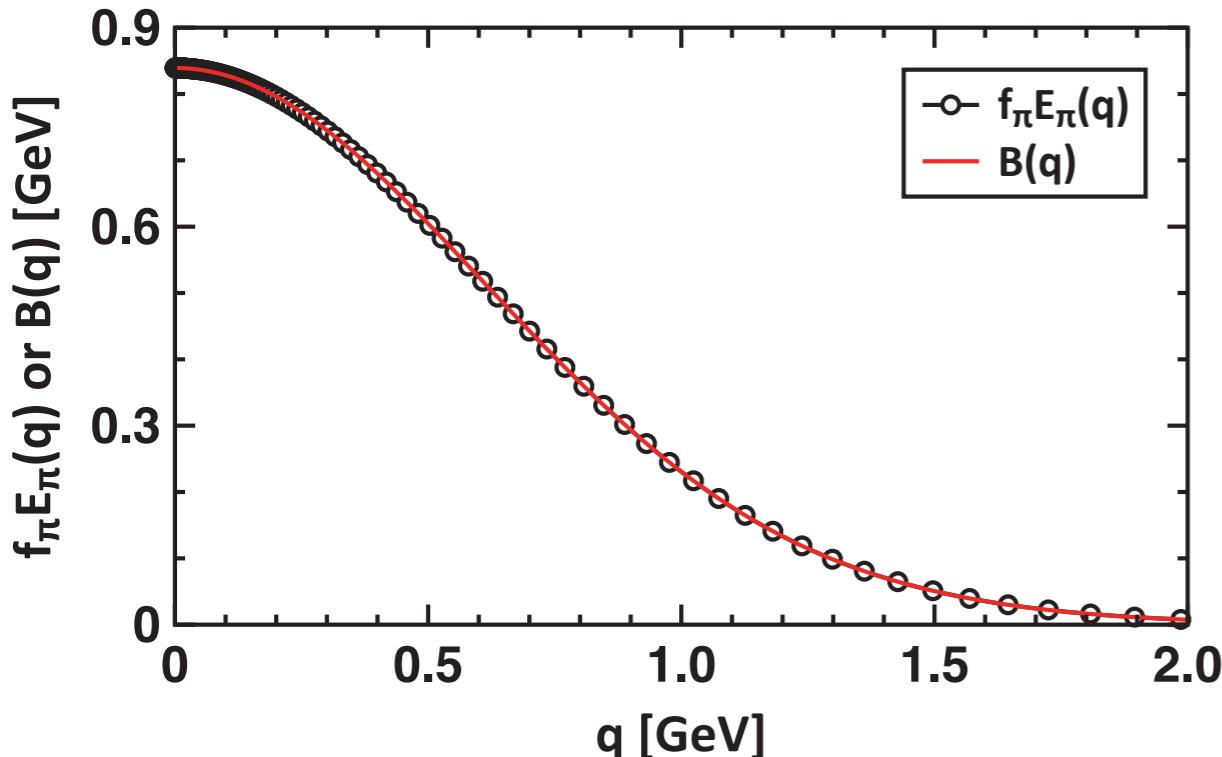
Meson spectrum

- **Solve BSE for the meson spectrum**

use top-down interaction kernel and

[Qin, Roberts,...](#)

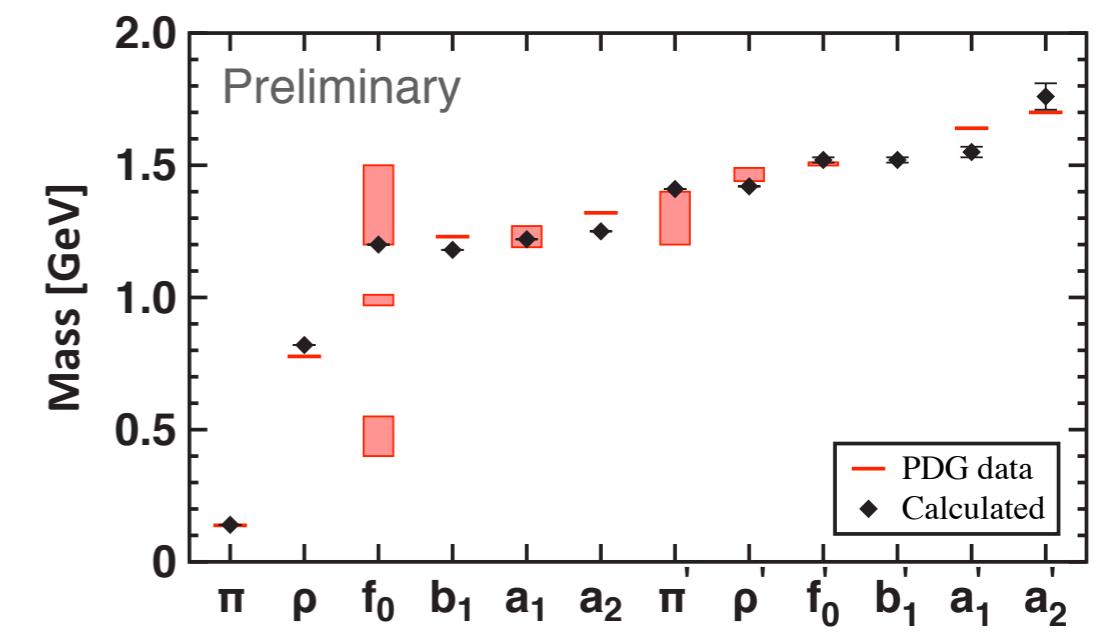
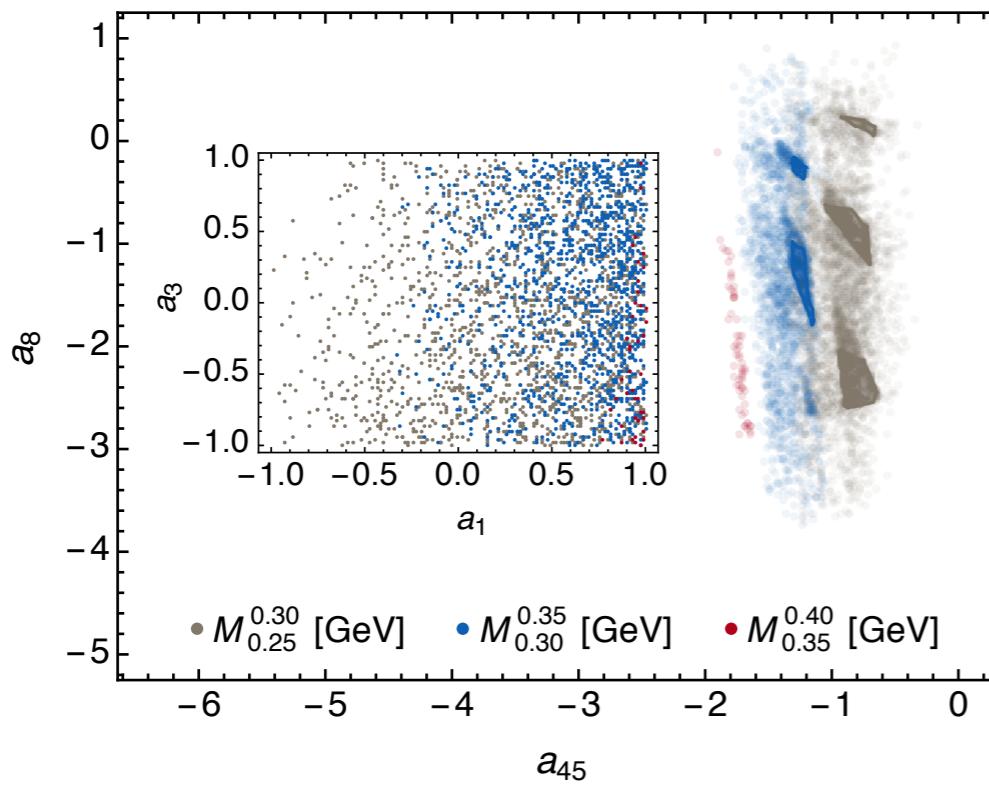
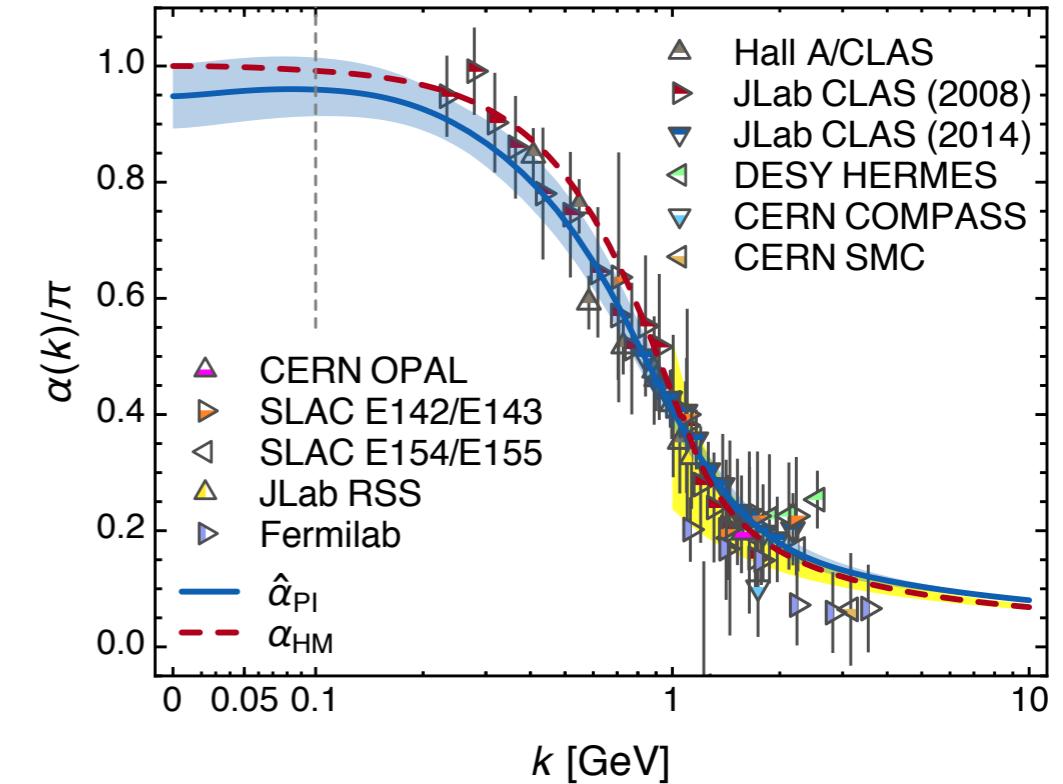
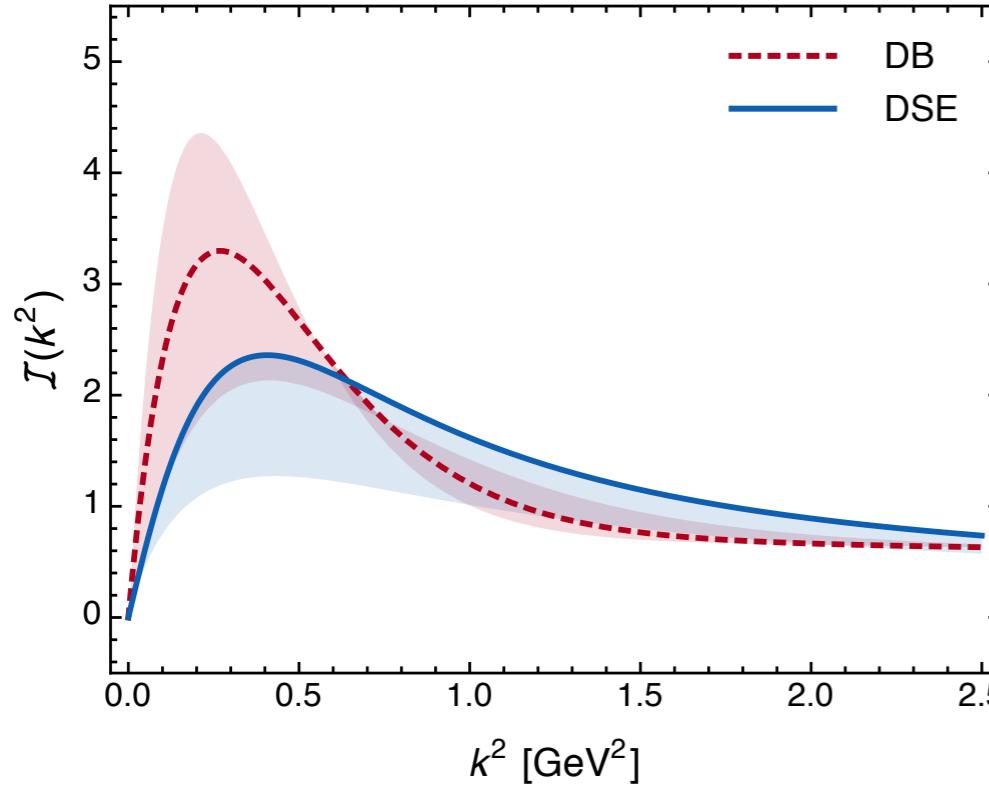
$$\begin{aligned}\Gamma_\mu(p, q) &= \Gamma_\mu^{\text{BC}}(p, q) + \eta \Gamma_\mu^T(p, q) & \tau_\mu^4 &= l_\mu^T \gamma \cdot k + i \gamma_\mu^T \sigma_{\nu\rho} l_\nu k_\rho, \\ \Gamma_\mu^T(p, q) &= \Delta_B \tau_\mu^8 + \Delta_A \tau_\mu^4 & \tau_\mu^8 &= 3 l_\mu^T \sigma_{\nu\rho} l_\nu k_\rho / (l^T \cdot l^T).\end{aligned}$$



	$-\langle \bar{q}q \rangle_0^{1/3}$	$\rho_\pi^{1/2}$	f_π	m_π	m_ρ	m_σ	m_{b_1}	m_{a_1}	m_{a_2}	$m_{\pi'}$	$m_{\rho'}$	$m_{\sigma'}$	$m_{b'_1}$	$m_{a'_1}$	$m_{a'_2}$
this work	0.283	0.493	0.093	0.14	0.82	1.20	1.18	1.22	1.25	1.41	1.42	1.52 ± 0.01	1.52 ± 0.01	1.55 ± 0.02	1.76 ± 0.05
PDG	-	-	0.092	0.14	0.78	0.50	1.24	1.26	1.32	1.30	1.45	-	-	1.64	1.70

TABLE I: The meson spectrum (Full vertex, $(D\omega)^{1/3} = 0.637$ GeV, $\omega = 0.60$ GeV, $\eta = 1.00$ and $m_q = 3.0$ MeV).

Take away



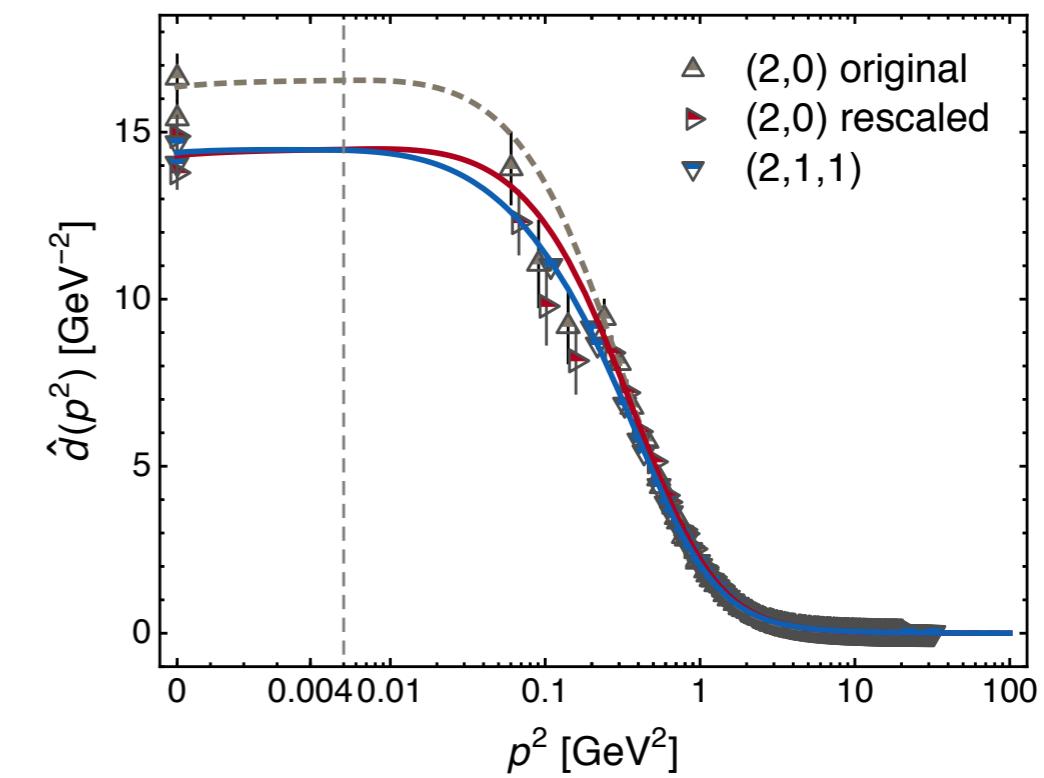
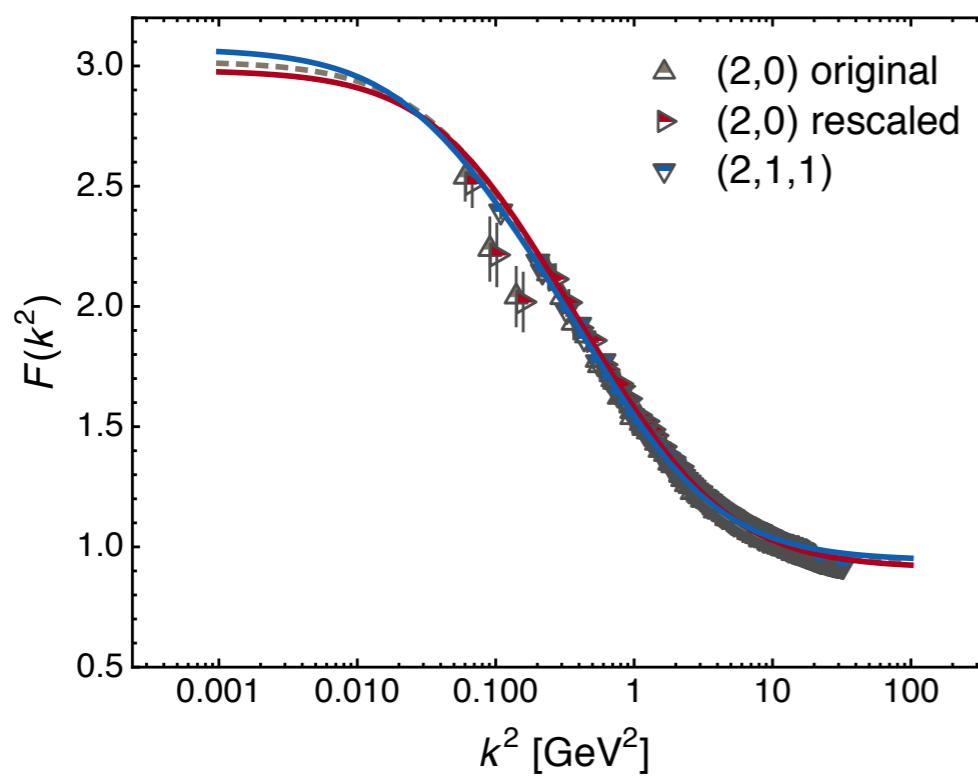
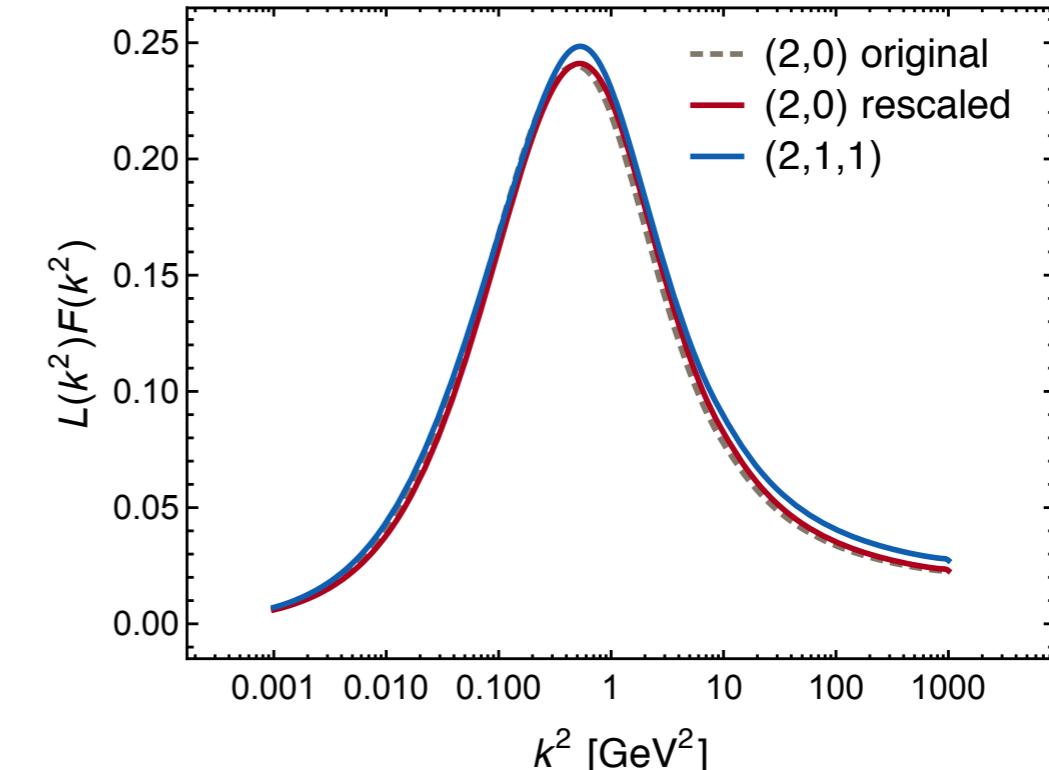
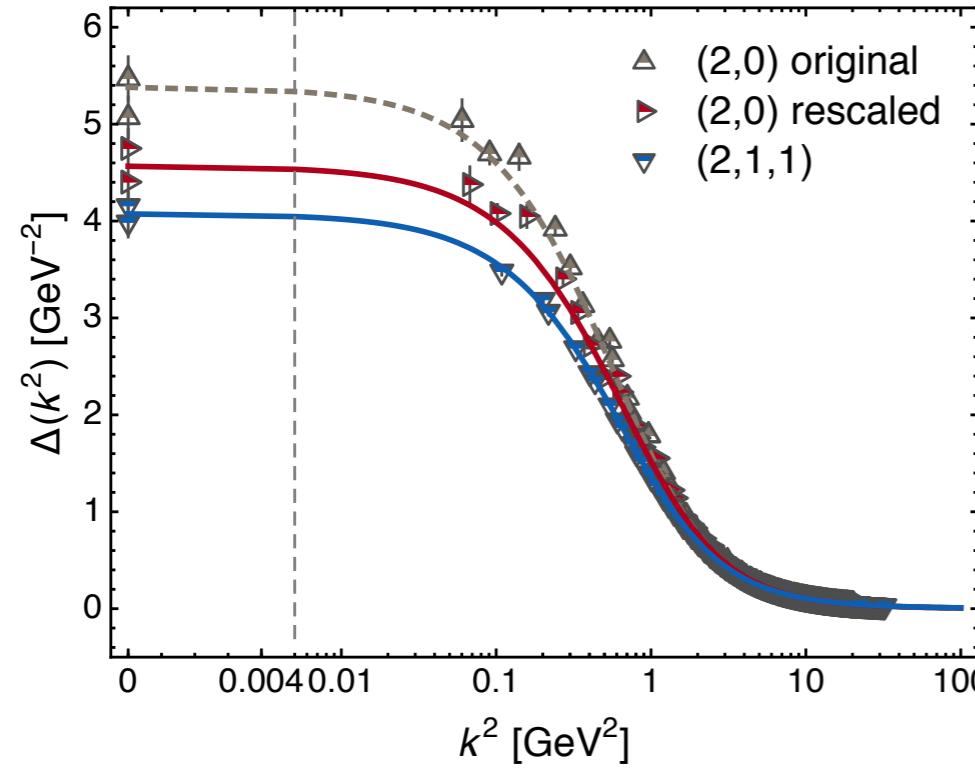


Flavour dependence

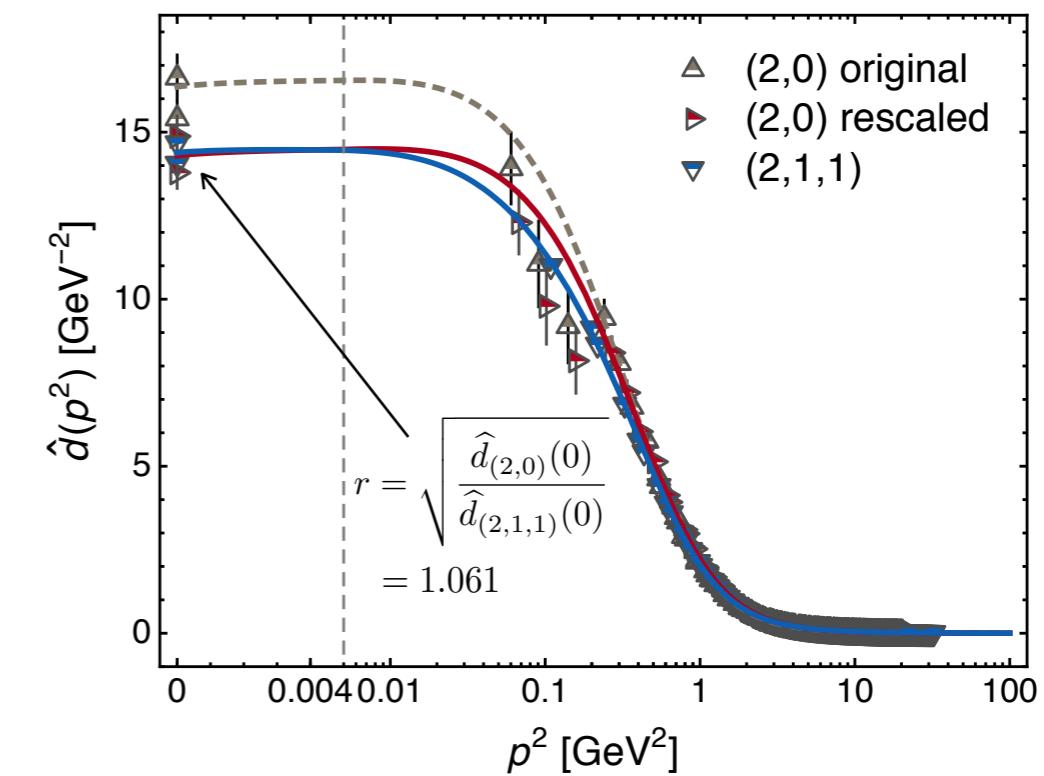
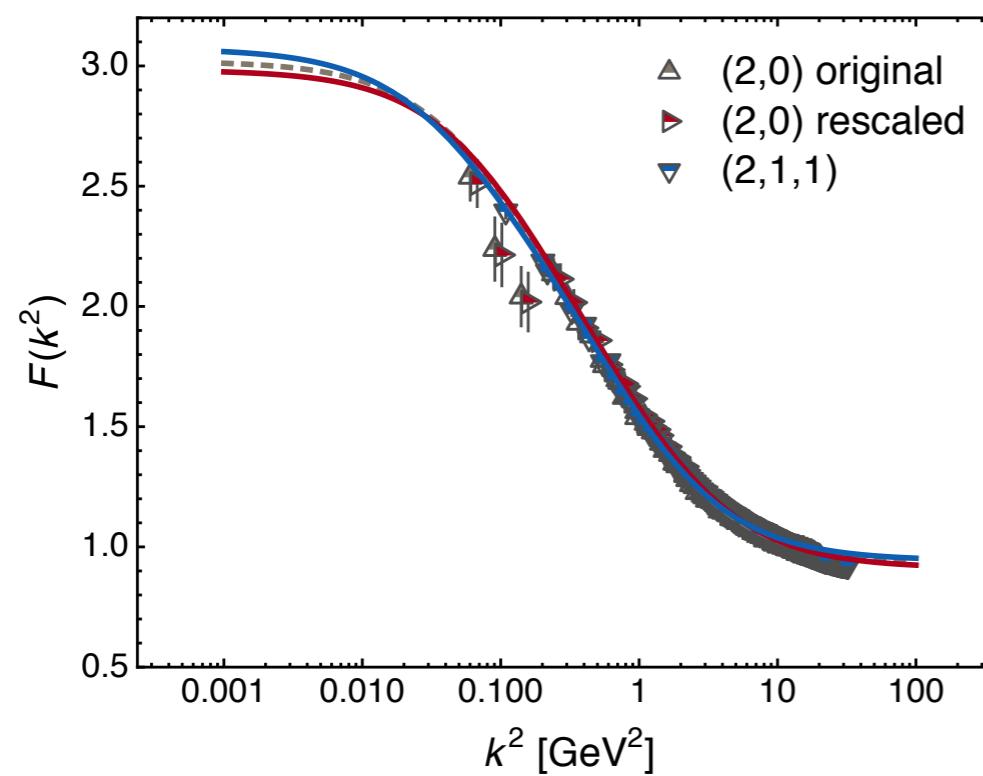
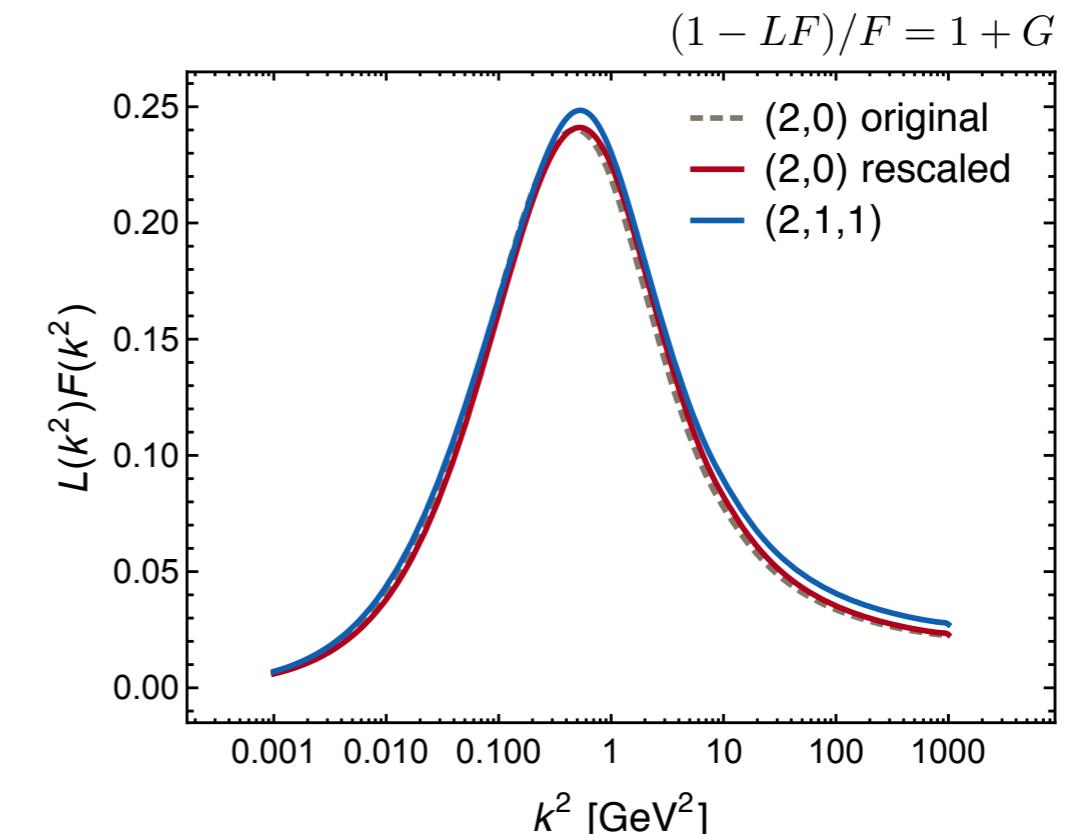
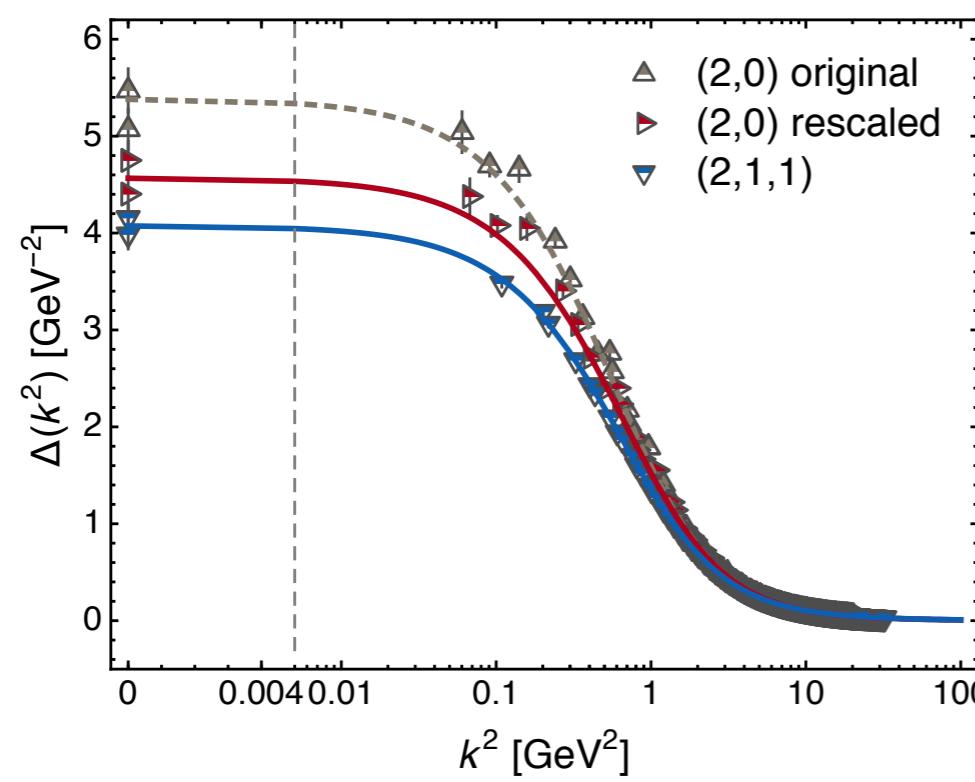
- **Top-down kernel:**
how does flavour dependence get communicated between gauge/quark sectors?
 - **Gauge sector**
only *quantitative* differences - lattice (2,0) and (2,1,1)
[Ayala, Bashir, DB, Cristoforetti, Rodriguez-Quintero, PRD 86 \(2012\)](#)
 - **Quark sector**
qualitative differences: chiral symmetry restoration
- **However:**
calculation of \mathcal{I} using directly lattice results yields $\mathcal{I}_{(2,0)} > \mathcal{I}_0 \sim \mathcal{I}_{(2,1,1)}$
- **Scale setting problem**
lattice spacing has not been correctly computed for quenched and (2,0) theories
- **Scale resetting**
use PT charge requiring decoupling of heavy flavors at low enough p
[DB, Roberts, Rodriguez-Quintero, 1611.03523](#)

$$\lim_{p^2 \rightarrow 0} \frac{\mathcal{I}_{n_f}(p^2)}{p^2} = \lim_{p^2 \rightarrow 0} \frac{\mathcal{I}_{N_f}(p^2)}{p^2} \qquad \Leftrightarrow \qquad \widehat{d}_{n_f}(0) = \widehat{d}_{N_f}(0)$$

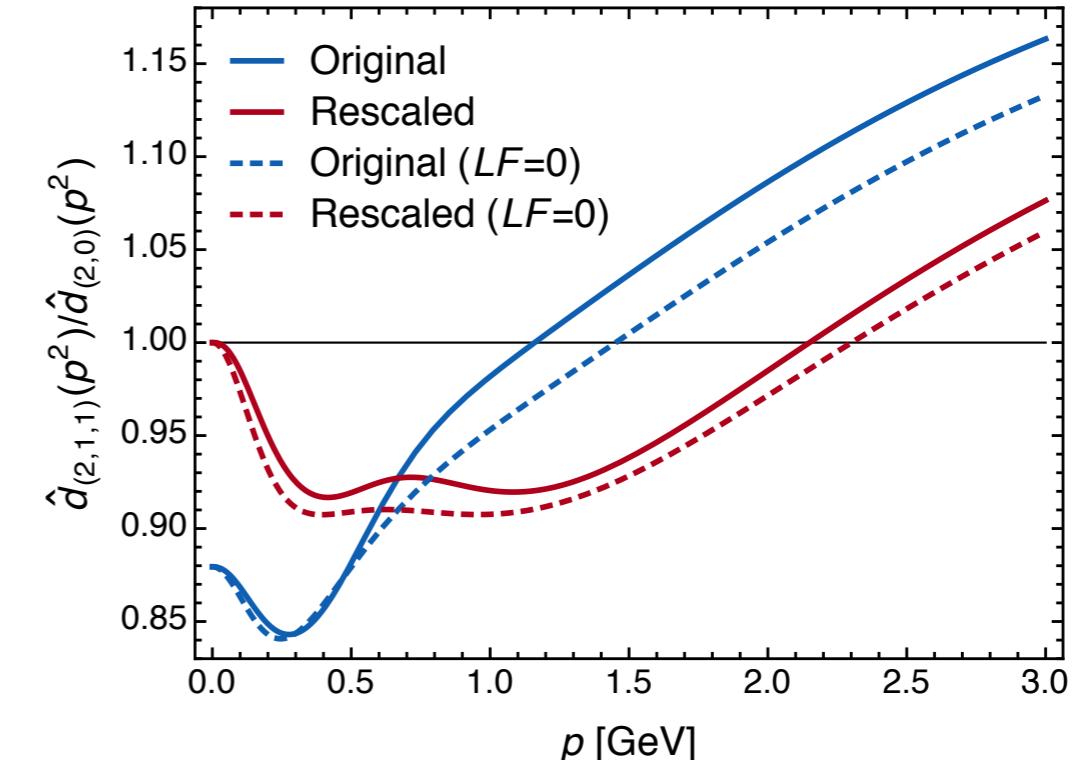
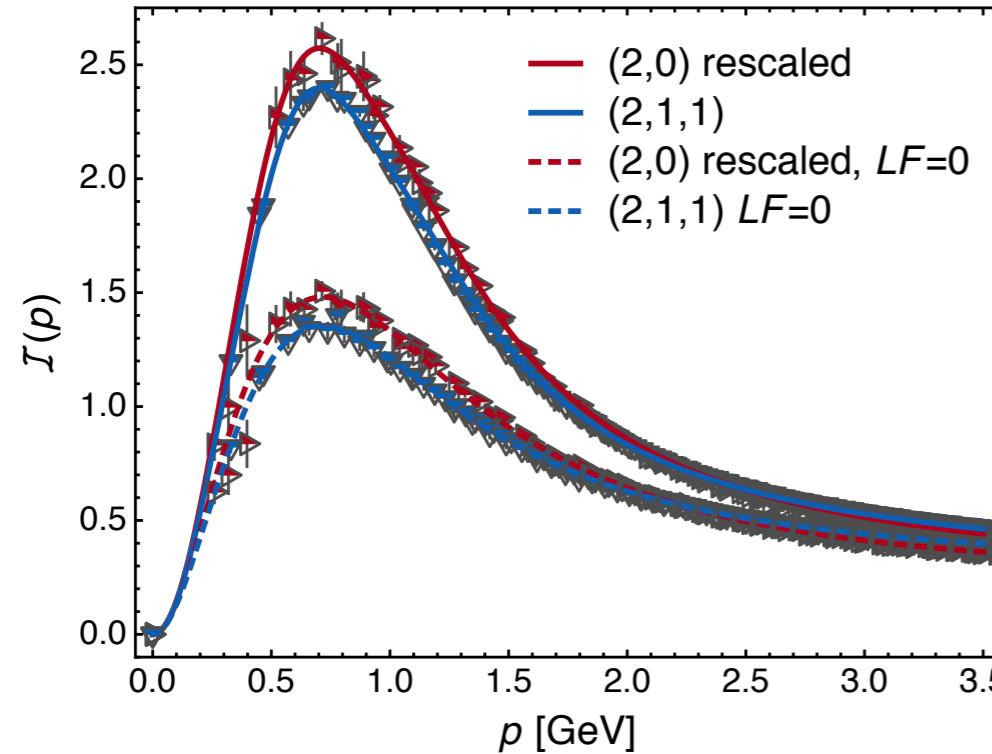
Flavour dependence



Flavour dependence



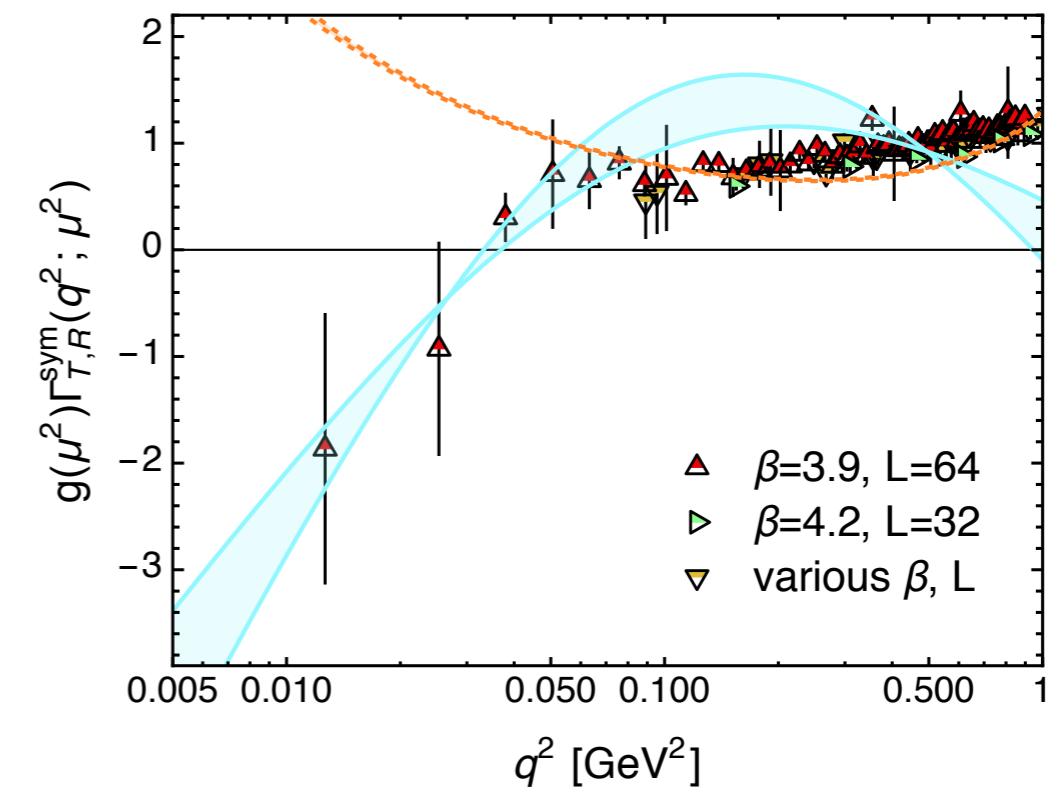
Flavour dependence



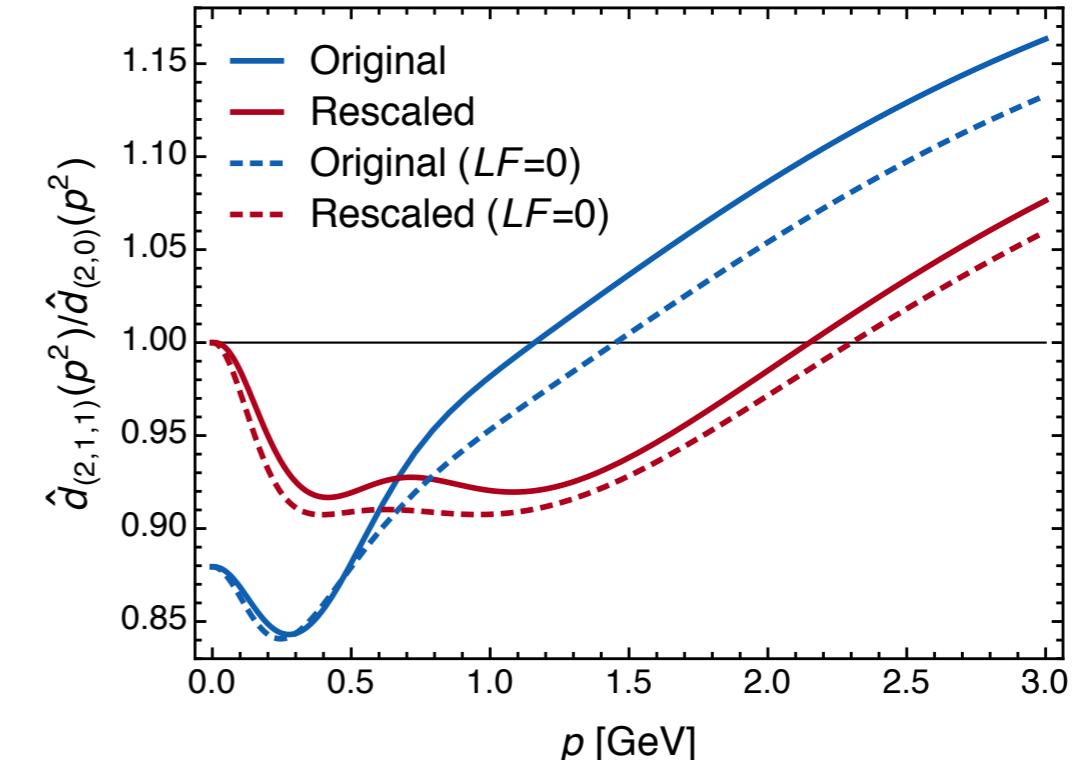
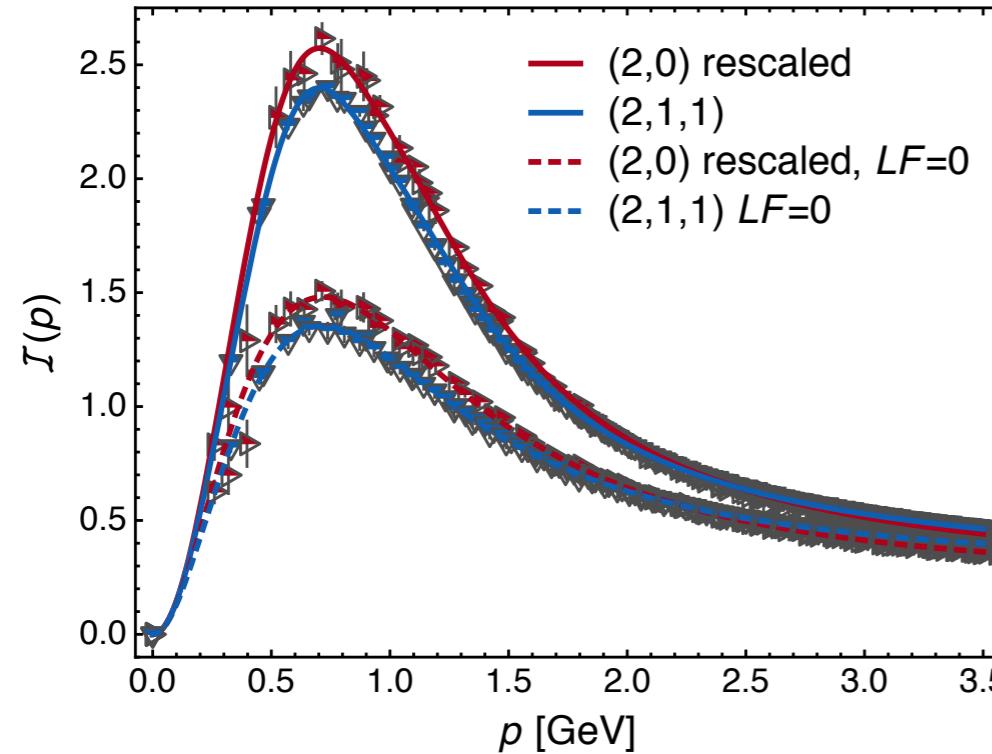
- **IR/UV asymptotics**
described by

$$I(p^2) \underset{p^2/\Lambda_T^2 \ll 1}{\sim} p^2 \hat{d}(0) \left[1 - \left(\frac{\hat{d}(0)}{8\pi} + \frac{c}{M^2} \right) p^2 \ln \frac{p^2}{\Lambda_T^2} \right]$$

$$I(p^2) \underset{p^2/\Lambda_T^2 \gg 1}{\sim} \alpha_T(p^2) \underset{p^2/\Lambda_T^2 \gg 1}{\sim} \frac{4\pi}{\beta_0 \ln(p^2/\Lambda_T^2)}$$



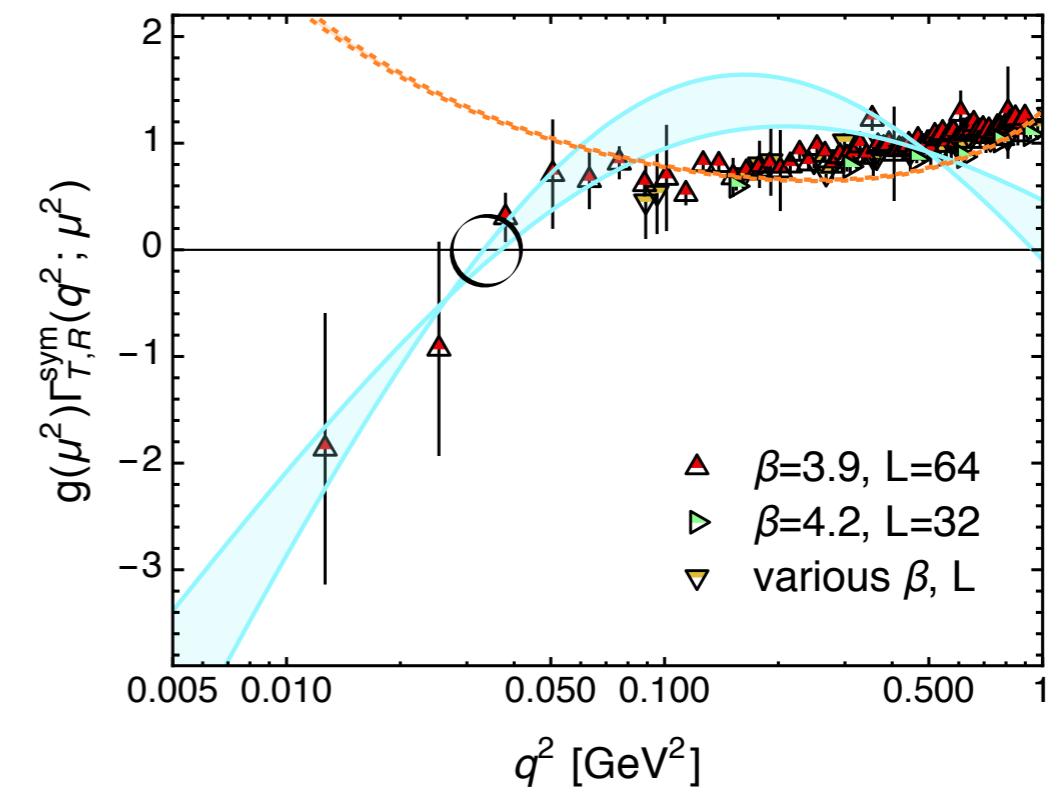
Flavour dependence



- **IR/UV asymptotics**
described by

$$I(p^2) \underset{p^2/\Lambda_T^2 \ll 1}{\sim} p^2 \hat{d}(0) \left[1 - \left(\frac{\hat{d}(0)}{8\pi} + \frac{\mathcal{C}}{M^2} \right) p^2 \ln \frac{p^2}{\Lambda_T^2} \right]$$

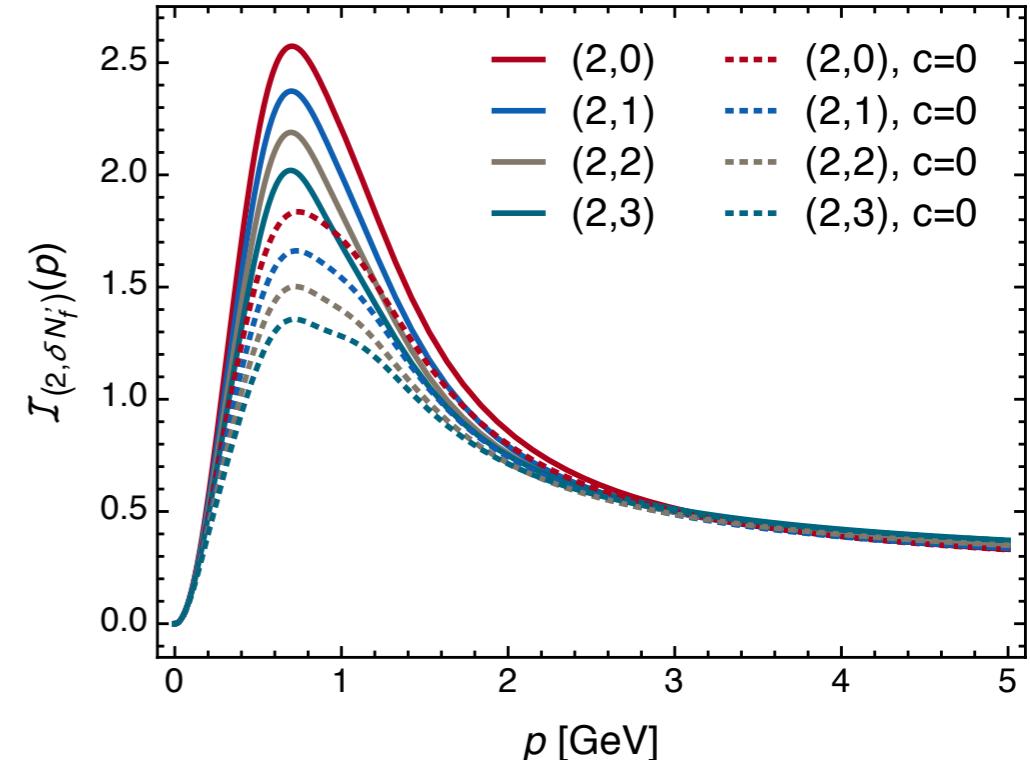
$$I(p^2) \underset{p^2/\Lambda_T^2 \gg 1}{\sim} \alpha_T(p^2) \underset{p^2/\Lambda_T^2 \gg 1}{\sim} \frac{4\pi}{\beta_0 \ln(p^2/\Lambda_T^2)}$$



Chiral symmetry restoration



- **Interaction bulk:**
located where strange quark is active (no charm)
 - **Suppression of (2,1,1) wrt (2,0)**
mainly due to the strange quark
 - **Expand interaction**
around $N_f + \delta N'_f$ where $m_{\delta N'_f} = 0.095\text{GeV}$
 - **Match with (2,1,1) theory**
for $\delta N'_f = 1$



- **Study chiral symmetry restoration**
solving the quark gap equation

- **Use previous vertex Ansatz**
set $a_1=a_3=0$
- **Use parameters in the common region**

$$\mathbb{V}_2 = \{(a_{45}, a_8) \mid a_{45} \in [-0.95, -0.7], a_8 \in [-1.3, -0.73]\}$$
- **Build ratio** ${}^q M_{(2,\delta N'_f)}(0) / {}^q M_{(2,0)}(0)$
and extrapolate linearly to x intercept
- **Critical number of flavours 9 ✓**
unacceptably low number (5) without ghost enhancement

